Computation Power of NFA's

We saw NFA's, which seem to have more flexibility than DFAs. But are NFAs more powerful? We show that in terms of languages recognized, NFAs and DFA's have the same power.

Defn: Two machines / automata are equivalent if they recognize the same language.

Theorem 1.39: Every NFA has an equivalent DFA.

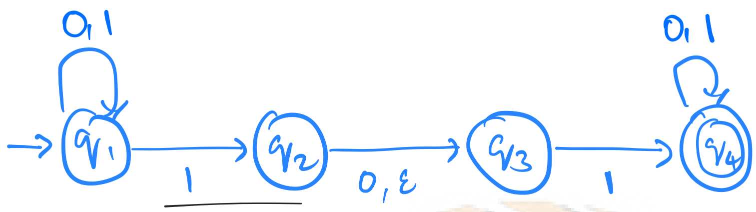
This implies that NFAs are only as powerful as DFAs.

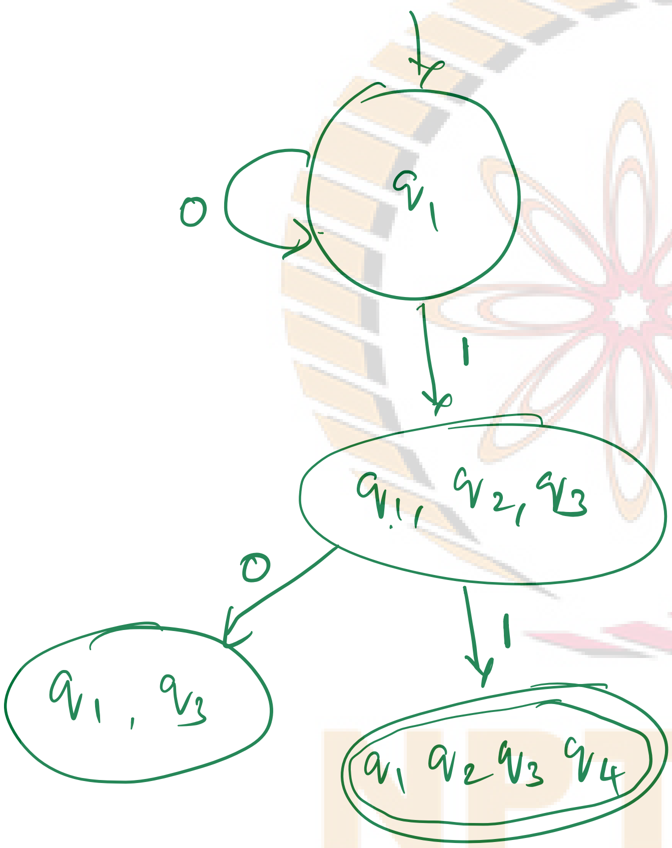
Corollary 1.40: A language is regular if and only if some nondeterministic finite automaton (NFA) recognizes it.

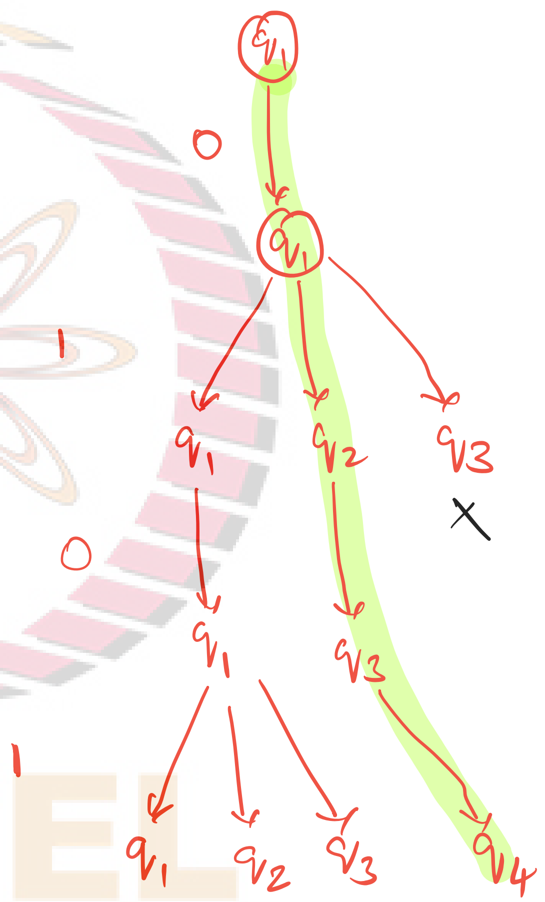
Proof (of Theorem 1.39)

IDEA: Keep track of the set of all the possible states the NFA could possibly be in.

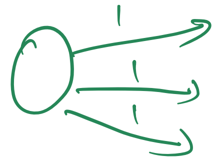
$\delta : Q \times \Sigma_\varepsilon \to P(Q)$







Proof: Suppose there are no ε transitions. Let $N=(Q,\Sigma,\delta,q_0,F)$ be the NFA, recognizing some language A. We want to construct a DFA recognizing the same language. Let our target DFA be $M=(Q',\Sigma,\delta',q_0',F')$.



1) $Q' = \mathcal{P}(Q) = \{R \mid R \subseteq Q\}$

↳ Power set of Q.

$q_1 \; q_2 \; q_3 \; q_4$

2) $q_0' = \{q_0\}$

$q_1 \; q_2 \; q_4$

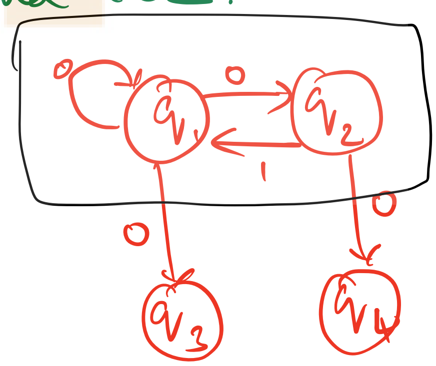
3) $F' = \{R \subseteq Q \mid \exists r \in R \text{ s.t. } r \in F\}$

$= \{\underbrace{R \subseteq Q}_{R \in Q'} \mid R \cap F \neq \phi\}$

4) $\delta'(R,a)$ where $R \subseteq Q$ and $a \in \Sigma$.

$\delta'(R,a) = \{q \in Q \mid q \in \delta(r,a)$ for some $r \in R\}$

$= \bigcup_{r \in R} \delta(r,a)$



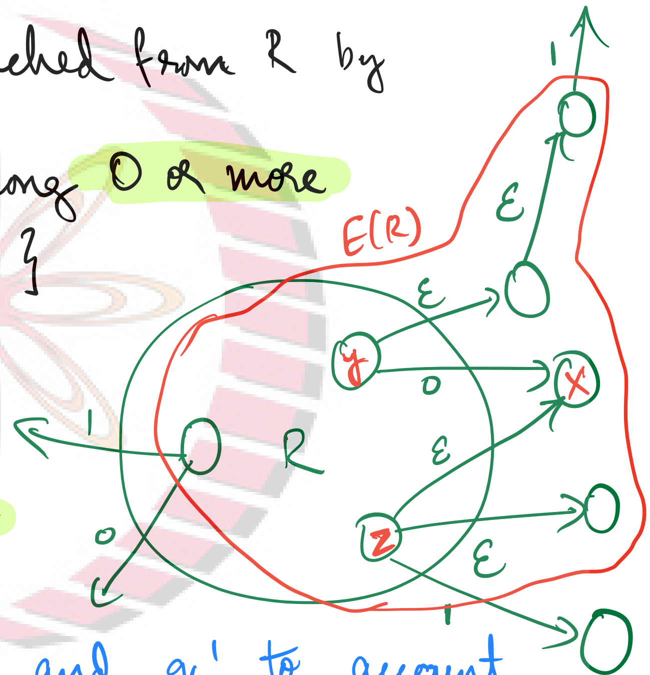
$\delta(\{q_1,q_2\},0) = \{q_1,q_2,q_3,q_4\}$

$r \in R \rightarrow \underline{\delta(r,a)}$

$\delta(\{q_1, q_2\}, 1) = \{q_1\}$

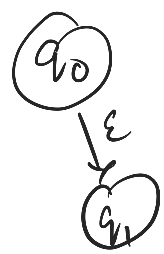
This takes care of all the transitions except the ε transitions. To handle ε transitions, we define $E(R)$ for every $R \subseteq Q$ (or equivalently $R \in Q'$).

$E(R) = \{q \mid q$ can be reached from R by travelling along 0 or more ε transitions $\}$



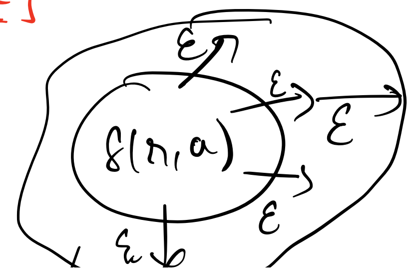
Notice that $R \subseteq E(R)$.

We need to redefine δ' and q_0' to account for ε transitions.



→ $\delta'(R,a) = \{q \in Q \mid q \in E(\delta(r,a))$ for some $\underline{r \in R}\}$

$= \bigcup_{r \in R} E(\delta(r,a))$.



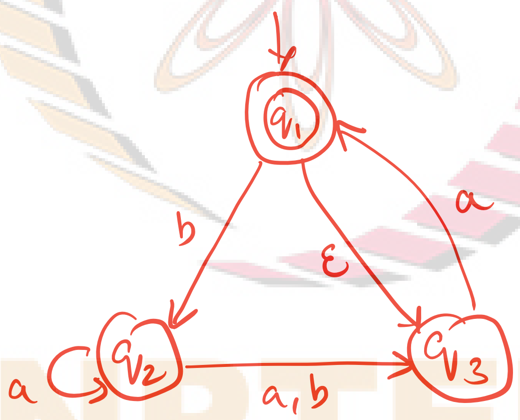
$\rightarrow \quad q_0' = E(\{q_0\})$.

$\overline{E(\delta(r,a))}$

After reading any string, if N can possibly reach the states $R \subseteq Q$, then M will reach $R \in \mathcal{P}(Q)$.

$r_1 \quad r_3 \quad r_{10}$

Read example 1.41 in the book, for an illustration of the construction in the above proof.



Eqvt DFA contains $2^3 = 8$ states $\rightarrow$ 6 states