Closure Properties of Regular Languages.

Theorem 1.39: Every NFA has an equivalent DFA.

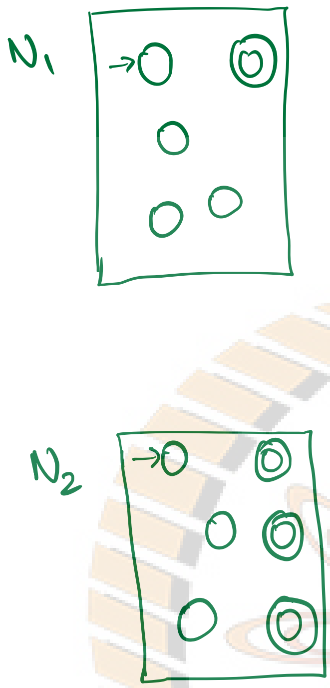
This implies that NFAs are only as powerful as DFAs.

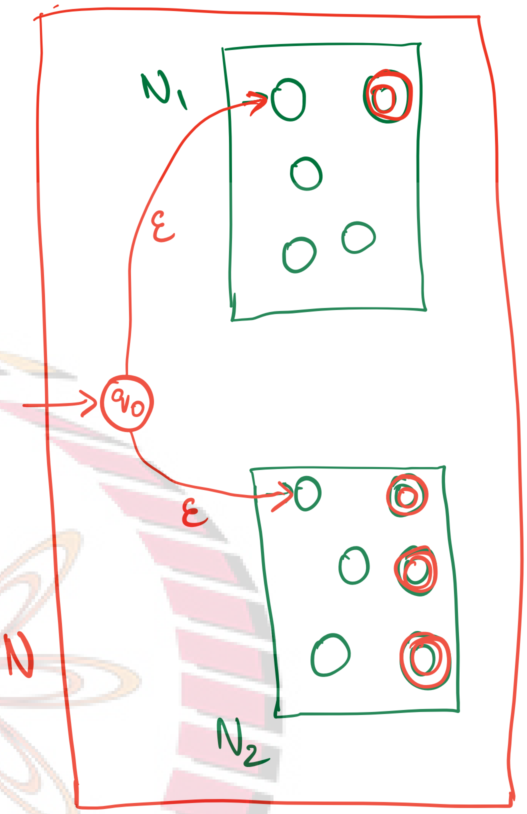
Corollary 1.40: A language is regular if and only if some nondeterministic finite automaton (NFA) recognizes it.

Now we can use this characterization of regular languages to show closure under the regular operations.

Theorem 1.45: The class of regular languages is closed under union.

Proof: Let A_1 & A_2 be two regular languages. We may assume that N_1 and N_2 respectively are NFA's that recognise A_1 and A_2.





$N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$

$N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$

$N = (Q, \Sigma, \delta, q_0, F)$.

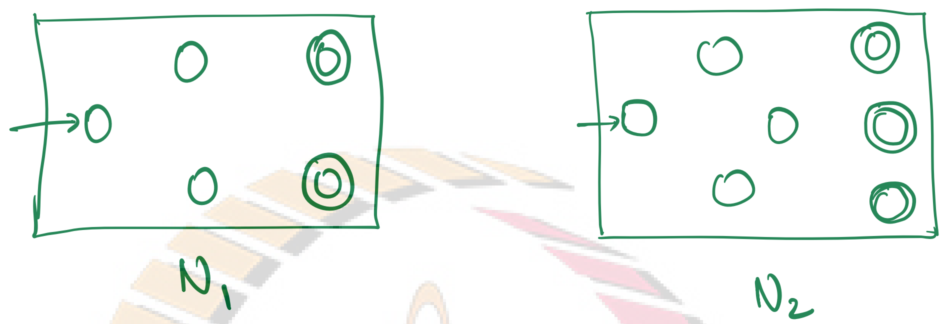
$Q = Q_1 \cup Q_2 \cup \{q_0\}$

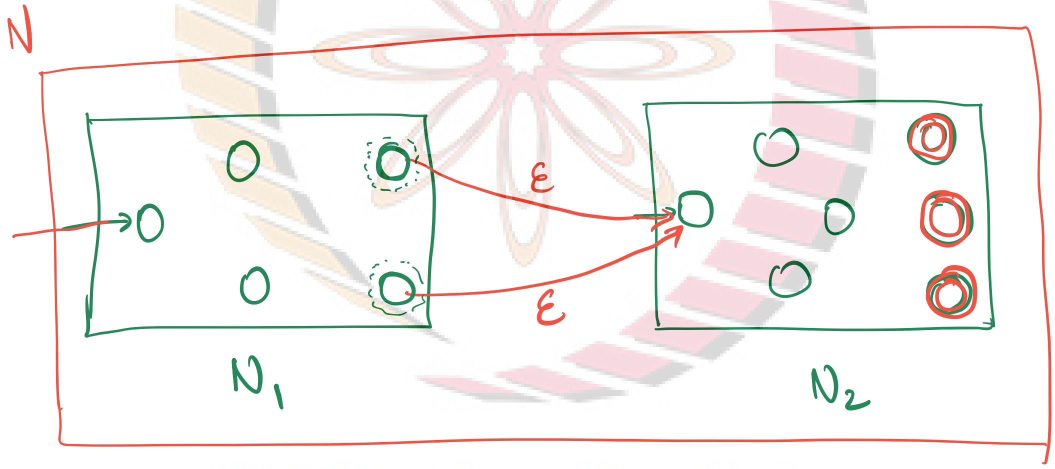
$F = F_1 \cup F_2$

Exercise: Read the details of this proof.

Theorem 1.47: The class of regular languages is closed under the concatenation operation.

Proof Sketch: Let A_1 and A_2 be regular languages recognized by NFA's N_1 and N_2 respectively.





Start state of N = Start state of N_1

Accept states of N = Accept states of N_2.

w is such that $w = w_1 w_2$ where $w_1 \in A_1$ and $w_2 \in A_2$

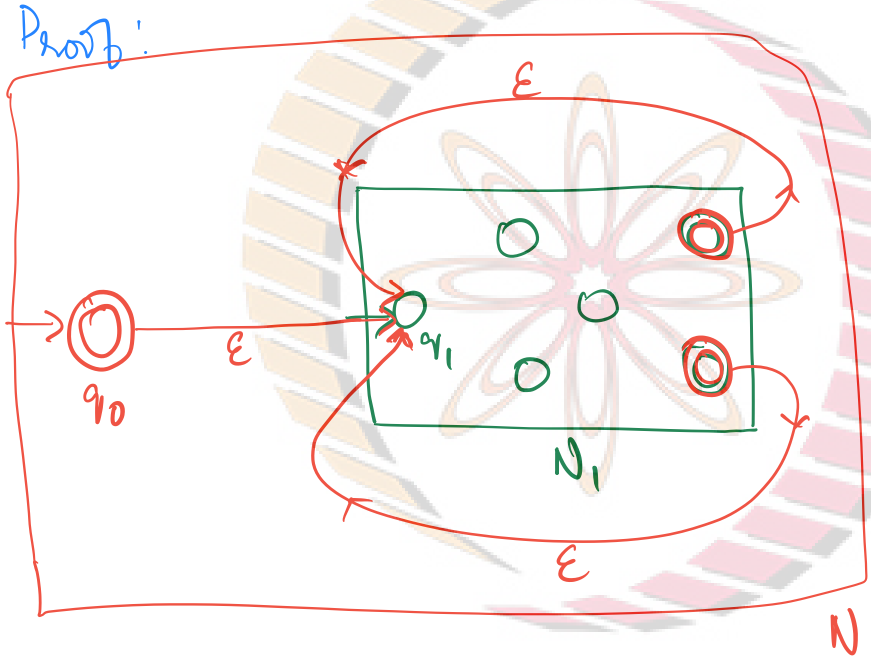
$$\Updownarrow$$

w is accepted by the above NFA.

Exercise: Read the details of the proof.

Theorem 1.49: The class of regular languages is closed under the star operation.

Proof:



A^* contains ε, for any A.

1) Added ε transitions from accept states of N_1 to the start state of N_1 (q_1)

2) Unless q_1 (start state of N_1) $\in F_1$, ε is not accepted. We add a new start state q_0 and have ε transition from $q_0 \longrightarrow q_1$.

Suppose A is recognized by the NFA N_1, where $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$. We will construct $N = (Q, \Sigma, \delta, q_0, F)$, such that

$L(N) = A^* = \{x_1 x_2 \ldots x_k \mid k \geq 0$, and $x_i \in A$ for each $i\}$

$Q = Q_1 \cup \{q_0\}$ where q_0 is the new start state.

$F = F_1 \cup \{q_0\}$

$\delta(q, a) = \delta_1(q, a)$ when $q \in Q_1$ and $a \neq \varepsilon$

$\delta(q, \varepsilon) = \delta_1(q, a)$ when $q \notin F_1$

$\delta(q, \varepsilon) = \delta_1(q, \varepsilon) \cup \{q_1\}$ when $q \in F_1$

$\delta(q_0, a) = \phi$ when $a \neq \varepsilon$

$\delta(q_0, \varepsilon) = \{q_1\}$