A language over Σ is a set of strings over Σ. That is, a language over Σ is a subset $A \subseteq \Sigma^*$.

Example: ① Set of all binary strings with an odd number of 1's.

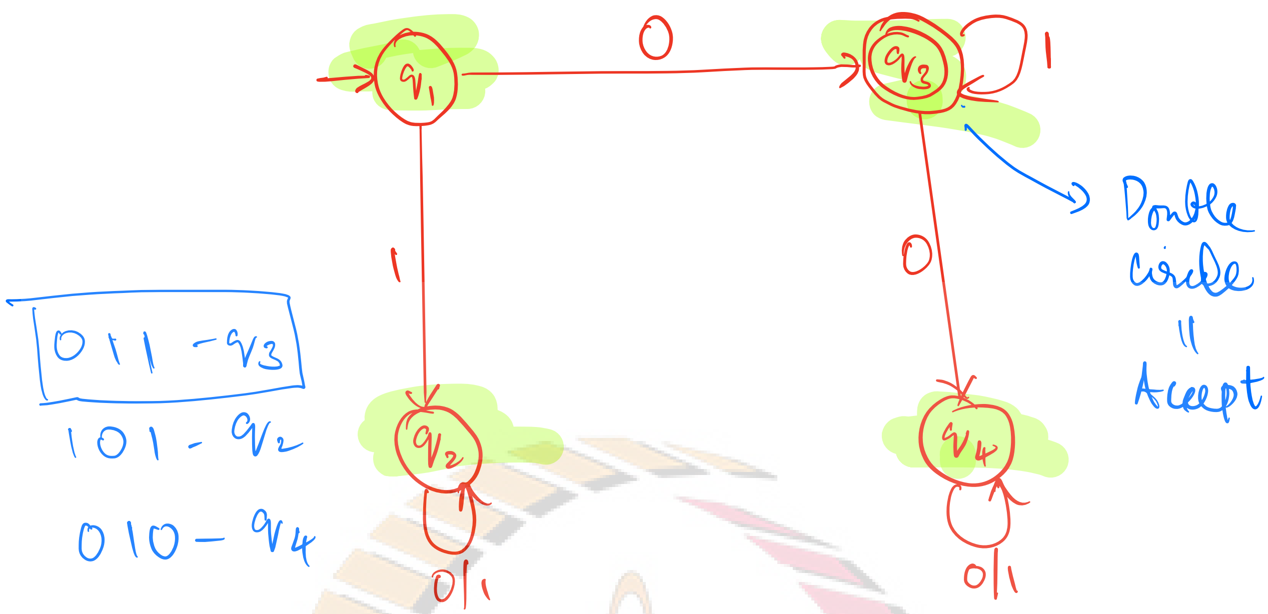
② Set of all binary strings that are palindromes.

Finite Automata

AUTOMATON : Singular
AUTOMATA : Plural

- Computers with a limited amount of memory.
- State based devices
- Examples like timers, door open | close controller, thermostat
- These abstract models help us gain understanding. States can be thought of as memory.

Deterministic Finite Automata (DFA)



→ States : At q_1, the start state

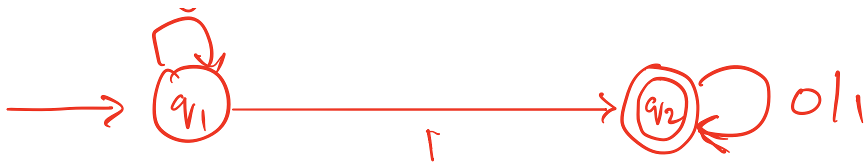
→ Moves : As per the arrows

→ Decides : Accept if ending at ◎

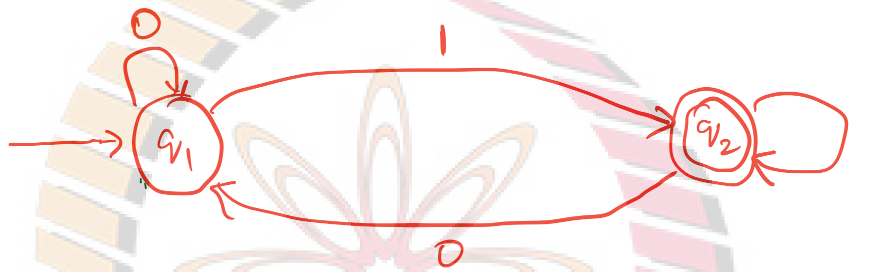
The language recognized by the above DFA is $A = \{0, 01, 011, 0111, \ldots\}$

<u>Deterministic</u> : Unique transition for each input symbol read.

<u>Finite</u> : Finite no. of states.

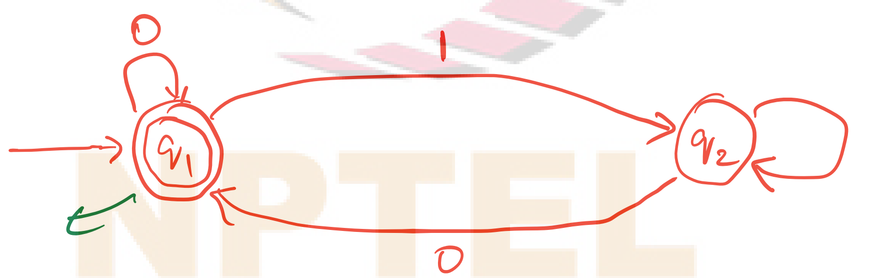


language of the above = All the strings that have at least one 1.



010 : ✗
101 : ✓

language = Strings that end in 1.



ε accepted

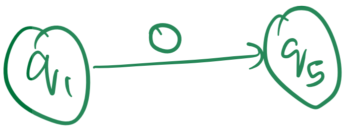
language = Strings that end in 0. $\cup \{\varepsilon\}$

Definition 1.5 : A deterministic finite automaton (DFA) is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$

where

1. Q is a finite set of states
2. Σ is a finite alphabet
3. $\delta : Q \times \Sigma \longrightarrow Q$ is the transition function
4. $q_0 \in Q$ is the start state
5. $F \subseteq Q$ is the set of accepting states.

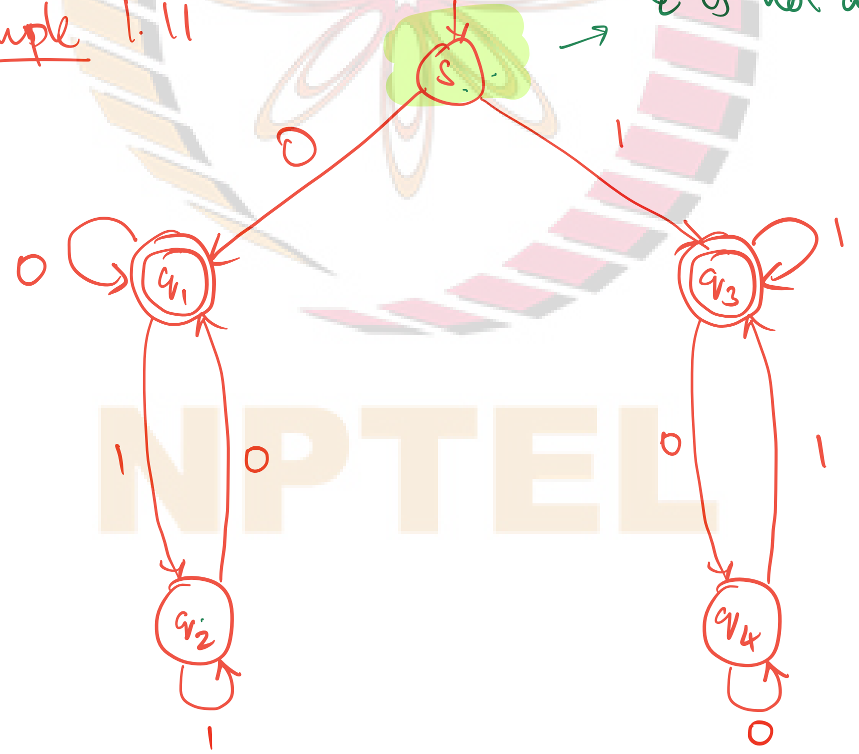
denoted by $\bigcirc$



$\delta(q_1, 0) = q_5$

Example 1.11

ε is not accepted



010 ✓
011 ✗
101 ✓
110 ✗

$Q = \{S, q_1, q_2, q_3, q_4\}$

$\Sigma = \{0\ 1$

$\Sigma = \{0, 1\}$

$\delta(s, 0) = q_1$ $\quad$ $\delta(s, 1) = q_3$

$\delta(q_1, 0) = q_1$ $\quad$ $\delta(q_1, 1) = q_2$

$\delta(q_2, 0) = q_1$ $\quad$ $\delta(q_2, 1) = q_2$

$\delta(q_3, 0) = q_4$ $\quad$ $\delta(q_3, 1) = q_3$

$\delta(q_4, 0) = q_4$ $\quad$ $\delta(q_4, 1) = q_3$

$q_0 = s$ $\qquad$ $F = \{q_1, q_3\}$

Language of this DFA is the set of all binary strings that start and end with 0 or start and end with 1.

$w = 1011$

We say $M = (Q, \Sigma, \delta, q_0, F)$ accepts a string $w = w_1 w_2 \cdots w_n$ if there is a sequence of states $r_0, r_1, r_2, \ldots r_n$ such that

STARTS

1. $r_0 = q_0$
2. $\delta(r_i, w_{i+1}) = r_{i+1}$

MOVES ← $\forall\ 0 \le i \le n-1$

ENDS at ACCEPT ← 3. $r_n \in F$.

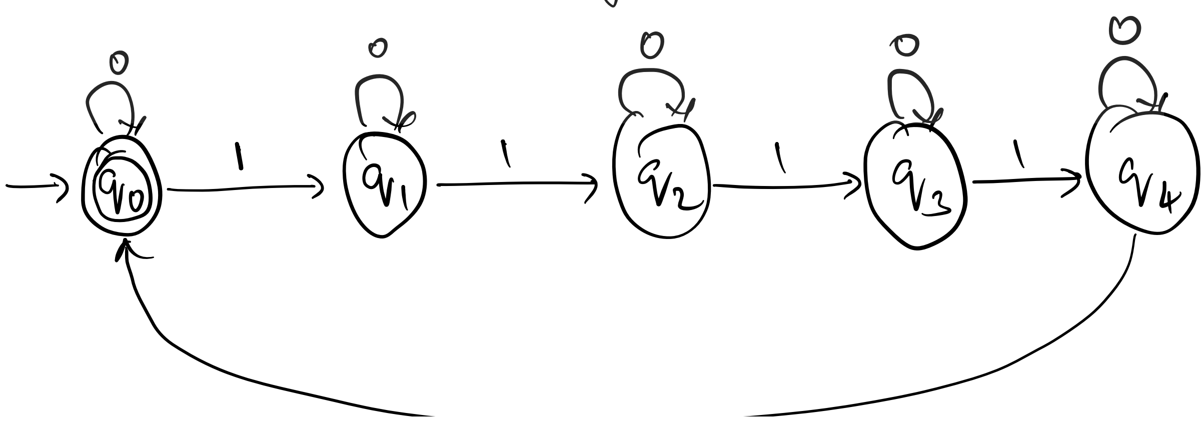
DFA M recognizes the language A if

$A = \{ w \mid M \text{ accepts } w \}$.

Also denoted $L(M) = A$.

Q: Construct a DFA that accepts all binary strings for which no. of 1's is divisible by 5.

- Need to keep track the number of 1's.
- We only need to keep track of the count modulo 5.



Q: let $A = \{ w \in \{0,1\}^* \mid \text{no. of 1's in } w \equiv 2 \bmod 5 \}$

Construct a DFA M such that $L(M) = A$.

Regular languages (Def 1.16): A language is regular if some finite automaton recognizes it.

- Soon we will see other definitions of regular languages.

Q: Construct a DFA for the following:

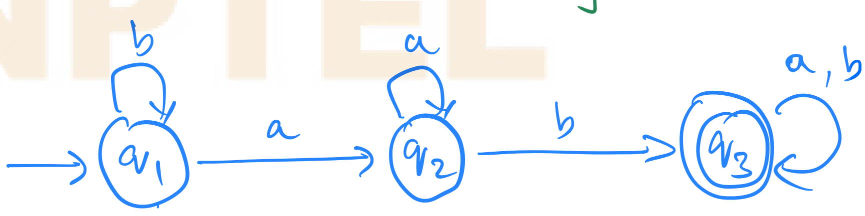
$\{ w \in \{a,b\}^* \mid w \text{ contains } ab \text{ as a substring} \}$

aabb ✓

bbaa ✗

baab ✓

abba ✓

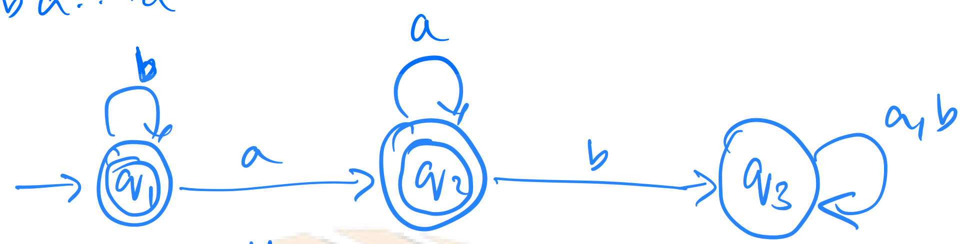


M_1

Q: DFA for $\{ w \in \{a,b\}^* \mid w \text{ does not contain } ab \text{ a substring} \}$

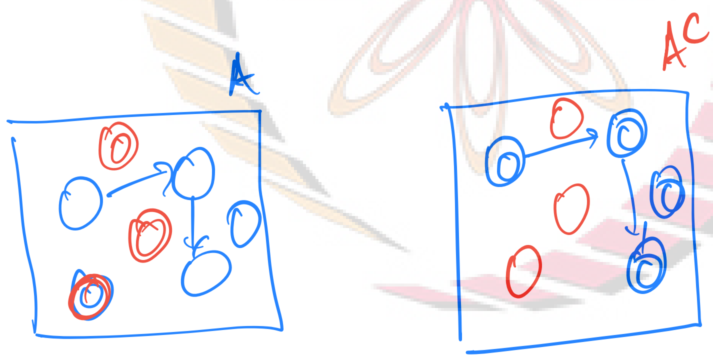
$b\,b\,b\ldots b = b^*$

$b\,b\,b\ldots b\,a\ldots a = b^* a^*$



M_2

Q: DFA for $\{ w \in \{0,1\}^* \mid w$ begins with 1 and ends in 0 $\}$



$A \qquad A^c$

Theorem: Regular languages are closed under complement.

Proof: Take the complement of F.

Next: Regular operations – Union,

Concatenation , Star

Closure under regular operations.