Week -1 Questions - Theory of Computation

January 21, 2023

1. (1 point) For a given language A, there could be multiple DFA's that recognize A.
   A. False
   B. True

2. (1 point) The minimum number of states in the DFA that is required to recognize the empty Language $L = \emptyset$ is equal to _____**1**_____.

3. (1 point) The minimum number of states in the DFA that is required to recognize the Language $L = \{\varepsilon\}$ is equal to _____**2**_____.

4. (1 point) The minimum number of states in the DFA that is required to recognize the following Language L is equal to _____**5**_____.

$$L = \{w \in \{0,1\}^* | w \text{ has a number of zeroes that is divisible by } 5\}$$

0.1 Flipping and Reversing

For questions 5, 6, 7 we define the following. Let $\Sigma = \{0, 1\}$. For every string w in Σ^*, we define $\text{Comp}(w)$ to be the string obtained by complementing each bit of w. For instance $\text{Comp}(00101) = 11010$. For a language A, We define $\text{FLIP}(A) = \{\text{Comp}(w) \ : \ w \in A\}$.

For every string w in Σ^*, we define $\text{Rev}(w)$ to be the string obtained by reversing the string w. For instance $\text{Rev}(00101) = 10100$. For a language A, We define $\text{REV}(A) = \{\text{Rev}(w) \ : \ w \in A\}$.

5. (2 points) Let A be recognized by a DFA $M = (Q, \Sigma, \delta, q_0, F)$. Suppose our goal is to construct a DFA $M' = (Q', \Sigma', \delta', q'_0, F')$ that recognizes $\text{FLIP}(A)$. Which of the following transformations yields such a DFA M'?
   A. Make the accept states non-accepting, and make the non-accepting states accepting. i.e $F' = Q \setminus F$
   B. Reverse the direction of every edge in the DFA M, i.e. if $\delta(q_1, a) = q_2$ then $\delta'(q_2, a) = q_1$
   C. Complement the value on every edge(arrow) of the DFA, i.e. if $\delta(q_1, a) = q_2$ then $\delta'(q_1, \bar{a}) = q_2$ (Here $\bar{0} = 1$ and $\bar{1} = 0$)
   D. None of the above are true. The language $\text{FLIP}(A)$ is not regular.

6. (2 points) Let A be recognized by a DFA with a single accept state $M = (Q, \Sigma, \delta, q_0, F)$, where $F = \{q_a\}$. Suppose our goal is to construct a DFA $M' = (Q', \Sigma', \delta', q'_0, F')$ that recognizes $\text{REV}(A)$. Which of the following transformations yields such a DFA M'?
   A. Make the accept states non-accepting, and make the non-accepting states accepting. i.e $F' = Q \setminus F$
   B. Reverse the direction of every edge in the DFA M, i.e. if $\delta(q_1, a) = q_2$ then $\delta'(q_2, a) = q_1$

C. Reverse the direction of every edge in the DFA M, i.e. if $\delta(q_1, a) = q_2$ then $\delta'(q_2, a) = q_1$ **and** $q'_0 = q_a$ (New start state is old accept state) and $F' = \{q_0\}$.

D. Complement the value on every edge(arrow) of the DFA, i.e. if $\delta(q_1, a) = q_2$ then $\delta'(q_1, \bar{a}) = q_2$ (Here $\bar{0} = 1$ and $\bar{1} = 0$)

E. Set $q'_0 = q_a$ (New start state is old accept state) and $F' = \{q_0\}$.

F. None of the above choices create a valid DFA that accept the language $REV(A)$

7. (2 points) Consider two DFA's $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$. Note that both the DFAs operate over the same alphabet $\Sigma = \{0, 1\}$. Consider a new DFA $M_3 = (Q_3, \Sigma, \delta_3, q_3, F_3)$ that is obtained in the following manner.

   1. $Q_3 = Q_1 \times Q_2$
   2. $\Sigma_3 = \Sigma = \{0, 1\}$
   3. For all states $(r_1, r_2) \in Q_3$ and $a \in \Sigma$, we define $\delta_3((r_1, r_2), a) = (\delta_1(r_1, \bar{a}), \delta_2(r_2, \bar{a}))$ where $\bar{a}$ denotes the complement of the bit a.
   4. $F_3 = \{(r_1, r_2) | r_1 \in F_1$ **and** $r_2 \in F_2\}$

   Then what is the language that is accepted by this DFA M_3?

   A. FLIP$(A \cup B)$

   B. FLIP$(A) \cup$ FLIP(B)

   C. FLIP$(A \cap B)$

   D. None, The language is not regular.

8. (2 points) The class of regular languages is closed under which of the below operations? Select all that is/are correct.

   ☐ subset i.e, subset of a regular language is also regular.

   ■ **union**

   ■ **complement**

   ■ **concatenation**

9. (2 points) Which of the following statements is/are true? Select all that is/are correct.

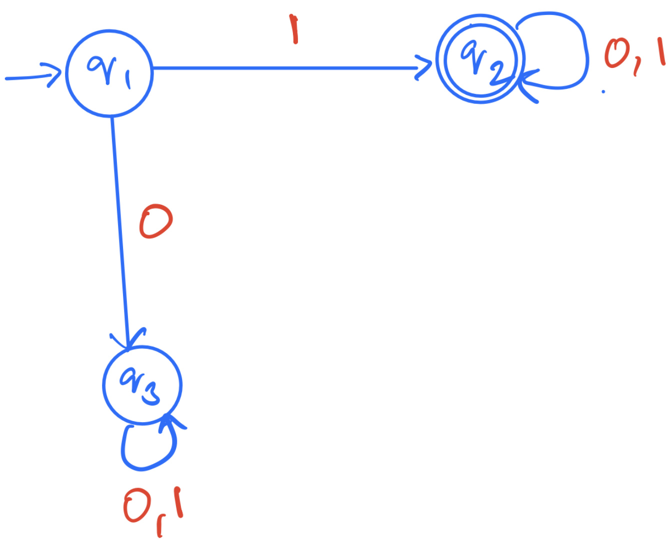
   ☐ All regular languages are finite in size.

   ■ **The class of regular languages is closed under intersection.**

   ☐ If A is a regular language, and $A \subseteq B$, then B is necessarily regular.

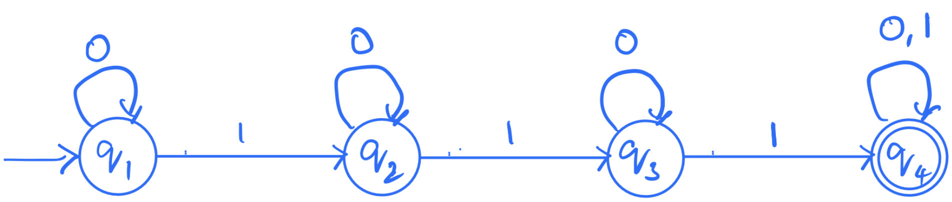
   ■ **All finite languages are regular.**

10. (2 points) What is the language recognized by the below DFA?



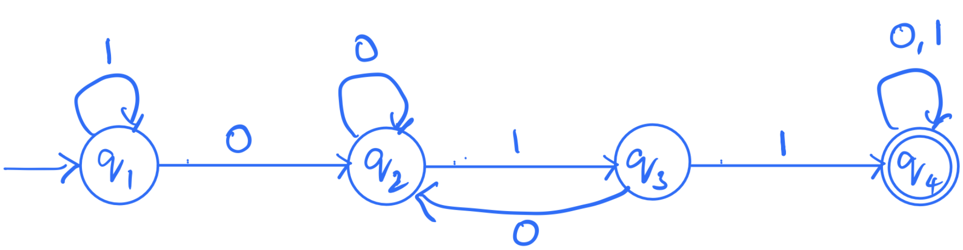
A. All binary strings that begin with a 1.
B. All binary strings that contain a 1.
C. All binary strings that end with 1.
D. All binary strings that have no 0's.

11. (2 points) What is the language recognized by the below DFA?



A. All binary strings that start with 111.
B. All binary strings that contain 111 as a substring.
C. All binary strings that contain at least three 1's.
D. All binary strings that contain exactly three 1's.

12. (2 points) What is the language recognized by the below DFA?



A. All binary strings that start with 011.

B. All binary strings that contain 011 as a substring.

C. All binary strings that contain a 0 followed by at least two 1's.

D. All binary strings that end with 011.