* Read Chapter 0 from Sipser.

  - Sets, Functions, Relations
  - Graphs
  - Mathematical logic, Theorem, Proofs

Every computational problem will be represented as a string input to a computer. String will be over an alphabet, denoted by Σ.

Example:

Q: Is the given number even?

Is the given number in $\{2, 4, 6, 8, 10, \ldots\}$

OR

language $\leftarrow$ $\{10, 100, 110, 1000, \ldots\}$

Q: Is the given binary number a palindrome? RADAR NOON

Is the given number in $\{0, 1, 11, 101, 111 \ldots\}$

Q: Is the given binary number a perfect square?

Is the number in $\{1, 100, 1001, 10000, \underline{11001}, \ldots\}$

Q: Does the given English word contain the letter 'a'?

Is the word in $\{ a, an, at, am, bat, cat \ldots \}$

Usually, strings contain "symbols" from an alphabet, Σ. We say that a string is over an alphabet, Σ.

Examples: (1) Binary alphabet $\Sigma_1 = \{0, 1\}$

(2) Decimal $\Sigma_2 = \{0, 1, 2, 3, 4, \ldots 9\}$

(3) English $\Sigma_3 = \{a, b, c, d, \ldots x, y, z\}$

(4) $\Sigma_4 = \{a, b, c, 1, 2\}$

$$\boxed{\text{Def: A string over an alphabet } \Sigma, \text{ is a finite sequence of symbols written one after another.}}$$

Example: 101101 is a binary string

consider is an English string

83262 is a decimal string

The length of the string w is denoted by $|w|$

$w = 110110 \implies |w| = 6$

$w = \text{hello} \implies |w| = 5$

$w = 253 \implies |w| = 3$

Q: Is "hello␣there!" a string over $\{a, b, c, \ldots, x, y, z\}$?

NO! → Two things that are not part of the alphabet. (1) Space after "hello" (2) Exclamation Mark.

Some Definitions

1. w^R denotes the reverse of the string w.

2. ε denotes the empty string. $|\varepsilon| = 0$.

3. Concatenation: If x, y are two strings, the concatenation is denoted by xy.

$x =$ "top" $\quad y =$ "hat"

Concatenation $\quad xy =$ "top hat"

$x^R y =$ "pot hat"

4. $x^k = \underbrace{x \cdot x \cdot x \cdots x}_{k \text{ times}}$, for $k \geq 0$ integer

$x^0 = \varepsilon$.

$x =$ "top" $\implies x^3 =$ "top top top"

5. Kleene star of x, denoted x^*, is defined as

$x^* =$ Set of all x^k

$= \{x^k \mid k \geq 0\}$

$x = 10$

$x^* = \{\varepsilon, 10, 1010, 101010, \ldots\}$

6. For a set of symbols, Σ, we define Σ^* as the set of all strings using symbols from Σ.

Example: If $\Sigma = \{0, 1\}$, then

$$\Sigma^* = \{ \varepsilon, 0, 1, 00, 01, 10, 11, 000, \ldots \}$$

7. Substring: v is a substring of w if there exist strings x and y such that $w = xvy$.

Example: The word "get" is a substring of the word "together"

The string 1100 is a substring of 10101100

8. A language over Σ is a set of strings over Σ. That is, a language over Σ is a subset $A \subseteq \Sigma^*$.

Empty set ϕ is a language

Σ^* is also a language

Example: ① Set of all binary strings with an odd number of 1's is a language over $\{0, 1\}$.

② Set of all dictionary words is

a language over the English alphabet.

We can describe languages in many ways.

1. Brute Force listing : $\{a, ab, abb, \ldots\}$

2. language operations : ab^*

3. Other set theoretic descriptions.

Example: $ab^* \cup a^*b$