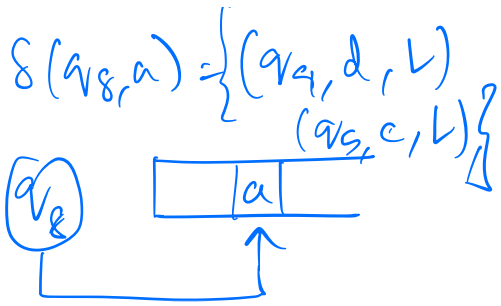
Nondeterministic TM's



$M = (Q, \Sigma, \Gamma, \delta, q_0, q_a, q_r)$

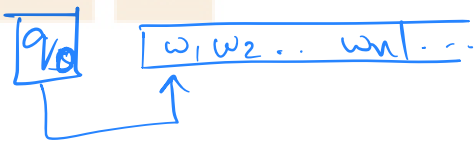
$\delta : Q \times \Gamma \longrightarrow \mathcal{P}(Q \times \Gamma \times \{L, R\})$

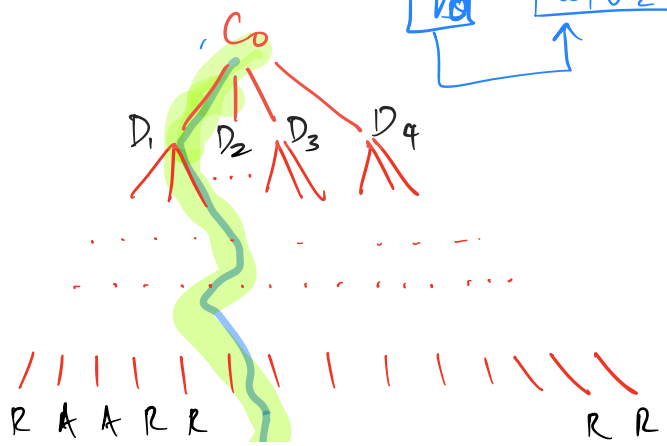
det TM
$\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$

There could be multiple computation paths possible. The TM accepts if any branch of computation leads to an accept.

M accepts $w = w_1 w_2 \ldots w_n$ iff $\exists$ a sequence of configurations $C_0, C_1, \ldots C_k$ such that

1. C_0 is the starting configuration
2. $C_i \to C_{i+1}$ is a valid move for each i
3. C_k is an accepting configuration.





A

We accept if there is at least computation path that leads to an accepting config.

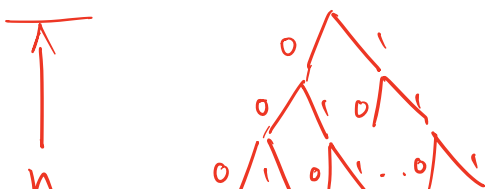
Compositeness

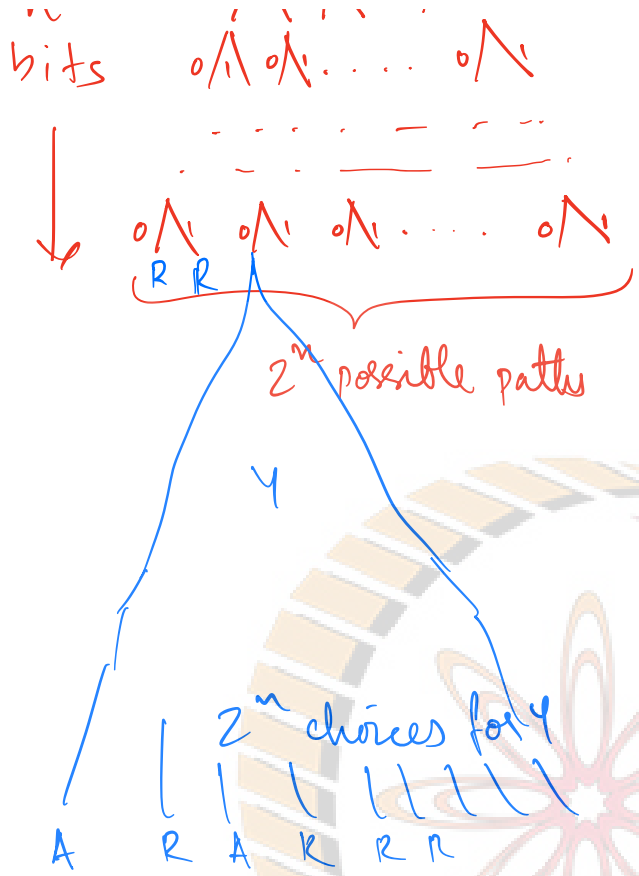
Given $w = w_1 w_2 \ldots w_n$. Need to decide if w is a composite number.

High level Idea: Guess $X, Y > 1$ and check if $XY = w$

0 1 1 0 . . 1	1 0 1 1 0 1 . . 0

1. Guess X and write on tape.
   - Nondeterministically write 0 | 1, n bits
2. If $X \leq 1$, reject.
3. Guess Y and write on tape.
   - Nondeterministically write n bits
4. If $Y \leq 1$, reject.
5. Check if $XY = w$
6. Accept if $XY = w$, else reject.





27

6 bits

10111 | 00101

23 5

$23 * 5 = 115 \neq 27$

Reject

01001 | 00011

9 3

Accept

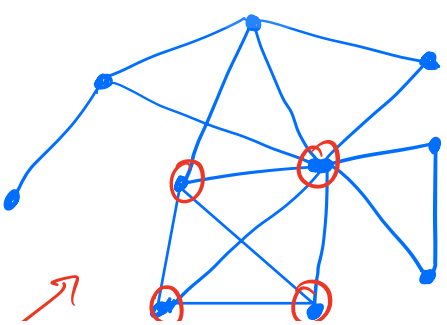
CLIQUE

Input: Given a graph with n vertices $V = \{1, 2, \ldots n\}$ and an integer k.

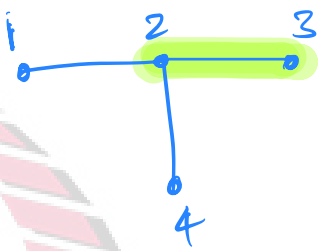
Output: Determine if G has a k-clique

$S \subseteq V$ is a set of vertices.

S is a clique if for all $u, v \in S$, (u, v) is an edge

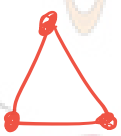


This graph has a 4-clique Graph G.
but no 5-clique.



The graph may be given as an adjacency matrix. If i, j is an edge, then $A_{i,j} = A_{ji} = 1$ else $A_{ij} = A_{ji} = 0$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$



NTM decider

1 2 3 4 ... n

1 ... 4 5 ... n

For $i = 1$ to n

Nondeterministically select / omit vertex i.

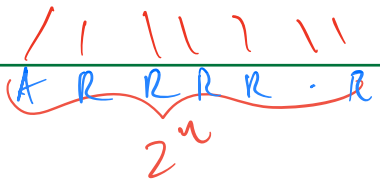
Each selected vertex is written on the tape.

If the number of selected vertices $\neq k$, reject.

Else, for each selected pair (i, j), check if $A_{ij} = 1$

If $A_{ij} = 0$ for any such pair, reject. (i, j) is an edge

Accept if not rejected.



Guess & Verify TM

In both the above NTM examples, all the non-deterministic choices happen at the beginning of the computation. After which, the guesses are "verified".

In all the NTM's, we can move all the nondeterministic choices to the beginning.