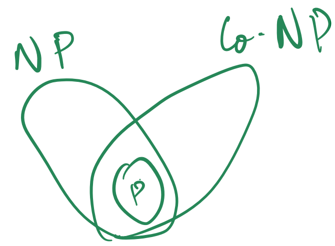
Other results in Space Complexity

Def: co-NL $= \{ A \mid \bar{A} \in NL \}$

We can similarly define co-NP.



Def: co-NP $= \{ A \mid \bar{A} \in NP \}$

In the case of NP and co-NP, we know that $P \subseteq NP \cap$ co-NP. But NP and co-NP are incomparable. Neither NP nor co-NP is known to be contained in the other.

But in the case of NL and co-NL, we have the following.

Theorem: NL = co-NL
(Neil Immerman and Robert Szelepcsényi - 1987)

This is a surprising result. It is proved by showing that $\overline{PATH} \in NL$. Since PATH is NL complete, we have that $\overline{PATH}$ is co-NL complete.

So $\overline{PATH} \in NL \Rightarrow$ co-NL $\subseteq$ NL.

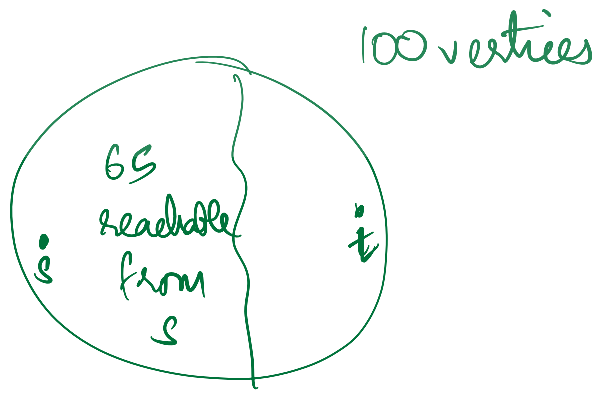
But $\overline{PATH} \in NL$ also implies that PATH $\in$ co-NL.
Since PATH is NL-complete, this implies that
NL $\subseteq$ co-NL.

Thus we get NL = co-NL.

$\overline{PATH} = \{\langle G, s, t\rangle \mid$ G has no directed s-t path $\}$

We need to build a NL-verifiable certificate for there not being a PATH.

This is accomplished by obtaining c, the number of reachable vertices from s, and identifying c vertices (none of them equal to t) that are reachable from s.



PSPACE - Completeness

$$PSPACE = \bigcup_{k=1}^{\infty} DSPACE(n^k)$$

Examples: (1) SAT $\in$ DSPACE (n)

(2) BFS $\in$ DSPACE (n)

By Savitch's theorem, we have that PSPACE = NPSPACE.

We also know that $L \subsetneq PSPACE$ by space hierarchy theorem.

Def: B is PSPACE-Complete if

(1) $B \in PSPACE$

(2) $\forall A \in PSPACE$, $A \leq_p B$

Poly time reduction

Examples

PSPACE Complete:

1) TQBF: True Fully Quantified Boolean Formula

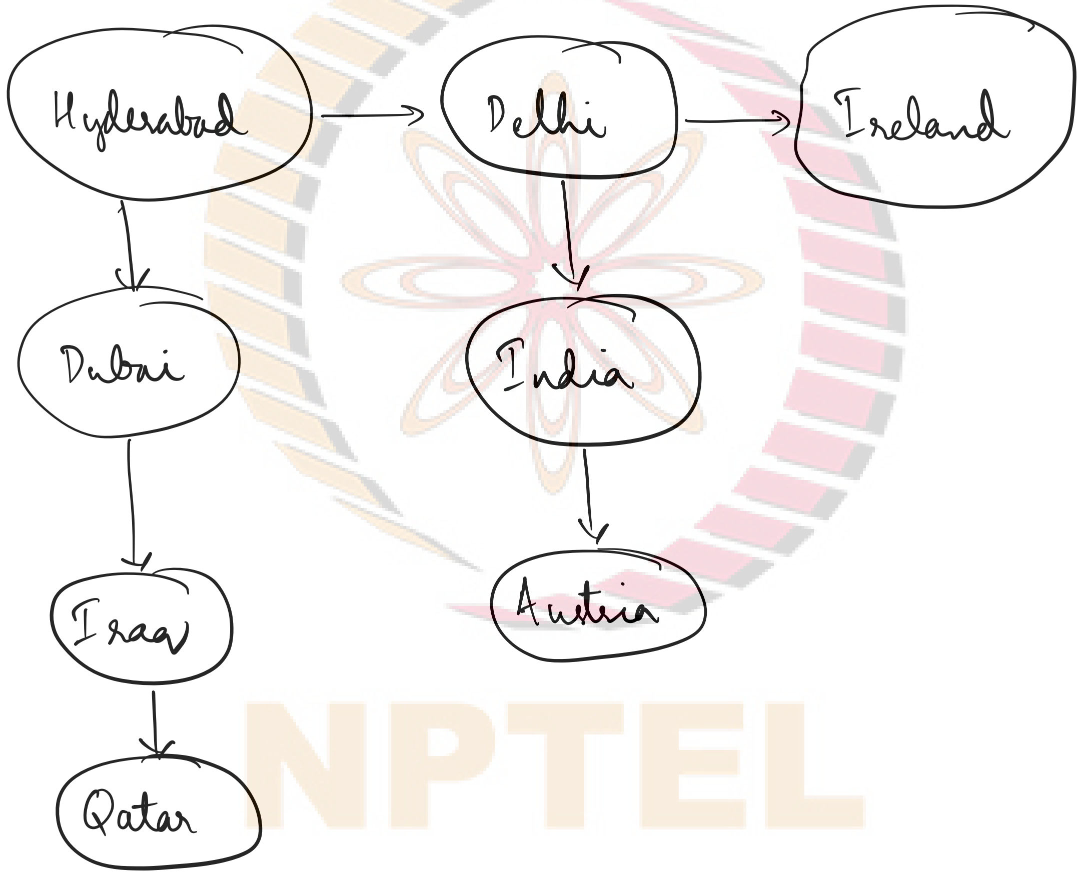
→ A Boolean formula with quantifiers $(\forall, \exists)$ in each variable.

SAT can be written as

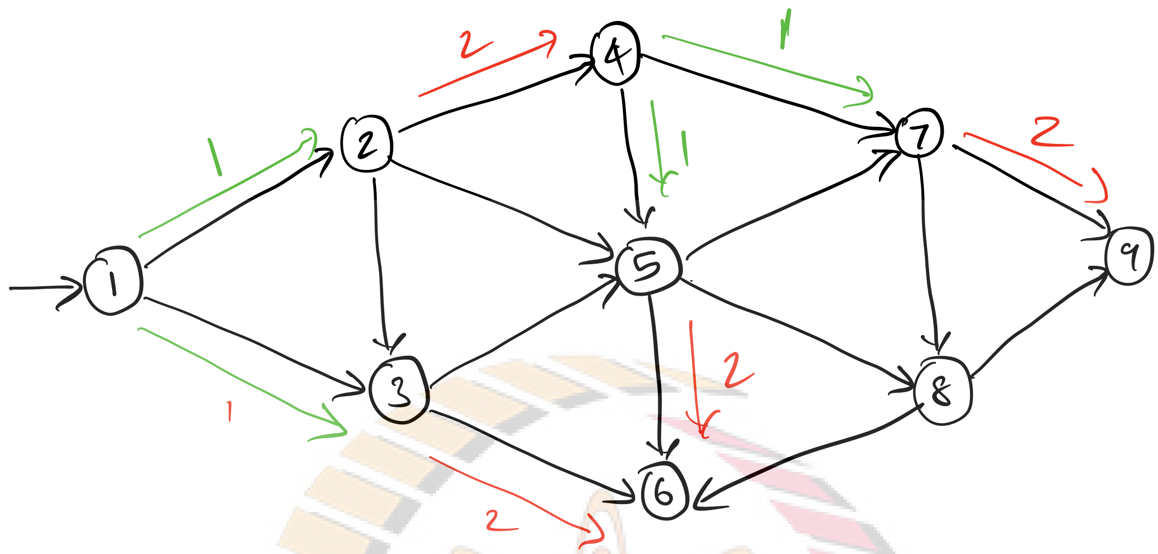
$$\exists x_1 \exists x_2 \ldots \exists x_n \; \phi(x_1, x_2 \ldots x_n) = \text{TRUE}$$

$$\forall x_1 \exists x_2 \exists x_3 \forall x_4 \ldots \; \psi(x_1, x_2 \ldots) = \text{TRUE?}$$

2) GENERALIZED GEOGRAPHY



GEN-GEOGRAPHY is PSPACE-complete.



Question : Does player 1 have a winning strategy?

In general, many "general" versions of games are PSPACE-complete. For instance, generalized chess (played on an $n \times n$ board) is PSPACE complete.

Classes ordering that we know

$$L \subseteq NL \subseteq P \subseteq NP \subseteq \ldots \subseteq PSPACE$$

$NL = coNL$, $\quad P \subseteq co\text{-}NP$, $\quad PSPACE = NPSPACE$

But we know $L \neq PSPACE$