exercise

1.5 $\mathrm{var}[f(x)]$ 1.39 만족함을 증명하라.

Sol) $E[(f(x) - E[f(x)])^2] = E[f(x)^2] - 2E[f(x)]E[f(x)] + E[f(x)]^2 = E[f(x)^2] - E[f(x)]^2$

$\hookrightarrow$ $f(x)^2 - 2E[f(x)]f(x) + E[f(x)]^2$

1.6 two variable x, y indepent, $\mathrm{cov}[x,y]$?

two random variable, cov Matrix

$$\mathrm{cov}[x,y] = E_{x,y}[\{x - E[x]\}\{y - E[y]\}] = E[xy] - E[x]E[y].$$

$$E[xy] = \sum_x \sum_y p(x,y)\,xy = \sum_x p(x)x \sum_y p(y)y = E[x]E[y]$$

$x, y \rightarrow$ Continuous variable?

1.9. Gaussian Distribution 1.46 is given by μ, Similarity multivariate Gaussian 1.52 given by μ.

Sol) For the univariate, $\frac{d}{dx}N(x|\mu,\sigma^2) = -N(x|\mu,\sigma^2)\frac{x-\mu}{\sigma^2}$

$$\frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left\{-\frac{1}{2\sigma^2}(x-\mu)^2\right\} \left(-\frac{1}{2\sigma^2}(x-\mu)^2\right)'$$

$$(2) - \frac{1}{2\sigma^2}(x-\mu)\cdot 1$$

$$= -\frac{1}{\sigma^2}(x-\mu)$$

defin

$$N(x|\mu,\sigma^2) = \frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left\{-\frac{1}{2\sigma^2}(x-\mu)^2\right\}$$

$\beta = 1/\sigma^2$

$\downarrow$

precision

mean

Variance

$N(x|\mu,\sigma^2) > 0$

$$\underbrace{\frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left\{-\frac{1}{2\sigma^2}(x-\mu)^2\right\}}_{N(x|\mu,\sigma^2)} \cdot -\frac{1}{\sigma^2}(x-\mu) = N(x|\mu,\sigma^2)\left(-\frac{(x-\mu)}{\sigma^2}\right)$$

$\rightarrow$ Contious.
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(1.52)

1.9 ex) (1.52) multivariate Gaussian Distribution parameters μ, Σ

$$N(x;\mu,\Sigma) = \frac{1}{(2\pi)^{D/2}} \frac{1}{|\Sigma|^{1/2}} \exp\left\{ -\frac{1}{2}(x-\mu)^t \Sigma^{-1} (x-\mu) \right\}$$

mean

D-dimentional vector x of continuous variable.

$x \sim N(\mu, V)$

Σ convariance.

D x D matrix Σ → covariance.

$\{x^{(1)}, x^{(2)}, \dots, x^{(m)}\}$ $\quad \mu = \frac{1}{m}\sum_{i=1}^{m} x^{(i)}$ $\quad \Sigma = \frac{1}{m}\sum_{i=1}^{m}(x^{(i)}-\mu)(x^{(i)}-\mu)^t$

$$\frac{\partial}{\partial x} N(x|\mu,\Sigma) = -\frac{1}{2} N(x|\mu,\Sigma)\, \nabla_x \left\{ (x-\mu)^t \Sigma^{-1} (x-\mu) \right\}$$

$$-N(x|\mu,\Sigma)\,\Sigma^{-1}(x-\mu)$$

Convariance matrix = $\begin{bmatrix} \sigma_1^2 & & 0 \\ & \sigma_2^2 & \\ & & \ddots \\ 0 & & \sigma_n^2 \end{bmatrix}$

1.46 → derrivate same,

$$-N(x|\mu,\Sigma)\cdot\Sigma^{-1}(x-\mu)$$

$$-N(x|\mu,\sigma^2)\cdot\frac{1}{\sigma^2}(x-\mu)$$

1.10 ex) suppose that variable x and z statistically indepedent. mean, variance. statisfies?

$$E[x+z] = E[x] + E[z]$$

$$Var[x+z] = Var[x] + var[z]$$

Joint distribution $p(x,z) = p(x)\,p(z)$

sol) $$E[x+z] = \iint (x+z)\,p(x)\,p(z)\,dx\,dz$$

$$= \int x\,p(x)\,dx + \int z\,p(z)\,dz = E[x] + E[z]$$

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the variance.

$$Var[x] = E[x^2] - (E[x])^2$$

$$(x+z - E[x+z])^2$$

$$= x^2 + xz - 2xE[x+z] + xz + z^2 - 2zE[x+z] + (E[x+z])^2$$

$$(x+z-E[x+z])(x+z-E[x+z])$$

$$E[x+z] = E[x] + E[z]$$

$$x^2 - 2x(E[x]+E[z]) + 2xz + z^2 - 2z(E[x]+E[z])$$

$$= (x-E[x])^2 + (z-E[z])^2 + 2xz - 2xE[z] - 2zE[x]$$

$$= \quad '' \quad + 2(x - E[z])(z - E[x])$$

$$Var[x+z] = \iint (x+z-E[x+z])^2 p(x)p(z)\,dx\,dz$$

$$= \int (x-E[x])^2 p(x)\,dx + \int (z-E[z])^2 p(z)\,dz$$

$$= Var[x] + Var[z]$$

1.11ex) deprivatives log likelihood function (1.54) μ, σ^2 equal zero result 1.55 and 1.56

$$\ln p(x|\mu,\sigma^2) = -\frac{1}{2\sigma^2}\sum_{n=1}^{N}(x_n-\mu)^2 - \frac{N}{2}\ln\sigma^2 - \frac{N}{2}\ln(2\pi)$$

$$\frac{\partial l}{\partial\mu} = -\frac{1}{2\sigma^2}\sum_{n=1}^{N}(x_n-\mu)\cdot 2(-1) = \frac{1}{\sigma^2}\sum_{n=1}^{N}(x_n-\mu)$$

$$\frac{1}{\sigma^2}\sum_{n=1}^{N}(x_n-\mu) = 0 \rightarrow \frac{1}{\sigma^2}\sum_{n=1}^{N}x_n = \frac{1}{\sigma^2}\sum_{n=1}^{N}\mu \Rightarrow$$

$$\frac{\sigma^2}{N} \ldots$$

$$\frac{\partial l}{\partial\sigma^2} = \frac{1}{2(\sigma^2)^2}\sum_{n=1}^{N}(x_n-\mu)^2 - \frac{N}{2}\frac{1}{\sigma^2}$$

$$\frac{\partial l}{\partial\sigma^2} = 0$$

$$\frac{1}{2\sigma^2}\sum_{n=1}^{N}(x_n-\mu)^2 = \frac{N}{2}\cdot\frac{1}{\sigma^2}$$

$$\hookrightarrow \sum_{n=1}^{N}(x_n-\mu)^2 = N$$

1.13 ex) Suppose that the variance of Gaussian estimated (1.56). but the maximum likelihood estimate μ_{ML} true value μ of the mean.

estimator the property that its expectation given by variance σ^2

Sol) $E[x] = \int_{-\infty}^{\infty} N(x|\mu,\sigma^2)\, x\, dx = \mu$

$$E[\sigma^2] = E\left[\frac{1}{N}\sum_{n=1}^{N}(x_n-\mu)^2\right] = E\left[\frac{1}{N}\sum_{n=1}^{N}(x_n^2 - 2\mu x_n + \mu^2)\right]$$

$$\sigma^2 = \frac{1}{N}\sum_{n=1}^{N}(x_n-\mu)^2$$

$$(x_n-\mu)^2 = x_n^2 - 2\mu x_n + \mu^2$$

$$\frac{1}{N}\sum_{n=1}^{N} E[x_n^2 - 2\mu x_n + \mu^2]$$

$$E_n[x] = \mu$$

$$= \frac{1}{N}\sum_{n=1}^{N}[\sigma^2 + \mu^2 - 2\mu\mu + \mu^2]$$

$$\sigma^2 = E[x^2] - (E[x])^2 \rightarrow \sigma^2 + (E[x])^2 = E[x^2]$$

$$= \frac{1}{N}\cdot N \cdot \sigma^2 = \sigma^2$$