Lecture 2: Proofs

1. By example/counterexample
2. Direct ($P \Rightarrow Q$)
3. Contrapositive (prove $\neg Q \Rightarrow \neg P$)
4. By contradiction
5. By cases
6. For all $n \in \mathbb{N}, n \geq 2$ (only for integers)

$a|b \Rightarrow c|d$ if $c|a$ and $d|b$
For all $a, b, c \in \mathbb{Z}$, if $a|b$ and $a|c$ then $a|(b+c)$.
$a|b \Rightarrow a|bc$ if $a \nmid c$ then $a|b$ and $\gcd(a,c) = 1$.
So $a|(b+c) \Rightarrow a|b$ and $a|c$.
There exists a number $N = ac \pm 1$ such that $a|N$.
Prove $\exists x \in \mathbb{Z}$ s.t. $11 \nmid x$
where x is the sum of alternating digits of $\bar{x}$.
For all $n \in \mathbb{D}_2, n = 100a + 10b + c \Rightarrow 11 \nmid n$.
An sum is $a + c \Rightarrow 11|(a+c)$ so let $k \in \mathbb{Z}$.
add $a + c + 11k \equiv b \equiv 11k + a + c + b$
$n = 100a + 10b + c = 11k + a + 9a + 11b$