Univariate Regression: Hypothesis Tests and Confidence Intervals (SW Ch. 5)

Part 2

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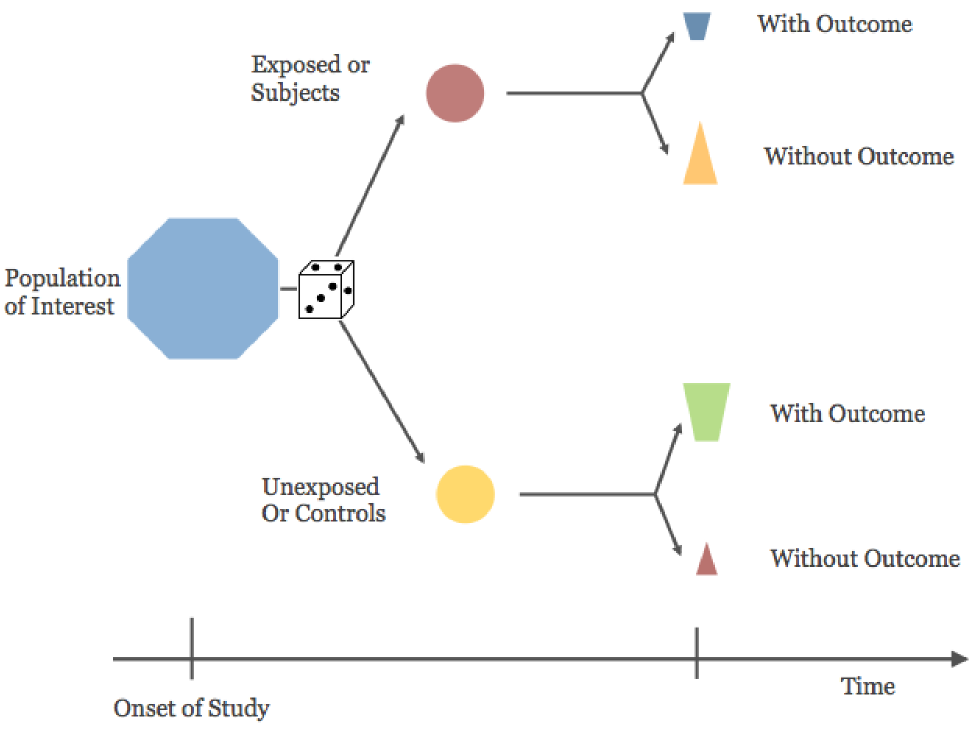
Outline

1. The standard error of $\hat{\beta}_1$
2. Hypothesis tests concerning β_1
3. Confidence intervals for β_1
4. **Regression when X is binary**
5. Heteroskedasticity and homoskedasticity
6. Efficiency of OLS and the Student t distribution

Experimental data (RCTs)

Randomized Control Trials:

- All participants are randomly assigned into **two** groups.
- The control group receives no treatment (or placebo)
- The experimental group receives the treatment.
- After a follow-up period, compare the two groups



Regression when X is Binary (Section 5.3)

Sometimes a regressor is binary:

- $X = 1$ if treated (experimental drug), $= 0$ if not
- $X = 1$ if small class size, $= 0$ if not
- $X = 1$ if female, $= 0$ if male

Binary regressors are sometimes called “dummy” variables.

- Oxford dictionary

  dummy /ˈdəmē/ - something designed to resemble and serve as a substitute for the real or usual thing

Interpreting regressions with a binary regressor $Y_i = \beta_0 + \beta_1 X_i + u_i$, where X is binary ($X_i = 0$ or 1):

(1 of 2)

So far, β_1 has been called a "slope," but that doesn't make sense if X is binary.

How do we interpret regression with a binary regressor?

When $X_i = 0$, $Y_i = \beta_0 + u_i$

- the mean of Y_i is β_0
- that is, $E(Y_i|X_i=0) = \beta_0$

When $X_i = 1$, $Y_i = \beta_0 + \beta_1 + u_i$

- the mean of Y_i is $\beta_0 + \beta_1$
- that is, $E(Y_i|X_i=1) = \beta_0 + \beta_1$

so: $\beta_1 = E(Y_i|X_i=1) - E(Y_i|X_i=0)$

= population difference in group means

Excel live session

Interpreting regressions with a binary regressor $Y_i = \beta_0 + \beta_1 X_i + u_i$, where X is binary ($X_i = 0$ or 1):

Example: Let $D_i = \begin{cases} 1 \text{ if } STR_i < 20 \\ 0 \text{ if } STR_i \geq 20 \end{cases}$

OLS regression: $\widehat{TestScore} = \underset{(1.3)}{650.0} + \underset{(1.8)}{7.4} \times D$

Tabulation of group means:

Class Size	Average score ($\bar{Y}$)	Std. dev. (s_Y)	N
Small (STR < 20)	657.4	19.4	238
Large ($STR \geq$ 20)	**650.0**	17.9	182

Difference in means: $\bar{Y}_{\text{small}} - \bar{Y}_{\text{large}} = 657.4 - 650.0 = \mathbf{7.4}$

Standard error $SE = \sqrt{\dfrac{S_s^2}{n_s} + \dfrac{S_l^2}{n_1}} = \sqrt{\dfrac{19.4^2}{238} + \dfrac{17.9^2}{182}} = \mathbf{1.8}$

Summary: regression when X_i is binary

$Y_i = \beta_0 + \beta_1 X_i + u_i$

- β_0 = mean of Y when X = 0
- $\beta_0 + \beta_1$ = mean of Y when X = 1
- β_1 = difference in group means, X =1 minus X = 0
- $SE(\hat{\beta}_1)$ has the usual interpretation
- t-statistics, confidence intervals constructed as usual
- This is another way (an easy way) to do difference-in-means analysis
- The regression formulation is especially useful when we have additional regressors (coming soon)