Erratum

• There is a fixable error in the proof of Lemma 8 where we falsely claimed $(1 - \frac{1}{n})^{\tau} \leq \frac{1}{2}$ (a counterexample would be the case when n = 4 and $\tau = 2$). We hereby provide another proof to show that $\frac{dg(\tau)}{\tau} \leq 0$.

To show that

$$(1 + \frac{\tau}{n-1})(1 - \frac{1}{n})^{\tau} - 1 \le 0,$$

we let $p(\tau) = (1 + \tau/(n-1))(1 - 1/n)^{\tau}$. Then, we need to prove $p(\tau) \le 1$ for all $1 \le \tau \le n-1$. Consider $q(\tau) = \log p(\tau)$, then we need to prove $q(\tau) \le 0$ for all $1 \le \tau \le n-1$. Note that q(1) = 0 (p(1) = 1). It is easy to derive

$$\frac{dq}{d\tau} = \frac{1}{n+\tau-1} + \log(1-\frac{1}{n}) \le \frac{1}{n+\tau-1} - \frac{1}{n} \le 0,$$

where we used that $\log(x) \leq x - 1$. This implies that $q(\tau)$ is decreasing. Along with the fact that q(1) = 0, we have $q(\tau) \leq 0$ for all $\tau \geq 1$.

• There is a typo in the last equation on the left column of page 7, where $\mathbb{P}(T_{1j} \leq t-1)$ should be $\mathbb{P}(T_{kj} \leq t-1)$.