Erratum

There is a fixable error in the proof of Lemma 8 where we falsely claimed $(1-\frac{1}{n})^\tau \leq \frac{1}{2}$ (a counterexample would be the case when $n = 4$ and $\tau = 2$). We hereby provide another proof to show that $\frac{dg(\tau)}{\tau} \leq 0$.

To show that

$$\big(1 + \frac{\tau}{n-1}\big)\big(1 - \frac{1}{n}\big)^\tau - 1 \leq 0,$$

we let $p(\tau) = (1 + \tau/(n-1))(1 - 1/n)^\tau$. Then, we need to prove $p(\tau) \leq 1$ for all $1 \leq \tau \leq n-1$. Consider $q(\tau) = \log p(\tau)$, then we need to prove $q(\tau) \leq 0$ for all $1 \leq \tau \leq n-1$. Note that $q(1) = 0$ ($p(1) = 1$). It is easy to derive

$$\frac{dq}{d\tau} = \frac{1}{n+\tau-1} + \log(1 - \frac{1}{n}) \leq \frac{1}{n+\tau-1} - \frac{1}{n} \leq 0,$$

where we used that $\log(x) \leq x - 1$. This implies that $q(\tau)$ is decreasing. Along with the fact that $q(1) = 0$, we have $q(\tau) \leq 0$ for all $\tau \geq 1$.