EECS545 Lecture 10 Quiz Solutions

1. It is given that the constrained optimization problem below is "convex". Which of the following statements are correct? (**Choose all options that apply**)

$$\begin{aligned} \min_{\mathbf{x}} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & g_i(\mathbf{x}) \leq 0, i = 1, ..., m \\ & h_i(\mathbf{x}) = 0, i = 1, ..., p \end{aligned}$$

   (a) The objective function f is convex.

   (b) All g_i are convex.

   (c) All h_i are affine.

   (d) All h_i are conex but not necessarily affine.

> **Solution:** (a), (b), (c) Please revisit Slide 16.

2. Which of the following is always true about $\star$? Note that $\mathcal{L}$ is the Lagrangian function corresponding to the optimization problem.

$$\star = \min_{\mathbf{x}} \max_{\boldsymbol{\nu}, \boldsymbol{\lambda}: \lambda_i \geq 0, \forall i} \mathcal{L}(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\nu}) - \max_{\boldsymbol{\nu}, \boldsymbol{\lambda}: \lambda_i \geq 0, \forall i} \min_{\mathbf{x}} \mathcal{L}(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\nu})$$

   (a) $\star = 0$

   (b) $\star \geq 0$

   (c) $\star \leq 0$

   (d) $\star > 0$

   (e) $\star < 0$

> **Solution:** (b). Primal optimization (the first term) is greater or equal to Dual optimization (the second term); weak duality. Those two terms are the same if conditions are met (= strong duality). See Slide 13-14 for details.

3. (True/False) Linear hard-margin SVM must have at least one support vector for positive margin and one support vector for negative margin that satisfies the margin constraint. (Let's assume we have trained linear hard-margin SVM with linearly separable data, for simplicity)

> **Solution:** True. If there is no support vector in either end, it means there is a linear hard-margin SVM that have a larger margin.

4. Let $(x^{(n)}, y^{(n)})$ be a data point that is a support vector in the Linear Soft SVM formulation. Which of the following equations does the support vector satisfy?

   (a) $y^{(n)} h\left(\mathbf{x}^{(n)}\right) = 1$

   (b) $y^{(n)} h\left(\mathbf{x}^{(n)}\right) = 1 - \xi^{(n)}$

   Solution: (b). Please see Slide 29. (a) is for the Linear hard-margin SVM case (as in Slide 27).

5. (True/False) The dual view of SVM shows that the objective function depends on $\phi(\mathbf{x})$ only via inner products $\left(\phi\left(\mathbf{x}\right)^\top \phi\left(\mathbf{x}\right)\right)$ and hence the kernel trick can be used.

   Solution: True. See Slide 25-26 for an example.