

## EECS545 Lecture 3 Quiz Solutions

1. Let  $D = \{(x_i, y_i)\}_{i=1,\dots,3} = \{(-1, -0.5), (0, 0), (1, 1)\}$ . Suppose that we want to fit a linear regression model with a 1-degree polynomial function with  $\hat{y} = w_0 + w_1x$ . After convergence, what would be the solution of  $w_0$  and  $w_1$ ? Hint: write down the objective function and set the derivative with respect to  $w_0$  and  $w_1$  to zero to find the optimal solution.

**Solution:** Plugging in for the objective function:

$$E(w) = \frac{1}{2} [(w_0 - w_1 + 0.5)^2 + (w_0)^2 + (w_0 + w_1 - 1)^2]$$

Partial derivatives:

$$\frac{\partial E(w)}{\partial w_0} = (w_0 - w_1 + 0.5) + w_0 + (w_0 + w_1 - 1) = 3w_0 - 0.5$$

$$\frac{\partial E(w)}{\partial w_1} = -(w_0 - w_1 + 0.5) + (w_0 + w_1 - 1) = 2w_1 - 1.5$$

Setting these to zero, we get  $w_0 = \frac{1}{6}, w_1 = \frac{3}{4}$ .

2. Continued. Suppose instead we used ridge linear regression with  $\lambda = 1$ . Find  $w_0$  and  $w_1$  after convergence.

**Solution:** Plugging in for the objective function:

$$E(w) = \frac{1}{2} [(w_0 - w_1 + 0.5)^2 + (w_0)^2 + (w_0 + w_1 - 1)^2] + \frac{1}{2}(w_0^2 + w_1^2)$$

Partial derivatives:

$$\frac{\partial E(w)}{\partial w_0} = (w_0 - w_1 + 0.5) + w_0 + (w_0 + w_1 - 1) + w_0 = 4w_0 - 0.5$$

$$\frac{\partial E(w)}{\partial w_1} = -(w_0 - w_1 + 0.5) + (w_0 + w_1 - 1) + w_1 = 3w_1 - 1.5$$

Setting these to zero, we get  $w_0 = \frac{1}{8}, w_1 = \frac{1}{2}$ .

3. Consider polynomial regression by optimizing the least-squares objective function with regularization term  $\lambda$ . **Choose all options that apply:**
- (a) As  $\lambda$  increases, the curve formed by polynomial regression will become flatter.
  - (b) Best practice indicates that we should keep tuning  $\lambda$  until we find a final value  $\lambda^*$  that minimizes error on the test set.

- (c) It is not valid to use the  $L1$  norm in place of the  $L2$  norm in the regularized objective function.

**Solution:**

- (a) True.
- (b) False.
- (c) False.