

Claims reserving with R: ChainLadder-0.1.5-3 Package Vignette DRAFT

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Abstract

The ChainLadder package provides various statistical methods which are typically used for the estimation of outstanding claims reserves in general insurance.

The package has implementations of the Mack-, Munich-, Bootstrap, and multi-variate chain-ladder methods, as well as the loss development factor curve fitting methods of Dave Clark and generalised linear model based reserving models.

This document is still in a draft stage. Any pointers which will help to iron out errors, clarify and make this document more helpful will be much appreciated.

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1 Introduction

1.1 Claims reserving in insurance

Unlike other industries the insurance industry does not sell products as such, but promises. An insurance policy is a promise by the insurer to the policyholder to pay for future claims for an upfront received premium.

As a result insurers don't know the upfront cost of their service, but rely on historical data analysis and judgement to derive a sustainable price for their offering. In General Insurance (or Non-Life Insurance, e.g. motor, property and casualty insurance) most policies run for a period of 12 months. However, the claims payment process can take years or even decades. Therefore often not even the delivery date of their product is known to insurers.

In particular claims arising from casualty insurance can take a long time to settle. Claims can take years to materialise. A complex and costly example are the claims from asbestos liabilities. A research report by a working party of the Institute of Actuaries has estimated that the undiscounted cost of UK mesothelioma-related claims to the UK Insurance Market for the period 2009 to 2050 could be around £10bn [GBB⁺09]. The cost for asbestos related claims in the US for the worldwide insurance industry was estimated to be around \$120bn in 2002 [Mic02].

Thus, it should come to no surprise that the biggest item on the liability side of an insurer's balance sheet is often the provision or reserves for future claims payments. Those reserves can be broken down in case reserves (or out-standings claims), which are losses already reported to the insurance company and incurred but not reported (IBNR) claims.

Over the years several methods have been developed to estimate reserves for insurance claims, see [Sch11], [PR02] for an overview. Changes in regulatory requirements, e.g. Solvency II¹ in Europe, have fostered further research into this topic, with a focus on stochastic and statistical techniques.

2 The ChainLadder package

2.1 Motivation

The ChainLadder [GMZ12] package provides various statistical methods which are typically used for the estimation of outstanding claims reserves in general insurance. The package started out of presentations given by Markus Gesmann at the Stochastic Reserving Seminar at the Institute of Actuaries in 2007 and 2008, followed by talks at Casualty Actuarial Society (CAS) meetings joined by Dan Murphy in 2008 and Wayne Zhang in 2010.

Implementing reserving methods in R has several advantages. R provides:

- a rich language for statistical modelling and data manipulations allowing fast prototyping
- a very active user base, which publishes many extensions
- many interfaces to data bases and other applications, such as MS Excel
- an established framework for documentation and testing

¹See http://ec.europa.eu/internal_market/insurance/solvency/index_en.htm

- workflows with version control systems
- code written in plain text files, allowing effective knowledge transfer
- an effective way to collaborate over the internet
- built in functions to create reproducible research reports²
- in combination with other tools such as L^AT_EX and Sweave easy to set up automated reporting facilities
- access to academic research, which is often first implemented in R

2.2 Brief package overview

This vignette will give the reader a brief overview of the functionality of the ChainLadder package. The functions are discussed and explained in more detail in the respective help files and examples.

The ChainLadder package has implementations of the Mack-, Munich- and Bootstrap chain-ladder methods [Mac93a], [Mac99], [QM04], [EV99]. Since version 0.1.3-3 it provides general multivariate chain ladder models by Wayne Zhang [Zha10]. Version 0.1.4-0 introduced new functions on loss development factor (LDF) fitting methods and Cape Cod by Daniel Murphy following a paper by David Clark [Cla03]. Version 0.1.5-0 has added loss reserving models within the generalized linear model framework following a paper by England and Verrall [EV99] implemented by Wayne Zhang.

The package also offers utility functions to convert quickly tables into triangles, triangles into tables, cumulative into incremental and incremental into cumulative triangles.

A set of demos is shipped with the packages and the list of demos is available via:

```
R> demo(package="ChainLadder")
```

and can be executed via

```
R> library(ChainLadder)
R> demo("demo name")
```

Additionally the ChainLadder package comes with example files which demonstrates how to the ChainLadder functions can be embedded in Excel and Word using the statconn interface[BN07].

For more information and examples see the project web site: <http://code.google.com/p/chainladder/>

2.3 Installation

We can install ChainLadder in the usual way from CRAN, e.g.:

```
R> install.packages('ChainLadder')
```

²For an example see the project: Formatted Actuarial Vignettes in R, <http://www.favir.net/>

For more details about installing packages see [Tea12b]. The installation was successful if the command `library(ChainLadder)` gives you the following message:

```
R> library(ChainLadder)
```

```
ChainLadder version 0.1.5-3 by:  
Markus Gesmann <markus.gesmann@gmail.com>  
Wayne Zhang <actuary_zhang@hotmail.com>  
Daniel Murphy <danielmarkmurphy@gmail.com>
```

```
Type library(help='ChainLadder') or ?ChainLadder  
to see overall documentation.
```

```
Type demo(ChainLadder) to get an idea of the functionality of this package.
```

```
See demo(package='ChainLadder') for a list of more demos.
```

Feel free to send us an email if you would like to be kept informed of new versions or if you have any feedback, ideas, suggestions or would like to collaborate.

More information is available on the ChainLadder project web-site:
<http://code.google.com/p/chainladder/>

To suppress this message use the statement:
`suppressPackageStartupMessages(library(ChainLadder))`

3 Using the ChainLadder package

3.1 Working with triangles

Historical insurance data is often presented in form of a triangle structure, showing the development of claims over time for each origin period. An origin period could be the year the policy was sold, or the accident year. Of course the frequency doesn't have to be yearly, e.g. quarterly or monthly origin periods are also often used. Most reserving methods of the ChainLadder package expect triangles as input data sets with development periods along the columns and the origin period in rows. The package comes with several example triangles. The following R command will list them all:

```
R> require(ChainLadder)  
R> data(package="ChainLadder")
```

Let's look at one example triangle more closely. The following triangle shows data from the Reinsurance Association of America (RAA):

```
R> ## Sample triangle
R> RAA
```

| | dev | | | | | | | | | | |
|--------|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--|
| origin | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | |
| 1981 | 5012 | 8269 | 10907 | 11805 | 13539 | 16181 | 18009 | 18608 | 18662 | 18834 | |
| 1982 | 106 | 4285 | 5396 | 10666 | 13782 | 15599 | 15496 | 16169 | 16704 | NA | |
| 1983 | 3410 | 8992 | 13873 | 16141 | 18735 | 22214 | 22863 | 23466 | NA | NA | |
| 1984 | 5655 | 11555 | 15766 | 21266 | 23425 | 26083 | 27067 | NA | NA | NA | |
| 1985 | 1092 | 9565 | 15836 | 22169 | 25955 | 26180 | NA | NA | NA | NA | |
| 1986 | 1513 | 6445 | 11702 | 12935 | 15852 | NA | NA | NA | NA | NA | |
| 1987 | 557 | 4020 | 10946 | 12314 | NA | NA | NA | NA | NA | NA | |
| 1988 | 1351 | 6947 | 13112 | NA | NA | NA | NA | NA | NA | NA | |
| 1989 | 3133 | 5395 | NA | NA | NA | NA | NA | NA | NA | NA | |
| 1990 | 2063 | NA | NA | NA | NA | NA | NA | NA | NA | NA | |

The objective of a reserving exercise is to forecast the future claims development in the bottom right corner of the triangle and potential further developments. Eventually all claims for a given origin period will be settled, but it is not always obvious to judge how many years or even decades it will take. We speak of long and short tail business depending on the time it takes to pay all claims.

3.1.1 Plotting triangles

The first thing you often want to do is to plot the data to get an overview. For a data set of class `triangle` the ChainLadder package provides default plotting methods to give a graphical overview of the data:

```
R> plot(RAA)
```

Setting the argument `lattice=TRUE` will produce individual plots for each origin period³, see Figure 2.

```
R> plot(RAA, lattice=TRUE)
```

You will notice from the plots in Figures 1 and 2 that the triangle RAA presents claims developments for the origin years 1981 to 1990 in a cumulative form. For more information on the triangle plotting functions see the help pages of `plot.triangle`, e.g. via

```
R> ?plot.triangle
```

3.1.2 Transforming triangles between cumulative and incremental representation

The ChainLadder packages comes with two helper functions, `cum2incr` and `incr2cum` to transform cumulative triangles into incremental triangles and vice versa:

³ChainLadder uses the `lattice` package for plotting the development of the origin years in separate panels.

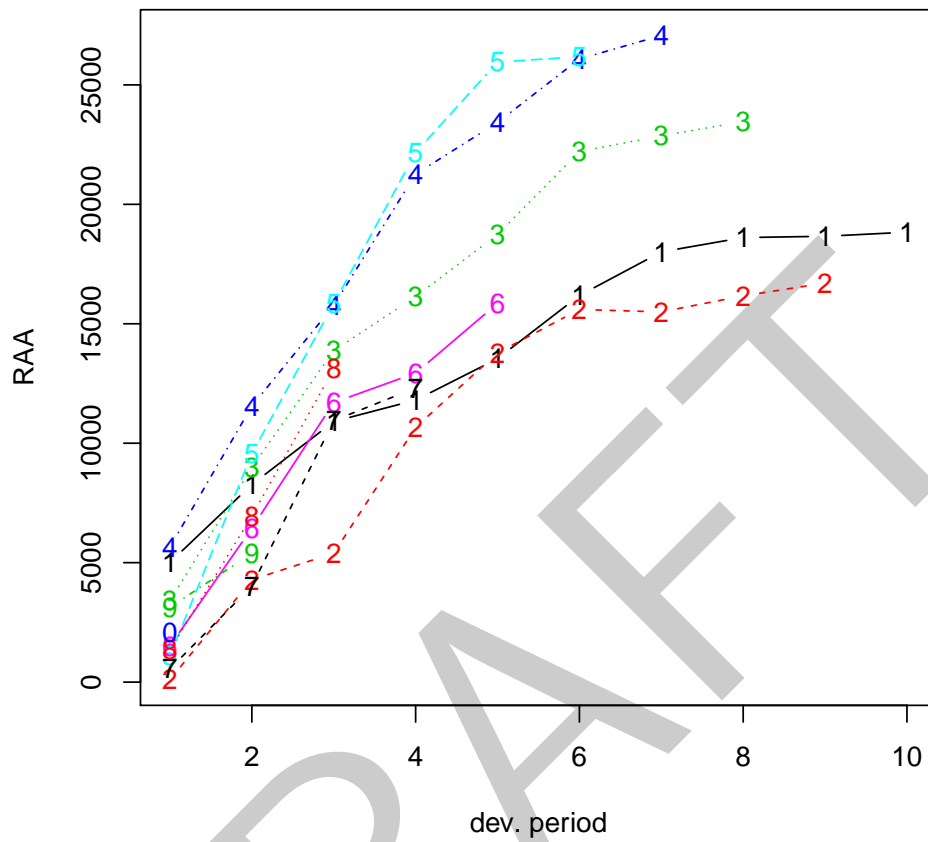


Figure 1: Claims development chart of the RAA triangle, with one line per origin period. Output of `plot(RAA)`

```
R> raa.inc <- cum2incr(RAA)
R> ## Show first origin period and its incremental development
R> raa.inc[1,]
```

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------|------|------|-----|------|------|------|-----|----|-----|----|
| 5012 | 3257 | 2638 | 898 | 1734 | 2642 | 1828 | 599 | 54 | 172 | |

```
R> raa.cum <- incr2cum(raa.inc)
R> ## Show first origin period and its cumulative development
R> raa.cum[1,]
```

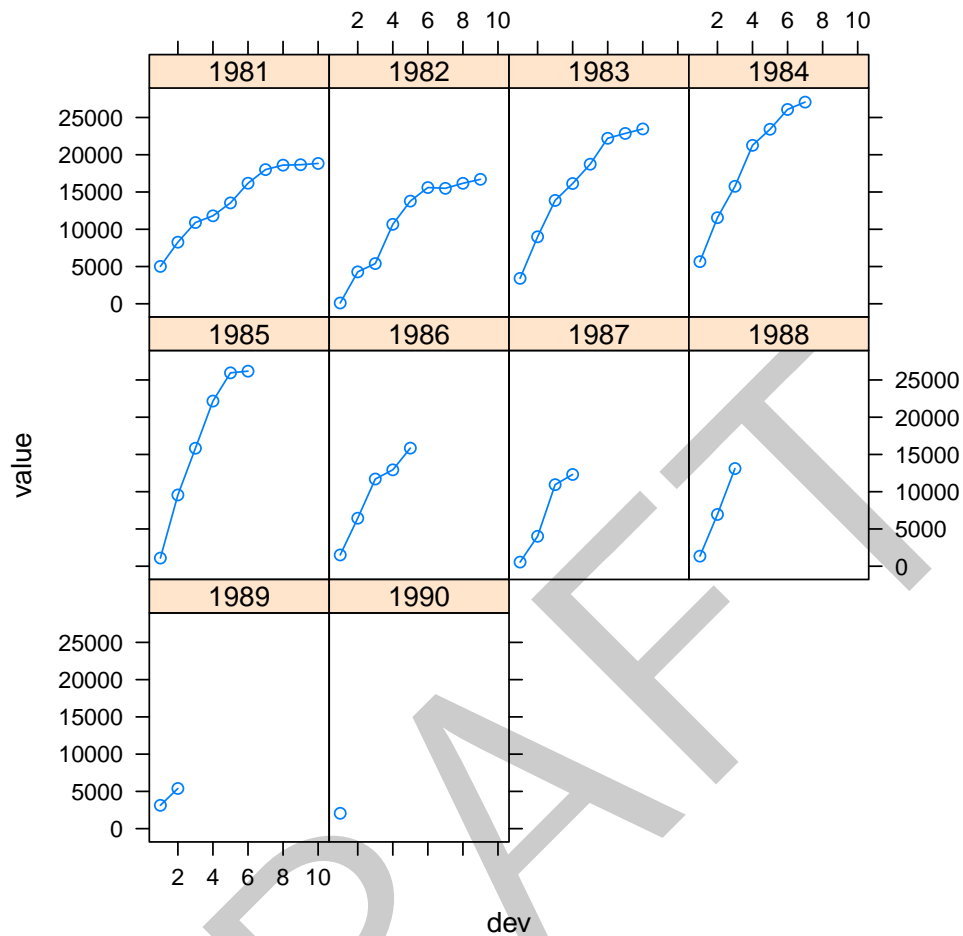



Figure 2: Claims development chart of the RAA triangle, with individual panels for each origin period. Output of `plot(RAA, lattice=TRUE)`

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------|------|-------|-------|-------|-------|-------|-------|-------|-------|
| 5012 | 8269 | 10907 | 11805 | 13539 | 16181 | 18009 | 18608 | 18662 | 18834 |

3.1.3 Importing triangles from external data sources

In most cases you want to analyse your own data, usually stored in data bases. R makes it easy to access data using SQL statements, e.g. via an ODBC connection⁴ and the `ChainLadder` packages includes a demo to showcase how data can be imported from a MS Access data base, see:

⁴See the `RODBC` package

```
R> demo(DatabaseExamples)
```

For more details see [Tea12a].

In this section we use data stored in a CSV-file⁵ to demonstrate some typical operations you will want to carry out with data stored in data bases. In most cases your triangles will be stored in tables and not in a classical triangle shape. The ChainLadder package contains a CSV-file with sample data in a long table format. We read the data into R's memory with the `read.csv` command and look at the first couple of rows and summarise it:

```
R> filename <- file.path(system.file("Database",
+                                   package="ChainLadder"),
+                         "TestData.csv")
R> myData <- read.csv(filename)
R> head(myData)
```

| | origin | dev | value | lob |
|---|--------|-----|--------|-----|
| 1 | 1977 | 1 | 153638 | ABC |
| 2 | 1978 | 1 | 178536 | ABC |
| 3 | 1979 | 1 | 210172 | ABC |
| 4 | 1980 | 1 | 211448 | ABC |
| 5 | 1981 | 1 | 219810 | ABC |
| 6 | 1982 | 1 | 205654 | ABC |

```
R> summary(myData)
```

| | origin | dev | value | lob |
|--------------|--------|---------------|-----------------|------------------------|
| Min. : | 1 | Min. : 1.00 | Min. : -17657 | AutoLiab :105 |
| 1st Qu.: | 3 | 1st Qu.: 2.00 | 1st Qu.: 10324 | GeneralLiab :105 |
| Median : | 6 | Median : 4.00 | Median : 72468 | M3IR5 :105 |
| Mean : | 642 | Mean : 4.61 | Mean : 176632 | ABC : 66 |
| 3rd Qu.:1979 | | 3rd Qu.: 7.00 | 3rd Qu.: 197716 | CommercialAutoPaid: 55 |
| Max. :1991 | | Max. :14.00 | Max. :3258646 | GenIns : 55 |
| | | | | (Other) :210 |

Let's focus on one subset of the data. We select the RAA data again:

```
R> raa <- subset(myData, lob %in% "RAA")
R> head(raa)
```

| | origin | dev | value | lob |
|----|--------|-----|-------|-----|
| 67 | 1981 | 1 | 5012 | RAA |
| 68 | 1982 | 1 | 106 | RAA |
| 69 | 1983 | 1 | 3410 | RAA |

⁵Please ensure that your CSV-file is free from formatting, e.g. characters to separate units of thousands, as those columns will be read as characters or factors rather than numerical values.

```
70  1984    1  5655 RAA
71  1985    1  1092 RAA
72  1986    1  1513 RAA
```

To transform the long table of the RAA data into a triangle we use the function `as.triangle`. The arguments we have to specify are the column names of the origin and development period and further the column which contains the values:

```
R> raa.tri <- as.triangle(raa,
+                          origin="origin",
+                          dev="dev",
+                          value="value")
R> raa.tri
```

| | dev | | | | | | | | | | |
|--------|------|------|------|------|------|------|------|-----|-----|-----|--|
| origin | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | |
| 1981 | 5012 | 3257 | 2638 | 898 | 1734 | 2642 | 1828 | 599 | 54 | 172 | |
| 1982 | 106 | 4179 | 1111 | 5270 | 3116 | 1817 | -103 | 673 | 535 | NA | |
| 1983 | 3410 | 5582 | 4881 | 2268 | 2594 | 3479 | 649 | 603 | NA | NA | |
| 1984 | 5655 | 5900 | 4211 | 5500 | 2159 | 2658 | 984 | NA | NA | NA | |
| 1985 | 1092 | 8473 | 6271 | 6333 | 3786 | 225 | NA | NA | NA | NA | |
| 1986 | 1513 | 4932 | 5257 | 1233 | 2917 | NA | NA | NA | NA | NA | |
| 1987 | 557 | 3463 | 6926 | 1368 | NA | NA | NA | NA | NA | NA | |
| 1988 | 1351 | 5596 | 6165 | NA | NA | NA | NA | NA | NA | NA | |
| 1989 | 3133 | 2262 | NA | NA | NA | NA | NA | NA | NA | NA | |
| 1990 | 2063 | NA | NA | NA | NA | NA | NA | NA | NA | NA | |

We note that the data has been stored as an incremental data set. As mentioned above, we could now use the function `incr2cum` to transform the triangle into a cumulative format.

We can transform a triangle back into a data frame structure:

```
R> raa.df <- as.data.frame(raa.tri, na.rm=TRUE)
R> head(raa.df)
```

| | origin | dev | value |
|--------|--------|-----|-------|
| 1981-1 | 1981 | 1 | 5012 |
| 1982-1 | 1982 | 1 | 106 |
| 1983-1 | 1983 | 1 | 3410 |
| 1984-1 | 1984 | 1 | 5655 |
| 1985-1 | 1985 | 1 | 1092 |
| 1986-1 | 1986 | 1 | 1513 |

This is particular helpful when you would like to store your results back into data base. Figure 3 gives you an idea of a potential data flow between R and data bases.

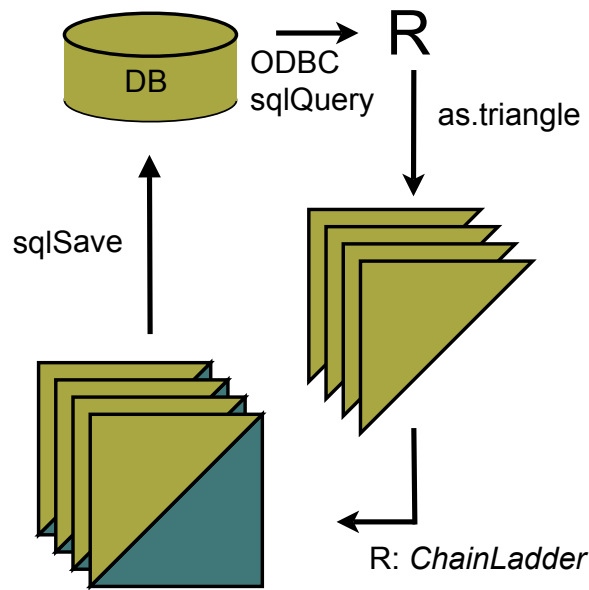


Figure 3: Flow chart of data between R and data bases.

3.1.4 Coping and pasting from MS Excel

Small data sets in Excel can be transferred to R backwards and forwards with via the clipboard under MS Windows.

Copying from Excel to R Select a data set in Excel and copy it into the clipboard, then go to R and type:

```
R> x <- read.table(file="clipboard", sep="\t", na.strings="")
```

Copying from R to Excel Suppose you would like to copy the RAA triangle into Excel, then the following statement would copy the data into the clipboard:

```
R> write.table(RAA, file="clipboard", sep="\t", na="")
```

Now you can paste the content into Excel. Please note that you can't copy lists structures from R to Excel.

3.2 Chain-ladder methods

The classical chain-ladder is a deterministic algorithm to forecast claims based on historical data. It assumes that the proportional developments of claims from one development period to the next are the same for all origin years.

3.2.1 Basic idea

The age-to-age link ratios are calculated as the volume weighted average development ratios of a cumulative loss development triangle from one development period to the next $C_{ik}, i, k = 1, \dots, n$.

$$f_k = \frac{\sum_{i=1}^{n-k} C_{i,k+1}}{\sum_{i=1}^{n-k} C_{i,k}} \quad (1)$$

```
R> n <- 10
R> f <- sapply(1:(n-1),
+           function(i){
+             sum(RAA[c(1:(n-i)),i+1])/sum(RAA[c(1:(n-i)),i])
+           }
+         )
R> f
```

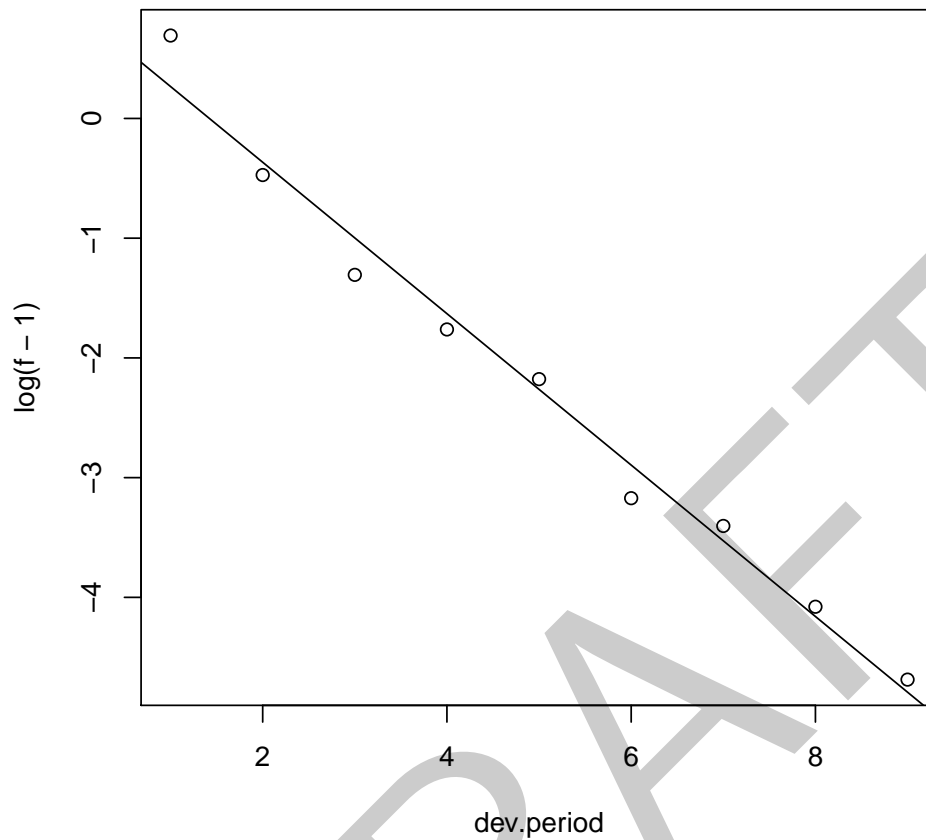
```
[1] 2.999 1.624 1.271 1.172 1.113 1.042 1.033 1.017 1.009
```

Often it is not suitable to assume that the oldest origin year is fully developed. A typical approach is to extrapolate the development ratios, e.g. assuming a log-linear model.

```
R> dev.period <- 1:(n-1)
R> plot(log(f-1) ~ dev.period, main="Log-linear extrapolation of age-to-age factors")
R> tail.model <- lm(log(f-1) ~ dev.period)
R> abline(tail.model)
R> co <- coef(tail.model)
R> ## extrapolate another 100 dev. period
R> tail <- exp(co[1] + c((n + 1):(n + 100)) * co[2]) + 1
R> f.tail <- prod(tail)
R> f.tail
```

```
[1] 1.005
```

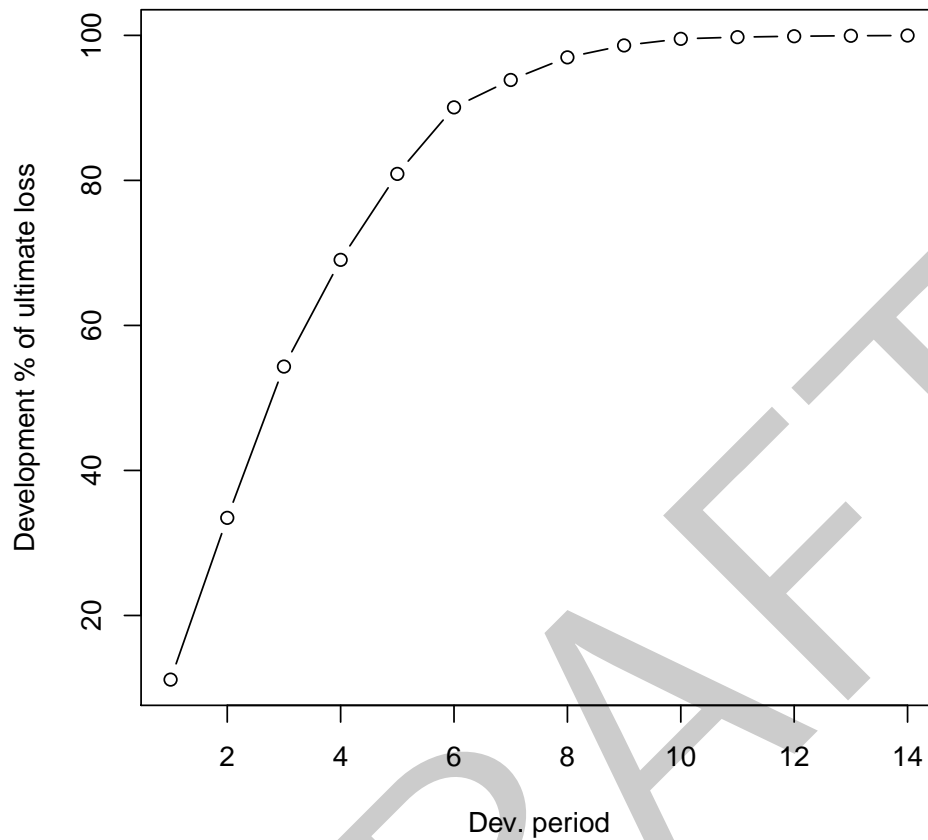
Log-linear extrapolation of age-to-age factors



The age-to-age factors allow us to plot the expected claims development patterns.

```
R> plot(100*(rev(1/cumprod(rev(c(f, tail[tail>1.0001]))))), t="b",  
+      main="Expected claims development pattern",  
+      xlab="Dev. period", ylab="Development % of ultimate loss")
```

Expected claims development pattern



The link ratios are then applied to the latest know cumulative claims amount to forecast the next development period.

```
R> f <- c(f, f.tail)
R> fullRAA <- RAA
R> for(k in 1:(n-1)){
+   fullRAA[(n-k+1):n, k+1] <- fullRAA[(n-k+1):n, k]*f[k]
+ }
R> fullRAA[,n] <- fullRAA[,n]*f[n]
R> round(fullRAA)
```

| | dev | | | | | | | | | | |
|--------|------|------|------|-------|-------|-------|-------|-------|-------|-------|-------|
| origin | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | |
| | 1981 | 5012 | 8269 | 10907 | 11805 | 13539 | 16181 | 18009 | 18608 | 18662 | 18928 |

| | | | | | | | | | | |
|------|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1982 | 106 | 4285 | 5396 | 10666 | 13782 | 15599 | 15496 | 16169 | 16704 | 16942 |
| 1983 | 3410 | 8992 | 13873 | 16141 | 18735 | 22214 | 22863 | 23466 | 23863 | 24204 |
| 1984 | 5655 | 11555 | 15766 | 21266 | 23425 | 26083 | 27067 | 27967 | 28441 | 28847 |
| 1985 | 1092 | 9565 | 15836 | 22169 | 25955 | 26180 | 27278 | 28185 | 28663 | 29072 |
| 1986 | 1513 | 6445 | 11702 | 12935 | 15852 | 17649 | 18389 | 19001 | 19323 | 19599 |
| 1987 | 557 | 4020 | 10946 | 12314 | 14428 | 16064 | 16738 | 17294 | 17587 | 17838 |
| 1988 | 1351 | 6947 | 13112 | 16664 | 19525 | 21738 | 22650 | 23403 | 23800 | 24139 |
| 1989 | 3133 | 5395 | 8759 | 11132 | 13043 | 14521 | 15130 | 15634 | 15898 | 16125 |
| 1990 | 2063 | 6188 | 10046 | 12767 | 14959 | 16655 | 17353 | 17931 | 18234 | 18495 |

This approach is also called Loss Development Factor (LDF) method.

Since the early 1990s several papers have been published to embed the simple chain-ladder method into a statistical framework. Ben Zehnwirth and Glenn Branett point out in [Zx00] that the age-to-age link ratios can be regarded as the coefficients of a weighted linear regression through the origin, see also [Mur94].

```
R> lmCL <- function(i, Triangle){
+   lm(y~x+0, weights=1/Triangle[,i],
+     data=data.frame(x=Triangle[,i], y=Triangle[,i+1]))
+ }
R> sapply(lapply(c(1:(n-1)), lmCL, RAA), coef)
```

| | x | x | x | x | x | x | x | x | x |
|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 2.999 | 1.624 | 1.271 | 1.172 | 1.113 | 1.042 | 1.033 | 1.017 | 1.009 |

3.2.2 Mack chain-ladder

Thomas Mack published in 1993 [Mac93b] an article which allows to estimate the standard errors of the chain-ladder forecast without assuming a distribution under certain constrain to the data.

Following Mack [Mac99] let C_{ik} denote the cumulative loss amounts of origin period (e.g. accident year) $i = 1, \dots, m$, with losses known for development period (e.g. development year) $k \leq n + 1 - i$.

In order to forecast the amounts C_{ik} for $k > n + 1 - i$ the Mack chain-ladder-model assumes:

$$\text{CL1: } E[F_{ik}|C_{i1}, C_{i2}, \dots, C_{ik}] = f_k \text{ with } F_{ik} = \frac{C_{i,k+1}}{C_{ik}} \quad (2)$$

$$\text{CL2: } \text{Var}\left(\frac{C_{i,k+1}}{C_{ik}}|C_{i1}, C_{i2}, \dots, C_{ik}\right) = \frac{\sigma_k^2}{w_{ik}C_{ik}^\alpha} \quad (3)$$

$$\text{CL3: } \{C_{i1}, \dots, C_{in}\}, \{C_{j1}, \dots, C_{jn}\}, \text{ are independent for origin period } i \neq j \quad (4)$$

with $w_{ik} \in [0; 1], \alpha \in \{0, 1, 2\}$. If these assumptions are hold, the Mack-chain-ladder-model gives an unbiased estimator for IBNR (Incurred But Not Reported) claims.

The Mack-chain-ladder model can be regarded as a weighted linear regression through the origin for each development period: $\text{lm}(y \sim x + 0, \text{weights}=w/x^{(2-\alpha)})$, where y is the vector of claims at development period $k + 1$ and x is the vector of claims at development period k .


```
R> mack <- MackChainLadder(RAA, est.sigma="Mack")
R> mack
```

```
MackChainLadder(Triangle = RAA, est.sigma = "Mack")
```

| | Latest | Dev.To.Date | Ultimate | IBNR | Mack.S.E | CV(IBNR) |
|------|--------|-------------|----------|--------|----------|----------|
| 1981 | 18,834 | 1.000 | 18,834 | 0 | 0 | NaN |
| 1982 | 16,704 | 0.991 | 16,858 | 154 | 206 | 1.339 |
| 1983 | 23,466 | 0.974 | 24,083 | 617 | 623 | 1.010 |
| 1984 | 27,067 | 0.943 | 28,703 | 1,636 | 747 | 0.457 |
| 1985 | 26,180 | 0.905 | 28,927 | 2,747 | 1,469 | 0.535 |
| 1986 | 15,852 | 0.813 | 19,501 | 3,649 | 2,002 | 0.549 |
| 1987 | 12,314 | 0.694 | 17,749 | 5,435 | 2,209 | 0.406 |
| 1988 | 13,112 | 0.546 | 24,019 | 10,907 | 5,358 | 0.491 |
| 1989 | 5,395 | 0.336 | 16,045 | 10,650 | 6,333 | 0.595 |
| 1990 | 2,063 | 0.112 | 18,402 | 16,339 | 24,566 | 1.503 |

```
Totals
Latest:    160,987.00
Dev:       0.76
Ultimate:  213,122.23
IBNR:      52,135.23
Mack S.E.: 26,909.01
CV(IBNR):  0.52
```

Access the loss development factors and the full triangle

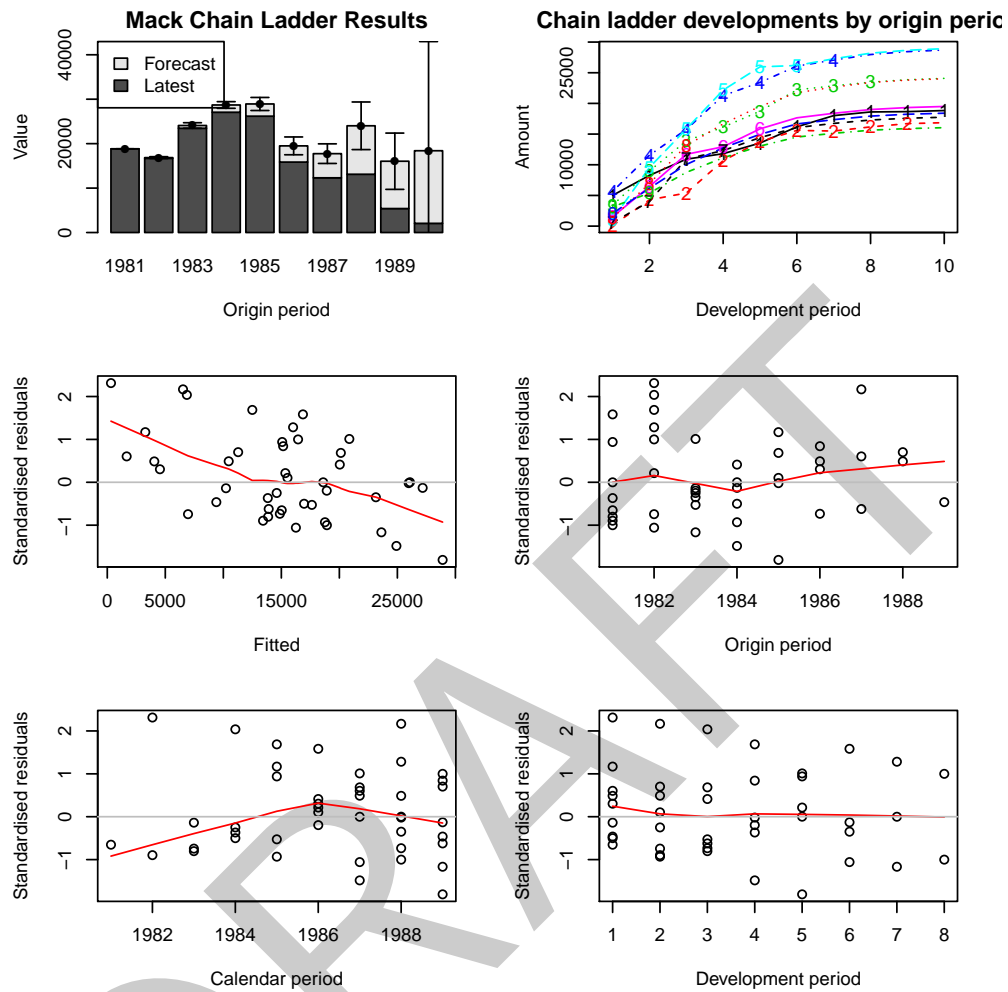
```
R> mack$f
```

```
[1] 2.999 1.624 1.271 1.172 1.113 1.042 1.033 1.017 1.009 1.000
```

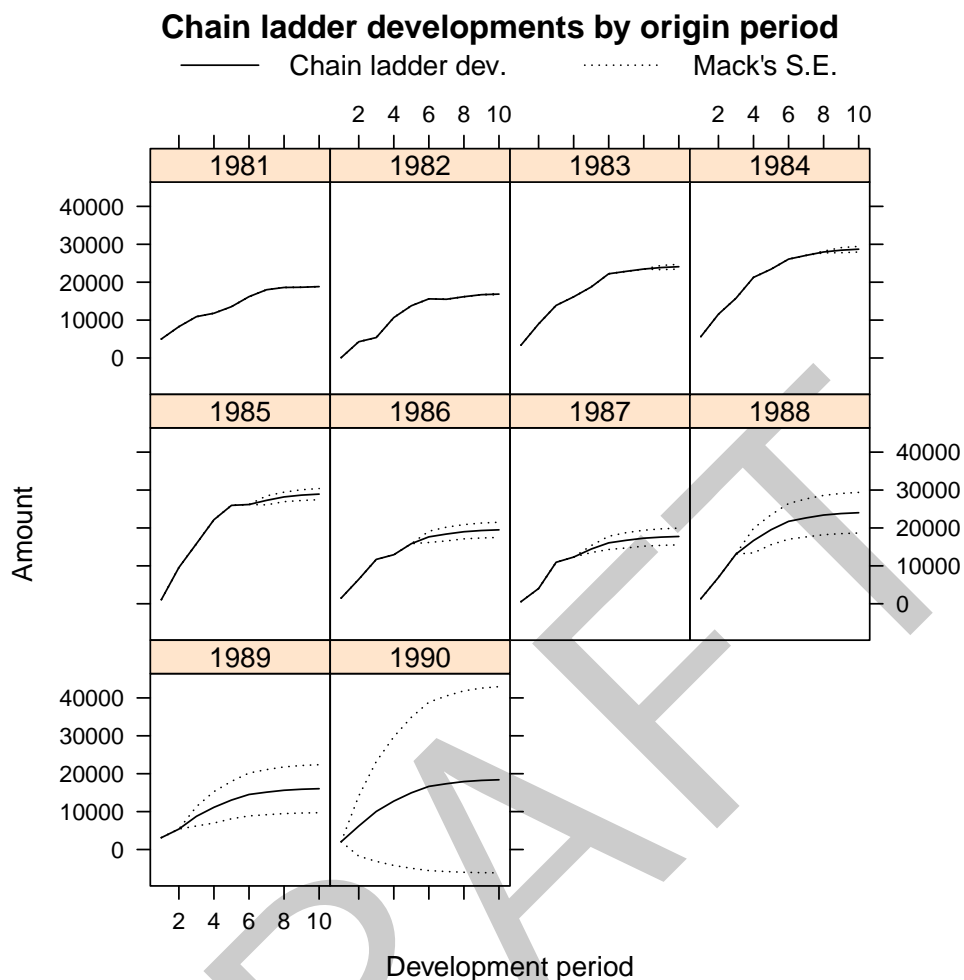
```
R> mack$FullTriangle
```

| | dev | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|--------|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----|
| origin | | | | | | | | | | | |
| 1981 | 5012 | 8269 | 10907 | 11805 | 13539 | 16181 | 18009 | 18608 | 18662 | 18834 | |
| 1982 | 106 | 4285 | 5396 | 10666 | 13782 | 15599 | 15496 | 16169 | 16704 | 16858 | |
| 1983 | 3410 | 8992 | 13873 | 16141 | 18735 | 22214 | 22863 | 23466 | 23863 | 24083 | |
| 1984 | 5655 | 11555 | 15766 | 21266 | 23425 | 26083 | 27067 | 27967 | 28441 | 28703 | |
| 1985 | 1092 | 9565 | 15836 | 22169 | 25955 | 26180 | 27278 | 28185 | 28663 | 28927 | |
| 1986 | 1513 | 6445 | 11702 | 12935 | 15852 | 17649 | 18389 | 19001 | 19323 | 19501 | |
| 1987 | 557 | 4020 | 10946 | 12314 | 14428 | 16064 | 16738 | 17294 | 17587 | 17749 | |
| 1988 | 1351 | 6947 | 13112 | 16664 | 19525 | 21738 | 22650 | 23403 | 23800 | 24019 | |
| 1989 | 3133 | 5395 | 8759 | 11132 | 13043 | 14521 | 15130 | 15634 | 15898 | 16045 | |
| 1990 | 2063 | 6188 | 10046 | 12767 | 14959 | 16655 | 17353 | 17931 | 18234 | 18402 | |

```
R> plot(mack)
```



```
R> plot(mack, lattice=TRUE)
```



3.2.3 Bootstrap chain-ladder

R> # See also the example in section 8 of England & Verrall (2002) on page 55.

R>

R> B <- BootChainLadder(RAA, R=999, process.distr="gamma")

R> B

`BootChainLadder(Triangle = RAA, R = 999, process.distr = "gamma")`

| | Latest | Mean | Ultimate | Mean | IBNR | SD | IBNR | IBNR | 75% | IBNR | 95% |
|------|--------|------|----------|------|------|----|-------|------|-------|------|-------|
| 1981 | 18,834 | | 18,834 | | 0 | | 0 | | 0 | | 0 |
| 1982 | 16,704 | | 16,862 | | 158 | | 675 | | 190 | | 1,365 |
| 1983 | 23,466 | | 24,117 | | 651 | | 1,300 | | 1,189 | | 3,102 |

| | | | | | | |
|------|--------|--------|--------|--------|--------|--------|
| 1984 | 27,067 | 28,855 | 1,788 | 2,083 | 2,796 | 5,707 |
| 1985 | 26,180 | 28,956 | 2,776 | 2,313 | 4,003 | 7,261 |
| 1986 | 15,852 | 19,481 | 3,629 | 2,514 | 4,904 | 8,363 |
| 1987 | 12,314 | 17,774 | 5,460 | 3,093 | 7,198 | 11,228 |
| 1988 | 13,112 | 24,227 | 11,115 | 5,054 | 14,207 | 20,451 |
| 1989 | 5,395 | 16,021 | 10,626 | 6,264 | 14,132 | 22,864 |
| 1990 | 2,063 | 19,800 | 17,737 | 13,740 | 24,881 | 43,148 |

```
Totals
Latest:      160,987
Mean Ultimate: 214,928
Mean IBNR:   53,941
SD IBNR:     19,078
Total IBNR 75%: 65,132
Total IBNR 95%: 87,062
```

```
R> plot(B)
R> # Compare to MackChainLadder
R> MackChainLadder(RAA)
```

```
MackChainLadder(Triangle = RAA)
```

| | Latest | Dev.To.Date | Ultimate | IBNR | Mack.S.E | CV(IBNR) |
|------|--------|-------------|----------|--------|----------|----------|
| 1981 | 18,834 | 1.000 | 18,834 | 0 | 0 | NaN |
| 1982 | 16,704 | 0.991 | 16,858 | 154 | 143 | 0.928 |
| 1983 | 23,466 | 0.974 | 24,083 | 617 | 592 | 0.959 |
| 1984 | 27,067 | 0.943 | 28,703 | 1,636 | 713 | 0.436 |
| 1985 | 26,180 | 0.905 | 28,927 | 2,747 | 1,452 | 0.529 |
| 1986 | 15,852 | 0.813 | 19,501 | 3,649 | 1,995 | 0.547 |
| 1987 | 12,314 | 0.694 | 17,749 | 5,435 | 2,204 | 0.405 |
| 1988 | 13,112 | 0.546 | 24,019 | 10,907 | 5,354 | 0.491 |
| 1989 | 5,395 | 0.336 | 16,045 | 10,650 | 6,332 | 0.595 |
| 1990 | 2,063 | 0.112 | 18,402 | 16,339 | 24,566 | 1.503 |

```
Totals
Latest:      160,987.00
Dev:         0.76
Ultimate:    213,122.23
IBNR:        52,135.23
Mack S.E.:   26,880.74
CV(IBNR):    0.52
```

```
R> quantile(B, c(0.75,0.95,0.99, 0.995))
```

```
$ByOrigin
IBNR 75% IBNR 95% IBNR 99% IBNR 99.5%
```

| | | | | |
|------|---------|-------|-------|-------|
| 1981 | 0.0 | 0 | 0 | 0 |
| 1982 | 189.9 | 1365 | 2379 | 3460 |
| 1983 | 1188.7 | 3102 | 4873 | 5407 |
| 1984 | 2795.7 | 5707 | 8327 | 9259 |
| 1985 | 4002.5 | 7261 | 9987 | 10526 |
| 1986 | 4903.9 | 8363 | 12105 | 13042 |
| 1987 | 7198.5 | 11228 | 14146 | 16520 |
| 1988 | 14207.1 | 20451 | 25664 | 27620 |
| 1989 | 14132.4 | 22864 | 28897 | 32299 |
| 1990 | 24881.5 | 43148 | 57603 | 63907 |

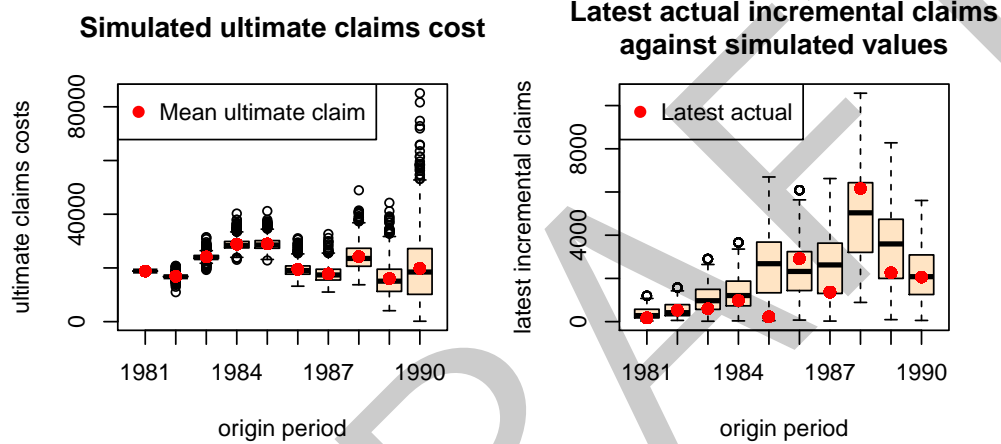
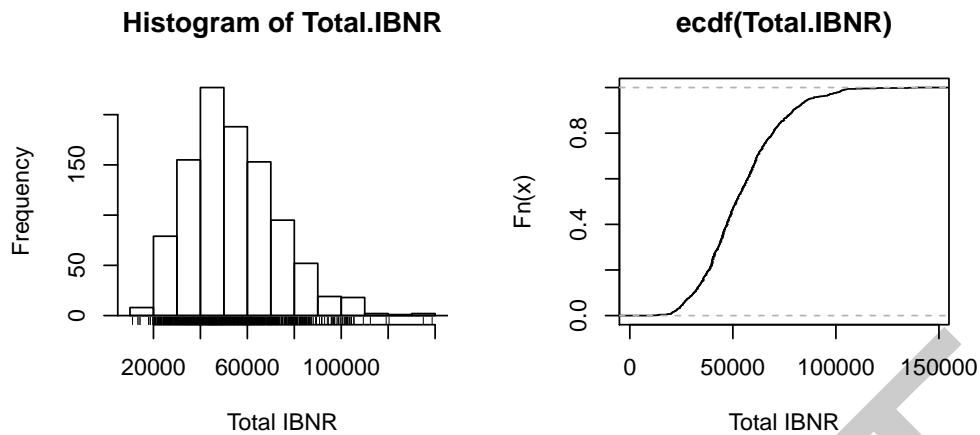
\$Totals

| | Totals |
|-------------|--------|
| IBNR 75%: | 65132 |
| IBNR 95%: | 87062 |
| IBNR 99%: | 103800 |
| IBNR 99.5%: | 109438 |

```
R> # fit a distribution to the IBNR
R> library(MASS)
R> plot(ecdf(B$IBNR.Totals))
R> # fit a log-normal distribution
R> fit <- fitdistr(B$IBNR.Totals[B$IBNR.Totals>0], "lognormal")
R> fit
```

| | |
|-------------|-------------|
| meanlog | sdlog |
| 10.831007 | 0.368602 |
| (0.011662) | (0.008246) |

```
R> curve(plnorm(x,fit$estimate["meanlog"], fit$estimate["sdlog"]),
+       col="red", add=TRUE)
```



3.2.4 Munich chain-ladder

R> MCLpaid

| | dev | | | | | | |
|--------|------|------|------|------|------|------|------|
| origin | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 576 | 1804 | 1970 | 2024 | 2074 | 2102 | 2131 |
| 2 | 866 | 1948 | 2162 | 2232 | 2284 | 2348 | NA |
| 3 | 1412 | 3758 | 4252 | 4416 | 4494 | NA | NA |
| 4 | 2286 | 5292 | 5724 | 5850 | NA | NA | NA |
| 5 | 1868 | 3778 | 4648 | NA | NA | NA | NA |
| 6 | 1442 | 4010 | NA | NA | NA | NA | NA |
| 7 | 2044 | NA | NA | NA | NA | NA | NA |

```
R> MCLincurred
```

```

      dev
origin 1    2    3    4    5    6    7
  1  978 2104 2134 2144 2174 2182 2174
  2 1844 2552 2466 2480 2508 2454   NA
  3 2904 4354 4698 4600 4644   NA   NA
  4 3502 5958 6070 6142   NA   NA   NA
  5 2812 4882 4852   NA   NA   NA   NA
  6 2642 4406   NA   NA   NA   NA   NA
  7 5022   NA   NA   NA   NA   NA   NA

```

```

R> op <- par(mfrow=c(1,2))
R> plot(MCLpaid)
R> plot(MCLincurred)
R> par(op)
R> # Following the example in Quarg's (2004) paper:
R> MCL <- MunichChainLadder(MCLpaid, MCLincurred, est.sigmaP=0.1, est.sigmaI=0.1)
R> MCL

```

```

MunichChainLadder(Paid = MCLpaid, Incurred = MCLincurred, est.sigmaP = 0.1,
  est.sigmaI = 0.1)

```

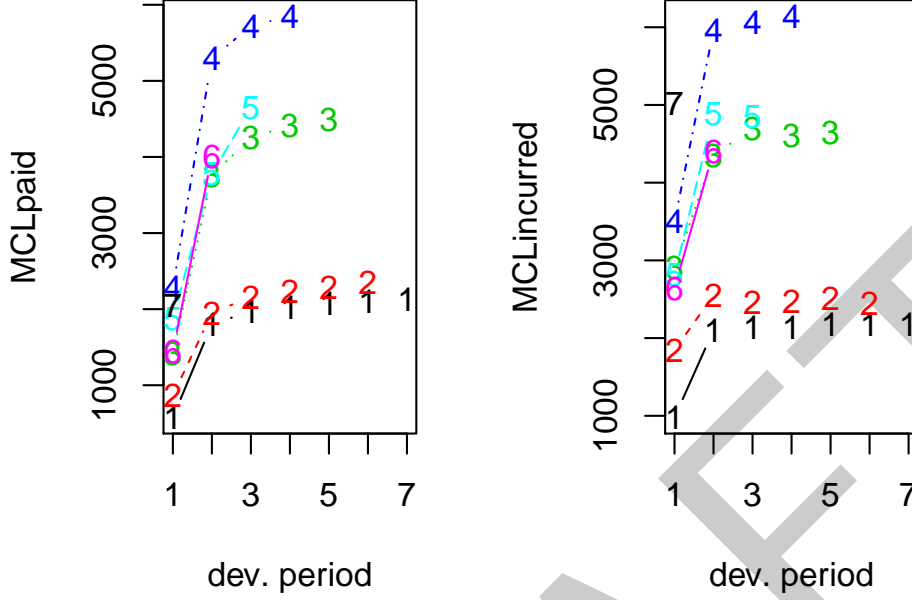
| | Latest Paid | Latest Incurred | Latest P/I Ratio | Ult. Paid | Ult. Incurred |
|---|-------------|-----------------|------------------|-----------|---------------|
| 1 | 2,131 | 2,174 | 0.980 | 2,131 | 2,174 |
| 2 | 2,348 | 2,454 | 0.957 | 2,383 | 2,444 |
| 3 | 4,494 | 4,644 | 0.968 | 4,597 | 4,629 |
| 4 | 5,850 | 6,142 | 0.952 | 6,119 | 6,176 |
| 5 | 4,648 | 4,852 | 0.958 | 4,937 | 4,950 |
| 6 | 4,010 | 4,406 | 0.910 | 4,656 | 4,665 |
| 7 | 2,044 | 5,022 | 0.407 | 7,549 | 7,650 |

| | Ult. P/I Ratio |
|---|----------------|
| 1 | 0.980 |
| 2 | 0.975 |
| 3 | 0.993 |
| 4 | 0.991 |
| 5 | 0.997 |
| 6 | 0.998 |
| 7 | 0.987 |

Totals

| | Paid | Incurred | P/I Ratio |
|-----------|--------|----------|-----------|
| Latest: | 25,525 | 29,694 | 0.86 |
| Ultimate: | 32,371 | 32,688 | 0.99 |

```
R> plot(MCL)
```



3.3 Multivariate chain-ladder

The Mack chain ladder technique can be generalized to the multivariate situation where multiple reserving triangles are modeled simultaneously. The advantage of the multivariate modeling is that correlations among different triangles can be modeled, which will lead to more accurate uncertainty assessment. There has been considerable development in this area, but most of the chain-ladder-based models can be summarized as sequential seemingly unrelated regressions (see Zhang 2010).

Denote $Y_{i,k} = (Y_{i,k}^{(1)}, \dots, Y_{i,k}^{(N)})$ as an $N \times 1$ vector of cumulative losses at accident year i and development year k where (n) refers to the n -th triangle. Zhang (2010) specifies the model in development period k as:

$$Y_{i,k+1} = A_k + B_k \cdot Y_{i,k} + \epsilon_{i,k}, \quad (5)$$

where A_k is a column of intercepts and B_k is the development matrix for development period k . Assumptions for this model are:

$$E(\epsilon_{i,k} | Y_{i,1}, \dots, Y_{i,I+1-k}) = 0. \quad (6)$$

$$\text{cov}(\epsilon_{i,k} | Y_{i,1}, \dots, Y_{i,I+1-k}) = D(Y_{i,k}^{-\delta/2}) \Sigma_k D(Y_{i,k}^{-\delta/2}). \quad (7)$$

$$\text{losses of different accident years are independent.} \quad (8)$$

$$\epsilon_{i,k} \text{ are symmetrically distributed.} \quad (9)$$

In the above, D is the diagonal operator, and δ is a known positive value that controls how the variance depends on the mean (as weights).

The simplest case where $A_k = 0$ and B_k 's are diagonal is a naive generalization of the chain ladder, often referred to as the multivariate chain ladder (see Pröhl and Schmidt 2005). The following shows an application of this case using the data set from Merz and Wüthrich (2008):

```
R> data(liab)
R> # this is a list of two triangles
R> length(liab)
```

```
[1] 2
```

```
R> dim(liab[[1]])
```

```
[1] 14 14
```

3.3.1 Separate chain ladder ignoring correlations

If we specify `fit.method = "OLS"`, the ordinary least squares will be used in estimating the development factors for each triangle independent of the others. In this case, the covariance matrix Σ is diagonal. As a result, the multivariate model is equivalent to running two Mack chain ladders separately.

```
R> fit1 <- MultiChainLadder(liab, fit.method = "OLS")
R> (s1 <- summary(fit1))
```

```
$`Summary Statistics for Triangle 1`
      Latest Dev.To.Date Ultimate IBNR S.E CV
1      549,589      1.0000   549,589     0  0 0.0000
2      562,795      0.9966   564,740  1,945 1,743 0.8961
3      602,710      0.9911   608,104  5,394 7,354 1.3633
4      784,632      0.9867   795,248 10,616 9,042 0.8518
5      768,373      0.9806   783,593 15,220 11,181 0.7346
6      811,100      0.9690   837,088 25,988 16,781 0.6457
7      896,728      0.9551   938,861 42,133 19,690 0.4673
8     1,022,241      0.9308  1,098,200 75,959 23,344 0.3073
9     1,019,303      0.8826  1,154,902 135,599 29,585 0.2182
10    1,141,750      0.7976  1,431,409 289,659 37,492 0.1294
11    1,174,196      0.6766  1,735,433 561,237 57,623 0.1027
12    1,032,684      0.4998  2,065,991 1,033,307 89,488 0.0866
13      772,971      0.2905  2,660,561 1,887,590 193,210 0.1024
14      204,325      0.0898  2,274,941 2,070,616 282,960 0.1367
Total 11,343,397      0.6482 17,498,658 6,155,261 427,289 0.0694
```

```
$`Summary Statistics for Triangle 2`
```

| | Latest | Dev.To.Date | Ultimate | IBNR | S.E | CV |
|-------|-----------|-------------|------------|-----------|---------|---------|
| 1 | 391,428 | 1.000 | 391,428 | 0 | 0 | 0.0000 |
| 2 | 483,974 | 1.000 | 483,839 | -135 | 604 | -4.4825 |
| 3 | 540,742 | 1.001 | 540,002 | -740 | 1,436 | -1.9416 |
| 4 | 485,016 | 0.998 | 486,227 | 1,211 | 2,912 | 2.4043 |
| 5 | 507,752 | 0.998 | 508,744 | 992 | 3,202 | 3.2284 |
| 6 | 549,693 | 0.994 | 552,825 | 3,132 | 5,418 | 1.7298 |
| 7 | 635,452 | 0.994 | 639,113 | 3,661 | 6,221 | 1.6993 |
| 8 | 648,365 | 0.985 | 658,410 | 10,045 | 7,483 | 0.7449 |
| 9 | 663,152 | 0.969 | 684,719 | 21,567 | 9,123 | 0.4230 |
| 10 | 790,901 | 0.935 | 845,543 | 54,642 | 16,191 | 0.2963 |
| 11 | 844,159 | 0.877 | 962,734 | 118,575 | 26,742 | 0.2255 |
| 12 | 915,109 | 0.783 | 1,169,260 | 254,151 | 36,736 | 0.1445 |
| 13 | 909,066 | 0.617 | 1,474,514 | 565,448 | 53,398 | 0.0944 |
| 14 | 394,997 | 0.277 | 1,426,060 | 1,031,063 | 126,613 | 0.1228 |
| Total | 8,759,806 | 0.809 | 10,823,418 | 2,063,612 | 162,872 | 0.0789 |

\$`Summary Statistics for Triangle 1+2`

| | Latest | Dev.To.Date | Ultimate | IBNR | S.E | CV |
|-------|------------|-------------|------------|-----------|---------|--------|
| 1 | 941,017 | 1.000 | 941,017 | 0 | 0 | 0.0000 |
| 2 | 1,046,769 | 0.998 | 1,048,579 | 1,810 | 1,845 | 1.0190 |
| 3 | 1,143,452 | 0.996 | 1,148,107 | 4,655 | 7,493 | 1.6097 |
| 4 | 1,269,648 | 0.991 | 1,281,475 | 11,827 | 9,499 | 0.8032 |
| 5 | 1,276,125 | 0.988 | 1,292,337 | 16,212 | 11,631 | 0.7174 |
| 6 | 1,360,793 | 0.979 | 1,389,913 | 29,120 | 17,634 | 0.6056 |
| 7 | 1,532,180 | 0.971 | 1,577,973 | 45,793 | 20,650 | 0.4509 |
| 8 | 1,670,606 | 0.951 | 1,756,610 | 86,004 | 24,514 | 0.2850 |
| 9 | 1,682,455 | 0.915 | 1,839,620 | 157,165 | 30,960 | 0.1970 |
| 10 | 1,932,651 | 0.849 | 2,276,952 | 344,301 | 40,838 | 0.1186 |
| 11 | 2,018,355 | 0.748 | 2,698,167 | 679,812 | 63,526 | 0.0934 |
| 12 | 1,947,793 | 0.602 | 3,235,251 | 1,287,458 | 96,735 | 0.0751 |
| 13 | 1,682,037 | 0.407 | 4,135,075 | 2,453,038 | 200,453 | 0.0817 |
| 14 | 599,322 | 0.162 | 3,701,001 | 3,101,679 | 309,995 | 0.0999 |
| Total | 20,103,203 | 0.710 | 28,322,077 | 8,218,874 | 457,278 | 0.0556 |

By default, the summary produces reserve statistics for all individual triangles, as well as for the portfolio that is assumed to be the sum of the two triangles. This behavior can be changed by supplying the portfolio argument. See the documentation for details.

We can verify if this is indeed the same as the univariate Mack chain ladder. For example, we can apply the MackChainLadder function on the first triangle:

```
R> fit0 <- MackChainLadder(liab[[1]], est.sigma = "Mack")
R> # just show the total estimates for minimize output
R> t(summary(fit0)$Totals) # the same as the first triangle above
```

```

Latest:   Dev: Ultimate:   IBNR: Mack S.E.: CV(IBNR):
Totals 11343397 0.6482 17498658 6155261      427289 0.06942

```

3.3.2 Multivariate chain ladder using seemingly unrelated regressions

To allow correlations to be incorporated, we employ the seemingly unrelated regressions (see the package `systemfit`) that simultaneously model the two triangles in each development period. This is invoked when we specify `fit.method = "SUR"`:

```

R> fit2 <- MultiChainLadder(liab, fit.method = "SUR")
R> s2 <- summary(fit2)
R> print(format(s2$report.summary[[3]], digits = 4, big.mark = ","))

```

| | Latest | Dev.To.Date | Ultimate | IBNR | S.E | CV |
|-------|------------|-------------|------------|-----------|---------|--------|
| 1 | 941,017 | 1.0000 | 941,017 | 0 | 0 | 0.0000 |
| 2 | 1,046,769 | 0.9983 | 1,048,579 | 1,810 | 1,851 | 1.0221 |
| 3 | 1,143,452 | 0.9960 | 1,148,032 | 4,580 | 7,859 | 1.7158 |
| 4 | 1,269,648 | 0.9909 | 1,281,370 | 11,722 | 9,545 | 0.8143 |
| 5 | 1,276,125 | 0.9874 | 1,292,393 | 16,268 | 12,133 | 0.7458 |
| 6 | 1,360,793 | 0.9789 | 1,390,091 | 29,298 | 18,913 | 0.6455 |
| 7 | 1,532,180 | 0.9704 | 1,578,868 | 46,688 | 22,448 | 0.4808 |
| 8 | 1,670,606 | 0.9505 | 1,757,679 | 87,073 | 25,913 | 0.2976 |
| 9 | 1,682,455 | 0.9140 | 1,840,846 | 158,391 | 33,294 | 0.2102 |
| 10 | 1,932,651 | 0.8482 | 2,278,572 | 345,921 | 45,253 | 0.1308 |
| 11 | 2,018,355 | 0.7476 | 2,699,816 | 681,461 | 72,050 | 0.1057 |
| 12 | 1,947,793 | 0.6021 | 3,235,135 | 1,287,342 | 112,187 | 0.0871 |
| 13 | 1,682,037 | 0.4070 | 4,132,660 | 2,450,623 | 222,927 | 0.0910 |
| 14 | 599,322 | 0.1624 | 3,691,189 | 3,091,867 | 342,127 | 0.1107 |
| Total | 20,103,203 | 0.7100 | 28,316,248 | 8,213,045 | 500,607 | 0.0610 |

To reduce the output, only the portfolio summary is printed out. We see that the portfolio prediction error is inflated to 500,607 from 457,278 in the separate development model ("OLS"). This is because of the positive correlation between the two triangles. The estimated correlation for each development period can be retrieved through the `residCor` function:

```

R> round(unlist(residCor(fit2)), 3)

[1] 0.247 0.495 0.682 0.446 0.487 0.451 -0.172 0.805 0.337 0.688
[11] -0.004 1.000 0.021

```

Similarly, most methods that work for linear models such as `coef`, `fitted`, `resid` and so on will also work. Since we have a sequence of models, the retrieved results from these methods are stored in a list. For example, we can retrieve the estimated development factors for each period as

```

R> do.call("rbind", coef(fit2))

```

| | eq1_x[[1]] | eq2_x[[2]] |
|-------|------------|------------|
| [1,] | 3.227 | 2.2224 |
| [2,] | 1.719 | 1.2688 |
| [3,] | 1.352 | 1.1200 |
| [4,] | 1.179 | 1.0665 |
| [5,] | 1.106 | 1.0356 |
| [6,] | 1.055 | 1.0168 |
| [7,] | 1.026 | 1.0097 |
| [8,] | 1.015 | 1.0002 |
| [9,] | 1.012 | 1.0038 |
| [10,] | 1.006 | 0.9994 |
| [11,] | 1.005 | 1.0039 |
| [12,] | 1.005 | 0.9989 |
| [13,] | 1.003 | 0.9997 |

The smaller-than-one development factors from the 10-th period for the second triangle result in the negative IBNR estimates for the first several accident years in the previous output.

The package also offers the `plot` method that produces various summary and diagnostic figures:

```
R> parold <- par(mfrow = c(4, 2), mar = c(4, 4, 2, 1),
+   mgp = c(1.3, 0.3, 0), tck = -0.02)
R> plot(fit2, which.triangle = 1:2, which.plot = 1:4)
R> par(parold)
```

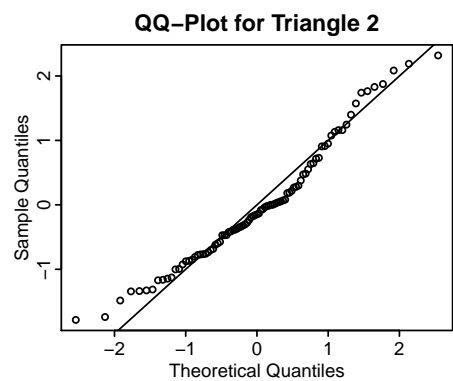
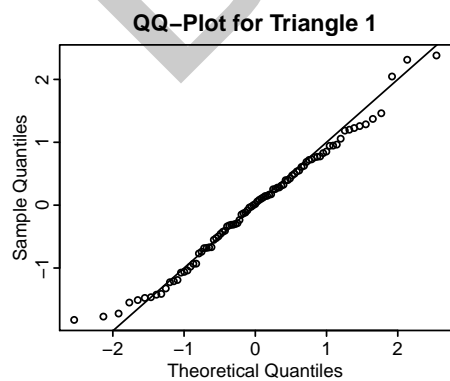
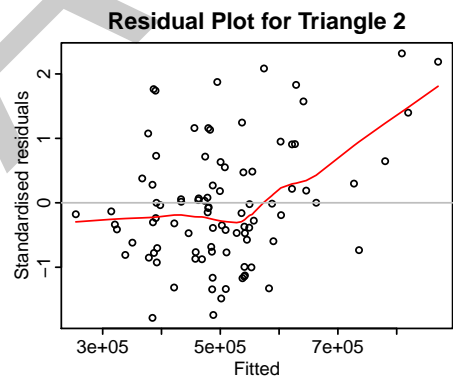
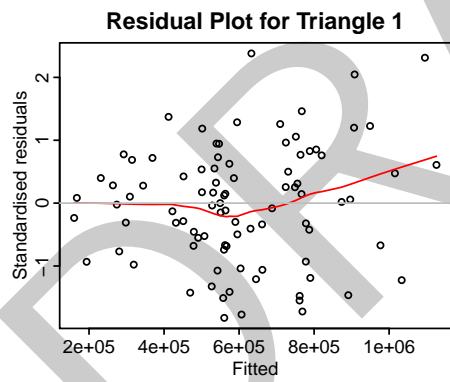
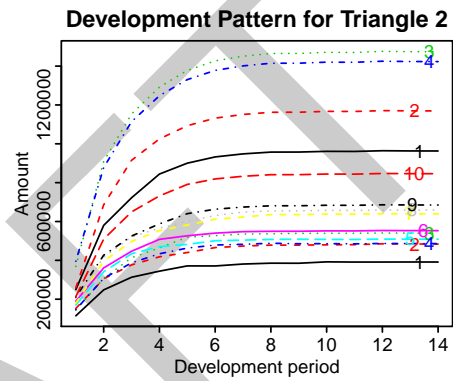
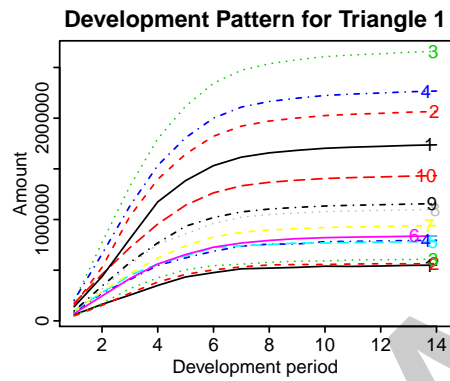
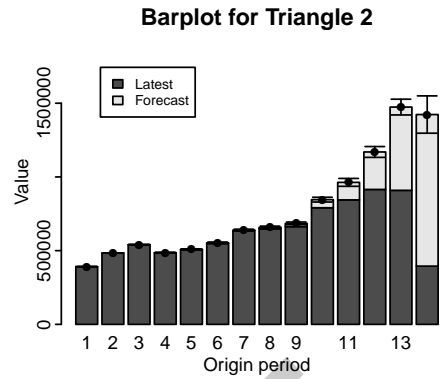
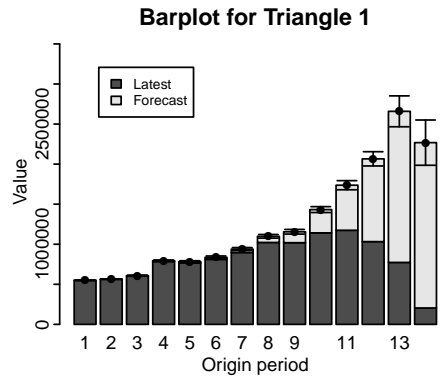
We use the `which.triangle` to suppress the plot for the portfolios, and use the `which.plot` to select the desired types of plots. See the documentation for possible values for these two arguments.

3.3.3 Other covariance estimation methods

Internally, the `MultiChainLadder` calls the `systemfit` function to fit the regression models period by period. When SUR models are specified, there are several ways to estimate the covariance matrix Σ . Available methods are "noDfCor", "geomean", "max", and "Theil" with the default as "geomean". The method "Theil" will produce unbiased covariance estimate, but the resulting estimate may not be positive semi-definite. This is also the estimator used by Merz and Wüthrich (2008). However, this method does not work out of the box for the `liab` data, and is perhaps why Merz and Wüthrich (2008) used extrapolation to get the estimate for the last several periods. Here, we simply split the data into two parts: on the first part we apply the multivariate chain ladder with the unbiased covariance estimate, and on the second part, we simply run a separate chain ladder approach.

impose parameter restriction in GMCL so that development matrix is diagonal

```
W1 <- MultiChainLadder2(liab, mse.method = "Independence", control = systemfit.control(methodResidCov =
"Theil"))
```



3.4 Clark's methods

3.4.1 Clark's Cap Cod method

3.4.2 Clark's LDF method

3.5 Generalised linear model methods

Recent years have also seen growing interest in using generalised linear models [GLM] for insurance loss reserving. The use of GLM in insurance loss reserving has many compelling aspects, e.g.,

- when over-dispersed Poisson model is used, it reproduces the estimates from Chain Ladder;
- it provides a more coherent modeling framework than the Mack method;
- all the relevant established statistical theory can be directly applied to perform hypothesis testing and diagnostic checking;

The `glmReserve` function takes an insurance loss triangle, converts it to incremental losses internally if necessary, transforms it to the long format (see `as.data.frame`) and fits the resulting loss data with a generalised linear model where the mean structure includes both the accident year and the development lag effects. The function also provides both analytical and bootstrapping method to compute the associated prediction errors. The bootstrapping approach also simulates the full predictive distribution, based on which the user can compute other uncertainty measures such as predictive intervals.

Only the Tweedie family of distributions are allowed, that is, the exponential family that admits a power variance function $V(\mu) = \mu^p$. The variance power p is specified in the `var.power` argument, and controls the type of the distribution. When the Tweedie compound Poisson distribution $1 < p < 2$ is to be used, the user has the option to specify `var.power = NULL`, where the variance power p will be estimated from the data using the `cplm` package.

For example, the following fits the over-dispersed Poisson model and spells out the estimated reserve information:

```
R> # load data
R> data(GenIns)
R> GenIns <- GenIns / 1000
R> # fit Poisson GLM
R> (fit1 <- glmReserve(GenIns))
```

| | Latest | Dev.To.Date | Ultimate | IBNR | S.E | CV |
|---|--------|-------------|----------|------|--------|--------|
| 2 | 5339 | 0.98252 | 5434 | 95 | 110.1 | 1.1589 |
| 3 | 4909 | 0.91263 | 5379 | 470 | 216.0 | 0.4597 |
| 4 | 4588 | 0.86599 | 5298 | 710 | 260.9 | 0.3674 |
| 5 | 3873 | 0.79725 | 4858 | 985 | 303.6 | 0.3082 |
| 6 | 3692 | 0.72235 | 5111 | 1419 | 375.0 | 0.2643 |
| 7 | 3483 | 0.61527 | 5661 | 2178 | 495.4 | 0.2274 |
| 8 | 2864 | 0.42221 | 6784 | 3920 | 790.0 | 0.2015 |
| 9 | 1363 | 0.24162 | 5642 | 4279 | 1046.5 | 0.2446 |

| | | | | | | |
|-------|-------|---------|-------|-------|--------|--------|
| 10 | 344 | 0.06922 | 4970 | 4626 | 1980.1 | 0.4280 |
| total | 30457 | 0.61982 | 49138 | 18681 | 2945.7 | 0.1577 |

We can also extract the underlying GLM model by specify `type = "model"` in the `summary` function:

```
R> summary(fit1, type = "model")
```

Call:

```
glm(formula = value ~ factor(origin) + factor(dev), family = fam,
     data = ldaFit, offset = offset)
```

Deviance Residuals:

| Min | 1Q | Median | 3Q | Max |
|---------|--------|--------|-------|--------|
| -14.701 | -3.913 | -0.688 | 3.675 | 15.633 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|------------------|----------|------------|---------|----------|
| (Intercept) | 5.59865 | 0.17292 | 32.38 | < 2e-16 |
| factor(origin)2 | 0.33127 | 0.15354 | 2.16 | 0.0377 |
| factor(origin)3 | 0.32112 | 0.15772 | 2.04 | 0.0492 |
| factor(origin)4 | 0.30596 | 0.16074 | 1.90 | 0.0650 |
| factor(origin)5 | 0.21932 | 0.16797 | 1.31 | 0.1999 |
| factor(origin)6 | 0.27008 | 0.17076 | 1.58 | 0.1225 |
| factor(origin)7 | 0.37221 | 0.17445 | 2.13 | 0.0398 |
| factor(origin)8 | 0.55333 | 0.18653 | 2.97 | 0.0053 |
| factor(origin)9 | 0.36893 | 0.23918 | 1.54 | 0.1317 |
| factor(origin)10 | 0.24203 | 0.42756 | 0.57 | 0.5749 |
| factor(dev)2 | 0.91253 | 0.14885 | 6.13 | 4.7e-07 |
| factor(dev)3 | 0.95883 | 0.15257 | 6.28 | 2.9e-07 |
| factor(dev)4 | 1.02600 | 0.15688 | 6.54 | 1.3e-07 |
| factor(dev)5 | 0.43528 | 0.18391 | 2.37 | 0.0234 |
| factor(dev)6 | 0.08006 | 0.21477 | 0.37 | 0.7115 |
| factor(dev)7 | -0.00638 | 0.23829 | -0.03 | 0.9788 |
| factor(dev)8 | -0.39445 | 0.31029 | -1.27 | 0.2118 |
| factor(dev)9 | 0.00938 | 0.32025 | 0.03 | 0.9768 |
| factor(dev)10 | -1.37991 | 0.89669 | -1.54 | 0.1326 |

(Dispersion parameter for Tweedie family taken to be 52.6)

Null deviance: 10699 on 54 degrees of freedom
 Residual deviance: 1903 on 36 degrees of freedom
 AIC: NA

Number of Fisher Scoring iterations: 4

Similarly, we can fit the Gamma and a compound Poisson GLM reserving model by changing the `var.power` argument:

```
R> # Gamma GLM
R> (fit2 <- glmReserve(GenIns, var.power = 2))
```

| | Latest | Dev.To.Date | Ultimate | IBNR | S.E | CV |
|-------|--------|-------------|----------|-------|---------|--------|
| 2 | 5339 | 0.98288 | 5432 | 93 | 45.17 | 0.4857 |
| 3 | 4909 | 0.91655 | 5356 | 447 | 160.56 | 0.3592 |
| 4 | 4588 | 0.88248 | 5199 | 611 | 177.62 | 0.2907 |
| 5 | 3873 | 0.79611 | 4865 | 992 | 254.47 | 0.2565 |
| 6 | 3692 | 0.71757 | 5145 | 1453 | 351.33 | 0.2418 |
| 7 | 3483 | 0.61440 | 5669 | 2186 | 526.29 | 0.2408 |
| 8 | 2864 | 0.43870 | 6529 | 3665 | 941.32 | 0.2568 |
| 9 | 1363 | 0.24854 | 5485 | 4122 | 1175.95 | 0.2853 |
| 10 | 344 | 0.07078 | 4860 | 4516 | 1667.39 | 0.3692 |
| total | 30457 | 0.62742 | 48543 | 18086 | 2702.71 | 0.1494 |

```
R> # compound Poisson GLM (variance function estimated from the data):
R> (fit3 <- glmReserve(GenIns, var.power = NULL))
```

| | Latest | Dev.To.Date | Ultimate | IBNR | S.E | CV |
|-------|--------|-------------|----------|-------|--------|--------|
| 2 | 5339 | 0.98270 | 5433 | 94 | 91.6 | 0.9745 |
| 3 | 4909 | 0.91331 | 5375 | 466 | 186.5 | 0.4003 |
| 4 | 4588 | 0.86780 | 5287 | 699 | 223.7 | 0.3201 |
| 5 | 3873 | 0.79709 | 4859 | 986 | 264.8 | 0.2685 |
| 6 | 3692 | 0.72164 | 5116 | 1424 | 333.2 | 0.2340 |
| 7 | 3483 | 0.61505 | 5663 | 2180 | 452.9 | 0.2078 |
| 8 | 2864 | 0.42365 | 6761 | 3897 | 754.6 | 0.1936 |
| 9 | 1363 | 0.24231 | 5626 | 4263 | 1019.5 | 0.2391 |
| 10 | 344 | 0.06943 | 4955 | 4611 | 1911.0 | 0.4144 |
| total | 30457 | 0.62058 | 49078 | 18621 | 2831.5 | 0.1521 |

By default, the formulaic approach is used to compute the prediction errors. We can also carry out bootstrapping simulations by specifying `mse.method = "bootstrap"` (note that this argument supports partial match):

```
R> set.seed(11)
R> (fit5 <- glmReserve(GenIns, mse.method = "boot"))
```

| | Latest | Dev.To.Date | Ultimate | IBNR | S.E | CV |
|---|--------|-------------|----------|------|--------|--------|
| 2 | 5339 | 0.98252 | 5434 | 95 | 105.4 | 1.1098 |
| 3 | 4909 | 0.91263 | 5379 | 470 | 216.1 | 0.4597 |
| 4 | 4588 | 0.86599 | 5298 | 710 | 266.6 | 0.3755 |
| 5 | 3873 | 0.79725 | 4858 | 985 | 307.5 | 0.3122 |
| 6 | 3692 | 0.72235 | 5111 | 1419 | 376.3 | 0.2652 |
| 7 | 3483 | 0.61527 | 5661 | 2178 | 496.1 | 0.2278 |
| 8 | 2864 | 0.42221 | 6784 | 3920 | 812.9 | 0.2074 |
| 9 | 1363 | 0.24162 | 5642 | 4279 | 1050.9 | 0.2456 |

| | | | | | | |
|-------|-------|---------|-------|-------|--------|--------|
| 10 | 344 | 0.06922 | 4970 | 4626 | 2004.1 | 0.4332 |
| total | 30457 | 0.61982 | 49138 | 18681 | 2959.4 | 0.1584 |

When bootstrapping is used, the resulting object has three additional components - "sims.par", "sims.reserve.mean", and "sims.reserve.pred" that store the simulated parameters, mean values and predicted values of the reserves for each year, respectively.

```
R> names(fit5)
```

```
[1] "call"           "summary"        "Triangle"
[4] "FullTriangle"   "model"          "sims.par"
[7] "sims.reserve.mean" "sims.reserve.pred"
```

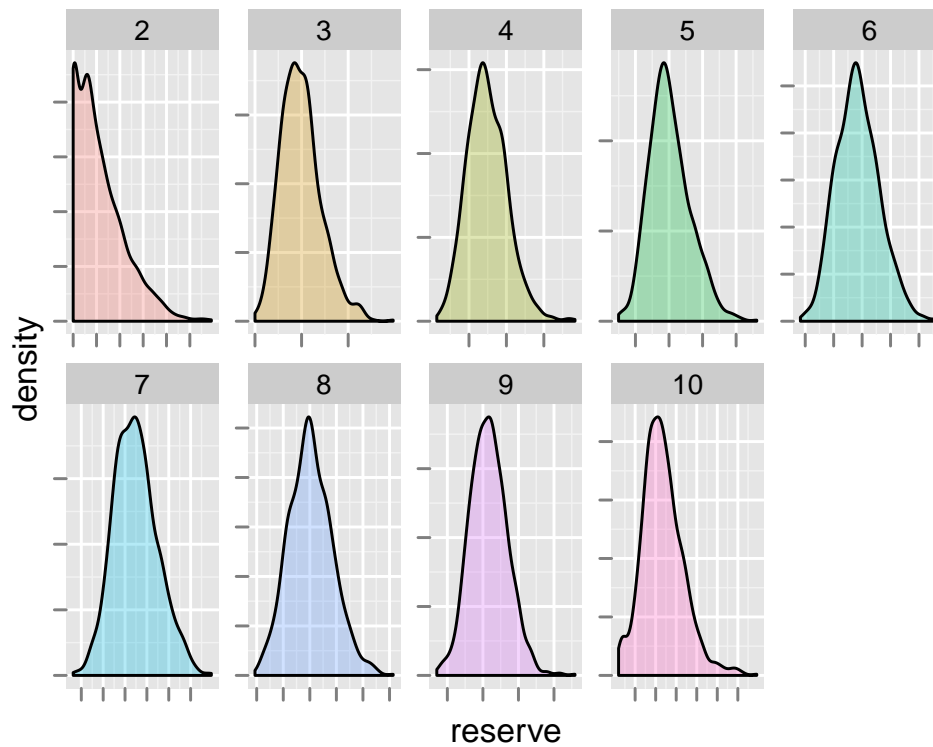
We can thus compute the quantiles of the predictions based on the simulated samples in the "sims.reserve.pred" element as:

```
R> pr <- as.data.frame(fit5$sims.reserve.pred)
R> qv <- c(0.025, 0.25, 0.5, 0.75, 0.975)
R> res.q <- t(apply(pr, 2, quantile, qv))
R> print(format(round(res.q), big.mark = ","), quote = FALSE)
```

| | 2.5% | 25% | 50% | 75% | 97.5% |
|----|-------|-------|-------|-------|-------|
| 2 | 0 | 34 | 82 | 170 | 376 |
| 3 | 136 | 337 | 470 | 615 | 987 |
| 4 | 279 | 556 | 719 | 917 | 1,302 |
| 5 | 506 | 797 | 972 | 1,197 | 1,674 |
| 6 | 774 | 1,159 | 1,404 | 1,666 | 2,203 |
| 7 | 1,329 | 1,877 | 2,210 | 2,547 | 3,303 |
| 8 | 2,523 | 3,463 | 3,991 | 4,572 | 5,713 |
| 9 | 2,364 | 3,593 | 4,310 | 5,013 | 6,531 |
| 10 | 913 | 3,354 | 4,487 | 5,774 | 9,165 |

The full predictive distribution of the simulated reserves for each year can be visualized easily:

```
R> library(ggplot2)
R> library(reshape2)
R> prm <- melt(pr)
R> names(prm) <- c("year", "reserve")
R> gg <- ggplot(prm, aes(reserve))
R> gg <- gg + geom_density(aes(fill = year), alpha = 0.3) +
+   facet_wrap(~year, nrow = 2, scales = "free") +
+   opts(axis.text.x = theme_blank(),
+         axis.text.y = theme_blank(),
+         legend.position = "none")
R> print(gg)
```



4 Using ChainLadder with RExcel and SWord

The spreadsheet is located in the Excel folder of the package. The R command

```
R> system.file("Excel", package="ChainLadder")
```

will tell you the exact path to the directory. To use the spreadsheet you will need the RExcel-Add-in [BN07]. The package also provides an example SWord file, demonstrating how the functions of the package can be integrated into a MS Word file via SWord [BN07]. Again you find the Word file via the command:

```
R> system.file("SWord", package="ChainLadder")
```

The package comes with several demos to provide you with an overview of the package functionality, see

```
R> demo(package="ChainLadder")
```

5 Further resources

Other useful documents and resources to get started with R in the context of actuarial work:

- Introduction to R for Actuaries [DS06].
- An Actuarial Toolkit [MSH⁺06].
- The book *Modern Actuarial Risk Theory – Using R* [KGDD01]
- Actuar package vignettes: <http://cran.r-project.org/web/packages/actuar/index.html>
- Mailing list **R-SIG-insurance**⁶: Special Interest Group on using R in actuarial science and insurance

5.1 Other insurance related R packages

Below is a list of further R packages in the context of insurance. The list is by no-means complete, and the CRAN Task Views '*Emperical Finance*' and '*Probability Distributions*' will provide links to additional resources. Please feel free to contact [us](#) with items to be added to the list.

- `cp1m`: Monte Carlo EM algorithms and Bayesian methods for fitting Tweedie compound Poisson linear models [Zha11].
- `lossDev`: A Bayesian time series loss development model. Features include skewed-t distribution with time-varying scale parameter, Reversible Jump MCMC for determining the functional form of the consumption path, and a structural break in this path [LS11].
- `favir`: Formatted Actuarial Vignettes in R. FAViR lowers the learning curve of the R environment. It is a series of peer-reviewed Sweave papers that use a consistent style [Esc11].
- `actuar`: Loss distributions modelling, risk theory (including ruin theory), simulation of compound hierarchical models and credibility theory [DGP08].
- `fitdistrplus`: Help to fit of a parametric distribution to non-censored or censored data [DMPDD10].
- `mondate`: R packackge to keep track of dates in terms of months [Mur11].
- `lifecontingencies`: Package to perform actuarial evaluation of life contingencies [Spe11].

5.2 Presentations

Over the years the contributors of the `ChainLadder` package have given numerous presentations and most of those are still available online:

- **Bayesian Hierarchical Models in Property-Casualty Insurance**, Wayne Zhang, 2011

⁶<https://stat.ethz.ch/mailman/listinfo/r-sig-insurance>

- ChainLadder at the Predictive Modelling Seminar, Institute of Actuaries, November 2010, Markus Gesmann, 2011
- Reserve variability calculations, CAS spring meeting, San Diego, Jimmy Curcio Jr., Markus Gesmann and Wayne Zhang, 2010
- The ChainLadder package, working with databases and MS Office interfaces, presentation at the "R you ready?" workshop , Institute of Actuaries, Markus Gesmann, 2009
- The ChainLadder package, London R user group meeting, Markus Gesmann, 2009
- Introduction to R, Loss Reserving with R, Stochastic Reserving and Modelling Seminar, Institute of Actuaries, Markus Gesmann, 2008
- Loss Reserving with R , CAS meeting, Vincent Goulet, Markus Gesmann and Daniel Murphy, 2008
- The ChainLadder package R-user conference Dortmund, Markus Gesmann, 2008

5.3 Further reading

Other papers and presentation which cited ChainLadder : [Orr07], [Nic09], [Zha10], [MNNV10], [Sch10], [MNV10], [Esc11], [Spe11]

6 Training and consultancy

Please contact [us](#) if you would like to discuss tailored training or consultancy.

References

- [BN07] Thomas Baier and Erich Neuwirth. Excel :: Com :: R. *Computational Statistics*, 22(1), April 2007. Physica Verlag.
- [Cla03] David R. Clark. *LDF Curve-Fitting and Stochastic Reserving: A Maximum Likelihood Approach*. Casualty Actuarial Society, 2003. CAS Fall Forum.
- [DGP08] C Dutang, V. Goulet, and M. Pigeon. actuar: An R package for actuarial science. *Journal of Statistical Software*, 25(7), 2008.
- [DMPDD10] Marie Laure Delignette-Muller, Regis Pouillot, Jean-Baptiste Denis, and Christophe Dutang. *fitdistrplus: help to fit of a parametric distribution to non-censored or censored data*, 2010. R package version 0.1-3.
- [DS06] Nigel De Silva. An introduction to r: Examples for actuaries. <http://toolkit.pbwiki.com/RToolkit>, 2006.
- [Esc11] Benedict Escoto. *favir: Formatted Actuarial Vignettes in R*, 0.5-1 edition, January 2011.

- [EV99] Peter England and Richard Verrall. Analytic and bootstrap estimates of prediction errors in claims reserving. *Mathematics and Economics*, Vol. 25:281 – 293, 1999.
- [GBB⁺09] Brian Gravelsons, Matthew Ball, Dan Beard, Robert Brooks, Naomi Couchman, Brian Gravelsons, Charlie Kefford, Darren Michaels, Patrick Nolan, Gregory Overton, Stephen Robertson-Dunn, Emiliano Ruffini, Graham Sandhouse, Jerome Schilling, Dan Sykes, Peter Taylor, Andy Whiting, Matthew Wilde, and John Wilson. B12: Uk asbestos working party update 2009. <http://www.actuaries.org.uk/research-and-resources/documents/b12-uk-asbestos-working-party-update-2009-5mb>, October 2009. Presented at the General Insurance Convention.
- [GMZ12] Markus Gesmann, Dan Murphy, and Wayne Zhang. *ChainLadder: Mack-, Bootstrap and Munich-chain-ladder methods for insurance claims reserving*, 2012. R package version 0.1.5-2.
- [KGDD01] R. Kaas, M. Goovaerts, J. Dhaene, and M. Denuit. *Modern actuarial risk theory*. Kluwer Academic Publishers, Dordrecht, 2001.
- [LS11] Christopher W. Laws and Frank A. Schmid. *lossDev: Robust Loss Development Using MCMC*, 2011. R package version 3.0.0-1.
- [Mac93a] Thomas Mack. Distribution-free calculation of the standard error of chain ladder reserve estimates. *Astin Bulletin*, Vol. 23:213 – 25, 1993.
- [Mac93b] Thomas Mack. Distribution-free calculation of the standard error of chain ladder reserve estimates. *ASTIN Bulletin*, 23:213–225, 1993.
- [Mac99] Thomas Mack. The standard error of chain ladder reserve estimates: Recursive calculation and inclusion of a tail factor. *Astin Bulletin*, Vol. 29(2):361 – 266, 1999.
- [Mic02] Darren Michaels. APH: how the love carnal and silicone implants nearly destroyed Lloyd's (slides). <http://www.actuaries.org.uk/research-and-resources/documents/aph-how-love-carnal-and-silicone-implants-nearly-destroyed-lloyds-s>, December 2002. Presented at the Younger Members' Convention.
- [MNNV10] Maria Dolores Martinez Miranda, Bent Nielsen, Jens Perch Nielsen, and Richard Verrall. *Cash flow simulation for a model of outstanding liabilities based on claim amounts and claim numbers*. CASS, September 2010.
- [MNV10] Maria Dolores Martinez Miranda, Jens Perch Nielsen, and Richard Verrall. *Double Chain Ladder*. ASTIN, Colloquia Madrid edition, 2010.
- [MSH⁺06] Trevor Maynard, Nigel De Silva, Richard Holloway, Markus Gesmann, Sie Lau, and John Harnett. An actuarial toolkit. introducing The Toolkit Manifesto. <http://www.actuaries.org.uk/sites/all/files/documents/pdf/actuarial-toolkit.pdf>, 2006. General Insurance Convention.
- [Mur94] Daniel Murphy. Unbiased loss development factors. *PCAS*, 81:154 – 222, 1994.
- [Mur11] Dan Murphy. *mondate: Keep track of dates in terms of months*, 2011. R package version 0.9.8.24.
- [Nic09] Luke Nichols. *Multimodel Inference for Reserving*. Australian Prudential Regulation Authority (APRA), December 2009.

- [Orr07] James Orr. *A Simple Multi-State Reserving Model*. ASTIN, Colloquia Orlando edition, 2007.
- [PR02] P.D.England and R.J.Verrall. Stochastic claims reserving in general insurance. *British Actuarial Journal*, 8:443–544, 2002.
- [QM04] Gerhard Quarg and Thomas Mack. Munich chain ladder. Munich Re Group, 2004.
- [Sch10] Ernesto Schirmacher. Reserve variability calculations, chain ladder, R, and Excel. <http://www.casact.org/affiliates/cane/0910/schirmacher.pdf>, September 2010. Presentation at the Casualty Actuaries of New England (CANE) meeting.
- [Sch11] Klaus D. Schmidt. A bibliography on loss reserving. <http://www.math.tu-dresden.de/sto/schmidt/dsvm/reserve.pdf>, 2011.
- [Spe11] Giorgio Alfredo Spedicato. *Introduction to lifecontingencies Package*. StatisticalAdvisor Inc, 0.0.4 edition, November 2011.
- [Tea12a] R Development Core Team. *R Data Import/Export*. R Foundation for Statistical Computing, 2012. ISBN 3-900051-10-0.
- [Tea12b] R Development Core Team. *R Installation and Administration*. R Foundation for Statistical Computing, 2012. ISBN 3-900051-09-7.
- [Zha10] Y. Zhang. A general multivariate chain ladder model. *Insurance: Mathematics and Economics*, 46:588 – 599, 2010.
- [Zha11] Wayne Zhang. *cplm: Tweedie compound Poisson linear models*, 2011. R package version 0.4-1.
- [Zx00] Ben Zehnwrith and Glenn xbBarnett. Best estimates for reserves. *Proceedings of the CAS*, LXXXVII(167), November 2000.