Claims reserving with R:
ChainLadder-0.1.5-2 Package Vignette

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Abstract

The `ChainLadder` package provides various statistical methods which are typically used for the calculation of outstanding claims reserves in general insurance.

The package has implementations of the Mack-, Munich-, Bootstrap, and multi-variate chain-ladder methods, as well as the loss development factor curve fitting methods of Dave Clark and generalised linear model based reserving models.

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Contents

1 Introduction

1.1 Claims reserving in insurance

Unlike other industries the insurance industry does not sell products as such, but promises. An insurance policy is a promise by the insurer to the policyholder to pay for future claims for an upfront received premium.

As a result insurers don't know the upfront cost of their service, but rely on data analysis and judgement to derive a sustainable price for their offering. In General Insurance (or Non-Life Insurance) most policies run for a period of 12 months, e.g. motor, property and casualty insurance. However, the claims payment process can take years or even decades.

In particular claims arising from casualty insurance can take a long time to settle, as claims can take years to materialise. A complex and costly example are the claims from asbestos liabilities. A research report by a Working Party of the Institute of Actuaries has estimated that the undiscounted cost of UK mesothelioma-related claims to the UK Insurance Market for the period 2009 to 2050 could be around £10bn [GBB+09]. The cost for Asbestos related claims in the US for the worldwide insurance industry was estimate to be around $120bn in 2002 [Mic02].

Thus, it should come to no surprise that the biggest item on the liability side of an insurer's balance sheet is often the provision or reserves for future claims payments. Those reserves can be broken down in case reserves (or out-standings claims), which are losses already reported to the insurance company and incurred but not reported (IBNR) claims.

Over the years several methods have been developed to estimate reserves for insurance claims, see [Sch11], [PR02] for an overview. Changes in the regulatory requirements, e.g. Solvency II[1] in Europe, have fostered further research into this topic, with a focus on stochastic and statistical techniques.

2 The ChainLadder package

2.1 Motivation

The ChainLadder [GMZ12] package provides various statistical methods which are typically used for the calculation of outstanding claims reserves in general insurance. The package started out of presentations given by Markus Gesmann at the Stochastic Reserving Seminar at the Institute of Actuaries in 2007 and 2008, followed by talks at Casualty Actuarial Society (CAS) meetings joined by Dan Murphy in 2008 and Wayne Zhang in 2010.

Implementing reserving methods in R has several advantages, as it provides:

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[1]See http://ec.europa.eu/internal_market/insurance/solvency/index_en.htm

- Rich language for statistical modelling and data manipulations allowing fast prototyping

- Very active user base, which publishes many extension

- Many interfaces to data bases and other applications, such as MS Excel

- Established framework for documentation and testing

- Works well with version control tools

- Code is written in plain text files, allowing effective knowledge transfer

- Easy to collaborate over the internet

- Built in functions to create reproducible research reports[2]

- In combination with other tools such as LaTeX and Sweave easy to set up automated reporting

- Academic research is often first implemented in R

2.2 Brief package overview

This vignette will give the reader a brief overview of the functionallity of the `Chain-Ladder` package. The functions are discussed and explained in more detail in the repective help files and examples. The help files contain the references to the research papers on which the methods are based.

The `ChainLadder` package has implementations of the Mack-, Munich- and Bootstrap chain-ladder methods [Mac93], [Mac99], [QM04], [EV99]. Since version 0.1.3-3 it provides general multivariate chain ladder models by Wayne Zhang [Zha10].

Version 0.1.4-0 introduced new functions on loss development factor (LDF) fitting and Cape Cod by Daniel Murphy following a paper by David Clark [Cla03]. Version 0.1.5-0 has added loss reserving models within the generalized linear model framework following a paper by England and Verrall [EV99] implemented by Wayne Zhang.

The package offers also some utility functions to convert quickly tables into triangles, triangles into tables, cumulative into incremental and incremental into cumulative triangles.

A set of demos is shipped with the packages and the list of demos names is available via:

```
R> demo(package="ChainLadder")
```

and can be executed via

---

[2]For an example see the project: Formatted Actuarial Vignettes in R, http://www.favir.net/

```
R> library(ChainLadder)
R> demo("demo name")
```

Further, the ChainLadder package comes with example files, which demonstrates how to the ChainLadder functions can be embedded in Excel and Word using the statconn interface[BN07].

For more information and examples see the project web site: http://code.google.com/p/chainladder/

2.3   Installation

We can install ChainLadder in the usual way from CRAN, e.g.:

```
R> install.packages('ChainLadder')
```

The installation was successful if the command library(ChainLadder) gives you the following message:

```
R> library(ChainLadder)
```

```
ChainLadder version 0.1.5-2 by:
Markus Gesmann <markus.gesmann@gmail.com>
Wayne Zhang <actuary_zhang@hotmail.com>
Daniel Murphy <danielmarkmurphy@gmail.com>

Type library(help='ChainLadder') or ?ChainLadder
to see overall documentation.

Type demo(ChainLadder) to get an idea of the functionality of this package.

See demo(package='ChainLadder') for a list of more demos.

Feel free to send us an email if you would like to keep informed of
new versions or if you have any feedback, ideas, suggestions or would
like to collaborate.

More information is available on the ChainLadder project web-site:
http://code.google.com/p/chainladder/

To suppress this message use the statement:
suppressPackageStartupMessages(library(ChainLadder))
```

3 Using the `ChainLadder` package

3.1 Working with triangles

Historical insurance data is often presented in form of a triangle structure, showing the development of claims over time for each origin period. An origin period could be the year the policy was sold, or the accident year. Most reserving methods of the `ChainLadder` package expect triangles as input data sets with development periods along the columns and the origin period in rows. The package comes with several example triangles. The following R command will list them all:

```
R> require(ChainLadder)
R> data(package="ChainLadder")
```

Let's look at one example triangle more closely. The following triangle shows data from the Reinsurance Association of America (RAA):

```
R> ## Sample triangle
R> RAA
```

```
      dev
origin    1     2     3     4     5     6     7     8     9    10
  1981 5012  8269 10907 11805 13539 16181 18009 18608 18662 18834
  1982  106  4285  5396 10666 13782 15599 15496 16169 16704    NA
  1983 3410  8992 13873 16141 18735 22214 22863 23466    NA    NA
  1984 5655 11555 15766 21266 23425 26083 27067    NA    NA    NA
  1985 1092  9565 15836 22169 25955 26180    NA    NA    NA    NA
  1986 1513  6445 11702 12935 15852    NA    NA    NA    NA    NA
  1987  557  4020 10946 12314    NA    NA    NA    NA    NA    NA
  1988 1351  6947 13112    NA    NA    NA    NA    NA    NA    NA
  1989 3133  5395    NA    NA    NA    NA    NA    NA    NA    NA
  1990 2063    NA    NA    NA    NA    NA    NA    NA    NA    NA
```

The objective of a reserving exercise is to forecast the future claims development in the bottom right corner of the triangle and potential further developments. Eventually all claims for a given origin period wil be settled, but it is not always obvious to judge how many years or even decades it will take. We speak of long and short tail business depending on the time it takes to finalise all claims reporting.

3.1.1 Plotting triangles

The first thing you often want to do is to plot the data to get an overview. For data set of class `triangle` `ChainLadder` provides default plotting methods to give a graphical overview of the data:

```
R> plot(RAA)
```
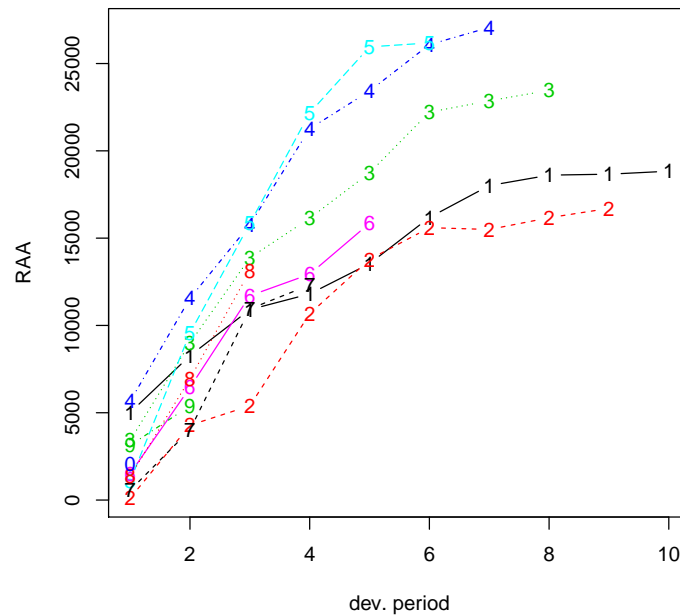

Figure 1: Claims development chart of the RAA triangle, with one line per origin period. Output of plot(RAA)

Setting the argument lattice=TRUE will produce individual plots for each origin period[3], see Figure 2.

```
R> plot(RAA, lattice=TRUE)
```

You will notice from the plots in Figures 1 and 2 that the triangle RAA presents claims developments for the origin years 1981 to 1990 in a cumulative form. For more information on the triangle plotting functions see the help pages of plot.triangle, e.g. via

```
R> ?plot.triangle
```

---

[3]ChainLadder uses the lattice package
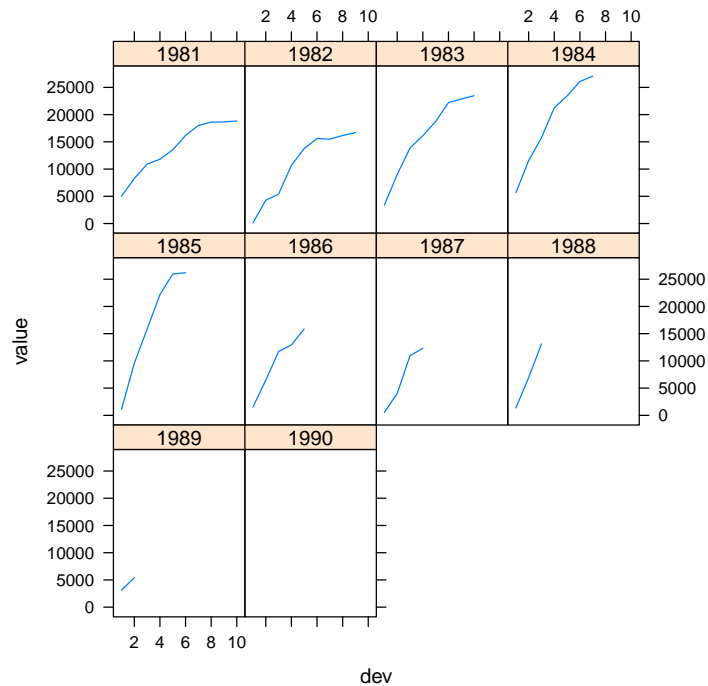
Figure 2: Claims development chart of the `RAA` triangle, with individual panels for each origin period. Output of `plot(RAA, lattice=TRUE)`

3.1.2 Transforming triangles between cumulative and incremental representation

The `ChainLadder` packages comes with two helper functions, `cum2incr` and `incr2cum` to transform cumulative triangles into incremental triangles and vis versa:

```
R> raa.inc <- cum2incr(RAA)
R> ## Show first origin period and its incremental development
R> raa.inc[1,]


   1    2    3    4    5    6    7    8    9   10
5012 3257 2638  898 1734 2642 1828  599   54  172


R> raa.cum <- incr2cum(raa.inc)
R> ## Show first origin period and its cumulative development
R> raa.cum[1,]
```

```
     1     2     3     4     5     6     7     8     9    10
  5012  8269 10907 11805 13539 16181 18009 18608 18662 18834
```

3.1.3 Importing triangles from external data sources

In most cases you want to analyse your own data, usually stored in data bases. R makes it easy to access data using SQL statements, e.g. via an ODBC connection[4] and the `ChainLadder` packages includes a demo to showcase how data can be imported from a MS Access data base, see:

```
R> demo(DatabaseExamples)
```

For more details see [Tea11].

In this section we use data stored in a CSV-file[5] to demonstrate some typical operations you will want to carry out with data stored in data bases. In most cases your triangles will be stored in tables and not in a classical triangle shape. The `ChainLadder` package contains a CSV-file with some sample data in a long table format. We can read the data into R's memory with the `read.csv` command and look at the first couple of rows and summarise it:

```
R> filename <-  file.path(system.file("Database",
+                                     package="ChainLadder"),
+                        "TestData.csv")
R> myData <- read.csv(filename)
R> head(myData)
```

```
  origin dev  value lob
1   1977   1 153638 ABC
2   1978   1 178536 ABC
3   1979   1 210172 ABC
4   1980   1 211448 ABC
5   1981   1 219810 ABC
6   1982   1 205654 ABC
```

```
R> summary(myData)
```

```
     origin           dev             value                 lob
 Min.   : 1    Min.   : 1.00   Min.   : -17657   AutoLiab    :105
 1st Qu.: 3    1st Qu.: 2.00   1st Qu.:  10324   GeneralLiab :105
 Median : 6    Median : 4.00   Median :  72468   M3IR5       :105
```

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[4]See the RODBC package

[5]Please ensure that your CSV-file is free from formatting, e.g. characters to separate units of thousands, as columns with those kind of formatting would be read as characters.

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```
Mean    : 642    Mean    : 4.61    Mean    : 176632    ABC                 : 66
3rd Qu.:1979    3rd Qu.: 7.00    3rd Qu.: 197716    CommercialAutoPaid: 55
Max.    :1991    Max.    :14.00    Max.    :3258646    GenIns              : 55
                                                       (Other)             :210
```

Let us focus on one subset of the data set. We select the RAA data again:

```
R> raa <- subset(myData, lob %in% "RAA")
R> head(raa)

   origin dev value lob
67   1981   1  5012 RAA
68   1982   1   106 RAA
69   1983   1  3410 RAA
70   1984   1  5655 RAA
71   1985   1  1092 RAA
72   1986   1  1513 RAA
```

To transform the long table of the RAA data into a triangle we use the function `as.triangle`. The arguments we have to specify are the column names of the origin and development period and further the column which contains the values:

```
R> raa.tri <- as.triangle(raa,
+                         origin="origin",
+                         dev="dev",
+                         value="value")
R> raa.tri

      dev
origin    1    2    3    4    5    6    7   8   9  10
  1981 5012 3257 2638  898 1734 2642 1828 599  54 172
  1982  106 4179 1111 5270 3116 1817 -103 673 535  NA
  1983 3410 5582 4881 2268 2594 3479  649 603  NA  NA
  1984 5655 5900 4211 5500 2159 2658  984  NA  NA  NA
  1985 1092 8473 6271 6333 3786  225   NA  NA  NA  NA
  1986 1513 4932 5257 1233 2917   NA   NA  NA  NA  NA
  1987  557 3463 6926 1368   NA   NA   NA  NA  NA  NA
  1988 1351 5596 6165   NA   NA   NA   NA  NA  NA  NA
  1989 3133 2262   NA   NA   NA   NA   NA  NA  NA  NA
  1990 2063   NA   NA   NA   NA   NA   NA  NA  NA  NA
```

We note that the data has been stored as an incremental data set. As mentioned above, we could now use the function `incr2cum` to transform the triangle into a cumulative format.

We can transform a triangle back into a data frame structure:

```
R> raa.df <- as.data.frame(raa.tri, na.rm=TRUE)
R> head(raa.df)
```

```
        origin dev value
1981-1    1981   1  5012
1982-1    1982   1   106
1983-1    1983   1  3410
1984-1    1984   1  5655
1985-1    1985   1  1092
1986-1    1986   1  1513
```

This is particular helpful when you would like to store your results back into data base. Figure 3 gives you an idea of a potential data flow between R and data bases.
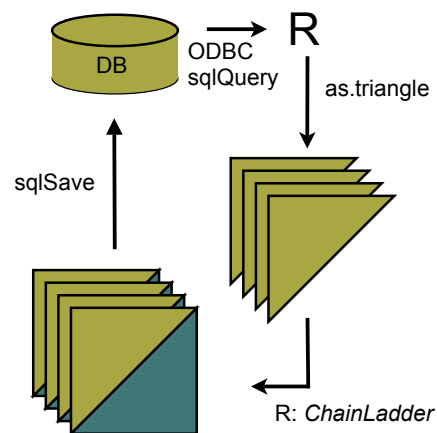
Figure 3: Flow chart of data between R and data bases.

3.1.4 Coping and pasting from MS Excel

Small data sets in Excel can be transfered to R backwards and forwards with via the clipboard under MS Windows.

Copying from Excel to R   Select a data set in Excel and copy it into the clipboard, then go to R and type:

```
R> x <- read.table(file="clipboard", sep="\t", na.strings="")
```

Copying from R to Excel Suppose you would like to copy the RAA triangle into Excel, then the following statement would copy the data into the clipboard:

```
R> write.table(RAA, file="clipboard", sep="\t", na="")
```

Now you can paste the content into Excel. Please note that you can not copy lists structures from R to Excel.

3.2 Chain-ladder methods

```
R> demo(ChainLadder)
```

3.2.1 Mack chain-ladder

```
R> mack <- MackChainLadder(RAA, est.sigma="Mack")
R> mack
```

```
MackChainLadder(Triangle = RAA, est.sigma = "Mack")
```

	Latest	Dev.To.Date	Ultimate	IBNR	Mack.S.E	CV(IBNR)
1981	18,834	1.000	18,834	0	0	NaN
1982	16,704	0.991	16,858	154	206	1.339
1983	23,466	0.974	24,083	617	623	1.010
1984	27,067	0.943	28,703	1,636	747	0.457
1985	26,180	0.905	28,927	2,747	1,469	0.535
1986	15,852	0.813	19,501	3,649	2,002	0.549
1987	12,314	0.694	17,749	5,435	2,209	0.406
1988	13,112	0.546	24,019	10,907	5,358	0.491
1989	5,395	0.336	16,045	10,650	6,333	0.595
1990	2,063	0.112	18,402	16,339	24,566	1.503

```
                Totals
Latest:      160,987.00
Dev:               0.76
Ultimate:    213,122.23
IBNR:         52,135.23
Mack S.E.:    26,909.01
CV(IBNR):          0.52
```

```
R> plot(mack)
```

R> plot(mack, lattice=TRUE)

Chain ladder developments by origin period
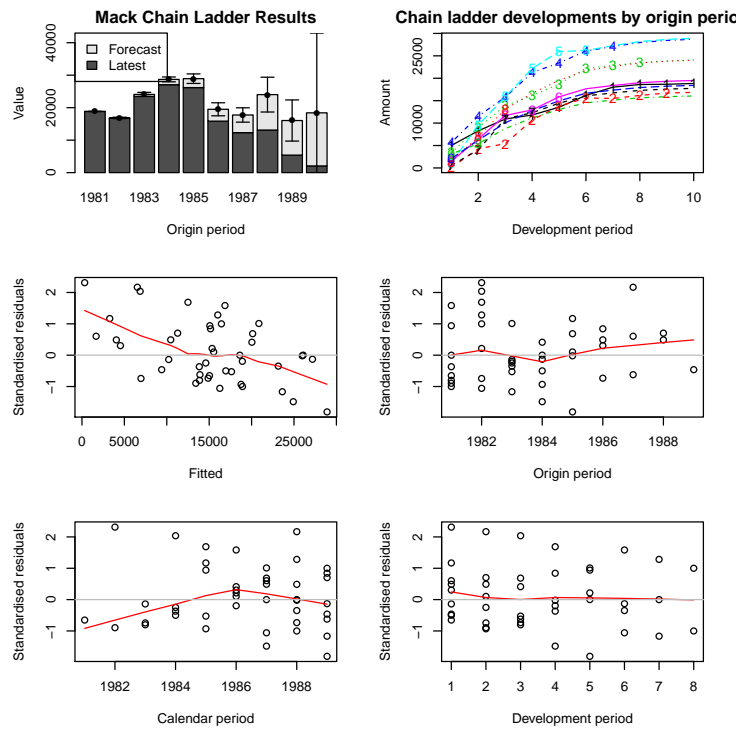
——— Chain ladder dev. ·········· Mack's S.E.

3.2.2 Bootstrap chain-ladder

```
R> # See also the example in section 8 of England & Verrall (2002) on page 55.
R>
R> B <- BootChainLadder(RAA, R=999, process.distr="gamma")
R> B


BootChainLadder(Triangle = RAA, R = 999, process.distr = "gamma")

     Latest Mean Ultimate Mean IBNR SD IBNR IBNR 75% IBNR 95%
1981 18,834       18,834         0       0        0        0
1982 16,704       16,865       161     671      191    1,416
1983 23,466       24,094       628   1,246    1,030    2,963
1984 27,067       28,825     1,758   1,949    2,758    5,439
1985 26,180       29,064     2,884   2,409    4,319    7,601
1986 15,852       19,636     3,784   2,469    5,166    8,102
1987 12,314       17,981     5,667   3,162    7,466   11,559
1988 13,112       24,255    11,143   5,060   13,988   20,197
1989  5,395       16,368    10,973   6,274   14,483   22,697
1990  2,063       19,713    17,650  13,555   25,363   43,594
```

```
                Totals
Latest:         160,987
Mean Ultimate:  215,636
Mean IBNR:       54,649
SD IBNR:         18,963
Total IBNR 75%:  65,835
Total IBNR 95%:  87,421


R> plot(B)
R> # Compare to MackChainLadder
R> MackChainLadder(RAA)


MackChainLadder(Triangle = RAA)

      Latest Dev.To.Date Ultimate    IBNR Mack.S.E CV(IBNR)
1981 18,834        1.000   18,834       0        0      NaN
1982 16,704        0.991   16,858     154      143    0.928
1983 23,466        0.974   24,083     617      592    0.959
1984 27,067        0.943   28,703   1,636      713    0.436
1985 26,180        0.905   28,927   2,747    1,452    0.529
1986 15,852        0.813   19,501   3,649    1,995    0.547
1987 12,314        0.694   17,749   5,435    2,204    0.405
1988 13,112        0.546   24,019  10,907    5,354    0.491
1989  5,395        0.336   16,045  10,650    6,332    0.595
1990  2,063        0.112   18,402  16,339   24,566    1.503

                Totals
Latest:      160,987.00
Dev:               0.76
Ultimate:    213,122.23
IBNR:         52,135.23
Mack S.E.:    26,880.74
CV(IBNR):          0.52


R> quantile(B, c(0.75,0.95,0.99, 0.995))


$ByOrigin
     IBNR 75% IBNR 95% IBNR 99% IBNR 99.5%
1981      0.0        0        0          0
1982    190.9     1416     2663       3284
1983   1030.5     2963     4617       5241
1984   2758.2     5439     7767       9004
1985   4319.1     7601     9666       9981
1986   5165.6     8102    11686      12326
1987   7465.9    11559    15167      15319
```
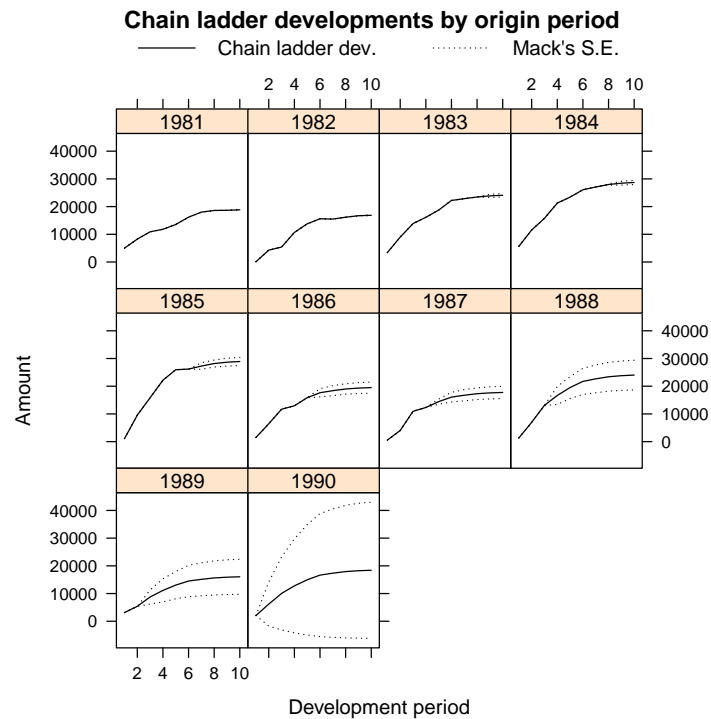
```
1988   13987.6      20197      26489        27254
1989   14482.7      22697      29008        31582
1990   25362.6      43594      56806        60296


$Totals
            Totals
IBNR 75%:     65835
IBNR 95%:     87421
IBNR 99%:    106253
IBNR 99.5%:  115272


R> # fit a distribution to the IBNR
R> library(MASS)
R> plot(ecdf(B$IBNR.Totals))
R> # fit a log-normal distribution
R> fit <- fitdistr(B$IBNR.Totals[B$IBNR.Totals>0], "lognormal")
R> fit


    meanlog       sdlog
  10.846301    0.363454
 ( 0.011499) ( 0.008131)


R> curve(plnorm(x,fit$estimate["meanlog"], fit$estimate["sdlog"]), col="red", add=TRUE)
R>
```

Histogram of Total.IBNR

ecdf(Total.IBNR)

Simulated ultimate claims cost

Latest actual incremental claims against simulated values
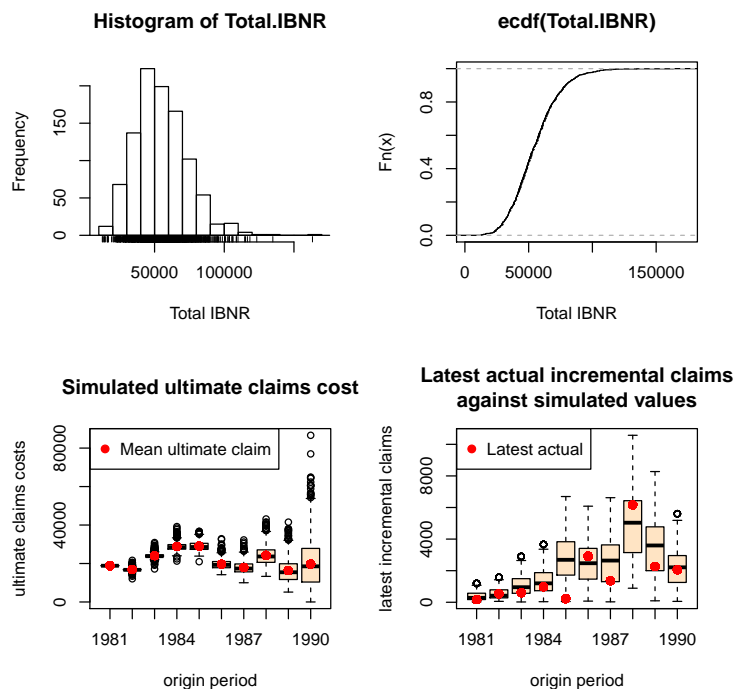
3.2.3 Munich chain-ladder

```
R> MCLpaid
```

```
      dev
origin   1    2    3    4    5    6    7
     1  576 1804 1970 2024 2074 2102 2131
     2  866 1948 2162 2232 2284 2348   NA
     3 1412 3758 4252 4416 4494   NA   NA
     4 2286 5292 5724 5850   NA   NA   NA
     5 1868 3778 4648   NA   NA   NA   NA
     6 1442 4010   NA   NA   NA   NA   NA
     7 2044   NA   NA   NA   NA   NA   NA
```

```
R> MCLincurred
```

```
      dev
origin    1    2    3    4    5    6    7
     1  978 2104 2134 2144 2174 2182 2174
     2 1844 2552 2466 2480 2508 2454   NA
```

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```
3 2904 4354 4698 4600 4644    NA    NA
4 3502 5958 6070 6142    NA    NA    NA
5 2812 4882 4852    NA    NA    NA    NA
6 2642 4406    NA    NA    NA    NA    NA
7 5022    NA    NA    NA    NA    NA    NA


R> op <- par(mfrow=c(1,2))
R> plot(MCLpaid)
R> plot(MCLincurred)
R> par(op)
R> # Following the example in Quarg's (2004) paper:
R> MCL <- MunichChainLadder(MCLpaid, MCLincurred, est.sigmaP=0.1, est.sigmaI=0.1)
R> MCL


MunichChainLadder(Paid = MCLpaid, Incurred = MCLincurred, est.sigmaP = 0.1,
    est.sigmaI = 0.1)

  Latest Paid Latest Incurred Latest P/I Ratio Ult. Paid Ult. Incurred
1       2,131           2,174            0.980     2,131        2,174
2       2,348           2,454            0.957     2,383        2,444
3       4,494           4,644            0.968     4,597        4,629
4       5,850           6,142            0.952     6,119        6,176
5       4,648           4,852            0.958     4,937        4,950
6       4,010           4,406            0.910     4,656        4,665
7       2,044           5,022            0.407     7,549        7,650
  Ult. P/I Ratio
1          0.980
2          0.975
3          0.993
4          0.991
5          0.997
6          0.998
7          0.987


Totals
           Paid Incurred P/I Ratio
Latest:   25,525   29,694      0.86
Ultimate: 32,371   32,688      0.99


R> plot(MCL)
```
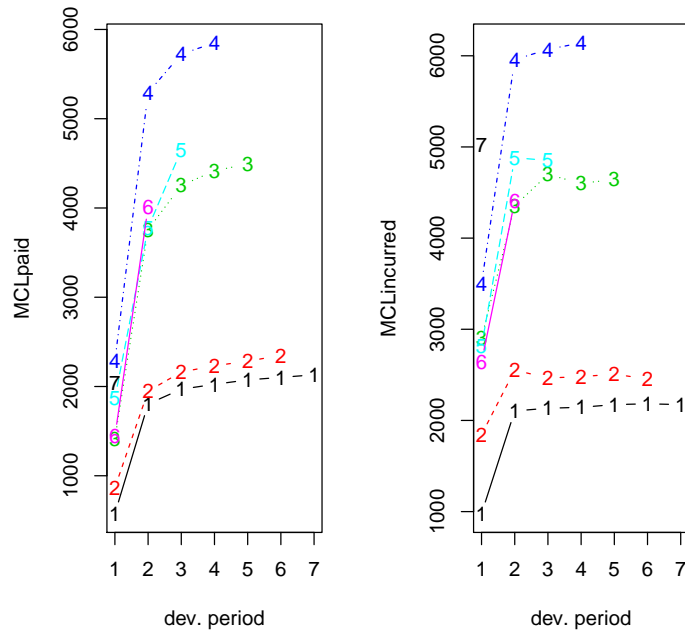
3.3 Multivariate chain-ladder

3.4 Clark's methods

3.4.1 Clark's Cap Cod method

3.4.2 Clark's LDF method

3.5 Generalised linear model methods

4 Using `ChainLadder` with RExcel and SWord

The spreadsheet is located in the Excel folder of the package. The R command

```
R> system.file("Excel", package="ChainLadder")
```

will tell you the exact path to the directory. To use the spreadsheet you will need the RExcel-Add-in [BN07]. The package also provides an example SWord file, demonstrating how the the functions of the package can be integrated into a MS Word file via SWord [BN07]. Again you find the Word file via the command:

```
R> system.file("SWord", package="ChainLadder")
```

The package comes with several demos to provide you with an overview of the package functionality, see

```
R> demo(package="ChainLadder")
```

5 Further resources

Other useful documents and resources to get started with R in the context of actuarial work:

- Introduction to R for Actuaries [DS06].

- An Actuarial Toolkit [MSH⁺06].

- Actuar package vignettes: http://cran.r-project.org/web/packages/actuar/index.html

- Mailing list R-SIG-insurance[6]: Special Interest Group on using R in actuarial science and insurance

5.1 Other insurance related R packages

Below is a list of further R packages in the context of insurance. The list is by no-means complete, and the CRAN Task Views *'Emperical Finance'* and *Probability Distributions* will provide links to additional resources. Please feel free to contact us with items to be added to the list.

- `cplm`: Monte Carlo EM algorithms and Bayesian methods for fitting Tweedie compound Poisson linear models [Zha11].

- `lossDev`: A Bayesian time series loss development model. Features include skewed-t distribution with time-varying scale parameter, Reversible Jump MCMC for determining the functional form of the consumption path, and a structural break in this path [LS11].

- `favir`: Formatted Actuarial Vignettes in R. FAViR lowers the learning curve of the R environment. It is a series of peer-reviewed Sweave papers that use a consistent style [Esc11].

- `actuar`: Loss distributions modelling, risk theory (including ruin theory), simulation of compound hierarchical models and credibility theory [DGP08].

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[6]https://stat.ethz.ch/mailman/listinfo/r-sig-insurance

- `fitdistrplus`: Help to fit of a parametric distribution to non-censored or censored data [DMPDD10].

- `mondate`: R packackge to keep track of dates in terms of months [Mur11].

- `lifecontingencies`: Package to perform actuarial evaluation of life contingencies [Spe11].

5.2 Presentations

Over the years the contributors of the `ChainLadder` package have given numerous presentations and most of those are still available online:

- Bayesian Hierarchical Models in Property-Casualty Insurance, Wayne Zhang, 2011

- ChainLadder at the Predictive Modelling Seminar, Institute of Actuaries, November 2010, Markus Gesmann, 2011

- Reserve variability calculations, CAS spring meeting, San Diego, Jimmy Curcio Jr., Markus Gesmann and Wayne Zhang, 2010

- The ChainLadder package, working with databases and MS Office interfaces, presentation at the "R you ready?" workshop , Institute of Actuaries, Markus Gesmann, 2009

- The ChainLadder package, London R user group meeting, Markus Gesmann, 2009

- Introduction to R, Loss Reserving with R, Stochastic Reserving and Modelling Seminar, Institute of Actuaries, Markus Gesmann, 2008

- Loss Reserving with R , CAS meeting, Vincent Goulet, Markus Gesmann and Daniel Murphy, 2008

- The ChainLadder package R-user conference Dortmund, Markus Gesmann, 2008

5.3 Further reading

Other papers and presentation which cited `ChainLadder` : [Orr07], [Nic09], [Zha10], [MNNV10], [Sch10], [MNV10], [Esc11], [Spe11]

6 Training and consultancy

Please contact us if you would like to discuss tailored training or consultancy.

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