

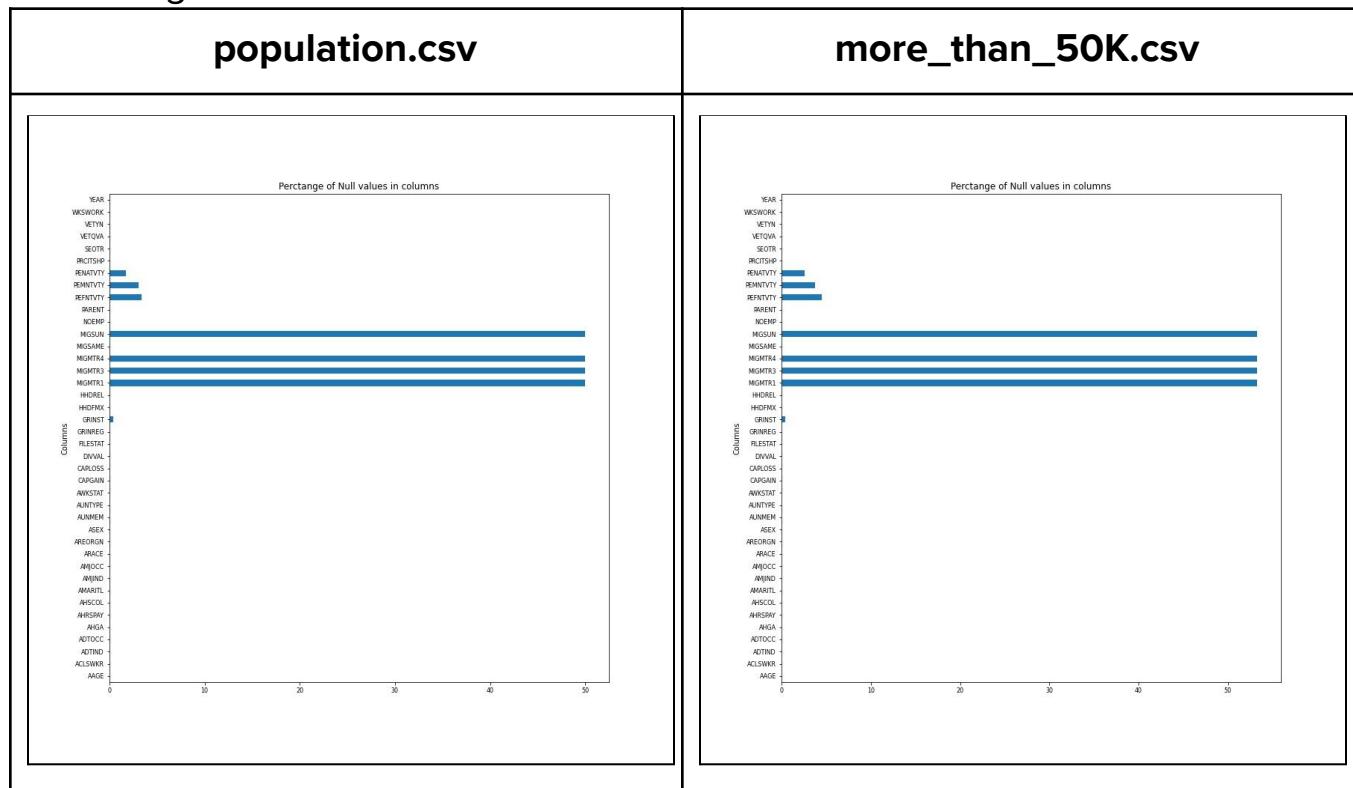
# Machine Learning : CSE343

## Assignment - 3

- Abhimanyu Gupta  
(2019226)

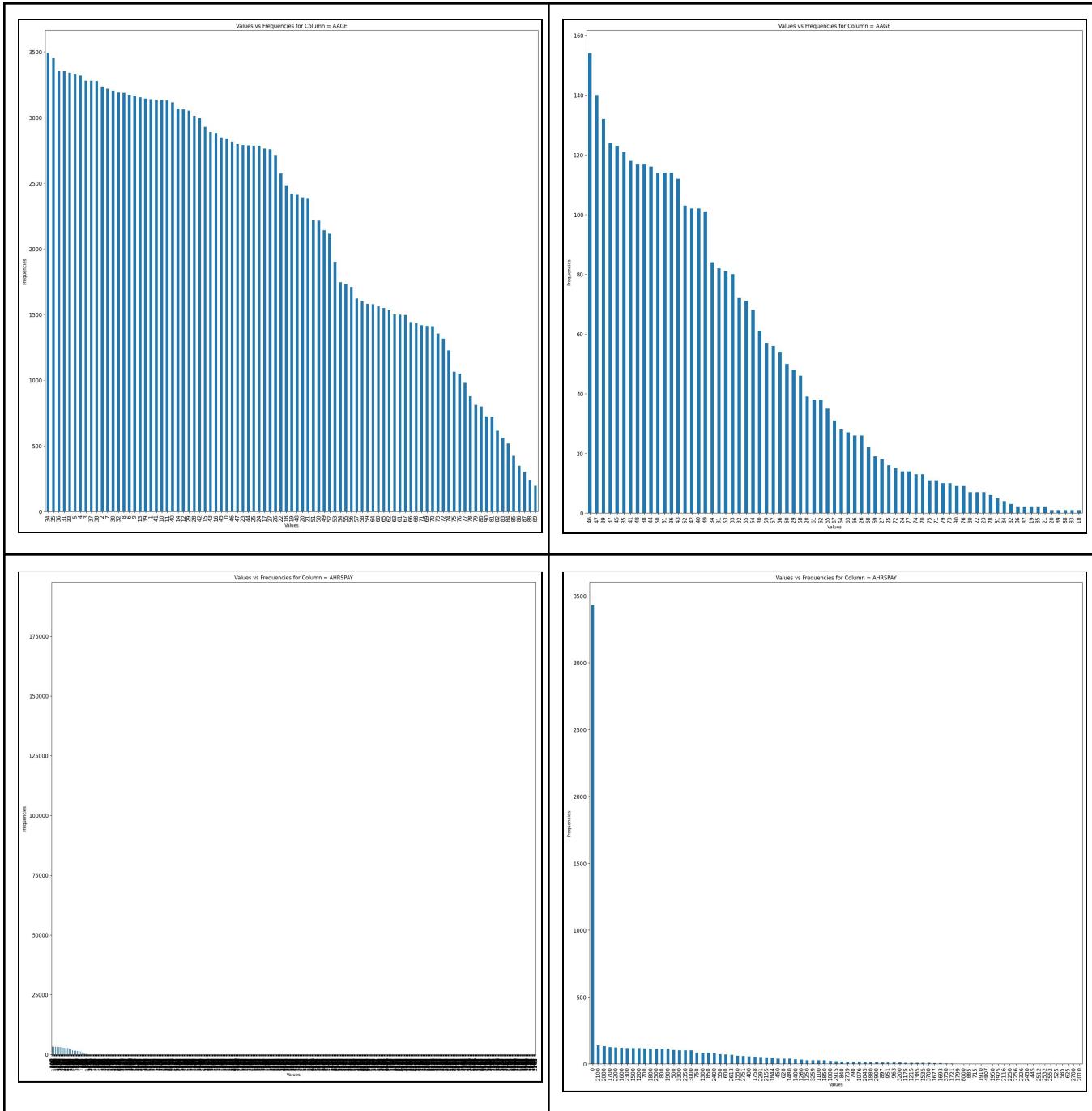
### Plots:

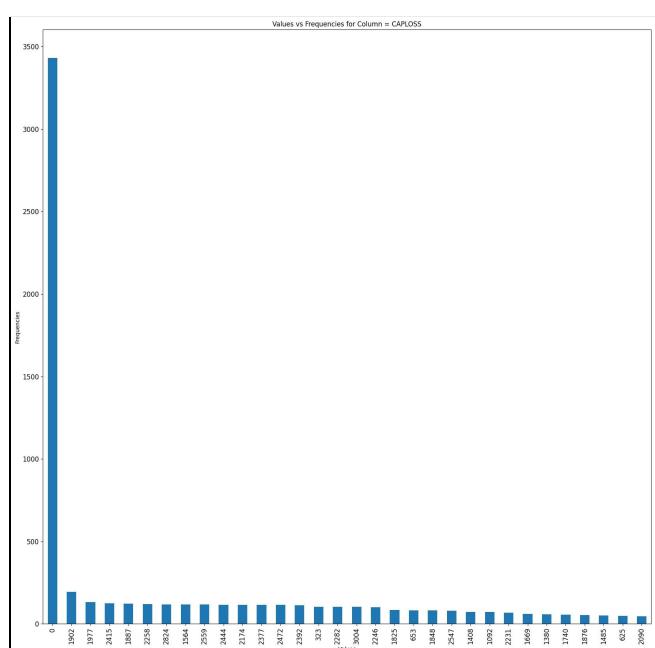
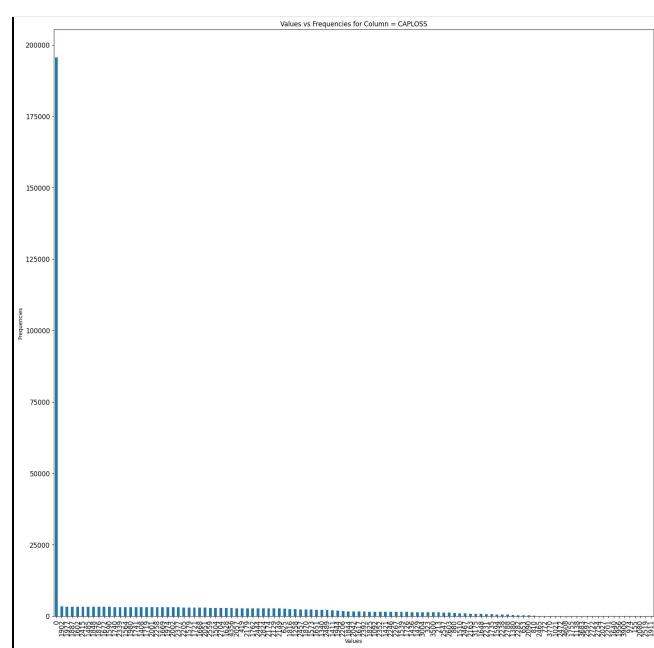
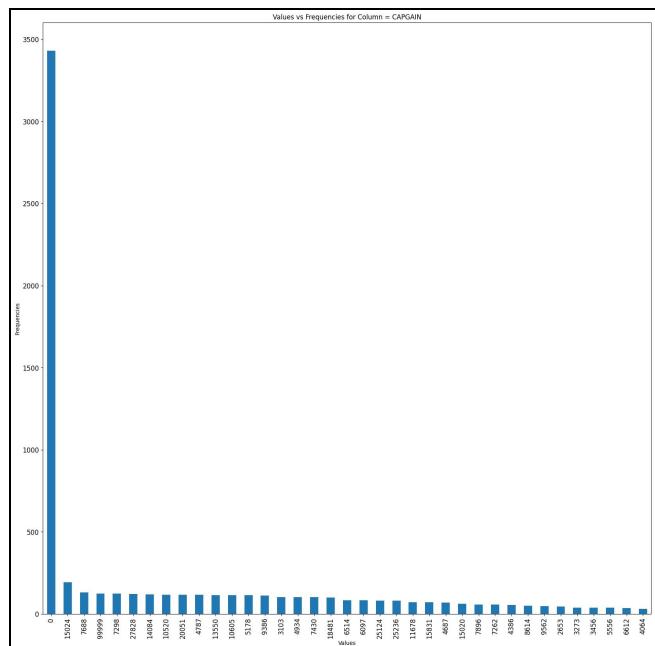
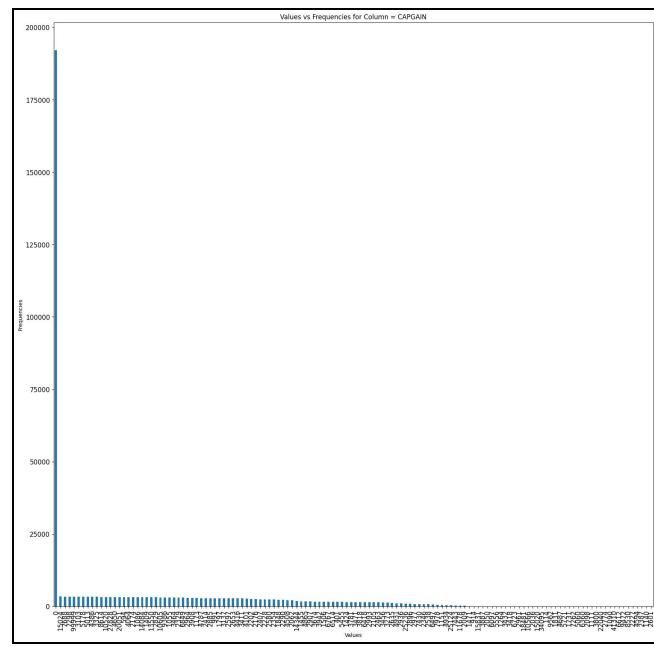
Percentage of Nulls

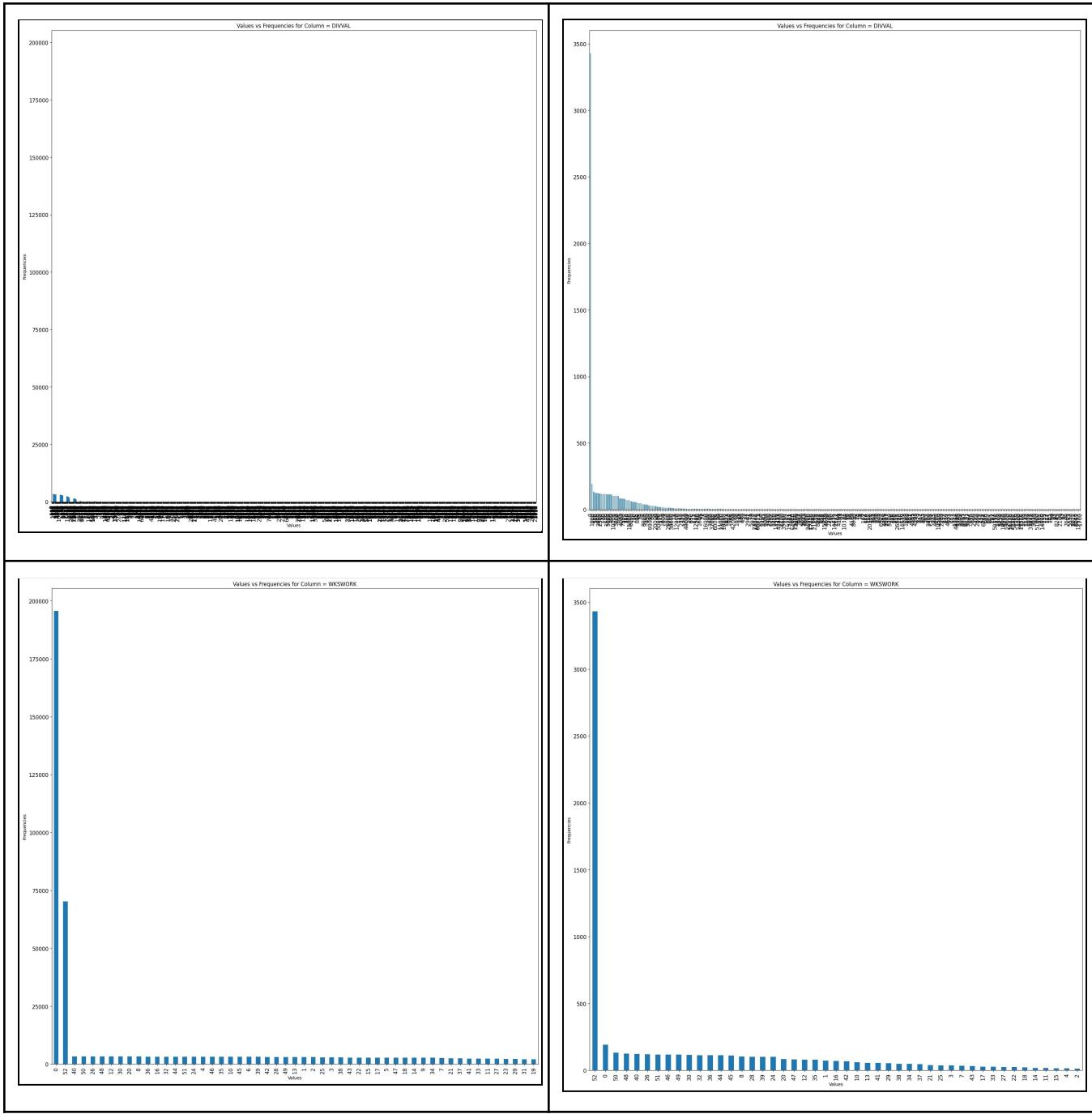


Numerical Data





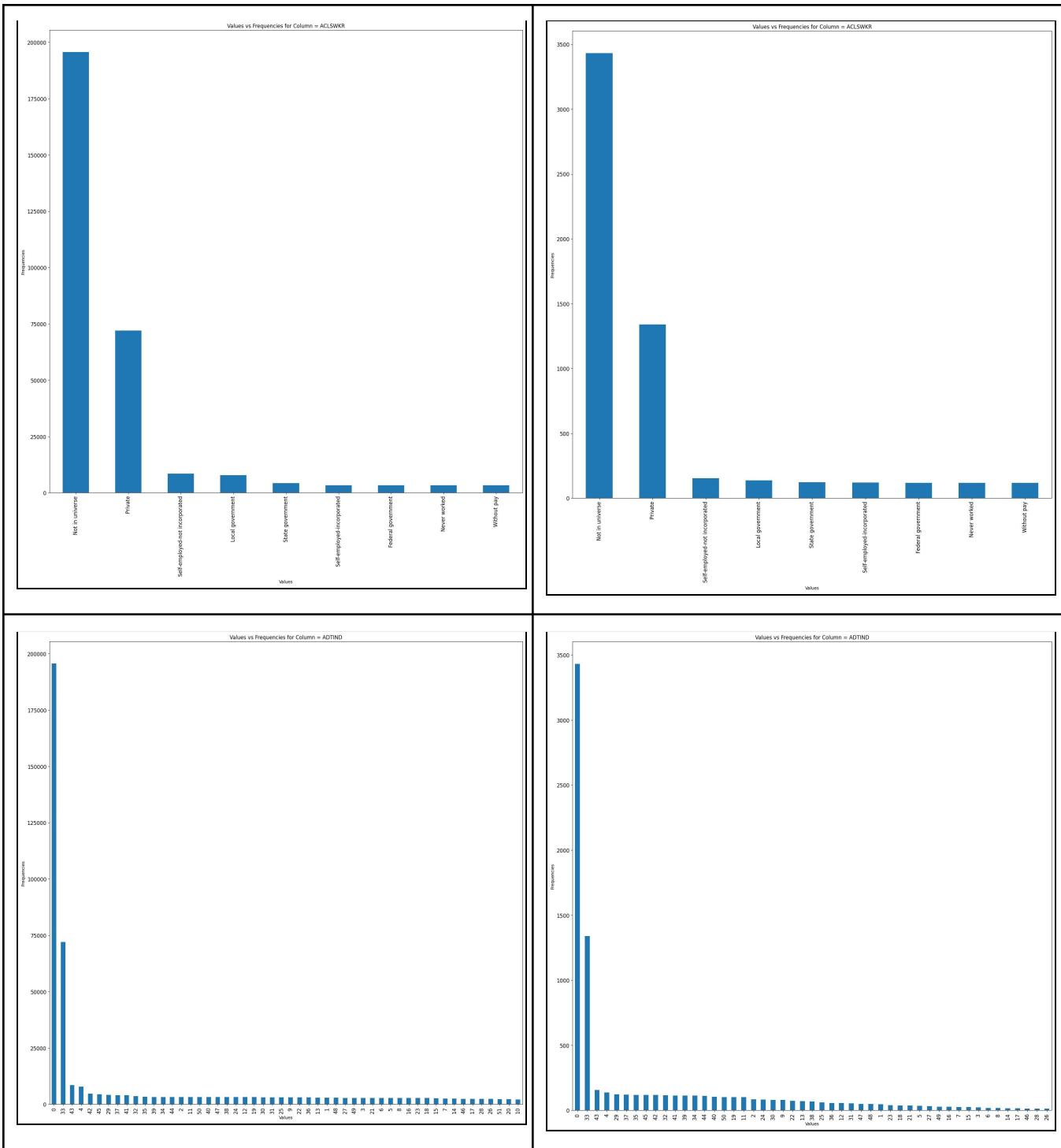


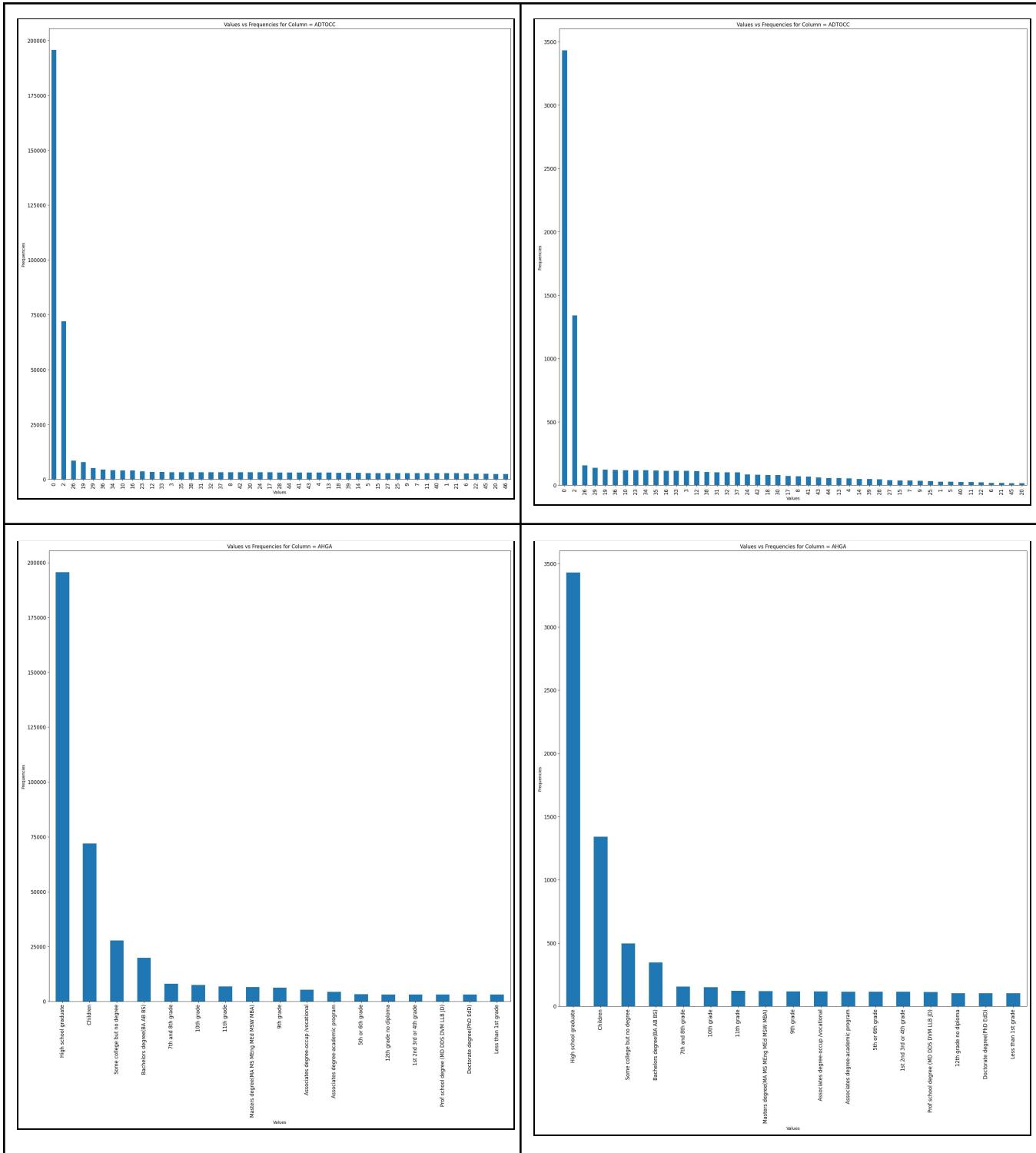


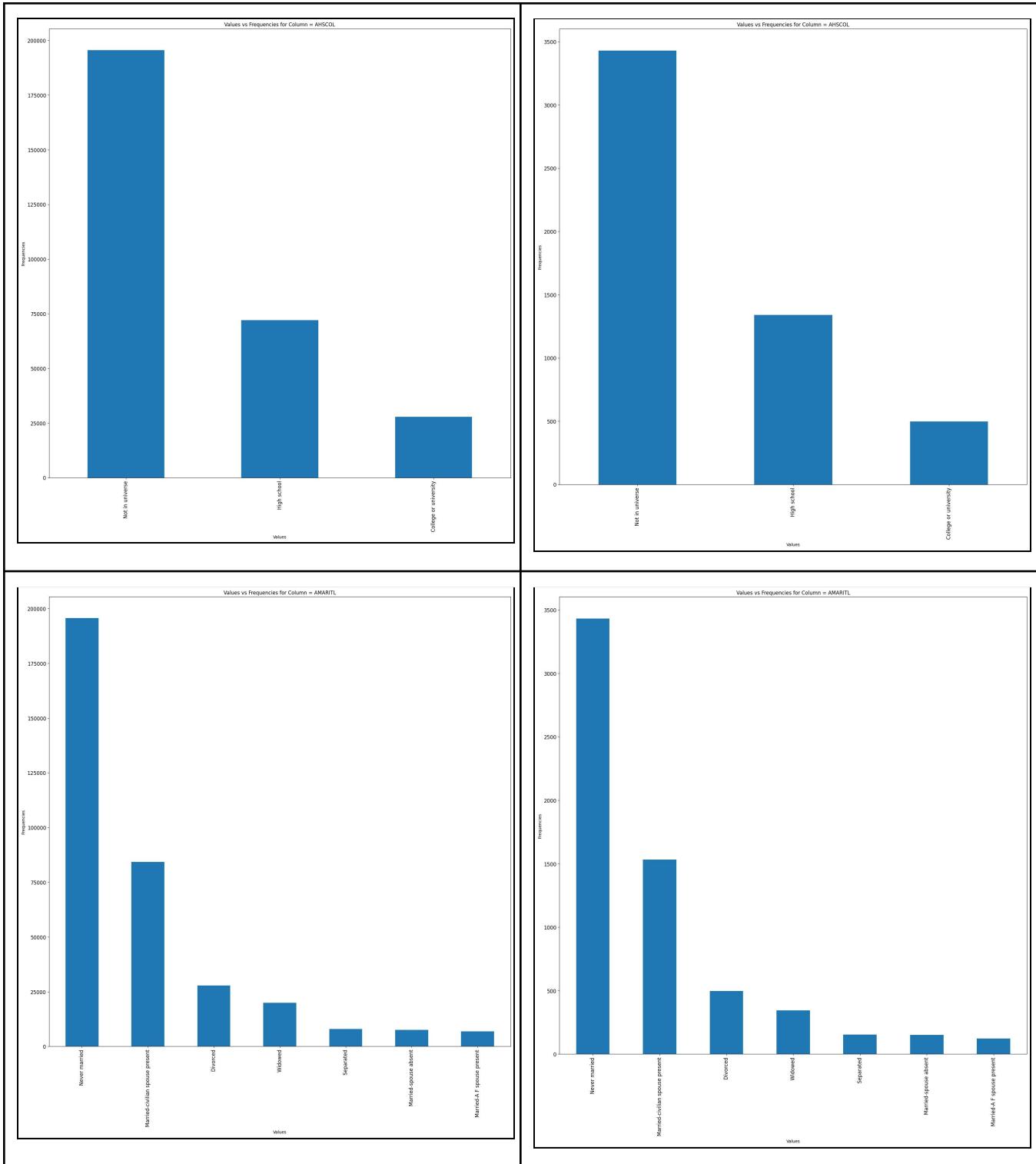
## Categorical Data

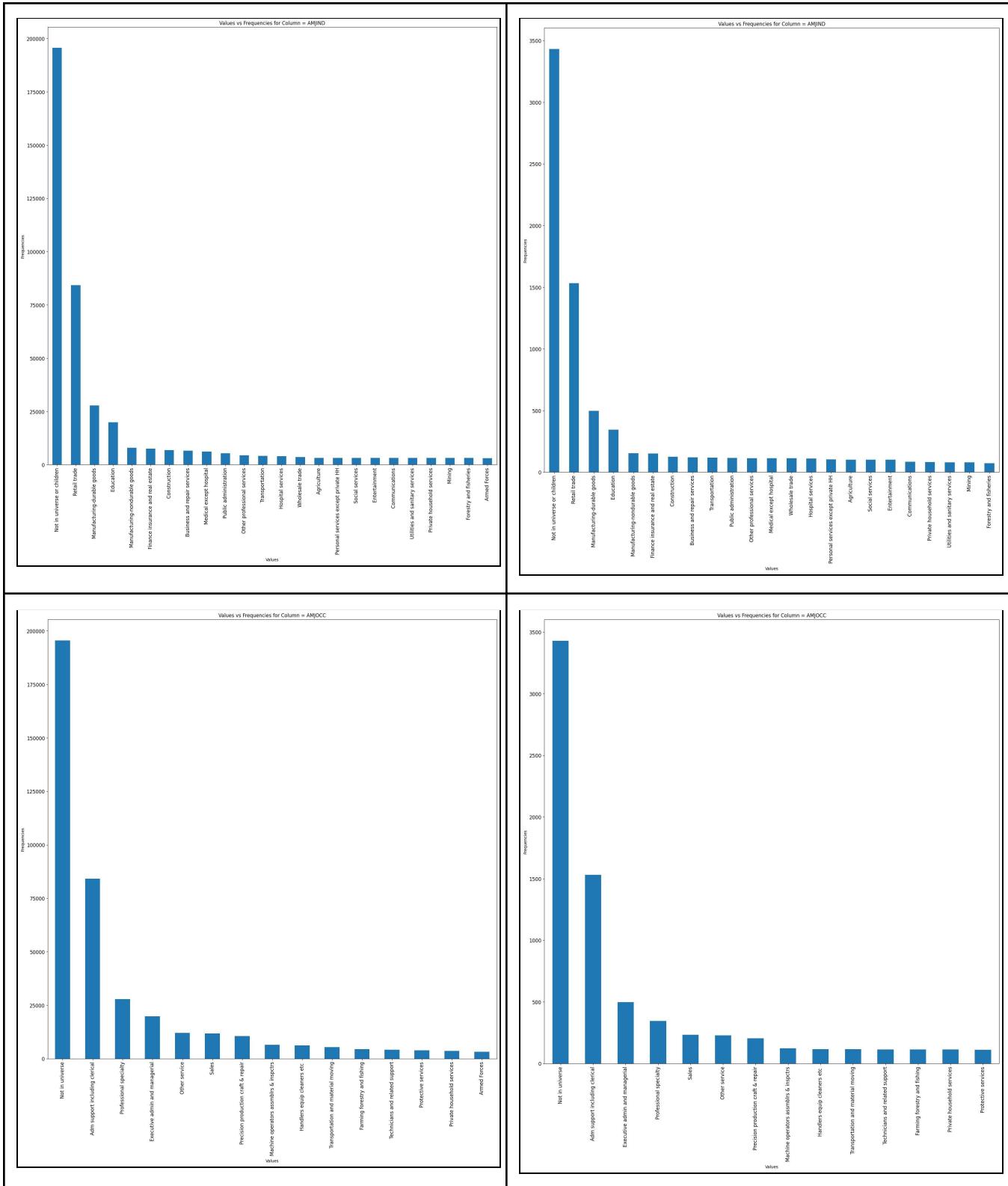
**population.csv**

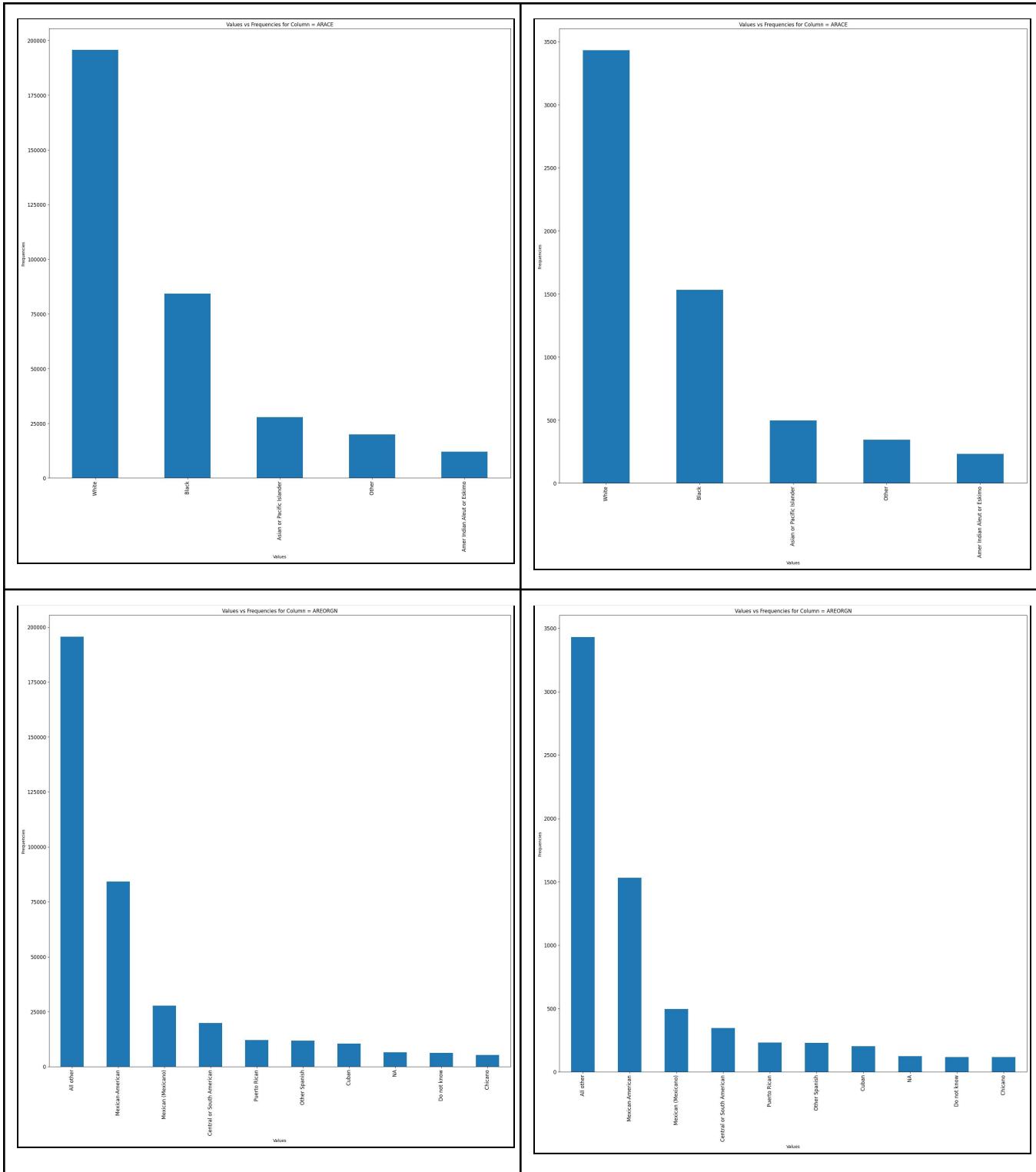
**more\_than\_50K.csv**

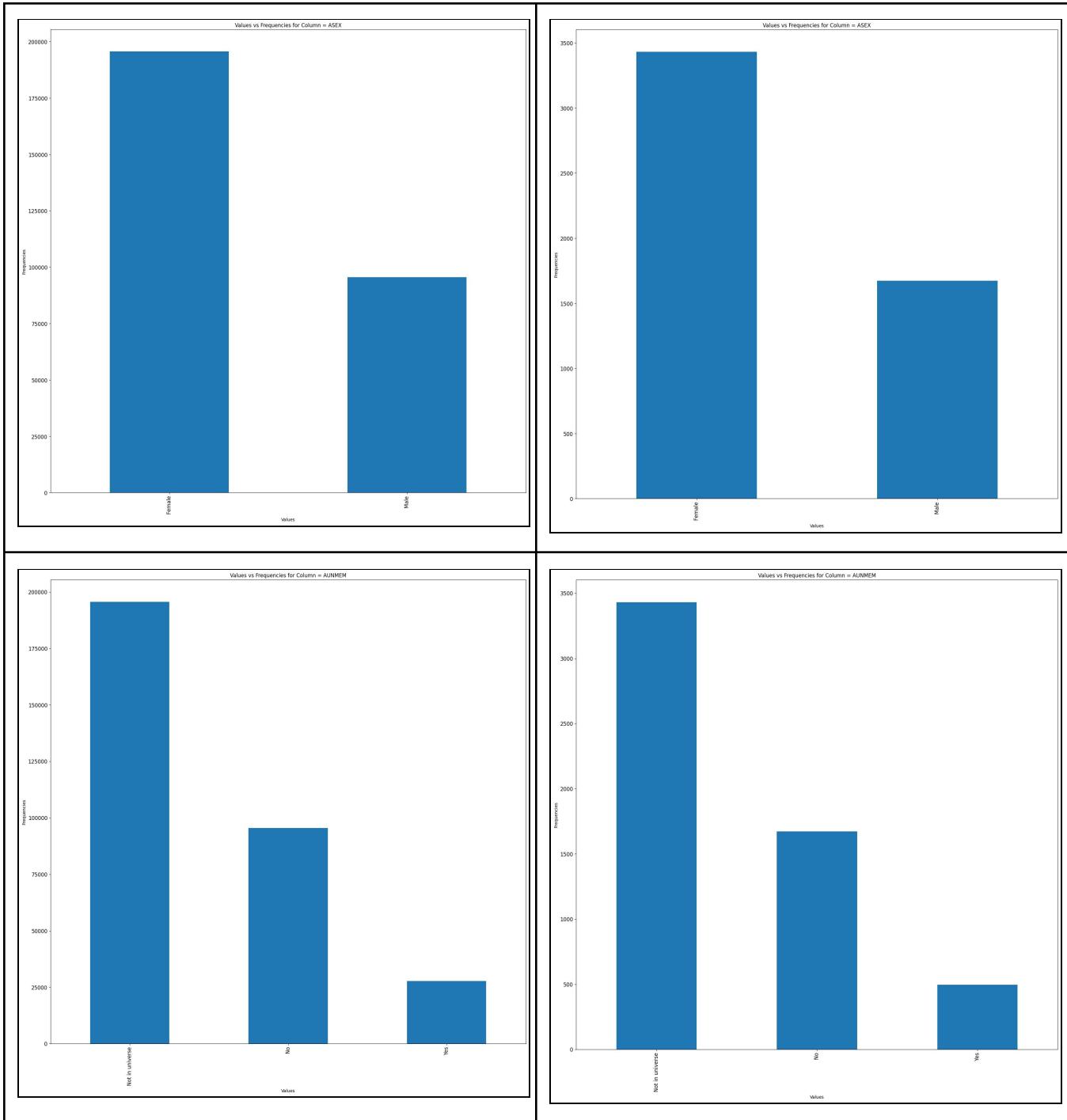


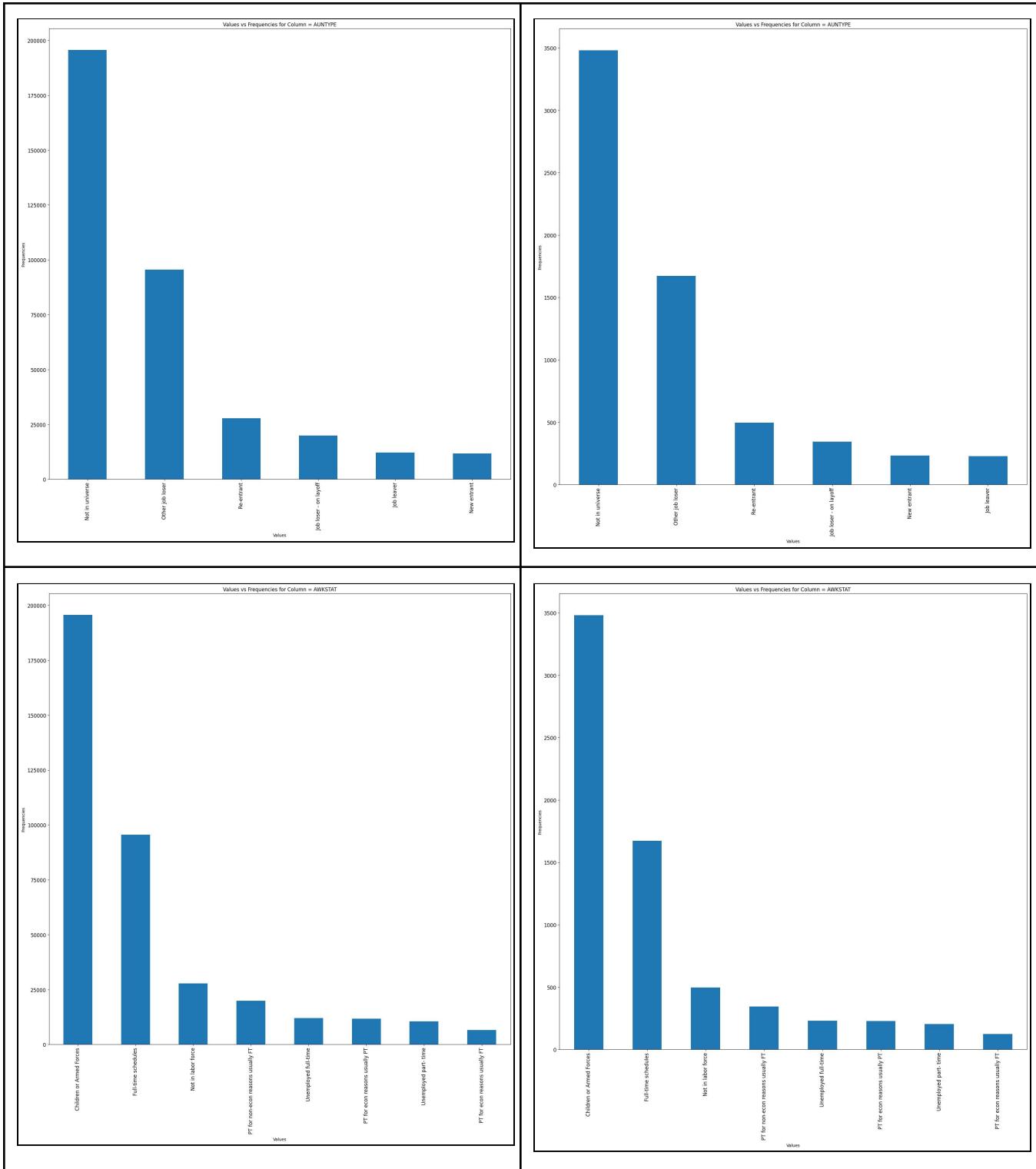


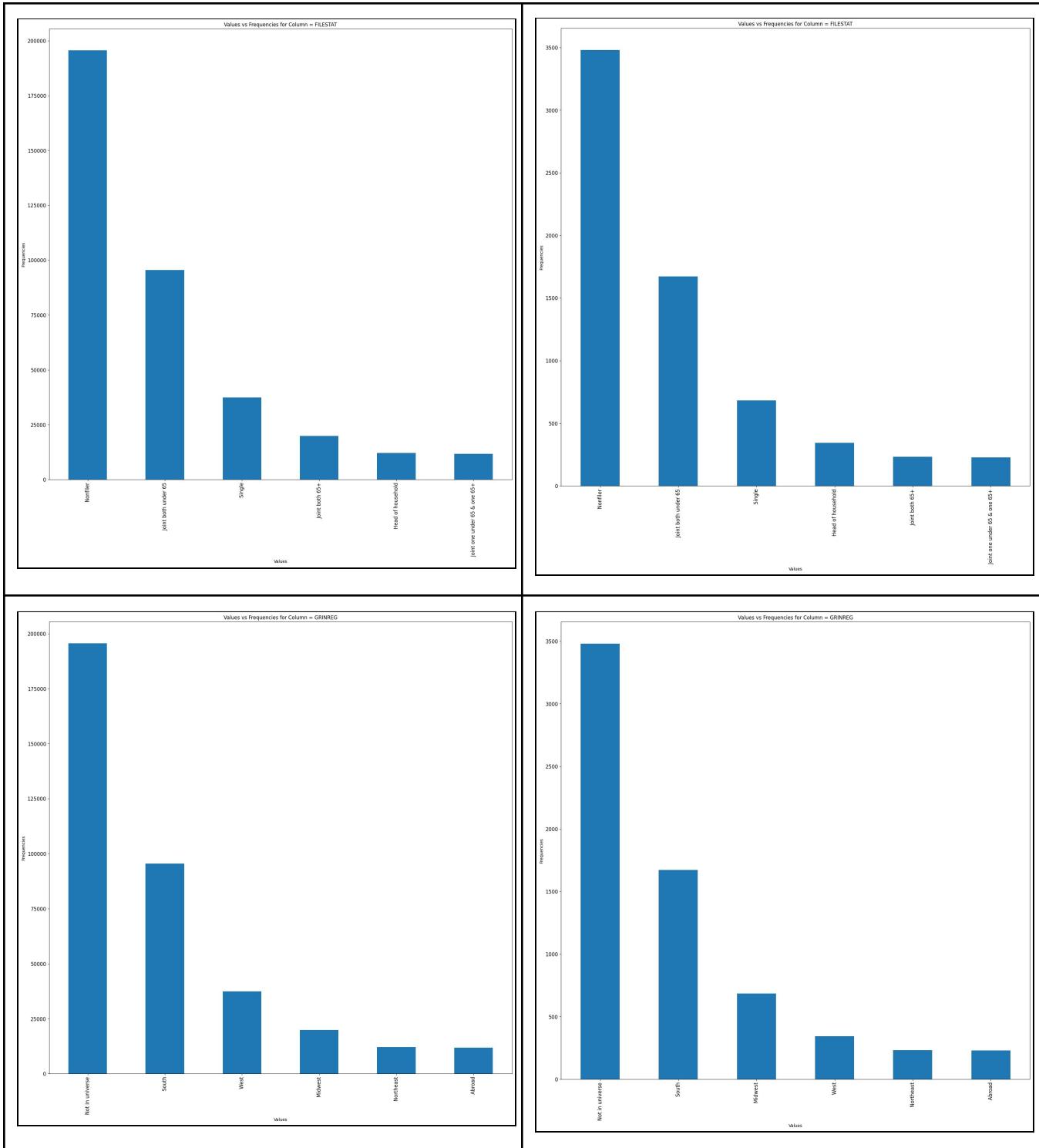


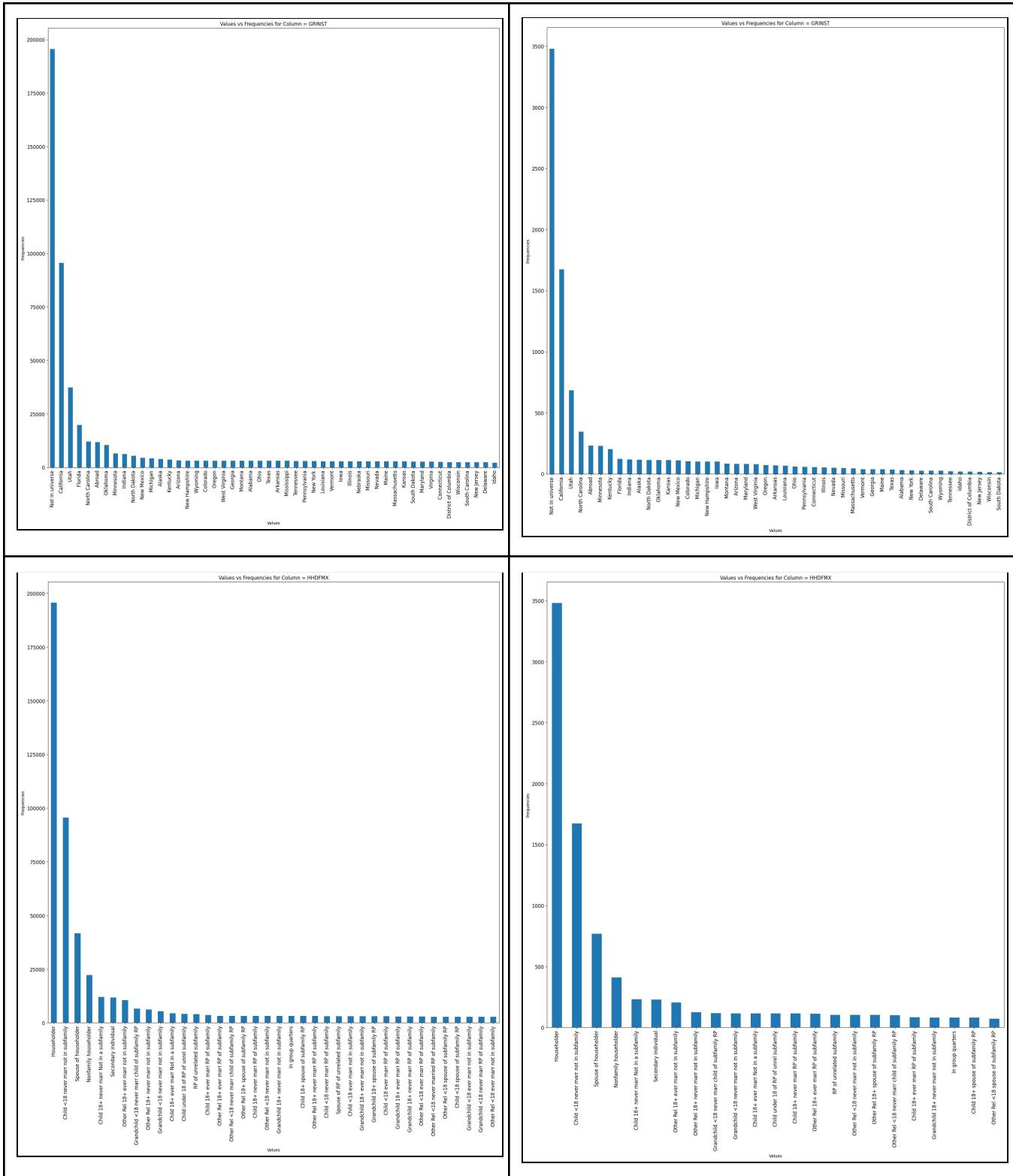


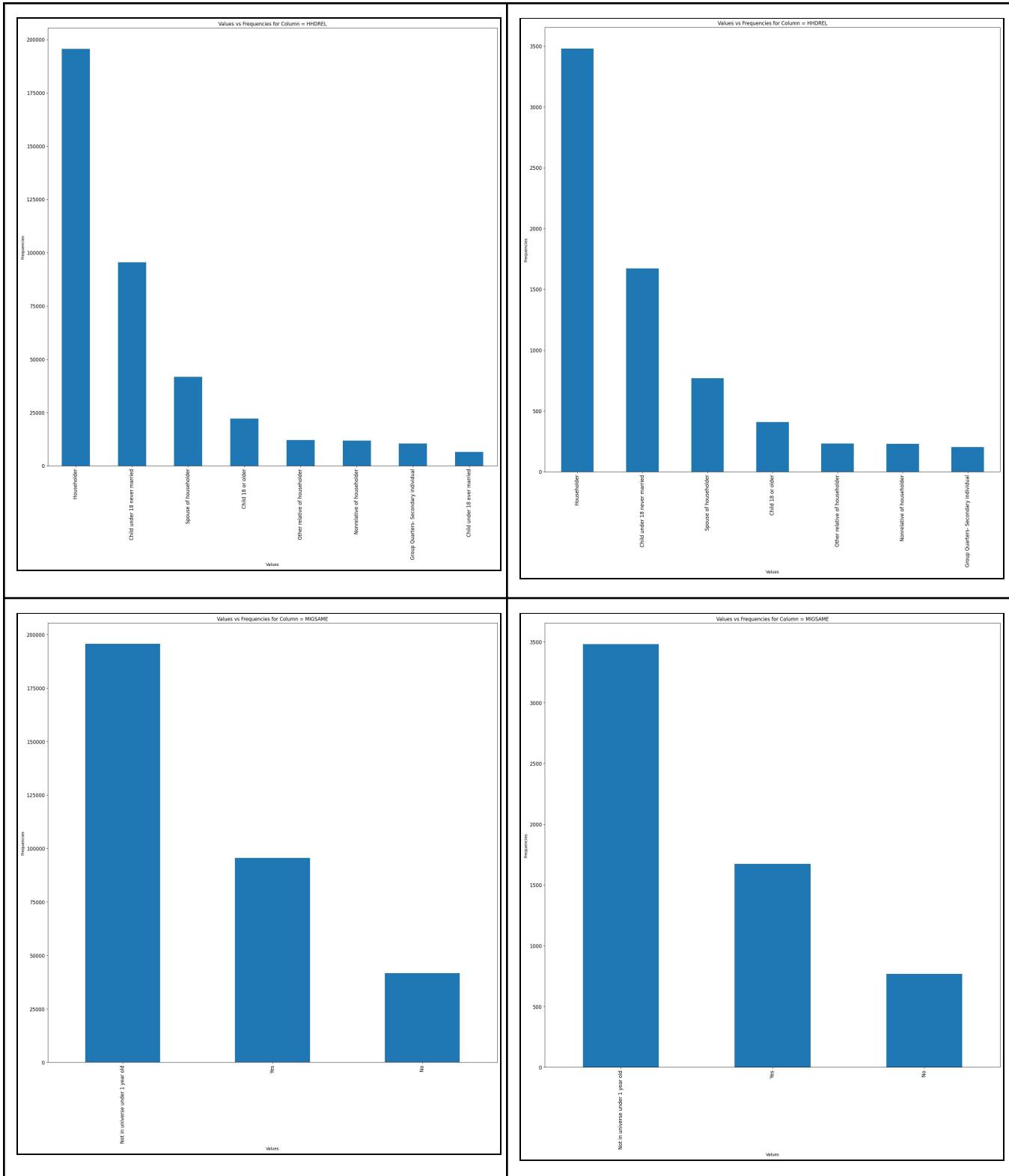


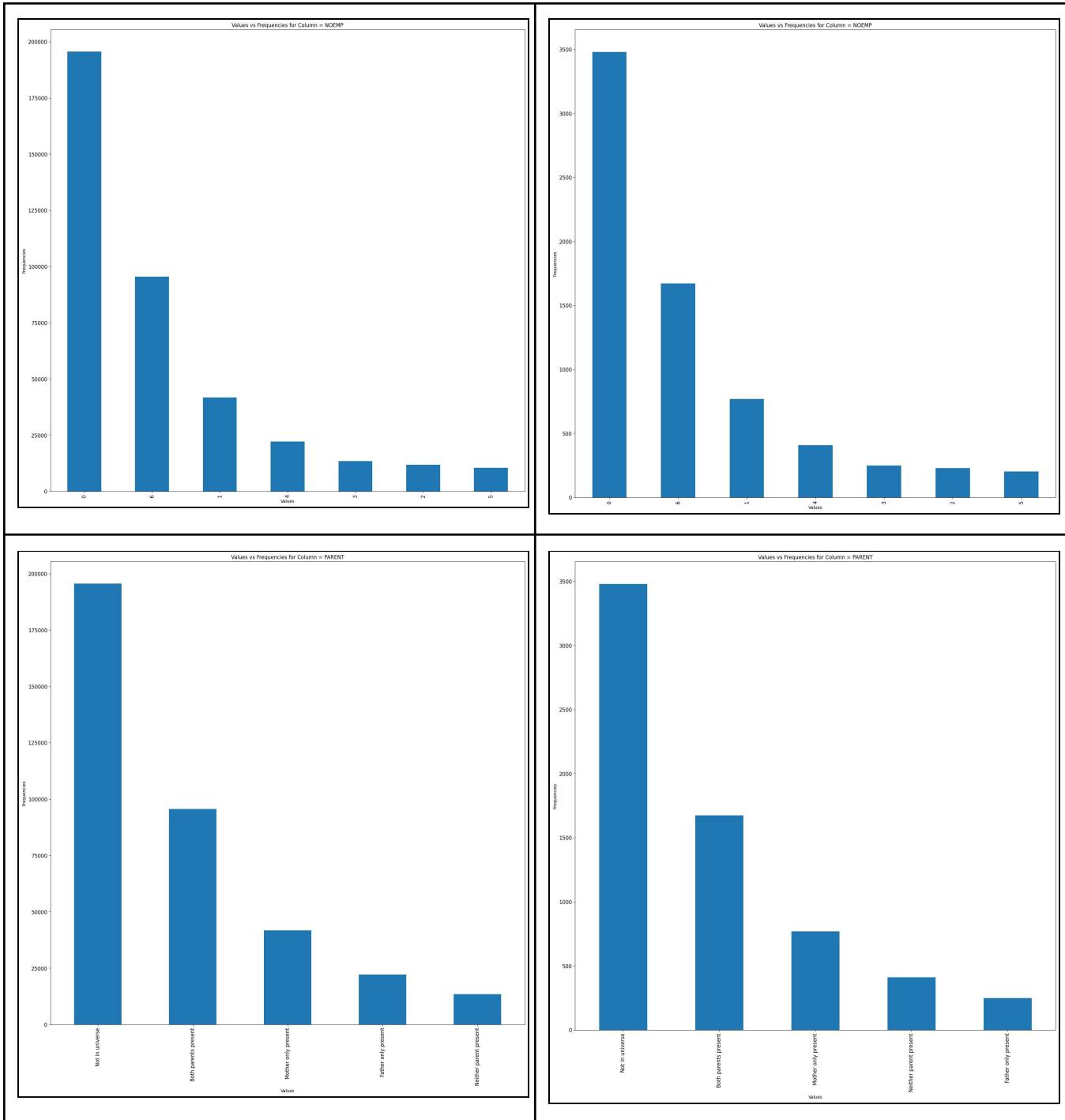


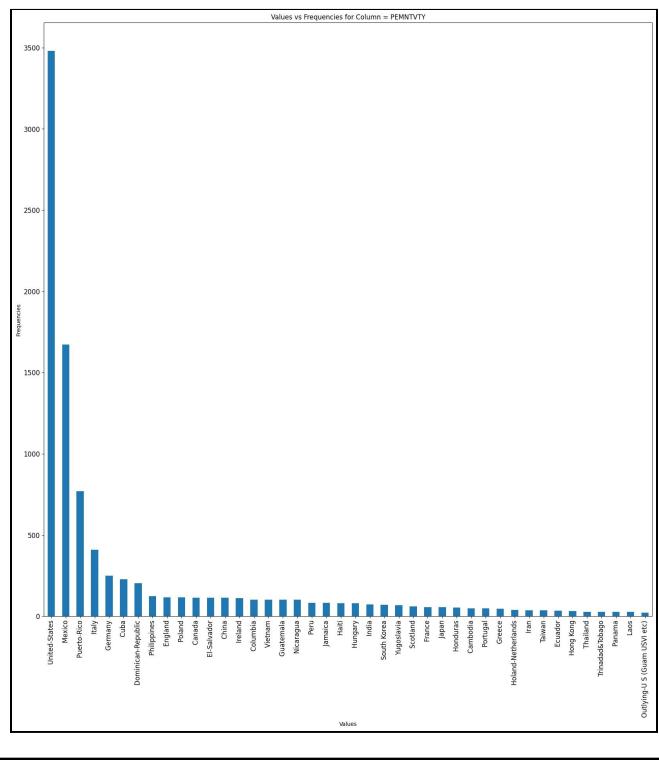
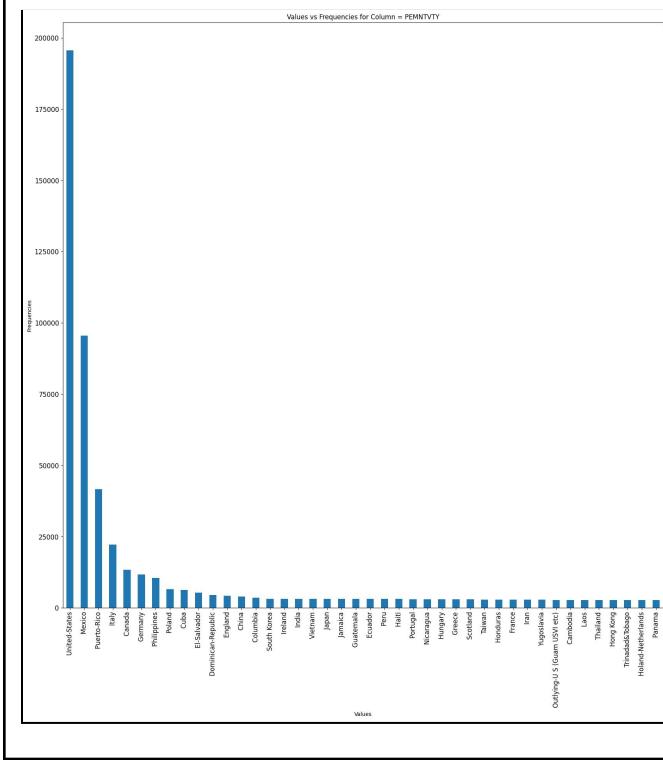
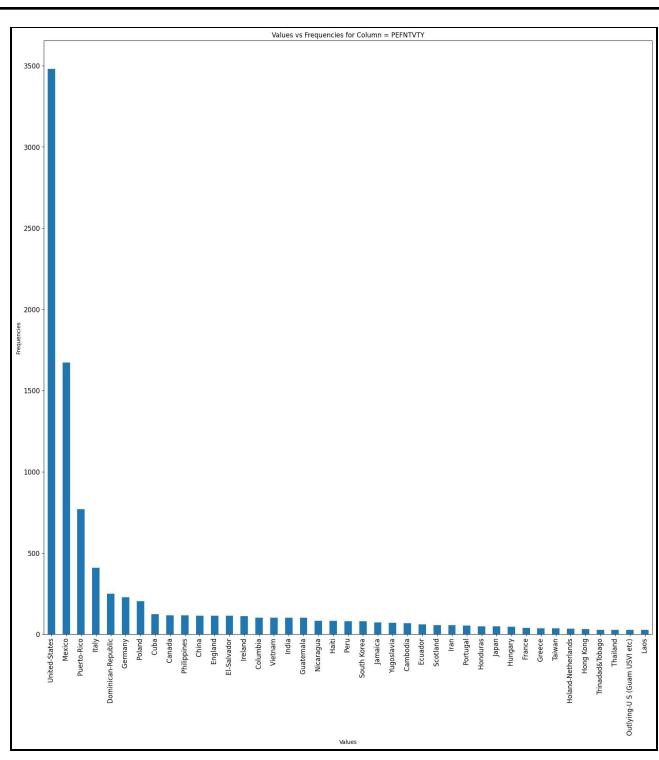
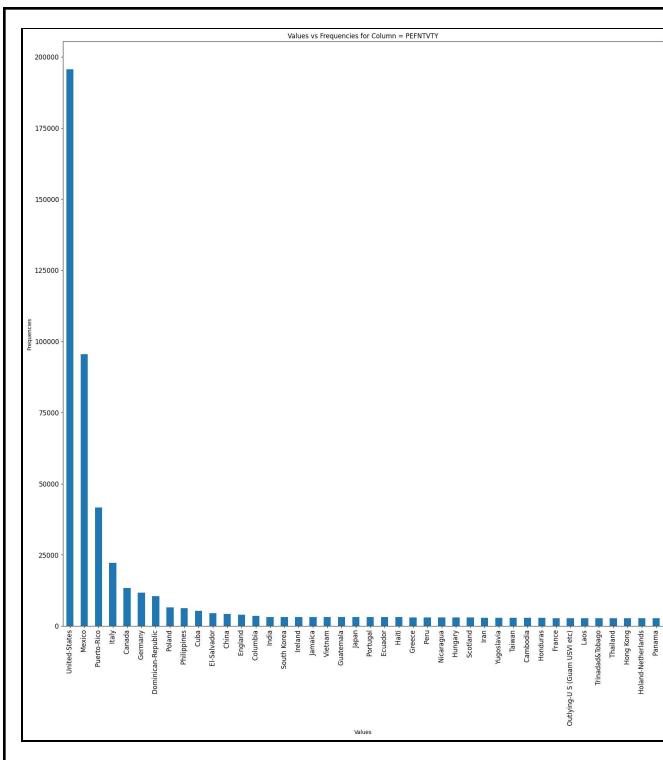


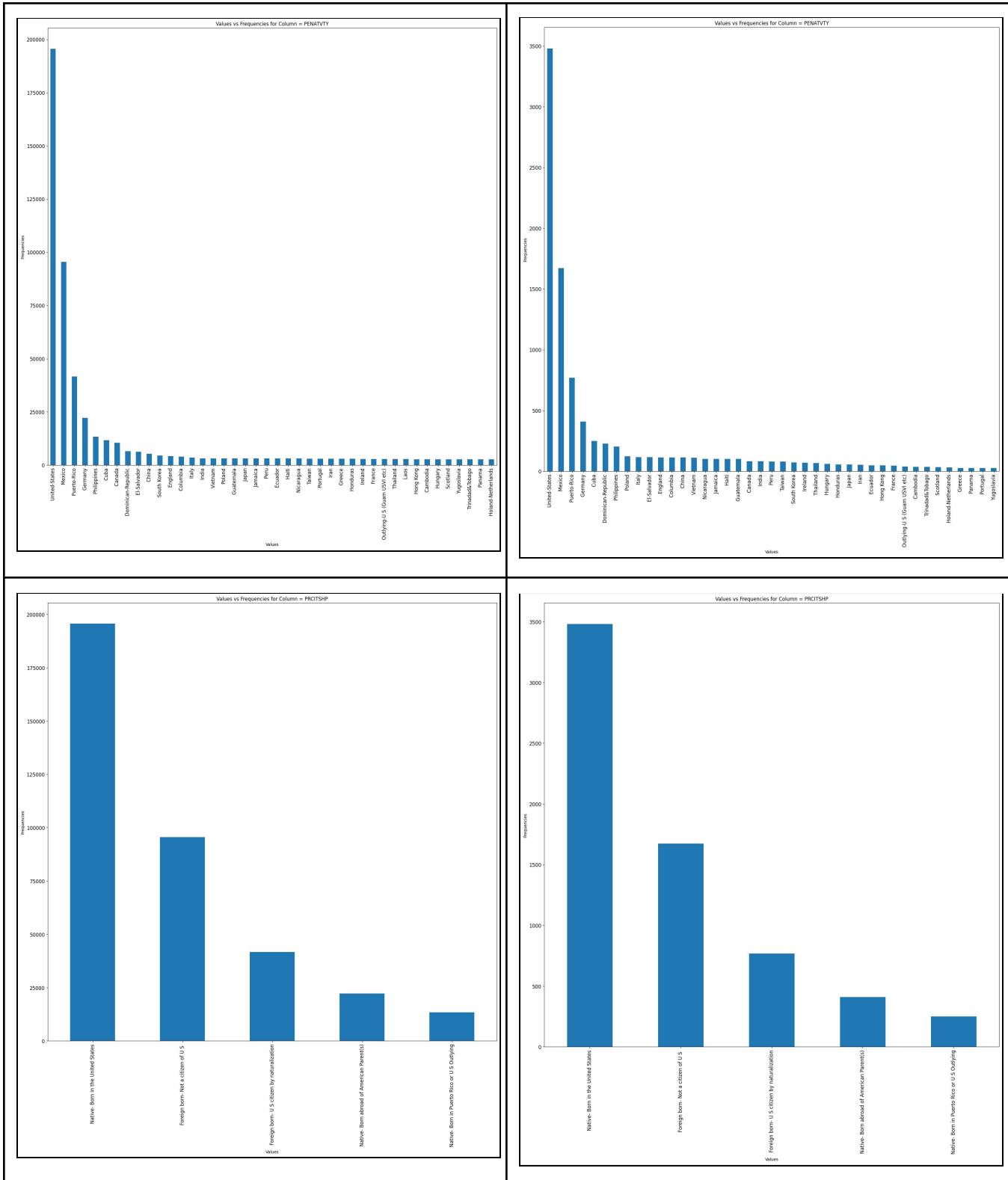


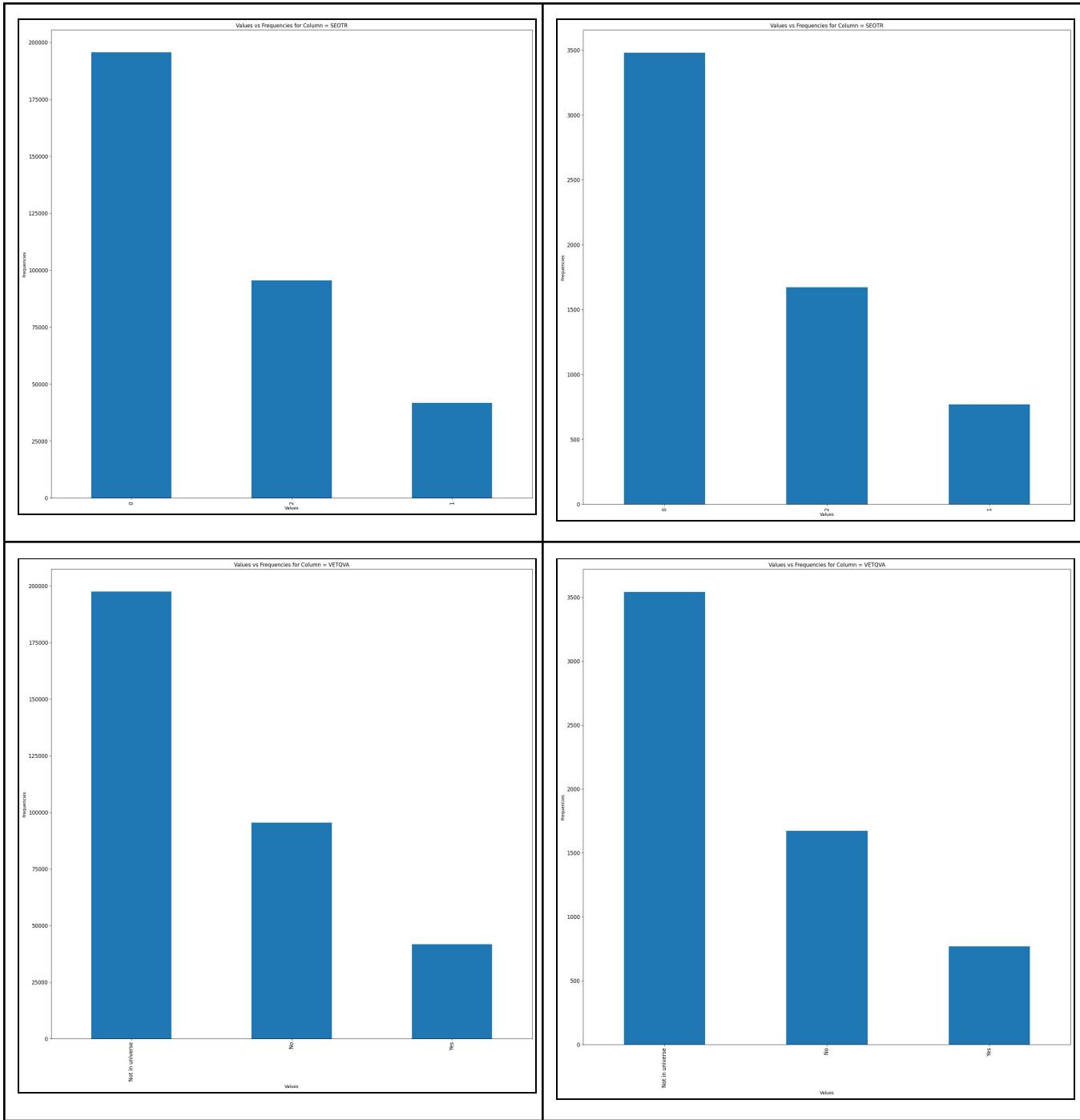


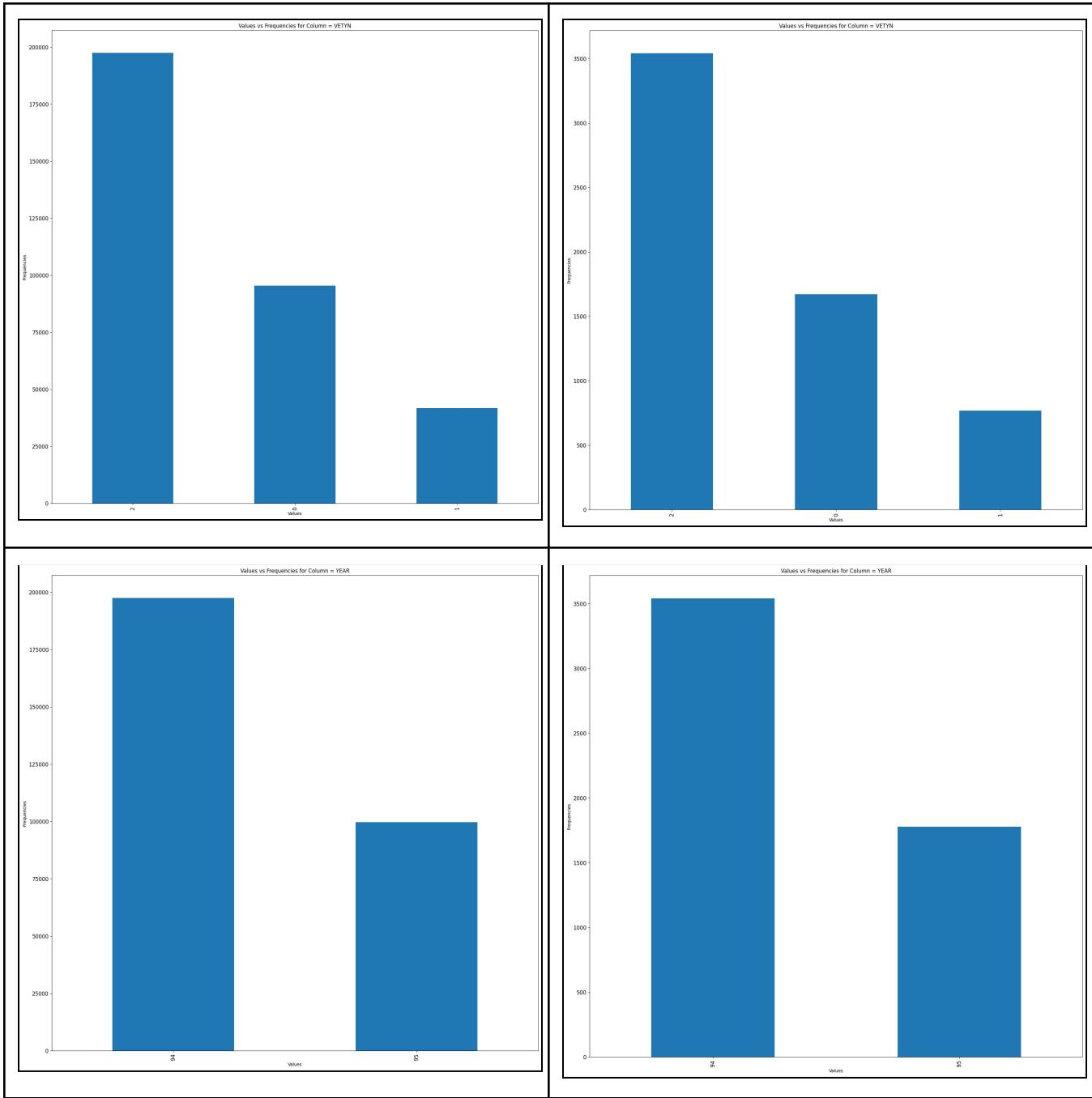








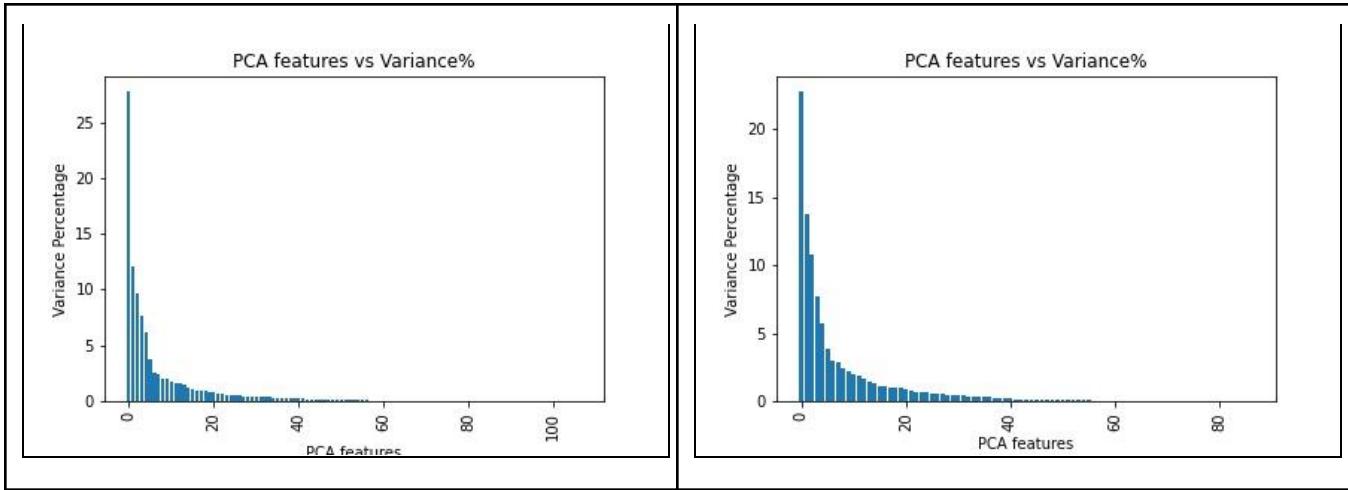




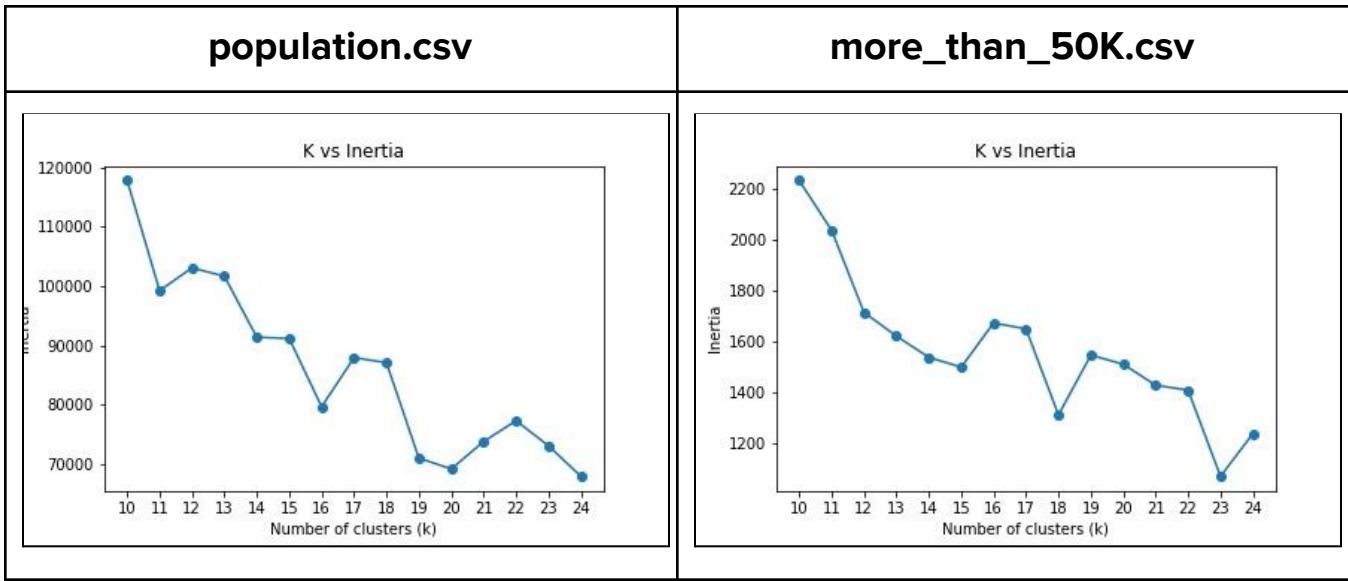
## PCA Features vs Variance Percentage

**population.csv**

**more\_than\_50K.csv**

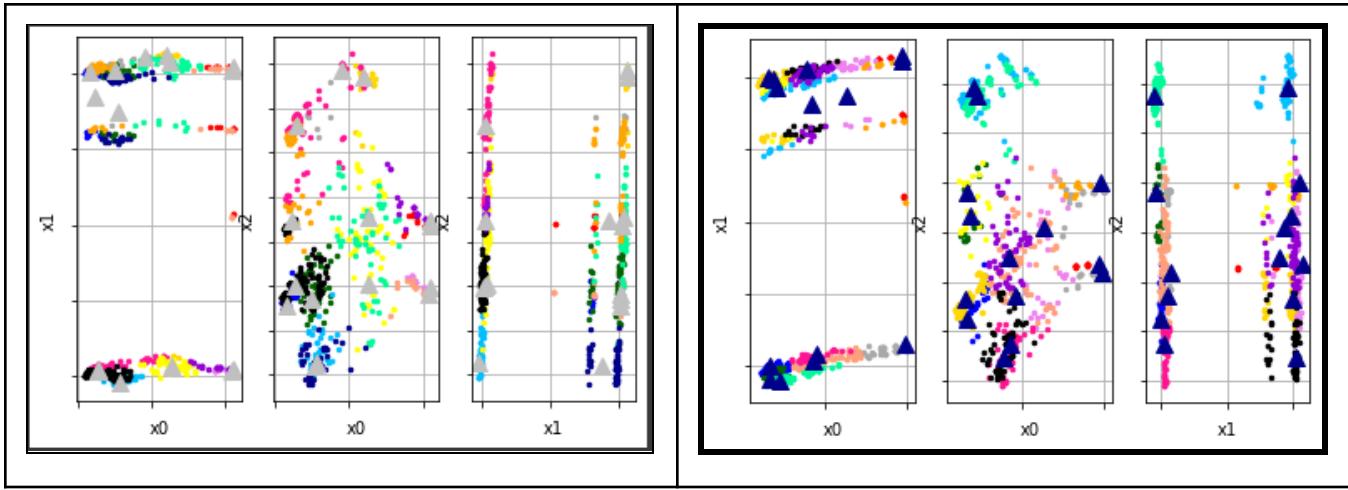


k vs Inertia for [10, 24]



Clusters for k = 20





**Question 6.2, 6.3, 6.4 :**

From the generated visuals over-represented clusters are

13 4 8 9 10 13 14 15 17 for General Population data. While all clusters are overrepresented in More than 50k data.

**Question 2 and 3 :**

## Question - 2

- a) Yes, we need a non-negativity constraint on  $\varepsilon$ .  
 To always hold the condition  $y^{(i)}(w^T x^{(i)} + b) \geq 1 - \varepsilon_i$ ,  
 $\varepsilon_i$  must be non-negative.  
 $\therefore \varepsilon_i$  should be defined as  $\max(\varepsilon_i, 0)$  to put  
 the non-negativity constraint on  $\varepsilon_i$ .

b) Lagrangian of above optimization problem -

$$L(w, b, \varepsilon, \alpha) = \frac{1}{2} \|w\|^2 + \frac{C}{2} \sum_{i=1}^m \varepsilon_i^2 + \sum \alpha_i (1 - y_i (w^T x_i + b) - \varepsilon_i)$$

c) Required Dual is -

Using Karush Kuhn Tucker condition,

$$\frac{\partial L}{\partial w} = 0$$

$$\Rightarrow w - \sum_{i=1}^m \alpha_i y_i x_i = 0$$

$$\Rightarrow \boxed{w = \sum_{i=1}^m \alpha_i y_i x_i} \quad \dots \textcircled{1}$$

~~$$\frac{\partial L}{\partial b} = 0$$~~

$$\Rightarrow \boxed{\sum \alpha_i y_i = 0} \quad \dots \textcircled{2}$$

$$\frac{\partial L}{\partial \varepsilon} = 0$$

$$\Rightarrow C\varepsilon - \alpha_i = 0$$

$$\Rightarrow \boxed{\varepsilon = \frac{\alpha_i}{C}} \quad \dots \textcircled{3}$$

From ①, ② & ③

Dual is

$$\max \sum_{i=1}^m \alpha_i - \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^m \alpha_i \alpha_j y_i y_j x^{(i)T} x^{(j)}$$

$$\text{satisfying } \sum \alpha_i y_i = 0 \text{ and } 0 \leq \alpha_i \leq C$$

### Question 3

$$a) f(x) = \sum_{i=1}^m \alpha_i y^{(i)} K(x^{(i)}, x) + b$$

For  $\alpha_i = 1$  &  $b = 0$  for  $1 \leq i \leq m$ , we have

$$\begin{aligned}
 |f(x^i) - y^i| &= \left| \sum_{j=1}^m y^j K(x^j, x^i) - y^i \right| \\
 &= \left| \sum_{j=1}^m y^j e^{-\frac{\|x^j - x^i\|^2}{\tau^2}} - y^i \right| \quad \left[ \text{Given } K(x, z) = e^{-\frac{\|x-z\|^2}{\tau^2}} \right] \\
 &= \left| y^i + \sum_{j \neq i} y^j e^{-\frac{\|x^j - x^i\|^2}{\tau^2}} - y^i \right| \\
 &= \left| \sum_{j \neq i} y^j e^{-\frac{\|x^j - x^i\|^2}{\tau^2}} \right| \\
 &\leq \sum_{j \neq i} |y^j| e^{-\frac{\|x^j - x^i\|^2}{\tau^2}} \quad [\text{As } |\sum a| \leq \sum |a|] \\
 &= \sum_{j \neq i} |y^j| e^{-\frac{\|x^j - x^i\|^2}{\tau^2}}
 \end{aligned}$$

$$= \sum_{j \neq i} e^{-\frac{\varepsilon^2}{\varepsilon^2}} \quad [ |y_j| e^{-\|x_j - x_i\|^2} = e^{-\varepsilon^2} ]$$

$$= (m-1) e^{-\frac{\varepsilon^2}{\varepsilon^2}}$$

Now  $(m-1) e^{-\frac{\varepsilon^2}{\varepsilon^2}} < 1$

$$\Rightarrow -\cancel{\frac{\varepsilon^2}{\varepsilon^2}} \ln \left[ (m-1), e^{-\frac{\varepsilon^2}{\varepsilon^2}} \right] < \ln 1$$

$$\Rightarrow \ln(m-1) - \left(\frac{\varepsilon}{\varepsilon}\right)^2 \ln e < 0$$

$$\Rightarrow \boxed{z < \frac{\varepsilon}{\ln(m-1)}}$$

b) To have a zero training error we would need ~~all~~ relative weights to be zero. Relative weights are controlled by ~~all~~  $\alpha$  and  $\|w\|$ .

Now, weights can never be zero  $[x^T w_i = 0 \text{ does not make sense}]$

In turn the relative weight  $\alpha$  and  $\|w\|$  won't be zero, i.e.

we can have close to zero training error but not zero exactly.