# Contents

	.T.								
一切的开									3
宏定》	义		 	 	 	 	 	 	 . 3
快速	卖		 	 	 	 	 	 	 . 3
对拍			 	 	 	 	 	 	 . 4
为什么	么 C++ 不自带)	文个?							. 5
/3/1/2	70 11117	_ ,	 	 	 	 	 	 	
数据结构	i								5
ST表									•
	对								
.,,,,,,,,,			 	 	 	 	 	 	 
***	夏杂度线段树								
持久化	化线段树		 	 	 	 	 	 	 . 8
K-D 7	Гree		 	 	 	 	 	 	 . 9
树状数	数组		 	 	 	 	 	 	 . 11
	对								
	-								
	对								
	р								
Treap	p-序列		 	 	 	 	 	 	 . 18
可回泡	衮并查集		 	 	 	 	 	 	 . 21
舞蹈镇	连		 	 	 	 	 	 	 . 22
CDO	分治								24
	表								
	尔树								
Trie			 	 	 	 	 	 	 . 26
pb_d	s		 	 	 	 	 	 	 . 27
- Link-	-Cut Tree		 	 	 	 	 	 	 . 28
草队			 	 	 	 	 	 	 . 29
2017			 	 	 	 	 	 	 /
数学									31
	云算								
	生筛								
	min_25		 	 	 	 	 	 	 . 32
;	杜教筛		 	 	 	 	 	 	 . 33
素数测	则试		 	 	 	 	 	 	 . 34
线性									
			 						2/
组合数	数						 		
	,,						 		
第二	类斯特灵数 .		 	 	 	 	 	 	 . 36
	类斯特灵数 .		 	 	 	  	 	  	 . 36 . 37
FFT 8	类斯特灵数 . & NTT & FWT		 	 	 	  	 	  	 . 36 . 37 . 37
FFT 8 simp	类斯特灵数 . & NTT & FWT son 自适应积分	····· [·····	 	   	 	   	 	 	 . 36 . 37 . 37
FFT { simp 快速៛	类斯特灵数 . & NTT & FWT son 自适应积分 乘	····· 「····· 分····	 	 	 	 	 	 	 . 36 . 37 . 37 . 38 . 39
FFT { simp 快速剩 快速網	类斯特灵数 . & NTT & FWT son 自适应积分 乘 幂	······ ······ ·····	 	 	 	 		 	 . 36 . 37 . 38 . 39
FFT { simp 快速剩 快速網	类斯特灵数 . & NTT & FWT son 自适应积分 乘	······ ······ ·····	 	 	 	 		 	 . 36 . 37 . 38 . 39
FFT { simp 快速 快速 高斯》	类斯特灵数 . & NTT & FWT son 自适应积分 乘 幂	 行 分 	 	 		 			 . 36 . 37 . 38 . 39 . 39
FFT { simp 快速  快速 高斯派  质因  数	类斯特灵数 & NTT & FWT son 自适应积分 乘 幂 肖元 数分解								 . 36 . 37 . 38 . 39 . 39 . 40
FFT { simp 快速 转速 高斯 质因 原根	类斯特灵数 & NTT & FWT son 自适应积分 乘 幂 肖元 数分解	·····································							 . 36 . 37 . 38 . 39 . 39 . 40 . 41
FFT { simp 快速等 高质根  原根  公	类斯特灵数 & NTT & FWT son 自适应积分 乘 幕 肖元 数分解	·····································							 . 36 . 37 . 38 . 39 . 40 . 41 . 42
FFT 8 simp 快速等 快速等 质质 人名英格兰 人名英格兰 人名英格兰人名	类斯特灵数 & NTT & FWT son 自适应积分 乘								. 36 . 37 . 38 . 39 . 40 . 41 . 42 . 43
FFT 8 simp 快速等 快速斯质原公二中国 以上国	类斯特灵数 & NTT & FWT son 自适应积分 乘								. 36 . 37 . 38 . 39 . 40 . 41 . 42 . 43 . 44
FFT Simp 快快高质原公二中伯 原公二中伯 等	类斯特灵数 & NTT & FWT son 自适应积分 乘								. 36 . 37 . 38 . 39 . 40 . 41 . 42 . 43 . 44 . 45
FFT Simp 快快高质原公二中伯 原公二中伯 等	类斯特灵数 & NTT & FWT son 自适应积分 乘								. 36 . 37 . 38 . 39 . 40 . 41 . 42 . 43 . 44 . 45
FFT Sim東東等 快度,所以二中伯单 原公二中伯单	类斯特灵数 & NTT & FWT son 自适应积分 乘								. 36 . 37 . 38 . 39 . 40 . 41 . 42 . 42 . 43 . 45
FFT Simp 快快高质原公二中伯 原公二中伯 等	类斯特灵数 & NTT & FWT son 自适应积分 乘								. 36 . 37 . 38 . 39 . 40 . 41 . 42 . 43 . 44 . 45
FFT Simp 等imp 快快高质原公二中伯单 企 图论	类斯特灵数 & NTT & FWT son 自适应积分 乘	· · · · · · · · · · · · · · · · · · ·							. 36. 37. 37. 38. 39. 39. 40. 41. 42. 42. 43. 44. 45. 45. 46.

	络流	 47
	上路径交	 50
	上点分治	 50
	· 链剖分	 53
	分图匹配	 57
	树	 59
	· 拉路径	 59
	·补排序	
	般图匹配(没有测试过)	
	MACH (KAMMA)	 01
计	1.何	62
	的反演	 62
	维	 63
	转卡壳	 66
	有测试过的	 67
	平面交	 68
字:	·	69
	缀自动机	
	文自动机	 72
	anacher	 72
	希	 73
	缀数组	 74
	MP 自动机	 77
	rie	 79
	C 自动机	 79
杂		80
	TL	
	期	 
	集枚举	
	值最大上升子序列	 82
	:位 DP	
	制 bitset	
	机	 83
	伪随机数	 83
	va	 84
	Regex	 84
	Decimal Format	 84
	Sort	 86
	栈(本地使用)	
	本崩了	 86

## 一切的开始

## 宏定义

● 需要 C++11

```
#include <bits/stdc++.h>
   using namespace std;
2
   using LL = long long;
   \#define\ FOR(i,\ x,\ y)\ for\ (decay< decltype(y)>::type\ i=(x),\ _\#\#i=(y);\ i<\_\#\#i;\ ++i)
   #define FORD(i, x, y) for (decay < decltype(x) > :: type i = (x), _##i = (y); i > _##i; --i)
   #define dbg(args...) do { cout << "\033[32;1m" << #args << " -> "; err(args); } while (0)
   #define dbg(...)
   #endif
10
   void err() { cout << "\033[39;0m" << endl; }</pre>
   template<template<typename...> class T, typename t, typename... Args>
12
   void err(T<t> a, Args... args) { for (auto x: a) cout << x << ' '; err(args...); }</pre>
   template<typename T, typename... Args>
14
   void err(T a, Args... args) { cout << a << ' '; err(args...); }</pre>
   // ----
       • 更多配色:
             - 33 黄色
             - 34 蓝色
             - 31 橙色
       • POJ/BZOJ version
   #include <cstdio>
   #include <iostream>
   #include <algorithm>
   #include <cmath>
   #include <string>
   #include <vector>
7 #include <set>
8 #include <queue>
   #include <cstring>
10
   #include <cassert>
using namespace std;
12 typedef long long LL;
   #define FOR(i, x, y) for (LL i = (x), _##i = (y); i < _##i; ++i)
13
   #define FORD(i, x, y) for (LL i = (x), _{-}##i = (y); i > _{-}##i; --i)
14
15
   #ifdef zerol
   #define dbq(args...) do { cout << "\033\[ \]32;1m" << #args<< " \rightarrow "; err(args); \} while (0)
16
17
   #else
   #define dbg(...)
18
   #endif
19
   void err() { cout << "\033[39;0m" << endl; }</pre>
20
   template<typename T, typename... Args>
21
22
   void err(T a, Args... args) {
        cout << a << ' '; err(args...);</pre>
23
24
   // -----
25
       • HDU Assert Patch
   #ifdef ONLINE_JUDGE
   #define assert(condition) if (!(condition)) { int x = 1, y = 0; cout << x / y << endl; }
   #endif
    快速读
    inline char next_char() {
2
        static char buf[100000], *p1 = buf, *p2 = buf;
        return p1 == p2 && (p2 = (p1 = buf) + fread(buf, 1, 100000, stdin), p1 == p2) ? EOF : *p1++;
3
   inline bool maybe_digit(char c) {
```

```
return c >= '0' && c <= '9';
7
    }
    template <typename T>
8
    void rn(T& _v) {
        static char ch;
        static bool negative = false;
11
12
        _v = 0;
        while (!maybe_digit(ch)) {
13
            negative = ch == '-';
14
            ch = next_char();
15
16
        do _{v} = (_{v} << 1) + (_{v} << 3) + ch - '0';
17
        while (maybe_digit(ch = next_char()));
18
        if (negative) _v = -_v;
19
    }
20
21
    template <typename T>
    void o(T p) {
23
24
        static int stk[70], tp;
        if (p == 0) {
25
            putchar('0');
26
27
            return;
28
        if (p < 0) { p = -p; putchar('-'); }</pre>
        while (p) stk[++tp] = p % 10, p /= 10;
30
31
        while (tp) putchar(stk[tp--] + '0');
    }
32
       • 需要初始化
       • 需要一次读入
        • 不支持负数
    const int MAXS = 100 * 1024 * 1024;
    char buf[MAXS];
    template<typename T>
    inline bool read(T& x) {
        static char* p = buf;
        x = 0;
        while (*p && !isdigit(*p)) ++p;
        if (!*p) return false;
        while (isdigit(*p)) x = x * 10 + *p++ - 48;
        return true;
10
11
12
    fread(buf, 1, MAXS, stdin);
    对拍
   #!/usr/bin/env bash
    g++ -o r main.cpp -02 -std=c++11
    g++ -o std std.cpp -02 -std=c++11
    while true; do
        python gen.py > in
        ./std < in > stdout
        ./r < in > out
        if test ? -ne 0; then
            exit 0
        fi
10
        if diff stdout out; then
11
            printf "AC\n"
13
            printf "GG\n"
14
15
            exit 0
16
    done
```

## 为什么 C++ 不自带这个?

```
1 LL bin(LL x, LL n, LL MOD) {
2     LL ret = MOD != 1;
3     for (x %= MOD; n; n >>= 1, x = x * x % MOD)
4         if (n & 1) ret = ret * x % MOD;
5     return ret;
6     }
7     inline LL get_inv(LL x, LL p) { return bin(x, p - 2, p); }
```

## 数据结构

## ST 表

二维

```
int f[maxn][maxn][10][10];
    inline int highbit(int x) { return 31 - __builtin_clz(x); }
    inline int calc(int x, int y, int xx, int yy, int p, int q) {
        return max(
            \max(f[x][y][p][q], f[xx - (1 << p) + 1][yy - (1 << q) + 1][p][q]),
            \max(f[xx - (1 << p) + 1][y][p][q], f[x][yy - (1 << q) + 1][p][q])
   }
    void init() {
        FOR (x, 0, highbit(n) + 1)
10
        FOR (y, \theta, highbit(m) + 1)
11
            FOR (i, 0, n - (1 << x) + 1)
12
            FOR (j, 0, m - (1 << y) + 1) {
13
                if (!x && !y) { f[i][j][x][y] = a[i][j]; continue; }
14
                f[i][j][x][y] = calc(
15
                    i, j,
16
                    i + (1 << x) - 1, j + (1 << y) - 1,
17
                    max(x - 1, 0), max(y - 1, 0)
18
19
                );
            }
20
21
    inline int get_max(int x, int y, int xx, int yy) {
22
23
        return calc(x, y, xx, yy, highbit(xx - x + 1), highbit(yy - y + 1));
24
       一维
    struct RMQ {
        int f[22][M];
2
        inline int highbit(int x) { return 31 - __builtin_clz(x); }
        void init(int* v, int n) {
            FOR (i, 0, n) f[0][i] = v[i];
            FOR (x, 1, highbit(n) + 1)
                FOR (i, 0, n - (1 << x) + 1)
                    f[x][i] = min(f[x - 1][i], f[x - 1][i + (1 << (x - 1))]);
        int get_min(int l, int r) {
            assert(l <= r);</pre>
11
12
            int t = highbit(r - l + 1);
            return min(f[t][l], f[t][r - (1 << t) + 1]);</pre>
13
        }
14
   } rmq;
```

## 线段树

● 普适

```
namespace sg {
struct Q {
LL setv;
```

```
explicit Q(LL setv = -1): setv(setv) {}
5
            void operator += (const Q& q) { if (q.setv != -1) setv = q.setv; }
        };
7
        struct P {
            LL min;
            explicit P(LL min = INF): min(min) {}
9
            void up(Q& q) { if (q.setv != -1) min = q.setv; }
10
11
        template<typename T>
12
13
        P operator & (T&& a, T&& b) {
            return P(min(a.min, b.min));
14
15
16
        P p[maxn << 2];
        Q q[maxn << 2];
17
    #define lson o * 2, l, (l + r) / 2
18
    #define rson o * 2 + 1, (l + r) / 2 + 1, r
19
20
        void up(int o, int l, int r) {
            if (l == r) p[o] = P();
21
22
            else p[o] = p[o * 2] & p[o * 2 + 1];
23
            p[o].up(q[o]);
24
        void down(int o, int l, int r) {
25
            q[o * 2] += q[o]; q[o * 2 + 1] += q[o];
26
            q[o] = Q();
            up(lson); up(rson);
28
29
30
        template<typename T>
        void build(T&& f, int o = 1, int l = 1, int r = n) {
31
            if (l == r) q[o] = f(l);
            else { build(f, lson); build(f, rson); q[o] = Q(); }
33
34
35
        P query(int ql, int qr, int 0 = 1, int l = 1, int r = n) {
36
37
            if (ql > r || l > qr) return P();
            if (ql <= l && r <= qr) return p[o];</pre>
38
            down(o, l, r);
39
            return query(ql, qr, lson) & query(ql, qr, rson);
40
41
42
        void update(int ql, int qr, const Q& v, int o = 1, int l = 1, int r = n) {
            if (ql > r || l > qr) return;
43
44
            if (ql <= l && r <= qr) q[o] += v;</pre>
45
            else {
                down(o, l, r);
46
47
                update(ql, qr, v, lson); update(ql, qr, v, rson);
48
49
            up(o, l, r);
        }
50
   }
       • SET + ADD
    struct IntervalTree {
    #define ls o * 2, l, m
    #define rs o * 2 + 1, m + 1, r
        static const LL M = maxn * 4, RS = 1E18 - 1;
        LL addv[M], setv[M], minv[M], maxv[M], sumv[M];
5
        void init() {
            memset(addv, 0, sizeof addv);
            fill(setv, setv + M, RS);
            memset(minv, 0, sizeof minv);
            memset(maxv, 0, sizeof maxv);
            memset(sumv, 0, sizeof sumv);
11
12
        void maintain(LL o, LL l, LL r) {
13
            if (l < r) {
14
                LL lc = 0 * 2, rc = 0 * 2 + 1;
15
                sumv[o] = sumv[lc] + sumv[rc];
16
                minv[o] = min(minv[lc], minv[rc]);
17
18
                maxv[o] = max(maxv[lc], maxv[rc]);
19
            } else sumv[o] = minv[o] = maxv[o] = 0;
            if (setv[o] != RS) { minv[o] = maxv[o] = setv[o]; sumv[o] = setv[o] * (r - l + 1); }
20
            if (addv[o]) { minv[o] += addv[o]; maxv[o] += addv[o]; sumv[o] += addv[o] * (r - l + 1); }
```

```
22
23
        void build(LL o, LL l, LL r) {
            if (l == r) addv[o] = a[l];
24
25
            else {
                 LL m = (l + r) / 2;
                 build(ls); build(rs);
27
28
            maintain(o, l, r);
29
30
        void pushdown(LL o) {
31
            LL lc = 0 * 2, rc = 0 * 2 + 1;
32
33
            if (setv[o] != RS) {
                 setv[lc] = setv[rc] = setv[o];
34
                 addv[lc] = addv[rc] = 0;
35
                 setv[o] = RS;
36
37
38
            if (addv[o]) {
                 addv[lc] += addv[o]; addv[rc] += addv[o];
39
                 addv[o] = 0;
            }
41
42
        void update(LL p, LL q, LL o, LL l, LL r, LL v, LL op) {
43
            if (p <= r && l <= q)
44
            if (p <= l && r <= q) {
45
                 if (op == 2) { setv[o] = v; addv[o] = 0; }
46
47
                 else addv[o] += v;
48
            } else {
                 pushdown(o);
49
50
                 LL m = (l + r) / 2;
                 update(p, q, ls, v, op); update(p, q, rs, v, op);
51
52
            maintain(o, l, r);
53
54
        }
55
        void query(LL p, LL q, LL o, LL l, LL r, LL add, LL& ssum, LL& smin, LL& smax) {
            if (p > r || l > q) return;
56
57
             if (setv[o] != RS) {
                 LL v = setv[o] + add + addv[o];
58
                 ssum += v * (min(r, q) - max(l, p) + 1);
59
                 smin = min(smin, v);
                 smax = max(smax, v);
61
62
            } else if (p <= l && r <= q) {</pre>
                 ssum += sumv[o] + add * (r - l + 1);
63
                 smin = min(smin, minv[o] + add);
64
65
                 smax = max(smax, maxv[o] + add);
66
            } else {
67
                 LL m = (l + r) / 2;
                 query(p, q, ls, add + addv[o], ssum, smin, smax);
68
                 query(p, q, rs, add + addv[o], ssum, smin, smax);
            }
70
71
    } IT;
```

#### 均摊复杂度线段树

● 区间取 max, 区间求和。

```
namespace R {
    #define lson o \star 2, l, (l + r) / 2
2
    #define rson o * 2 + 1, (l + r) / 2 + 1, r
        int m1[N], m2[N], cm1[N];
        LL sum[N];
        void up(int o) {
            int lc = o * 2, rc = lc + 1;
            m1[o] = max(m1[lc], m1[rc]);
            sum[o] = sum[lc] + sum[rc];
            if (m1[lc] == m1[rc]) {
                cm1[o] = cm1[lc] + cm1[rc];
11
                m2[o] = max(m2[lc], m2[rc]);
12
13
            } else {
```

```
cm1[o] = m1[lc] > m1[rc] ? cm1[lc] : cm1[rc];
14
15
                 m2[o] = max(min(m1[lc], m1[rc]), max(m2[lc], m2[rc]));
            }
16
17
        }
        void mod(int o, int x) {
            if (x >= m1[o]) return;
19
20
            assert(x > m2[o]);
            sum[o] = 1LL * (m1[o] - x) * cm1[o];
21
            m1[o] = x;
22
23
        void down(int o) {
24
25
            int lc = 0 * 2, rc = lc + 1;
            mod(lc, m1[o]); mod(rc, m1[o]);
26
27
        void build(int o, int l, int r) {
28
            if (l == r) { int t; read(t); sum[o] = m1[o] = t; m2[o] = -1; cm1[o] = 1; }
29
30
            else { build(lson); build(rson); up(o); }
31
32
        void update(int ql, int qr, int x, int o, int l, int r) {
33
            if (r < ql || qr < l || m1[o] <= x) return;</pre>
             if (ql <= l && r <= qr && m2[o] < x) { mod(o, x); return; }</pre>
34
            down(o);
35
            update(ql, qr, x, lson); update(ql, qr, x, rson);
36
            up(o);
38
39
        int qmax(int ql, int qr, int o, int l, int r) {
            if (r < ql || qr < l) return -INF;</pre>
40
             if (ql <= l && r <= qr) return m1[o];</pre>
41
            down(o);
42
            return max(qmax(ql, qr, lson), qmax(ql, qr, rson));
43
44
        LL qsum(int ql, int qr, int o, int l, int r) {
45
            if (r < ql || qr < l) return 0;</pre>
46
47
            if (ql <= l && r <= qr) return sum[o];</pre>
            down(o);
48
            return qsum(ql, qr, lson) + qsum(ql, qr, rson);
        }
50
   }
51
```

#### 持久化线段树

ADD

```
namespace tree {
    #define mid ((l + r) >> 1)
    #define lson ql, qr, l, mid
    #define rson ql, qr, mid + 1, r
        struct P {
5
            LL add, sum;
            int ls, rs;
        } tr[maxn * 45 * 2];
8
        int sz = 1;
        int N(LL add, int l, int r, int ls, int rs) {
10
            tr[sz] = \{add, tr[ls].sum + tr[rs].sum + add * (len[r] - len[l - 1]), ls, rs\};
            return sz++;
12
13
        int update(int o, int ql, int qr, int l, int r, LL add) {
14
            if (ql > r \mid \mid l > qr) return o;
15
            const P& t = tr[o];
            if (ql <= l && r <= qr) return N(add + t.add, l, r, t.ls, t.rs);</pre>
17
            return N(t.add, l, r, update(t.ls, lson, add), update(t.rs, rson, add));
18
19
        LL query(int o, int ql, int qr, int l, int r, LL add = 0) {
20
21
            if (ql > r || l > qr) return 0;
            const P& t = tr[o];
22
23
            if (ql <= l && r <= qr) return add * (len[r] - len[l - 1]) + t.sum;</pre>
            return query(t.ls, lson, add + t.add) + query(t.rs, rson, add + t.add);
24
        }
25
   }
26
```

#### K-D Tree

#### 最优化问题一定要用全局变量大力剪枝,而且左右儿子先递归潜力大的

- 维护信息
- 带重构(适合在线)
- 插入时左右儿子要标记为 null。

```
namespace kd {
        const int K = 2, inf = 1E9, M = N;
2
        const double lim = 0.7;
3
        struct P {
4
            int d[K], l[K], r[K], sz, val;
            LL sum;
            P *ls, *rs;
            P* up() {
                 sz = ls -> sz + rs -> sz + 1;
                 sum = ls -> sum + rs -> sum + val;
                 FOR (i, 0, K) {
11
                     l[i] = min(d[i], min(ls->l[i], rs->l[i]));
                     r[i] = max(d[i], max(ls->r[i], rs->r[i]));
13
14
15
                 return this;
            }
16
        } pool[M], *null = new P, *pit = pool;
17
        static P *tmp[M], **pt;
18
        void init() {
19
            null->ls = null->rs = null;
20
            FOR (i, 0, K) null->l[i] = inf, null->r[i] = -inf;
21
22
            null->sum = null->val = 0;
            null->sz = 0;
23
24
25
        P* build(P** l, P** r, int d = 0) { // [l, r)
27
            if (d == K) d = 0;
            if (l >= r) return null;
28
            P** m = l + (r - l) / 2; assert(l <= m && m < r);
29
            nth_element(l, m, r, [&](const P* a, const P* b){
30
31
                 return a\rightarrow d[d] < b\rightarrow d[d];
            });
32
33
            P* o = *m;
            o->ls = build(l, m, d + 1); o->rs = build(m + 1, r, d + 1);
34
            return o->up();
35
        P* Build() {
37
            pt = tmp; FOR (it, pool, pit) *pt++ = it;
38
39
            return build(tmp, pt);
40
        inline bool inside(int p[], int q[], int l[], int r[]) {
41
            FOR (i, 0, K) if (r[i] < q[i] \mid \mid p[i] < l[i]) return false;
42
            return true;
43
44
        LL query(P* o, int l[], int r[]) {
45
            if (o == null) return 0;
46
47
            FOR (i, 0, K) if (o->r[i] < l[i] || r[i] < o->l[i]) return 0;
48
            if (inside(o->l, o->r, l, r)) return o->sum;
            return query(o->ls, l, r) + query(o->rs, l, r) +
49
                    (inside(o->d, o->d, l, r) ? o->val : 0);
51
52
        void dfs(P* o) {
            if (o == null) return;
53
             *pt++ = o; dfs(o->ls); dfs(o->rs);
54
        P* ins(P* o, P* x, int d = 0) {
56
            if (d == K) d = 0;
57
58
            if (o == null) return x->up();
            P * \& oo = x - > d[d] <= o - > d[d] ? o - > ls : o - > rs;
59
            if (oo->sz > o->sz * lim) {
                 pt = tmp; dfs(o); *pt++ = x;
61
62
                 return build(tmp, pt, d);
            }
63
```

```
oo = ins(oo, x, d + 1);
64
65
            return o->up();
        }
66
67
   }
        • 维护信息
        ● 带修改(适合离线)
    namespace kd {
1
        const int K = 3, inf = 1E9, M = N << 3;</pre>
2
3
        extern struct P* null;
        struct P {
4
            int d[K], l[K], r[K], val;
5
            int Max;
            P *ls, *rs, *fa;
            P* up() {
                Max = max(val, max(ls->Max, rs->Max));
10
                FOR (i, 0, K) {
                    l[i] = min(d[i], min(ls->l[i], rs->l[i]));
11
12
                    r[i] = max(d[i], max(ls->r[i], rs->r[i]));
                }
13
                return ls->fa = rs->fa = this;
15
            }
        } pool[M], *null = new P, *pit = pool;
16
17
        void upd(P* o, int val) {
            o->val = val;
18
            for (; o != null; o = o->fa)
19
                o->Max = max(o->Max, val);
20
21
        static P *tmp[M], **pt;
22
        void init() {
23
            null->ls = null->rs = null;
24
            FOR (i, 0, K) null->l[i] = inf, null->r[i] = -inf;
25
            null->Max = null->val = 0;
26
27
        P* build(P** l, P** r, int d = 0) { // [l, r)
28
29
            if (d == K) d = 0;
            if (l >= r) return null;
30
            P** m = l + (r - l) / 2; assert(l <= m && m < r);
31
            nth_element(l, m, r, [&](const P* a, const P* b){
32
                return a->d[d] < b->d[d];
33
34
            });
35
            P * o = *m;
            o->ls = build(l, m, d + 1); o->rs = build(m + 1, r, d + 1);
36
            return o->up();
37
38
        P* Build() {
39
            pt = tmp; FOR (it, pool, pit) *pt++ = it;
40
41
            P* ret = build(tmp, pt); ret->fa = null;
            return ret;
42
43
        inline bool inside(int p[], int q[], int l[], int r[]) {
44
45
            FOR (i, 0, K) if (r[i] < q[i] || p[i] < l[i]) return false;
46
            return true;
47
        int query(P* o, int l[], int r[]) {
48
            if (o == null) return 0;
49
            FOR (i, 0, K) if (o->r[i] < l[i] || r[i] < o->l[i]) return 0;
50
            if (inside(o->l, o->r, l, r)) return o->Max;
51
            int ret = 0;
52
            if (o->val > ret && inside(o->d, o->d, l, r)) ret = max(ret, o->val);
            if (o->ls->Max > ret) ret = max(ret, query(o->ls, l, r));
54
            if (o->rs->Max > ret) ret = max(ret, query(o->rs, l, r));
55
56
            return ret:
        }
57
58
   }
        • 最近点对
        要用全局变量大力剪枝
    namespace kd {
        const int K = 3;
```

```
const int M = N;
3
4
        const int inf = 1E9 + 100;
        struct P {
5
             int d[K];
             int l[K], r[K];
            P *ls, *rs;
8
             P* up() {
                 FOR (i, 0, K) {
10
                     l[i] = min(d[i], min(ls->l[i], rs->l[i]));
11
12
                     r[i] = max(d[i], max(ls->r[i], rs->r[i]));
13
14
                 return this;
            }
15
        } pool[M], *null = new P, *pit = pool;
16
17
        static P *tmp[M], **pt;
        void init() {
18
19
            null->ls = null->rs = null;
            FOR (i, 0, K) null->l[i] = inf, null->r[i] = -inf;
20
21
        P* build(P** l, P** r, int d = 0) { // [l, r)
22
             if (d == K) d = 0;
23
            if (l >= r) return null;
24
            P** m = l + (r - l) / 2;
25
            nth_element(l, m, r, [&](const P* a, const P* b){
27
                 return a\rightarrow d[d] < b\rightarrow d[d];
28
            });
            P * o = *m;
29
             o->ls = build(l, m, d + 1); o->rs = build(m + 1, r, d + 1);
30
31
             return o->up();
32
        LL eval(P* o, int d[]) {
33
34
            // ...
        }
35
        LL dist(int d1[], int d2[]) {
36
37
38
        LL S;
39
        LL query(P* o, int d[]) {
40
             if (o == null) return 0;
41
             S = max(S, dist(o->d, d));
42
43
             LL mdl = eval(o->ls, d), mdr = eval(o->rs, d);
             if (mdl < mdr) {</pre>
44
                 if (S > mdl) S = max(S, query(o->ls, d));
45
46
                 if (S > mdr) S = max(S, query(o->rs, d));
47
             } else {
48
                 if (S > mdr) S = max(S, query(o->rs, d));
                 if (S > mdl) S = max(S, query(o->ls, d));
49
             return S;
51
52
        P* Build() {
53
            pt = tmp; FOR (it, pool, pit) *pt++ = it;
54
             return build(tmp, pt);
56
57
    }
```

#### 树状数组

● 注意: 0 是无效下标

```
namespace bit {
   LL c[M];
   inline int lowbit(int x) { return x & -x; }

void add(int x, LL v) {
   for (; x < M; x += lowbit(x))
        c[x] += v;
}

LL sum(int x) {
   LL ret = 0;</pre>
```

```
for (; x > 0; x -= lowbit(x))
10
11
               ret += c[x];
            return ret;
12
13
        }
14
        int kth(LL k) {
            int ret = 0;
15
            LL cnt = 0;
16
            FORD (i, 20, -1) {
17
                ret += 1 << i;
18
19
                if (ret >= M || cnt + c[ret] >= k)
                   ret -= 1 << i;
20
21
                else cnt += c[ret];
            }
22
            return ret + 1;
23
24
        }
   }
25

    区间修改 & 区间查询(单点修改,查询前缀和的前缀和)

    namespace bit {
        int c[maxn], cc[maxn];
2
        inline int lowbit(int x) { return x & -x; }
3
        void add(int x, int v) {
4
            for (int i = x; i <= n; i += lowbit(i)) {</pre>
5
                c[i] += v; cc[i] += x * v;
8
        void add(int l, int r, int v) { add(l, v); add(r + 1, -v); }
10
        int sum(int x) {
            int ret = 0;
11
12
            for (int i = x; i > 0; i -= lowbit(i))
                ret += (x + 1) * c[i] - cc[i];
13
            return ret;
14
15
        int sum(int l, int r) { return sum(r) - sum(l - 1); }
16
   }
17
       • 单点修改, 查询前缀和的前缀和的前缀和(有用才怪)
    namespace bit {
        LL c[N], cc[N], ccc[N];
2
        inline LL lowbit(LL x) { return x & -x; }
3
        void add(LL x, LL v) {
4
            for (LL i = x; i < N; i += lowbit(i)) {</pre>
5
                c[i] = (c[i] + v) \% MOD;
                cc[i] = (cc[i] + x * v) % MOD;
                ccc[i] = (ccc[i] + x * x % MOD * v) % MOD;
8
            }
10
11
        void add(LL l, LL r, LL v) { add(l, v); add(r + 1, -v); }
        LL sum(LL x) {
12
            static LL INV2 = (MOD + 1) / 2;
13
            LL ret = 0;
14
            for (LL i = x; i > 0; i -= lowbit(i))
15
                ret += (x + 1) * (x + 2) % MOD * c[i] % MOD
16
                         - (2 * x + 3) * cc[i] % MOD
17
18
                        + ccc[i];
            return ret % MOD * INV2 % MOD;
19
21
        LL sum(LL l, LL r) { return sum(r) - sum(l - 1); }
   }
22
       三维
    inline int lowbit(int x) { return x & -x; }
    void update(int x, int y, int z, int d) {
        for (int i = x; i <= n; i += lowbit(i))</pre>
            for (int j = y; j <= n; j += lowbit(j))</pre>
                for (int k = z; k <= n; k += lowbit(k))</pre>
5
                    c[i][j][k] += d;
    LL query(int x, int y, int z) {
        LL ret = 0;
```

```
for (int i = x; i > 0; i -= lowbit(i))
10
11
            for (int j = y; j > 0; j -= lowbit(j))
                for (int k = z; k > 0; k = lowbit(k))
12
                    ret += c[i][j][k];
13
        return ret;
14
   }
15
   LL solve(int x, int y, int z, int xx, int yy, int zz) {
16
        return query(xx, yy, zz)
17
                - query(xx, yy, z - 1)
18
19
                - query(xx, y - 1, zz)
                - query(x - 1, yy, zz)
20
21
                + query(xx, y - 1, z - 1)
                + query(x - 1, yy, z - 1)
22
                + query(x - 1, y - 1, zz)
23
24
                - query(x - 1, y - 1, z - 1);
    主席树
       ● 正常主席树
   namespace tree {
    #define mid ((l + r) >> 1)
2
    #define lson l, mid
    #define rson mid + 1, r
        const int MAGIC = M * 30;
        struct P {
7
           int sum, ls, rs;
8
        } tr[MAGIC] = {{0, 0, 0}};
        int sz = 1;
        int N(int sum, int ls, int rs) {
            if (sz == MAGIC) assert(0);
11
            tr[sz] = {sum, ls, rs};
12
13
            return sz++;
14
15
        int ins(int o, int x, int v, int l = 1, int r = ls) {
            if (x < l \mid | x > r) return o;
16
17
            const P& t = tr[o];
            if (l == r) return N(t.sum + v, 0, 0);
18
19
            return N(t.sum + v, ins(t.ls, x, v, lson), ins(t.rs, x, v, rson));
20
        int query(int o, int ql, int qr, int l = 1, int r = ls) {
21
22
            if (ql > r || l > qr) return 0;
            const P& t = tr[o];
23
            if (ql <= l && r <= qr) return t.sum;</pre>
            return query(t.ls, ql, qr, lson) + query(t.rs, ql, qr, rson);
25
        }
26
   }
27
       ● 第k大
    struct TREE {
1
    #define mid ((l + r) >> 1)
2
    #define lson l, mid
    #define rson mid + 1, r
        struct P {
            int w, ls, rs;
        } tr[maxn * 20];
        int sz = 1;
        TREE() { tr[0] = \{0, 0, 0\}; \}
        int N(int w, int ls, int rs) {
            tr[sz] = {w, ls, rs};
11
12
            return sz++;
13
        int ins(int tt, int l, int r, int x) {
14
            if (x < l \mid | r < x) return tt;
15
            const P& t = tr[tt];
16
17
            if (l == r) return N(t.w + 1, 0, 0);
            return N(t.w + 1, ins(t.ls, lson, x), ins(t.rs, rson, x));
18
```

int query(int pp, int qq, int l, int r, int k) { // (pp, qq]

19 20

```
if (l == r) return l;
21
22
            const P &p = tr[pp], &q = tr[qq];
            int w = tr[q.ls].w - tr[p.ls].w;
23
24
            if (k <= w) return query(p.ls, q.ls, lson, k);</pre>
25
            else return query(p.rs, q.rs, rson, k - w);
        }
26
   } tree;
       • 树状数组套主席树
   typedef vector<int> VI;
    struct TREE {
2
    #define mid ((l + r) >> 1)
    #define lson l, mid
    #define rson mid + 1, r
        struct P {
            int w, ls, rs;
        } tr[maxn * 20 * 20];
        int sz = 1;
        TREE() { tr[0] = \{0, 0, 0\}; \}
        int N(int w, int ls, int rs) {
11
            tr[sz] = {w, ls, rs};
12
13
            return sz++;
14
        int add(int tt, int l, int r, int x, int d) {
            if (x < l \mid | r < x) return tt;
16
17
            const P& t = tr[tt];
            if (l == r) return N(t.w + d, 0, 0);
18
            return N(t.w + d, add(t.ls, lson, x, d), add(t.rs, rson, x, d));
19
20
21
        int ls sum(const VI& rt) {
            int ret = 0;
22
            FOR (i, 0, rt.size())
23
                ret += tr[tr[rt[i]].ls].w;
24
25
            return ret;
26
27
        inline void ls(VI\& rt)  { transform(rt.begin(), rt.begin(), [&](int x)->int{ return } tr[x].ls; }); }
        inline void rs(VI& rt) { transform(rt.begin(), rt.end(), rt.begin(), [\&](int x)->int{ return tr[x].rs; }); }
28
        int query(VI& p, VI& q, int l, int r, int k) {
29
            if (l == r) return l;
30
            int w = ls_sum(q) - ls_sum(p);
31
32
            if (k <= w) {
                ls(p); ls(q);
33
                 return query(p, q, lson, k);
34
            }
35
36
            else {
37
                rs(p); rs(q);
                return query(p, q, rson, k - w);
38
39
            }
40
   } tree;
41
42
    struct BIT {
        int root[maxn];
43
44
        void init() { memset(root, 0, sizeof root); }
        inline int lowbit(int x) { return x & -x; }
45
        void update(int p, int x, int d) {
46
            for (int i = p; i <= m; i += lowbit(i))</pre>
47
48
                root[i] = tree.add(root[i], 1, m, x, d);
49
        int query(int l, int r, int k) {
50
            VI p, q;
51
            for (int i = l - 1; i > 0; i -= lowbit(i)) p.push_back(root[i]);
52
            for (int i = r; i > 0; i -= lowbit(i)) q.push_back(root[i]);
53
54
            return tree.query(p, q, 1, m, k);
55
        }
56
   } bit;
57
    void init() {
59
        m = 10000:
        tree.sz = 1;
60
61
        bit.init();
        FOR (i, 1, m + 1)
```

```
bit.update(i, a[i], 1);
63
   }
    左偏树
    namespace LTree {
        extern struct P* null, *pit;
        queue<P*> trash;
        const int M = 1E5 + 100;
        struct P {
5
            P *ls, *rs;
            LL v;
8
            int d;
            void operator delete (void* ptr) {
                trash.push((P*)ptr);
10
            void* operator new(size_t size) {
12
                if (trash.empty()) return pit++;
13
                void* ret = trash.front(); trash.pop(); return ret;
14
            }
15
            void prt() {
17
                if (this == null) return;
18
                cout << v << ' ';
19
                ls->prt(); rs->prt();
20
        } pool[M], *pit = pool, *null = new P{0, 0, -1, -1};
22
        P* N(LL v) {
23
            return new P{null, null, v, 0};
24
25
        P* merge(P* a, P* b) {
            if (a == null) return b;
27
28
            if (b == null) return a;
            if (a->v > b->v) swap(a, b);
29
            a->rs = merge(a->rs, b);
30
            if (a->ls->d < a->rs->d) swap(a->ls, a->rs);
31
            a->d = a->rs->d + 1;
32
33
            return a;
        }
34
35
        LL pop(P*& o) {
36
            LL ret = o->v;
37
38
            P* t = o;
            o = merge(o->ls, o->rs);
39
            delete t;
41
            return ret;
42
        }
    }
43
    可持久化
    namespace LTree {
1
2
        extern struct P* null, *pit;
        queue<P*> trash;
3
        const int M = 1E6 + 100;
        struct P {
            P *ls, *rs;
            LL v;
            int d:
            void operator delete (void* ptr) {
                trash.push((P*)ptr);
11
12
            void* operator new(size_t size) {
                if (trash.empty()) return pit++;
13
                void* ret = trash.front(); trash.pop(); return ret;
14
15
16
        } pool[M], *pit = pool, *null = new P\{0, 0, -1, -1\};
17
        P* N(LL v, P* ls = null, P* rs = null) {
            if (ls->d < rs->d) swap(ls, rs);
18
            return new P{ls, rs, v, rs->d + 1};
```

```
20
21
        P* merge(P* a, P* b) {
            if (a == null) return b;
22
            if (b == null) return a;
23
            if (a->v < b->v)
                 return N(a->v, a->ls, merge(a->rs, b));
25
26
                 return N(b->v, b->ls, merge(b->rs, a));
27
        }
28
29
        LL pop(P*& o) {
30
31
            LL ret = o->v;
32
            o = merge(o->ls, o->rs);
            return ret;
33
34
        }
    }
35
```

## **Treap**

- 非旋 Treap
- v 小根堆
- 模板题 bzoj 3224
- lower 第一个大于等于的是第几个 (0-based)
- upper 第一个大于的是第几个 (0-based)

extern struct P\* const null, \*pit;

struct P {

P \*ls, \*rs;

- split 左侧分割出 rk 个元素
- 树套树略

```
namespace treap {
        const int M = maxn * 17;
        extern struct P* const null;
        struct P {
4
           P *ls, *rs;
            int v, sz;
            unsigned rd;
            P(int v): ls(null), rs(null), v(v), sz(1), rd(rnd()) {}
            P(): sz(0) {}
            P* up() { sz = ls->sz + rs->sz + 1; return this; }
11
            int lower(int v) {
                if (this == null) return 0;
13
                return this->v >= v ? ls->lower(v) : rs->lower(v) + ls->sz + 1;
15
            int upper(int v) {
16
                if (this == null) return 0;
17
                return this->v > v ? ls->upper(v) : rs->upper(v) + ls->sz + 1;
18
19
        } *const null = new P, pool[M], *pit = pool;
20
21
        P* merge(P* l, P* r) {
22
            if (l == null) return r; if (r == null) return l;
23
24
            if (l->rd < r->rd) { l->rs = merge(l->rs, r); return l->up(); }
            else { r->ls = merge(l, r->ls); return r->up(); }
25
26
27
        void split(P* o, int rk, P*& l, P*& r) {
28
29
            if (o == null) { l = r = null; return; }
            if (o->ls->sz >= rk) { split(o->ls, rk, l, o->ls); r = o->up(); }
30
            else { split(o->rs, rk - o->ls->sz - 1, o->rs, r); l = o->up(); }
        }
32
   }
33
       ● 持久化 Treap
   namespace treap {
        const int M = \max n * 17 * 12;
2
```

```
int v, sz;
7
            LL sum;
            P(P* ls, P* rs, int v): ls(ls), rs(rs), v(v), sz(ls->sz + rs->sz + 1),
8
                                                           sum(ls->sum + rs->sum + v) {}
11
            void* operator new(size_t _) { return pit++; }
12
            template<tvpename T>
13
            int rk(int v, T&& cmp) {
14
                if (this == null) return 0;
15
                return cmp(this->v, v) ? ls->rk(v, cmp) : rs->rk(v, cmp) + ls->sz + 1;
16
17
            int lower(int v) { return rk(v, greater_equal<int>()); }
18
            int upper(int v) { return rk(v, greater<int>()); }
19
20
        } pool[M], *pit = pool, *const null = new P;
        P* merge(P* l, P* r) {
21
22
            if (l == null) return r; if (r == null) return l;
            if (rnd() % (l->sz + r->sz) < l->sz) return new P{l->ls, merge(l->rs, r), l->v};
23
            else return new P{merge(l, r->ls), r->rs, r->v};
24
25
        void split(P* o, int rk, P*& l, P*& r) {
26
            if (o == null) { l = r = null; return; }
27
            if (o->ls->sz >= rk) { split(o->ls, rk, l, r); r = new P{r, o->rs, o->v}; }
28
            else { split(o->rs, rk - o->ls->sz - 1, l, r); l = new P{o->ls, l, o->v}; }
        }
30
   }
31
       • 带 pushdown 的持久化 Treap
       ● 注意任何修改操作前一定要 FIX
    int now:
    namespace Treap {
        const int M = 100000000:
3
        extern struct P* const null, *pit;
4
5
        struct P {
            P *ls, *rs;
            int sz, time;
            LL cnt, sc, pos, add;
8
            bool rev:
10
            P* up() { sz = ls->sz + rs->sz + 1; sc = ls->sc + rs->sc + cnt; return this; } // MOD
11
12
            P* check() {
13
                if (time == now) return this;
                P* t = new(pit++) P; *t = *this; t->time = now; return t;
14
15
            P* \_do\_rev()  { rev ^-1; add *=-1; pos *=-1; swap(ls, rs); return this; } // MOD
            P* _do_add(LL v) { add += v; pos += v; return this; } // MOD
17
            P* do_rev() { if (this == null) return this; return check()->_do_rev(); } // FIX & MOD
18
            P* do_add(LL v) { if (this == null) return this; return check()->_do_add(v); } // FIX & MOD
19
            P* _down() { // MOD
20
                if (rev) { ls = ls->do_rev(); rs = rs->do_rev(); rev = 0; }
                if (add) { ls = ls->do_add(add); rs = rs->do_add(add); add = 0; }
22
                return this;
23
24
            P* down() { return check()->_down(); } // FIX & MOD
25
            void _split(LL p, P*& l, P*& r) { // MOD
26
                if (pos >= p) { ls->split(p, l, r); ls = r; r = up(); }
27
                               { rs->split(p, l, r); rs = l; l = up(); }
28
29
            void split(LL p, P*& l, P*& r) { // FIX & MOD
30
                if (this == null) l = r = null;
                else down()->_split(p, l, r);
32
33
        } pool[M], *pit = pool, *const null = new P;
34
        P* merge(P* a, P* b) {
35
            if (a == null) return b; if (b == null) return a;
            if (rand() % (a->sz + b->sz) < a->sz) { a = a->down(); a->rs = merge(a->rs, b); return a->up(); }
37
38
            else
                                                   { b = b \rightarrow down(); b \rightarrow ls = merge(a, b \rightarrow ls); return b \rightarrow up(); }
        }
39
   }
40
```

## Treap-序列

● 区间 ADD, SUM

```
namespace treap {
        const int M = 8E5 + 100;
2
        extern struct P*const null;
3
        struct P {
4
             P *ls, *rs;
             int sz, val, add, sum;
             P(int \ v, \ P* \ ls = null, \ P* \ rs = null): \ ls(ls), \ rs(rs), \ sz(1), \ val(v), \ add(0), \ sum(v) \ \{\}
             P(): sz(0), val(0), add(0), sum(0) {}
             P* up() {
                 assert(this != null);
11
                 sz = ls -> sz + rs -> sz + 1;
12
13
                 sum = ls \rightarrow sum + rs \rightarrow sum + val + add * sz;
14
                 return this;
             }
15
             void upd(int v) {
16
                 if (this == null) return;
                 add += v;
18
                 sum += sz * v;
19
20
             P* down() {
21
                 if (add) {
                      ls->upd(add); rs->upd(add);
23
24
                      val += add;
                      add = 0;
25
26
27
                 return this;
             }
28
29
             P* select(int rk) {
30
                 if (rk == ls->sz + 1) return this;
31
                 return ls->sz >= rk ? ls->select(rk) : rs->select(rk - ls->sz - 1);
32
33
34
         } pool[M], *pit = pool, *const null = new P, *rt = null;
35
         P* merge(P* a, P* b) {
36
37
             if (a == null) return b->up();
             if (b == null) return a->up();
38
39
             if (rand() % (a->sz + b->sz) < a->sz) {
                 a->down()->rs = merge(a->rs, b);
40
                 return a->up();
             } else {
42
43
                 b->down()->ls = merge(a, b->ls);
44
                 return b->up();
45
             }
        }
47
         void split(P* o, int rk, P*& l, P*& r) {
48
             if (o == null) { l = r = null; return; }
49
             o->down();
50
             if (o->ls->sz >= rk) {
                 split(o->ls, rk, l, o->ls);
52
53
                 r = o \rightarrow up();
54
             } else {
                 split(o->rs, rk - o->ls->sz - 1, o->rs, r);
55
                 l = o \rightarrow up();
             }
57
58
        }
59
60
         inline void insert(int k, int v) {
61
             P *l, *r;
             split(rt, k - 1, l, r);
62
63
             rt = merge(merge(l, new (pit++) P(v)), r);
        }
64
        inline void erase(int k) {
66
             P *1, *r, *_, *t;
67
```

```
split(rt, k - 1, l, t);
68
             split(t, 1, _, r);
69
             rt = merge(l, r);
70
71
        }
72
        P* build(int l, int r, int* a) {
73
74
             if (l > r) return null;
             if (l == r) return new(pit++) P(a[l]);
75
             int m = (l + r) / 2;
76
             return (new(pit++) P(a[m], build(l, m - 1, a), build(m + 1, r, a)))->up();
77
        }
78
79
    };
        • 区间 REVERSE, ADD, MIN
    namespace treap {
1
        extern struct P*const null;
2
         struct P {
             P *ls, *rs;
4
             int sz, v, add, m;
             bool flip;
             P(int \ v, P* \ ls = null, P* \ rs = null): \ ls(ls), \ rs(rs), \ sz(1), \ v(v), \ add(0), \ m(v), \ flip(0) \ \{\}
             P(): sz(0), v(INF), m(INF) {}
             void upd(int v) {
                 if (this == null) return;
11
12
                 add += v; m += v;
             }
13
             void rev() {
14
15
                 if (this == null) return;
                 swap(ls, rs);
16
17
                 flip ^= 1;
18
             P* up() {
19
                 assert(this != null);
20
                 sz = ls \rightarrow sz + rs \rightarrow sz + 1;
21
22
                 m = min(min(ls->m, rs->m), v) + add;
                 return this;
23
24
             P* down() {
25
26
                 if (add) {
27
                      ls->upd(add); rs->upd(add);
                     v += add;
28
                      add = 0;
29
30
                 if (flip) {
31
                      ls->rev(); rs->rev();
32
                      flip = 0;
33
34
                 }
                 return this;
35
36
37
             P* select(int k) {
38
39
                 if (ls->sz + 1 == k) return this;
                 if (ls->sz >= k) return ls->select(k);
40
41
                 return rs->select(k - ls->sz - 1);
             }
42
43
        } pool[M], *const null = new P, *pit = pool, *rt = null;
44
45
46
         P* merge(P* a, P* b) {
             if (a == null) return b;
47
             if (b == null) return a;
48
             if (rnd() % (a->sz + b->sz) < a->sz) {
49
                 a->down()->rs = merge(a->rs, b);
50
51
                 return a->up();
52
             } else {
                 b->down()->ls = merge(a, b->ls);
53
                 return b->up();
54
55
             }
56
        }
```

57

```
void split(P* o, int k, P*& l, P*& r) {
58
59
             if (o == null) { l = r = null; return; }
             o->down();
60
             if (o->ls->sz >= k) {
61
62
                 split(o->ls, k, l, o->ls);
                 r = o->up();
63
             } else {
64
                 split(o->rs, k - o->ls->sz - 1, o->rs, r);
65
                 l = o \rightarrow up();
66
             }
67
         }
68
69
         P* build(int l, int r, int* v) {
70
             if (l > r) return null;
71
72
             int m = (l + r) >> 1;
             return (new (pit++) P(v[m], build(l, m - 1, v), build(m + 1, r, v)))->up();
73
74
75
         void go(int x, int y, void f(P*\&)) {
76
77
             P *l, *m, *r;
             split(rt, y, l, r);
78
79
             split(l, x - 1, l, m);
80
             f(m);
             rt = merge(merge(l, m), r);
82
83
    }
84
    using namespace treap;
    int a[maxn], n, x, y, Q, v, k, d;
85
    char s[100];
87
    int main() {
88
89
         cin >> n;
         FOR (i, 1, n + 1) scanf("%d", &a[i]);
90
91
         rt = build(1, n, a);
         cin >> Q;
92
         while (Q--) {
93
             scanf("%s", s);
94
             if (s[0] == 'A') {
95
                 scanf("%d%d%d", &x, &y, &v);
                 go(x, y, [](P*& o){ o->upd(v); });
97
             } else if (s[0] == 'R' && s[3] == 'E') {
98
                 scanf("%d%d", &x, &y);
99
                 go(x, y, [](P*& o){ o->rev(); });
100
101
             } else if (s[0] == 'R' && s[3] == '0') {
                 scanf("%d%d%d", &x, &y, &d);
102
                 d \%= y - x + 1;
103
                 go(x, y, [](P*& o){
104
105
                      P *1, *r;
                      split(o, o->sz - d, l, r);
106
                      o = merge(r, l);
107
108
                 });
             } else if (s[0] == 'I') {
109
                 scanf("%d%d", &k, &v);
                 go(k + 1, k, [](P*\& o){o = new (pit++) P(v);});
111
             } else if (s[0] == 'D') {
112
                 scanf("%d", &k);
113
                 go(k, k, [](P*\& o){ o = null; });
114
115
             } else if (s[0] == 'M') {
                 scanf("%d%d", &x, &y);
116
                 go(x, y, [](P*& o) {
117
                     printf("%d\n", o->m);
118
                 });
119
120
             }
         }
121
122
    }
        • 持久化
    namespace treap {
2
         struct P;
3
         extern P*const null;
         P* N(P* ls, P* rs, LL v, bool fill);
```

```
struct P {
6
            P *const ls, *const rs;
            const int sz, v;
            const LL sum;
            bool fill;
            int cnt;
10
11
            void split(int k, P*& l, P*& r) {
12
                 if (this == null) { l = r = null; return; }
13
                 if (ls->sz >= k) {
14
                     ls->split(k, l, r);
15
16
                     r = N(r, rs, v, fill);
17
                 } else {
                     rs->split(k - ls->sz - fill, l, r);
18
19
                     l = N(ls, l, v, fill);
                 }
20
21
            }
22
23
        } *const null = new P{0, 0, 0, 0, 0, 0, 1};
24
25
        P* N(P* ls, P* rs, LL v, bool fill) {
26
            ls->cnt++; rs->cnt++;
27
28
            return new P{ls, rs, ls->sz + rs->sz + fill, v, ls->sum + rs->sum + v, fill, 1);
        }
29
30
        P* merge(P* a, P* b) {
31
            if (a == null) return b;
32
33
            if (b == null) return a;
            if (rand() % (a->sz + b->sz) < a->sz)
34
                return N(a->ls, merge(a->rs, b), a->v, a->fill);
35
            else
36
37
                 return N(merge(a, b->ls), b->rs, b->v, b->fill);
38
39
40
        void go(P* o, int x, int y, P*& l, P*& m, P*& r) {
            o->split(y, l, r);
41
            l->split(x - 1, l, m);
42
43
        }
    }
44
```

#### 可回滚并查集

- 注意这个不是可持久化并查集
- 查找时不进行路径压缩
- 复杂度靠按秩合并解决

```
namespace uf {
        int fa[maxn], sz[maxn];
        int undo[maxn], top;
3
        void init() { memset(fa, -1, sizeof fa); memset(sz, 0, sizeof sz); top = 0; }
        int findset(int x) { while (fa[x] != -1) x = fa[x]; return x; }
5
        bool join(int x, int y) {
            x = findset(x); y = findset(y);
            if (x == y) return false;
8
            if (sz[x] > sz[y]) swap(x, y);
            undo[top++] = x;
10
            fa[x] = y;
11
12
            sz[y] += sz[x] + 1;
            return true;
13
14
        inline int checkpoint() { return top; }
15
        void rewind(int t) {
16
            while (top > t) {
17
                int x = undo[--top];
18
19
                sz[fa[x]] = sz[x] + 1;
                fa[x] = -1;
20
            }
```

```
22 }
23 }
```

#### 舞蹈链

- 注意 link 的 y 的范围是 [1, n]
- 注意在某些情况下替换掉 memset
- 精确覆盖

```
struct P {
        P *L, *R, *U, *D;
2
        int x, y;
4
    };
    const int INF = 1E9;
    struct DLX {
    #define TR(i, D, s) for (P*i = s->D; i != s; i = i->D)
10
        static const int M = 2E5;
        P pool[M], *h[M], *r[M], *pit;
11
12
        int sz[M];
        bool solved;
13
14
        stack<int> ans;
        void init(int n) {
15
            pit = pool;
16
17
             ++n;
             solved = false;
18
19
             while (!ans.empty()) ans.pop();
             memset(r, 0, sizeof r);
20
             memset(sz, 0, sizeof sz);
21
             FOR (i, 0, n)
22
                h[i] = new (pit++) P;
23
             FOR (i, 0, n) {
24
                 h[i] \rightarrow L = h[(i + n - 1) \% n];
25
                 h[i] \rightarrow R = h[(i + 1) \% n];
26
                 h[i] -> U = h[i] -> D = h[i];
27
                 h[i]->y = i;
28
29
             }
30
31
        void link(int x, int y) {
32
            sz[y]++;
33
34
             auto p = new (pit++) P;
             p->x = x; p->y = y;
35
             p->U = h[y]->U; p->D = h[y];
36
             p->D->U = p->U->D = p;
37
             if (!r[x]) r[x] = p->L = p->R = p;
38
39
             else {
                 p->L = r[x]; p->R = r[x]->R;
40
41
                 p->L->R = p->R->L = p;
             }
42
        }
43
44
         void remove(P* p) {
45
             p->L->R = p->R; p->R->L = p->L;
             TR (i, D, p)
47
48
                 TR (j, R, i) {
                     j->D->U = j->U; j->U->D = j->D;
49
                     sz[j->y]--;
50
                 }
51
        }
52
53
54
         void recall(P* p) {
             p->L->R = p->R->L = p;
55
56
             TR (i, U, p)
57
                 TR (j, L, i) {
58
                     j->D->U = j->U->D = j;
                      sz[j->y]++;
59
```

```
}
61
62
         bool dfs(int d) {
63
64
             if (solved) return true;
             if (h[0] \rightarrow R == h[0]) return solved = true;
65
66
             int m = INF;
             P* c;
67
             TR (i, R, h[0])
68
                 if (sz[i->y] < m) { m = sz[i->y]; c = i; }
69
             remove(c);
70
71
             TR (i, D, c) {
                 ans.push(i->x);
72
                 TR (j, R, i) remove(h[j->y]);
73
                 if (dfs(d + 1)) return true;
74
                 TR (j, L, i) recall(h[j->y]);
75
                 ans.pop();
77
             recall(c);
             return false;
79
        }
80
    } dlx;
81
        ● 可重复覆盖
    struct P {
1
2
        P *L, *R, *U, *D;
        int x, y;
    };
    const int INF = 1E9;
    struct DLX {
    #define TR(i, D, s) for (P*i = s->D; i != s; i = i->D)
        static const int M = 2E5;
10
        P pool[M], *h[M], *r[M], *pit;
11
12
        int sz[M], vis[M], ans, clk;
        void init(int n) {
13
             clk = 0;
14
             ans = INF;
15
16
             pit = pool;
17
             ++n;
             memset(r, 0, sizeof r);
18
             memset(sz, 0, sizeof sz);
             memset(vis, -1, sizeof vis);
20
             FOR (i, 0, n)
21
22
                h[i] = new (pit++) P;
             FOR (i, 0, n) {
23
                 h[i] \rightarrow L = h[(i + n - 1) \% n];
24
                 h[i] -> R = h[(i + 1) \% n];
25
                 h[i] -> U = h[i] -> D = h[i];
26
27
                 h[i] \rightarrow y = i;
             }
28
29
        }
30
31
         void link(int x, int y) {
            sz[y]++;
32
             auto p = new (pit++) P;
33
34
             p->x = x; p->y = y;
             p->U = h[y]->U; p->D = h[y];
35
36
             p->D->U = p->U->D = p;
             if (!r[x]) r[x] = p->L = p->R = p;
37
38
                 p->L = r[x]; p->R = r[x]->R;
39
                 p->L->R = p->R->L = p;
40
             }
41
        }
42
43
        void remove(P* p) {
44
             TR (i, D, p) {
45
46
                 i->L->R = i->R;
                 i->R->L = i->L;
47
```

```
}
48
49
50
         void recall(P* p) {
51
52
             TR (i, U, p)
                 i->L->R = i->R->L = i;
53
54
55
         int eval() {
56
57
             ++clk;
             int ret = 0;
58
59
             TR (i, R, h[0])
                 if (vis[i->y] != clk) {
60
                      ++ret;
61
                      vis[i->y] = clk;
62
                      TR (j, D, i)
63
64
                          TR (k, R, j)
                               vis[k->y] = clk;
65
                 }
             return ret;
67
        }
68
69
        void dfs(int d) {
70
             if (h[0] \rightarrow R == h[0]) { ans = min(ans, d); return; }
             if (eval() + d >= ans) return;
72
73
             P* c;
             int m = INF;
74
             TR (i, R, h[0])
75
                 if (sz[i->y] < m) { m = sz[i->y]; c = i; }
             TR (i, D, c) {
77
                 remove(i);
78
                 TR (j, R, i) remove(j);
79
80
                 dfs(d + 1);
81
                 TR (j, L, i) recall(j);
                 recall(i);
82
83
        }
84
    } dlx;
    CDQ 分治
    const int maxn = 2E5 + 100;
1
    struct P {
        int x, y;
        int* f;
        bool d1, d2;
    } a[maxn], b[maxn], c[maxn];
    int f[maxn];
    void go2(int l, int r) {
        if (l + 1 == r) return;
10
11
         int m = (l + r) >> 1;
        go2(l, m); go2(m, r);
12
        FOR (i, l, m) b[i].d2 = 0;
13
        FOR (i, m, r) b[i].d2 = 1;
        merge(b + l, b + m, b + m, b + r, c + l, [](\textbf{const} \ P\& \ a, \ \textbf{const} \ P\& \ b) -> \textbf{bool} \ \{
15
                 if (a.y != b.y) return a.y < b.y;</pre>
16
                 return a.d2 > b.d2;
17
             });
18
19
        int mx = -1;
         FOR (i, l, r) {
20
21
             if (c[i].d1 && c[i].d2) *c[i].f = max(*c[i].f, mx + 1);
             if (!c[i].d1 && !c[i].d2) mx = max(mx, *c[i].f);
22
23
        FOR (i, l, r) b[i] = c[i];
24
    }
25
    void go1(int l, int r) { // [l, r)
27
        if (l + 1 == r) return;
```

```
int m = (l + r) >> 1;
29
30
        go1(l, m);
        FOR (i, l, m) a[i].d1 = 0;
31
32
        FOR (i, m, r) a[i].d1 = 1;
        copy(a + l, a + r, b + l);
        sort(b + l, b + r, [](const P& a, const P& b)->bool {
34
                 if (a.x != b.x) return a.x < b.x;</pre>
35
                 return a.d1 > b.d1;
36
            });
37
38
        go2(l, r);
        go1(m, r);
39
40
   }
        • k维LIS
    struct P {
1
        int v[K];
2
        LL f;
        bool d[K];
   } o[N << 10];
   P* a[K][N << 10];
    int k;
    void go(int now, int l, int r) {
        if (now == 0) {
            if (l + 1 == r) return;
            int m = (l + r) / 2;
11
12
            go(now, l, m);
            FOR (i, l, m) a[now][i]->d[now] = 0;
13
            FOR (i, m, r) a[now][i]->d[now] = 1;
14
            copy(a[now] + l, a[now] + r, a[now + 1] + l);
            sort(a[now + 1] + l, a[now + 1] + r, [now](const P* a, const P* b){}
16
                 if (a->v[now] != b->v[now]) return a->v[now] < b->v[now];
17
                 return a->d[now] > b->d[now];
18
            });
19
20
            go(now + 1, l, r);
            go(now, m, r);
21
22
        } else {
            if (l + 1 == r) return;
23
            int m = (l + r) / 2;
24
25
            go(now, l, m); go(now, m, r);
            FOR (i, l, m) a[now][i]->d[now] = 0;
26
27
            FOR (i, m, r) a[now][i] -> d[now] = 1;
            merge(a[now] + l, a[now] + m, a[now] + m, a[now] + r, a[now + 1] + l, [now](const P* a, const P* b){
28
                 if (a->v[now] != b->v[now]) return a->v[now] < b->v[now];
29
                 return a->d[now] > b->d[now];
30
31
            });
            copy(a[now + 1] + l, a[now + 1] + r, a[now] + l);
32
            if (now < k - 2) {
33
34
                 go(now + 1, l, r);
            } else {
35
                 LL sum = 0;
36
37
                 FOR (i, l, r) {
                     dbg(a[now][i]->v[0], a[now][i]->v[1], a[now][i]->f,
38
39
                                        a[now][i]->d[0], a[now][i]->d[1]);
                     int cnt = 0;
40
                     FOR (j, 0, now + 1) cnt += a[now][i]->d[j];
41
                     if (cnt == 0) {
42
                         sum += a[now][i]->f;
43
44
                     } else if (cnt == now + 1) {
                         a[now][i] \rightarrow f = (a[now][i] \rightarrow f + sum) % MOD;
45
                }
47
            }
48
        }
49
   }
50
```

## 哈希表

必须初始化

• 备选素数 1572869, 3145739, 6291469, 12582917, 25165843, 50331653

```
const LL HASH MOD=1572869;
1
    LL key[HASH_MOD], val[HASH_MOD];
    int head[HASH_MOD], next[HASH_MOD];
    struct Hash {
        int sz;
        void init() {
            memset(head, -1, sizeof head);
            sz = 0;
        LL insert(LL x, LL y) {
10
11
            int k = x % HASH_MOD;
            key[sz] = x;
12
            val[sz] = y;
13
            next[sz] = head[k];
            head[k] = sz++;
15
16
17
        LL find(LL x) {
            int k = x % HASH_MOD;
18
19
            for (int i = head[k]; i != -1; i = next[i])
                if (key[i] == x)
20
21
                    return val[i];
            return -1;
22
        }
23
    };
```

## 笛卡尔树

```
void build(const vector<int>& a) {
        static P *stack[M], *x, *last;
2
        int p = 0;
3
        FOR (i, 0, a.size()) {
            x = new P(i + 1, a[i]);
5
            last = null;
            while (p && stack[p - 1]->v > x->v) {
                stack[p - 1]->maintain();
                last = stack[--p];
10
            if (p) stack[p - 1]->rs = x;
11
            x->ls = last;
12
            stack[p++] = x;
14
        while (p)
15
           stack[--p]->maintain();
16
        rt = stack[0];
17
18
   }
    void build() {
1
        static int s[N], last;
2
        int p = 0;
3
        FOR (x, 1, n + 1) {
4
            last = 0;
            while (p \&\& val[s[p - 1]] > val[x]) last = s[--p];
            if (p) G[s[p - 1]][1] = x;
            if (last) G[x][0] = last;
            s[p++] = x;
11
        rt = s[0];
   }
12
```

## Trie

- Trie 二进制版
- M 为二进制的位数
- 使用前必须初始化

```
struct Trie2 {
1
2
        int ch[N * M][2], sz;
        void init() {
3
            memset(ch, 0, sizeof ch);
            sz = 1;
        void insert(LL x) {
            int u = 0;
            FORD (i, M, -1) {
                bool b = x & (1LL << i);
                if (!ch[u][b])
11
12
                     ch[u][b] = sz++;
13
                u = ch[u][b];
            }
14
        }
15
   } trie;
```

# pb\_ds

- 优先队列
- binary\_heap\_tag
- pairing\_heap\_tag 支持修改
- thin\_heap\_tag 如果修改只有 increase 则较快,不支持 join

```
#include<ext/pb_ds/priority_queue.hpp>
    template<typename _Tv,</pre>
       typename Cmp_Fn = std::less<_Tv>,
       typename Tag = pairing_heap_tag,
       typename _Alloc = std::allocator<char> >
   class priority_queue;
   #include<ext/pb_ds/priority_queue.hpp>
   using namespace __gnu_pbds;
   typedef __gnu_pbds::priority_queue<LL, less<LL>, pairing_heap_tag> PQ;
    __gnu_pbds::priority_queue<int, cmp, pairing_heap_tag>::point_iterator it;
   PQ pq, pq2;
   int main() {
       auto it = pq.push(2);
10
        pq.push(3);
        assert(pq.top() == 3);
11
12
        pq.modify(it, 4);
        assert(pq.top() == 4);
13
       pq2.push(5);
        pq.join(pq2);
15
        assert(pq.top() == 5);
16
17
   }
```

- 树
- ov\_tree\_tag
- rb\_tree\_tag
- splay\_tree\_tag
- mapped: null\_type 或 null\_mapped\_type (旧版本) 为空
- Node\_Update 为 tree\_order\_statistics\_node\_update 时才可以 find\_by\_order & order\_of\_key
- find\_by\_order 找 order + 1 小的元素(其实都是从 0 开始计数),或者有 order 个元素比它小的 key
- order\_of\_key 有多少个比 r\_key 小的元素
- join & split

```
template<typename Key, typename Mapped, typename Cmp_Fn = std::less<Key>,
       typename Tag = rb_tree_tag,
2
       template<typename Node_CItr, typename Node_Itr,</pre>
                typename Cmp_Fn_, typename _Alloc_>
       class Node_Update = null_node_update,
       typename _Alloc = std::allocator<char> >
   class tree
   #include <ext/pb_ds/assoc_container.hpp>
   using namespace __gnu_pbds;
   using Tree = tree<int, null_type, less<int>, rb_tree_tag, tree_order_statistics_node_update>;
11
   Tree t;

    hash table

   #include<ext/pb_ds/assoc_container.hpp>
   #include<ext/pb_ds/hash_policy.hpp>
   using namespace __gnu_pbds;
   gp_hash_table<int, int> mp;
   cc_hash_table<int, int> mp;
```

## **Link-Cut Tree**

- 图中相邻的结点在伸展树中不一定是父子关系
- 遇事不决 make\_root
- 跑左右儿子的时候不要忘记 down

```
namespace lct {
        extern struct P *const null;
        const int M = N;
3
        struct P {
4
            P *fa, *ls, *rs;
            int v, maxv;
            bool rev;
            bool has_fa() { return fa->ls == this || fa->rs == this; }
            bool d() { return fa->ls == this; }
            P * \& c(bool x) \{ return x ? ls : rs; \}
11
            void do_rev() {
                if (this == null) return;
13
                rev ^= 1;
14
                swap(ls, rs);
15
16
17
            P* up() {
                maxv = max(v, max(ls->maxv, rs->maxv));
18
                return this;
19
20
            void down() {
21
22
                if (rev) {
                     rev = 0:
23
24
                     ls->do_rev(); rs->do_rev();
                }
25
26
            void all_down() { if (has_fa()) fa->all_down(); down(); }
27
28
        } *const null = new P{0, 0, 0, 0, 0, 0}, pool[M], *pit = pool;
        void rot(P* o) {
30
            bool dd = o->d();
            P *f = o > fa, *t = o > c(!dd);
32
            if (f->has_fa()) f->fa->c(f->d()) = o; o->fa = f->fa;
33
34
            if (t != null) t->fa = f; f->c(dd) = t;
            o->c(!dd) = f->up(); f->fa = o;
35
        void splay(P* o) {
37
38
            o->all_down();
            while (o->has_fa()) {
39
                if (o->fa->has_fa())
40
41
                     rot(o->d() ^ o->fa->d() ? o : o->fa);
```

```
rot(o);
42
43
            }
            o->up();
44
45
        }
        void access(P* u, P* v = null) {
            if (u == null) return;
47
48
            splay(u); u->rs = v;
            access(u->up()->fa, u);
49
50
        void make_root(P* o) {
51
            access(o); splay(o); o->do_rev();
52
53
        void split(P* o, P* u) {
54
            make_root(o); access(u); splay(u);
55
        }
56
        void link(P* u, P* v) {
57
58
            make_root(u); u->fa = v;
59
        void cut(P* u, P* v) {
            split(u, v);
61
            u->fa = v->ls = null; v->up();
62
63
        bool adj(P* u, P* v) {
64
            split(u, v);
            return v->ls == u && u->ls == null && u->rs == null;
66
67
        bool linked(P* u, P* v) {
68
            split(u, v);
69
            return u == v || u->fa != null;
71
        P* findrt(P* o) {
72
            access(o); splay(o);
73
74
            while (o->ls != null) o = o->ls;
75
            return o;
76
        P* findfa(P* rt, P* u) {
77
            split(rt, u);
78
            u = u \rightarrow ls;
79
            while (u->rs != null) {
                 u = u \rightarrow rs;
81
82
                 u->down();
            }
83
            return u;
84
85
        }
   }
86
        • 维护子树大小
    P* up() {
        sz = ls->sz + rs->sz + _sz + 1;
2
        return this;
3
4
    void access(P* u, P* v = null) {
        if (u == null) return;
        splay(u);
        u->_sz += u->rs->sz - v->sz;
        u \rightarrow rs = v;
10
        access(u->up()->fa, u);
11
    }
    void link(P* u, P* v) {
12
13
        split(u, v);
        u->fa = v; v->\_sz += u->sz;
14
15
        v->up();
    }
16
    莫队
        • [1, r)
    while (l > q.l) mv(--l, 1);
```

```
while (r < q.r) mv(r++, 1);</pre>
    while (l < q.l) mv(l++, -1);
3
    while (r > q.r) mv(--r, -1);
        • 树上莫队
        ● 注意初始状态 u = v = 1, flip(1)
    struct Q {
        int u, v, idx;
2
        bool operator < (const Q& b) const {</pre>
3
             const Q& a = *this;
             return blk[a.u] < blk[b.u] || (blk[a.u] == blk[b.u] && in[a.v] < in[b.v]);</pre>
5
        }
    };
7
    void dfs(int u = 1, int d = 0) {
        static int S[maxn], sz = 0, blk_cnt = 0, clk = 0;
10
11
        in[u] = clk++;
        dep[u] = d;
12
        int btm = sz;
13
        for (int v: G[u]) {
14
             if (v == fa[u]) continue;
            fa[v] = u;
16
            dfs(v, d + 1);
17
             if (sz - btm >= B) {
18
                 while (sz > btm) blk[S[--sz]] = blk_cnt;
19
                 ++blk_cnt;
20
            }
21
22
        S[sz++] = u;
23
        if (u == 1) while (sz) blk[S[--sz]] = blk_cnt - 1;
24
26
27
    void flip(int k) {
28
        dbg(k);
        if (vis[k]) {
29
30
            // ...
        } else {
31
32
            // ...
33
        vis[k] ^= 1;
34
    }
35
36
37
    void go(int& k) {
        if (bug == -1) {
38
             if (vis[k] && !vis[fa[k]]) bug = k;
39
            if (!vis[k] && vis[fa[k]]) bug = fa[k];
40
41
42
        flip(k);
        k = fa[k];
43
44
    }
45
46
    void mv(int a, int b) {
47
        bug = -1;
        if (vis[b]) bug = b;
48
49
        if (dep[a] < dep[b]) swap(a, b);</pre>
        while (dep[a] > dep[b]) go(a);
50
        while (a != b) {
51
            go(a); go(b);
52
53
54
        go(a); go(bug);
    }
55
    for (Q& q: query) {
57
        mv(u, q.u); u = q.u;
58
59
        mv(v, q.v); v = q.v;
        ans[q.idx] = Ans;
60
61
    }
```

# 数学

## 矩阵运算

```
struct Mat {
1
        static const LL M = 2;
        LL v[M][M];
3
        Mat() { memset(v, 0, sizeof v); }
        void eye() { FOR (i, 0, M) v[i][i] = 1; }
5
        LL* operator [] (LL x) { return v[x]; }
        const LL* operator [] (LL x) const { return v[x]; }
        Mat operator * (const Mat& B) {
            const Mat& A = *this;
            Mat ret;
10
            FOR (i, 0, M)
11
                FOR (j, 0, M)
12
                     FOR (k, 0, M)
13
                           ret[i][j] = (ret[i][j] + A[i][k] * B[k][j]) % MOD;
14
15
            return ret;
16
        Mat pow(LL n) const {
17
            Mat A = *this, ret; ret.eye();
18
            for (; n; n >>= 1, A = A \star A)
19
                if (n & 1) ret = ret * A;
20
21
            return ret;
22
        Mat operator + (const Mat& B) {
23
24
            const Mat& A = *this;
            Mat ret;
25
26
            FOR (i, 0, M)
                FOR (j, \Theta, M)
27
28
                     ret[i][j] = (A[i][j] + B[i][j]) % MOD;
            return ret;
29
30
31
        void prt() const {
            FOR (i, 0, M)
32
                FOR (j, ⊙, M)
                     printf("%lld%c", (*this)[i][j], j == M - 1 ? '\n' : ' ');
34
35
   };
    筛
       线性筛
    const LL p_max = 1E6 + 100;
    LL pr[p_max], p_sz;
    void get_prime() {
        static bool vis[p_max];
        FOR (i, 2, p_max) {
            if (!vis[i]) pr[p_sz++] = i;
            FOR (j, 0, p_sz) {
                if (pr[j] * i >= p_max) break;
                vis[pr[j] * i] = 1;
9
                if (i % pr[j] == 0) break;
            }
11
12
        }
   }
13
       • 线性筛+欧拉函数
    const LL p_max = 1E5 + 100;
    LL phi[p_max] = \{-1, 1\};
    void get_phi() {
        static bool vis[p_max];
        static LL prime[p_max], p_sz, d;
5
        FOR (i, 2, p_max) {
            if (!vis[i]) {
```

```
prime[p_sz^{++}] = i;
8
                phi[i] = i - 1;
10
            for (LL j = 0; j < p_sz && (d = i * prime[j]) < p_max; ++j) {</pre>
11
                vis[d] = 1;
                if (i % prime[j] == 0) {
13
                     phi[d] = phi[i] * prime[j];
14
15
                    break:
16
                else phi[d] = phi[i] * (prime[j] - 1);
17
            }
18
19
        }
   }
20
       • 线性筛+莫比乌斯函数
    const LL p_max = 1E5 + 100;
1
    LL mu[p_max] = \{-1, 1\};
    void get_mu() {
        static bool vis[p_max];
5
        static LL prime[p_max], p_sz, d;
        mu[1] = 1;
6
        FOR (i, 2, p_max) {
            if (!vis[i]) {
                prime[p_sz++] = i;
                mu[i] = -1;
10
11
            for (LL j = 0; j < p_sz && (d = i * prime[j]) < p_max; ++j) {</pre>
12
                vis[d] = 1;
13
                if (i % prime[j] == 0) {
                     mu[d] = 0;
15
                     break;
17
                else mu[d] = -mu[i];
18
            }
19
        }
20
21
   }
    亚线性筛
    min_25
    namespace min25 {
1
2
        const int M = 1E6 + 100;
        LL B, N;
3
        // g(x)
        inline LL pg(LL x) { return 1; }
6
        inline LL ph(LL x) { return x % MOD; }
        // Sum[g(i), \{x, 2, x\}]
        inline LL psg(LL x) { return x % MOD - 1; }
        inline LL psh(LL x) {
10
11
            static LL inv2 = (MOD + 1) / 2;
12
            x = x \% MOD;
            return x * (x + 1) % MOD * inv2 % MOD - 1;
13
        // f(pp=p^k)
15
        inline LL fpk(LL p, LL e, LL pp) { return (pp - pp / p) % MOD; }
16
17
        // f(p) = fgh(g(p), h(p))
        inline LL fgh(LL g, LL h) { return h - g; }
18
19
        LL pr[M], pc, sg[M], sh[M];
20
21
        void get_prime(LL n) {
            static bool vis[M]; pc = 0;
22
23
            FOR (i, 2, n + 1) {
                if (!vis[i]) {
24
                    pr[pc++] = i;
25
                     sg[pc] = (sg[pc - 1] + pg(i)) % MOD;
26
                     sh[pc] = (sh[pc - 1] + ph(i)) % MOD;
27
```

```
28
29
                 FOR (j, 0, pc) {
                     if (pr[j] * i > n) break;
30
                     vis[pr[j] * i] = 1;
31
                     if (i % pr[j] == 0) break;
                 }
33
            }
34
        }
35
36
        LL w[M];
37
        LL id1[M], id2[M], h[M], g[M];
38
39
        inline LL id(LL x) { return x \le B ? id1[x] : id2[N / x]; }
40
        LL go(LL x, LL k) {
41
            if (x <= 1 || (k >= 0 && pr[k] > x)) return 0;
42
            LL t = id(x);
43
44
            LL ans = fgh((g[t] - sg[k + 1]), (h[t] - sh[k + 1]));
            FOR (i, k + 1, pc) {
45
                 LL p = pr[i];
                 if (p * p > x) break;
47
                 ans -= fgh(pg(p), ph(p));
48
49
                 for (LL pp = p, e = 1; pp \le x; ++e, pp = pp * p)
                     ans += fpk(p, e, pp) * (1 + go(x / pp, i)) % MOD;
50
            }
            return ans % MOD;
52
53
        }
54
        LL solve(LL _N) {
55
            N = N;
            B = sqrt(N + 0.5);
57
            get_prime(B);
58
            int sz = 0;
59
            for (LL l = 1, v, r; l <= N; l = r + 1) {
60
                 v = N / l; r = N / v;
                w[sz] = v; g[sz] = psg(v); h[sz] = psh(v);
62
                 if (v <= B) id1[v] = sz; else id2[r] = sz;</pre>
63
64
                 sz++;
65
            FOR (k, 0, pc) {
                 LL p = pr[k];
67
68
                 FOR (i, 0, sz) {
                     LL v = w[i]; if (p * p > v) break;
69
                     LL t = id(v / p);
70
                     g[i] = (g[i] - (g[t] - sg[k]) * pg(p)) % MOD;
71
                     h[i] = (h[i] - (h[t] - sh[k]) * ph(p)) % MOD;
72
73
                }
74
            return (go(N, -1) % MOD + MOD + 1) % MOD;
        }
76
77
   }
    杜教筛
    namespace dujiao {
1
2
        const int M = 5E6;
        LL f[M] = \{0, 1\};
3
        void init() {
4
            static bool vis[M];
            static LL pr[M], p_sz, d;
            FOR (i, 2, M) {
                 if (!vis[i]) { pr[p_sz++] = i; f[i] = -1; }
                 FOR (j, 0, p_sz) {
                     if ((d = pr[j] * i) >= M) break;
                     vis[d] = 1;
11
                     if (i % pr[j] == 0) {
12
                         f[d] = 0;
13
                         break;
14
                     } else f[d] = -f[i];
15
                 }
16
            }
17
```

```
FOR (i, 2, M) f[i] += f[i - 1];
18
19
        inline LL s_fg(LL n) { return 1; }
20
        inline LL s_g(LL n) { return n; }
21
        LL N, rd[M];
23
        bool vis[M];
24
        LL go(LL n) {
25
             if (n < M) return f[n];</pre>
26
27
            LL id = N / n;
            if (vis[id]) return rd[id];
28
             vis[id] = true;
            LL& ret = rd[id] = s_fg(n);
30
            for (LL l = 2, v, r; l <= n; l = r + 1) {
31
                 v = n / l; r = n / v;
32
                 ret -= (s_g(r) - s_g(l - 1)) * go(v);
33
34
            return ret;
35
        LL solve(LL n) {
37
             N = n;
38
39
            memset(vis, 0, sizeof vis);
             return go(n);
40
    }
42
```

#### 素数测试

- 前置: 快速乘、快速幂
- int 范围内只需检查 2, 7, 61
- long long 范围 2, 325, 9375, 28178, 450775, 9780504, 1795265022
- 3E15 内 2, 2570940, 880937, 610386380, 4130785767
- 4E13 内 2, 2570940, 211991001, 3749873356
- http://miller-rabin.appspot.com/

```
bool checkQ(LL a, LL n) {
        if (n == 2 || a >= n) return 1;
        if (n == 1 || !(n & 1)) return 0;
        LL d = n - 1;
        while (!(d & 1)) d >>= 1;
        LL t = bin(a, d, n); // 不一定需要快速乘
        while (d != n - 1 && t != 1 && t != n - 1) {
            t = mul(t, t, n);
8
            d <<= 1;
10
        return t == n - 1 || d & 1;
   }
12
13
14
   bool primeQ(LL n) {
        static vector<LL> t = {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
15
        if (n <= 1) return false;</pre>
        for (LL k: t) if (!checkQ(k, n)) return false;
17
        return true;
18
   }
19
```

## 线性递推

```
FORD (i, k - 2, - 1) {
10
11
                 FOR (j, 0, k)
                    up(r[i + j], r[i + k] * m[j]);
12
13
                 r.pop_back();
14
            }
            return r;
15
16
17
        V pow(LL n, const V& m) {
18
            int k = (int)m.size() - 1; assert(m[k] == -1 || m[k] == MOD - 1);
19
            V r(k), x(k); r[0] = x[1] = 1;
20
21
            for (; n; n >>= 1, x = mul(x, x, m, k))
                if (n & 1) r = mul(x, r, m, k);
22
            return r;
23
        }
24
25
26
        LL go(const V& a, const V& x, LL n) {
            // a: (-1, a1, a2, ..., ak).reverse
27
            // x: x1, x2, ..., xk
            // x[n] = sum[a[i]*x[n-i],{i,1,k}]
29
            int k = (int)a.size() - 1;
30
            if (n \le k) return x[n - 1];
31
            V r = pow(n - 1, a);
32
            LL ans = 0;
            FOR (i, 0, k)
34
35
                 up(ans, r[i] * x[i]);
36
            return ans;
        }
37
        V BM(const V& x) {
39
            V a = \{-1\}, b = \{233\};
40
            FOR (i, 1, x.size()) {
41
                 b.push_back(0);
42
43
                 LL d = 0, la = a.size(), lb = b.size();
                 FOR (j, 0, la) up(d, a[j] * x[i - la + 1 + j]);
44
45
                 if (d == 0) continue;
                 V t; for (auto& v: b) t.push_back(d * v % MOD);
46
                 FOR (j, 0, a.size()) up(t[lb - 1 - j], a[la - 1 - j]);
47
48
                 if (lb > la) {
                     b = a;
49
50
                     LL inv = -get_inv(d, MOD);
                     for (auto& v: b) v = v * inv % MOD;
51
                 }
52
53
                 a.swap(t);
54
55
            for (auto& v: a) up(v, MOD);
            return a;
56
        }
   }
58
```

## 扩展欧几里得

- 求 ax + by = gcd(a, b) 的一组解
- 如果 a 和 b 互素,那么 x 是 a 在模 b 下的逆元
- 注意 x 和 y 可能是负数

#### 类欧几里得

- $m = \lfloor \frac{an+b}{c} \rfloor$ .
- $f(a,b,c,n) = \sum_{i=0}^{n} \lfloor \frac{ai+b}{c} \rfloor$ : 当  $a \geq c$  or  $b \geq c$  时, $f(a,b,c,n) = (\frac{a}{c})n(n+1)/2 + (\frac{b}{c})(n+1) + f(a \bmod c, b \bmod c, c, n)$ ; 否则 f(a,b,c,n) = nm f(c,c-b-1,a,m-1)。
- $g(a,b,c,n) = \sum_{i=0}^{n} i \lfloor \frac{ai+b}{c} \rfloor$ : 当  $a \geq c$  or  $b \geq c$  时,  $g(a,b,c,n) = (\frac{a}{c})n(n+1)(2n+1)/6 + (\frac{b}{c})n(n+1)/2 + g(a \mod c,b \mod c,c,n)$ ;否则  $g(a,b,c,n) = \frac{1}{2}(n(n+1)m-f(c,c-b-1,a,m-1)-h(c,c-b-1,a,m-1))$ 。
- $h(a,b,c,n) = \sum_{i=0}^{n} \lfloor \frac{ai+b}{c} \rfloor^2$ : 当  $a \geq c$  or  $b \geq c$  时, $h(a,b,c,n) = (\frac{a}{c})^2 n(n+1)(2n+1)/6 + (\frac{b}{c})^2 (n+1) + (\frac{a}{c})(\frac{b}{c})n(n+1) + h(a \bmod c, b \bmod c, c, n) + 2(\frac{a}{c})g(a \bmod c, b \bmod c, c, n) + 2(\frac{b}{c})f(a \bmod c, b \bmod c, c, n)$ ; 否则 h(a,b,c,n) = nm(m+1) 2g(c,c-b-1,a,m-1) 2f(c,c-b-1,a,m-1) f(a,b,c,n)。

## 逆元

- $ax \equiv 1 \pmod{p}$
- 如果 p 不是素数, 使用拓展欧几里得
- 模数是素数, 求一个数的逆元
- 前置模板: 快速幂

```
inline LL get_inv(LL x, LL p) { return bin(x, p - 2, p); }
```

预处理

1 - n

的逆元

```
LL invf[M], fac[M] = {1};

void fac_inv_init(LL n, LL p) {

FOR (i, 1, n)

fac[i] = i * fac[i - 1] % p;

invf[n - 1] = bin(fac[n - 1], p - 2, p);

FORD (i, n - 2, -1)

invf[i] = invf[i + 1] * (i + 1) % p;
```

#### 组合数

- 如果数较小,模较大时使用逆元
- 前置模板: 逆元-预处理阶乘及其逆元

```
inline LL C(LL n, LL m) { // n >= m >= 0
return n < m || m < 0 ? 0 : fac[n] * invf[m] % MOD * invf[n - m] % MOD;
}</pre>
```

- 如果模数较小,数字较大,使用 Lucas 定理
- 前置模板可选 1: 求组合数(如果使用阶乘逆元,需 fac\_inv\_init(MOD, MOD);)
- 前置模板可选 2: 模数不固定下使用, 无法单独使用。

```
return ret;
7
8
   }
   LL Lucas(LL n, LL m) { // m >= n >= 0
       return m ? C(n % MOD, m % MOD) * Lucas(n / MOD, m / MOD) % MOD : 1;
2
   }
3
      • 组合数预处理
   LL C[M][M];
   void init_C(int n) {
2
       FOR (i, 0, n) {
           C[i][0] = C[i][i] = 1;
5
           FOR (j, 1, i)
              C[i][j] = (C[i - 1][j] + C[i - 1][j - 1]) % MOD;
       }
   }
   第二类斯特灵数
   S[0][0] = 1;
1
       FOR (i, 1, N)
2
           FOR (j, 1, i + 1) S[i][j] = (S[i - 1][j - 1] + j * S[i - 1][j]) % MOD;
   FFT & NTT & FWT
      • NTT
      ● 前置: 快速幂
   LL wn[N << 2], rev[N << 2];
```

```
2
    int NTT_init(int n_) {
        int step = 0; int n = 1;
        for ( ; n < n_; n <<= 1) ++step;</pre>
4
        FOR (i, 1, n)
            rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (step - 1));
        int g = bin(G, (MOD - 1) / n, MOD);
        wn[0] = 1;
        for (int i = 1; i <= n; ++i)</pre>
           wn[i] = wn[i - 1] * g % MOD;
        return n:
11
    }
12
13
    void NTT(LL a[], int n, int f) {
14
        FOR (i, 0, n) if (i < rev[i])
15
            std::swap(a[i], a[rev[i]]);
16
17
        for (int k = 1; k < n; k <<= 1) {
            for (int i = 0; i < n; i += (k << 1)) {</pre>
18
                 int t = n / (k << 1);</pre>
19
20
                 FOR (j, 0, k) {
                     LL w = f == 1 ? wn[t * j] : wn[n - t * j];
21
                     LL x = a[i + j];
                     LL y = a[i + j + k] * w % MOD;
23
                     a[i + j] = (x + y) \% MOD;
24
                     a[i + j + k] = (x - y + MOD) \% MOD;
25
                 }
26
27
            }
28
        if (f == -1) {
29
            LL ninv = get_inv(n, MOD);
30
             FOR (i, 0, n)
31
                 a[i] = a[i] * ninv % MOD;
32
        }
33
34
    }
```

- FFT
- n 需补成 2 的幂 (n 必须超过 a 和 b 的最高指数之和)

```
typedef double LD;
1
2
    const LD PI = acos(-1);
    struct C {
        LD r, i;
        C(LD r = 0, LD i = 0): r(r), i(i) {}
    };
6
    C operator + (const C& a, const C& b) {
        return C(a.r + b.r, a.i + b.i);
    C operator - (const C& a, const C& b) {
10
        return C(a.r - b.r, a.i - b.i);
11
12
    C operator \star (const C& a, const C& b) {
13
        return C(a.r * b.r - a.i * b.i, a.r * b.i + a.i * b.r);
14
15
16
17
    void FFT(C x[], int n, int p) {
        for (int i = 0, t = 0; i < n; ++i) {
18
19
            if (i > t) swap(x[i], x[t]);
            for (int j = n >> 1; (t ^= j) < j; j >>= 1);
20
21
        for (int h = 2; h <= n; h <<= 1) {</pre>
22
            C wn(cos(p * 2 * PI / h), sin(p * 2 * PI / h));
23
            for (int i = 0; i < n; i += h) {</pre>
                C w(1, 0), u;
25
                 for (int j = i, k = h >> 1; j < i + k; ++j) {
26
                     u = x[j + k] * w;
27
                     x[j + k] = x[j] - u;
28
                     x[j] = x[j] + u;
                     w = w * wn;
30
                }
31
            }
32
33
        if (p == -1)
34
            FOR (i, 0, n)
35
                x[i].r /= n;
36
    }
37
38
    void conv(C a[], C b[], int n) {
39
        FFT(a, n, 1);
40
41
        FFT(b, n, 1);
        FOR (i, 0, n)
42
            a[i] = a[i] * b[i];
43
44
        FFT(a, n, -1);
   }
45
        • FWT
    template<typename T>
    void fwt(LL a[], int n, T f) {
2
        for (int d = 1; d < n; d *= 2)</pre>
3
            for (int i = 0, t = d * 2; i < n; i += t)
                FOR (j, 0, d)
                    f(a[i + j], a[i + j + d]);
    }
    void AND(LL& a, LL& b) { a += b; }
    void OR(LL& a, LL& b) { b += a; }
    void XOR (LL& a, LL& b) {
        LL x = a, y = b;
12
        a = (x + y) \% MOD;
        b = (x - y + MOD) \% MOD;
14
    }
15
    simpson 自适应积分
    LD simpson(LD l, LD r) {
1
        LD c = (l + r) / 2;
        return (f(l) + 4 * f(c) + f(r)) * (r - l) / 6;
    }
```

```
6
   LD asr(LD l, LD r, LD eps, LD S) {
       LD m = (l + r) / 2;
        LD L = simpson(l, m), R = simpson(m, r);
       if (fabs(L + R - S) < 15 * eps) return L + R + (L + R - S) / 15;</pre>
       return asr(l, m, eps / 2, L) + asr(m, r, eps / 2, R);
10
11
12
   LD asr(LD l, LD r, LD eps) { return asr(l, r, eps, simpson(l, r)); }
13
       • FWT
    template<typename T>
    void fwt(LL a[], int n, T f) {
2
        for (int d = 1; d < n; d *= 2)</pre>
           for (int i = 0, t = d * 2; i < n; i += t)
                FOR (j, 0, d)
5
                     f(a[i + j], a[i + j + d]);
   }
   auto f = [](LL& a, LL& b) { // xor
           LL x = a, y = b;
10
           a = (x + y) \% MOD;
11
           b = (x - y + MOD) \% MOD;
12
   };
    快速乘
   LL mul(LL a, LL b, LL m) {
       LL ret = 0;
2
       while (b) {
3
           if (b & 1) {
4
               ret += a;
5
                if (ret >= m) ret -= m;
           }
           a += a;
           if (a >= m) a -= m;
           b >>= 1;
10
11
       return ret;
12
   }
       • O(1)
   LL mul(LL u, LL v, LL p) {
       return (u * v - LL((long double) u * v / p) * p + p) % p;
2
   }
    快速幂
       • 如果模数是素数,则可在函数体内加上 n %= MOD - 1; (费马小定理)。
   LL bin(LL x, LL n, LL MOD) {
2
       LL ret = MOD != 1;
        for (x \% = MOD; n; n >>= 1, x = x * x \% MOD)
3
           if (n & 1) ret = ret * x % MOD;
        return ret;
   }
       防爆 LL
       • 前置模板: 快速乘
   LL bin(LL x, LL n, LL MOD) {
       LL ret = MOD != 1;
2
        for (x \%= MOD; n; n >>= 1, x = mul(x, x, MOD))
3
           if (n & 1) ret = mul(ret, x, MOD);
       return ret;
5
   }
```

#### 高斯消元

- n 方程个数, m 变量个数, a 是 n \* (m + 1) 的增广矩阵, free 是否为自由变量
- 返回自由变量个数、-1 无解、-2 无整数解
- 浮点数版本

```
typedef double LD;
    const LD eps = 1E-10;
    const int maxn = 2000 + 10;
    int n, m;
    LD a[maxn][maxn], x[maxn];
    bool free_x[maxn];
    inline int sgn(LD x) { return (x > eps) - (x < -eps); }</pre>
10
11
    int guass(LD a[maxn][maxn], int n, int m) {
12
        memset(free_x, 1, sizeof free_x); memset(x, 0, sizeof x);
13
14
        int r = 0, c = 0;
        while (r < n && c < m) {
15
             int m_r = r;
            FOR (i, r + 1, n)
17
                 if (fabs(a[i][c]) > fabs(a[m_r][c])) m_r = i;
18
            if (m_r != r)
19
                 FOR (j, c, m + 1)
20
                      swap(a[r][j], a[m_r][j]);
            \textbf{if} \ (!sgn(a[r][c])) \ \{\\
22
                 a[r][c] = 0;
23
24
                 ++c;
                 continue;
25
             FOR (i, r + 1, n)
27
28
                 if (a[i][c]) {
                     LD t = a[i][c] / a[r][c];
29
                     FOR (j, c, m + 1) a[i][j] = a[r][j] * t;
30
                 }
31
            ++r; ++c;
32
33
              FOR (i, 0, n)
    //
                  FOR (j, 0, m + 1)
34
                       printf("%.2f%c", a[i][j], j == _j - 1 ? '\n' : ' '); puts("");
35
    //
36
        FOR (i, r, n)
37
38
            if (sgn(a[i][m])) return -1;
        if (r < m) {
39
            FORD (i, r - 1, -1) {
                 int f_{cnt} = 0, k = -1;
41
42
                 FOR (j, ⊕, m)
43
                     if (sgn(a[i][j]) && free_x[j]) {
                          ++f_cnt;
44
                          k = j;
                     }
46
47
                 if(f_cnt > 0) continue;
48
                 LD s = a[i][m];
                 FOR (j, 0, m)
49
                     if (j != k) s -= a[i][j] * x[j];
                 x[k] = s / a[i][k];
51
                 free_x[k] = 0;
52
            }
53
54
             return m - r;
55
        FORD (i, m - 1, -1) \{
56
57
             LD s = a[i][m];
            FOR (j, i + 1, m)
58
                s = a[i][j] * x[j];
60
            x[i] = s / a[i][i];
        }
61
62
        return 0;
   }
63
```

```
数据
```

```
3 4
1 1 -2 2
2 -3 5 1
4 -1 1 5
5 0 -1 7
// many
3 4
1 1 -2 2
2 -3 5 1
4 -1 -1 5
5 0 -1 0 2
// no
3 4
1 1 -2 2
2 -3 5 1
4 -1 1 5
5 0 1 0 7
// one
```

## 质因数分解

- 前置模板:素数筛
- 带指数

```
LL factor[30], f_sz, factor_exp[30];
1
    void get_factor(LL x) {
2
        f_sz = 0;
3
        LL t = sqrt(x + 0.5);
        for (LL i = 0; pr[i] <= t; ++i)</pre>
5
             if (x % pr[i] == 0) {
                 factor_exp[f_sz] = 0;
                 while (x % pr[i] == 0) {
                     x /= pr[i];
                     ++factor_exp[f_sz];
10
11
                 factor[f_sz++] = pr[i];
12
            }
13
        if (x > 1) {
14
             factor_exp[f_sz] = 1;
factor[f_sz++] = x;
15
16
        }
17
18
   }
        • 不带指数
    LL factor[30], f_sz;
    void get_factor(LL x) {
2
        f_sz = 0;
        LL t = sqrt(x + 0.5);
5
        for (LL i = 0; pr[i] <= t; ++i)</pre>
            if (x % pr[i] == 0) {
                 factor[f_sz++] = pr[i];
                 while (x % pr[i] == 0) x /= pr[i];
```

if (x > 1) factor $[f_sz++] = x$ ;

}

## 原根

- 前置模板:素数筛,快速幂,分解质因数
- 要求 p 为质数

```
LL find_smallest_primitive_root(LL p) {
        get_factor(p - 1);
        FOR (i, 2, p) {
            bool flag = true;
            FOR (j, 0, f_sz)
                if (bin(i, (p - 1) / factor[j], p) == 1) {
                    flag = false;
            if (flag) return i;
10
        assert(0); return -1;
12
13
```

### 公式

• 当  $x > \phi(p)$  时有  $a^x \equiv a^{x \mod \phi(p) + \phi(p)} \pmod{p}$ 

#### 斐波那契数列性质

$$\bullet \ F_{a+b} = F_{a-1} \cdot F_b + F_a \cdot F_{b+1}$$

### 常见生成函数

$$\bullet (1+ax)^n = \sum_{k=0}^n \binom{n}{k} a^k x^k$$

• 
$$\frac{1-x^{r+1}}{1-x} = \sum_{k=0}^{n} x^k$$

$$\bullet \ \frac{1}{1-ax} = \sum_{k=0}^{\infty} a^k x^k$$

• 
$$\frac{1}{(1-x)^2} = \sum_{k=0}^{\infty} (k+1)x^k$$

• 
$$(1 + ax)^n = \sum_{k=0}^n \binom{n}{k} a^k x^k$$
  
•  $\frac{1 - x^{r+1}}{1 - x} = \sum_{k=0}^n x^k$   
•  $\frac{1}{1 - ax} = \sum_{k=0}^\infty a^k x^k$   
•  $\frac{1}{(1 - x)^2} = \sum_{k=0}^\infty (k+1) x^k$   
•  $\frac{1}{(1 - x)^n} = \sum_{k=0}^\infty \binom{n+k-1}{k} x^k$   
•  $e^x - \sum_{k=0}^\infty \frac{x^k}{k}$ 

$$\bullet \ e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

• 
$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$
  
•  $\ln(1+x) = \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{k} x^k$ 

# 佩尔方程

若一个丢番图方程具有以下的形式:  $x^2 - ny^2 = 1$ 。且 n 为正整数,则称此二元二次不定方程为**佩尔方程**。

若 n 是完全平方数,则这个方程式只有平凡解  $(\pm 1,0)$ (实际上对任意的 n, $(\pm 1,0)$  都是解)。对于其余情况,拉格朗日证明了佩尔方 程总有非平凡解。而这些解可由  $\sqrt{n}$  的连分数求出。

程总有非平凡解。而这些解可由 
$$\sqrt{n}$$
 的连分数求出。 
$$x=[a_0;a_1,a_2,a_3]=x=a_0+\cfrac{1}{a_1+\cfrac{1}{a_2+\cfrac{1}{a_3+\cfrac{1}{\ddots}}}}$$

设  $\frac{p_i}{q_i}$  是  $\sqrt{n}$  的连分数表示:  $[a_0;a_1,a_2,a_3,\ldots]$  的渐近分数列,由连分数理论知存在 i 使得  $(p_i,q_i)$  为佩尔方程的解。取其中最小的 i,将 对应的  $(p_i,q_i)$  称为佩尔方程的基本解, 或最小解, 记作  $(x_1,y_1)$ , 则所有的解  $(x_i,y_i)$  可表示成如下形式:  $x_i+y_i\sqrt{n}=(x_1+y_1\sqrt{n})^i$ 。 或者由以下的递回关系式得到:

 $x_{i+1} = x_1 x_i + n y_1 y_i, y_{i+1} = x_1 y_i + y_1 x_i$ 

### **Burnside & Polya**

•  $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$ 

注:  $X^g$  是 g 下的不动点数量,也就是说有多少种东西用 g 作用之后可以保持不变。

• 
$$|Y^X/G| = \frac{1}{|G|} \sum_{g \in G} m^{c(g)}$$

注: 用m种颜色染色,然后对于某一种置换g,有c(g)个置换环,为了保证置换后颜色仍然相同,每个置换环必须染成同色。

### 二次剩余

```
LL q1, q2;
    LL w;
    struct Point { //x + y*sqrt(w)
        LL x;
        LL y;
    };
    LL mod(LL a, LL p) {
        a %= p;
10
        if (a < 0) {
11
            a += p;
12
14
        return a;
15
    }
16
    LL quick_mul(LL x, LL y, LL p) {
17
       x = mod(x, p);
        y = mod(y, p);
19
20
       LL ans = 0;
       while (y > 0) {
21
            if (y & 1) {
22
                ans = ans + x;
                if (ans >= p)ans -= p;
24
            }
25
            x = x + x;
26
            if (x >= p)x -= p;
27
28
            y >>= 1;
        }
29
30
        return ans;
    }
31
32
    Point point_mul(Point a, Point b, LL p) {
33
        Point res;
34
35
        res.x = quick_mul(a.x, b.x, p);
        res.x += quick_mul(w, quick_mul(a.y, b.y, p), p);
36
       res.x = mod(res.x, p);
37
38
       res.y = quick_mul(a.x, b.y, p);
        res.y += quick_mul(a.y, b.x, p);
39
        res.y = mod(res.y, p);
40
        return res;
41
43
    Point power(Point a, LL b, LL p) {
44
45
       Point res;
       res.x = 1;
46
       res.y = 0;
        while (b) {
48
            if (b & 1) {
                 res = point_mul(res, a, p);
```

```
51
52
             a = point_mul(a, a, p);
             b = b >> 1;
53
54
55
         return res;
56
57
    }
58
    LL quick_power(LL a, LL b, LL p)//(a^b)%p
59
60
         LL res = 1;
61
         while (b) {
62
            if (b & 1) {
63
                 res = quick_mul(res, a, p);
64
             }
65
66
67
             a = quick_mul(a, a, p);
             b = b >> 1;
68
70
71
         return res;
72
73
    LL Legendre(LL a, LL p) { // a^{\wedge}((p-1)/2)
75
76
         return quick_power(a, (p - 1) >> 1, p);
77
78
79
    LL equation_solve(LL b, LL p) { //求解 x^2=b(\%p) 方程解
         if ((Legendre(b, p) + 1) % p == 0) {
80
81
             return -1; //表示没有解
82
83
84
         LL a, t;
         while (true) {
85
86
             a = rand() % p;
             t = quick_mul(a, a, p) - b;
87
             t = mod(t, p);
88
             if ((Legendre(t, p) + 1) % p == 0) {
89
                 break;
90
91
         }
92
93
94
         w = t;
         Point temp, res;
95
96
         temp.x = a;
         temp.y = 1;
97
         res = power(temp, (p + 1) >> 1, p);
         return res.x;
99
100
    }
101
    bool getInv(LL p) {
102
         LL x = equation_solve(p - 3, p);
         if (x == -1) return false;
104
105
         LL inv2 = (p + 1) >> 1;
         q1 = quick_mul(p - 1 - x, inv2, p), q2 = quick_mul(x - 1, inv2, p);
106
         return true;
107
108
    }
```

### 中国剩余定理

- 无解返回 -1
- 前置模板: 拓展欧几里得

### 伯努利数和等幂求和

```
● 预处理逆元
```

```
• 预处理组合数
```

•  $\sum_{i=0}^{n} i^k = \frac{1}{k+1} \sum_{i=0}^{k} {k+1 \choose i} B_{k+1-i} (n+1)^i$ .

• 也可以  $\sum_{i=0}^{n} i^k = \frac{1}{k+1} \sum_{i=0}^{k} {k+1 \choose i} B_{k+1-i}^+ n^i$ 。区别在于  $B_1^+ = 1/2$ 。(心态崩了)

```
namespace Bernoulli {
2
        const int M = 100;
        LL inv[M] = {-1, 1};
3
        void inv_init(LL n, LL p) {
4
            FOR (i, 2, n)
                inv[i] = (p - p / i) * inv[p % i] % p;
        LL C[M][M];
        void init_C(int n) {
10
            FOR (i, 0, n) {
11
                C[i][0] = C[i][i] = 1;
                 FOR (j, 1, i)
13
                    C[i][j] = (C[i - 1][j] + C[i - 1][j - 1]) % MOD;
            }
15
16
17
        LL B[M] = \{1\};
18
19
        void init() {
20
            inv_init(M, MOD);
            init_C(M);
21
            FOR (i, 1, M - 1) \{
22
                 LL& s = B[i] = 0;
23
24
                 FOR (j, 0, i)
                  s += C[i + 1][j] * B[j] % MOD;
25
                 s = (s \% MOD * -inv[i + 1] \% MOD + MOD) \% MOD;
            }
27
28
29
        LL p[M] = \{1\};
30
        LL go(LL n, LL k) {
            n %= MOD;
32
            if (k == 0) return n;
33
34
            FOR (i, 1, k + 2)
               p[i] = p[i - 1] * (n + 1) % MOD;
35
            LL ret = 0;
            FOR (i, 1, k + 2)
37
                ret += C[k + 1][i] * B[k + 1 - i] % MOD * p[i] % MOD;
            ret = ret % MOD * inv[k + 1] % MOD;
39
            return ret;
41
        }
   }
42
```

### 单纯形

- 要求有基本解,也就是 x 为零向量可行
- v 要初始化为 0, n 表示向量长度, m 表示约束个数

```
1  // min{ b x } / max { c x }
2  // A x >= c / A x <= b</pre>
```

```
// x >= 0
3
    namespace lp {
        int n, m;
        double a[M][N], b[M], c[N], v;
        void pivot(int l, int e) {
8
            b[l] /= a[l][e];
            FOR (j, 0, n) if (j != e) a[l][j] /= a[l][e];
10
            a[l][e] = 1 / a[l][e];
11
            FOR (i, 0, m)
13
                if (i != l && fabs(a[i][e]) > 0) {
14
                    b[i] -= a[i][e] * b[l];
15
                     FOR (j, 0, n)
16
                        if (j != e) a[i][j] -= a[i][e] * a[l][j];
17
                     a[i][e] = -a[i][e] * a[l][e];
18
19
                }
            v += c[e] * b[l];
20
            FOR (j, 0, n) if (j != e) c[j] -= c[e] * a[l][j];
21
            c[e] = -c[e] * a[l][e];
22
23
        double simplex() {
24
25
            while (1) {
                v = 0;
                int e = -1, l = -1;
27
28
                FOR (i, 0, n) if (c[i] > eps) { e = i; break; }
                if (e == -1) return v;
29
                double t = INF;
30
                FOR (i, 0, m)
                     if (a[i][e] > eps && t > b[i] / a[i][e]) {
32
                         t = b[i] / a[i][e];
33
                         l = i;
34
35
                if (l == -1) return INF;
                pivot(l, e);
37
38
            }
        }
39
   }
40
```

# 图论

# **LCA**

● 倍增 void dfs(int u, int fa) { pa[u][0] = fa; dep[u] = dep[fa] + 1;FOR (i, 1, SP) pa[u][i] = pa[pa[u][i - 1]][i - 1]; for (int& v: G[u]) { if (v == fa) continue; dfs(v, u); } } int lca(int u, int v) { 10 11 if (dep[u] < dep[v]) swap(u, v);</pre> 12 int t = dep[u] - dep[v]; FOR (i, 0, SP) **if** (t & (1 << i)) u = pa[u][i]; 13 14 FORD (i, SP - 1, -1) { int uu = pa[u][i], vv = pa[v][i]; 15 if (uu != vv) { u = uu; v = vv; } 16 17 return u == v ? u : pa[u][0]; 18 19 }

# 最短路

```
bool BF() {
        queue<int> q;
2
        FOR (i, 1, n) d[i] = INF;
3
        d[0] = 0; inq[0] = true; q.push(0);
        while (!q.empty()) {
            int u = q.front(); q.pop();
            inq[u] = false;
             for (E& e: G[u]) {
                 int v = e.to;
10
                 if (d[u] < INF && d[v] > d[u] + e.d) {
                     d[v] = d[u] + e.d;
11
                     if (!inq[v]) {
12
13
                         q.push(v); inq[v] = true;
                         if (++cnt[v] > n) return false;
14
15
                     }
16
                 }
            }
17
18
        return true;
19
    网络流
        ● 最大流
    struct E {
        int to, cp;
2
        E(int to, int cp): to(to), cp(cp) {}
3
    };
    struct Dinic {
        static const int M = 1E5 * 5;
        int m, s, t;
        vector<E> edges;
        vector<int> G[M];
10
        int d[M];
11
        int cur[M];
12
13
        void init(int n, int s, int t) {
14
            this->s = s; this->t = t;
15
            for (int i = 0; i <= n; i++) G[i].clear();</pre>
16
            edges.clear(); m = 0;
17
18
19
        void addedge(int u, int v, int cap) {
            edges.emplace_back(v, cap);
21
             edges.emplace_back(u, 0);
22
23
            G[u].push_back(m++);
            G[v].push_back(m++);
24
26
27
        bool BFS() {
            memset(d, 0, sizeof d);
28
            queue<int> Q;
29
            Q.push(s); d[s] = 1;
            while (!Q.empty()) {
31
                 int x = Q.front(); Q.pop();
32
                 for (int& i: G[x]) {
33
                     E &e = edges[i];
34
                     if (!d[e.to] && e.cp > 0) {
35
                         d[e.to] = d[x] + 1;
36
37
                         Q.push(e.to);
                     }
38
                 }
39
            }
40
41
            return d[t];
42
```

```
43
44
        int DFS(int u, int cp) {
            if (u == t || !cp) return cp;
45
            int tmp = cp, f;
46
47
            for (int& i = cur[u]; i < G[u].size(); i++) {</pre>
                E& e = edges[G[u][i]];
48
49
                 if (d[u] + 1 == d[e.to]) {
                    f = DFS(e.to, min(cp, e.cp));
50
                     e.cp -= f;
51
                     edges[G[u][i] ^ 1].cp += f;
52
                     cp -= f;
53
                     if (!cp) break;
54
                }
55
56
57
            return tmp - cp;
58
        int go() {
60
            int flow = 0;
            while (BFS()) {
62
                memset(cur, 0, sizeof cur);
63
64
                flow += DFS(s, INF);
65
            }
            return flow;
67
   } DC;
68
       ● 费用流
    struct E {
        int from, to, cp, v;
2
        E(int f, int t, int cp, int v) : from(f), to(t), cp(cp), v(v) {}
5
   };
    struct MCMF {
        int n, m, s, t;
        vector<E> edges;
        vector<int> G[maxn];
10
                            //是否在队列
        bool inq[maxn];
11
12
        int d[maxn];
                             //Bellman_ford 单源最短路径
                             //p[i] 表从 s 到 i 的最小费用路径上的最后一条弧编号
13
        int p[maxn];
                             //a[i] 表示从 s 到 i 的最小残量
        int a[maxn];
14
15
        void init(int _n, int _s, int _t) {
16
            n = _n; s = _s; t = _t;
17
            FOR (i, 0, n + 1) G[i].clear();
18
            edges.clear(); m = 0;
19
20
21
        void addedge(int from, int to, int cap, int cost) {
22
23
            edges.emplace_back(from, to, cap, cost);
            edges.emplace_back(to, from, 0, -cost);
24
25
            G[from].push_back(m++);
            G[to].push_back(m++);
26
27
28
        bool BellmanFord(int &flow, int &cost) {
29
30
            FOR (i, 0, n + 1) d[i] = INF;
            memset(inq, \theta, sizeof inq);
31
32
            d[s] = 0, a[s] = INF, inq[s] = true;
            queue<int> Q; Q.push(s);
33
            while (!Q.empty()) {
34
35
                int u = Q.front(); Q.pop();
                inq[u] = false;
36
37
                 for (int& idx: G[u]) {
                    E &e = edges[idx];
38
                     if (e.cp && d[e.to] > d[u] + e.v) {
39
40
                         d[e.to] = d[u] + e.v;
                         p[e.to] = idx;
41
42
                         a[e.to] = min(a[u], e.cp);
                         if (!inq[e.to]) {
43
```

```
Q.push(e.to);
44
45
                             inq[e.to] = true;
                         }
46
                    }
47
                }
49
50
            if (d[t] == INF) return false;
            flow += a[t];
51
            cost += a[t] * d[t];
52
53
            int u = t;
            while (u != s) {
54
55
                 edges[p[u]].cp -= a[t];
                 edges[p[u] ^ 1].cp += a[t];
56
                 u = edges[p[u]].from;
57
            }
58
59
            return true;
60
        }
61
62
        int go() {
            int flow = 0, cost = 0;
63
            while (BellmanFord(flow, cost));
64
65
            return cost;
66
    } MM;
       ● zkw 费用流(代码长度没有优势)
        • 不允许有负权边
    struct E {
1
2
        int to, cp, v;
        E() {}
3
        E(int to, int cp, int v): to(to), cp(cp), v(v) {}
5
    };
    struct MCMF {
        int n, m, s, t, cost, D;
        vector<E> edges;
        vector<int> G[maxn];
10
        bool vis[maxn];
11
12
        void init(int _n, int _s, int _t) {
13
14
            n = _n; s = _s; t = _t;
            FOR (i, 0, n + 1) G[i].clear();
15
16
            edges.clear(); m = 0;
17
18
        void addedge(int from, int to, int cap, int cost) {
19
            edges.emplace_back(to, cap, cost);
20
21
            edges.emplace_back(from, 0, -cost);
            G[from].push_back(m++);
22
            G[to].push_back(m++);
24
25
        int aug(int u, int cp) {
26
            if (u == t) {
27
                 cost += D * cp;
28
                 return cp;
29
30
            vis[u] = true;
31
            int tmp = cp;
32
            for (int idx: G[u]) {
                 E& e = edges[idx];
34
35
                 if (e.cp && !e.v && !vis[e.to]) {
                     int f = aug(e.to, min(cp, e.cp));
36
                     e.cp -= f;
37
38
                     edges[idx ^ 1].cp += f;
                     cp -= f;
39
40
                     if (!cp) break;
                }
41
42
43
            return tmp - cp;
```

```
}
44
45
         bool modlabel() {
46
              int d = INF;
47
48
              FOR (u, 0, n + 1)
                  if (vis[u])
49
50
                       for (int& idx: G[u]) {
                           E& e = edges[idx];
51
                            if (e.cp && !vis[e.to]) d = min(d, e.v);
52
53
                       }
             if (d == INF) return false;
54
              FOR (u, 0, n + 1)
55
                  if (vis[u])
56
                       for (int% idx: G[u]) {
57
                            edges[idx].v -= d;
58
                            edges[idx ^ 1].v += d;
59
60
              D += d;
61
              return true;
         }
63
64
         int go(int k) {
65
             cost = D = 0;
66
              int flow = 0;
              while (true) {
68
69
                  memset(vis, 0, sizeof vis);
70
                  int t = aug(s, INF);
                  if (!t && !modlabel()) break;
71
72
                  flow += t;
73
74
              return cost;
75
    } MM;
    树上路径交
    int intersection(int x, int y, int xx, int yy) {
1
2
         int t[4] = {lca(x, xx), lca(x, yy), lca(y, xx), lca(y, yy)};
         sort(t, t + 4);
         int r = lca(x, y), rr = lca(xx, yy);
         \textbf{if} \ (\mathsf{dep}[\mathsf{t}[\theta]] \ < \ \mathsf{min}(\mathsf{dep}[\mathsf{r}], \ \mathsf{dep}[\mathsf{rr}]) \ | \ | \ \mathsf{dep}[\mathsf{t}[2]] \ < \ \mathsf{max}(\mathsf{dep}[\mathsf{r}], \ \mathsf{dep}[\mathsf{rr}]))
              return 0;
         int tt = lca(t[2], t[3]);
         int ret = 1 + dep[t[2]] + dep[t[3]] - dep[tt] * 2;
         return ret;
10
    }
    树上点分治
    int get_sz(int u, int fa) {
         int& s = sz[u] = 1;
2
         for (E& e: G[u]) {
              int v = e.to;
              if (vis[v] || v == fa) continue;
5
              s += get_sz(v, u);
         }
         return s;
    }
9
10
    void get_rt(int u, int fa, int s, int& m, int& rt) {
11
         int t = s - sz[u];
12
13
         for (E& e: G[u]) {
              int v = e.to;
14
15
              if (vis[v] || v == fa) continue;
              get_rt(v, u, s, m, rt);
16
              t = max(t, sz[v]);
17
         }
18
```

```
if (t < m) { m = t; rt = u; }</pre>
19
20
    }
21
    void dfs(int u) {
22
23
        int tmp = INF; get_rt(u, -1, get_sz(u, -1), tmp, u);
        vis[u] = true;
24
25
        get_dep(u, -1, 0);
26
        // ...
        for (E& e: G[u]) {
27
28
            int v = e.to;
             if (vis[v]) continue;
29
30
            dfs(v);
31
        }
32
   }
33
        • 动态点分治
    const int maxn = 15E4 + 100, INF = 1E9;
    struct E {
        int to, d;
3
4
    };
    vector<E> G[maxn];
    int n, Q, w[maxn];
    LL A, ans;
    bool vis[maxn];
    int sz[maxn];
10
11
12
    int get_rt(int u) {
13
          dba(u):
14
        static int q[N], fa[N], sz[N], mx[N];
        int p = 0, cur = -1;
15
        q[p++] = u; fa[u] = -1;
16
        while (++cur < p) {</pre>
17
            u = q[cur]; mx[u] = 0; sz[u] = 1;
18
19
             for (int& v: G[u])
                 if (!vis[v] && v != fa[u]) fa[q[p++] = v] = u;
20
21
        FORD (i, p - 1, -1) \{
22
23
            u = q[i];
24
            mx[u] = max(mx[u], p - sz[u]);
            if (mx[u] * 2 <= p) return u;</pre>
25
             sz[fa[u]] += sz[u];
            mx[fa[u]] = max(mx[fa[u]], sz[u]);
27
28
29
        assert(0);
    }
30
31
    int get_sz(int u, int fa) {
32
        int& s = sz[u] = 1;
33
        for (E& e: G[u]) {
34
             int v = e.to;
35
            if (vis[v] || v == fa) continue;
36
37
             s += get_sz(v, u);
38
        return s;
39
40
    }
41
    void get_rt(int u, int fa, int s, int& m, int& rt) {
42
43
        int t = s - sz[u];
        for (E& e: G[u]) {
44
             int v = e.to;
45
            if (vis[v] || v == fa) continue;
46
47
             get_rt(v, u, s, m, rt);
48
             t = max(t, sz[v]);
49
        if (t < m) { m = t; rt = u; }</pre>
    }
51
52
53
    int dep[maxn], md[maxn];
    void get_dep(int u, int fa, int d) {
```

```
dep[u] = d; md[u] = 0;
55
56
         for (E& e: G[u]) {
             int v = e.to;
57
             if (vis[v] || v == fa) continue;
58
             get_dep(v, u, d + e.d);
59
             md[u] = max(md[u], md[v] + 1);
60
61
    }
62
63
    struct P {
64
         int w;
65
         LL s;
67
    };
    using VP = vector<P>;
68
69
    struct R {
         VP *rt, *rt2;
70
71
         int dep;
72
73
    VP pool[maxn << 1], *pit = pool;</pre>
    vector<R> tr[maxn];
74
75
    void go(int u, int fa, VP* rt, VP* rt2) {
         tr[u].push_back({rt, rt2, dep[u]});
77
         for (E& e: G[u]) {
78
             int v = e.to;
79
80
             if (v == fa || vis[v]) continue;
81
             go(v, u, rt, rt2);
         }
82
    }
84
    void dfs(int u) {
85
         int tmp = INF; get_rt(u, -1, get_sz(u, -1), tmp, u);
86
87
         vis[u] = true;
88
         get_dep(u, -1, 0);
         VP* rt = pit++; tr[u].push_back({rt, nullptr, 0});
89
         for (E& e: G[u]) {
90
             int v = e.to:
91
             if (vis[v]) continue;
92
93
             go(v, u, rt, pit++);
94
             dfs(v);
95
    }
96
97
98
    bool cmp(const P& a, const P& b) { return a.w < b.w; }</pre>
99
100
    LL query(VP& p, int d, int l, int r) {
         l = lower_bound(p.begin(), p.end(), P{l, -1}, cmp) - p.begin();
101
102
         r = upper_bound(p.begin(), p.end(), P{r, -1}, cmp) - p.begin() - 1;
         return p[r].s - p[l - 1].s + 1LL * (r - l + 1) * d;
103
    }
104
105
    int main() {
106
         cin >> n >> Q >> A;
107
         FOR (i, 1, n + 1) scanf("%d", &w[i]);
108
         FOR (_, 1, n) {
109
             int u, v, d; scanf("%d%d%d", &u, &v, &d);
110
             G[u].push_back({v, d}); G[v].push_back({u, d});
111
112
         dfs(1);
113
         FOR (i, 1, n + 1)
114
115
             for (R& x: tr[i]) {
                 x.rt->push_back({w[i], x.dep});
116
117
                 if (x.rt2) x.rt2->push_back({w[i], x.dep});
118
119
         FOR (it, pool, pit) {
             it->push_back({-INF, 0});
120
             sort(it->begin(), it->end(), cmp);
121
122
             FOR (i, 1, it->size())
                 (*it)[i].s += (*it)[i - 1].s;
123
124
         while (Q--) {
125
```

```
int u; LL a, b; scanf("%d%lld%lld", &u, &a, &b);
126
127
             a = (a + ans) % A; b = (b + ans) % A;
             int l = min(a, b), r = max(a, b);
128
             ans = 0;
129
             for (R& x: tr[u]) {
                  ans += query(*(x.rt), x.dep, l, r);
131
                  if (x.rt2) ans -= query(*(x.rt2), x.dep, l, r);
132
133
             printf("%lld\n", ans);
134
135
         }
    }
136
     树链剖分
     int fa[N], dep[N], idx[N], out[N], ridx[N];
     namespace hld {
         int sz[N], son[N], top[N], clk;
         void predfs(int u, int d) {
4
             dep[u] = d; sz[u] = 1;
             int& maxs = son[u] = -1;
              for (int& v: G[u]) {
                  if (v == fa[u]) continue;
                  fa[v] = u;
                  predfs(v, d + 1);
10
                  sz[u] += sz[v];
11
                  if (maxs == -1 \mid \mid sz[v] > sz[maxs]) maxs = v;
             }
13
14
15
         void dfs(int u, int tp) {
             top[u] = tp; idx[u] = ++clk; ridx[clk] = u;
16
17
             if (son[u] != -1) dfs(son[u], tp);
             for (int& v: G[u])
18
19
                  if (v != fa[u] && v != son[u]) dfs(v, v);
             out[u] = clk;
20
21
22
         template<typename T>
         int go(int u, int v, T&& f = [](int, int) {}) {
23
24
              int uu = top[u], vv = top[v];
             while (uu != vv) {
25
                  if (dep[uu] < dep[vv]) { swap(uu, vv); swap(u, v); }</pre>
26
27
                  f(idx[uu], idx[u]);
                  u = fa[uu]; uu = top[u];
28
29
             if (dep[u] < dep[v]) swap(u, v);</pre>
30
31
             // f(idx[v], idx[u]);
             // if (u != v) f(idx[v] + 1, idx[u]);
32
             return v;
33
34
         int up(int u, int d) {
35
             while (d) {
                  \textbf{if} \ (\text{dep}[u] \ - \ \text{dep}[\text{top}[u]] \ \leq \ d) \ \{
37
38
                      d -= dep[u] - dep[top[u]];
39
                      u = top[u];
                  } else return ridx[idx[u] - d];
40
41
                  u = fa[u]; --d;
             }
42
43
             return u;
         }
44
    }
45
        • HDU 3966
    # include <bits/stdc++.h>
    using namespace std;
    typedef long long LL;
          \#define \ FOR(i,\ x,\ y) \ for \ (decay < decltype(y) > :: type \ i \ = \ (x), \ \_\#\#i \ = \ (y); \ i \ < \ \_\#\#i; \ ++i) 
    #define FORD(i, x, y) for (decay < decltype(x) > :: type i = (x), _##i = (y); i > _##i; --i)
    #define dbg(args...) do { cout << "\033[32;1m" << \#args<< " -> "; <math>err(args); } while (0)
```

```
#define dbg(...)
10
    #endif
    void err() { cout << "\033[39;0m" << endl; }</pre>
11
    template<typename T, typename... Args>
    void err(T a, Args... args) { cout << a << ' '; err(args...); }</pre>
    // --
14
    const int maxn = 5E4 + 100;
15
    vector<int> G[maxn];
16
    int dep[maxn], sz[maxn], son[maxn], fa[maxn], idx[maxn], top[maxn];
17
    int clk, n, Q;
18
19
    struct IntervalTree {
    #define ls o * 2, l, (l + r) >> 1
21
    #define rs o * 2 + 1, ((l + r) >> 1) + 1, r
22
        static const int M = maxn << 2;</pre>
23
        int addv[M];
24
25
        void init() { memset(addv, 0, sizeof addv); }
        int query(int k, int o, int l, int r, int add = 0) {
26
             if (k < l || r < k) return 0;
             if (l == r) return add + addv[o];
28
             return query(k, ls, add + addv[o]) + query(k, rs, add + addv[o]);
29
30
        void update(int p, int q, int o, int l, int r, int add) {
31
             assert(l <= r && r <= n);
             if (q < l | | r < p) return;
33
34
             if (p <= l && r <= q) addv[o] += add;
             else { update(p, q, ls, add); update(p, q, rs, add); }
35
        }
36
37
    } IT;
38
    void predfs(int u, int d) {
39
        dep[u] = d;
40
        sz[u] = 1;
41
42
        int& maxs = son[u] = -1;
        for (int v: G[u])
43
             if (v != fa[u]) {
44
                 fa[v] = u;
45
                 predfs(v, d + 1);
46
47
                 sz[u] += sz[v];
                 if (maxs == -1 \mid | sz[v] > sz[maxs])
48
49
                     maxs = v;
            }
50
51
52
    void dfs(int u, int tp) {
53
54
        top[u] = tp;
        idx[u] = ++clk;
55
        if (son[u] != -1) dfs(son[u], tp);
        for (int v: G[u])
57
58
             if (v != son[u] && v != fa[u])
                 dfs(v, v);
59
    }
60
    void update(int u, int v, int add) {
62
63
        int uu = top[u], vv = top[v];
        while (uu != vv) {
64
             if (dep[uu] < dep[vv]) { swap(uu, vv); swap(u, v); }</pre>
65
66
            IT.update(idx[uu], idx[u], 1, 1, n, add);
            u = fa[uu];
67
             uu = top[u];
68
69
70
        if (dep[u] < dep[v]) swap(u, v);</pre>
71
        dbg(u, v, idx[u], idx[v]);
        IT.update(idx[v], idx[u], 1, 1, n, add);
72
73
    }
74
75
    int a[maxn];
76
    void init();
    int main() {
77
78
        int u, v, l, r, k, d;
        char s[100];
79
```

```
while (cin >> n >> Q >> Q) {
80
81
             init();
             FOR (i, 1, n + 1) scanf("%d", &a[i]);
82
83
             FOR (i, 1, n) {
84
                  scanf("%d%d", &u, &v);
                  G[u].push_back(v);
85
                  G[v].push_back(u);
86
             }
87
             predfs(1, 1);
88
89
             dfs(1, 1);
             while (Q--) {
90
91
                  scanf("%s", s);
                  if (s[0] == 'I') {
92
                      scanf("%d%d%d", &l, &r, &d);
93
94
                      update(l, r, d);
                  } else if (s[0] == 'D') {
95
                      scanf("%d%d%d", &l, &r, &d);
96
                      update(l, r, -d);
97
                  } else {
98
                      scanf("%d", &k);
99
                      printf("%d\n", a[k] + IT.query(idx[k], 1, 1, n));
100
101
             }
102
         }
103
    }
104
105
    void init() {
106
         clk = 0;
107
108
         fa[1] = 0;
         IT.init();
109
         FOR (i, 0, n + 1) G[i].clear();
110
    }
111
        • SPOJ QTREE
    #include <bits/stdc++.h>
1
    using namespace std;
    typedef long long LL;
    #define FOR(i, x, y) for (decay < decltype(y) > :: type i = (x), _##i = (y); i < _##i; ++i)
    \#define\ FORD(i,\ x,\ y)\ for\ (decay< decltype(x)>::type\ i\ =\ (x),\ \_\#i\ =\ (y);\ i\ >\ \_\#i;\ --i)
    #define dbg(args...) do { cout << "\033[32;1m" << #args<< " -> "; err(args); } while (0)
    #else
    #define dbg(...)
    #endif
10
    void err() { cout << "\033[39m" << endl; }</pre>
11
12
    template<typename T, typename... Args>
    void err(T a, Args... args) {
13
14
         cout << a << ' ';
         err(args...);
15
    }
16
17
    const int maxn = 10000 \times 2 \times 4 + 100;
18
19
    struct Edge {
         int from, to, c;
20
         Edge(int u, int v, int c): from(u), to(v), c(c) {}
21
    };
22
    vector<Edge> edge;
23
    vector<int> G[maxn];
    int fa[maxn], dep[maxn], sz[maxn], son[maxn], top[maxn], idx[maxn], w[maxn], val[maxn];
25
    LL sum[maxn];
    int n, clk, len;
27
28
    struct IntervalTree {
29
    #define lson p, q, o \star 2, l, m
30
    #define rson p, q, o \star 2 + 1, m + 1, r
31
32
         int maxv[maxn];
         void init() { memset(maxv, 0, sizeof maxv); }
33
34
         int query(int p, int q, int o, int l, int r) {
35
    //
               dbg(p, q);
36
             assert(p <= q);</pre>
             if (p > r \mid | q < l) return 0;
37
```

```
if (p <= l && r <= q) return maxv[o];</pre>
38
39
             int m = (l + r) / 2;
             return max(query(lson), query(rson));
40
41
         }
42
         void maintain(int o, int l, int r) {
             if (l < r)
43
44
                  \max v[o] = \max(\max v[o * 2], \max v[o * 2 + 1]);
45
         void update(int p, int q, int o, int l, int r, int v) {
46
47
               dbg(p, q, o, l, r, v);
             assert(p <= q);</pre>
48
49
             if (p > r || q < l) return;
             if (p <= l && r <= q) maxv[o] = v;
50
             else {
51
                  int m = (l + r) / 2;
52
                  update(lson, v); update(rson, v);
53
54
                  maintain(o, l, r);
             }
55
         }
    } IT;
57
58
    void dfs1(int u, int d) {
59
60
         dep[u] = d;
61
         sz[u] = 1;
         FOR (i, \theta, G[u].size()) {
62
             Edge& e = edge[G[u][i]];
63
             int v = e.to;
64
             if (v == fa[u]) continue;
65
             val[v] = e.c;
               dbg(v, e.from, e.to, e.c);
    //
67
             fa[v] = u;
68
             dfs1(v, d + 1);
69
             sz[u] += sz[v];
70
             if (son[u] == -1 \mid \mid sz[v] > sz[son[u]])
71
                  son[u] = v;
72
73
    }
74
75
    void dfs2(int u, int tp) {
76
         top[u] = tp;
77
78
         idx[u] = ++clk;
         w[idx[u]] = tp;
79
         if (son[u] == -1) return;
80
81
         dfs2(son[u], tp);
82
         FOR (i, 0, G[u].size()) {
83
             int v = edge[G[u][i]].to;
             if (v != son[u] && v != fa[u])
84
85
                  dfs2(v, v);
         }
86
87
    }
88
     int query(int u, int v) {
89
         dbg(u, v);
90
         int uu = top[u], vv = top[v], ret = 0;
91
92
         while (uu != vv) {
             if (dep[uu] < dep[vv]) { swap(u, v); swap(uu, vv); }</pre>
93
    //
               dbg(u, v, uu, vv, dep[uu], dep[vv], idx[uu], idx[u]);
94
             ret = max(ret, IT.query(idx[uu], idx[u], 1, 1, len));
95
             u = fa[uu];
96
             uu = top[u];
97
98
99
         if (dep[u] < dep[v]) swap(u, v);</pre>
100
           dbg(idx[v], idx[u]);
         if (u != v) ret = max(ret, IT.query(idx[v] + 1, idx[u], 1, 1, len));
101
102
         return ret;
    }
103
104
105
    void init();
    void add_edge(int u, int v, int c);
106
107
    int main() {
108
```

```
#ifdef zerol
109
110
         freopen("in", "r", stdin);
     #endif
111
         int T, u, v, c;
112
113
         char s[100];
         cin >> T;
114
         while (T--) {
115
             cin >> n;
116
             for (len = 1; len < n; len *= 2);</pre>
117
118
             init();
             FOR (i, 1, n) {
119
                  scanf("%d%d%d", &u, &v, &c);
120
121
                  add_edge(u, v, c);
                  add_edge(v, u, c);
122
123
             }
             dfs1(1, 0);
124
125
             dfs2(1, 1);
               FOR (i, 1, n + 1) dbg(idx[i], w[i]);
126
             FOR (i, 2, n + 1)
127
                  IT.update(idx[i], idx[i], 1, 1, len, val[i]);
128
             while (scanf("%s", s) && s[0] != 'D') {
129
                  scanf("%d%d", &u, &v);
130
                  if (s[0] == 'C') {
131
                      Edge& e = edge[u * 2 - 1];
132
                      dbg(u, e.from, e.to);
133
                      int t = max(idx[e.from], idx[e.to]);
134
135
                      IT.update(t, t, 1, 1, len, v);
                      dbg("upd", t, v);
136
137
                  if (s[0] == 'Q') printf("%d\n", query(u, v));
138
139
             FOR (i, 1, n + 1) if (idx[i] == 2) dbg(i, idx[i]);
140
             dbg(IT.query(idx[2], idx[2], 1, 1, len));
141
142
             dbg(IT.query(idx[6], idx[6], 1, 1, len));
143
    }
144
145
     void init() {
146
147
         edge.clear();
         memset(son, -1, sizeof son);
148
         memset(sum, 0, sizeof sum);
149
         IT.init();
150
         FOR (i, 0, n + 1) G[i].clear();
151
152
         clk = 0;
         fa[1] = 0;
153
154
         sum[0] = sum[1] = 0;
    }
155
156
     void add_edge(int u, int v, int c) {
157
         edge.emplace_back(u, v, c);
158
159
         G[u].push_back(edge.size() - 1);
    }
160
```

# 二分图匹配

- 最小覆盖数 = 最大匹配数
- 最大独立集 = 顶点数 二分图匹配数
- DAG 最小路径覆盖数 = 结点数 拆点后二分图最大匹配数

```
struct MaxMatch {
    int n;
    vector<int> G[maxn];
    int vis[maxn], left[maxn], clk;

void init(int n) {
    this->n = n;
    FOR (i, 0, n + 1) G[i].clear();
    memset(left, -1, sizeof left);
    memset(vis, -1, sizeof vis);
```

```
}
11
12
        bool dfs(int u) {
13
             for (int v: G[u])
14
15
                 if (vis[v] != clk) {
                      vis[v] = clk;
16
17
                      if (left[v] == -1 || dfs(left[v])) {
                          left[v] = u;
18
                          return true;
19
20
                      }
                 }
21
22
             return false;
        }
23
24
        int match() {
25
             int ret = 0;
26
27
             for (clk = 0; clk <= n; ++clk)</pre>
                 if (dfs(clk)) ++ret;
28
29
             return ret;
        }
30
    } MM;
31
        ● 二分图最大权完美匹配 KM
    namespace R {
        const int maxn = 300 + 10;
2
3
         int n, m;
         int left[maxn], L[maxn], R[maxn];
4
         int w[maxn][maxn], slack[maxn];
5
        bool visL[maxn], visR[maxn];
        bool dfs(int u) {
8
             visL[u] = true;
             FOR (v, 0, m) {
10
11
                 if (visR[v]) continue;
                 int t = L[u] + R[v] - w[u][v];
12
13
                 if (t == 0) {
                      visR[v] = true;
14
                      if (left[v] == -1 || dfs(left[v])) {
15
                          left[v] = u;
16
17
                          return true;
18
                      }
                 } else slack[v] = min(slack[v], t);
19
             }
20
             return false;
21
22
        }
23
         int go() {
24
25
             memset(left, -1, sizeof left);
             memset(R, 0, sizeof R);
26
             memset(L, 0, sizeof L);
27
28
             FOR (i, 0, n)
                 FOR (j, 0, m)
29
30
                     L[i] = max(L[i], w[i][j]);
31
             FOR (i, 0, n) {
32
                 memset(slack, 0x3f, sizeof slack);
33
                 while (1) {
34
                      memset(visL, \ 0, \ \textbf{sizeof} \ visL); \ memset(visR, \ 0, \ \textbf{sizeof} \ visR);
35
                      if (dfs(i)) break;
36
37
                      int d = 0x3f3f3f3f;
                      FOR (j, \theta, m) if (!visR[j]) d = min(d, slack[j]);
38
                      FOR (j, 0, n) if (visL[j]) L[j] -= d;
39
                      FOR (j, 0, m) if (visR[j]) R[j] += d; else slack[j] -= d;
40
                 }
41
             }
42
43
             int ret = 0;
             FOR (i, 0, m) if (left[i] != -1) ret += w[left[i]][i];
44
45
             return ret;
46
        }
47
    }
```

# 虚树

```
void go(vector<int>& V, int& k) {
        int u = V[k]; f[u] = 0;
2
        dbg(u, k);
3
        for (auto& e: G[u]) {
            int v = e.to;
            if (v == pa[u][0]) continue;
            while (k + 1 < V.size()) {
                 int to = V[k + 1];
                 if (in[to] <= out[v]) {</pre>
                     go(V, ++k);
                     if (key[to]) f[u] += w[to];
11
                     else f[u] += min(f[to], (LL)w[to]);
12
13
                 } else break;
            }
14
15
        }
        dbg(u, f[u]);
16
17
    inline bool cmp(int a, int b) { return in[a] < in[b]; }</pre>
18
    LL solve(vector<int>& V) {
19
        static vector<int> a; a.clear();
        for (int& x: V) a.push_back(x);
21
        sort(a.begin(), a.end(), cmp);
22
23
        FOR (i, 1, a.size())
            a.push_back(lca(a[i], a[i - 1]));
24
        a.push_back(1);
        sort(a.begin(), a.end(), cmp);
26
27
        a.erase(unique(a.begin(), a.end());
        dbg(a);
28
29
        int tmp; go(a, tmp = 0);
30
        return f[1];
    }
31
    欧拉路径
    int S[N << 1], top;</pre>
    Edge edges[N << 1];</pre>
   set<int> G[N];
    void DFS(int u) {
       S[top++] = u;
        for (int eid: G[u]) {
            int v = edges[eid].get_other(u);
            G[u].erase(eid);
10
            G[v].erase(eid);
            DFS(v);
11
            return;
        }
13
    }
14
15
    void fleury(int start) {
16
        int u = start;
        top = 0; path.clear();
18
19
        S[top++] = u;
        while (top) {
20
            u = S[--top];
21
22
            if (!G[u].empty())
                DFS(u);
23
24
            else path.push_back(u);
        }
25
    }
```

## 强连通分量与 2-SAT

```
int n, m;
vector<int> G[N], rG[N], vs;
```

```
int used[N], cmp[N];
    void add_edge(int from, int to) {
        G[from].push_back(to);
        rG[to].push_back(from);
    }
8
    void dfs(int v) {
10
        used[v] = true;
11
        for (int u: G[v]) {
12
            if (!used[u])
13
14
                 dfs(u);
15
        vs.push_back(v);
16
    }
17
18
19
    void rdfs(int v, int k) {
        used[v] = true;
20
21
        cmp[v] = k;
        for (int u: rG[v])
22
            if (!used[u])
23
24
                rdfs(u, k);
25
    }
27
    int scc() {
28
        memset(used, 0, sizeof(used));
29
        vs.clear();
        for (int v = 0; v < n; ++v)
30
           if (!used[v]) dfs(v);
        memset(used, 0, sizeof(used));
32
        int k = 0;
33
        for (int i = (int) vs.size() - 1; i >= 0; --i)
34
35
            if (!used[vs[i]]) rdfs(vs[i], k++);
        return k;
    }
37
38
    int main() {
39
        cin >> n >> m;
40
        n *= 2;
41
        for (int i = 0; i < m; ++i) {</pre>
42
            int a, b; cin >> a >> b;
43
            add_edge(a - 1, (b - 1) ^ 1);
44
            add_edge(b - 1, (a - 1) ^ 1);
45
46
        }
47
        scc();
        for (int i = 0; i < n; i += 2) {
48
            if (cmp[i] == cmp[i + 1]) {
49
                 puts("NIE");
                 return 0;
51
52
53
        for (int i = 0; i < n; i += 2) {</pre>
54
            if (cmp[i] > cmp[i + 1]) printf("%d\n", i + 1);
            else printf("%d\n", i + 2);
56
57
        }
    }
58
    拓补排序
    vector<int> toporder(int n) {
        vector<int> orders;
2
        queue<int> q;
3
        for (int i = 0; i < n; i++)</pre>
            if (!deg[i]) {
                 q.push(i);
                 orders.push_back(i);
        while (!q.empty()) {
            int u = q.front(); q.pop();
```

```
for (int v: G[u])
11
12
                if (!--deg[v]) {
13
                    q.push(v);
14
                    orders.push_back(v);
15
16
17
        return orders;
   }
18
    一般图匹配(没有测试过)
    // maximum matching (graph not necessarily bipartite)
    // whatever code uses this needs to set N in main()
    // author claims N^4 running time
    #define PB push_back
    #define SZ(x) ((int)(x).size())
    #define REP(i, n) for(int i=0; i<n; ++i)</pre>
    #define FOR(i, b, e) for(typeof(e) i=b; i!=e; ++i)
    int N; // the number of vertices in the graph
11
    typedef vector<int> VI;
12
    typdef vector<vector<int>> VVI;
13
14
    VI match;
15
   VI vis;
16
17
    void couple(int n, int m) {
18
        match[n] = m;
19
20
        match[m] = n;
   }
21
22
   // returns true if something interesting has been found, thus a
23
   // augmenting path or a blossom (if blossom is non-empty).
24
   // the dfs returns true from the moment the stem of the flower is
   // reached and thus the base of the blossom is an unmatched node.
26
27
    // blossom should be empty when dfs is called and
    // contains the nodes of the blossom when a blossom is found.
28
    bool dfs(int n, VVI &conn, VI &blossom) {
29
30
        vis[n] = 0;
        REP(i, N) if (conn[n][i]) {
31
32
                if (vis[i] == -1) {
                    vis[i] = 1;
33
34
                    if (match[i] == -1 || dfs(match[i], conn, blossom)) {
35
                         couple(n, i);
                         return true;
36
                    }
37
38
                if (vis[i] == 0 || SZ(blossom)) { // found flower
                    blossom.PB(i);
40
41
                    blossom.PB(n);
                    if (n == blossom[0]) {
42
                        match[n] = -1;
43
44
                         return true;
                    }
45
                    return false;
46
                }
47
            }
48
49
        return false;
   }
50
   // search for an augmenting path.
52
   // if a blossom is found build a new graph (newconn) where the
   // (free) blossom is shrunken to a single node and recurse.
   // if a augmenting path is found it has already been augmented
   // except if the augmented path ended on the shrunken blossom.
   // in this case the matching should be updated along the appropriate
   // direction of the blossom.
```

```
bool augment(VVI & conn) {
59
60
         REP(m, N) if (match[m] == -1) {
                 VI blossom;
61
                 vis = VI(N, -1);
62
                 if (!dfs(m, conn, blossom)) continue;
                 if (SZ(blossom) == 0) return true; // augmenting path found
64
65
                 // blossom is found so build shrunken graph
66
                 int base = blossom[0], S = SZ(blossom);
67
                 VVI newconn = conn;
68
                 FOR(i, 1, S - 1) REP(j, N)
69
                     newconn[base][j] = newconn[j][base] |= conn[blossom[i]][j];
71
                 FOR(i, 1, S - 1) REP(j, N)
                     newconn[blossom[i]][j] = newconn[j][blossom[i]] = 0;
72
                 newconn[base][base] = 0; // is now the new graph
73
                 if (!augment(newconn)) return false;
74
75
                 int n = match[base];
76
77
                 // if n!=-1 the augmenting path ended on this blossom
                 if (n != -1)
78
                     REP(i, S) if (conn[blossom[i]][n]) {
79
80
                          couple(blossom[i], n);
81
                          if (i & 1) for (int j = i + 1; j < S; j += 2)
                             couple(blossom[j], blossom[j + 1]);
                          else for (int j = 0; j < i; j += 2)
83
84
                             couple(blossom[j], blossom[j + 1]);
85
                         break;
                     }
86
87
                 return true;
            }
88
         return false;
89
    }
90
91
    // conn should have N VI's, each of length N. conn[i][j] = 1 if (i,j) is an
    // edge, and conn[i][j] = 0 otherwise. returns the number of edges in a max
93
    // matching.
94
    int edmonds(VVI & conn) {
95
         int res = 0;
96
         match = VI(N, -1);
97
        while (augment(conn)) res++;
98
99
         return res;
    }
100
```

# 计算几何

#### 圆的反演

```
typedef double LD;
   const LD PI = 3.14159265358979323846;
   const LD eps = 1E-10;
   const LD R2 = 1.0;
    int sgn(LD x) \{ return fabs(x) < eps ? 0 : (x > 0 ? 1 : -1); \}
    struct P {
        LD x, y;
        P(LD x = 0, LD y = 0): x(x), y(y) {}
        P operator * (LD k) { return P(x * k, y * k); }
        P operator / (LD k) { return P(x / k, y / k); }
10
11
        string prt() const {
12
            char s[100];
            sprintf(s, "(%.2f, %.2f)", x, y);
13
            return string(s);
14
        }
15
16
   };
   typedef P V;
   P operator - (const P& a, const P& b) { return P(a.x - b.x, a.y - b.y); }
   P operator + (const P& a, const P& b) { return P(a.x + b.x, a.y + b.y); }
19
   struct C {
       P p;
21
```

```
LD r;
22
23
        C(LD x = 0, LD y = 0, LD r = 0): p(x, y), r(r) {}
    };
24
    LD dist(V v) { return sqrt(v.x * v.x + v.y * v.y); }
25
    C inv(C c, const P& o) {
27
        LD d = dist(c.p - o);
28
        assert(sgn(d) != 0);
29
        LD a = 1 / (d - c.r);
30
31
        LD b = 1 / (d + c.r);
        c.r = (a - b) / 2 * R2;
32
33
        c.p = o + (c.p - o) * ((a + b) * R2 / 2 / d);
34
        return c;
    }
35
    二维
       ● nxt 宏要求多边形变量名为 s
        ● L 可隐式转换为 V(P)
        • 可以自定义结构体 PP, 可隐式转换为 P
    #define y1 yy1
1
    #define nxt(i) ((i + 1) % s.size())
    typedef double LD;
    const LD PI = 3.14159265358979323846;
    const LD eps = 1E-10;
    int sgn(LD x) { return fabs(x) < eps ? 0 : (x > 0 ? 1 : -1); }
    struct L:
    struct P;
    //struct PP;
    typedef P V;
10
    struct P {
11
        LD x, y;
12
        explicit P(LD x = 0, LD y = 0): x(x), y(y) {}
13
        P(const L& l);
14
          P(const PP& pp);
15
    };
16
    struct L {
17
        Ps, t;
18
        L() {}
19
        L(P s, P t): s(s), t(t) {}
20
21
22
    P operator + (const P& a, const P& b) { return P(a.x + b.x, a.y + b.y); }
    P operator - (const P& a, const P& b) { return P(a.x - b.x, a.y - b.y); }
24
    P operator \star (const P& a, LD k) { return P(a.x \star k, a.y \star k); }
25
    P operator / (const P& a, LD k) { return P(a.x / k, a.y / k); }
26
    inline int operator < (const P& a, const P& b) {</pre>
        return sgn(a.x - b.x) < 0 \mid | (sgn(a.x - b.x) == 0 && sgn(a.y - b.y) < 0);
29
    bool operator == (const P& a, const P& b) { return !sgn(a.x - b.x) && !sgn(a.y - b.y); }
30
    P::P(const L& l) { *this = l.t - l.s; }
31
    ostream &operator << (ostream &os, const P &p) {</pre>
32
        return (os << "(" << p.x << "," << p.y << ")");
    }
34
    istream &operator >> (istream &is, P &p) {
35
        return (is >> p.x >> p.y);
36
37
38
39
40
    //struct PP {
41
        Р р;
42
    //
          LD v, l;
43
44
    //P::P(const PP& pp) { *this = pp.p; }
    typedef P PP;
46
    typedef vector<PP> S;
```

```
49
50
    LD dist(const P& p) { return sqrt(p.x * p.x + p.y * p.y); }
51
    LD dot(const V& a, const V& b) { return a.x * b.x + a.y * b.y; }
    LD det(const V& a, const V& b) { return a.x * b.y - a.y * b.x; }
    LD cross(const P& s, const P& t, const P& o = P()) { return det(s - o, t - o); }
54
    // 如需支持 unique, 需要加 eps
56
    bool cmp_xy(const P& a, const P& b) { return a.x < b.x || a.x == b.x && a.y < b.y; }</pre>
57
58
    // 象限
59
60
    int quad(P p) {
61
        int x = sgn(p.x), y = sgn(p.y);
         if (x > 0 && y >= 0) return 1;
62
        if (x <= 0 \&\& y > 0) return 2;
63
         if (x < 0 && y <= 0) return 3;
64
        if (x >= 0 && y < 0) return 4;
        assert(0);
66
68
    // 仅适用于参照点在所有点一侧的情况
69
    struct cmp_angle {
        P p;
71
         bool operator () (const P& a, const P& b) {
72
    //
              int qa = quad(a), qb = quad(b);
73
74
    //
              if (qa != qb) return qa < qb;
            int d = sgn(cross(a, b, p));
75
            if (d) return d > 0;
76
            return dist(a - p) < dist(b - p);</pre>
78
    };
79
80
    // -----线------线------
81
    // 是否平行
83
    bool parallel(const L& a, const L& b) {
84
        return !sgn(det(a, b));
85
86
    // 直线是否相等
87
    bool l_eq(const L& a, const L& b) {
88
89
        return parallel(a, b) && parallel(L(a.s, b.t), L(b.s, a.t));
90
    P rotation(const P& p, const LD& r) { return P(p.x * cos(r) - p.y * sin(r), p.x * sin(r) + p.y * cos(r)); }
    P RotateCCW90(const P& p) { return P(-p.y, p.x); }
93
    P RotateCW90(const P& p) { return P(p.y, -p.x); }
95
    V normal(const V& v) { return V(-v.y, v.x) / dist(v); }
97
98
    // ------点和线------
99
100
    // 点在线段上 <= 0 包含端点 < 0 则不包含
    bool p_on_seg(const P& p, const L& seg) {
102
        P a = seg.s, b = seg.t;
103
        return !sgn(det(p - a, b - a)) && sgn(dot(p - a, p - b)) <= 0;</pre>
104
105
    // 点到直线距离
106
    LD dist_to_line(const P& p, const L& l) {
107
        return fabs(cross(l.s, l.t, p)) / dist(l);
108
109
    // 点到线段距离
110
    LD dist_to_seg(const P& p, const L& l) {
111
         if (l.s == l.t) return dist(p - l);
112
         V vs = p - l.s, vt = p - l.t;
113
         if (sgn(dot(l, vs)) < 0) return dist(vs);</pre>
114
         else if (sgn(dot(l, vt)) > 0) return dist(vt);
115
116
         else return dist_to_line(p, l);
    }
117
118
```

119

```
// -----线和线-----
120
121
    // 求直线交 需要事先保证有界
122
    P l_intersection(const L& a, const L& b) {
123
124
        LD s1 = det(a, b.s - a.s), s2 = det(a, b.t - a.s);
        return (b.s * s2 - b.t * s1) / (s2 - s1);
125
126
    // 向量夹角的弧度
127
    LD angle(const V& a, const V& b) {
128
129
        LD r = asin(fabs(det(a, b)) / dist(a) / dist(b));
        if (sgn(dot(a, b)) < 0) r = PI - r;</pre>
130
131
        return r;
132
    }
    // 线段和直线是否有交 1 = 规范, 2 = 不规范
133
134
    int s_l_cross(const L& seg, const L& line) {
        int d1 = sgn(cross(line.s, line.t, seg.s));
135
136
        int d2 = sgn(cross(line.s, line.t, seg.t));
        if ((d1 ^ d2) == -2) return 1; // proper
137
138
        if (d1 == 0 || d2 == 0) return 2;
139
        return 0;
    }
140
    // 线段的交 1 = 规范, 2 = 不规范
141
    int s_cross(const L& a, const L& b, P& p) {
142
        int d1 = sgn(cross(a.t, b.s, a.s)), d2 = sgn(cross(a.t, b.t, a.s));
143
        int d3 = sgn(cross(b.t, a.s, b.s)), d4 = sgn(cross(b.t, a.t, b.s));
144
        if ((d1 \land d2) == -2 \&\& (d3 \land d4) == -2) { p = l_intersection(a, b); return 1; }
145
        if (!d1 && p_on_seg(b.s, a)) { p = b.s; return 2; }
146
        if (!d2 && p_on_seg(b.t, a)) { p = b.t; return 2; }
147
        if (!d3 && p_on_seg(a.s, b)) { p = a.s; return 2; }
        if (!d4 && p_on_seg(a.t, b)) { p = a.t; return 2; }
149
        return 0;
150
151
152
153
    // -----多边形-----
154
155
    // 点是否在多边形中 0 = 在外部 1 = 在内部 -1 = 在边界上
156
    int inside(const S& s, const P& p) {
157
158
        int cnt = 0;
        FOR (i, 0, s.size()) {
159
160
            P = s[i], b = s[nxt(i)];
161
            if (p_on_seg(p, L(a, b))) return -1;
            if (sgn(a.y - b.y) <= 0) swap(a, b);
162
163
            if (sgn(p.y - a.y) > 0) continue;
            if (sgn(p.y - b.y) <= 0) continue;</pre>
164
            cnt += sgn(cross(b, a, p)) > 0;
165
166
167
        return bool(cnt & 1);
168
    }
    // 多边形面积, 有向面积可能为负
169
    LD polygon_area(const S& s) {
170
        LD ret = 0;
171
        FOR (i, 1, (LL)s.size() - 1)
172
            ret += cross(s[i], s[i + 1], s[0]);
173
        return ret / 2;
174
175
    // 构建凸包 点不可以重复 < 0 边上可以有点, <= 0 则不能
176
    // 会改变输入点的顺序
178
    const int MAX_N = 1000;
    S convex_hull(S& s) {
179
180
         assert(s.size() >= 3);
        sort(s.begin(), s.end(), cmp_xy);
181
        S ret(MAX_N * 2);
182
        int sz = 0;
183
184
        FOR (i, 0, s.size()) {
            while (sz > 1 && sgn(cross(ret[sz - 1], s[i], ret[sz - 2])) < 0) --sz;
185
            ret[sz++] = s[i];
186
187
        int k = sz;
188
        FORD (i, (LL)s.size() - 2, -1) {
189
            while (sz > k && sgn(cross(ret[sz - 1], s[i], ret[sz - 2])) < 0) --sz;</pre>
190
```

```
ret[sz++] = s[i];
191
192
         ret.resize(sz - (s.size() > 1));
193
         return ret;
194
195
196
    P ComputeCentroid(const vector<P> &p) {
197
        P c(0, 0);
198
         LD scale = 6.0 * polygon_area(p);
199
200
         for (unsigned i = 0; i < p.size(); i++) {</pre>
             unsigned j = (i + 1) % p.size();
201
202
             c = c + (p[i] + p[j]) * (p[i].x * p[j].y - p[j].x * p[i].y);
203
        return c / scale;
204
205
    }
206
    207
208
    P ComputeCircleCenter(P a, P b, P c) {
209
210
        b = (a + b) / 2;
        c = (a + c) / 2;
211
         return l_intersection({b, b + RotateCW90(a - b)}, {c , c + RotateCW90(a - c)});
212
213
    vector<P> CircleLineIntersection(P a, P b, P c, LD r) {
214
        vector<P> ret;
215
        b = b - a;
216
        a = a - c;
217
        LD A = dot(b, b), B = dot(a, b), C = dot(a, a) - r * r;
218
         LD D = B \star B - A \star C;
         if (sgn(D) < 0) return ret;</pre>
220
         ret.push_back(c + a + b \star (-B + sqrt(D + eps)) / A);
221
         if (sgn(D) > 0) ret.push_back(c + a + b * (-B - sqrt(D)) / A);
222
        return ret;
223
224
    vector<P> CircleCircleIntersection(P a, P b, LD r, LD R) {
225
         vector<P> ret;
226
         LD d = dist(a - b);
227
         if (sgn(d) == 0 \mid | sgn(d - (r + R)) > 0 \mid | sgn(d + min(r, R) - max(r, R)) < 0) return ret;
228
229
        LD x = (d * d - R * R + r * r) / (2 * d);
        LD y = sqrt(r * r - x * x);
230
231
        P v = (b - a) / d;
         ret.push_back(a + v * x + RotateCCW90(v) * y);
232
         if (sgn(y) > 0) ret.push_back(a + v * x - RotateCCW90(v) * y);
233
234
         return ret;
    }
235
236
    // -----模板结束-----
237
    旋转卡壳
    LD rotatingCalipers(S& qs) {
 1
 2
         int n = qs.size();
         if (n == 2)
 3
            return dist(qs[0] - qs[1]);
        int i = 0, j = 0;
        FOR (k, 0, n) {
             if (!(qs[i] < qs[k])) i = k;</pre>
             if (qs[j] < qs[k]) j = k;
        }
        LD res = 0;
        int si = i, sj = j;
11
         while (i != sj || j != si) {
12
             res = max(res, dist(qs[i] - qs[j]));
13
             if (sgn(cross(qs[(i+1)%n] - qs[i], qs[(j+1)%n] - qs[j])) < 0)
14
15
                 i = (i + 1) \% n;
             else j = (j + 1) \% n;
16
17
        return res;
18
```

}

```
20
21
    int main() {
22
        int n;
        while (cin >> n) {
23
            S v(n);
24
            FOR (i, 0, n) cin >> v[i].x >> v[i].y;
25
            convex_hull(v);
26
            printf("%.0f\n", rotatingCalipers(v));
27
28
    }
    没有测试过的
    int relation(Point p, Circle a) {//点和圆的关系
        //0: 圆外 1: 圆上 2: 圆内
2
        double d = dis(p, a.p);
3
        if (dcmp(d - a.r) == 0) return 1;
        return (dcmp(d - a.r) < 0 ? 2 : 0);</pre>
5
    int relation(Line a, Circle b) {//直线和圆的关系
        //0: 相离 1: 相切 2: 相交
        double p = point_to_line(b.p, a);
10
        if (dcmp(p - b.r) == 0) return 1;
11
        return (dcmp(p - b.r) < 0 ? 2 : 0);
12
13
14
    int relation(Circle a, Circle v) {//两圆的位置关系
15
        //1: 内含 2: 内切 3: 相交 4: 外切 5: 相离
16
        double d = dis(a.p, v.p);
17
18
        if (dcmp(d - a.r - v.r) > 0) return 5;
        if (dcmp(d - a.r - v.r) == 0) return 4;
19
20
        double l = fabs(a.r - v.r);
        if (dcmp(d - a.r - v.r) < 0 && dcmp(d - l) > 0) return 3;
21
        if (dcmp(d - l) == 0) return 2;
22
        if (dcmp(d - l) < 0) return 1;</pre>
23
        assert (0);
24
25
    }
26
    double circle_traingle_area(Point a, Point b, Circle c) {//圆心三角形的面积
27
28
        //a.output (), b.output (), c.output ();
        Point p = c.p;
29
        double r = c.r; //cout << cross (p-a, p-b) << endl;</pre>
30
        if (dcmp(cross(p - a, p - b)) == 0) return 0;
31
        Point q[5];
32
        int len = 0;
33
        q[len++] = a;
34
35
        Line l(a, b);
        Point p1, p2;
36
        if (line_cirlce_intersection(l, c, q[1], q[2]) == 2) {
             if (dcmp(dot(a - q[1], b - q[1])) < 0) q[len++] = q[1];</pre>
38
            if (dcmp(dot(a - q[2], b - q[2])) < 0) q[len++] = q[2];</pre>
39
40
        q[len++] = b;
41
        if (len == 4 && dcmp(dot(q[0] - q[1], q[2] - q[1])) > 0)
42
            swap(q[1], q[2]);
43
        double res = 0;
44
        for (int i = 0; i < len - 1; i++) {</pre>
45
            if (relation(q[i], c) == 0 \mid \mid relation(q[i + 1], c) == 0) {
46
47
                 double arg = rad(q[i] - p, q[i + 1] - p);
                 res += r * r * arg / 2.0;
48
49
            } else {
                 res += fabs(cross(q[i] - p, q[i + 1] - p)) / 2;
50
            }
51
52
        return res:
53
54
    }
```

#### 半平面交

```
struct Line {
        PT p, v;
2
        double ang;
3
        Line() {}
        Line(PT from, PT to) : p(from), v(to - from) { ang = atan2(v.y, v.x); }
        friend bool operator<(Line a, Line b) {</pre>
             return a.ang < b.ang;</pre>
10
    };
11
    bool OnLeft(Line L, PT p) {
12
13
        return dcmp(cross(L.v, p - L.p)) >= 0;
14
15
16
    PT GetIntersection(Line a, Line b) {
        PT u = a.p - b.p;
17
        ld t = cross(b.v, u) / cross(a.v, b.v);
18
        return a.p + a.v * t;
19
21
    vector<PT> HalfplaneIntersection(vector<Line>& L) {
22
23
        int n = L.size();
        sort(L.begin(), L.end());
24
        int first, last;
26
27
        vector<PT> p(n);
        vector<Line> q(n);
28
        q[first = last = 0] = L[0];
29
30
        for (int i = 1; i < n; i++) {
             while (first < last && !OnLeft(L[i], p[last - 1])) last--;</pre>
31
             while (first < last && !OnLeft(L[i], p[first])) first++;</pre>
32
             q[++last] = L[i];
33
             if (dcmp(cross(q[last].v, q[last - 1].v)) == 0) {
34
35
                 last--:
                 if (OnLeft(q[last], L[i].p)) q[last] = L[i];
36
37
             if (first < last) p[last - 1] = GetIntersection(q[last - 1], q[last]);</pre>
38
39
        while (first < last && !OnLeft(q[first], p[last - 1])) last--;</pre>
40
        if (last - first <= 1) return vector<PT>();
41
42
        p[last] = GetIntersection(q[last], q[first]);
43
44
        return vector<PT>(p.begin() + first, p.begin() + last + 1);
    }
45
46
    vector<PT> convexIntersection(const vector<PT> &v1, const vector<PT> &v2) {
47
48
        vector<Line> h;
        int n = v1.size(), m = v2.size();
        for (int i = 0; i < n; ++i)</pre>
50
            h.push_back(Line(v1[i], v1[(i + 1) % n]));
51
        for (int i = 0; i < m; ++i)</pre>
52
            h.push_back(Line(v2[i], v2[(i + 1) % m]));
53
        return HalfplaneIntersection(h);
55
    }
    ld ComputeSignedArea(const vector<PT> &p) {
57
        ld area = 0;
58
        for (unsigned i = 0; i < p.size(); i++) {</pre>
59
            unsigned j = (i + 1) % p.size();
60
61
             area += p[i].x * p[j].y - p[j].x * p[i].y;
62
        return area / 2.0;
63
    }
64
65
    ld ComputeArea(const vector<PT> &p) {
        return fabs(ComputeSignedArea(p));
67
    }
```

# 字符串

### 后缀自动机

- 广义后缀自动机如果直接使用以下代码的话会产生一些冗余状态(置 last 为 1),所以要用拓扑排序。用 len 基数排序不能。
- 字符集大的话要使用 map。
- 树上 dp 时注意边界 (root 和 null)。
- rsort 需要初始化

```
namespace sam {
        const int M = N << 1;</pre>
2
        int t[M][26], len[M] = {-1}, fa[M], sz = 2, last = 1;
        void ins(int ch) {
            int p = last, np = last = sz++;
            len[np] = len[p] + 1;
            for (; p && !t[p][ch]; p = fa[p]) t[p][ch] = np;
            if (!p) { fa[np] = 1; return; }
            int q = t[p][ch];
            if (len[p] + 1 == len[q]) fa[np] = q;
            else {
11
                int nq = sz++; len[nq] = len[p] + 1;
12
13
                memcpy(t[nq], t[q], sizeof t[0]);
                fa[nq] = fa[q];
14
15
                fa[np] = fa[q] = nq;
                for (; t[p][ch] == q; p = fa[p]) t[p][ch] = nq;
16
17
            }
        }
18
19
        int c[M] = {1}, a[M];
20
        void rsort() {
21
           FOR (i, 1, sz) c[i] = 0;
22
            FOR (i, 1, sz) c[len[i]]++;
23
            FOR (i, 1, sz) c[i] += c[i - 1];
            FOR (i, 1, sz) a[--c[len[i]]] = i;
25
26
        }
27
   }
       真·广义后缀自动机
   int t[M][26], len[M] = {-1}, fa[M], sz = 2, last = 1;
    LL cnt[M][2];
    void ins(int ch, int id) {
        int p = last, np = 0, nq = 0, q = -1;
        if (!t[p][ch]) {
            np = sz++;
            len[np] = len[p] + 1;
            for (; p && !t[p][ch]; p = fa[p]) t[p][ch] = np;
        if (!p) fa[np] = 1;
10
        else {
11
12
            q = t[p][ch];
            if (len[p] + 1 == len[q]) fa[np] = q;
13
14
                nq = sz++; len[nq] = len[p] + 1;
15
                memcpy(t[nq], t[q], sizeof t[0]);
16
17
                fa[nq] = fa[q];
                fa[np] = fa[q] = nq;
18
                for (; t[p][ch] == q; p = fa[p]) t[p][ch] = nq;
            }
20
21
        last = np ? np : nq ? nq : q;
22
        cnt[last][id] = 1;
23
24
   }
       • 按字典序建立后缀树注意逆序插入
    void ins(int ch, int pp) {
        int p = last, np = last = sz++;
```

```
len[np] = len[p] + 1; one[np] = pos[np] = pp;
for (; p && !t[p][ch]; p = fa[p]) t[p][ch] = np;
```

```
if (!p) { fa[np] = 1; return; }
6
        int q = t[p][ch];
        if (len[q] == len[p] + 1) fa[np] = q;
        else {
8
            int nq = sz++; len[nq] = len[p] + 1; one[nq] = one[q];
            memcpy(t[nq],\ t[q],\ \textbf{sizeof}\ t[\theta]);
10
            fa[nq] = fa[q];
11
            fa[q] = fa[np] = nq;
12
            for (; p && t[p][ch] == q; p = fa[p]) t[p][ch] = nq;
13
14
        }
    }
15
16
    int up[M], c[256] = {2}, a[M];
17
    void rsort2() {
18
        FOR (i, 1, 256) c[i] = 0;
19
        FOR (i, 2, sz) up[i] = s[one[i] + len[fa[i]]];
20
21
        FOR (i, 2, sz) c[up[i]]++;
        FOR (i, 1, 256) c[i] += c[i - 1];
22
        FOR (i, 2, sz) a[--c[up[i]]] = i;
23
        FOR (i, 2, sz) G[fa[a[i]]].push_back(a[i]);
24
   }
25
        • 广义后缀自动机建后缀树, 必须反向插入
    int t[M][26], len[M] = {0}, fa[M], sz = 2, last = 1;
    char* one[M];
2
    void ins(int ch, char* pp) {
        int p = last, np = 0, nq = 0, q = -1;
        if (!t[p][ch]) {
            np = sz++; one[np] = pp;
7
            len[np] = len[p] + 1;
            for (; p && !t[p][ch]; p = fa[p]) t[p][ch] = np;
        if (!p) fa[np] = 1;
10
11
        else {
            q = t[p][ch];
12
13
            if (len[p] + 1 == len[q]) fa[np] = q;
            else {
14
                nq = sz^{++}; len[nq] = len[p] + 1; one[nq] = one[q];
15
16
                memcpy(t[nq], t[q], sizeof t[0]);
17
                fa[nq] = fa[q];
18
                fa[np] = fa[q] = nq;
                for (; t[p][ch] == q; p = fa[p]) t[p][ch] = nq;
19
            }
        }
21
22
        last = np ? np : nq ? nq : q;
23
    int up[M], c[256] = {2}, aa[M];
24
    vector<int> G[M];
    void rsort() {
26
        FOR (i, 1, 256) c[i] = 0;
27
28
        FOR (i, 2, sz) up[i] = *(one[i] + len[fa[i]]);
        FOR (i, 2, sz) c[up[i]]++;
29
30
        FOR (i, 1, 256) c[i] += c[i - 1];
        FOR (i, 2, sz) aa[--c[up[i]]] = i;
31
        FOR (i, 2, sz) G[fa[aa[i]]].push_back(aa[i]);
32
   }
33
        匹配
    int u = 1, l = 0;
1
    FOR (i, 0, strlen(s)) {
        int ch = s[i] - 'a';
        while (u && !t[u][ch]) { u = fa[u]; l = len[u]; }
        ++l; u = t[u][ch];
        if (!u) u = 1;
        // do something...
    }
        • 获取子串状态
    int get_state(int l, int r) {
        int u = rpos[r], s = r - l + 1;
```

```
FORD (i, SP - 1, -1) if (len[pa[u][i]] >= s) u = pa[u][i];
3
4
        return u;
   }
5
        • 配合 LCT
    namespace lct_sam {
1
        extern struct P *const null;
        const int M = N;
3
        struct P {
4
            P *fa, *ls, *rs;
5
6
            int last;
            bool has_fa() { return fa->ls == this || fa->rs == this; }
            bool d() { return fa->ls == this; }
            P*\& c(bool x) { return x ? ls : rs; }
10
            P* up() { return this; }
11
            void down() {
12
                 if (ls != null) ls->last = last;
13
                 if (rs != null) rs->last = last;
15
            }
            void all_down() { if (has_fa()) fa->all_down(); down(); }
16
        } *const null = new P{0, 0, 0, 0}, pool[M], *pit = pool;
17
        P* G[N];
18
        int t[M][26], len[M] = {-1}, fa[M], sz = 2, last = 1;
20
21
        void rot(P* o) {
            bool dd = o->d();
22
            P *f = o \rightarrow fa, *t = o \rightarrow c(!dd);
23
            if (f->has_fa()) f->fa->c(f->d()) = o; o->fa = f->fa;
25
            if (t != null) t->fa = f; f->c(dd) = t;
            o->c(!dd) = f->up(); f->fa = o;
26
27
        void splay(P* o) {
28
29
            o->all_down();
            while (o->has_fa()) {
30
31
                 if (o->fa->has_fa())
                    rot(o->d() ^ o->fa->d() ? o : o->fa);
32
                 rot(o);
33
            }
34
35
            o->up();
36
        void access(int last, P* u, P* v = null) {
37
            if (u == null) { v->last = last; return; }
            splay(u);
39
40
            P *t = u;
41
            while (t->ls != null) t = t->ls;
            int L = len[fa[t - pool]] + 1, R = len[u - pool];
42
43
            if (u->last) bit::add(u->last - R + 2, u->last - L + 2, 1);
44
            else bit::add(1, 1, R - L + 1);
45
46
            bit::add(last - R + 2, last - L + 2, -1);
47
48
            u->rs = v;
            access(last, u->up()->fa, u);
49
50
        void insert(P* u, P* v, P* t) {
51
            if (v != null) { splay(v); v->rs = null; }
52
53
            splay(u);
            u->fa = t; t->fa = v;
54
55
56
        void ins(int ch, int pp) {
57
58
            int p = last, np = last = sz++;
            len[np] = len[p] + 1;
59
60
            for (; p && !t[p][ch]; p = fa[p]) t[p][ch] = np;
61
            if (!p) fa[np] = 1;
62
            else {
                 int q = t[p][ch];
63
                 if (len[p] + 1 == len[q]) { fa[np] = q; G[np]->fa = G[q]; }
64
65
                 else {
                     int nq = sz++; len[nq] = len[p] + 1;
```

```
68
                     insert(G[q], G[fa[q]], G[nq]);
                     G[nq]->last = G[q]->last;
69
                     fa[nq] = fa[q];
70
                     fa[np] = fa[q] = nq;
                     G[np] \rightarrow fa = G[nq];
72
                     for (; t[p][ch] == q; p = fa[p]) t[p][ch] = nq;
73
                 }
74
75
            access(pp + 1, G[np]);
        }
77
78
        void init() {
79
             ++pit;
80
            FOR (i, 1, N) \{
81
                 G[i] = pit++;
82
83
                 G[i]->ls = G[i]->rs = G[i]->fa = null;
84
            G[1] = null;
85
        }
86
87
    }
    回文自动机
    namespace pam {
1
        int t[N][26], fa[N], len[N], rs[N], cnt[N];
3
        int sz, n, last;
        int _new(int l) {
4
            memset(t[sz], 0, sizeof t[0]);
            len[sz] = l; cnt[sz] = 0;
            return sz++;
        }
8
        void init() {
            rs[n = sz = 0] = -1;
10
            last = _{new(0)};
11
12
            fa[last] = _new(-1);
13
        int get_fa(int x) {
14
            while (rs[n - 1 - len[x]] != rs[n]) x = fa[x];
15
            return x;
16
17
        void ins(int ch) {
18
19
            rs[++n] = ch;
            int p = get_fa(last);
20
21
            if (!t[p][ch]) {
                 int np = _new(len[p] + 2);
22
                 fa[np] = t[get_fa(fa[p])][ch];
23
24
                 t[p][ch] = np;
            }
25
             ++cnt[last = t[p][ch]];
        }
27
28
    }
    manacher
    int RL[N];
    void manacher(int* a, int n) { // "abc" => "#a#b#a#"
2
        int r = 0, p = 0;
3
        FOR (i, 0, n) {
            if (i < r) RL[i] = min(RL[2 * p - i], r - i);</pre>
            else RL[i] = 1;
            while (i - RL[i] >= 0 \&\& i + RL[i] < n \&\& a[i - RL[i]] == a[i + RL[i]])
                RL[i]++;
            if (RL[i] + i - 1 > r) { r = RL[i] + i - 1; p = i; }
        }
10
        FOR (i, 0, n) --RL[i];
11
   }
12
```

memcpy(t[nq], t[q], sizeof t[ $\theta$ ]);

67

### 哈希

```
内置了自动双哈希开关(小心 TLE)。
    #include <bits/stdc++.h>
    using namespace std;
2
    #define ENABLE_DOUBLE_HASH
    typedef long long LL;
    typedef unsigned long long ULL;
    const int x = 135;
    const int N = 4e5 + 10;
    const int p1 = 1e9 + 7, p2 = 1e9 + 9;
11
    ULL xp1[N], xp2[N], xp[N];
12
13
14
    void init_xp() {
        xp1[0] = xp2[0] = xp[0] = 1;
15
        for (int i = 1; i < N; ++i) {</pre>
16
            xp1[i] = xp1[i - 1] * x % p1;
            xp2[i] = xp2[i - 1] * x % p2;
18
            xp[i] = xp[i - 1] * x;
19
20
        }
    }
21
    struct String {
23
24
        char s[N];
        int length, subsize;
25
        bool sorted;
26
27
        ULL h[N], hl[N];
28
        ULL hash() {
29
            length = strlen(s);
30
            ULL res1 = 0, res2 = 0;
31
            h[length] = 0; // ATTENTION!
32
            for (int j = length - 1; j >= 0; --j) {
33
34
            #ifdef ENABLE_DOUBLE_HASH
                res1 = (res1 * x + s[j]) % p1;
35
                res2 = (res2 * x + s[j]) % p2;
36
37
                h[j] = (res1 << 32) | res2;
            #else
38
39
                res1 = res1 * x + s[j];
                h[j] = res1;
40
            #endif
                // printf("%llu\n", h[j]);
42
43
44
            return h[0];
45
        }
        // 获取子串哈希, 左闭右开区间
47
        ULL get_substring_hash(int left, int right) const {
48
            int len = right - left;
49
        #ifdef ENABLE_DOUBLE_HASH
50
            // get hash of s[left...right-1]
            unsigned int mask32 = \sim(0u);
52
53
            ULL left1 = h[left] >> 32, right1 = h[right] >> 32;
54
            ULL left2 = h[left] & mask32, right2 = h[right] & mask32;
            return (((left1 - right1 * xp1[len] % p1 + p1) % p1) << 32) |</pre>
55
                    (((left2 - right2 * xp2[len] % p2 + p2) % p2));
57
        #else
58
            return h[left] - h[right] * xp[len];
        #endif
59
60
61
62
        void get_all_subs_hash(int sublen) {
63
            subsize = length - sublen + 1;
            for (int i = 0; i < subsize; ++i)</pre>
64
                hl[i] = get_substring_hash(i, i + sublen);
            sorted = 0;
66
        }
67
```

```
68
69
         void sort_substring_hash() {
             sort(hl, hl + subsize);
70
             sorted = 1;
71
72
73
74
         bool match(ULL key) const {
             if (!sorted) assert (0);
75
             if (!subsize) return false;
76
77
             return binary_search(hl, hl + subsize, key);
         }
78
79
         void init(const char *t) {
80
             length = strlen(t);
81
82
             strcpy(s, t);
         }
83
84
    };
85
     int LCP(const String &a, const String &b, int ai, int bi) {
         // Find LCP of a[ai...] and b[bi...]
87
         int l = 0, r = min(a.length - ai, b.length - bi);
88
         while (l < r) {
89
             int mid = (l + r + 1) / 2;
90
             if (a.get_substring_hash(ai, ai + mid) == b.get_substring_hash(bi, bi + mid))
                  l = mid;
92
93
             else r = mid - 1;
94
         return l;
95
    }
97
     int check(int ans) {
98
         if (T.length < ans) return 1;</pre>
99
         T.get_all_subs_hash(ans); T.sort_substring_hash();
100
101
         for (int i = 0; i < S.length - ans + 1; ++i)</pre>
              if (!T.match(S.get_substring_hash(i, i + ans)))
102
                  return 1;
103
         return 0;
104
    }
105
106
     int main() {
107
108
         init_xp(); // DON'T FORGET TO DO THIS!
109
         for (int tt = 1; tt <= kases; ++tt) {</pre>
110
             scanf("%d", &n); scanf("%s", str);
111
             S.init(str);
112
113
             S.hash(); T.hash();
114
115
    }
```

#### 后缀数组

构造时间: $O(L \log L)$ ; 查询时间  $O(\log L)$ 。suffix 数组是排好序的后缀下标,suffix 的反数组是后缀数组。

```
#include <bits/stdc++.h>
   using namespace std;
   const int N = 2e5 + 10;
   const int Nlog = 18;
   struct SuffixArray {
        const int L;
        vector<vector<int> > P;
        vector<pair<int, int>, int> > M;
10
11
        int s[N], sa[N], rank[N], height[N];
        // s: raw string
12
13
        // sa[i]=k: s[k...L-1] ranks i (0 based)
       // rank[i]=k: the rank of s[i...L-1] is k (0 based)
14
       // height[i] = lcp(sa[i-1], sa[i])
15
16
```

```
SuffixArray(const string &raw_s) : L(raw_s.length()), P(1, vector<int>(L, 0)), M(L) {
17
18
             for (int i = 0; i < L; i++)
                 P[0][i] = this->s[i] = int(raw_s[i]);
19
            for (int skip = 1, level = 1; skip < L; skip *= 2, level++) {</pre>
20
                 P.push_back(vector<int>(L, 0));
21
                 for (int i = 0; i < L; i++)
22
                     M[i] = make_pair(make_pair(P[level - 1][i], i + skip < L ? P[level - 1][i + skip] : -1000), i);
23
                 sort(M.begin(), M.end());
24
                 for (int i = 0; i < L; i++)</pre>
25
                      P[[evel][M[i].second] = (i > 0 \&\& M[i].first == M[i - 1].first) ? P[[evel][M[i - 1].second] : i; 
27
28
            for (unsigned i = 0; i < P.back().size(); ++i) {</pre>
                 rank[i] = P.back()[i];
29
                 sa[rank[i]] = i;
30
            }
31
        }
32
33
        // This is a traditional way to calculate LCP
34
        void getHeight() {
35
            memset(height, 0, sizeof height);
36
             int k = 0;
37
             for (int i = 0; i < L; ++i) {</pre>
38
                 if (rank[i] == 0) continue;
39
                 if (k) k--;
                 int j = sa[rank[i] - 1];
41
                 while (i + k < L \&\& j + k < L \&\& s[i + k] == s[j + k]) ++k;
42
43
                 height[rank[i]] = k;
44
45
            rmq_init(height, L);
        }
46
47
        int f[N][Nlog];
48
        inline int highbit(int x) {
49
50
            return 31 - __builtin_clz(x);
        }
51
52
        int rmq_query(int x, int y) {
53
54
            int p = highbit(y - x + 1);
            return min(f[x][p], f[y - (1 << p) + 1][p]);
55
        }
56
57
        // arr has to be 0 based
58
        void rmq_init(int *arr, int length) {
59
60
            for (int x = 0; x <= highbit(length); ++x)</pre>
                 for (int i = 0; i <= length - (1 << x); ++i) {</pre>
61
62
                     if (!x) f[i][x] = arr[i];
                     else f[i][x] = min(f[i][x - 1], f[i + (1 << (x - 1))][x - 1]);
63
                 }
        }
65
66
        #ifdef NEW
67
        // returns the length of the longest common prefix of s[i...L-1] and s[i...L-1]
68
        int LongestCommonPrefix(int i, int j) {
             int len = 0;
70
71
             if (i == j) return L - i;
            for (int k = (int) P.size() - 1; k >= 0 && i < L && j < L; k--) {
72
                 if (P[k][i] == P[k][j]) {
73
                     i += 1 << k;
74
                     j += 1 << k;
75
                     len += 1 << k;
76
77
                 }
            }
78
79
            return len;
        }
80
81
        int LongestCommonPrefix(int i, int j) {
82
83
            // getHeight() must be called first
84
            if (i == j) return L - i;
            if (i > j) swap(i, j);
85
            return rmq_query(i + 1, j);
        }
87
```

```
#endif
88
89
         int checkNonOverlappingSubstring(int K) {
90
             // check if there is two non-overlapping identical substring of length K
91
92
             int minsa = 0, maxsa = 0;
             for (int i = 0; i < L; ++i) {</pre>
93
                 if (height[i] < K) {</pre>
94
                     minsa = sa[i]; maxsa = sa[i];
95
96
                 } else {
97
                     minsa = min(minsa, sa[i]);
98
                     maxsa = max(maxsa, sa[i]);
99
                     if (maxsa - minsa >= K) return 1;
                 }
100
             }
101
102
             return 0;
103
104
         int checkBelongToDifferentSubstring(int K, int split) {
105
106
             int minsa = 0, maxsa = 0;
             for (int i = 0; i < L; ++i) {
107
                 if (height[i] < K) {</pre>
108
                     minsa = sa[i]; maxsa = sa[i];
                 } else {
110
                     minsa = min(minsa, sa[i]);
111
112
                     maxsa = max(maxsa, sa[i]);
                     if (maxsa > split && minsa < split) return 1;</pre>
113
114
                 }
             }
115
             return 0;
117
118
119
    } *S;
120
121
    int main() {
         string s, t;
122
         cin >> s >> t;
123
         int sp = s.length();
124
         s += "*" + t;
125
         S = new SuffixArray(s);
126
         S->getHeight();
127
128
         int left = 0, right = sp;
         while (left < right) {</pre>
129
             int mid = (left + right + 1) / 2;
130
131
             if (S->checkBelongToDifferentSubstring(mid, sp))
                 left = mid;
132
133
             else right = mid - 1;
134
135
         printf("%d\n", left);
    }
136
        • SA-IS
        ● 仅在后缀自动机被卡内存或者卡常且需要 O(1) LCA 的情况下使用(比赛中敲这个我觉得不行)
    // rk [0..len-1] -> [1..len], sa/ht [1..len]
    // s[i] > 0 \&\& s[len] = 0
2
    template<size_t size>
    struct SuffixArray {
         bool type[size << 1];</pre>
         int bucket[size], bucket1[size];
         int sa[size], rk[size], ht[size];
         inline bool isLMS(const int i, const bool *type) { return i > 0 && type[i] && !type[i - 1]; }
         template<class T>
         inline void inducedSort(T s, int *sa, const int len, const int sigma, const int bucketSize, bool *type, int
10
         ⇒ *bucket, int *cntbuf, int *p) {
             memset(bucket, 0, sizeof(int) * sigma);
11
             memset(sa, -1, sizeof(int) * len);
12
             for (int i = 0; i < len; i++) bucket[s[i]]++;</pre>
13
14
             cntbuf[0] = bucket[0];
             for (int i = 1; i < sigma; i++) cntbuf[i] = cntbuf[i - 1] + bucket[i];</pre>
15
             for (int i = bucketSize - 1; i >= 0; i--) sa[--cntbuf[s[p[i]]]] = p[i];
16
             for (int i = 1; i < sigma; i++) cntbuf[i] = cntbuf[i - 1] + bucket[i - 1];</pre>
17
```

```
for (int i = 0; i < len; i++) if (sa[i] > 0 && !type[sa[i] - 1]) sa[cntbuf[s[sa[i] - 1]]++] = sa[i] - 1;
18
19
            cntbuf[0] = bucket[0];
            for (int i = 1; i < sigma; i++) cntbuf[i] = cntbuf[i - 1] + bucket[i];</pre>
20
            for (int i = len - 1; i >= 0; i--) if (sa[i] > 0 && type[sa[i] - 1]) sa[--cntbuf[s[sa[i] - 1]]] = sa[i] - 1;
21
22
        template<typename T>
23
        inline void sais(T s, int *sa, int len, bool *type, int *bucket, int *bucket1, int sigma) {
24
            int i, j, bucketSize = 0, cnt = 0, p = -1, x, *cntbuf = bucket + sigma;
25
            type[len - 1] = 1;
26
27
            for (i = len - 2; i >= 0; i--) type[i] = s[i] < s[i + 1] || (s[i] == s[i + 1] && type[i + 1]);
            for (i = 1; i < len; i++) if (type[i] && !type[i - 1]) bucket1[bucketSize++] = i;</pre>
28
29
            inducedSort(s, sa, len, sigma, bucketSize, type, bucket, cntbuf, bucket1);
            for (i = bucketSize = 0; i < len; i++) if (isLMS(sa[i], type)) sa[bucketSize++] = sa[i];</pre>
30
            for (i = bucketSize; i < len; i++) sa[i] = -1;</pre>
31
            for (i = 0; i < bucketSize; i++) {</pre>
32
                x = sa[i];
33
34
                for (j = 0; j < len; j++) {</pre>
                    35
36
                    else if (j > 0 \&\& (isLMS(x + j, type) || isLMS(p + j, type))) break;
                }
37
                x = (~x \& 1 ? x >> 1 : x - 1 >> 1), sa[bucketSize + x] = cnt - 1;
38
39
            for (i = j = len - 1; i >= bucketSize; i--) if (sa[i] >= 0) sa[j--] = sa[i];
40
            int *s1 = sa + len - bucketSize, *bucket2 = bucket1 + bucketSize;
            if (cnt < bucketSize) sais(s1, sa, bucketSize, type + len, bucket, bucket1 + bucketSize, cnt);</pre>
42
            else for (i = 0; i < bucketSize; i++) sa[s1[i]] = i;</pre>
43
            for (i = 0; i < bucketSize; i++) bucket2[i] = bucket1[sa[i]];</pre>
44
            inducedSort(s, sa, len, sigma, bucketSize, type, bucket, cntbuf, bucket2);
45
46
        template<typename T>
47
        inline void getHeight(T s, const int len, const int *sa) {
48
            for (int i = 0, k = 0; i < len; i++) {</pre>
49
                if (rk[i] == 0) k = 0;
50
51
                else {
                    if (k > 0) k--;
52
                    int j = sa[rk[i] - 1];
53
                    while (i + k < len \&\& j + k < len \&\& s[i + k] == s[j + k]) k++;
54
55
56
                ht[rk[i]] = k;
            }
57
58
        template<class T>
59
        inline void init(T s, int len, int sigma) {
60
61
            sais(s, sa, ++len, type, bucket, bucket1, sigma);
            for (int i = 1; i < len; i++) rk[sa[i]] = i;</pre>
62
            getHeight(s, len, sa);
63
64
   };
    KMP 自动机
    int m; int pat[N];
1
    namespace kmp {
2
        int f[N]; // f[i] 表示已匹配成功 i 个, 失配要去哪里
        template<typename T>
5
        int go(int stat, T c, bool& acc) {
            // stat 是当前态 (表示已经匹配了 stat 个字符), c 是要走的边
            while (stat && c != pat[stat]) stat = f[stat];
            if (c == pat[stat]) stat++;
            if (stat == m) acc = true;
10
            return stat;
11
12
13
14
        void getFail() {
            static int f2[N];
15
16
            f[0] = f[1] = 0;
            f2[0] = f2[1] = 0;
```

17

18

FOR (i, 1, m) {

```
int j = f2[i];
19
20
                 while (j && pat[i] != pat[j]) j = f2[j];
                 f2[i+1] = f[i+1] = (pat[i] == pat[j]) ? j+1 : 0;
21
                 if (f[i+1] == j+1 \&\& pat[i+1] == pat[j+1]) f[i+1] = f[j+1];
22
            FOR (i, 0, m) dbg(i, f[i]);
24
25
    }
26
        ● 拓展 KMP
    #include <bits/stdc++.h>
1
2
    using namespace std;
    Define template S, pattern T, len(S)=n, len(T)=m
5
     Find the longest common prefix of T and every suffix of S
     ex[i]: the LCP between T and S[i..n-1]
    const int maxn = 1e6 + 10;
10
    int nt[maxn], ex[maxn];
11
12
    char s[maxn], t[maxn];
13
    void get_next(char *str) {
        int i = 0, j, po, len = strlen(str);
15
16
        nt[0] = len;
        while (str[i] == str[i + 1] && i + 1 < len)</pre>
17
            i++;
18
19
        nt[1] = i;
20
        po = 1;
        for (i = 2; i < len; i++) {</pre>
21
            if (nt[i - po] + i < nt[po] + po)</pre>
22
                 nt[i] = nt[i - po];
23
24
            else {
                 j = nt[po] + po - i;
25
26
                 if (j < 0) j = 0;
                 while (i + j < len \&\& str[j] == str[j + i])
27
28
                     j++;
                 nt[i] = j;
29
30
                 po = i;
31
            }
        }
32
33
34
35
    void exkmp(char *s1, char *s2) {
        int i = 0, j, po, len = strlen(s1), l2 = strlen(s2);
36
        get_next(s2);
37
        while (s1[i] == s2[i] && i < l2 && i < len)
38
            i++;
39
        ex[0] = i;
40
41
        po = 0;
        for (i = 1; i < len; i++) {</pre>
42
43
            if (nt[i - po] + i < ex[po] + po)
                 ex[i] = nt[i - po];
44
             else {
45
                 j = ex[po] + po - i;
46
47
                 if (j < 0) j = 0;
                 while (i + j < len \&\& j < l2 \&\& s1[j + i] == s2[j])
48
                     j++;
49
                 ex[i] = j;
50
                 po = i;
51
            }
52
        }
53
    }
54
55
56
    int main() {
        const int modn = 1e9 + 7;
57
        int T; scanf("%d", &T);
58
59
        while (T--) {
60
            memset(nt, 0, sizeof nt);
            memset(ex, 0, sizeof ex);
61
```

```
scanf("%s", s); scanf("%s", t);
62
63
            int slen = strlen(s), tlen = strlen(t);
            reverse(s, s + slen);
64
            reverse(t, t + tlen);
65
            exkmp(s, t);
            int ans = 0;
67
68
            for (int i = 0; i < slen; ++i)</pre>
                 ans = (ans + 1LL * ex[i] * (ex[i] + 1) / 2) % modn;
69
            printf("%d\n", ans);
70
71
        }
   }
72
    Trie
    namespace trie {
        int t[N][26], sz, ed[N];
        void init() { sz = 2; memset(ed, 0, sizeof ed); }
        int _new() { memset(t[sz], 0, sizeof t[sz]); return sz++; }
4
        void ins(char* s, int p) {
            int u = 1;
            FOR (i, 0, strlen(s)) {
                 int c = s[i] - 'a';
                 if (!t[u][c]) t[u][c] = _new();
                 u = t[u][c];
11
            ed[u] = p;
13
        }
   }
14
    AC 自动机
    const int N = 1e6 + 100, M = 26;
    int mp(char ch) { return ch - 'a'; }
    struct ACA {
5
        int ch[N][M], danger[N], fail[N];
        int sz;
        void init() {
8
            sz = 1;
            memset(ch[\theta], \theta, sizeof ch[\theta]);
10
            memset(danger, 0, sizeof danger);
11
12
        void insert(const string &s, int m) {
13
14
            int n = s.size(); int u = 0, c;
            FOR (i, 0, n) {
15
                 c = mp(s[i]);
                 if (!ch[u][c]) {
17
                     memset(ch[sz], 0, sizeof ch[sz]);
18
19
                     danger[sz] = 0; ch[u][c] = sz++;
                 }
20
                 u = ch[u][c];
22
            danger[u] |= 1 << m;
23
24
        void build() {
25
            queue<int> Q;
            fail[0] = 0;
27
            for (int c = 0, u; c < M; c++) {</pre>
28
29
                u = ch[0][c];
                 if (u) { Q.push(u); fail[u] = 0; }
30
31
            while (!Q.empty()) {
32
33
                 int r = Q.front(); Q.pop();
                 danger[r] |= danger[fail[r]];
34
                 for (int c = 0, u; c < M; c^{++}) {
35
                     u = ch[r][c];
36
```

```
if (!u) {
37
38
                         ch[r][c] = ch[fail[r]][c];
                         continue;
39
40
                    fail[u] = ch[fail[r]][c];
41
                    Q.push(u);
42
43
                }
            }
44
45
        }
   } ac;
47
48
    char s[N];
49
    int main() {
50
        int n; scanf("%d", &n);
51
        ac.init();
52
53
        while (n--) {
            scanf("%s", s);
54
            ac.insert(s, 0);
        }
56
57
        ac.build();
58
        scanf("%s", s);
59
        int u = 0; n = strlen(s);
        FOR (i, 0, n) {
61
62
            u = ac.ch[u][mp(s[i])];
            if (ac.danger[u]) {
63
                puts("YES");
64
65
                return 0;
            }
66
67
        puts("NO");
68
69
        return 0;
    杂项
   STL

    copy

    template <class InputIterator, class OutputIterator>
      OutputIterator copy (InputIterator first, InputIterator last, OutputIterator result);
       • merge (如果相等, 第一个优先)
    template <class InputIterator1, class InputIterator2,</pre>
1
              class OutputIterator, class Compare>
      OutputIterator merge (InputIterator1 first1, InputIterator1 last1,
3
                             InputIterator2 first2, InputIterator2 last2,
                             OutputIterator result, Compare comp);
       • for_each
    template <class InputIterator, class Function>
       Function for_each (InputIterator first, InputIterator last, Function fn);

    transform

    template <class InputIterator, class OutputIterator, class UnaryOperation>
      OutputIterator transform (InputIterator first1, InputIterator last1,
                                 OutputIterator result, UnaryOperation op);
       • numeric_limits
   template <class T> numeric_limits;
       iota
```

```
template< class ForwardIterator, class T >
   void iota( ForwardIterator first, ForwardIterator last, T value );
    日期
   // Routines for performing computations on dates. In these routines,
   // months are expressed as integers from 1 to 12, days are expressed
   // as integers from 1 to 31, and years are expressed as 4-digit
   // integers.
   string dayOfWeek[] = {"Mo", "Tu", "We", "Th", "Fr", "Sa", "Su"};
   // converts Gregorian date to integer (Julian day number)
   int DateToInt (int m, int d, int y){
10
11
       1461 * (y + 4800 + (m - 14) / 12) / 4 +
12
        367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
13
        3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
14
        d - 32075;
15
17
   // converts integer (Julian day number) to Gregorian date: month/day/year
18
19
   void IntToDate (int jd, int &m, int &d, int &y){
20
21
      int x, n, i, j;
22
     x = jd + 68569;
23
     n = 4 * x / 146097;
24
     x = (146097 * n + 3) / 4;
25
     i = (4000 * (x + 1)) / 1461001;
     x = 1461 * i / 4 - 31;
27
28
      j = 80 * x / 2447;
     d = x - 2447 * j / 80;
29
     x = j / 11;
     m = j + 2 - 12 * x;
     y = 100 * (n - 49) + i + x;
32
33
34
   // converts integer (Julian day number) to day of week
35
36
   string IntToDay (int jd){
37
38
      return dayOfWeek[jd % 7];
39
    子集枚举
       ● 枚举真子集
   for (int s = (S - 1) \& S; s; s = (s - 1) \& S)
       ● 枚举大小为 k 的子集
   template<typename T>
   void subset(int k, int n, T&& f) {
        int t = (1 << k) - 1;
        while (t < 1 << n) {
           f(t);
5
            int x = t \& -t, y = t + x;
            t = ((t \& ~y) / x >> 1) | y;
   }
```

### 权值最大上升子序列

```
const LL maxn = 1E5 + 10;
    const LL INF = 1E10;
2
    struct P {
3
       LL k, v;
        bool operator < (const P& rhs) const {</pre>
            return k < rhs.k || (k == rhs.k && v < rhs.v);
8
    };
    LL k[maxn], v[maxn], n, T;
9
10
    set<P> s;
11
12
    int main() {
13
        cin >> T;
        while (T--) {
14
            s.clear();
15
16
            s.insert({-INF, 0});
            cin >> n;
17
            FOR (i, 0, n) scanf("%lld", &k[i]);
18
            FOR (i, 0, n) scanf("%lld", &v[i]);
19
            FOR (i, 0, n) {
                auto it = s.lower_bound({k[i], INF});
21
                LL vv = (--it)->v + v[i];
22
                ++it;
23
                while (it != s.end() && it->v <= vv)
24
                    it = s.erase(it);
                if (it == s.end() || it->k != k[i]) s.insert({k[i], vv});
26
27
            }
            cout << s.rbegin()->v << endl;</pre>
28
29
        }
   }
    数位 DP
    LL dfs(LL base, LL pos, LL len, LL s, bool limit) {
        if (pos == -1) return s ? base : 1;
2
        if (!limit && dp[base][pos][len][s] != -1) return dp[base][pos][len][s];
        LL ret = 0;
        LL ed = limit ? a[pos] : base - 1;
5
        FOR (i, 0, ed + 1) {
            tmp[pos] = i;
            if (len == pos)
                ret += dfs(base, pos - 1, len - (i == 0), s, limit && i == a[pos]);
            else if (s &&pos < (len + 1) / 2)
10
                ret += dfs(base, pos - 1, len, tmp[len - pos] == i, limit && i == a[pos]);
11
12
                ret += dfs(base, pos - 1, len, s, limit && i == a[pos]);
14
        if (!limit) dp[base][pos][len][s] = ret;
15
16
        return ret;
17
    LL solve(LL x, LL base) {
19
        LL sz = 0;
20
        while (x) {
21
            a[sz++] = x \% base;
22
23
            x /= base;
24
25
        return dfs(base, sz - 1, sz - 1, 1, true);
    }
26
    土制 bitset
```

```
1 // M 要开大至少 1 个 64
  const int M = (1E4 + 200) / 64;
   typedef unsigned long long ULL;
```

```
const ULL ONE = 1;
5
   struct Bitset {
        ULL a[M];
        void go(int x) {
            int offset = x / 64; x %= 64;
            for (int i = offset, j = 0; i + 1 < M; ++i, ++j) {</pre>
10
                a[j] |= a[i] >> x;
11
                if (x) a[j] |= a[i + 1] << (64 - x); // 不能左移 64 位
12
13
        }
14
15
        void init() { memset(a, 0, sizeof a); }
        void set(int x) {
16
            int offset = x / 64; x %= 64;
17
            a[offset] \mid = (ONE << x);
18
19
            FOR (i, 0, M) FOR (j, 0, 64) putchar((a[i] & (ONE << j)) ? '1' : '0');
21
            puts("");
22
23
        int lowbit() {
24
            FOR (i, 0, M) if (a[i]) return i * 64 + __builtin_ctzll(a[i]);
25
            assert (0);
26
        int highbit(int x) {
28
29
            // [0,x) 的最高位
            int offset = x / 64; x %= 64;
30
            FORD (i, offset, -1) {
31
                if (!a[i]) continue;
                if (i == offset) {
33
                    FORD (j, x - 1, -1) if ((ONE << j) \& a[i]) \{ return i * 64 + j; \}
34
                } else return i * 64 + 63 - __builtin_clzll(a[i]);
35
36
37
            assert (0);
38
   };
   随机
       ● 不要使用 rand()。
       • chrono::steady_clock::now().time_since_epoch().count()可用于计时。
       • 64 位可以使用 mt19937_64。
   int main() {
        mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
2
        vector<int> permutation(N);
3
        for (int i = 0; i < N; i++)
            permutation[i] = i;
        shuffle(permutation.begin(), permutation.end(), rng);
        for (int i = 0; i < N; i++)</pre>
           permutation[i] = i;
10
        for (int i = 1; i < N; i++)</pre>
            swap(permutation[i], permutation[uniform_int_distribution<int>(0, i)(rng)]);
12
    伪随机数
   unsigned rnd() {
        static unsigned A = 1 << 16 | 3, B = 33333331, C = 2341;</pre>
2
3
        return C = A * C + B;
   }
```

#### Java

## Regex

```
// Code which demonstrates the use of Java's regular expression libraries.
   // This is a solution for
   //
   //
         Loglan: a logical language
        http://acm.uva.es/p/v1/134.html
   import java.util.*;
   import java.util.regex.*;
   public class LogLan {
10
11
        public static void main(String args[]) {
12
13
            String regex = BuildRegex();
14
            Pattern pattern = Pattern.compile(regex);
15
16
            Scanner s = new Scanner(System.in);
17
            while (true) {
19
                // In this problem, each sentence consists of multiple lines, where the last
20
                // line is terminated by a period. The code below reads lines until
21
                // encountering a line whose final character is a '.'. Note the use of
22
                      s.length() to get length of string
24
                //
                      s.charAt() to extract characters from a Java string
25
                      s.trim() to remove whitespace from the beginning and end of Java string
26
                //
27
                // Other useful String manipulation methods include
                //
29
30
                      s.compareTo(t) < 0 if s < t, lexicographically
                      s.indexOf("apple") returns index of first occurrence of "apple" in s
31
                //
                      s.lastIndexOf("apple") returns index of last occurrence of "apple" in s
32
33
                      s.replace(c,d) replaces occurrences of character c with d
                      s.startsWith("apple) returns (s.indexOf("apple") == 0)
34
35
                      s.toLowerCase() / s.toUpperCase() returns a new lower/uppercased string
                //
36
                      Integer.parseInt(s) converts s to an integer (32-bit)
                //
37
38
                      Long.parseLong(s) converts s to a long (64-bit)
                      Double.parseDouble(s) converts s to a double
39
40
                String sentence = "";
41
42
                while (true) {
                    sentence = (sentence + " " + s.nextLine()).trim();
43
                    if (sentence.equals("#")) return;
44
                    if (sentence.charAt(sentence.length() - 1) == '.') break;
45
46
47
                // now, we remove the period, and match the regular expression
48
49
                String removed_period = sentence.substring(0, sentence.length() - 1).trim();
50
                if (pattern.matcher(removed_period).find()) {
51
                    System.out.println("Good");
                } else {
53
                    System.out.println("Bad!");
54
55
            }
56
57
        }
   }
58
```

#### **Decimal Format**

```
// examples for printing floating point numbers
import java.util.*;
import java.io.*;
```

```
import java.text.DecimalFormat;
   public class DecFormat {
        public static void main(String[] args) {
            DecimalFormat fmt;
10
            // round to at most 2 digits, leave of digits if not needed
11
            fmt = new DecimalFormat("#.##");
12
            System.out.println(fmt.format(12345.6789)); // produces 12345.68
13
            System.out.println(fmt.format(12345.0)); // produces 12345
14
            System.out.println(fmt.format(0.0)); // produces 0
15
            System.out.println(fmt.format(0.01)); // produces .1
17
            // round to precisely 2 digits
18
            fmt = new DecimalFormat("#.00");
19
            System.out.println(fmt.format(12345.6789)); // produces 12345.68
20
21
            System.out.println(fmt.format(12345.0)); // produces 12345.00
            System.out.println(fmt.format(0.0)); // produces .00
22
23
            // round to precisely 2 digits, force leading zero
24
            fmt = new DecimalFormat("0.00");
25
            System.out.println(fmt.format(12345.6789)); // produces 12345.68
26
            System.out.println(fmt.format(12345.0)); // produces 12345.00
27
            System.out.println(fmt.format(0.0)); // produces 0.00
29
            // round to precisely 2 digits, force leading zeros
30
            fmt = new DecimalFormat("000000000.00");
31
            System.out.println(fmt.format(12345.6789)); // produces 000012345.68
32
            System.out.println(fmt.format(12345.0)); // produces 000012345.00
33
            System.out.println(fmt.format(0.0)); // produces 000000000.00
34
35
            // force leading '+'
36
            fmt = new DecimalFormat("+0;-0");
37
38
            System.out.println(fmt.format(12345.6789)); // produces +12346
            System.out.println(fmt.format(-12345.6789)); // produces -12346
39
            System.out.println(fmt.format(0)); // produces +0
40
41
42
            // force leading positive/negative, pad to 2
            fmt = new DecimalFormat("positive 00; negative 0");
43
            System.out.println(fmt.format(1)); // produces "positive 01"
44
45
            System.out.println(fmt.format(-1)); // produces "negative 01"
46
47
            // goute special chars (#)
48
            fmt = new DecimalFormat("text with '#' followed by #");
            System.out.println(fmt.format(12.34)); // produces "text with # followed by 12"
49
50
            // always show "."
51
            fmt = new DecimalFormat("#.#");
            fmt.setDecimalSeparatorAlwaysShown(true);
53
            System.out.println(fmt.format(12.34)); // produces "12.3"
54
            System.out.println(fmt.format(12)); // produces "12."
55
            System.out.println(fmt.format(0.34)); // produces "0.3"
56
            // different grouping distances:
58
59
            fmt = new DecimalFormat("#,####.##");
            System.out.println(fmt.format(123456789.123)); // produces "1,2345,6789.123"
60
61
            // scientific:
62
            fmt = new DecimalFormat("0.000E00");
63
            System.out.println(fmt.format(123456789.123)); // produces "1.235E08"
64
            System.out.println(fmt.format(-0.000234)); // produces "-2.34E-04"
65
66
            // using variable number of digits:
67
            fmt = new DecimalFormat("0");
68
            System.out.println(fmt.format(123.123)); // produces "123"
70
            fmt.setMinimumFractionDigits(8):
71
            System.out.println(fmt.format(123.123)); // produces "123.12300000"
72
            fmt.setMaximumFractionDigits(0);
            System.out.println(fmt.format(123.123)); // produces "123"
73
74
            // note: to pad with spaces, you need to do it yourself:
75
```

```
// String out = fmt.format(...)
77
             // while (out.length() < targlength) out = " "+out;</pre>
78
   }
79
    Sort
    import java.util.ArrayList;
    import java.util.Collections;
    import java.util.List;
    public class Employee implements Comparable<Employee> {
        private int id;
        private String name;
        private int age;
        public Employee(int id, String name, int age) {
10
            this.id = id;
11
             this.name = name;
            this.age = age;
13
14
15
        @Override
16
        public int compareTo(Employee o) {
17
            if (id > o.id) {
18
                 return 1;
19
            } else if (id < o.id) {</pre>
20
                 return -1;
21
             }
23
             return 0:
24
25
        public static void main(String[] args) {
26
27
             List<Employee> list = new ArrayList<Employee>();
             list.add(new Employee(2, "Java", 20));
list.add(new Employee(1, "C", 30));
28
29
             list.add(new Employee(3, "C#", 10));
30
             Collections.sort(list);
31
32
    }
33
    扩栈 (本地使用)
    #include <sys/resource.h>
    void init_stack(){
        const rlim_t kStackSize = 512 * 1024 * 1024;
        struct rlimit rl;
4
        int result;
        result = getrlimit(RLIMIT_STACK, &rl);
        if (result == 0) {
             if (rl.rlim_cur < kStackSize) {</pre>
                 rl.rlim_cur = kStackSize;
                 result = setrlimit(RLIMIT_STACK, &rl);
10
                 if (result != 0) {
11
                      fprintf(stderr, "setrlimit returned result = %d\n", result);
13
                 }
             }
14
15
        }
16
    }
```

# 心态崩了

- (int)v.size()
- 1LL << k
- 递归函数用全局或者 static 变量要小心

- 预处理组合数注意上限
- 想清楚到底是要 multiset 还是 set
- 提交之前看一下数据范围, 测一下边界
- 数据结构注意数组大小(2倍,4倍)
- 字符串注意数据集
- 如果函数中使用了默认参数的话,注意调用时的参数个数。
- 注意要读完
- 构造参数无法使用自己
- 树链剖分/dfs 序,初始化或者询问不要忘记 idx, ridx
- 排序时注意结构体的所有属性是不是考虑了
- 不要把 while 写成 if
- 不要把 int 开成 char
- 清零的时候全部用 0~n+1。
- 模意义下不要用除法
- 哈希不要自然溢出
- 最短路不要 SPFA,乖乖写 Dijkstra
- 上取整以及 GCD 小心负数
- mid 用 l + (r l) / 2 可以避免溢出和负数的问题