Problem 5

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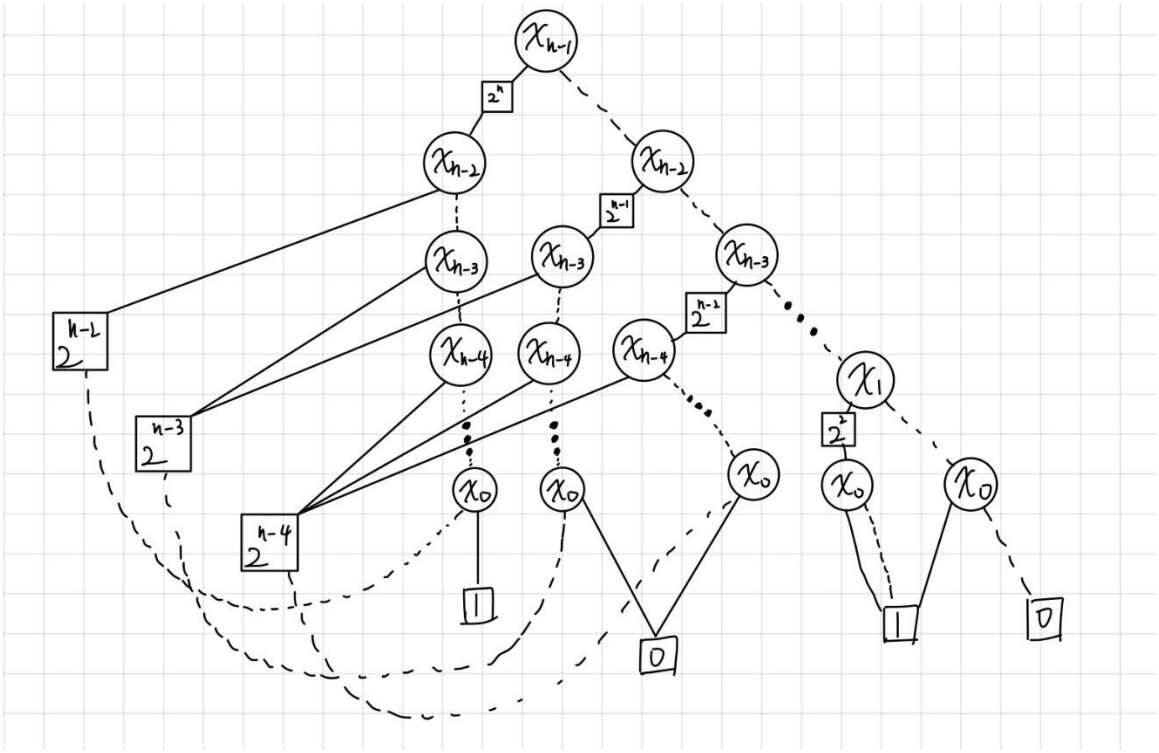
(a)



(b) First, define the branch of i-th bit x_i is a sub-*BMD consists of the positive sub-*BMD of node x_i and node x_i itself. Here the node x_i should stay in the negative sub-*BMD of every node with higher index. The weight of a set of branch is the sum of all weight of branches in the set. After some definition above, we can dive into the pseudo part:

Def. Function SubRoot(Y,A,t,k,w_0,s):
If($k - w_0 \geq 0$) then
If($Y \leq t + 2^{(k-w_0)} * weight(A) + 2^{2(k-w_0)}$) then
$t = t + 2^{(k-w_0)} * weight(A) + 2^{2(k-w_0)}$
$s = k - w_0$
add the branch of x_s to set A
return (A,t,s, $weight(A)$)
else
(A,t,s, $weight(A)$) = SubRoot (Y,A,t,k-1,w_0,s)
return (A,t,s,$weight(A)$)
end If
else
$s = k - w_0$
return (A,t,s,$weight(A)$)
end If
End function

The SubRoot function find the partial root of Y and help us solve the problem recursively. The main algorithm is presented below:

Initialization
assume y_{k_0} is the first not zero significant bit in Y
let $s = \lfloor k_0/2 \rfloor$, $\mathbf{A} = \{\}$, $t = 2^{2s}$, $s_0 = \lfloor k_0/2 \rfloor$, $w_0 = weight$(the branch of x_{s_0}), and $\mathbf{w} = 0$
add the branch of x_s to set **A**
end Initialization

While($s \geq 0$) ***do***
find the first non-zero significant bit y'_k for $y' = Y - t$.
(A,t,s,w) = SubRoot (Y,A,t,k,w_0,s)
End While

Then the integer part of the square root of Y would be $w/2$.