Homework #2 for MATH 6395

Due at the beginning of class, Sep. 30th, 2013

1. Consider Burgers' equation
$$u_t + (u^2/2)_x = 0, \quad x \in \mathcal{R}$$
with general Riemann initial data consisting of two constant states u_l and u_r,
$$u_0(x) = \begin{cases} u_l, & x < 0 \\ u_r, & x > 0 \end{cases} \tag{1}$$
Write down the exact solution $u(x,t)$. *Hint: separate discussion as two cases of* $u_l < u_r$ *and* $u_l \geq u_r$.

2. Consider linear advection equation
$$u_t - u_x = 0, \quad x \in [0,1] \tag{2}$$
with initial condition $u_0(x) = \sin(2\pi x)$ and periodic boundary condition. For the following two schemes:

   (a)
$$u_j^{n+1} = u_j^n + \frac{\Delta t}{\Delta x}(u_j^n - u_{j-1}^n),$$

   (b)
$$u_j^{n+1} = u_j^n + \frac{\Delta t}{\Delta x}(u_{j+1}^n - u_j^n).$$

   - Please analyze the accuracy and stability;
   - Implement the scheme in Fortran/Matlab/your preferred computer language;
   - If the scheme is convergent, test for accuracy and convergence by putting down an error table for numerical solutions at $T = 3$ with $N = 10, 20, 40, 80$ for spatial meshes and $CFL = \frac{\Delta t}{\Delta x} = 0.5$. If the scheme is not convergent, explain why.