第9章

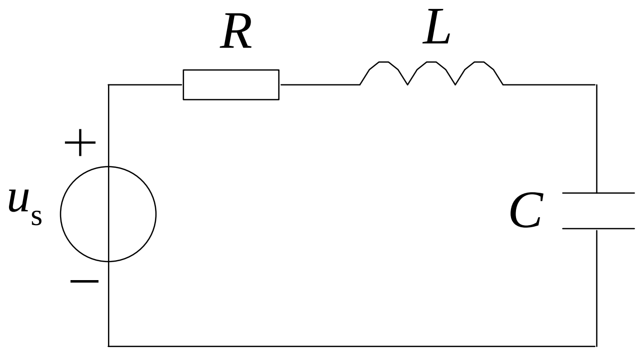
二阶电路的暂态分析

9.1 二阶电路

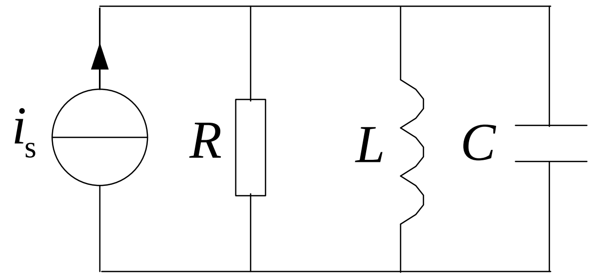
9.2 零输入响应（自然响应）

9.3 直流电源激励下的响应

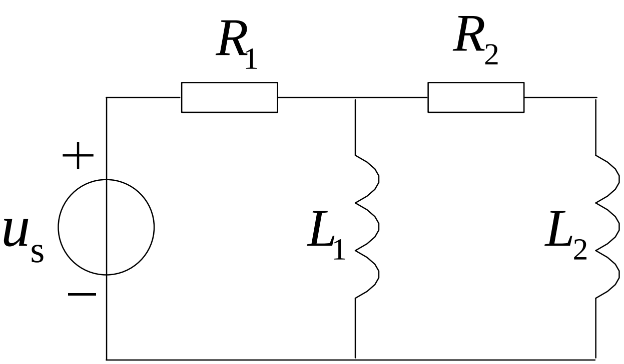
9.1 二阶电路



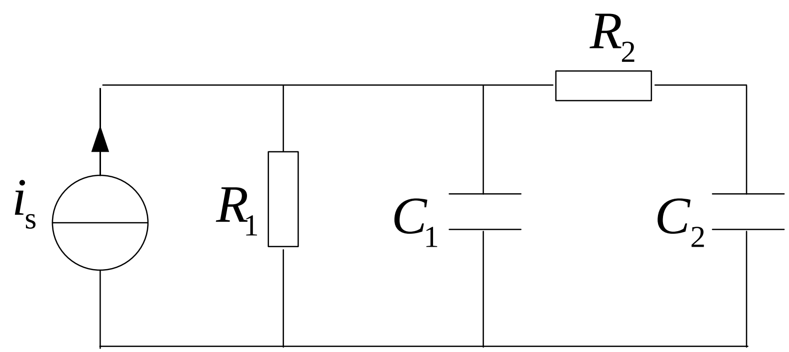
RLC串联电路



RLC并联电路

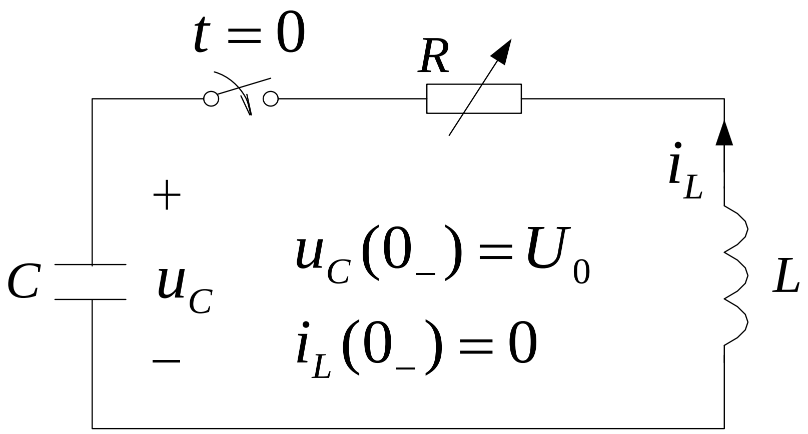


一般二阶RLL电路



一般二阶RCC电路

9.2 二阶电路的零输入响应



$$\begin{cases} LC\dfrac{\mathrm{d}^2u_C}{\mathrm{d}t^2}+RC\dfrac{\mathrm{d}u_C}{\mathrm{d}t}+u_C=0 \\ u_C(0_+)=u_C(0_-)=U_0 \\ \left.\dfrac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{0_+}=\dfrac{i_C(0_+)}{C}=\dfrac{i_L(0_+)}{C}=0 \end{cases}$$

特征方程：

$$LCs^2+RCs+1=0$$

特征根：

$$s_{1,2}=\frac{-RC\pm\sqrt{(RC)^2-4LC}}{2LC}=-\frac{R}{2L}\pm\sqrt{(\frac{R}{2L})^2-\frac{1}{LC}}$$

令$\alpha=\dfrac{R}{2L}$ (衰减系数), $\omega_0=\sqrt{\dfrac{1}{LC}}$ (谐振角频率)

$$=-\alpha\pm\sqrt{\alpha^2-\omega_0^2}$$

9.2 二阶电路的零输入响应

特征根：

$$s_{1,2}=\frac{-RC\pm\sqrt{(RC)^2-4LC}}{2LC}=-\frac{R}{2L}\pm\sqrt{(\frac{R}{2L})^2-\frac{1}{LC}}=-\alpha\pm\sqrt{\alpha^2-\omega_0^2}$$

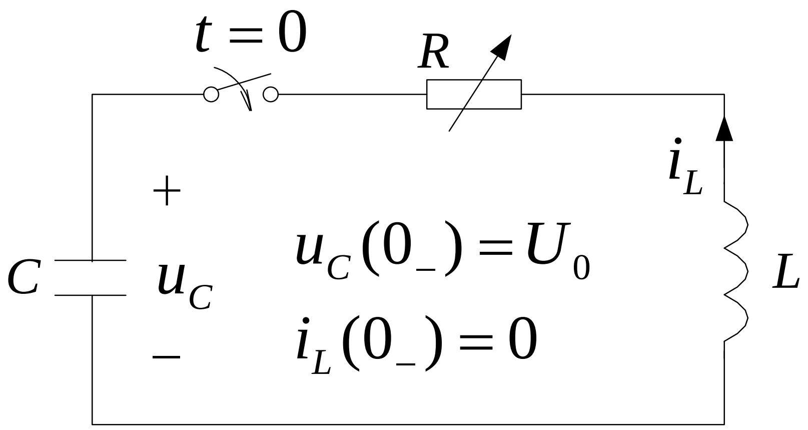
零状态响应的三种情况

（1） $\alpha>\omega_0 \quad \rightarrow R>2\sqrt{\frac{L}{C}}$ **两个不相等负实根** $\boxed{\text{过阻尼}}$

（2） $\alpha<\omega_0 \quad \rightarrow R<2\sqrt{\frac{L}{C}}$ **两个共轭复根** $\boxed{\text{欠阻尼}}$

（3） $\alpha=\omega_0 \quad \rightarrow R=2\sqrt{\frac{L}{C}}$ **两个相等负实根** $\boxed{\text{临界阻尼}}$

9.2 二阶电路的零输入响应



Overdamped——过阻尼

(1) $\alpha > \omega_0 \quad \rightarrow R > 2\sqrt{\dfrac{L}{C}}$

$$s_{1,2}=-\alpha \pm \sqrt{\alpha^2-\omega_0^2}$$

$$u_C = k_1 e^{s_1 t} + k_2 e^{s_2 t}$$

$$u_C(0_+)=U_0 \rightarrow k_1+k_2=U_0$$

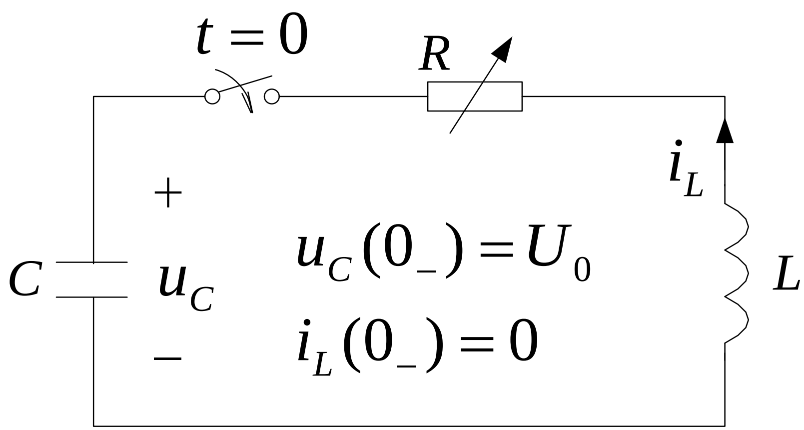
$$\left.\frac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{(0_+)} \rightarrow s_1k_1+s_2k_2=0$$

$$\begin{cases} k_1 = \dfrac{s_2}{s_2-s_1}U_0 \\ k_2 = \dfrac{-s_1}{s_2-s_1}U_0 \end{cases}$$

$$\boxed{u_C = \frac{U_0}{s_2-s_1}(s_2 e^{s_1 t} - s_1 e^{s_2 t})}$$

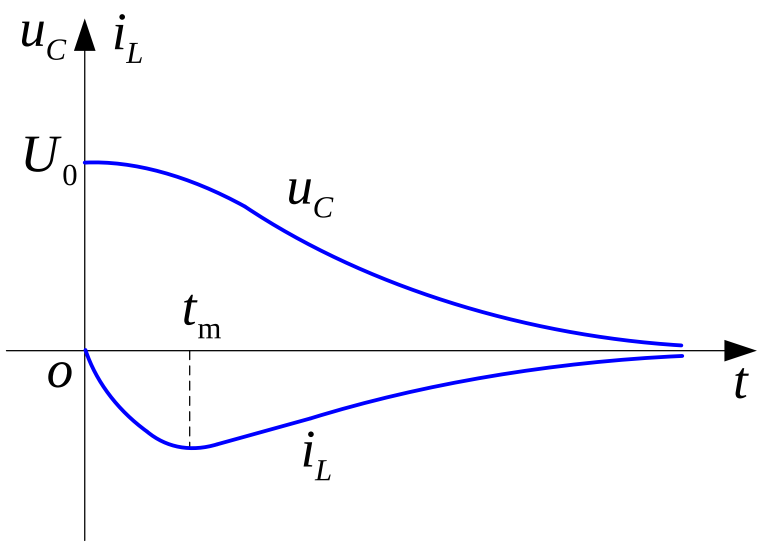
$$\boxed{i_L = C\frac{\mathrm{d}u_c}{\mathrm{d}t} = \frac{U_0}{L(s_2-s_1)}(e^{s_1 t} - e^{s_2 t})}$$

9.2 二阶电路的零输入响应



$$\boxed{u_C = \frac{U_0}{s_2 - s_1}(s_2 e^{s_1 t} - s_1 e^{s_2 t})}$$

$$\boxed{i_\mathrm{L} = C\frac{\mathrm{d}u_c}{\mathrm{d}t} = \frac{U_0}{L(s_2 - s_1)}(e^{s_1 t} - e^{s_2 t})}$$

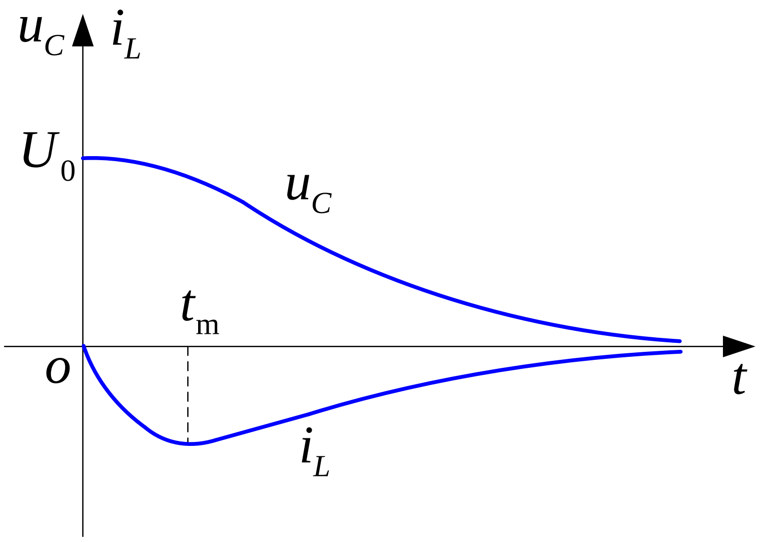


由 $\mathrm{d}i_L/\mathrm{d}t$ **可确定** i_L **为极小时的** t_m .

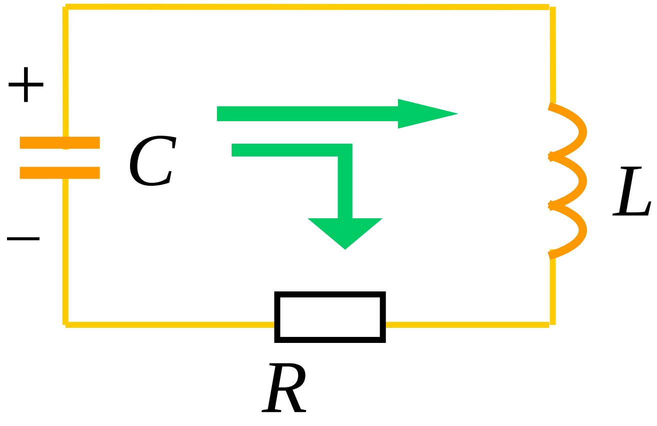
$$s_1 e^{s_1 t_m} - s_2 e^{s_2 t_m} = 0$$

$$t_m = \frac{\ell n \frac{s_2}{s_1}}{s_1 - s_2}$$

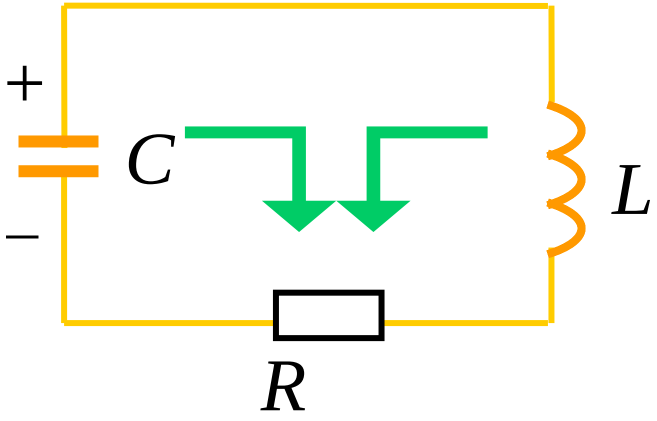
能量转换关系



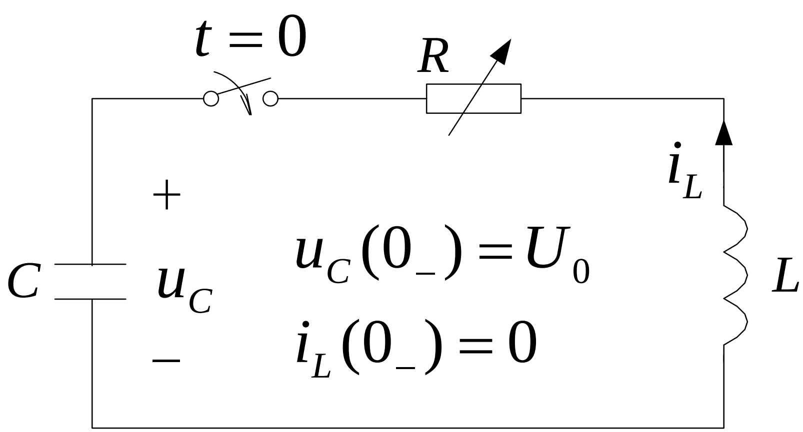
$0 < t < t_m$　u_C减小，i 增加。



$t > t_m$　u_C减小，i 减小.



9.2 二阶电路的零输入响应



underdamped——欠阻尼

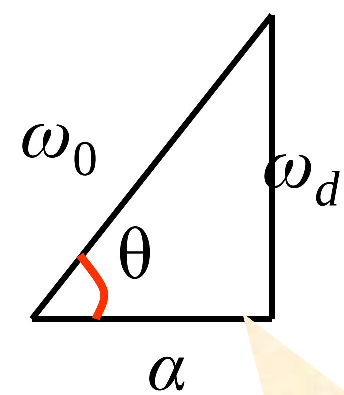
（2） $\alpha < \omega_0 \quad \rightarrow R < 2\sqrt{\dfrac{L}{C}}$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} \quad \text{(固有振荡角频率)}$$

$$s_{1,2} = -\alpha \pm \mathrm{j}\sqrt{\omega_0^2 - \alpha^2} = -\alpha \pm \mathrm{j}\omega_\mathrm{d}$$

$$u_C = k_1 \mathrm{e}^{s_1 t} + k_2 \mathrm{e}^{s_2 t} = k\mathrm{e}^{-\alpha t}\sin(\omega_\mathrm{d} t + \theta)$$

$$\text{初始条件}\begin{cases} u_C(0^+) = U_0 \rightarrow k\sin\theta = U_0 \\ \dfrac{du_C}{dt}(0^+) = 0 \rightarrow k(-\alpha)\sin\theta + k\omega_d\cos\theta = 0 \end{cases}$$



$$k = \frac{U_0}{\sin\theta}\text{，}\quad \theta = arctg\frac{\omega_d}{\alpha}$$

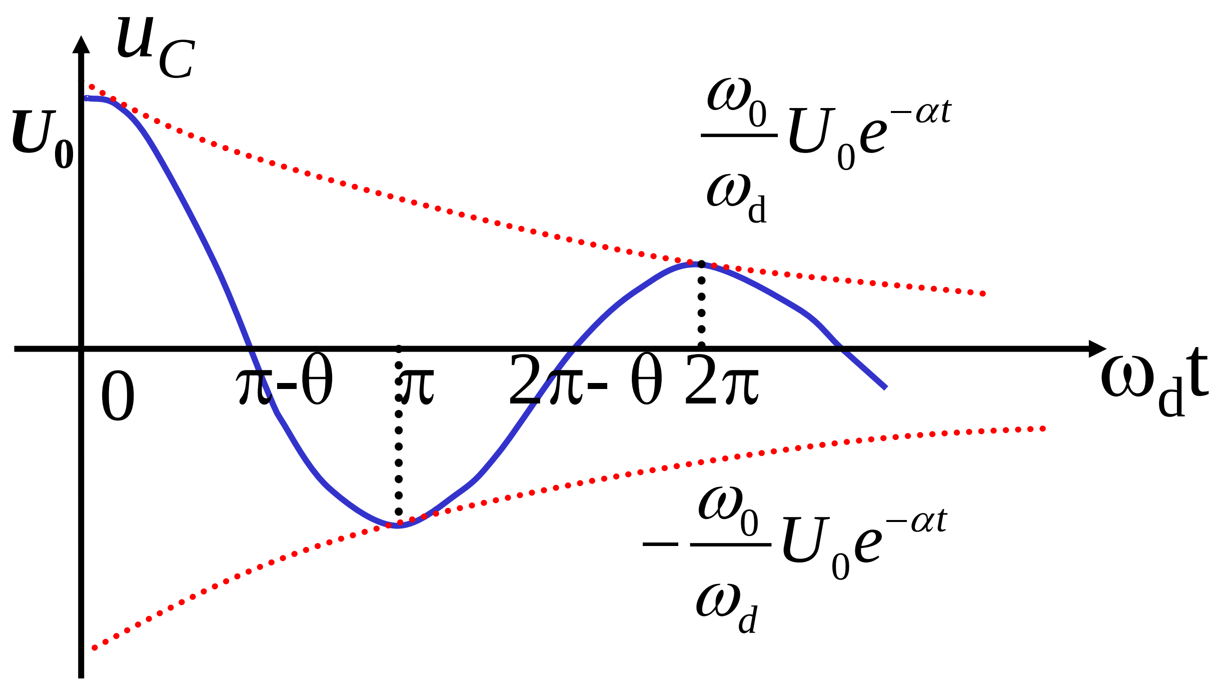
$$\sin\theta = \frac{\omega_d}{\omega_0} \qquad k = \frac{\omega_0}{\omega_d}U_0$$

ω_d，ω_0，α 的关系

$$u_C = \frac{\omega_0}{\omega_d} U_0 e^{-\alpha t} \sin(\omega_d t + \theta)$$

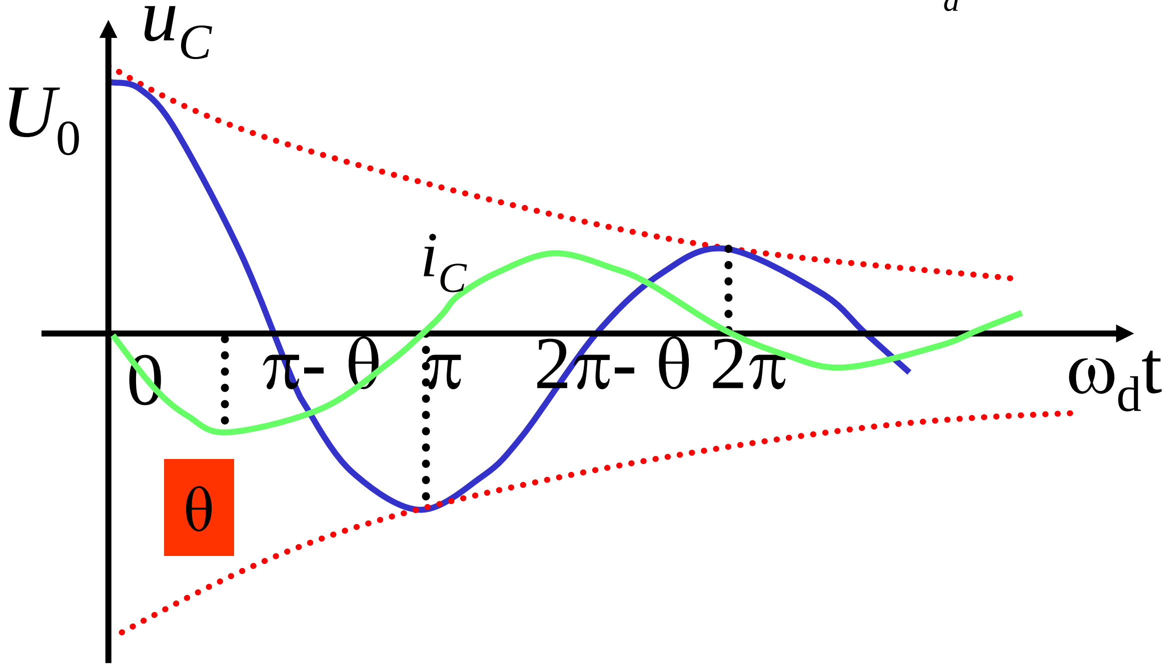
u_C是振幅以$\pm\frac{\omega_0}{\omega}U_0$为包络线依指数衰减的正弦函数

t=0 **时** u_c=U_0 u_C=0：$\omega_d t = \pi-\theta$，$2\pi-\theta$ ... $n\pi-\theta$



$$u_C = \frac{\omega_0}{\omega_d} U_0 e^{-\alpha t} \sin(\omega_d t + \theta)$$

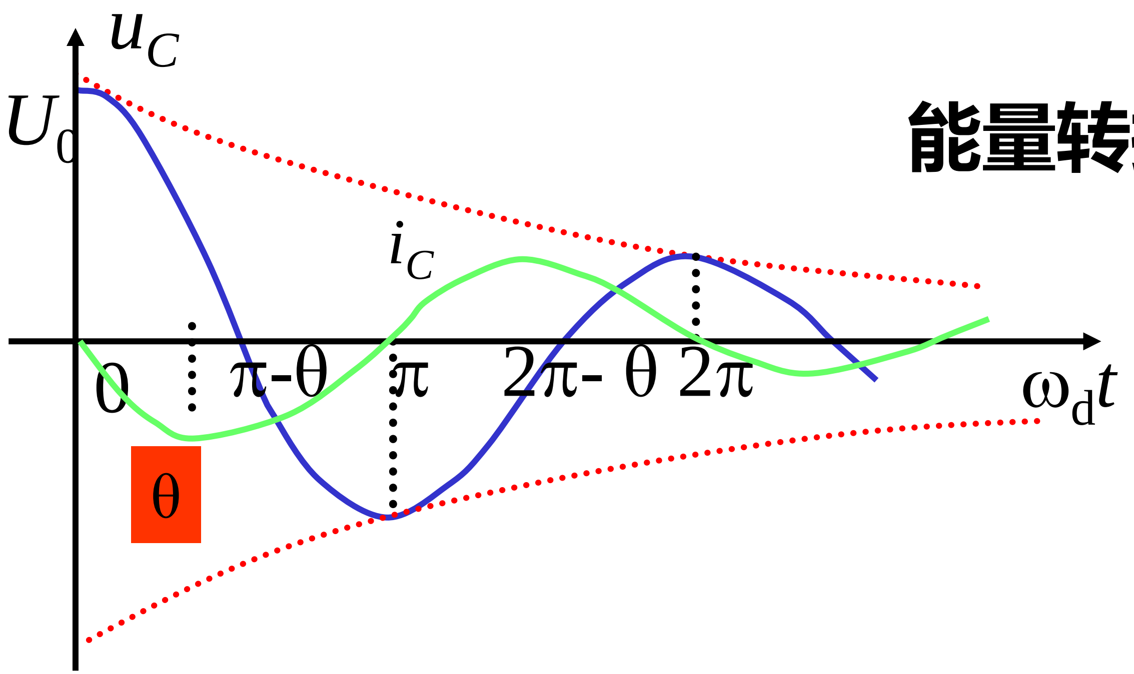
$$i_C = C\frac{\mathrm{d}u_C}{\mathrm{d}t} = -\frac{U_0}{\omega_d L} e^{-\alpha t} \sin \omega_d t$$

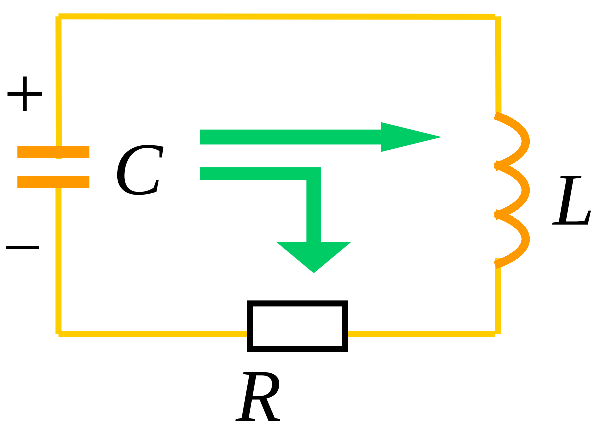


i_c=0： $\omega_d t$ =0， π， 2π ... nπ ，**为 u_c极值点，**

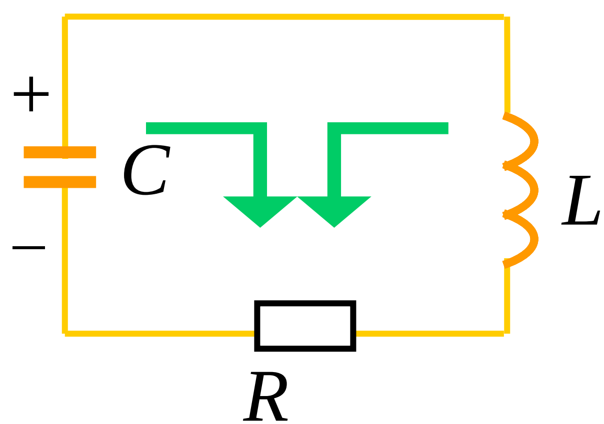
i_c **的第一个极值点为**$\omega_d t$ =θ。

能量转换关系：

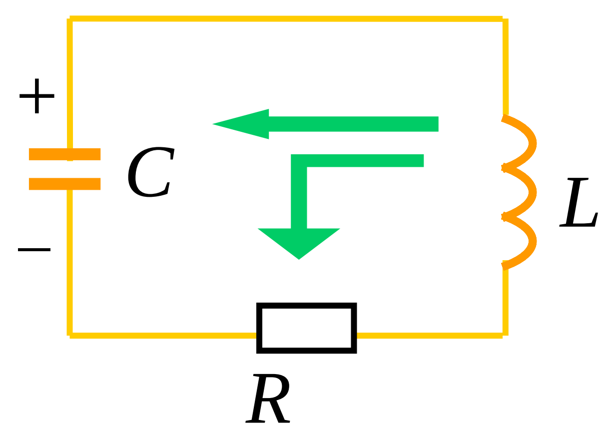




$0 < \omega_d t < \theta$



$\theta < \omega_d t < \pi - \theta$



$\pi - \theta < \omega_d t < \pi$

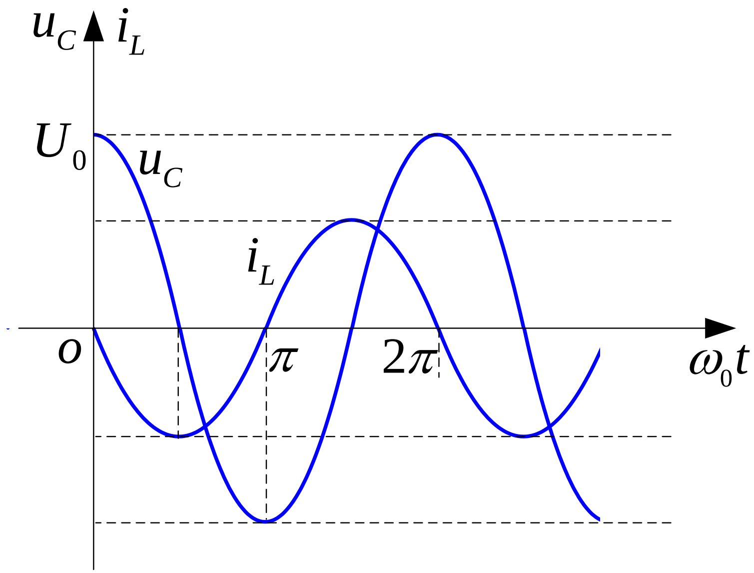
特例：R=0 时

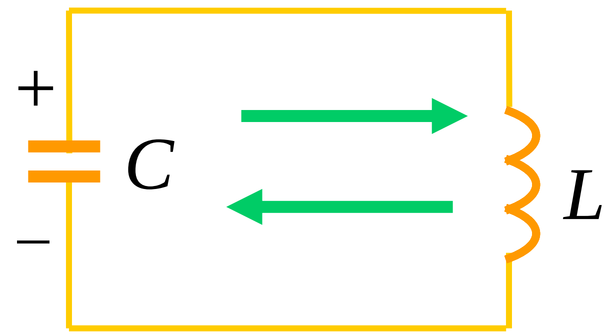
$$\alpha = 0 \;，\; \omega_d = \omega_0 = \frac{1}{\sqrt{LC}} \;，\theta = \frac{\pi}{2}$$

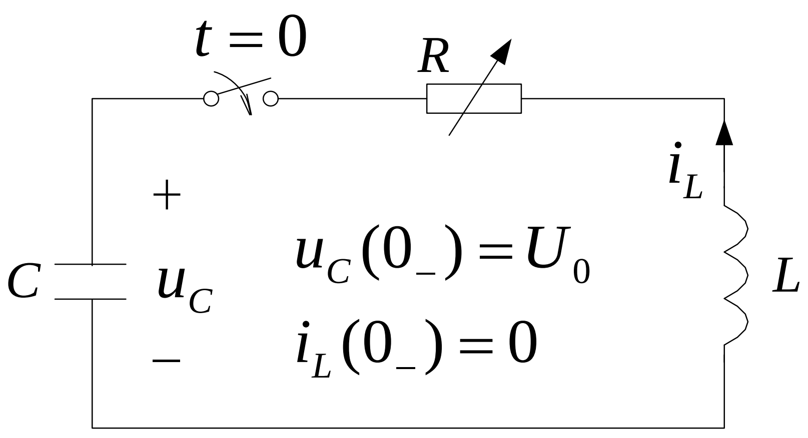
$$u_C = U_0 \sin(\omega_0 t + 90^0) = u_L$$

$$i = -\frac{U_0}{\omega_0 L} \sin \omega_0 t$$

→ **等幅振荡**







Critically damped——临界阻尼

（3） $\alpha=\omega_0 \quad \rightarrow R=2\sqrt{\dfrac{L}{C}}$

$$s_{1,2}=-\alpha$$

$$u_C=(k_1+k_2t)\mathrm{e}^{-\alpha t}$$

$$\text{初始条件}\begin{cases} u_c(0^+)=U_0 \rightarrow k_1=U_0 \\ \dfrac{\mathrm{d}u_c}{\mathrm{d}t}(0^+)=0 \rightarrow k_1(-\alpha)+k_2=0 \end{cases}$$

$$\begin{cases} k_1=U_0 \\ k_2=U_0\alpha \end{cases}$$

$$u_C=U_0e^{-\alpha t}(1+\alpha t)$$

$$i_C=C\frac{\mathrm{d}u_C}{\mathrm{d}t}=-\frac{U_0}{L}te^{-\alpha t}$$

非振荡电路

小结

可推广应用于一般二阶电路

$R > 2\sqrt{\frac{L}{C}}$ 过阻尼，非振荡放电

$$u_C = k_1 e^{s_1 t} + k_2 e^{s_2 t}$$

$R = 2\sqrt{\frac{L}{C}}$ 临界阻尼，非振荡放电

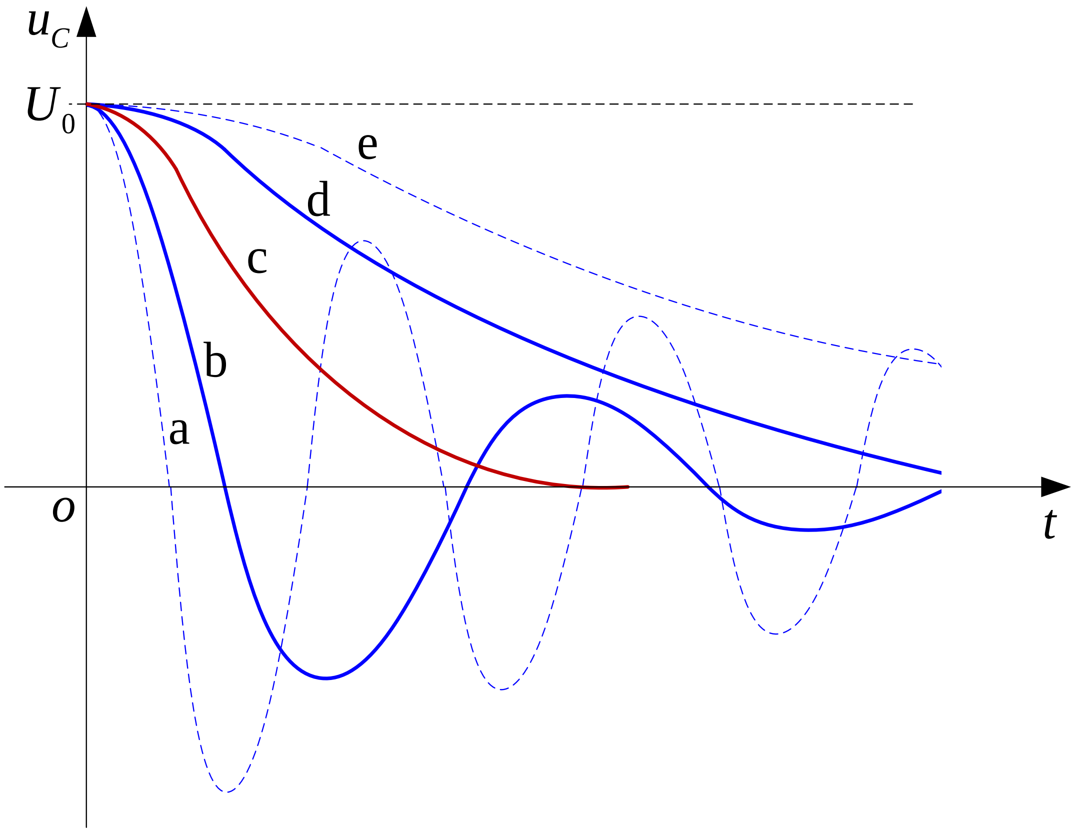
$$u_C = k_1 e^{-\alpha t} + k_2 t e^{-\alpha t}$$

$R < 2\sqrt{\frac{L}{C}}$ 欠阻尼，振荡放电

$$u_C = k e^{-\alpha t} \sin(\omega_d t + \theta)$$

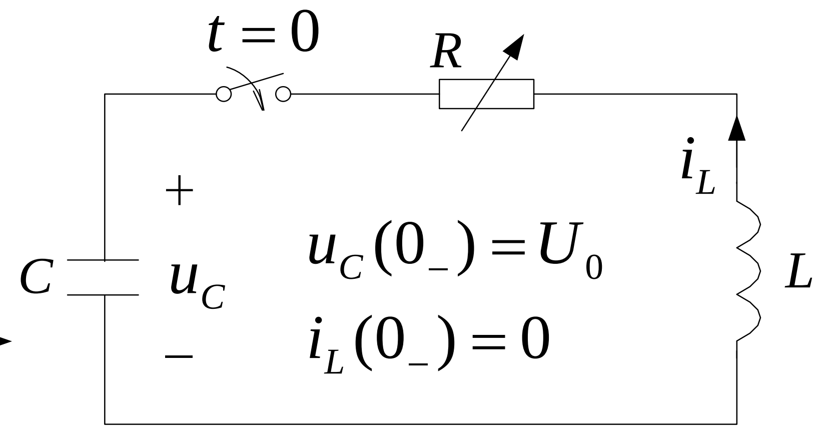
初始条件 $\begin{cases} u_C(0_+) \\ \dfrac{\mathrm{d}u_C}{\mathrm{d}t}(0_+) \end{cases}$ **定常数**

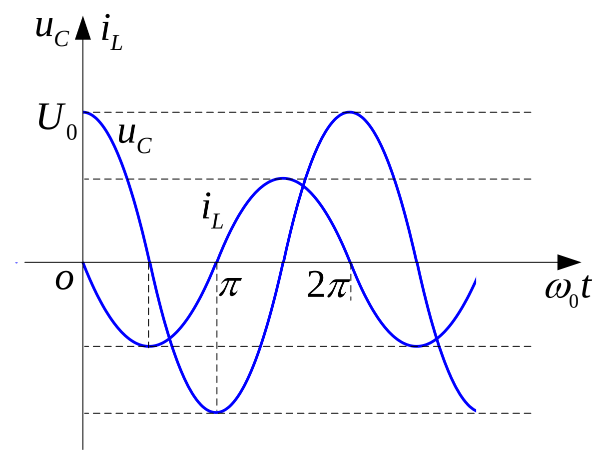
9.2 二阶电路的零输入响应



$\boxed{R_e > R_d} > \boxed{R_c} > \boxed{R_b > R_a}$

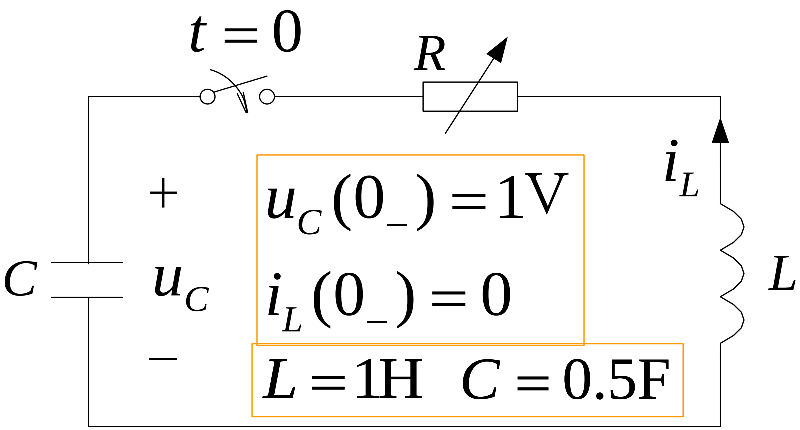
过阻尼　　临界阻尼　　欠阻尼





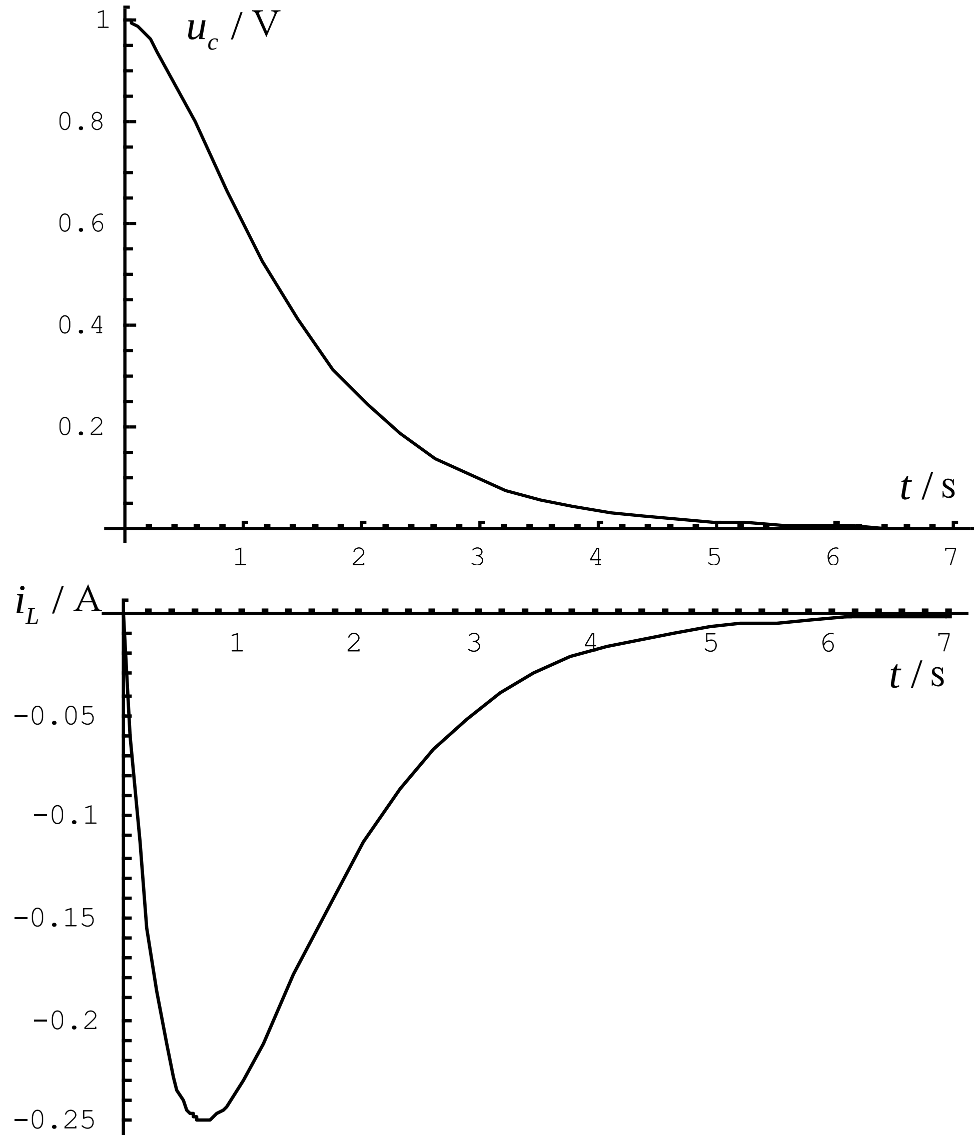
欠阻尼特例 $R=0$

9.2 二阶电路的零输入响应

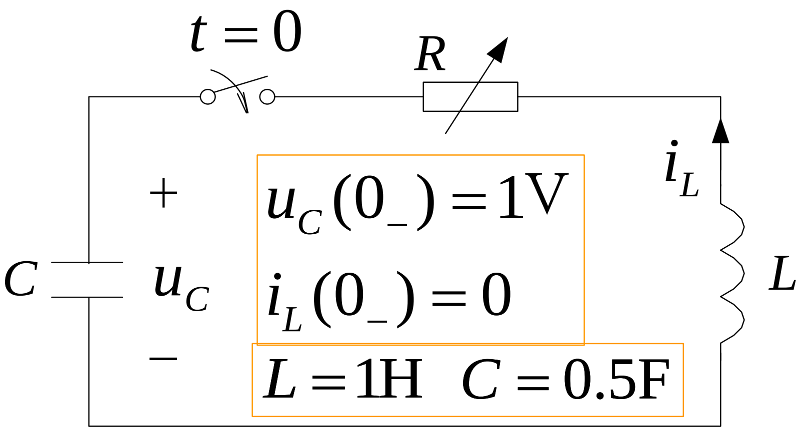


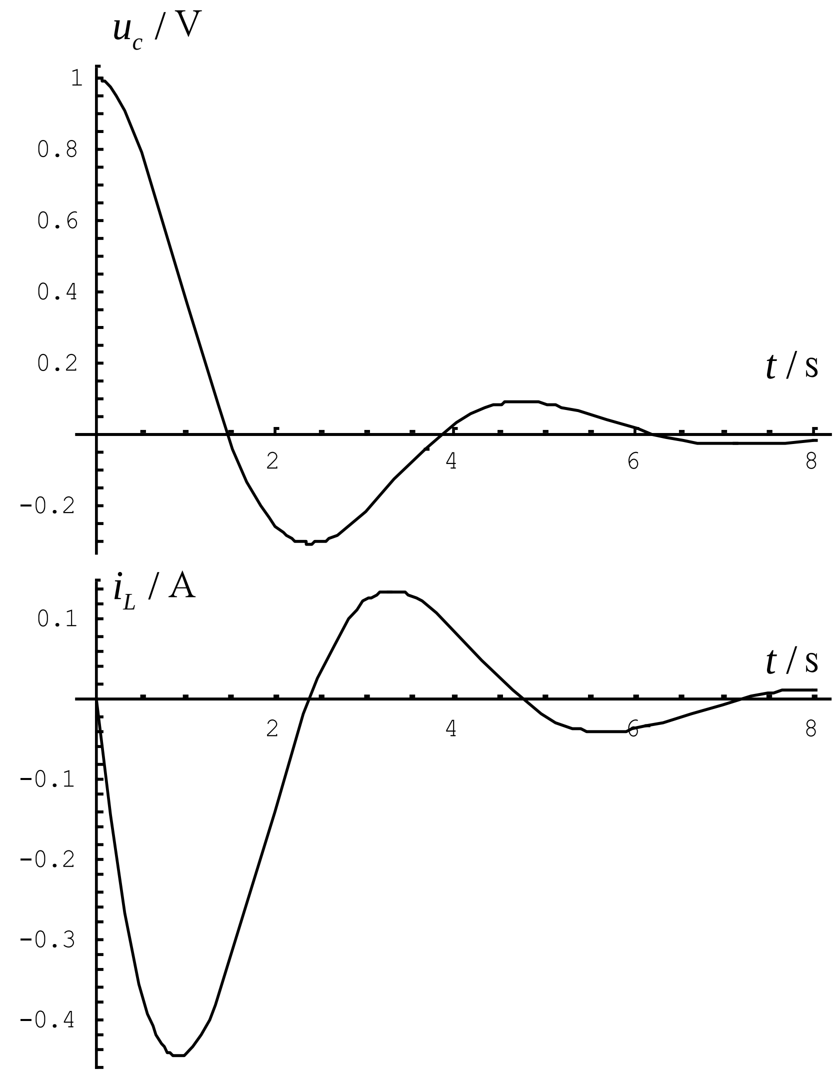
$\boxed{R = 3\Omega}$ **过阻尼**

$$\begin{bmatrix} u_c \\ i_L \end{bmatrix} = \begin{bmatrix} (2e^{-t} - e^{-2t})\text{V} \\ (-e^{-t} + e^{-2t})\text{A} \end{bmatrix}$$



9.2 二阶电路的零输入响应

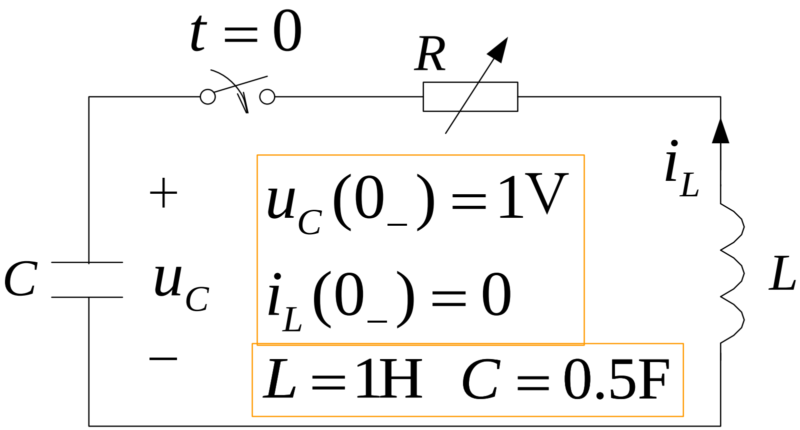




$R = 1\Omega$ **欠阻尼**

$$\begin{bmatrix} u_C \\ i_L \end{bmatrix} = \begin{bmatrix} e^{-0.5t}(\cos\frac{\sqrt{7}}{2}t + \frac{1}{\sqrt{7}}\sin\frac{\sqrt{7}}{2}t)\mathrm{V} \\ (-\frac{2}{\sqrt{7}}e^{-0.5t}\sin\frac{\sqrt{7}}{2}t)\mathrm{A} \end{bmatrix}$$

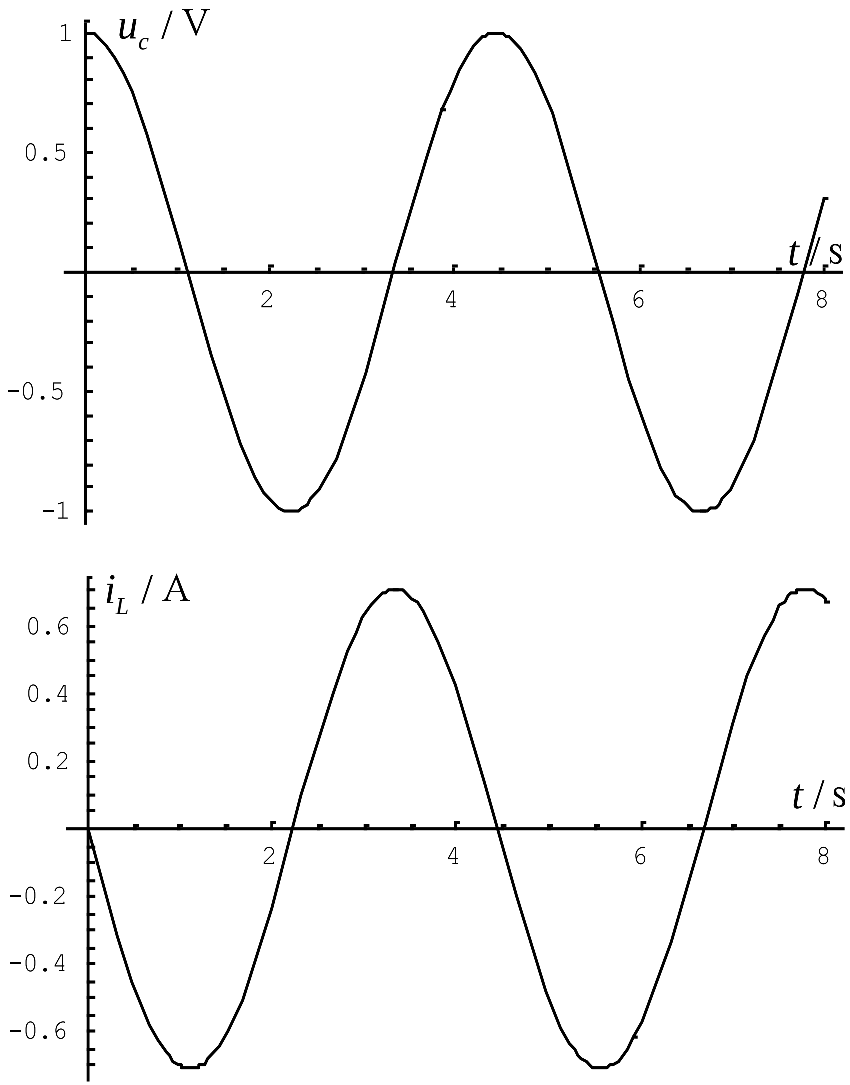
9.2 二阶电路的零输入响应



$u_C(0_-)=1\text{V}$

$i_L(0_-)=0$

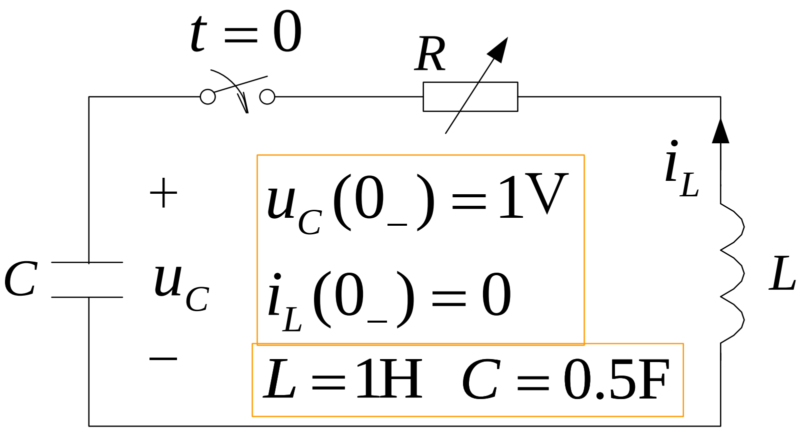
$L=1\text{H}\quad C=0.5\text{F}$



$R=0$ **无阻尼**

$$\begin{bmatrix} u_C \\ i_L \end{bmatrix} = \begin{bmatrix} (\cos\sqrt{2}t)\text{V} \\ (-\dfrac{1}{\sqrt{2}}\sin\sqrt{2}t)\text{A} \end{bmatrix}$$

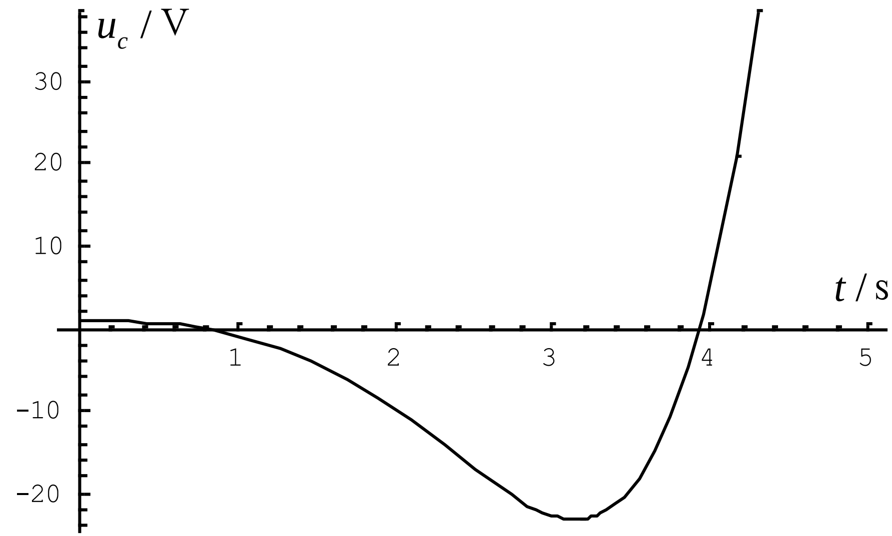
9.2 二阶电路的零输入响应



$u_C(0_-)=1\text{V}$

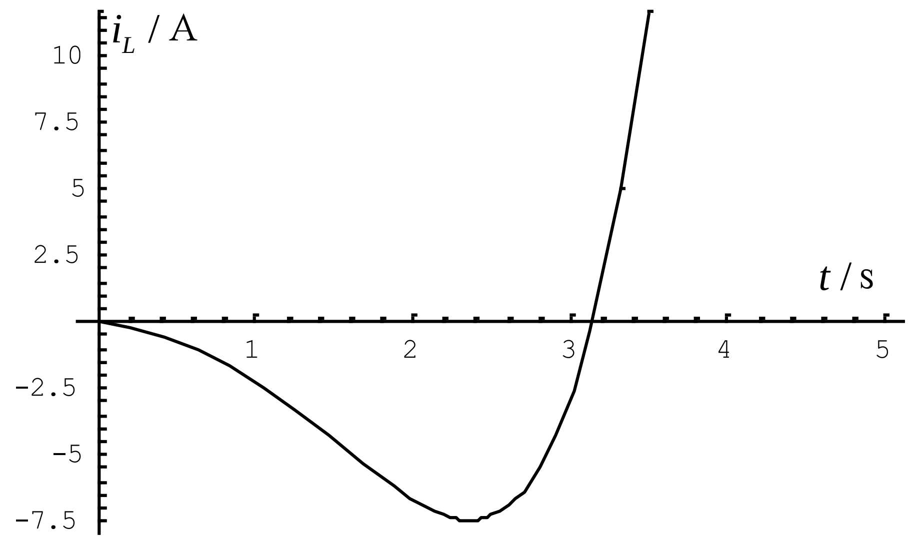
$i_L(0_-)=0$

$L=1\text{H} \quad C=0.5\text{F}$



$\boxed{R=-2\Omega}$ 负阻尼

$$\begin{bmatrix} u_C \\ i_L \end{bmatrix} = \begin{bmatrix} e^t(\cos t - \sin t)\text{V} \\ (-e^t \sin t)\text{A} \end{bmatrix}$$

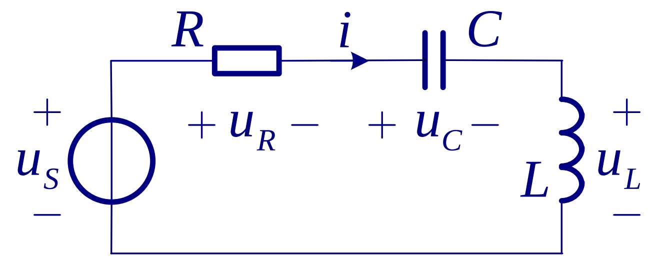


9.3 二阶电路的零状态响应

以阶跃响应为例来分析二阶*RLC*电路的零状态响应。

一、*RLC*串联电路的阶跃响应

根据KVL和支路电压–电流关系,可得



$$u_s = U_0\varepsilon(t)$$

$$LC\frac{d^2u_C}{dt^2} + RC\frac{du_C}{dt} + u_C = U_0$$

二阶常系数线性非齐次微分方程

初始条件为: $u_C(0_+)=u_C(0_-)=0$

$i_L(0_+)=i_L(0_-)=0$

方程的解为 $u_C = u_{Ch} + u_{Cp}$

齐次解为 $u_{Ch} = K_1 e^{s_1 t} + K_2 e^{s_2 t}$

特征方程 $LCs^2 + RCs + 1 = 0$

特征根(固有频率)
$$\begin{cases} s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0} \\ s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0} \end{cases}$$

与*RLC*串联电路零输入响应一样，*RLC*串联电路的固有频率*s*1和*s*2也可以是两个不相等的负实数，两个相等的负实数，一对共轭复数和一对共轭虚数。

阶跃激励下的稳态分量u_{Cp}=U_0

$$u_C = u_{Ch} + u_{Cp} = K_1 e^{s_1 t} + K_2 e^{s_2 t} + U_0$$

根据初始条件,有 $\begin{cases} u_C(0_+) = K_1 + K_2 + U_0 = 0 \\ \left.\dfrac{du_C}{dt}\right|_{t=0_+} = K_1 s_1 + K_2 s_2 = 0 \end{cases}$

$$\Rightarrow \quad K_1 = \frac{s_2}{s_1 - s_2} U_0, \quad K_2 = \frac{s_1}{s_2 - s_1} U_0$$

电容电压为

$$u_C = \left[\frac{1}{s_1 - s_2} (s_2 e^{s_1 t} - s_1 e^{s_2 t}) + 1 \right] U_0 \varepsilon(t)$$

*RLC*串联充电电路也可以区分为:

1.过阻尼 $\alpha > \omega_0$ 电路参数满足 $R > 2\sqrt{L/C}$

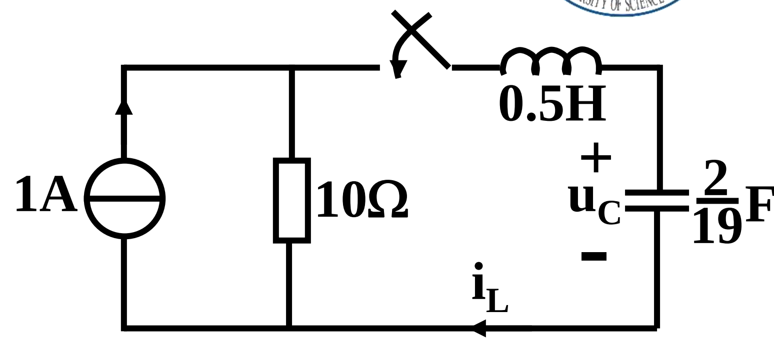
2.临界阻尼 $\alpha = \omega_0$ $R = 2\sqrt{L/C}$

3.欠阻尼 $\alpha < \omega_0$ $R < 2\sqrt{L/C}$

4.无阻尼α=0 (即R=0)

9.3 直流激励下的响应

全响应——二阶电路响应计算



$u_c(0-)=15V$， $i_L(0-)=0$

$$u_C(0_+)=u_C(0_-)=15\text{V} \qquad i_L(0_+)=i_L(0_-)=0$$

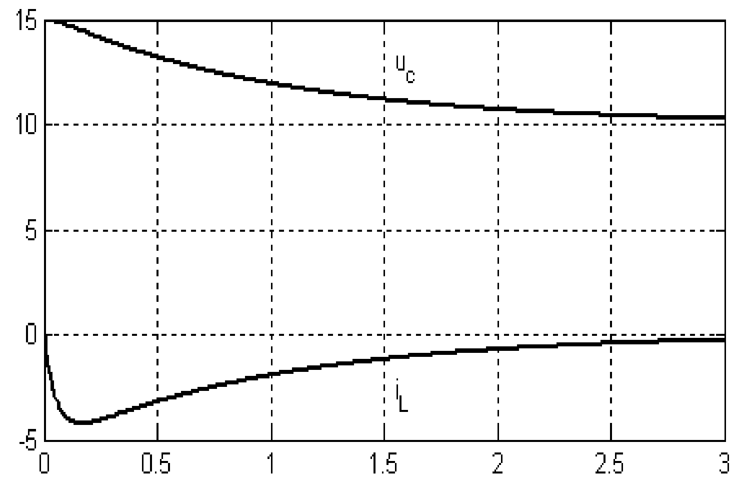
$$\left.\frac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{0_+}=\frac{i_C(0_+)}{C}=\frac{i_L(0_+)}{C}=0$$

$$\text{KVL:}\ \ 0.5\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{2}{19}\frac{\mathrm{d}u_C}{\mathrm{d}t}\right)+10\left(\frac{2}{19}\frac{\mathrm{d}u_C}{\mathrm{d}t}-1\right)+u_C=0$$

$$\frac{\mathrm{d}^2u_C}{\mathrm{d}t^2}+20\frac{\mathrm{d}u_C}{\mathrm{d}t}+19u_C=190$$

$$p_{1,2}=-10\pm\sqrt{100-19}=\begin{cases}-1\\-19\end{cases}$$

$$u_C=k_1\mathrm{e}^{-t}+k_2\mathrm{e}^{-19t}+10$$



$$k_1=\frac{95}{18} \qquad k_2=-\frac{5}{18} \qquad i_L=C\frac{\mathrm{d}u_C}{\mathrm{d}t}=-\frac{95}{18}\mathrm{e}^{-t}+\frac{95}{18}\mathrm{e}^{-19t}$$

作 业

- 9.2节：9-5，9-7
- 9.3节：9-13