Chapter 15

周期性非正弦稳态电路

15.1 周期性函数的傅里叶级数

Trigonometric Fourier Series

15.2 平均功率和有效值

Average Power and RMS Values

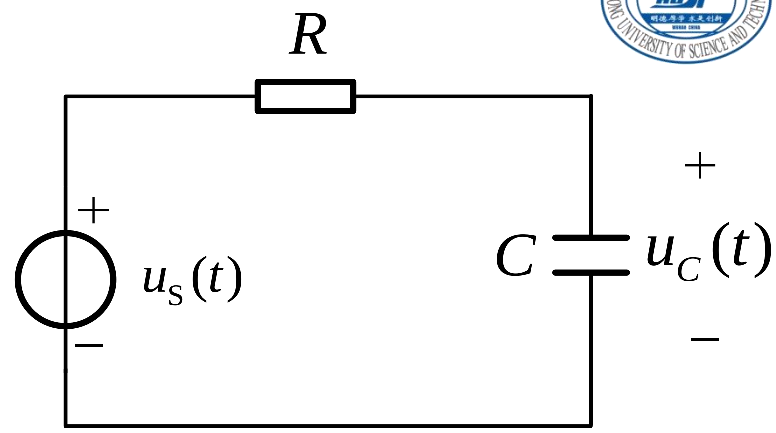
15.3 周期性非正弦电源激励下的稳态响应

Steady-state Response under Nonsinasoidal Input

目标：

利用傅里叶级数和叠加原理计算周期电源下的稳态响应

问题：求电路的稳定响应。



三要素法：

$$u_C = E + (U_1 - E)e^{-\frac{t}{RC}} \quad 0 \le t \le 0.5T$$

$$u_C = -E + (U_2 + E)e^{-\frac{t-0.5T}{RC}} \quad 0.5T \le t \le T$$

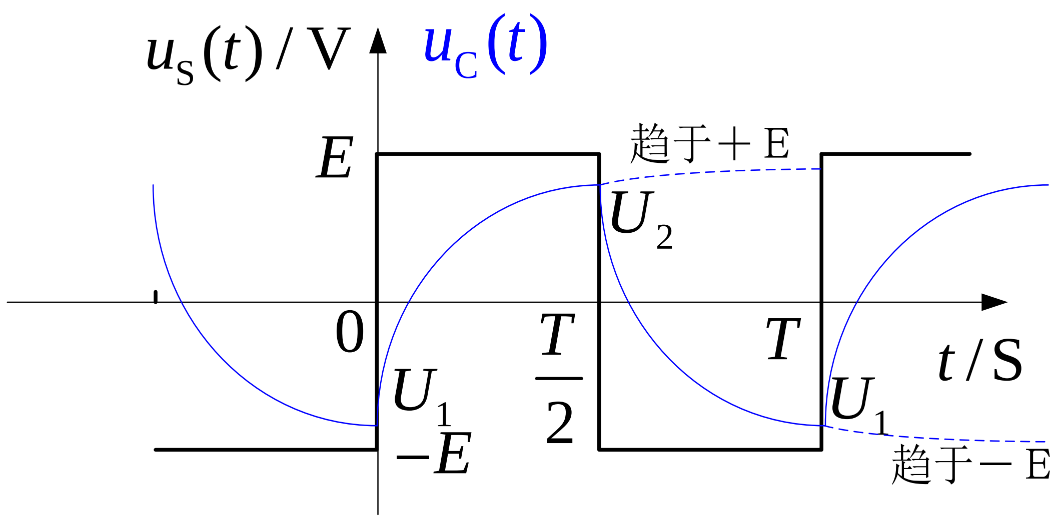
⇩

$$U_2 = E + (U_1 - E)e^{-\frac{0.5T}{RC}}$$

$$U_1 = -E + (U_2 + E)e^{-\frac{T-0.5T}{RC}}$$

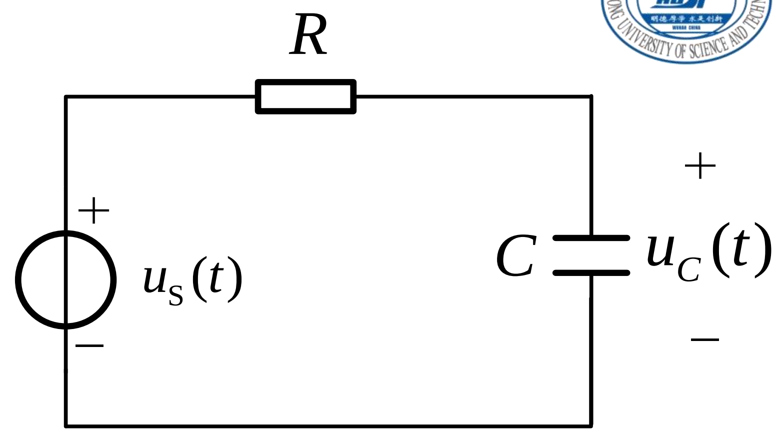
⇩

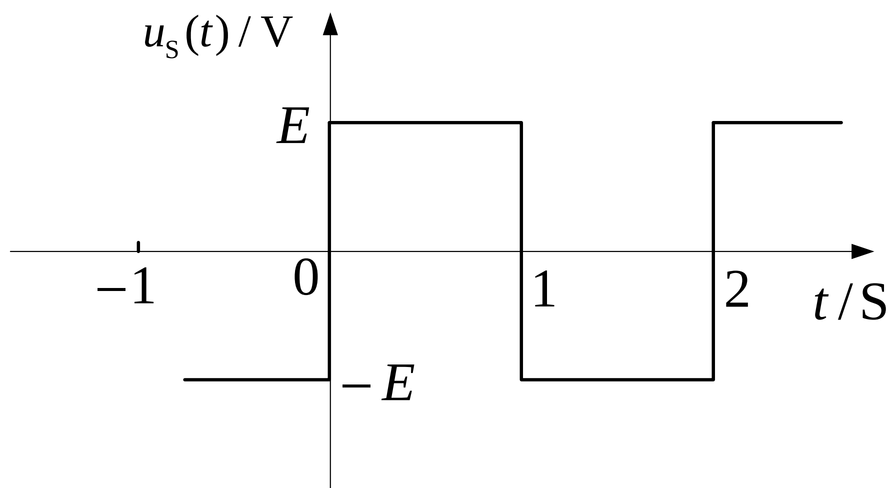
$$U_1、U_2$$



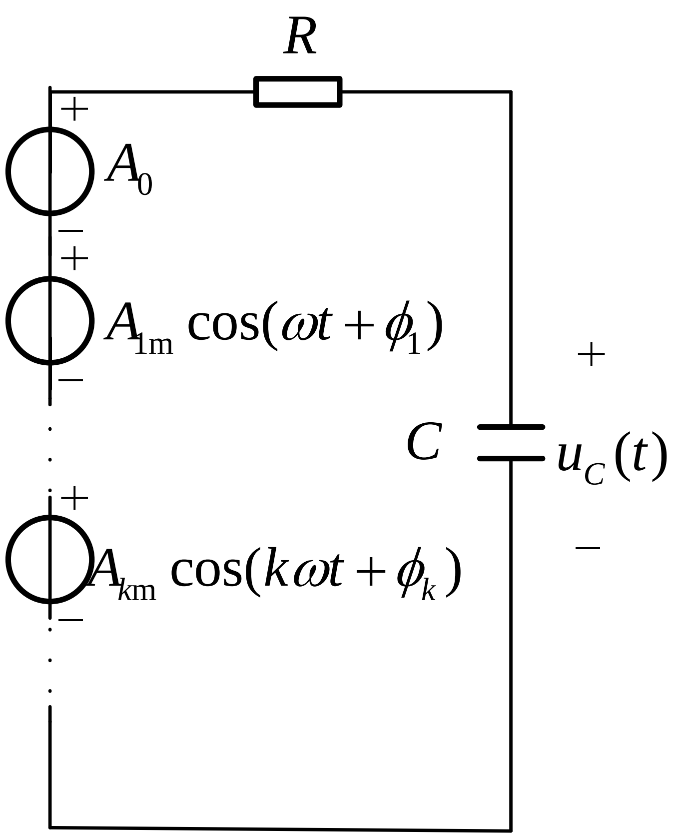
问题：求电路的稳定响应。

$$u_S(t) = A_0 + \sum_{k=1}^{\infty} A_{km}\cos(k\omega t + \phi_k)$$





周期非正弦输入



叠加定理

15. 1周期性函数的傅里叶级数Fourier Series

1.周期函数 Periodic function

$$f(t)=f(t\pm T) \qquad \omega T=2\pi$$

$$f(t)=a_0+\sum_{k=1}^{\infty}(a_k\cos k\omega t+b_k\sin k\omega t) = \underbrace{A_0}_{\text{dc}}+\sum_{k=1}^{\infty}\underbrace{A_{km}\cos(k\omega t+\phi_k)}_{\text{ac— harmonics}}$$

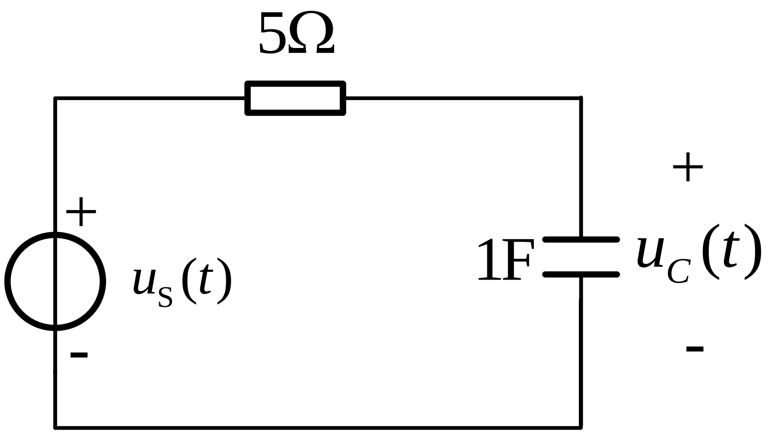
ω—基频 fundamental frequency

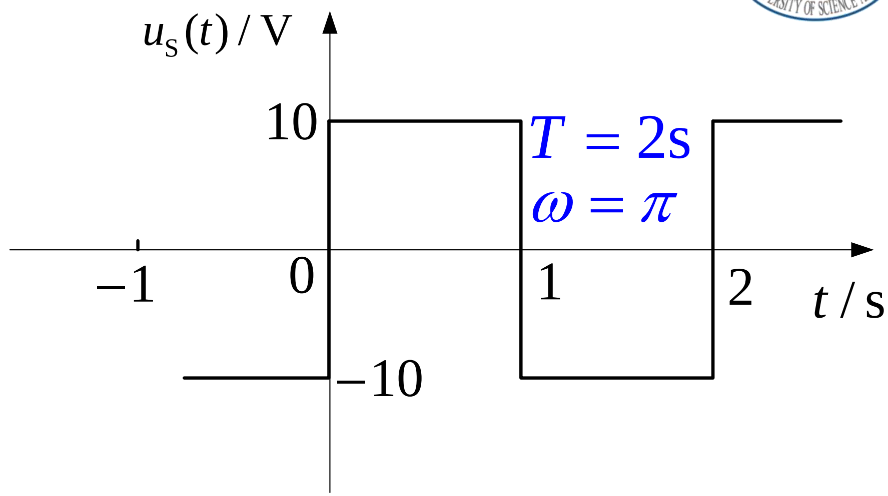
$k\omega$ —k次谐波频率 harmonic frequency

$$\begin{cases} a_0=\dfrac{1}{T}\displaystyle\int_0^T f(t)\mathrm{d}t \\ a_k=\dfrac{2}{T}\displaystyle\int_0^T f(t)\cos k\omega t\mathrm{d}t \\ b_k=\dfrac{2}{T}\displaystyle\int_0^T f(t)\sin k\omega t\mathrm{d}t \end{cases}$$

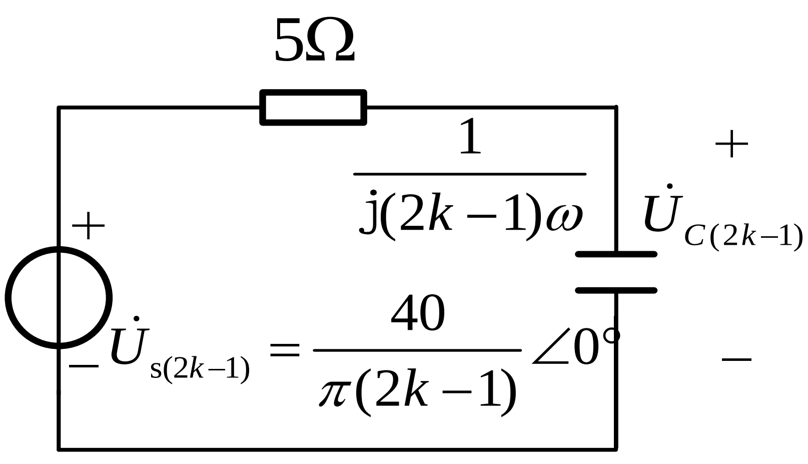
$$\boxed{A_{km}\angle\varphi_k=a_k-\mathrm{j}b_k} \qquad \Longleftrightarrow$$

$$\begin{cases} A_0=a_0 \\ A_{km}=\sqrt{a_k^2+b_k^2} \\ \phi_k=-\arctan\dfrac{b_k}{a_k} \end{cases}$$





$$u_S = \frac{40}{\pi}\sum_{k=1}^{\infty}\frac{1}{2k-1}\sin(2k-1)\omega t$$



$$\dot{U}_{C(2k-1)} = \frac{-\mathrm{j}\dfrac{1}{(2k-1)\pi}}{5-\mathrm{j}\dfrac{1}{(2k-1)\pi}} \times \frac{40}{\pi(2k-1)}\angle 0^\circ$$

$$= A_{(2k-1)}\angle\theta_{(2k-1)}$$

$$u_C(t) = \sum_{k=1}^{\infty} A_{(2k-1)}\sin[(2k-1)\pi t + \theta_{(2k-1)}]$$

无法获得稳态响应的具体波形——

只能通过有限项获得近似响应！取多少项是可以接受的近似呢？

15.3非正弦量的有效值与平均功率

1. RMS values

$$u(t)=U_0+\sum_1^{\infty}U_{km}\cos(k\omega t+\phi_k)$$

$$\boxed{U=\sqrt{\frac{1}{T}\int_0^T u^2\mathrm{d}t}}\qquad =U_0+\sum_1^{\infty}\sqrt{2}U_k\cos(k\omega t+\phi_k)$$

$$U=\sqrt{\frac{1}{T}\int_0^T[U_0+\sum_1^{\infty}\sqrt{2}U_k\cos(k\omega t+\phi_k)]^2\mathrm{d}t}$$

$$\frac{1}{T}\int_0^T U_0^2\mathrm{d}t=U_0^2\qquad \frac{1}{T}\int_0^T[U_0\cdot\sqrt{2}U_k\cos(k\omega t+\phi_k)]\mathrm{d}t=0$$

$$\frac{1}{T}\int_0^T[\sqrt{2}U_k\cos(k\omega t+\phi_k)\cdot\sqrt{2}U_q\cos(q\omega t+\phi_q)]\mathrm{d}t=0\qquad k\neq q$$

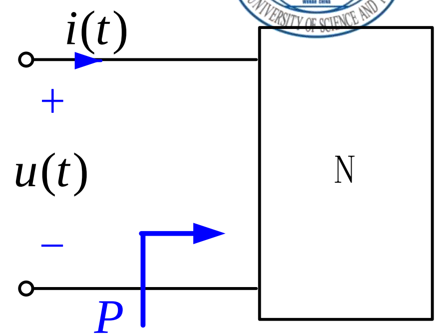
$$\frac{1}{T}\int_0^T[\sqrt{2}U_k\cos(k\omega t+\phi_k)]^2\mathrm{d}t=U_k^2$$

$$\boxed{U=\sqrt{U_0^2+\sum_1^{\infty}U_k^2}}$$

2. Average power

$$u(t)=U_0+\sum_{1}^{\infty}\sqrt{2}U_k\cos(k\omega t+\phi_{uk})$$

$$i(t)=I_0+\sum_{1}^{\infty}\sqrt{2}I_k\cos(k\omega t+\phi_{ik})$$



$$\boxed{P=\frac{1}{T}\int_0^T ui\mathrm{d}t}$$

$$\frac{1}{T}\int_0^T U_0I_0\mathrm{d}t=U_0I_0=P_0 \qquad \frac{1}{T}\int_0^T U_0\sum_{1}^{\infty}\sqrt{2}I_k\cos(k\omega t+\phi_{ik})\mathrm{d}t=0$$

$$\frac{1}{T}\int_0^T I_0\sum_{1}^{\infty}\sqrt{2}U_k\cos(k\omega t+\phi_{uk})\mathrm{d}t=0$$

$$\frac{1}{T}\int_0^T[\sqrt{2}U_k\cos(k\omega t+\phi_{uk})][\sqrt{2}I_k\cos(k\omega t+\phi_{ik})]\mathrm{d}t=U_kI_k\cos(\phi_{uk}-\phi_{ik})=P_k$$

$$\frac{1}{T}\int_0^T[\sqrt{2}U_k\cos(k\omega t+\phi_{uk})][\sqrt{2}I_q\cos(q\omega t+\phi_{iq})]\mathrm{d}t=0 \qquad (k\neq q)$$

$$\boxed{P=U_0I_0+\sum_{1}^{\infty}U_kI_k\cos(\phi_{uk}-\phi_{ik})} \quad \boxed{=P_0+P_1+P_2+P_3+\cdots}$$

功率符合叠加原理

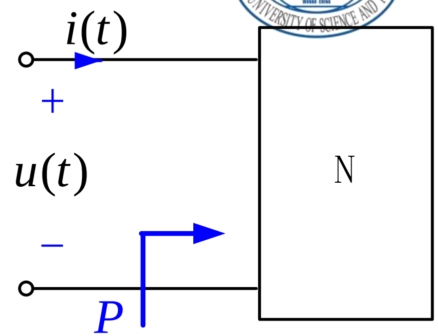
15.3非正弦量的有效值与平均功率

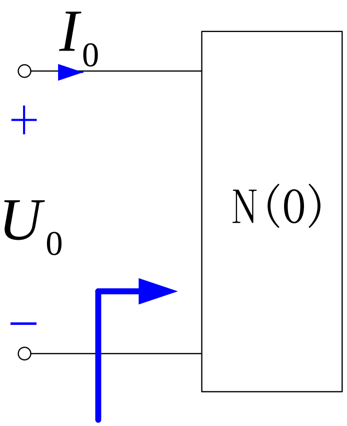
2. Average power

$$P = \frac{1}{T}\int_0^T ui\,dt$$

$$u(t) = U_0 + \sum_1^{\infty} \sqrt{2} U_k \cos(k\omega t + \phi_{uk})$$

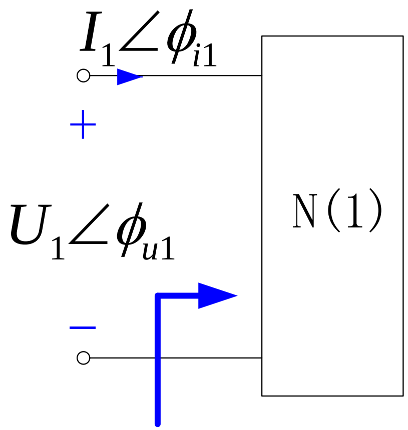
$$i(t) = I_0 + \sum_1^{\infty} \sqrt{2} I_k \cos(k\omega t + \phi_{ik})$$





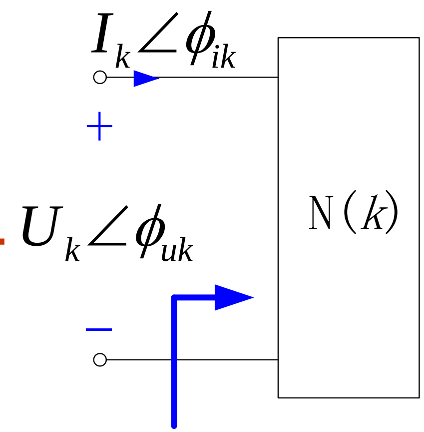
$P_0 = U_0 I_0$

+



$P_1 = U_1 I_1 \cos(\phi_{u1} - \phi_{i1})$

+…+



$P_k = U_k I_k \cos(\phi_{uk} - \phi_{ik})$

+…

$$P = U_0 I_0 + \sum_1^{\infty} U_k I_k \cos(\phi_{uk} - \phi_{ik}) = P_0 + P_1 + P_2 + P_3 + \cdots$$

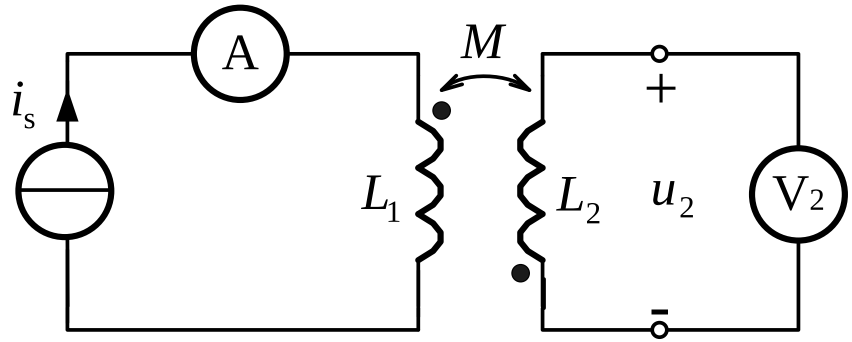
功率符合叠加原理

Practice 1

$$i_s = [5 + 10\cos(10t - 20°) - 5\sin(30t + 60°)]\text{A},$$

$$L_1 = L_2 = 2\text{H}，\ M = 0.5\text{H}。$$

Find the reading of each meter.



$$i_{s(0)} = 5\text{A}$$

$$u_{2(0)} = 0$$

$$\dot{I}_{s(1)} = 10\angle -20°\text{A}$$

$$\dot{U}_{2(1)} = -\text{j}\omega M\dot{I}_{s(1)} = -\text{j}10 \times 0.5 \times 10\angle -20° = 50\angle -110°\text{V}$$

$$\dot{I}_{s(3)} = 5\angle 60°\text{A}$$

$$\dot{U}_{2(3)} = -\text{j}3\omega M\dot{I}_{s(3)} = -\text{j}30 \times 0.5 \times 5\angle 60° = 75\angle -30°\text{V}$$

$$u_2 = [50\cos(10t - 110°) - 75\sin(30t - 30°)]\text{V}$$

$$I_s = \sqrt{5^2 + (10^2 + 5^2)/2} = 9.4\text{A}$$

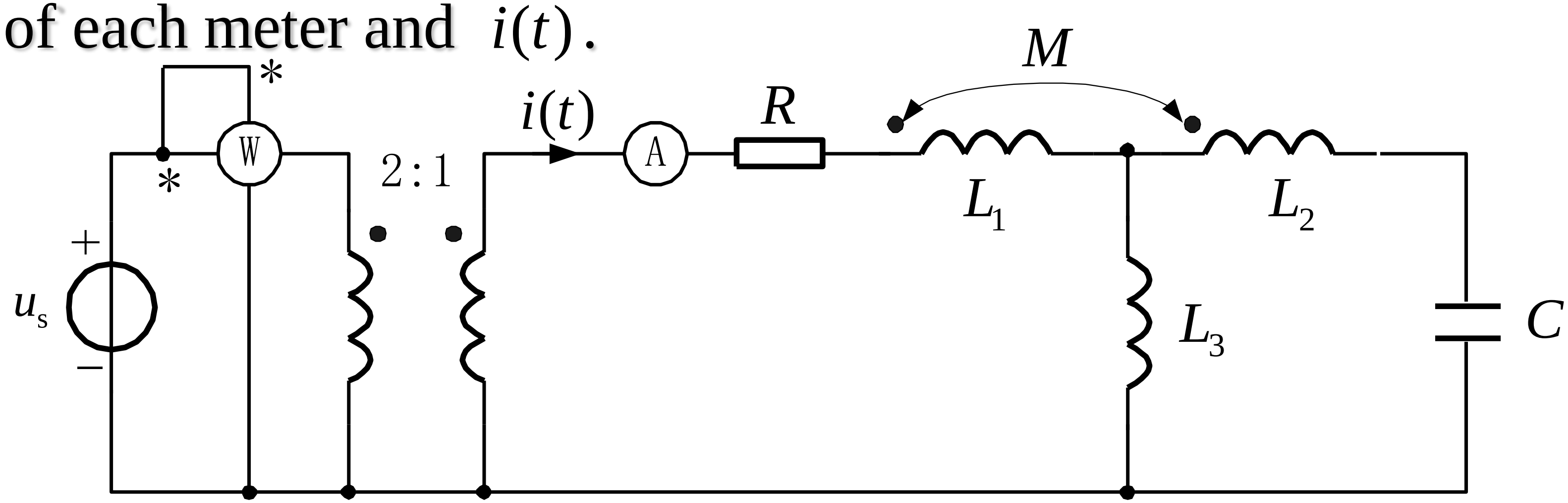
$$U_2 = \sqrt{(50^2 + 75^2)/2} = 63.7\text{V}$$

Practice 2

$$u_s(t) = (300\sqrt{2}\sin\omega t + 200\sqrt{2}\sin 3\omega t) \quad R = 50\Omega$$

$$\omega L_1 = 60\Omega \quad \omega L_2 = 50\Omega \quad \omega M = 40\Omega \quad \omega L_3 = 20\Omega$$

There is not fundamental component in the current of L_3 . Find the reading of each meter and $i(t)$.

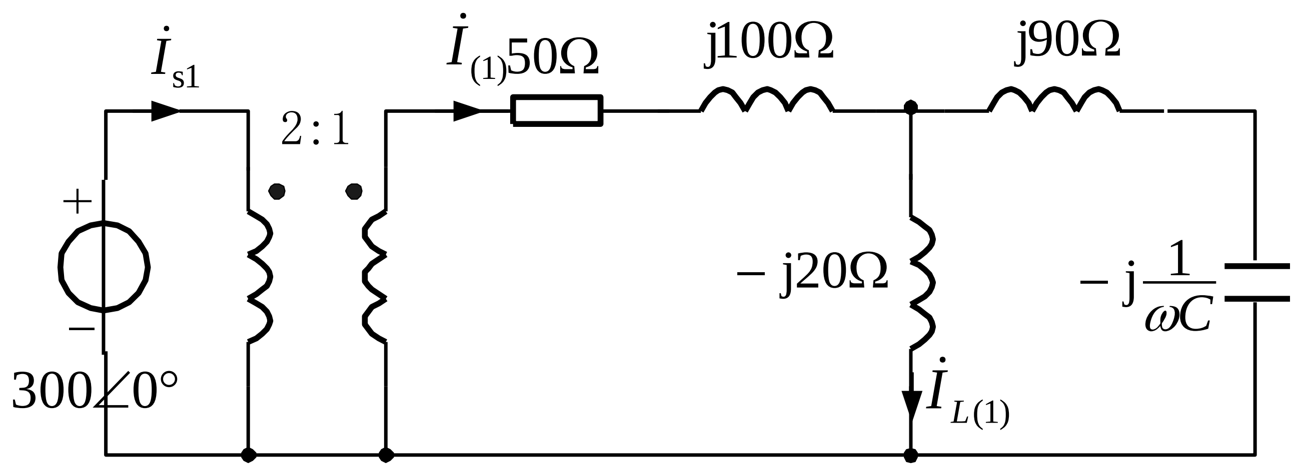


$$\because \dot{I}_{L(1)} = 0$$

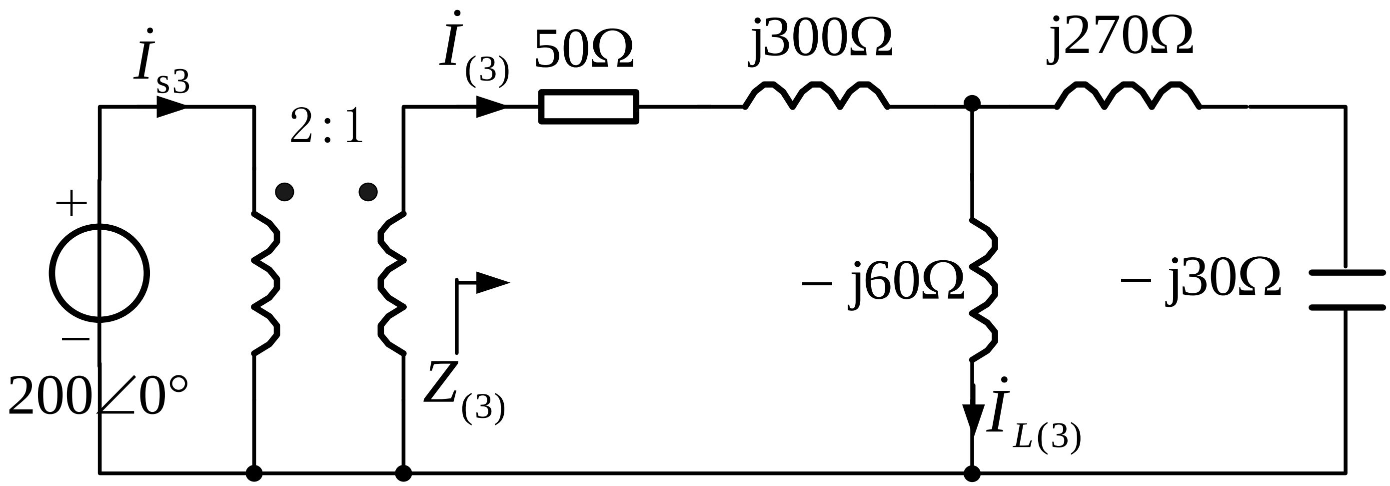
$$\therefore \frac{1}{\omega C} = 90$$

$$\dot{I}_{(1)} = \frac{150}{50 + j100}$$

$$= 1.34\angle -63.4°$$



Be continued



$$Z_{(3)} = 50 + \mathrm{j}300 + \frac{-\mathrm{j}240 \times \mathrm{j}60}{\mathrm{j}240 - \mathrm{j}60} = 50 + j220$$

$$\dot{I}_{(3)} = \frac{100}{Z_{(3)}} = 0.44\angle -77.2°$$

$$I = \sqrt{I_{(1)}{}^2 + I_{(3)}{}^2} = 1.41\mathrm{A} \qquad P = 50I^2 = 99.5\mathrm{W}$$

$$i(t) = 1.34\sqrt{2}\sin(\omega t - 63.4°) + 0.44\sqrt{2}\sin(3\omega t - 77.7°)\mathrm{A}$$

作业

- **15.4节：15-8、15-12**