

Chapter 14

正弦稳态电路的频率响应

14.2 传递函数与频率响应

14.3 谐振电路

➤ 串联谐振

➤ 并联谐振

14.4 滤波器

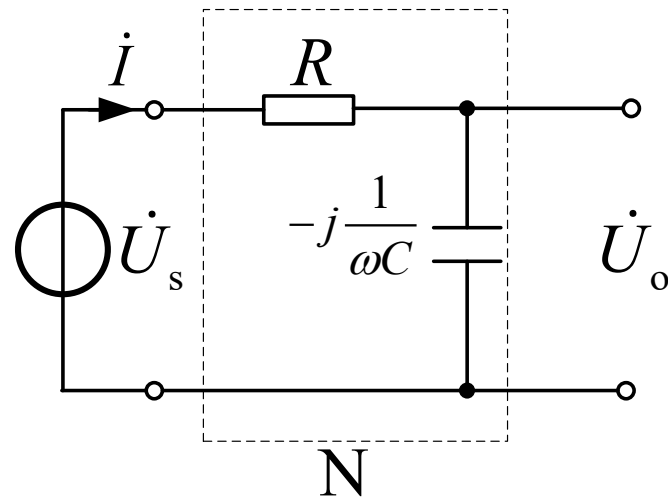
频率响应：正弦稳态响应随激励频率的变化规律。

$$Z_{\text{in}} = R - j\frac{1}{\omega C}$$

$$\dot{I} = \frac{\dot{U}_s}{R - j\frac{1}{\omega C}} = \frac{j\omega C}{1 + j\omega CR} \dot{U}_s$$

$$\dot{U}_R = \frac{j\omega CR}{1 + j\omega CR} \dot{U}_s$$

$$\dot{U}_C = \frac{1}{1 + j\omega CR} \dot{U}_s$$



不同频率下工作状态不同！

$$\omega \rightarrow 0 \quad \frac{1}{\omega C} \rightarrow \infty, I \rightarrow 0, U_R \rightarrow 0, U_C \rightarrow U_s$$

$$\omega \rightarrow \infty \quad \frac{1}{\omega C} \rightarrow 0, I \rightarrow \frac{U_s}{R}, U_R \rightarrow U_s, U_C \rightarrow 0$$

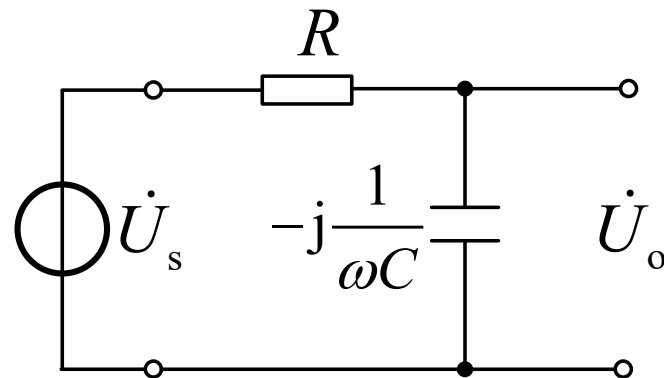
用什么手段来描述频率响应？

分析频率响应有何意义？

14.2 传递函数与频率响应

如何描述响应与激励频率之间的关系？ $\dot{U}_o \sim \dot{U}_s, \omega$

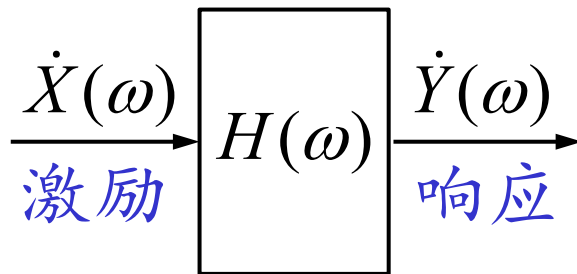
$$H(\omega) = \frac{\dot{U}_o(\omega)}{\dot{U}_s(\omega)} = \frac{-j\frac{1}{\omega C}}{R - j\frac{1}{\omega C}} = \frac{1}{1 + j\omega RC}$$



只包含频率

1. 传递函数 响应相量与激励相量的比值

$$H(\omega) = \frac{\dot{Y}(\omega)}{\dot{X}(\omega)} = \frac{\text{响应}}{\text{激励}}$$



- 传递函数与电路结构、参数、输入与输出变量类型、端口对的相对位置有关。

相同端口

$$H(\omega) = \frac{\dot{U}_s(\omega)}{\dot{I}_s(\omega)}$$

$$H(\omega) = \frac{\dot{I}_s(\omega)}{\dot{U}_s(\omega)}$$

不同端口

$$H(\omega) = \frac{\dot{U}_o(\omega)}{\dot{U}_s(\omega)}$$

$$H(\omega) = \frac{\dot{U}_o(\omega)}{\dot{I}_s(\omega)}$$

$$H(\omega) = \frac{\dot{I}_o(\omega)}{\dot{I}_s(\omega)}$$

$$H(\omega) = \frac{\dot{I}_o(\omega)}{\dot{U}_s(\omega)}$$

- 传递函数是一个复数，频率特性分为幅频特性和相频特性。

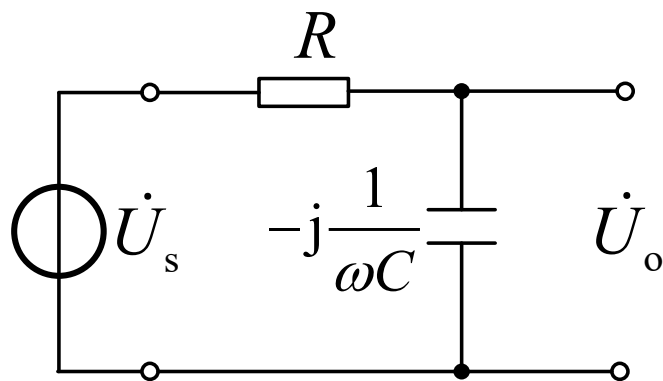
$$H(\omega) = |H(\omega)| \angle \varphi(\omega)$$

响应幅值与激励幅值之比

幅频响应 相频响应

响应初相与激励初相之差

2. 频率响应 掌握电路对信号的频率选择性



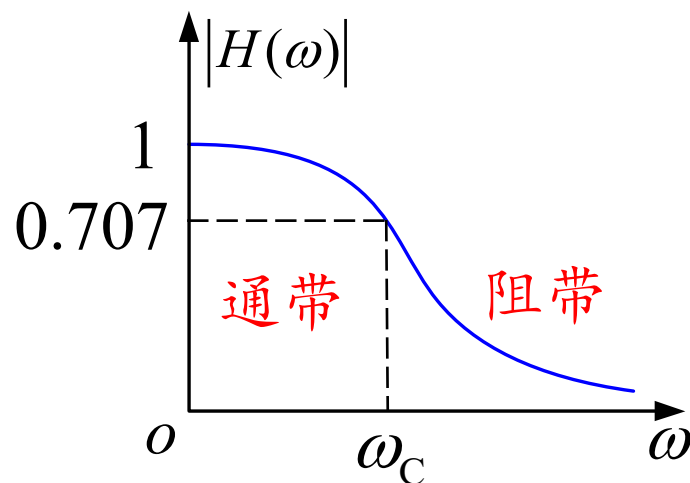
$$H_C(\omega) = \frac{\dot{U}_o(\omega)}{\dot{U}_s(\omega)} = \frac{1}{1 + j\omega RC}$$

$$= \frac{1}{\sqrt{1 + (\omega CR)^2}} \angle -\arctan(\omega CR)$$

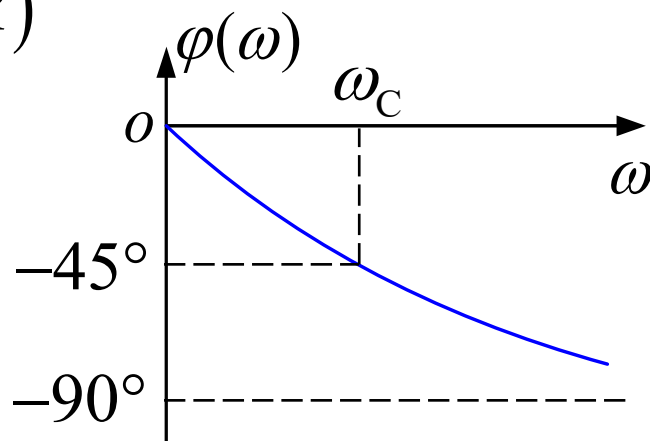
$$H_C(0) = 1 \angle 0^\circ \quad H_C(\infty) = 0 \angle -90^\circ$$

$$H_C\left(\frac{1}{RC}\right) = 0.707 \angle -45^\circ$$

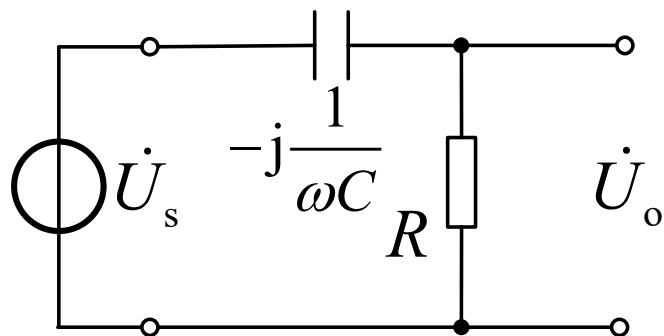
低通滤波器



$$\omega_c = \frac{1}{RC}$$



2. 频率响应



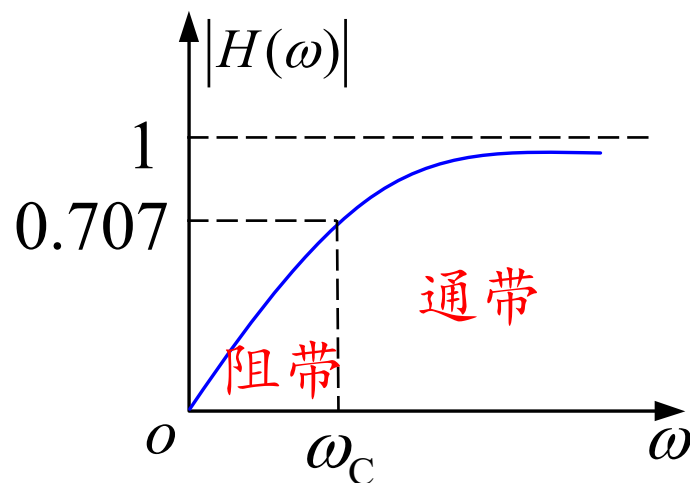
$$H_R(\omega) = \frac{\dot{U}_o(\omega)}{\dot{U}_s(\omega)} = \frac{j\omega RC}{1 + j\omega RC}$$

$$= \frac{\omega RC}{\sqrt{1 + (\omega CR)^2}} \angle 90^\circ - \arctan(\omega CR)$$

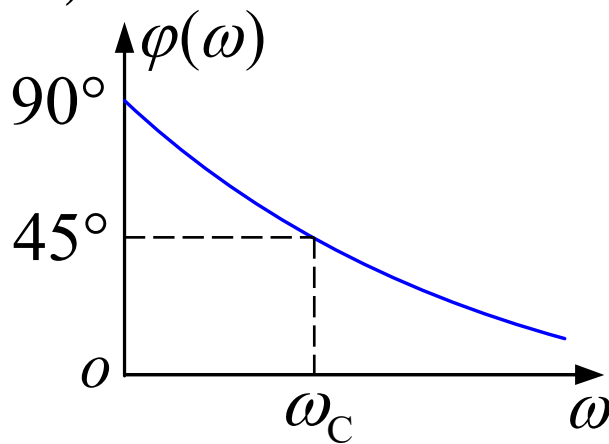
$$H_R(0) = 0 \angle 90^\circ \quad H_R(\infty) = 1 \angle 0^\circ$$

$$H_R\left(\frac{1}{RC}\right) = 0.707 \angle 45^\circ$$

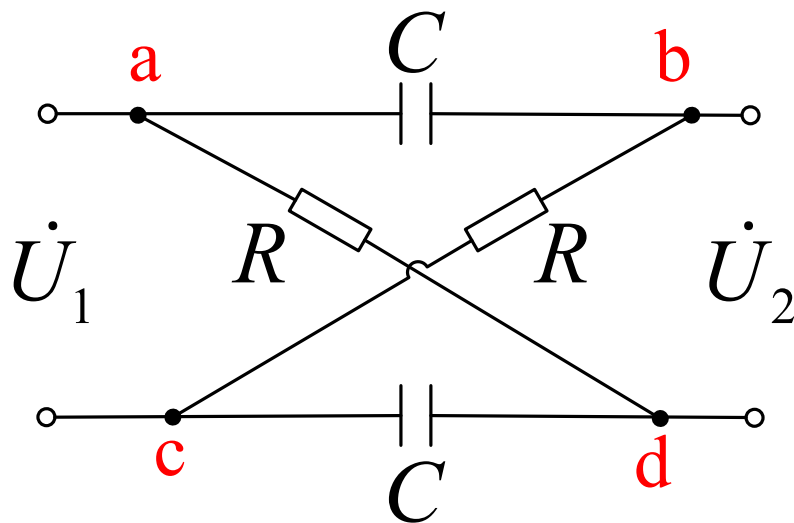
高通滤波器



$$\omega_c = \frac{1}{RC}$$



例 图示电路中 $RC=1\text{ s}$ 。求电压增益 \dot{U}_2/\dot{U}_1



解 根据分压公式，有：

$$\dot{U}_2 = \frac{R}{R + \frac{1}{j\omega C}} \dot{U}_1 - \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \dot{U}_1 = \frac{j\omega CR - 1}{1 + j\omega CR} \dot{U}_1 = \frac{j\omega - 1}{1 + j\omega} \dot{U}_1$$

14.3 谐振电路

1. 谐振 正弦稳态下，电感和电容的阻抗完全互补

$$Z_{\text{in}} = R(\omega) + jX(\omega) \quad X(\omega) = 0$$

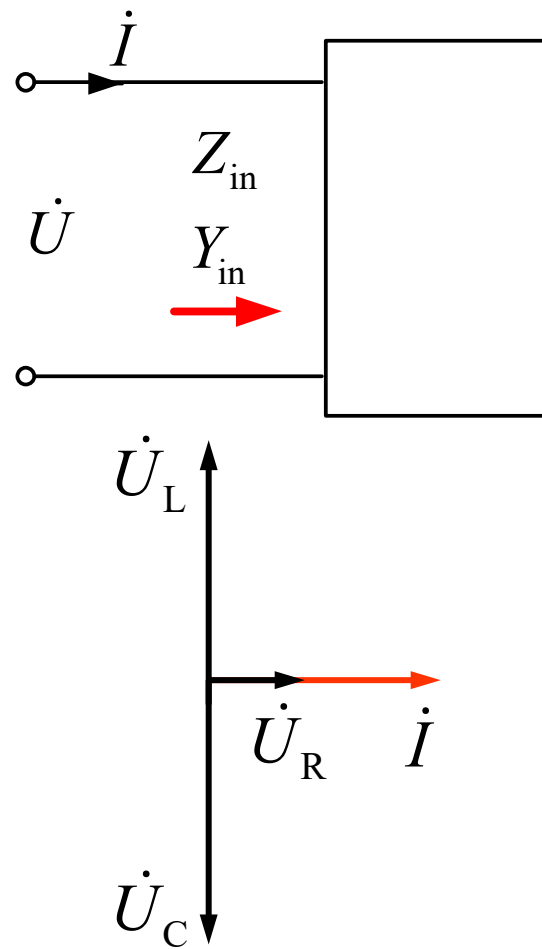
$$Y_{\text{in}} = G(\omega) + jB(\omega) \quad B(\omega) = 0$$

\dot{U} 和 \dot{I} 同相位

$$P = UI \quad Q = 0$$

改变电源频率

改变电路L、C参数值



2. RLC串联谐振

$$Z = R + j(\omega L - \frac{1}{\omega C})$$

$$\omega = \frac{1}{\sqrt{LC}} = \omega_0 \quad \text{谐振频率}$$

谐振特点：

$$(1) |Z(\omega_0)| = R = |Z_{\min}(\omega)|$$

$$(2) \dot{U}_s \text{ 和 } \dot{I}_0 \text{ 同相位}$$

$$(3) |\dot{I}_0| = \left| \frac{\dot{U}_s}{R} \right| = |\dot{I}_{\max}(\omega)|$$

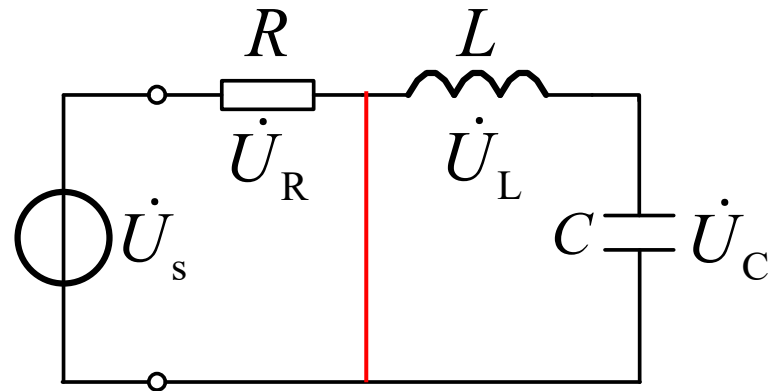
$$(4) \dot{U}_{R0} = \dot{U}_s$$

$$(5) \dot{U}_{L0} = j\omega_0 L \dot{I}_0 = j \frac{\omega_0 L}{R} \dot{U}_s$$

$$\dot{U}_{C0} = -j \frac{1}{\omega_0 C} \dot{I}_0 = -j \frac{1}{\omega_0 CR} \dot{U}_s$$

$$U_{L0} = U_{C0} = Q U_s$$

Q: 品质因数



对外相当于短路！

收音机

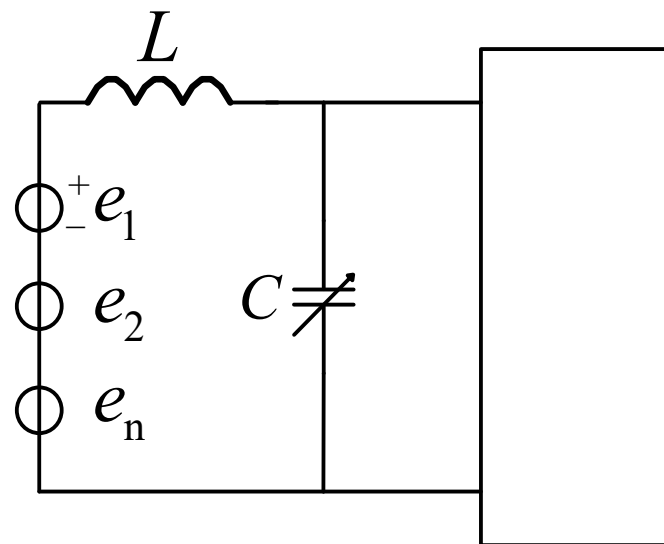
品质因数

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

$$U_{L0} = U_{C0} = \textcolor{blue}{Q} U_s$$

当 $Q \gg 1$ 时，电容或电感上电压将远大于电源电压，称为过电压现象。

应用：电信系统的信号放大



例 某收音机输入回路 $L=0.3\text{mH}$, $R=10\Omega$, 为收到中央电台 560kHz 信号, 求 (1) 调谐电容 C ; (2) 如输入电压为 $1.5\mu\text{V}$, 求谐振电流和电容电压。

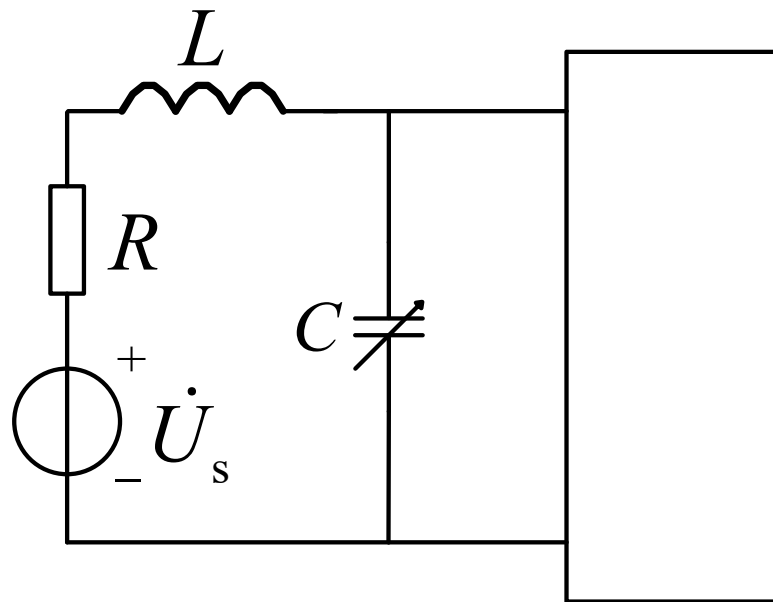
解 (1) 调谐电容 C

$$C = \frac{1}{(2\pi f)^2 L} = 269 \text{ pF}$$

(2) 谐振电流和电容电压

$$I_0 = \frac{U}{R} = 0.15 \mu\text{A}$$

$$U_C = I_0 X_C = 158.5 \mu\text{V} \gg 1.5 \mu\text{V}$$



(6) 能量关系

储能 $w_0 = w_{L0} + w_{C0} = \frac{1}{2}Li_0^2 + \frac{1}{2}Cu_{C0}^2$

$$= LI_0^2(\cos \omega_0 t)^2 + C\left(\frac{I_0}{\omega_0 C}\right)^2[\cos(\omega_0 t - 90^\circ)]^2$$
$$= LI_0^2(\cos \omega_0 t)^2 + LI_0^2(\sin \omega_0 t)^2 = LI_0^2$$

- 电感和电容能量按正弦规律变化，且最大值相等。 L 、 C 的电场能量和磁场能量作周期振荡性的交换，而不与电源进行能量交换；
- 总能量是不随时间变化的常量，且等于最大值。

(6) 能量关系 储能 $w_0 = LI_0^2$

耗能 $w_{R0} = \int_0^{T_0} i_0^2 R dt = I_0^2 R T_0 = I_0^2 R \frac{2\pi}{\omega_0} = 2\pi I_0^2 R \sqrt{LC}$

$$\frac{w_0}{w_{R0}} = \frac{1}{2\pi} \frac{\sqrt{L/C}}{R} = \frac{Q}{2\pi} \quad Q = 2\pi \frac{\text{谐振电路存储的能量}}{\text{一个周期内消耗的能量}}$$

Q值反映了谐振回路中电磁振荡的程度，Q越大，总能量就越大，维持振荡所消耗的能量越小，振荡程度越剧烈，则振荡电路的“品质”越好。一般要求在发生谐振的回路中尽可能的提高Q值。

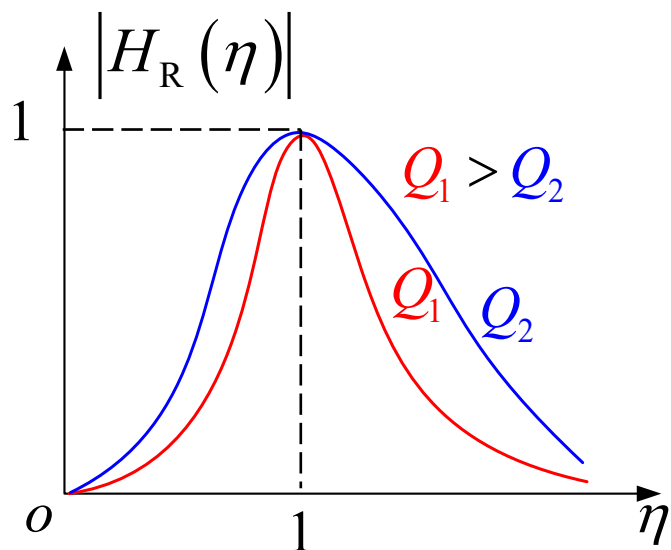
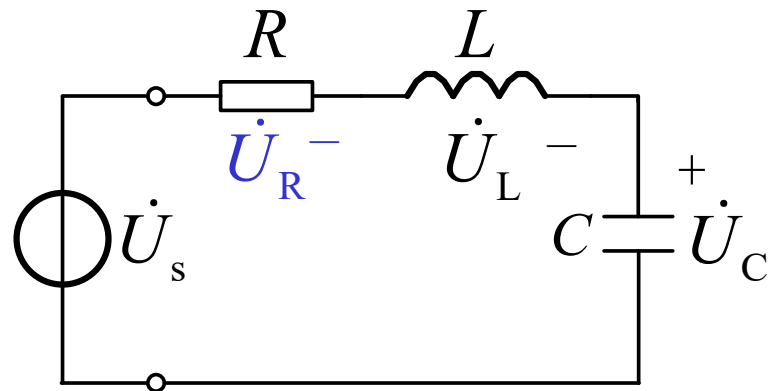
$$|H_R(\omega)| = \left| \frac{\dot{U}_R(\omega)}{\dot{U}_S(\omega)} \right| = \frac{R}{\left| R + j\left(\omega L - \frac{1}{\omega C}\right) \right|}$$

$$= \frac{1}{\sqrt{1 + \left(\frac{\omega L}{R} - \frac{1}{R\omega C} \right)^2}}$$

$$= \frac{1}{\sqrt{1 + Q^2 \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^2}}$$

$$\frac{\omega}{\omega_0} = \eta \quad |H_R(\eta)| = \frac{1}{\sqrt{1 + Q^2 \left(\eta - \frac{1}{\eta} \right)^2}}$$

$$|H_R(0)| = 0 \quad |H_R(\infty)| = 0 \quad |H_R(1)| = 1$$



**Q值越大，频率
选择性越好**

➤ 谐振电路具有选择性

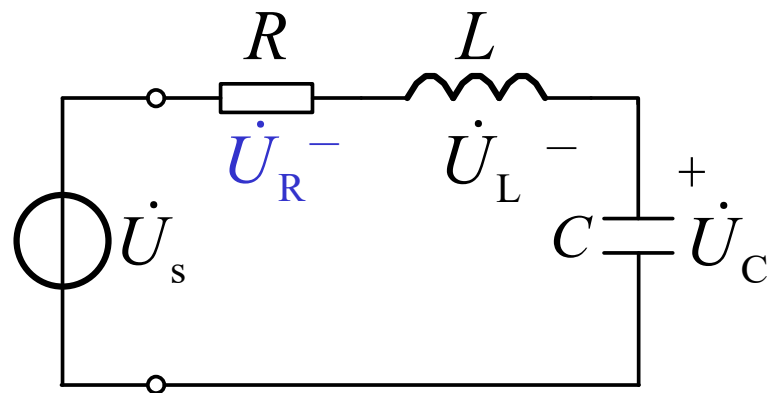
在谐振点响应出现峰值，当 ω 偏离 ω_0 时，输出下降。即**串联谐振电路对不同频率信号有不同的响应，对谐振信号响应最大**，而对远离谐振频率的信号具有抑制能力。这种对不同输入信号的选择能力称为“选择性”。

➤ 谐振电路的选择性与Q成正比

Q越大，谐振曲线越陡。电路对非谐振频率信号的抑制能力强，所以选择性好。因此，**Q是反映谐振电路性质的一个重要指标。**

➤ 在谐振点电路消耗的有功功率最大；

➤ 当 $|H_R(\eta)|$ 下降至0.707倍时，信号就被截止了。



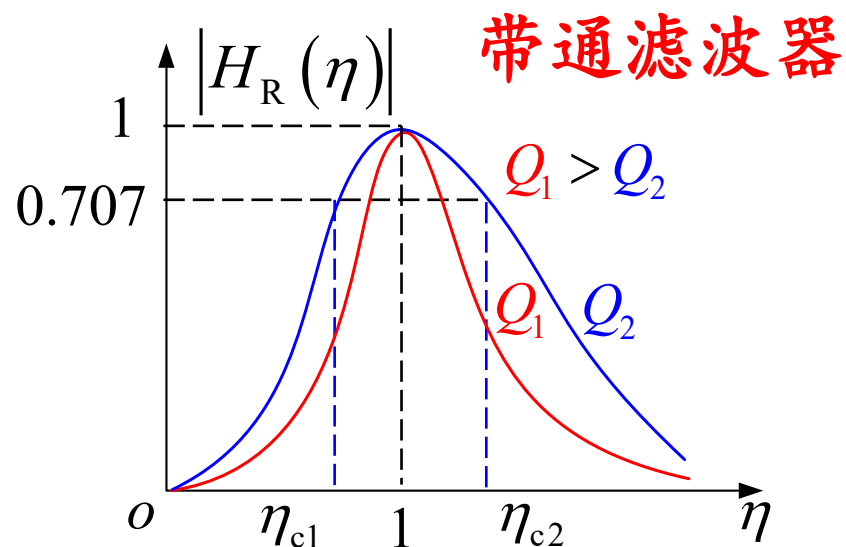
$$|H_R(\eta_{c1,c2})| = \frac{1}{\sqrt{2}} = 0.707$$

截止频率（半功率频率）

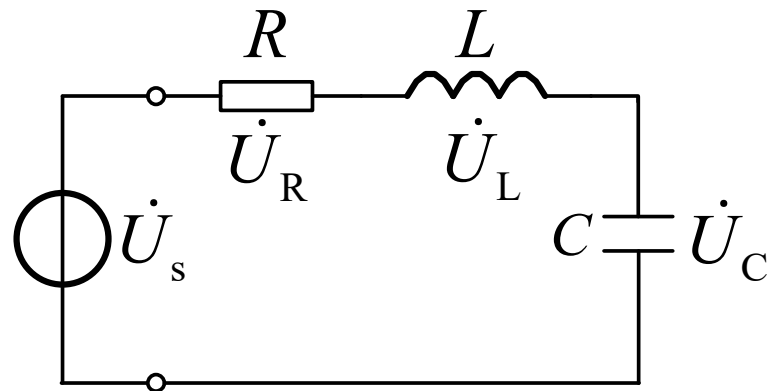
$$\omega_{c1,c2} = \eta_{c1,c2} \omega_0$$

从有功功率角度：

$$P(\omega_{c1,c2}) = \frac{U_R^2}{R} = \frac{(U_s / \sqrt{2})^2}{R} = \frac{1}{2} \frac{U_s^2}{R} = \frac{1}{2} P(\omega_0)$$



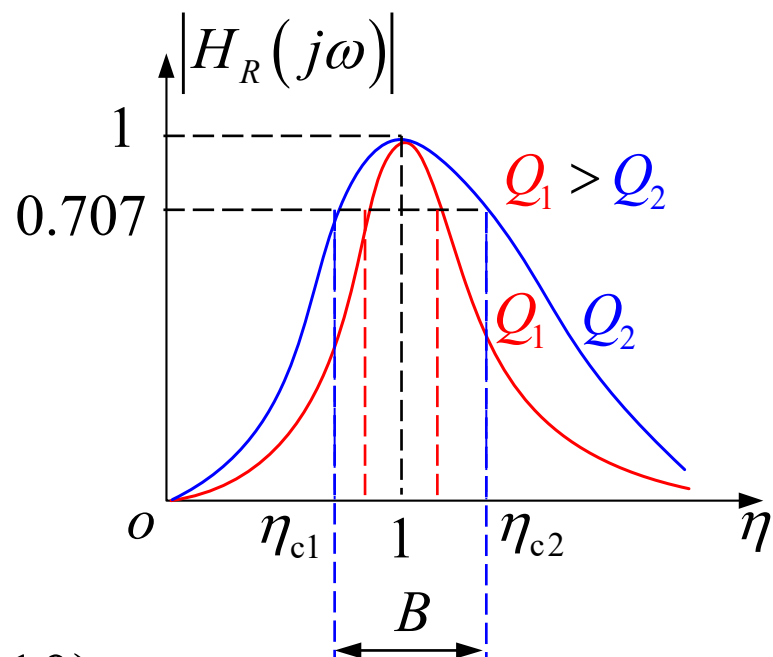
$$|H_R(\eta)| = \frac{1}{\sqrt{1 + Q^2 \left(\eta - \frac{1}{\eta} \right)^2}} = \frac{\sqrt{2}}{2}$$



$$\begin{cases} \eta_{c1} = -\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \\ \eta_{c2} = \frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \end{cases}$$

$$\text{带宽 } B = (\eta_{c2} - \eta_{c1})\omega_0 = \frac{\omega_0}{Q}$$

$$\text{截止频率 } \omega_{c1,c2} \approx \omega_0 \mp \frac{1}{2}B \quad (Q \geq 10)$$



*RLC*串联谐振的频率响应

谐振频率 $\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{\omega_{c1}\omega_{c2}}$

品质因数 $Q = \frac{X_{L0}(X_{C0})}{R} = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{U_{L0}(U_{C0})}{U_S} = 2\pi \frac{W_0}{W_{R0}} = \frac{\omega_0}{B}$

截止频率
$$\omega_{c1,c2} = \mp \frac{\omega_0}{2Q} + \omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2} = \mp \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$
$$\omega_{c1,c2} \approx \omega_0 \mp \frac{B}{2} \quad (Q \geq 10)$$

带宽 $B = \omega_{c2} - \omega_{c1} = \frac{\omega_0}{Q} = \frac{R}{L}$

例 设计一个 RLC 串联电路，使得带宽 $B=20 \text{ rad/s}$ ， $\omega_0 = 1000 \text{ rad/s}$ 。（1）求该电路的 Q 值；（2）若 $C=5 \mu\text{F}$ ，求 L 和 R 的值；（3）求截止频率。

解
$$B = \frac{\omega_0}{Q} \rightarrow Q = \frac{\omega_0}{B} = \frac{1000}{20} = 50$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 1000 \rightarrow L = 200 \text{ mH}$$

$$Q = \frac{\omega_0 L}{R} \rightarrow R = \frac{\omega_0 L}{Q} = 4 \Omega$$

$$\omega_{c1,c2} \approx \omega_0 \mp \frac{B}{2} = 990 \text{ rad/s}, 1010 \text{ rad/s}$$

3. RLC 并联谐振

$$Y = G + j(\omega C - \frac{1}{\omega L})$$

$$\omega = \frac{1}{\sqrt{LC}} = \omega_0 \quad \text{谐振频率}$$

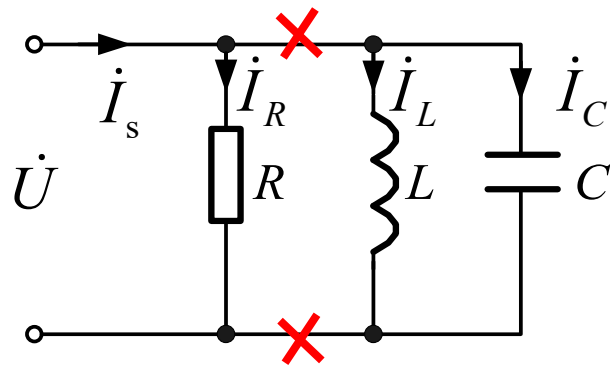
谐振特点：

$$(1) |Y(\omega_0)| = G = |Y_{\min}(\omega)|$$

$$(2) \dot{U}_s \text{ 和 } \dot{I}_0 \text{ 同相位}$$

$$(3) |\dot{U}_0| = \left| \frac{\dot{I}_s}{G} \right| = |\dot{U}_{\max}(\omega)|$$

$$(4) \dot{U}_{R0} = \dot{I}_s / G$$



对外相当于开路！

$$(5) \dot{I}_{L0} = -j \frac{1}{\omega_0 L} \dot{U}_0 = -j \frac{1}{G \omega_0 L} \dot{I}_s$$

$$\dot{I}_{C0} = j \omega_0 C \dot{U}_0 = j \frac{\omega_0 C}{G} \dot{I}_s$$

$$I_{L0} = I_{C0} = Q I_s$$

Q ：品质因数

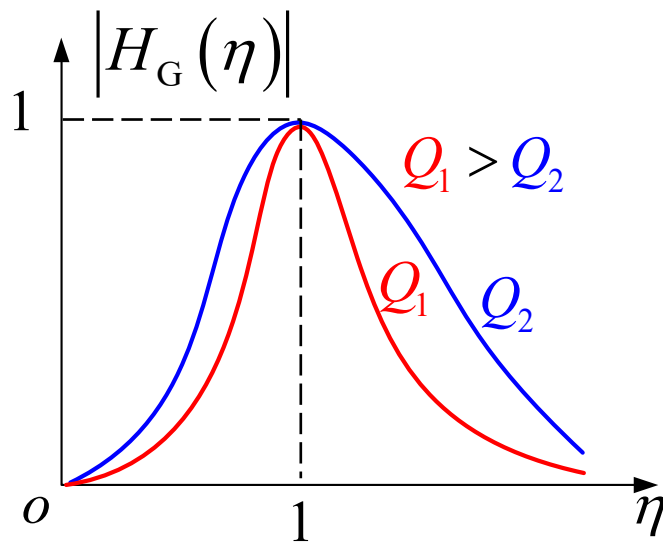
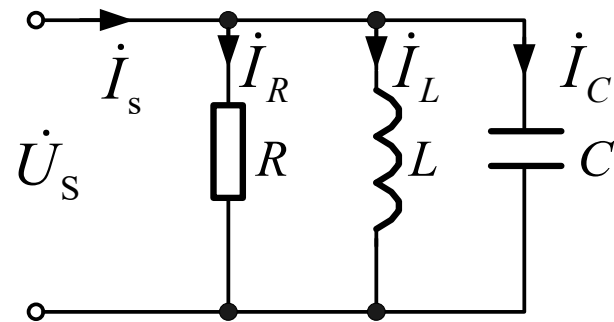
$$|H_G(\omega)| = \left| \frac{\dot{I}_G(\omega)}{\dot{I}_s(\omega)} \right| = \frac{G}{\left| G + j\left(\omega C - \frac{1}{\omega L}\right) \right|}$$

$$= \frac{1}{\sqrt{1 + \left(\frac{\omega C}{G} - \frac{1}{G\omega L} \right)^2}}$$

$$= \frac{1}{\sqrt{1 + Q^2 \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^2}}$$

$$\frac{\omega}{\omega_0} = \eta \quad |H_G(\eta)| = \frac{1}{\sqrt{1 + Q^2 \left(\eta - \frac{1}{\eta} \right)^2}}$$

$$|H_G(0)| = 0 \quad |H_G(\infty)| = 0 \quad |H_G(1)| = 1$$



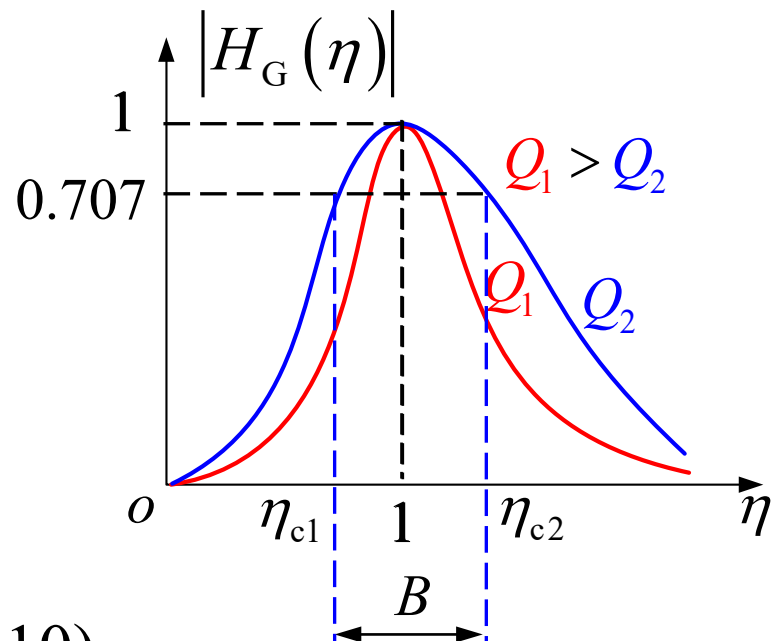
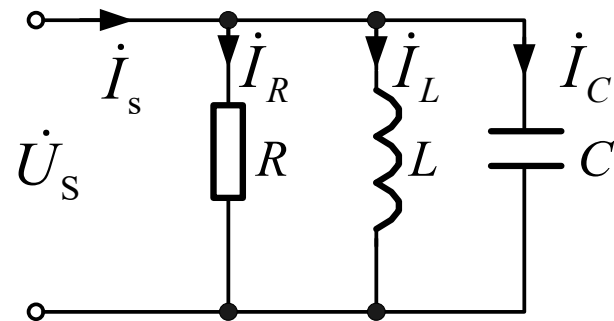
**Q值越大，频率
选择性越好**

$$|H_G(\eta)| = \frac{1}{\sqrt{1 + Q^2 \left(\eta - \frac{1}{\eta} \right)^2}} = \frac{\sqrt{2}}{2}$$

$$\begin{cases} \eta_{c1} = -\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \\ \eta_{c2} = \frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \end{cases}$$

$$\text{带宽 } B = (\eta_{c2} - \eta_{c1})\omega_0 = \frac{\omega_0}{Q}$$

$$\text{截止频率 } \omega_{c1,c2} \approx \omega_0 \mp \frac{1}{2}B \quad (Q \geq 10)$$



RLC 串联谐振

GCL 并联谐振

谐振频率

$$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{\omega_{c1}\omega_{c2}}$$

品质因数

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} = \frac{\omega_0}{B}$$

$$Q = \frac{1}{\omega_0 LG} = \frac{\omega_0 C}{G} = \frac{\omega_0}{B}$$

截止频率

$$\omega_{c1,c2} = \mp \frac{\omega_0}{2Q} + \omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2}$$

$$\omega_{c1,c2} \approx \omega_0 \mp \frac{B}{2} \quad (Q \geq 10)$$

带宽

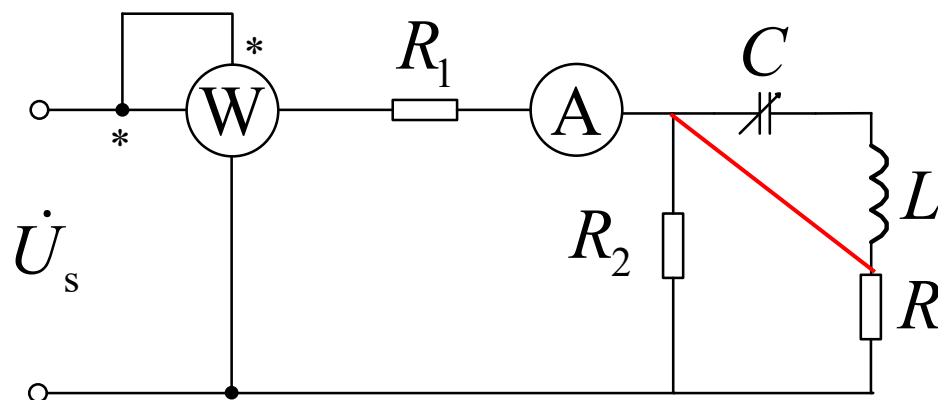
$$B = \omega_{c2} - \omega_{c1} = \frac{\omega_0}{Q}$$

例 电源频率 $f=100/\pi$ Hz, $R_1=6\ \Omega$, $R_2=20\ \Omega$, 当改变电容 $C=1000\ \mu\text{F}$ 时, 电流表读数 I 最大为 1 A, 功率表读数为 10 W, 试计算电阻 R 和电感 L 。

解 电流最大 \rightarrow 总阻抗
最小 \rightarrow LC 串联谐振

$$\omega_0 = 2\pi f = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow L = 25\ \text{mH}$$



当 LC 发生串联谐振时, 对外相当于短路。

$$P = U_s I = I^2 R_{\text{eq}} = I^2 (R_1 + R_2 // R) \Rightarrow R = 5\ \Omega$$

例 求如图所示电路发生谐振时的谐振角频率 ω_0

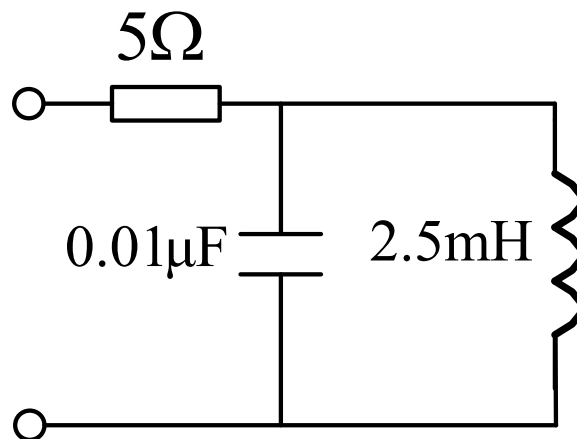
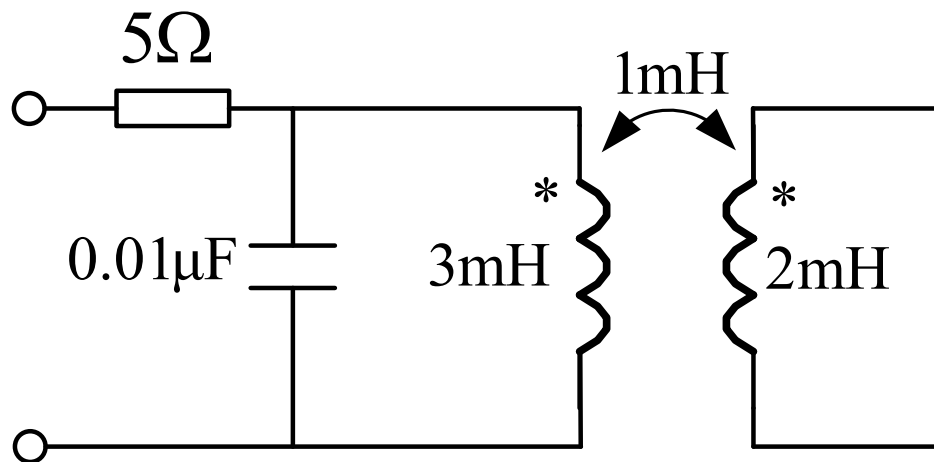
解 映射阻抗法

$$Z_{\text{eq}} = j\omega L_1 + Z_{\text{ref}}$$

$$= j\omega L_1 + \frac{(\omega M)^2}{j\omega L_2}$$

$$L_{\text{eq}} = L_1 - \frac{M^2}{L_2} = 2.5 \text{ mH}$$

$$\omega_0 = \frac{1}{\sqrt{L_{\text{eq}} C}} = 200 \text{ kHz}$$



例 求图示电路中各支路电流。

解 先去耦。

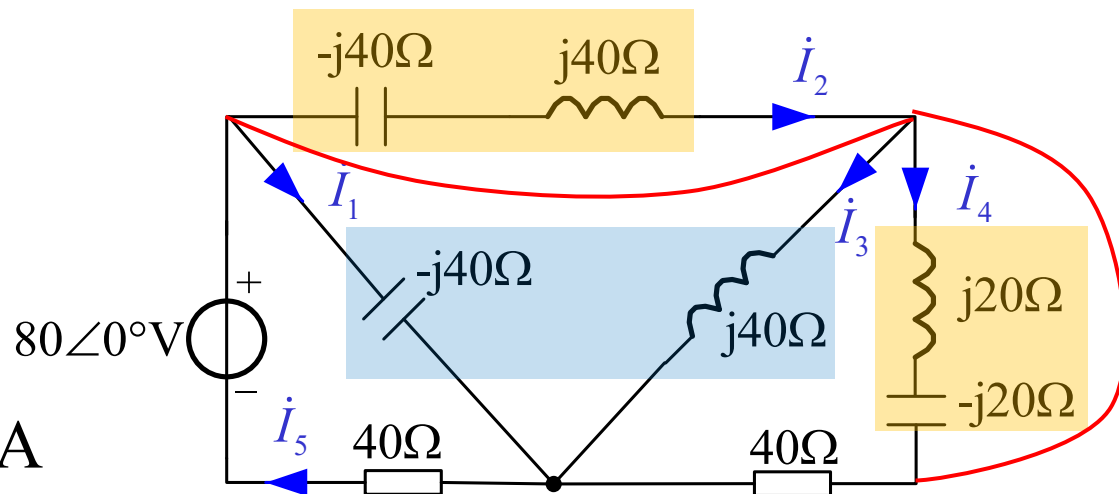
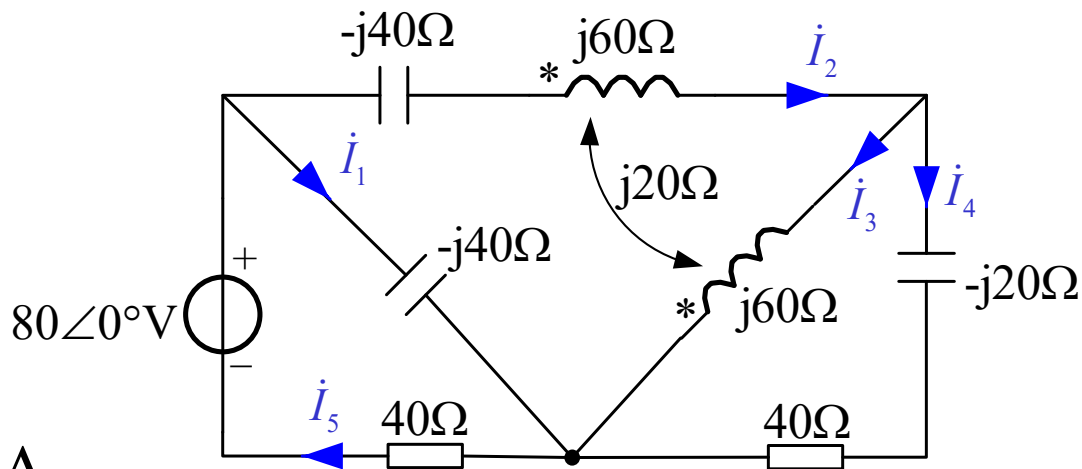
$$\dot{I}_1 + \dot{I}_3 = 0$$

$$\dot{I}_4 = \dot{I}_5 = \frac{80\angle 0^\circ}{40 + 40} = 1\angle 0^\circ \text{ A}$$

$$\dot{I}_3 = \frac{40\dot{I}_4}{j40} = 1\angle -90^\circ \text{ A}$$

$$\dot{I}_1 = -\dot{I}_3 = 1\angle 90^\circ \text{ A}$$

$$\dot{I}_2 = \dot{I}_3 + \dot{I}_4 = \sqrt{2}\angle -45^\circ \text{ A}$$



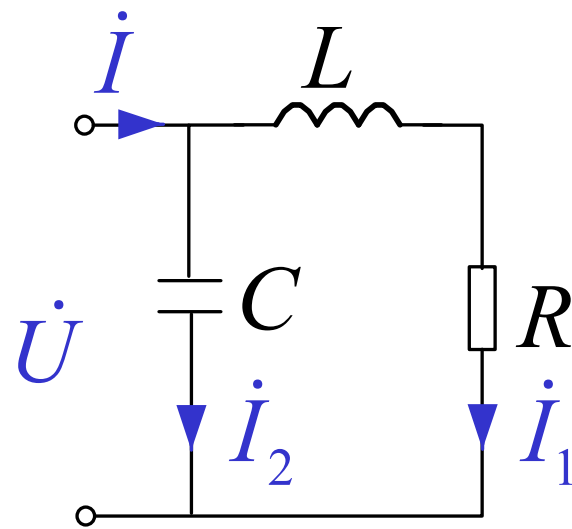
例 图示电路的谐振角频率 $\omega_0=1\text{krad/s}$ ，谐振时端口等效阻抗 $Z_0=1\text{k}\Omega$ ，品质因数 $Q=10$ 。（1）求参数 R 、 L 、 C 。（2）若在端口接频率为 ω_0 、 $R_p=2\text{k}\Omega$ 、 $I_s=1\text{A}$ 的电流源，求品质因数及 I 、 I_1 、 I_2 。

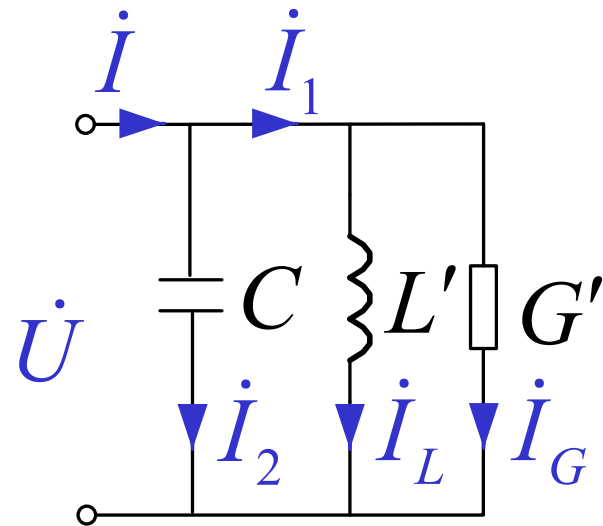
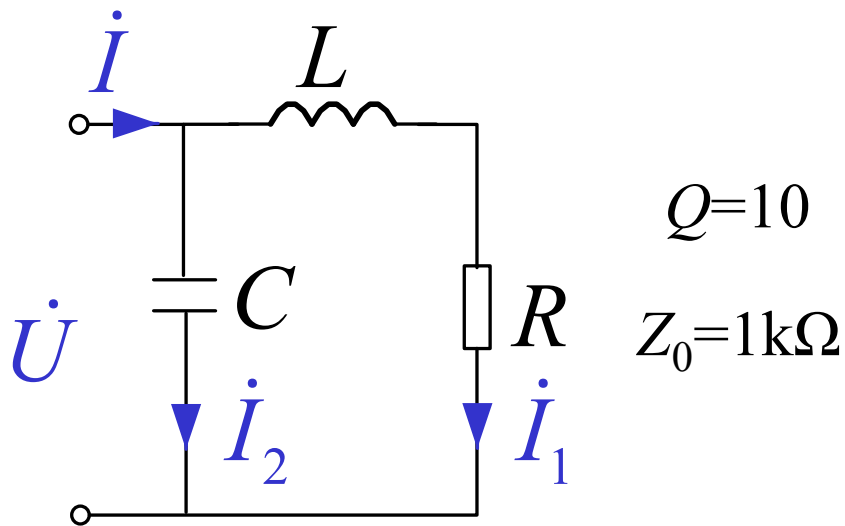
解 （1）等效为 RLC 并联形式

$$\frac{1}{Z_0} = j\omega_0 C + \frac{1}{R + j\omega_0 L}$$

$$= \frac{R}{R^2 + (\omega_0 L)^2} + j \left[\omega_0 C - \frac{\omega_0 L}{R^2 + (\omega_0 L)^2} \right]$$

$$= G' + j(B_C - B_{L'})$$





$$\frac{1}{Z_0} = \frac{R}{R^2 + (\omega_0 L)^2} + j \left[\omega_0 C - \frac{\omega_0 L}{R^2 + (\omega_0 L)^2} \right] = G' + j(B_C - B_{L'})$$

$$\frac{1}{Z_0} = G' = \frac{R}{R^2 + (\omega_0 L)^2} = \frac{1}{1000} \quad \rightarrow \quad R = \frac{1000}{101} \Omega \quad L = \frac{10}{101} \text{H}$$

$$Q = \frac{B_{L'}}{G'} = \frac{\omega_0 L}{R} \rightarrow \omega_0 L = 10R$$

$$B_C = B_{L'} \rightarrow \omega_0 C = \frac{\omega_0 L}{R^2 + (\omega_0 L)^2} \quad \rightarrow \quad C = 10 \mu\text{F}$$

(2) 若在端口接频率为 ω_0 、 $R_p=2k\Omega$ 、 $I_s=1A$ 的电流源，求品质因数及 I 、 I_1 、 I_2 。

$$Q' = \frac{B_C}{G' + G_p} = \frac{\omega_0 C}{G' + G_p} = \frac{20}{3}$$

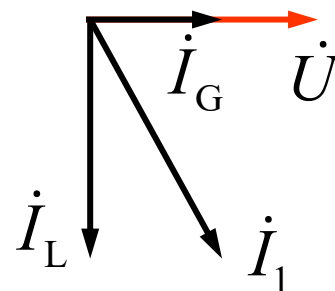
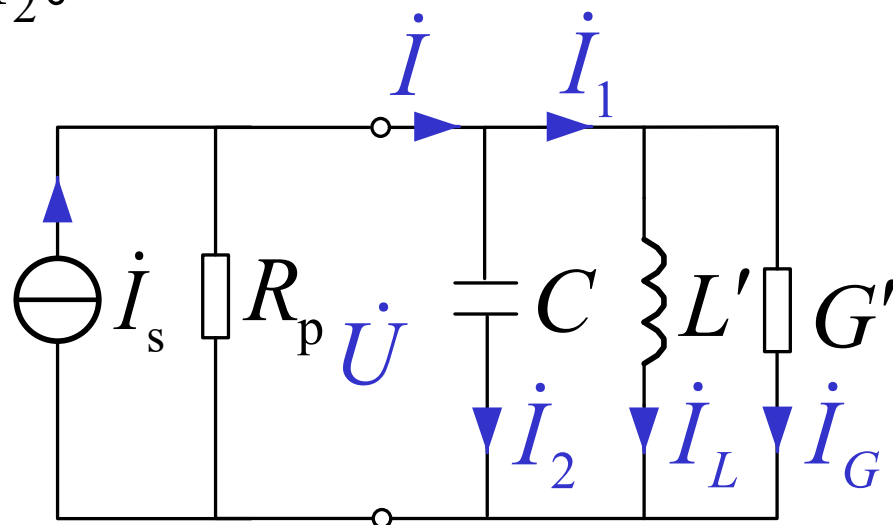
信号源内阻会影响性能指标

$$I = \frac{G'}{G' + G_p} I_s = \frac{2}{3} \text{ A}$$

$$I_2 = Q' I_s = \frac{20}{3} \text{ A} \quad (\text{或} \quad I_2 = QI = 10 \times \frac{2}{3} = \frac{20}{3} \text{ A})$$

$$\dot{I}_1 = \dot{I}_L + \dot{I}_G \rightarrow I_1 = QI + I = \frac{22}{3} \text{ A} \quad \times$$

$$\dot{I}_1 = \dot{I}_L + \dot{I}_G \rightarrow I_1 = \sqrt{(QI)^2 + I^2} = 6.7 \text{ A}$$

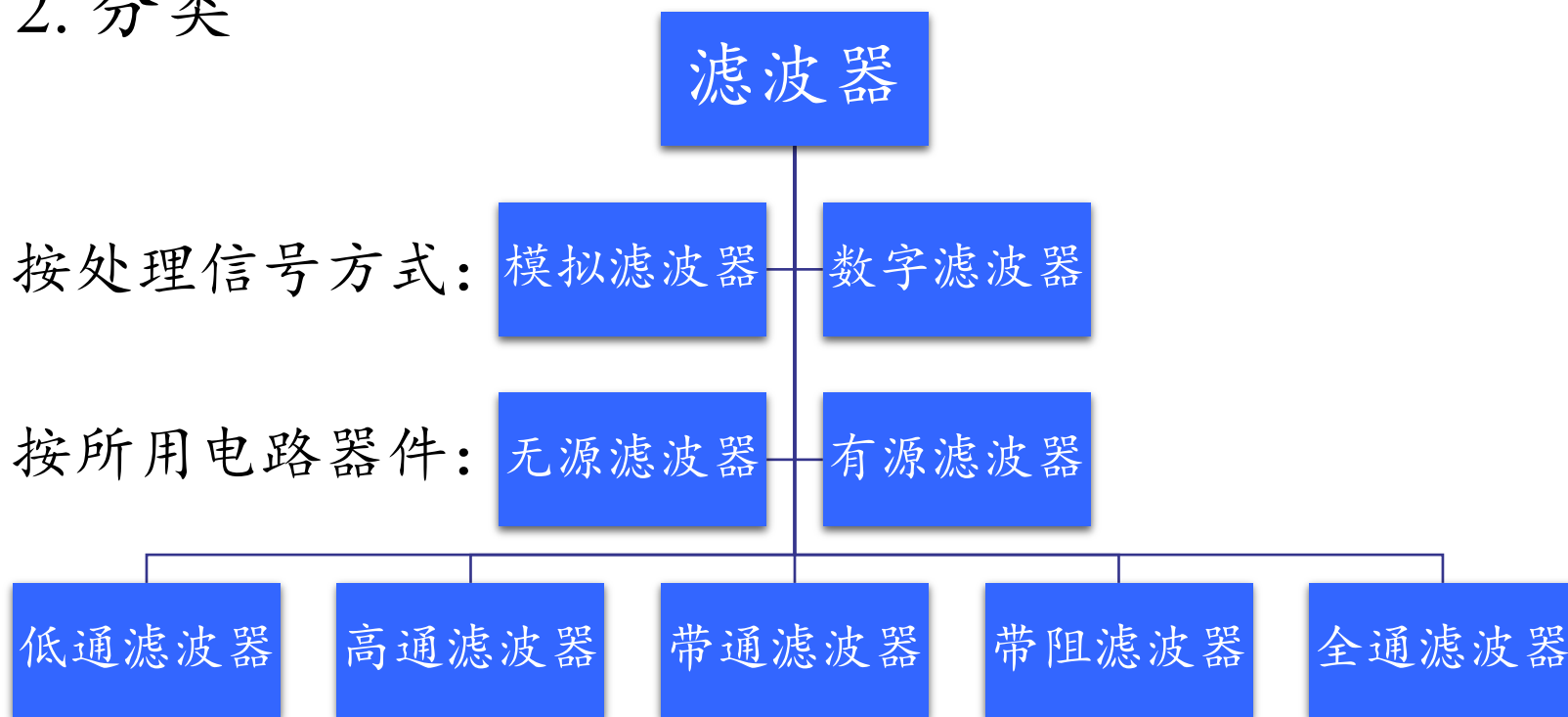


14.4 滤波器

1. 定义

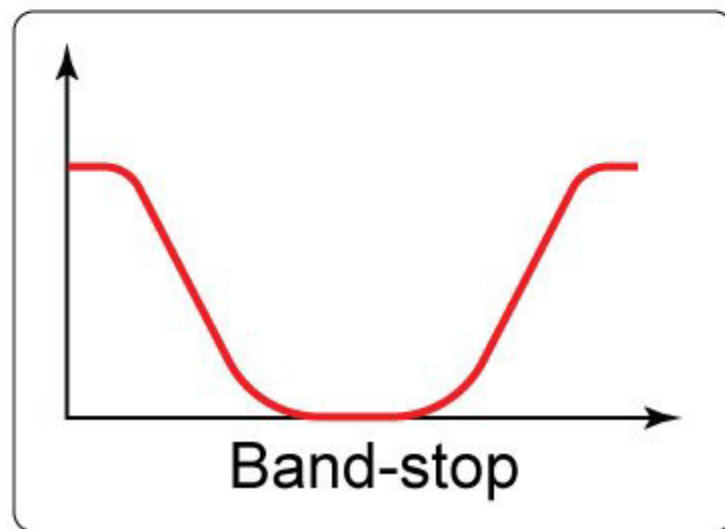
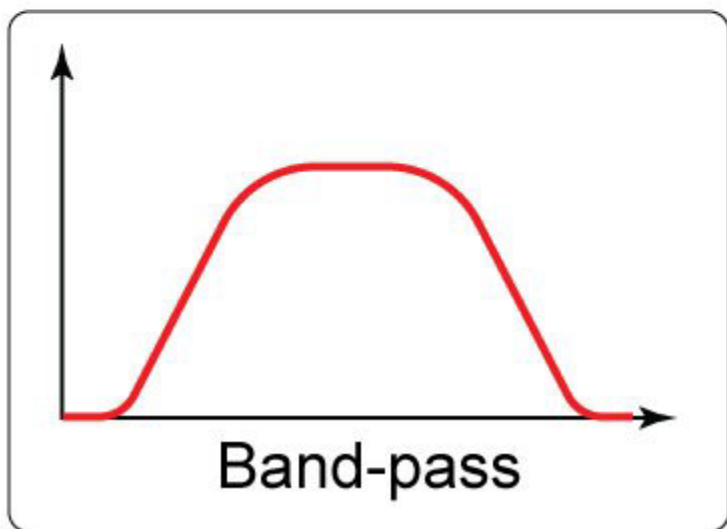
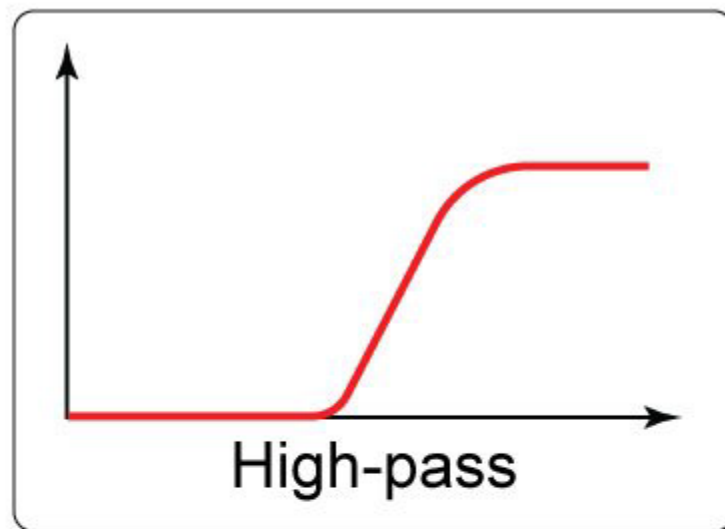
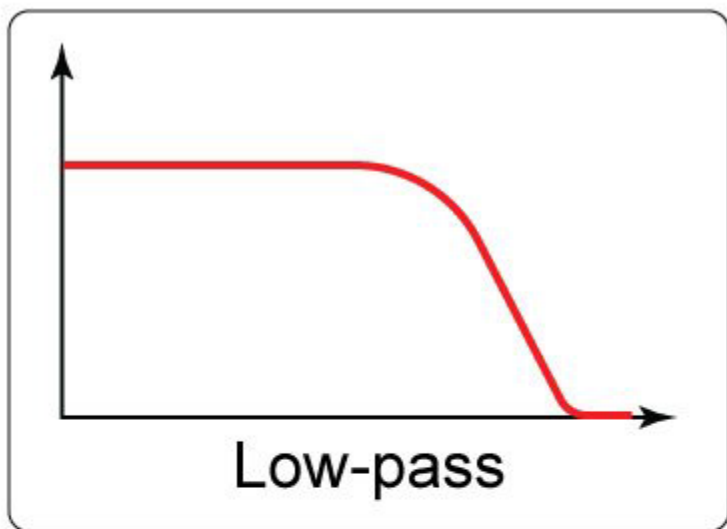
滤波器是对输入信号频率具有选择功能的电路，广泛应用于通信领域。

2. 分类



滤波器

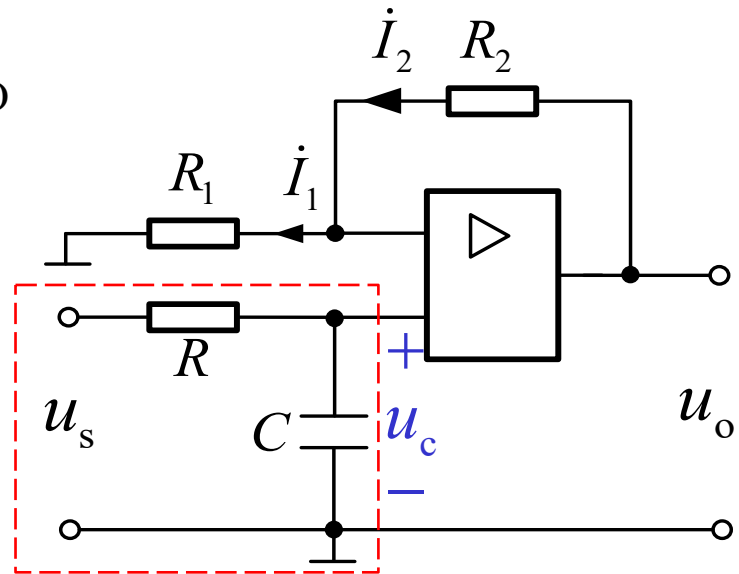
- **低通滤波器**：它允许信号中的低频或直流分量通过，抑制高频分量或干扰和噪声；
- **高通滤波器**：它允许信号中的高频分量通过，抑制低频或直流分量；
- **带通滤波器**：它允许一定频段的信号通过，抑制低于或高于该频段的信号、干扰和噪声；
- **带阻滤波器**：它抑制一定频段内的信号，允许该频段以外的信号通过，又称为陷波滤波器。
- **全通滤波器**：指在全频带范围内，信号的幅值不会改变，也就是全频带内幅值增益恒等于1。全通滤波器常用于移相。



例 求图示电路中输出电压 \dot{U}_O

解 红框内为低通滤波器

$$\dot{U}_C = \frac{1}{R + \frac{1}{j\omega C}} \dot{U}_s = \frac{1}{1 + j\omega RC} \dot{U}_s$$



一阶有源低通滤波器

应用运算放大器“虚短、虚断”特性：

$$\dot{I}_1 = \dot{U}_C / R_1 \quad \dot{I}_1 = \dot{I}_2$$

则输出电压：

$$\dot{U}_O = R_2 \dot{I}_2 + R_1 \dot{I}_1 = \frac{R_1 + R_2}{R_1} \dot{U}_C = \left(1 + \frac{R_2}{R_1} \right) \frac{1}{1 + j\omega RC} \dot{U}_s$$

作业

- 14.3节：14-9, 14-10
- 14.4节：14-14