第2章
电阻电路等效变换

1.等效变换的概念 Concept of Equivalence

2.串联与并联 Series and Parallel Connections

3.对称电路 Symmetric Circuits

4.电桥 Bridge Circuits

5.星－三角互换 Wye-Delta Transformation

6.电源变换 Source transformation

第2章

电阻电路等效变换

目标： 1.熟练应用支路电流分析法分析简单电路。

2.综合应用各种等效化简方法获得等效电路。

难点： 1.含受控源电路的等效化简。

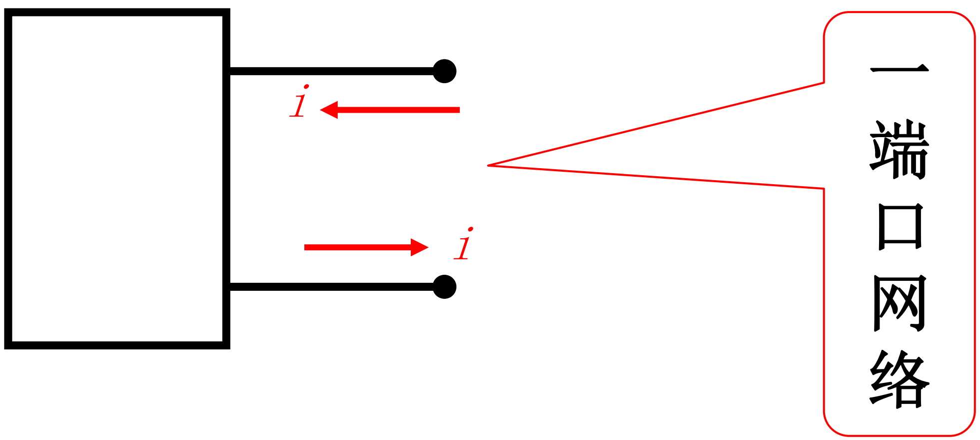
2.综合应用多种等效变换方法化简复杂电路。

讲授学时： 4　　**讨论学时：** 1

2.1等效变换的概念

二端子网络（一端口网络）

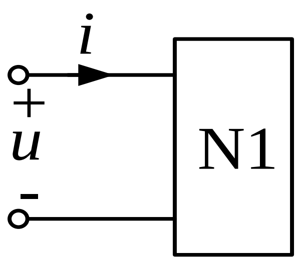
任何一个复杂的电路，向外引出两个端子，且从一个端子流入的电流等于从另一端子流出的电流，则称这一电路为二端子网络（或一端口网络）。



2.1等效变换的概念

1.等效的概念 Concept of Equivalence

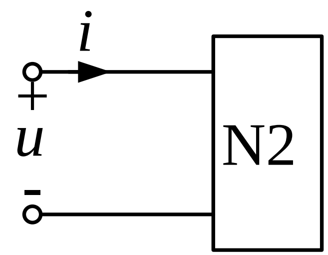
两个一端口网络，端口具有相同的电压、电流关系,则称它们是等效的电路。



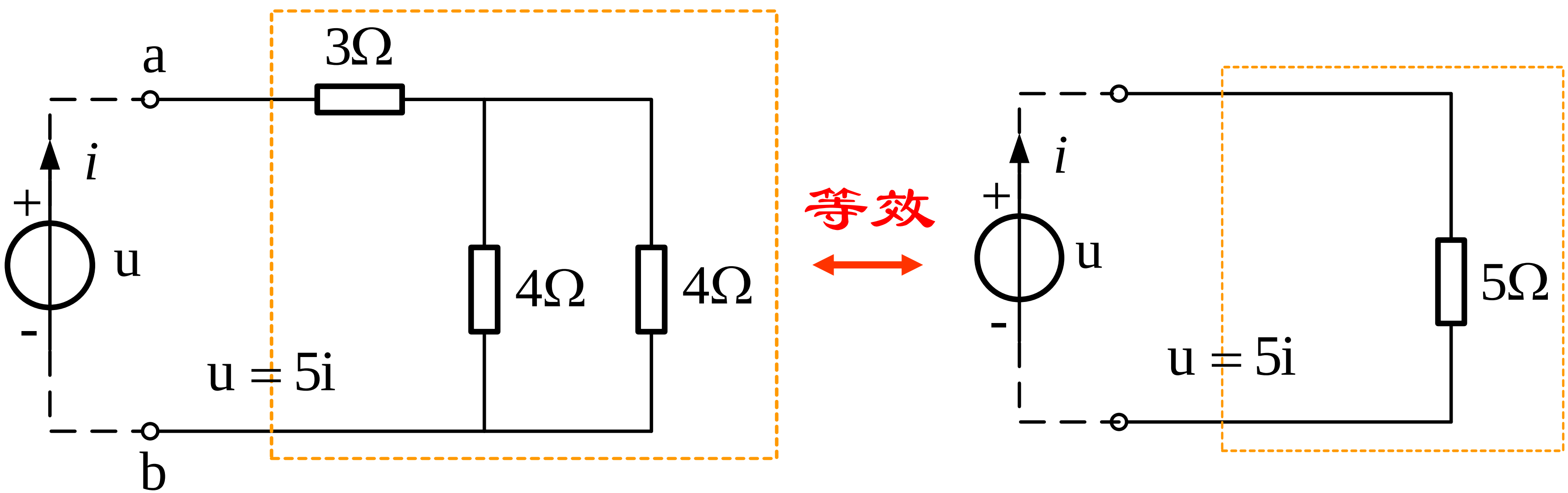
$f(u,i)=0$

Equivalent networks

u-i 关系相同



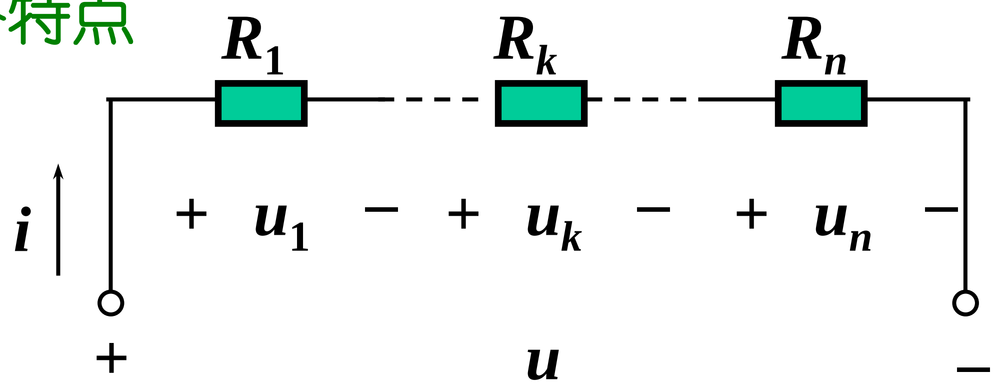
$f(u,i)=0$



2.2 串联与并联

1. 电阻串联（*series connection*）

（1）电路特点



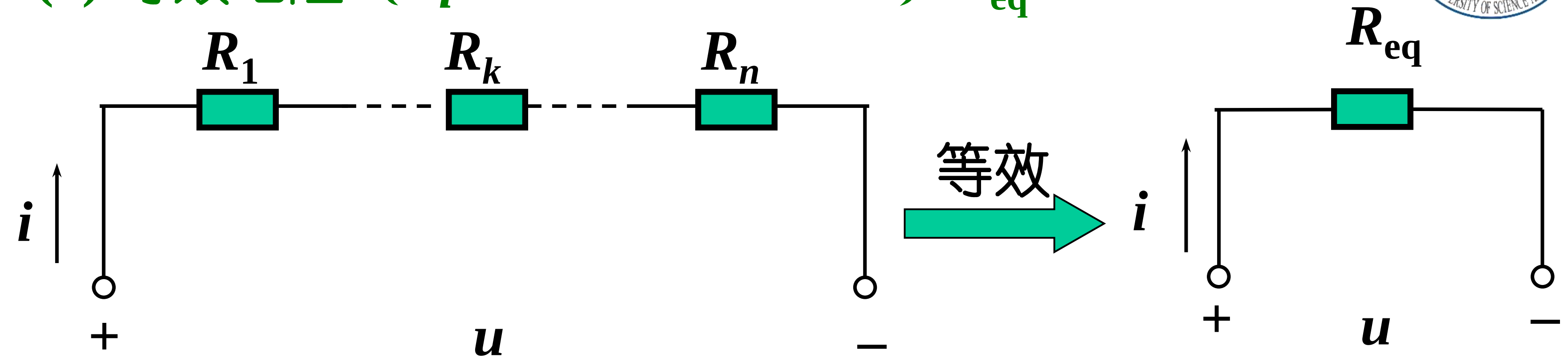
(a) 各电阻顺序连接，流过同一电流 （**KCL**）；

(b) 总电压等于各串联电阻上的电压之和 （**KVL**）：

$$u = u_1 + \cdots + u_k + \cdots + u_n$$

$$u = (R_1 + \cdots + R_k + \cdots + R_n)\ i$$

(2) 等效电阻（*equivalent resistance*）R_{eq}

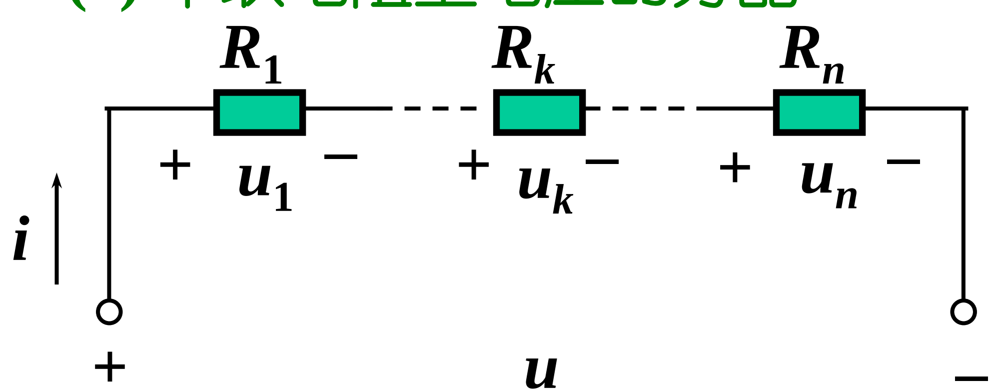


$$u = (R_1 + \cdots + R_k + \cdots + R_n)\ i \qquad u = R_{eq}\, i$$

$$R_{eq} = (R_1 + R_2 + \ldots + R_n) = \sum R_k$$

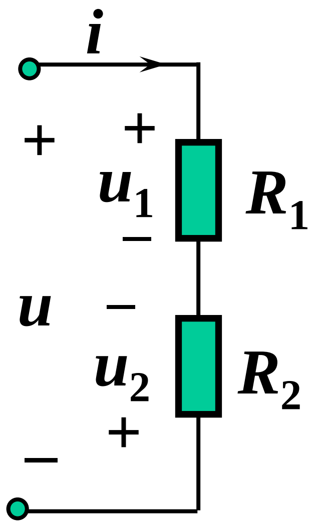
等效电阻等于串联的各电阻之和

(3) 串联电阻上电压的分配



$$u_k = \frac{R_k}{R_{eq}} u$$

例　两个电阻分压（*voltage division*），如下图所示。



$$u_1 = \frac{R_1}{R_1 + R_2} u$$

$$u_2 = -\frac{R_2}{R_1 + R_2} u$$

（注意方向！）

（4）功率关系　$p_1 = R_1 i^2$，$p_2 = R_2 i^2$，$\cdots$，$p_n = R_n i^2$

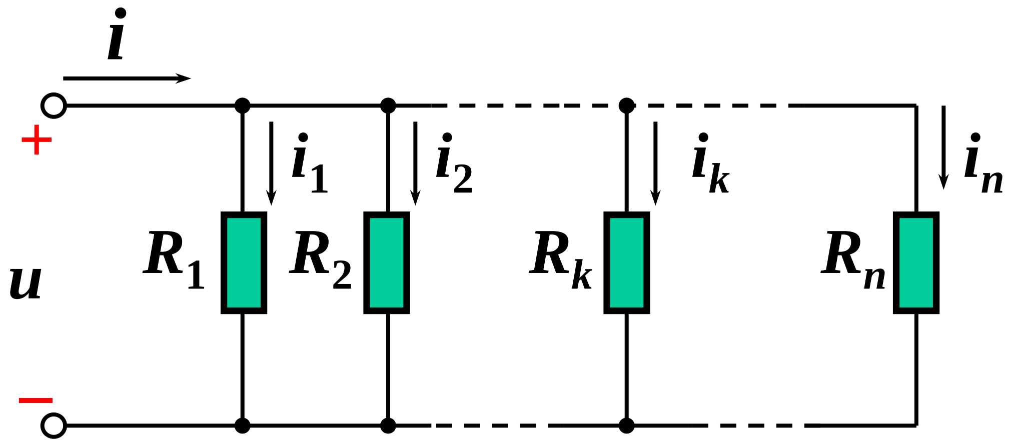
$$p_1 : p_2 : \cdots : p_n = R_1 : R_2 : \cdots : R_n$$

总功率　$p = R_{eq} i^2 = (R_1 + R_2 + \cdots + R_n)\, i^2$

$$= R_1 i^2 + R_2 i^2 + \cdots + R_n i^2$$

$$= p_1 + p_2 + \cdots + p_n$$

2. 电阻并联 （*parallel connection*）



(1) 电路特点

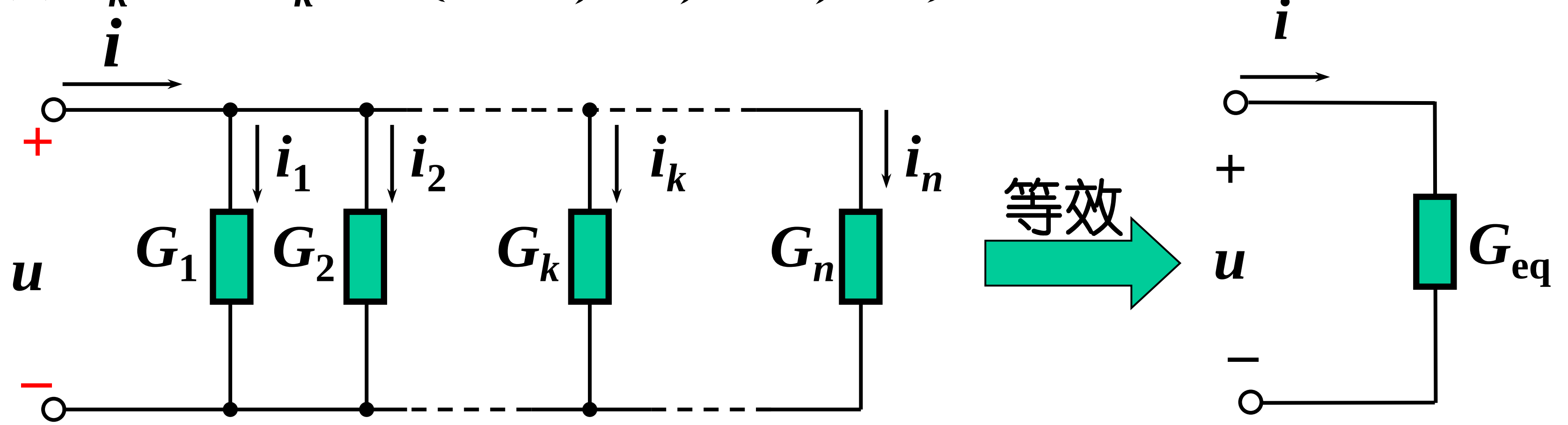
(a) 各电阻两端分别接在一起，端电压为同一电压 （**KVL**）；

(b) 总电流等于流过各并联电阻的电流之和 （**KCL**）：

$$i = i_1 + i_2 + \cdots + i_k + \cdots + i_n$$

（2） 等效电导（*equivalent conductance*）G_{eq}

设 $G_k = 1/R_k \qquad (k = 1,\ 2,\ \cdots,\ n)$



由**KCL** $\qquad i = i_1 + i_2 + \cdots + i_k + \cdots + i_n = u\,G_{eq}$

故有 $uG_{eq} = i = uG_1 + uG_2 + \cdots + uG_n = u\,(G_1 + G_2 + \cdots + G_n)$

即 $\quad G_{eq} = G_1 + G_2 + \cdots + G_k + \cdots + G_n = \sum G_k = \sum 1/R_k$

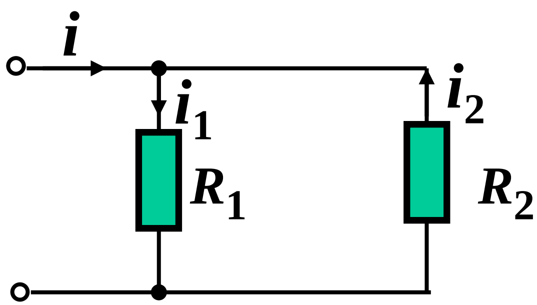
等效电导等于并联的各电导之和。

(3) 并联电阻的分流（*current division*）

由 $$\frac{i_k}{i}=\frac{u/R_k}{u/R_{\mathrm{eq}}}=\frac{G_k}{G_{\mathrm{eq}}}$$ 电流分配与电导成正比

得 $$\boxed{i_k=\frac{G_k}{G_{\mathrm{eq}}}i}$$

对于两电阻并联， 有



$$\boxed{i_1=\frac{1/R_1}{1/R_1+1/R_2}i=\frac{R_2}{R_1+R_2}i}$$

$$\boxed{i_2=\frac{-1/R_2}{1/R_1+1/R_2}i=-\frac{R_1}{R_1+R_2}i}$$

（4） 功率关系

$$p_1=G_1u^2,\quad p_2=G_2u^2,\quad \cdots,\quad p_n=G_nu^2$$

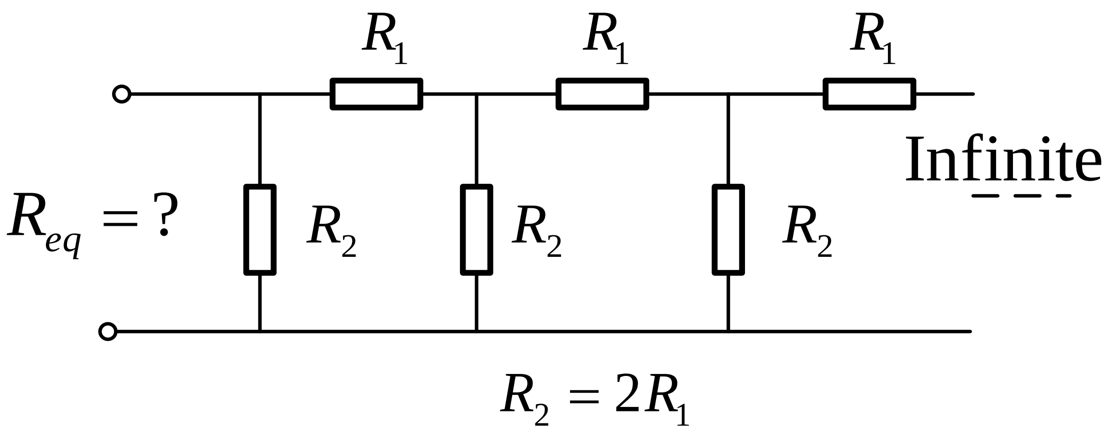
$$p_1:p_2:\cdots:p_n=G_1:G_2:\cdots:G_n$$

总功率 $p=G_{eq}u^2=(G_1+G_2+\cdots+G_n)\,u^2$

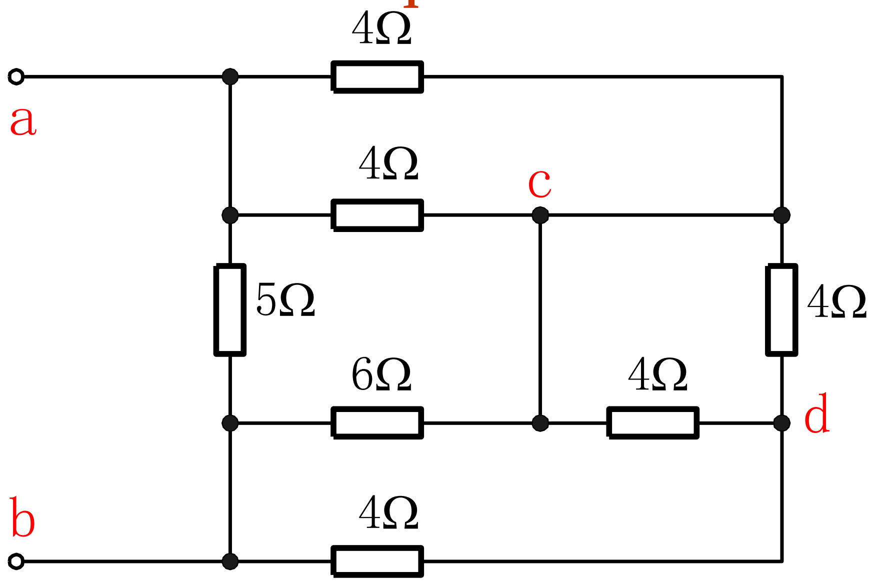
$$=G_1u^2+G_2u^2+\cdots+G_nu^2$$

$$=p_1+p_2+\cdots+p_n$$

例1: Find the equivalent resistance for the circuit.

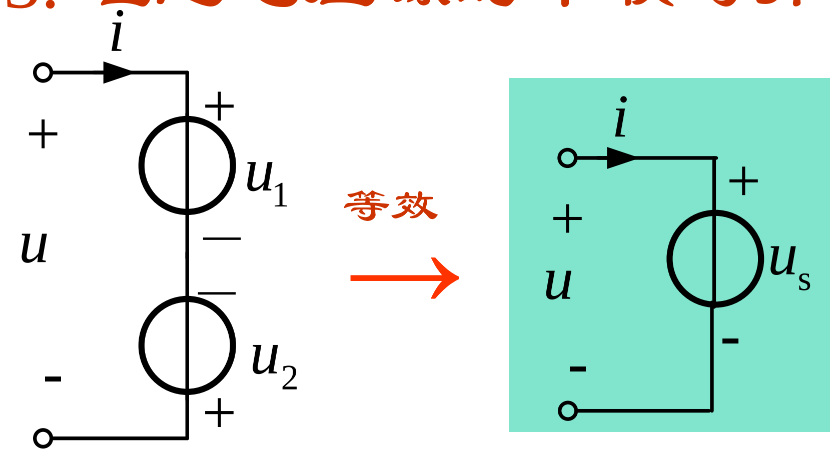


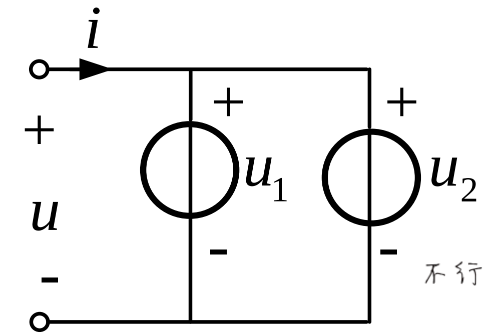
例2: Find the equivalent resistance for the circuit.



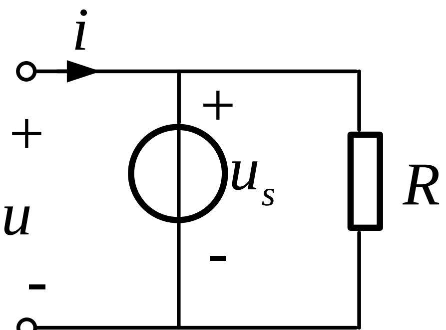
$$R_{ab} = 2.5\Omega$$

3. 理想电压源的串联与并联

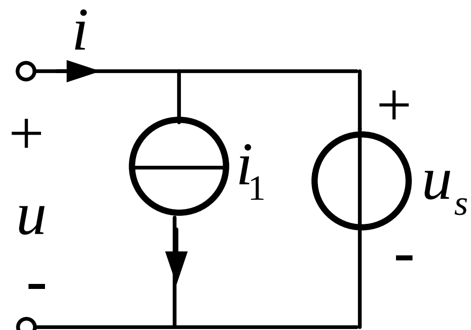




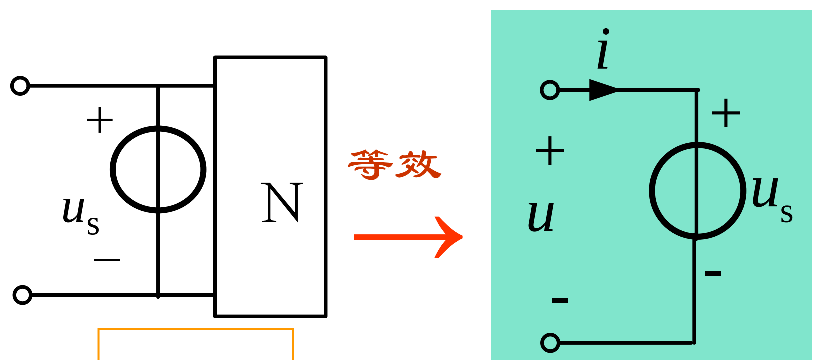
$u_s = u_1 - u_2$　　电压源串联等效为电压源。



$u = u_s$



$u = u_s$

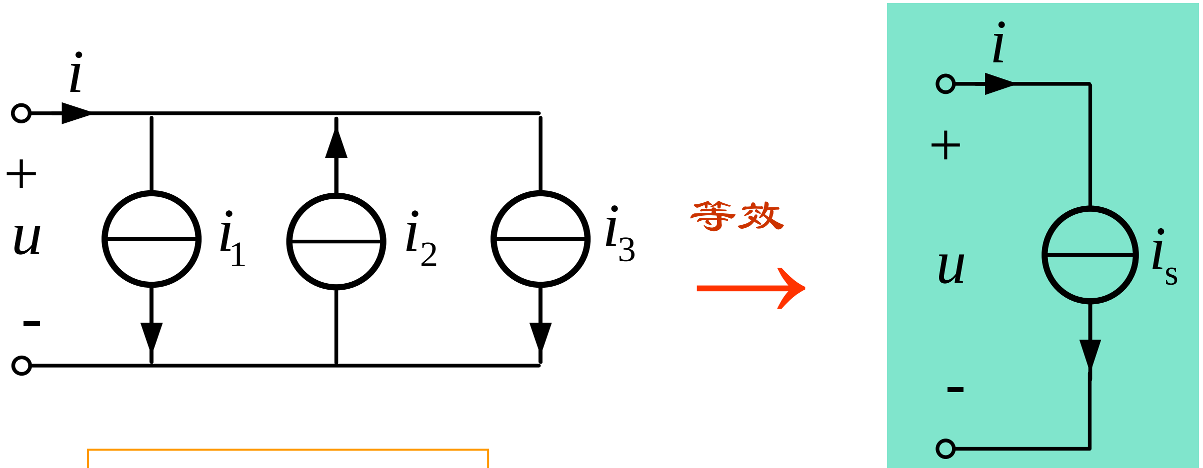


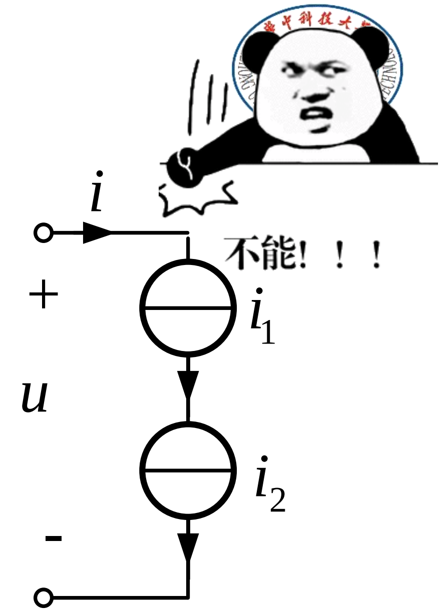
$u = u_s$

电压源并联任何元件、或并联一端口网络，均等效为电压源。

注意：对外等效（对内不等效）

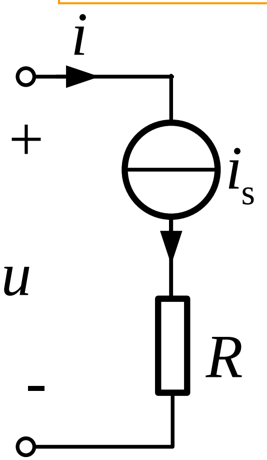
3. 理想电流源的串联与并联



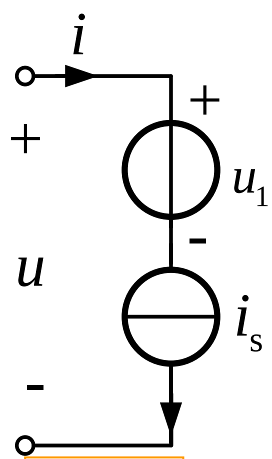


$$i_s = i_1 - i_2 + i_3$$

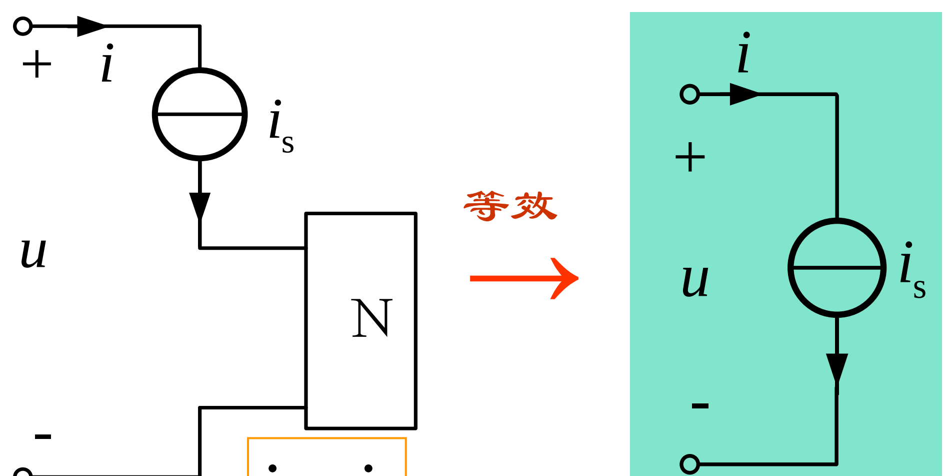
电流源并联等效为电流源。



$$i = i_s$$



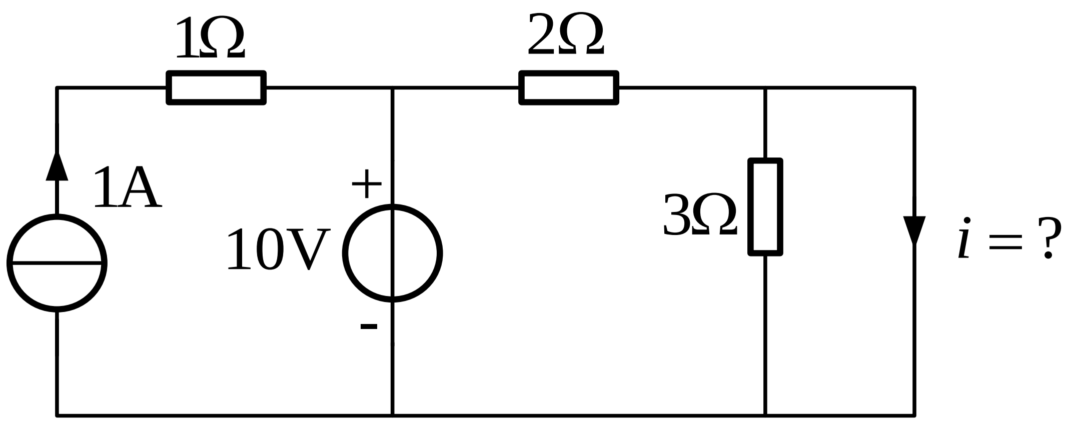
$$i = i_s$$



$$i = i_s$$

电流源串联任何元件、或串联一端口网络，均等效为电流源。

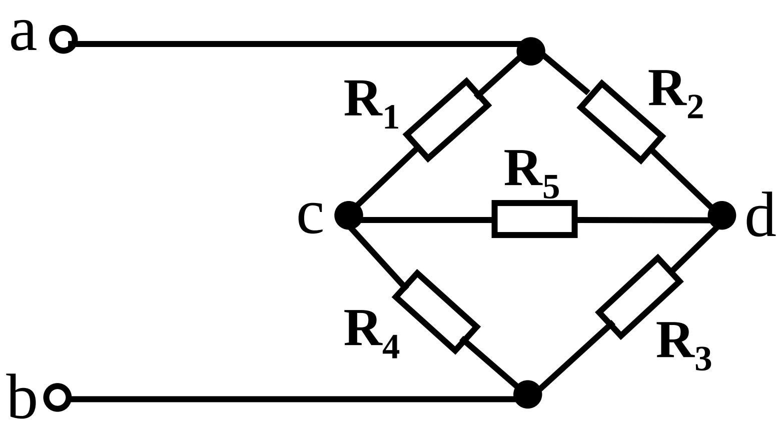
同样注意：只对外等效



i=5A

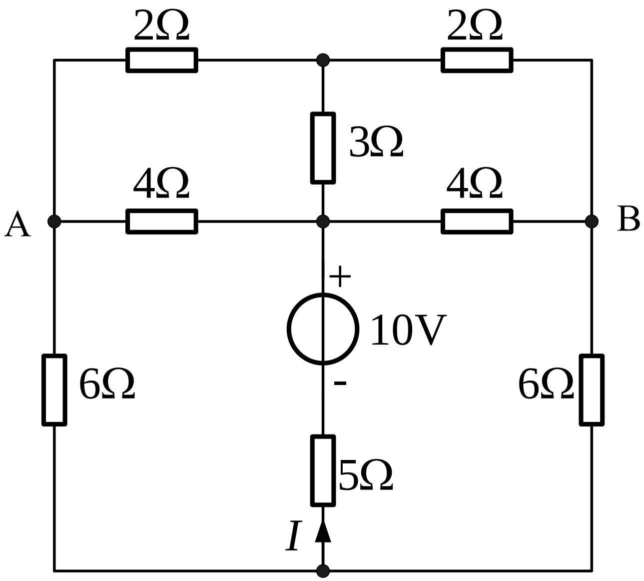
Find the current i.

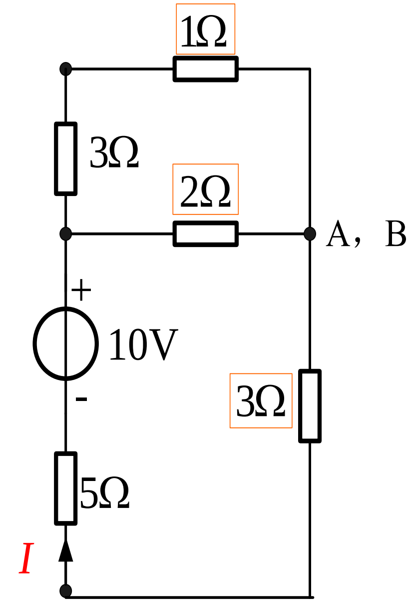
2.3 对称电路

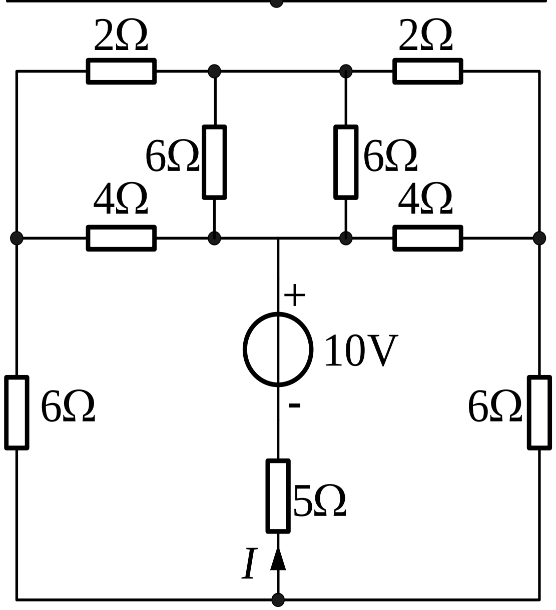


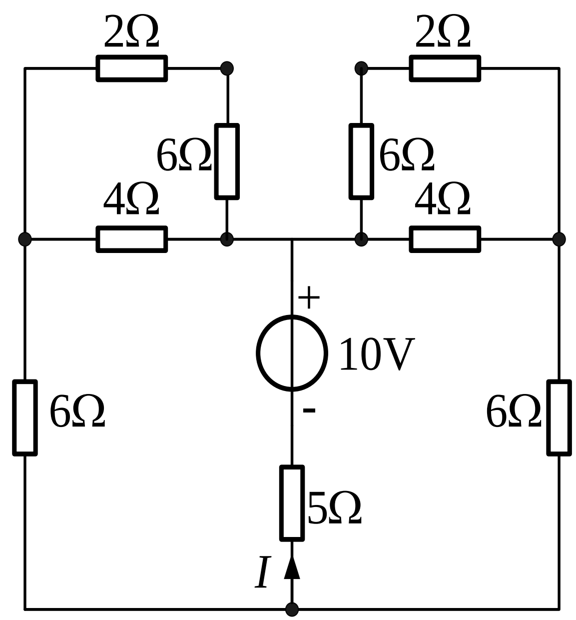
- 求解对称电路的核心，是观察电路的对称关系，寻找对称面和等电位点。
- 如果对称面通过端口，垂直对称面的支路的电流为0，可断开该支路，对称节点为等位点，短接等电位点；
- 如果对称面垂直端口，对称面上的结点为等位点，可短接这些结点。

例4. Find the current *I* in the circuit.



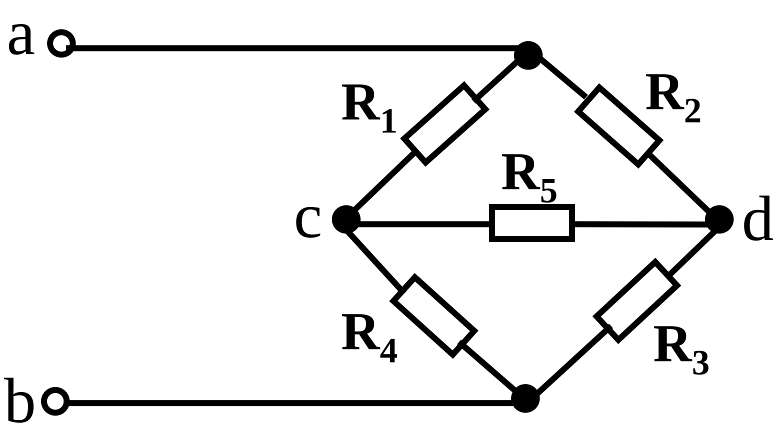






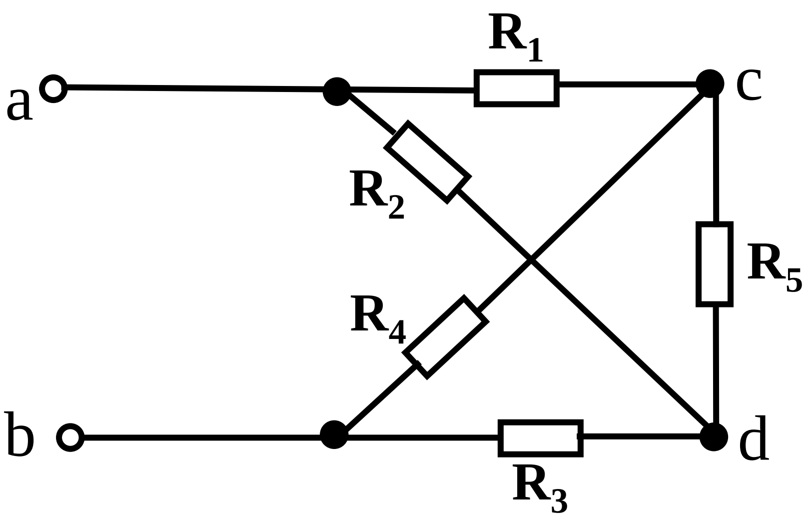
2.4 电桥

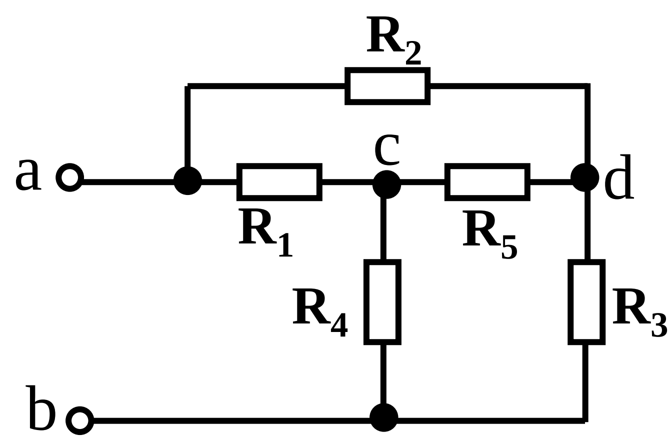
电桥Bridge Circuits



（1）结构特点：

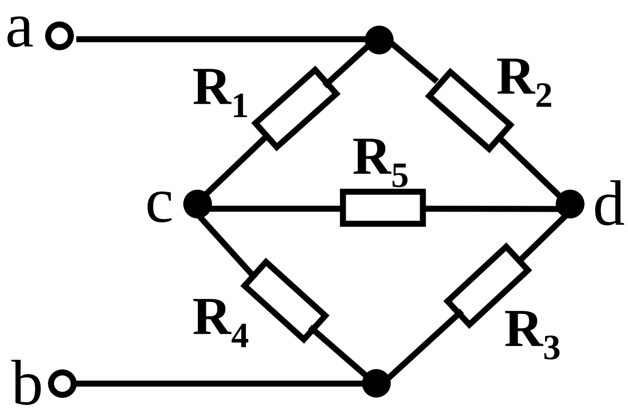
各电阻为非串并联, R_5为桥支路,其余4条为桥臂





2.4 电桥

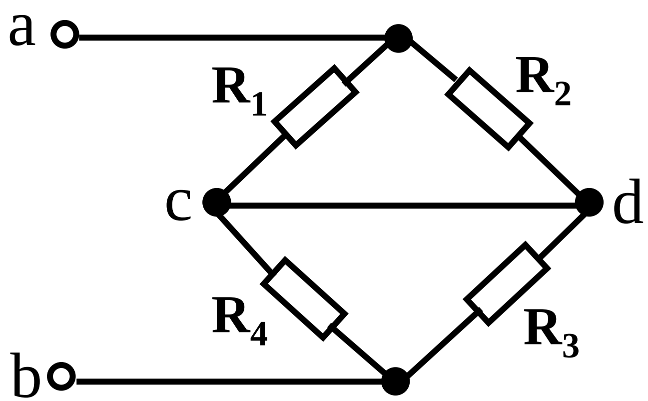
平衡电桥Balanced Bridge

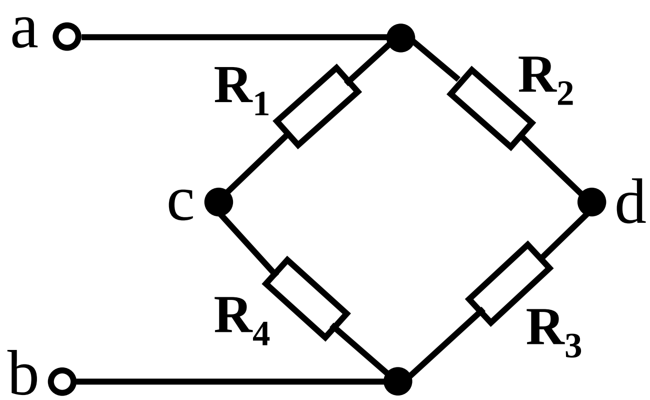


1、平衡条件

$R_1R_3=R_2R_4 \rightarrow u_{cd}=0$，$i_{cd}=u_{cd}/R_5=0$

2、端口的等效电阻





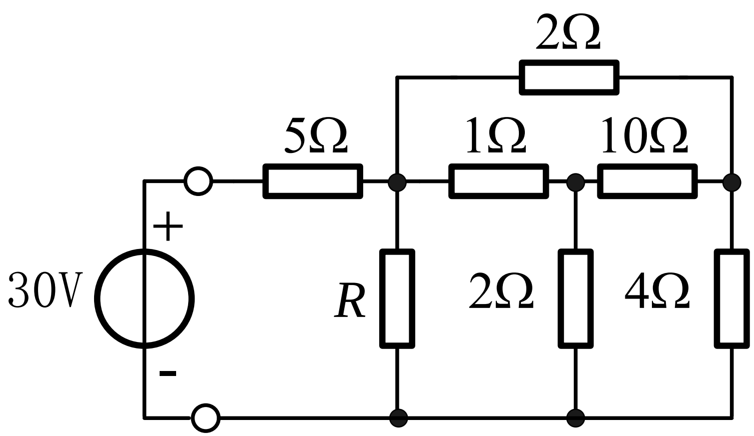
3、两种简单的等效变换

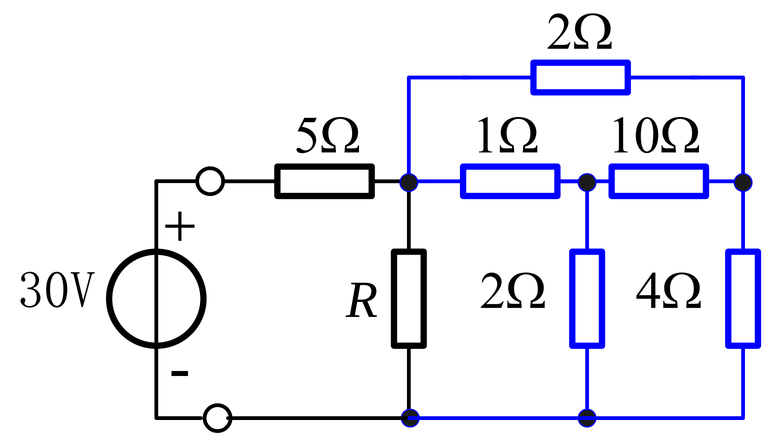
（1）电路中电位相等的点 ⟶ 短接

（2）电路中电流为零的支路 ⟶ 断开

2.4 电桥

例2. The voltage source supplies 150W. Find the resistance R.



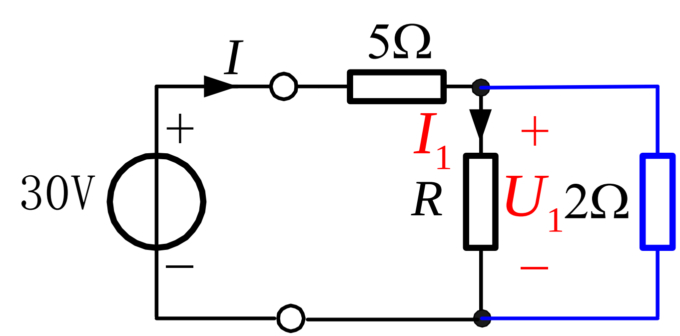


$$I = 5\text{A}$$

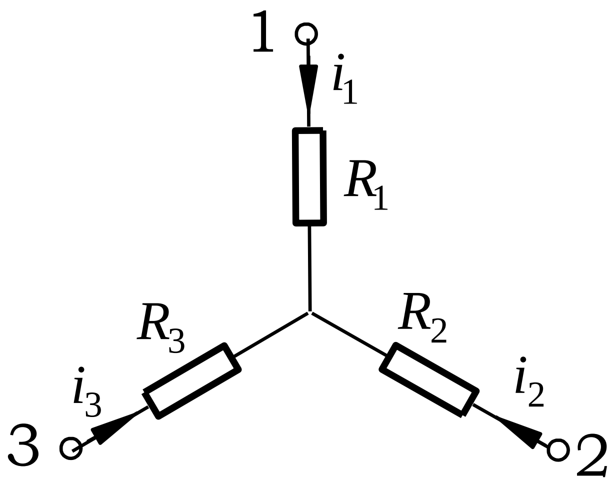
$$U_1 = 5\text{V}$$

$$I_1 = 2.5\text{A}$$

$$R = 2\Omega$$

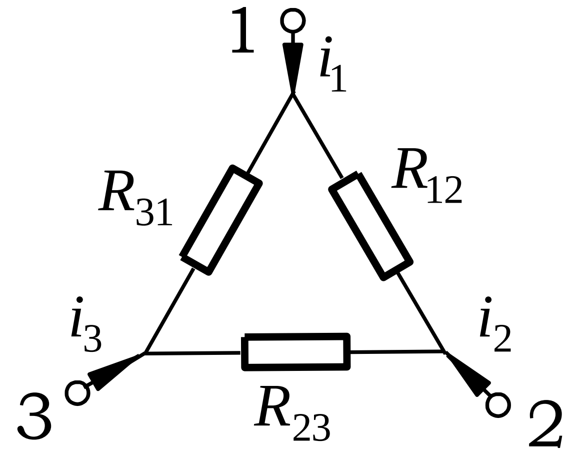


2.5 星形与三角形电路



$$u_{ij} = f_k(i_1, i_2, i_3)$$

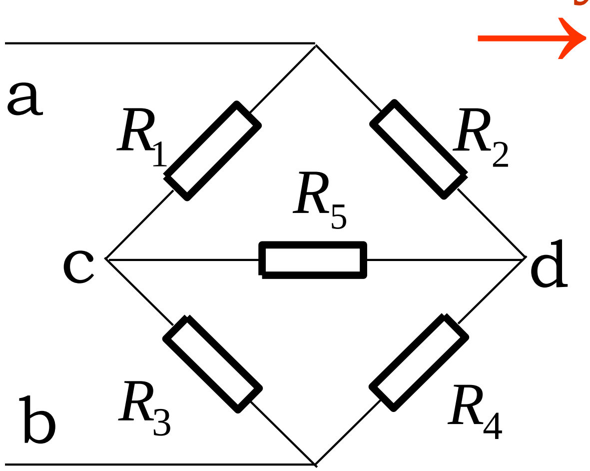
u-*i* 关系相同

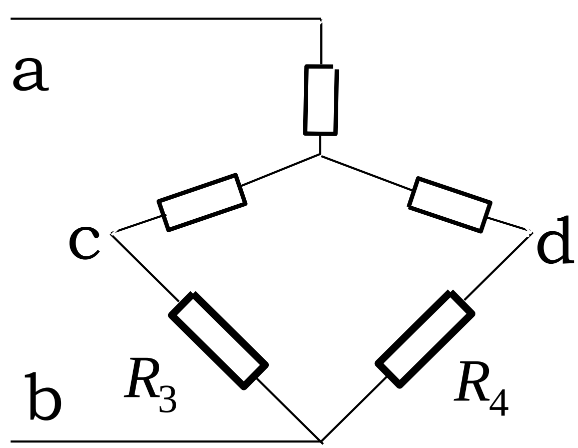


星形电路

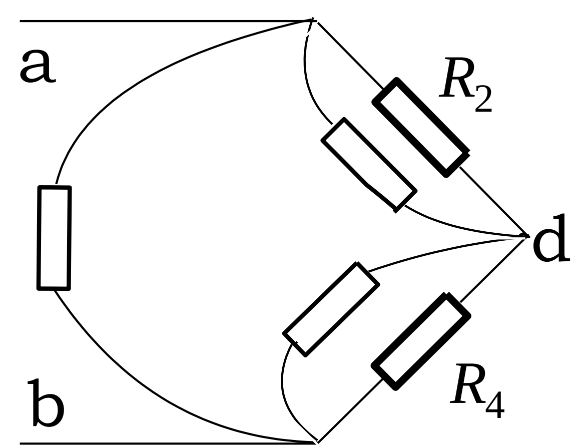
三角形电路

星－三角变换 Wye-Delta Transformation





三角形→星形



星形→三角形

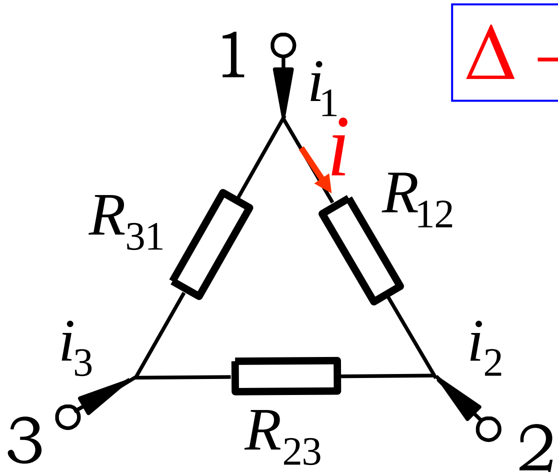
2.5 星形与三角形电路

星－三角变换 Wye-Delta Transformation

KVL、KCL确定i： $R_{12}i + R_{23}(i + i_2) - R_{31}(-i + i_1) = 0$

$$i = \frac{R_{31}i_1 - R_{23}i_2}{R_{12} + R_{23} + R_{31}}$$

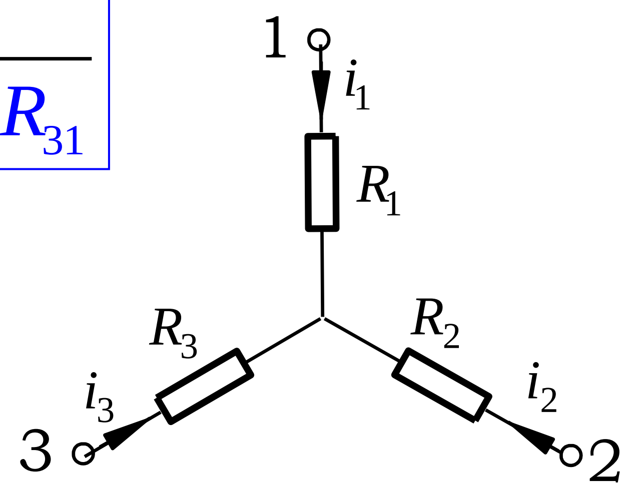
$$u_{12} = R_1 i_1 - R_2 i_2 \qquad = R_{12}i = \frac{R_{12}R_{31}i_1 - R_{12}R_{23}i_2}{R_{12} + R_{23} + R_{31}}$$



$\Delta \rightarrow Y$ $\boxed{R_1 = \frac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}}}$

$$u_{ij} = f_k(i_1, i_2, i_3)$$

u-i 关系相同



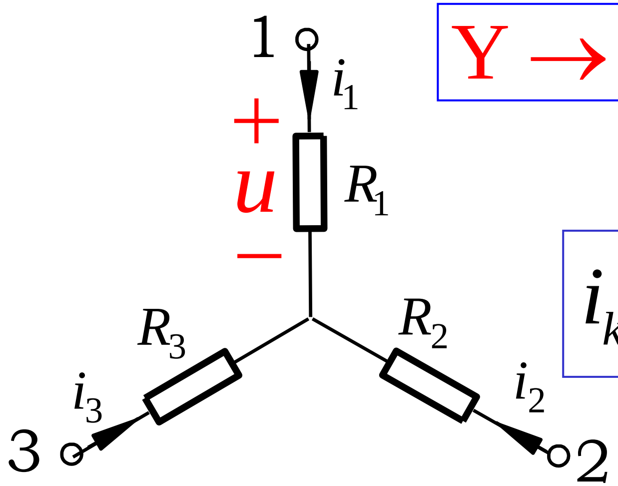
2.5 星形与三角形电路

星－三角变换 Wye-Delta Transformation

KVL、KCL确定u： $G_1 u + G_2(u - u_{12}) + G_3(u + u_{31}) = 0$

$$u = \frac{G_2 u_{12} - G_3 u_{31}}{G_1 + G_2 + G_3}$$

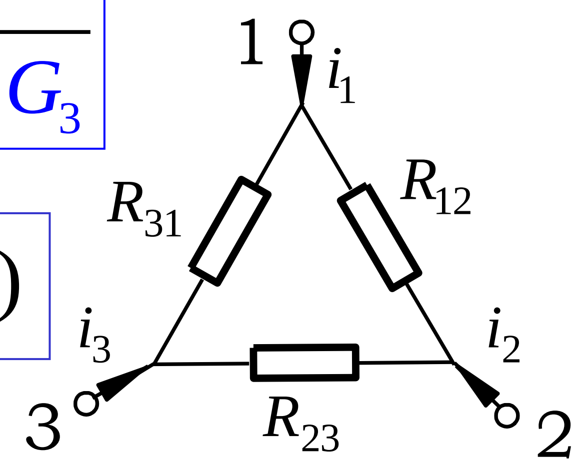
$$i_1 = G_1 u = \frac{G_1 G_2 u_{12} - G_1 G_3 u_{31}}{G_1 + G_2 + G_3} = G_{12} u_{12} - G_{31} u_{31}$$



$Y \rightarrow \Delta$ $\boxed{G_{12} = \dfrac{G_1 G_2}{G_1 + G_2 + G_3}}$

$$\boxed{i_k = g_k(u_{12}, u_{23}, u_{31})}$$

u-i 关系相同



2.5 星形与三角形电路

星－三角变换 Wye-Delta Transformation

$\Delta \rightarrow Y$

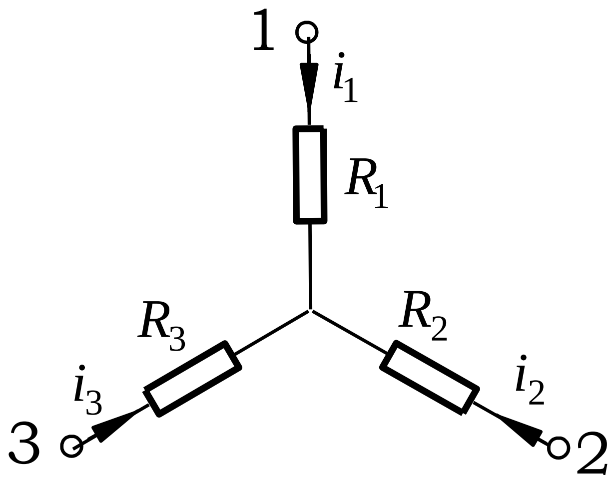
$$R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}$$

$Y \rightarrow \Delta$

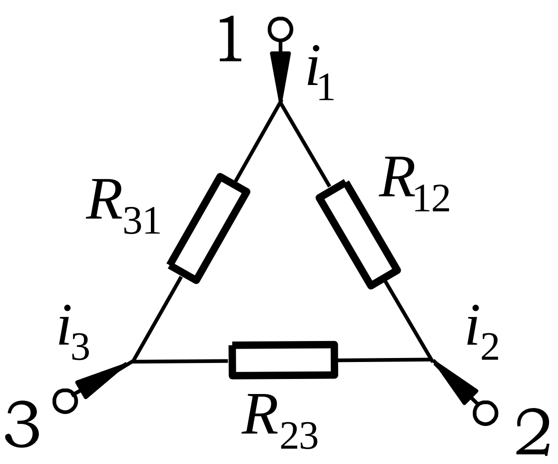
$$G_{12} = \frac{G_1 G_2}{G_1+G_2+G_3}$$

$$R_{12} = \frac{R_1R_2+R_2R_3+R_3R_1}{R_3}$$

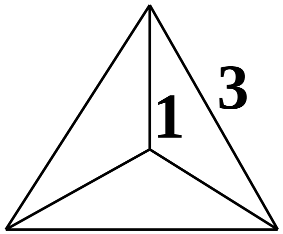
规律： 一个臂的电阻（导）＝ $\dfrac{\text{夹臂电阻（导）之积}}{\text{三个臂的电阻（导）之和}}$



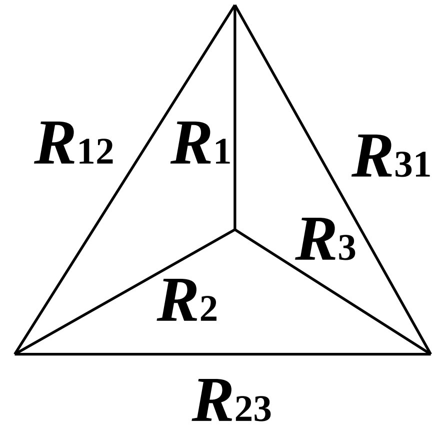
u-i 关系相同



特例　若三个电阻相等(对称)，则有



$$R_{\Delta} = 3R_{Y}$$

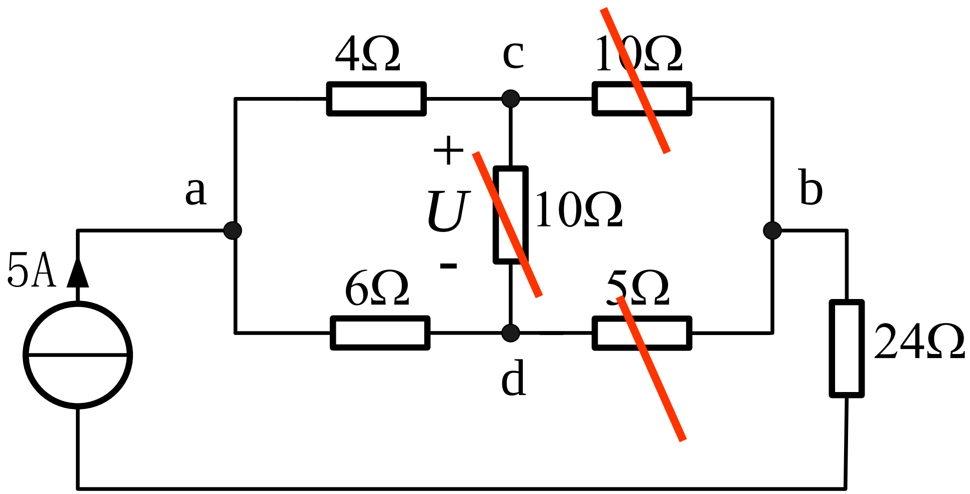


注意

(1) 等效是指对外部（端子以外）电路而言，对内不成立；

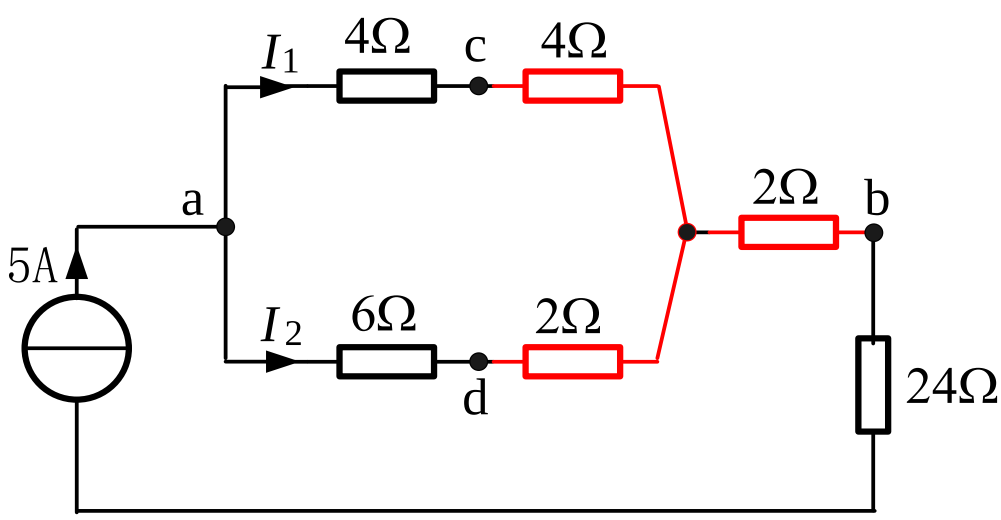
(2) 等效电路与外部电路无关。

例5. Find the voltage U in the circuit.



$$I_1 = I_2 = \frac{5}{2}\text{A}$$

(Current division)

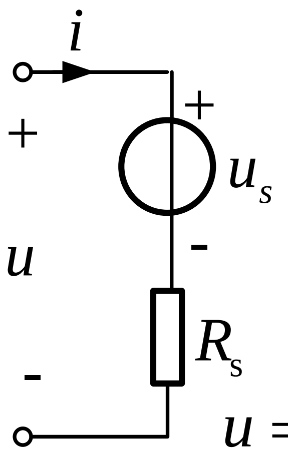


$$U = 4I_1 - 2I_2 = 5\text{V}$$

（KVL）

2.6电源变换Source transformation

独立电源变换

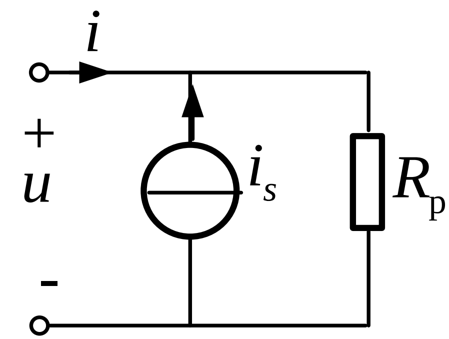


$$u = u_s + R_s i$$

$$R_s = R_p = R$$

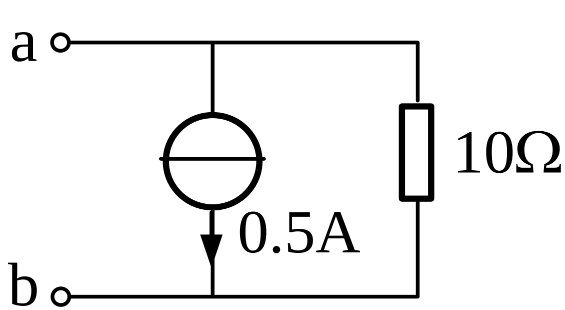
$$u_s = R i_s$$

等效

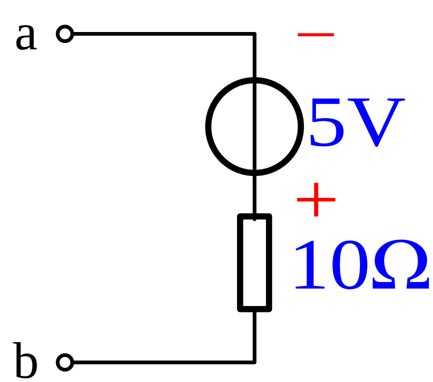


$$u = R_p i_s + R_p i$$

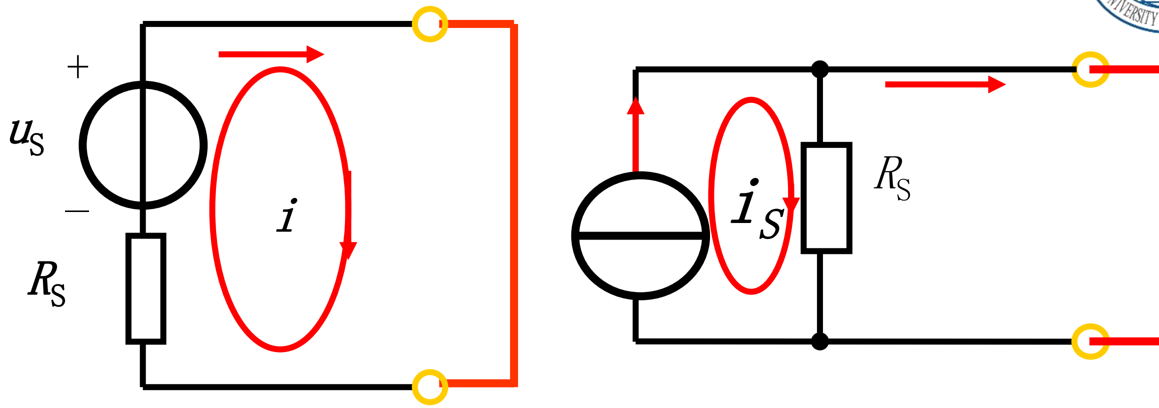
一个电压为u_s的独立电压源与一个电阻为R电阻器串联而成的支路，可用一个独立电流源与一个电阻器并联而成的支路等效。



→



注意电压源与电流源间的参考方向



① **变换关系**

数值关系

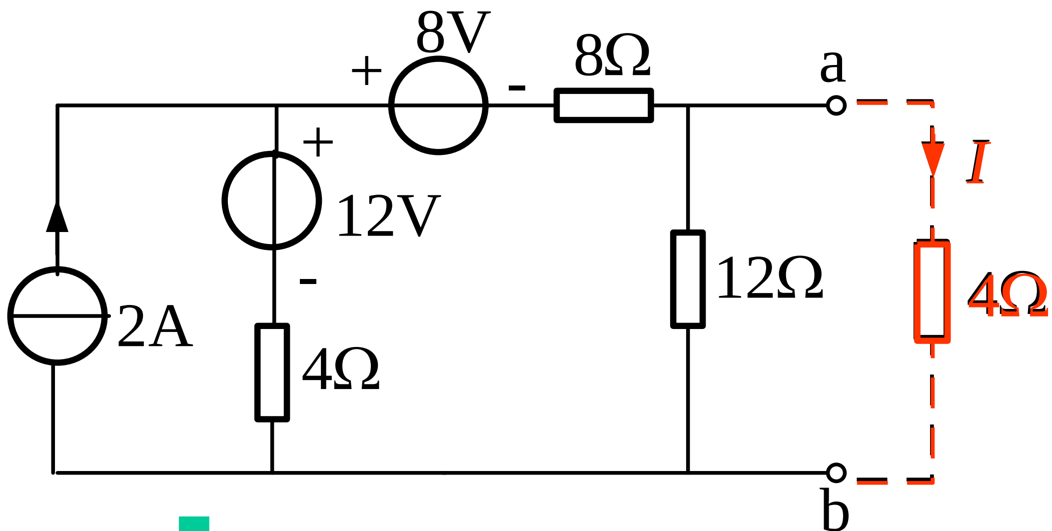
方向：变换前后电源电路有相同的向外提供电压/电流的“趋势”。

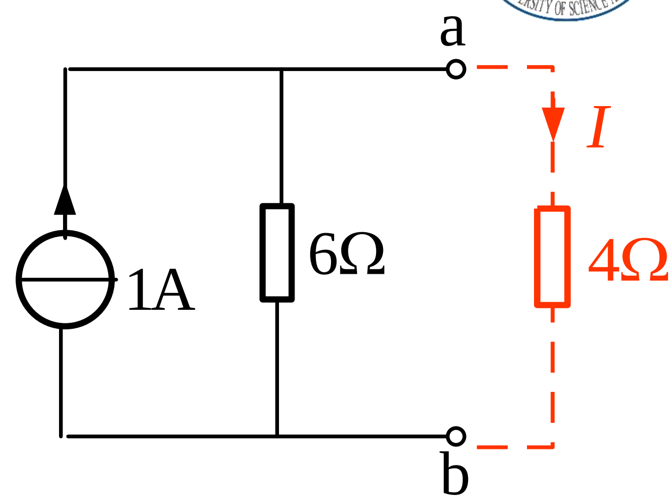
② **等效是对外部电路等效，对内部电路是不等效的。**

③ **理想电压源与理想电流源不能相互转换。**

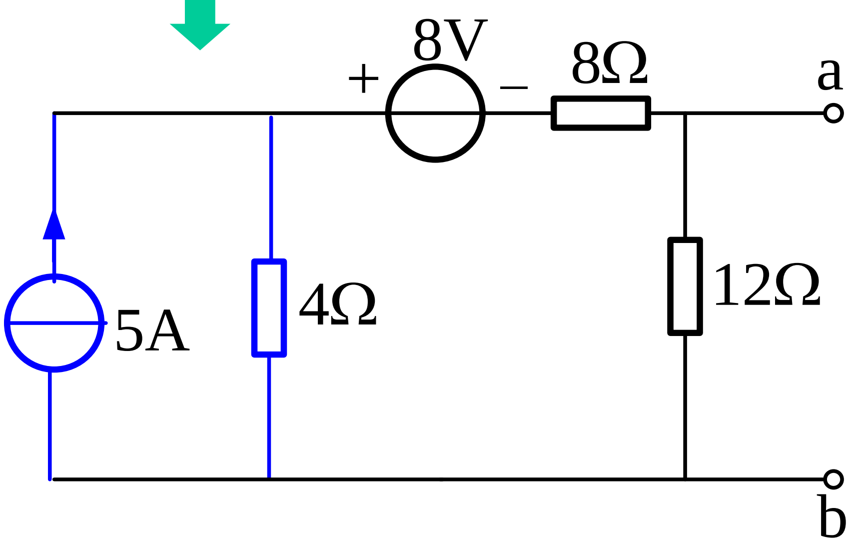
2.6电源变换Source transformation

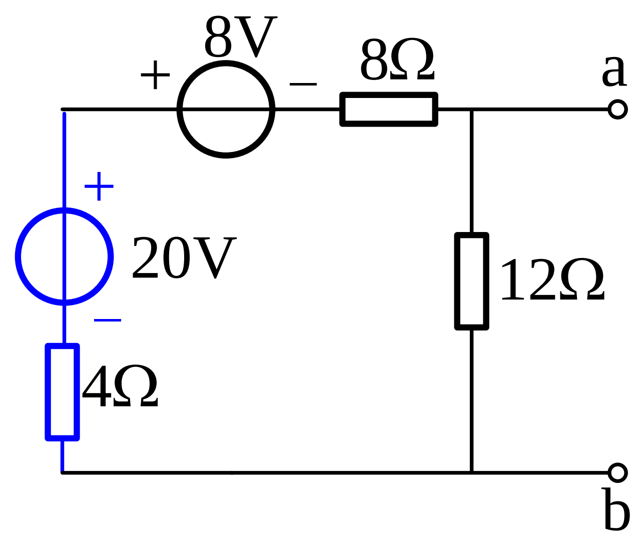
例6.Find the current *I*.





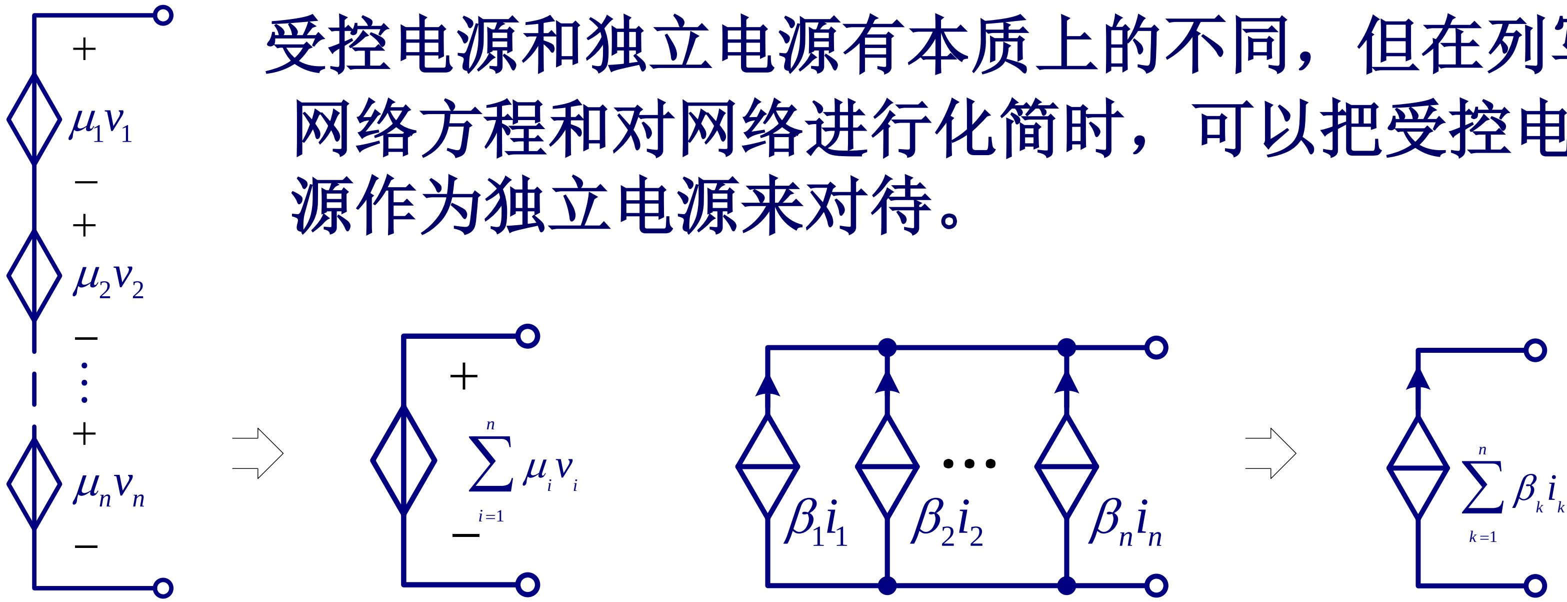
I=0.6A

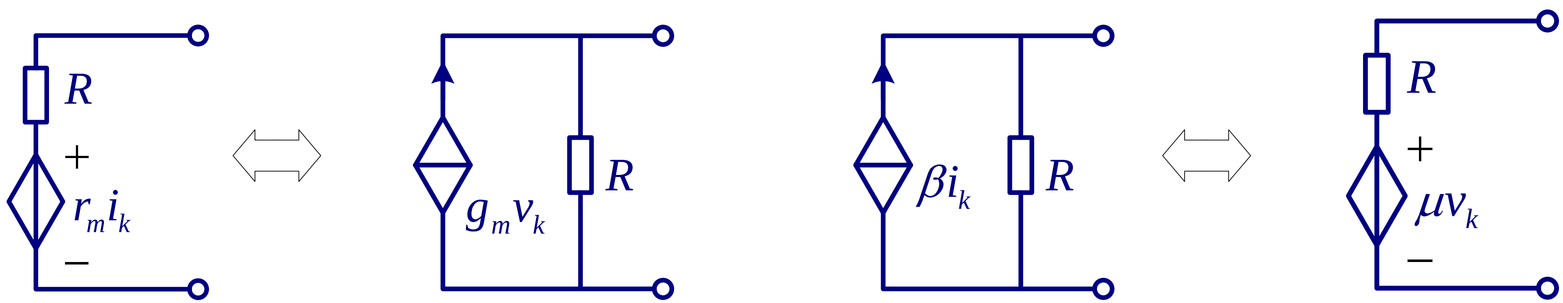




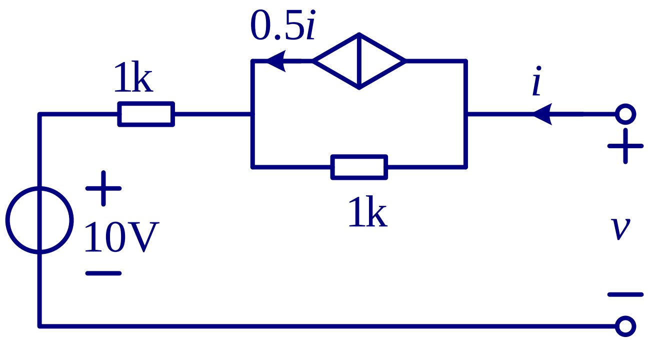
受控电源变换

受控电源和独立电源有本质上的不同，但在列写网络方程和对网络进行化简时，可以把受控电源作为独立电源来对待。

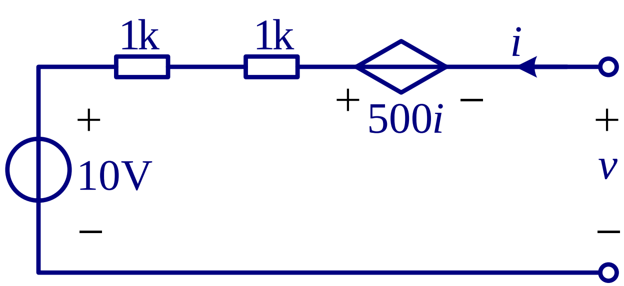


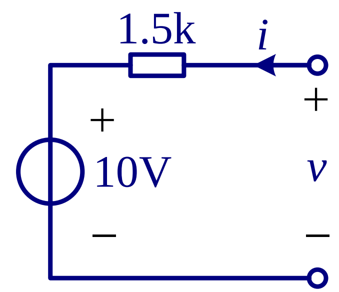


例 将网络化简成最简网络



将图中受控电流源当独立电源处理，转换成电压源形式

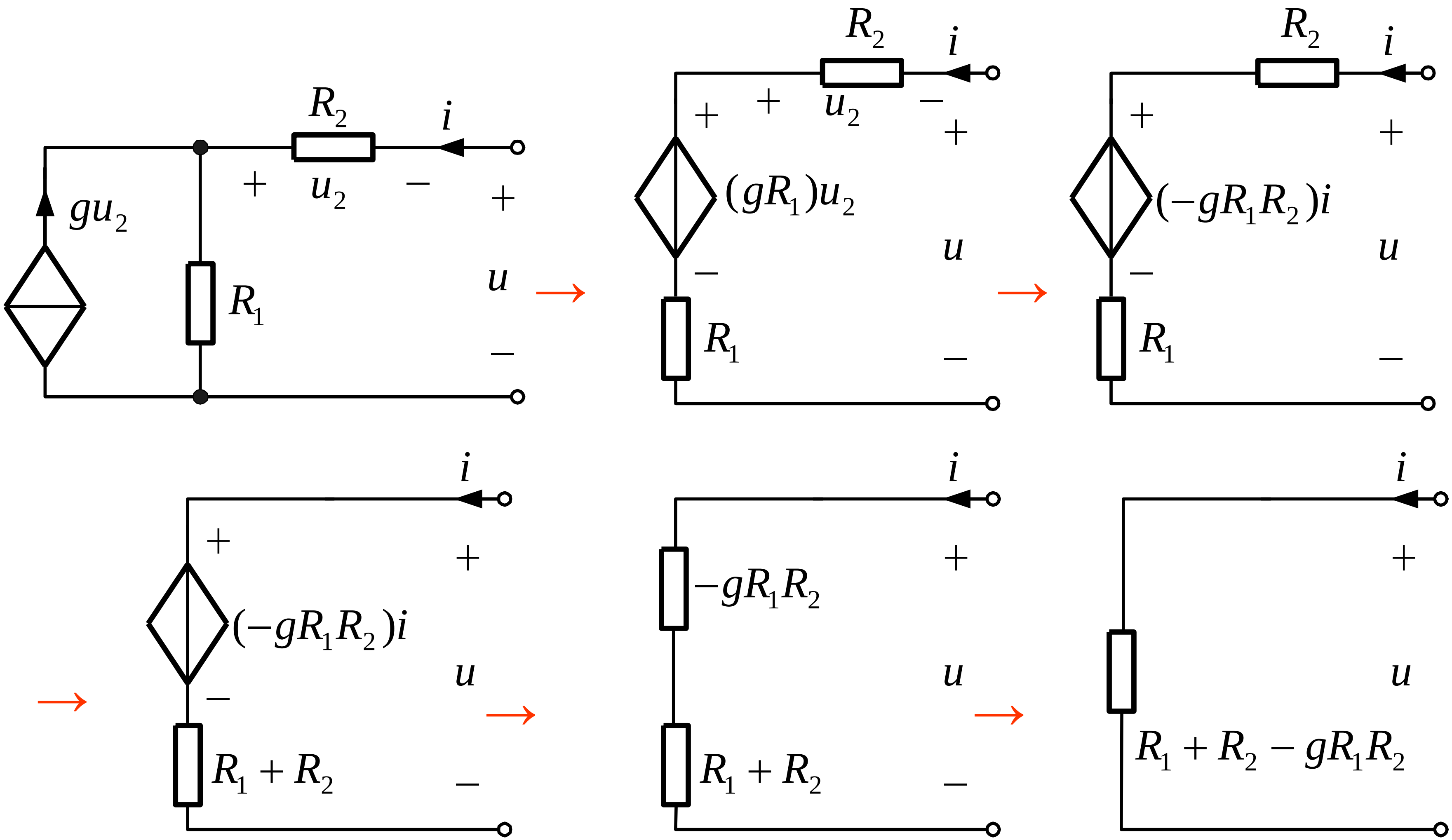




- 在此例中，受控源好比一个负500Ω的电阻。
- 当受控电源和控制量在同一条戴维南或者诺顿支路上时，受控电源可以变换为线性电阻。

2.6电源变换Source transformation

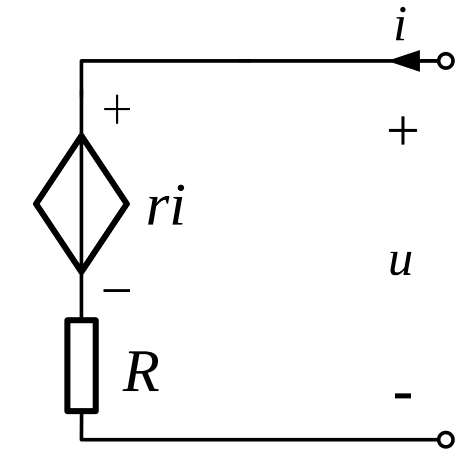
受控电源变换



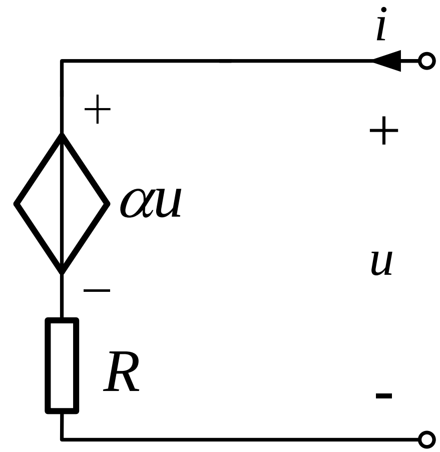
Method2: $u-i$ 关系: $u=R_2 i+R_1(gu_2+i)=(R_1+R_2-gR_1R_2)i$

2.6电源变换Source transformation

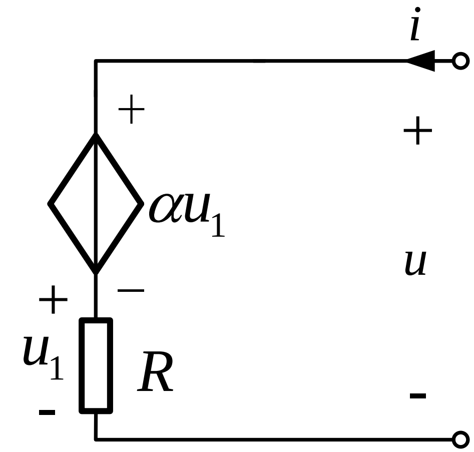
例7.Find the equivalent resistance of each circuit.



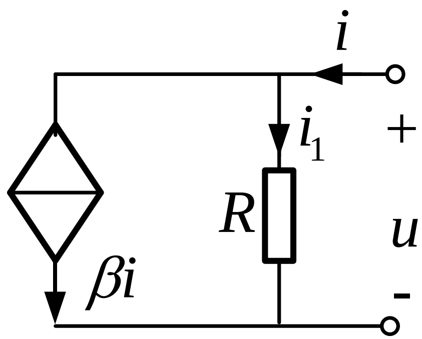
$$R_{\text{eq}} = R + r$$



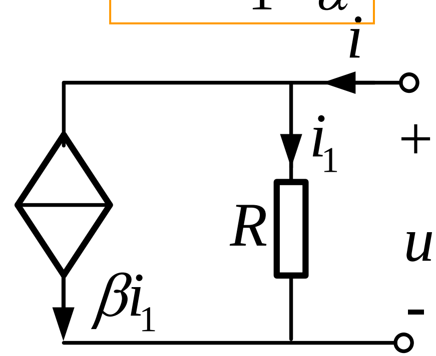
$$R_{\text{eq}} = \frac{R}{1-\alpha}$$



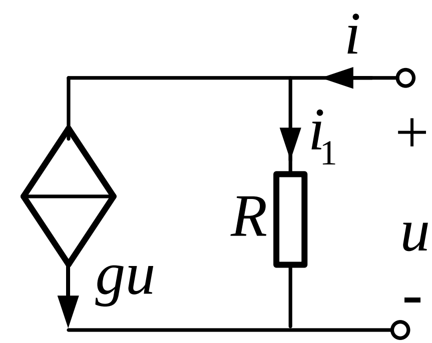
$$R_{\text{eq}} = (1+\alpha)R$$



$$R_{\text{eq}} = (1-\beta)R$$

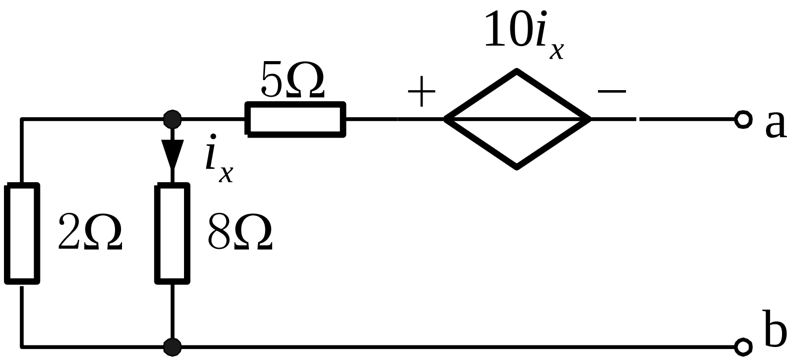


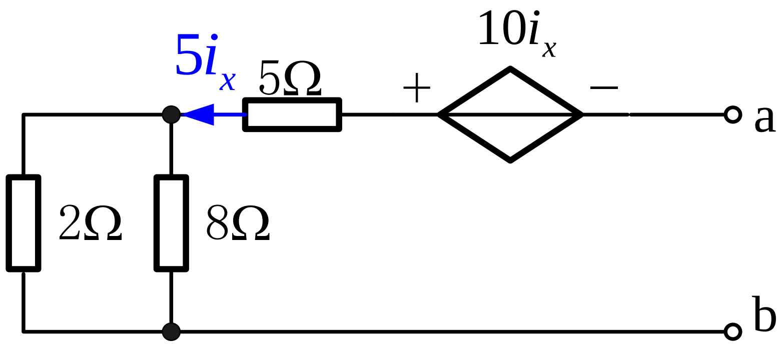
$$R_{\text{eq}} = \frac{R}{1+\beta}$$

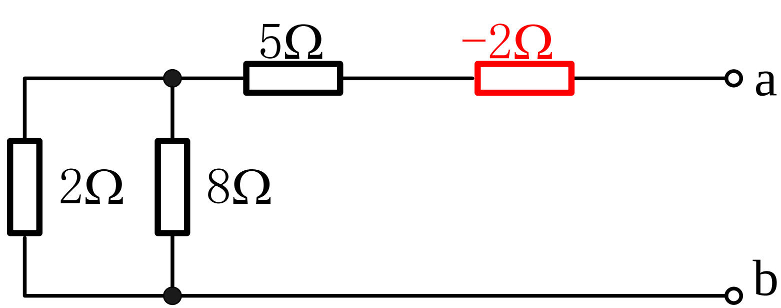


$$R_{\text{eq}} = \frac{R}{1+gR}$$

例8.Find the equivalent circuit.



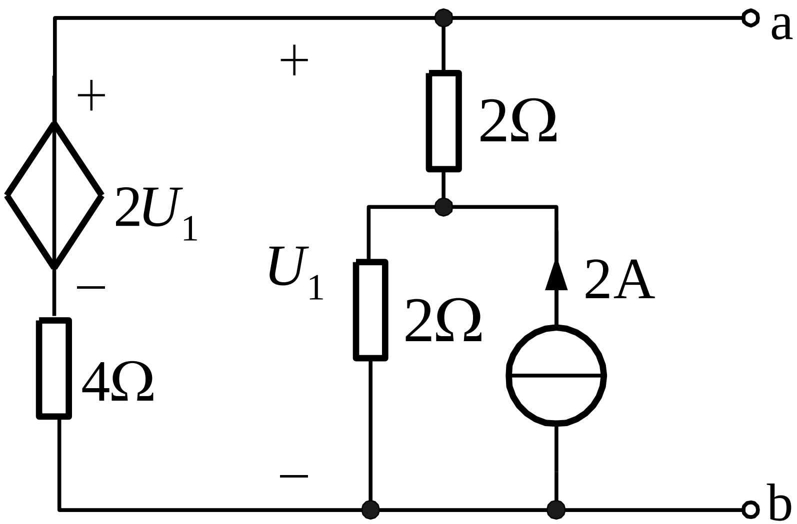


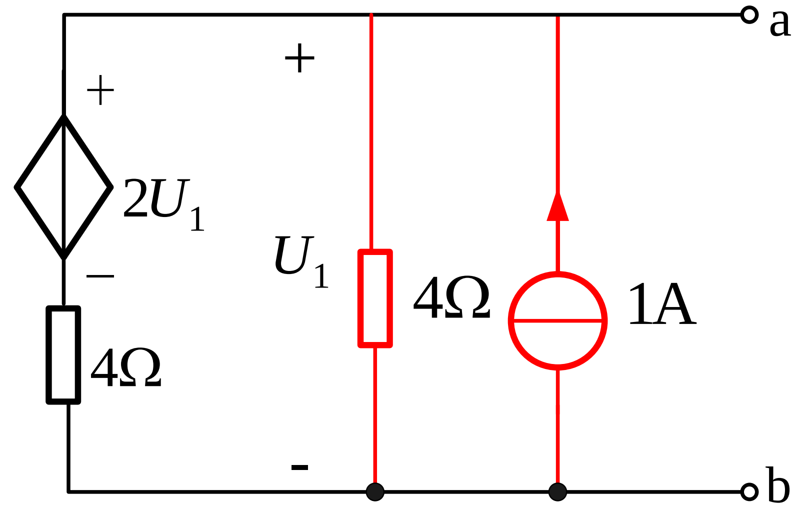


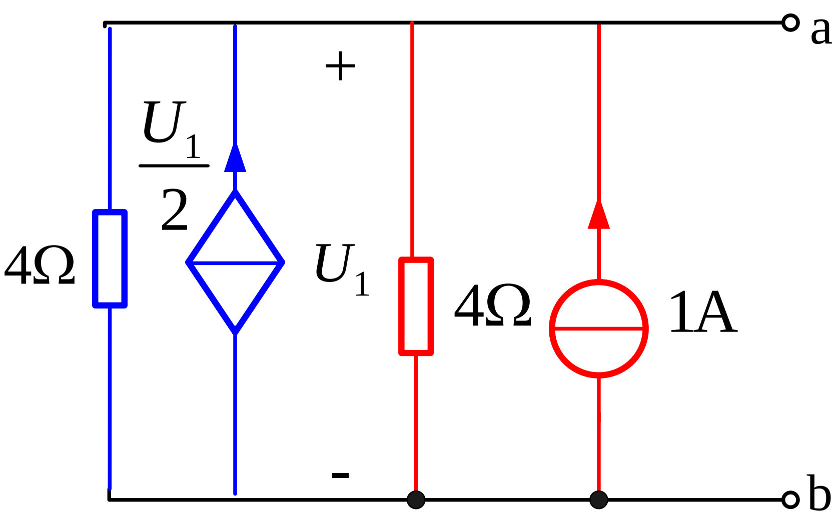
$$R_{ab} = 4.6\Omega$$

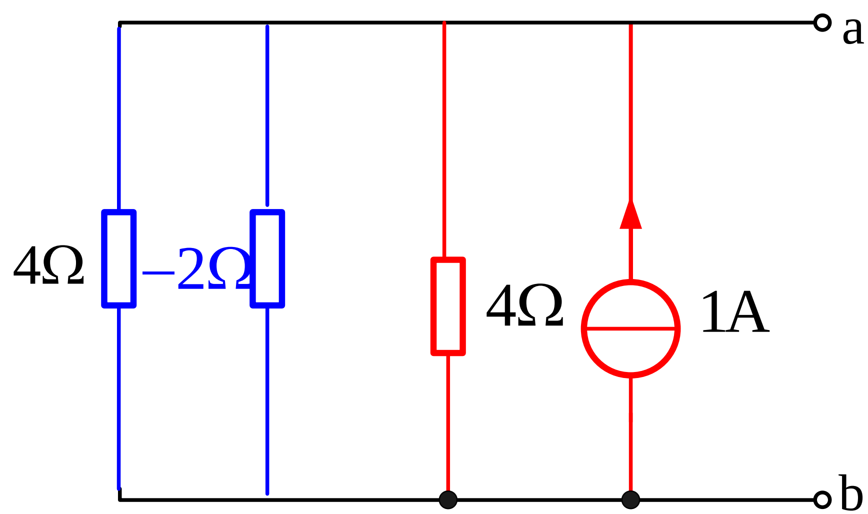
等效电路综合应用

例9.Find the equivalent circuit.

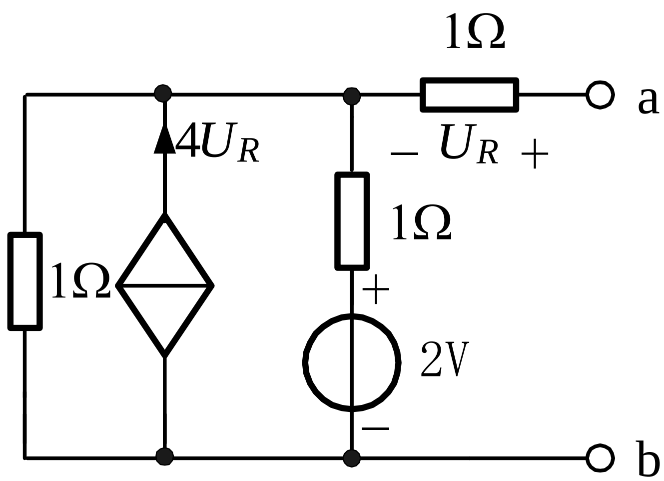


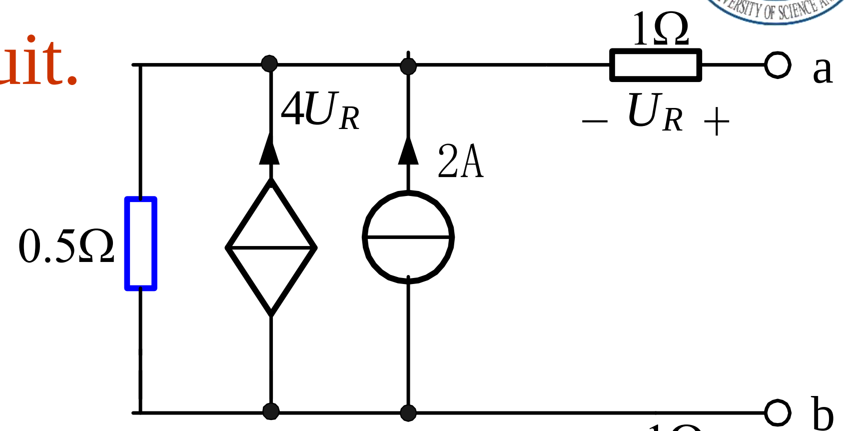


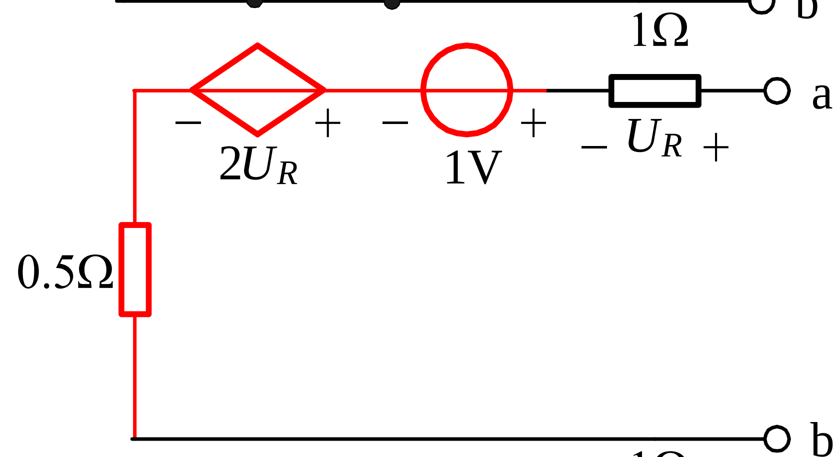


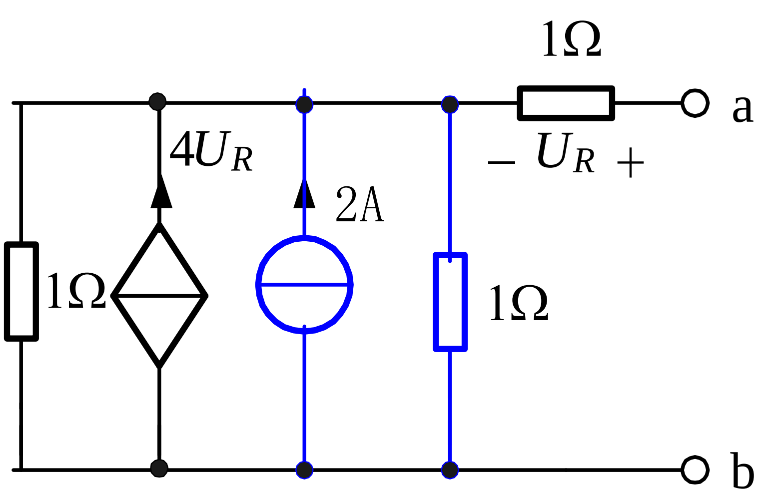


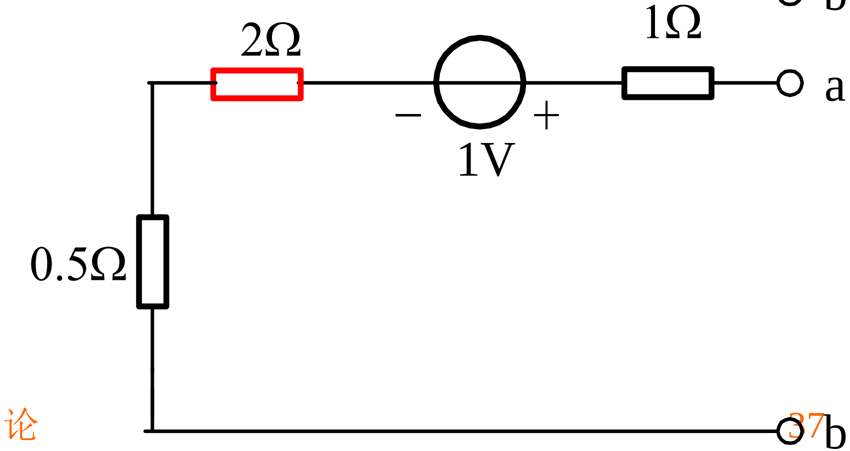
例10.Find the equivalent circuit.











作 业

- 2.2节：2-12，2-14
- 2.3节：2-16，2-20
- 2.4节：2-26，2-32
- 综合：2-36