# Chapter 14

# 正弦稳态电路的频率响应

14.1 网络函数与频率响应

Network Function and Frequency Response

14.2 谐振电路的频率响应

Resonance

目标：

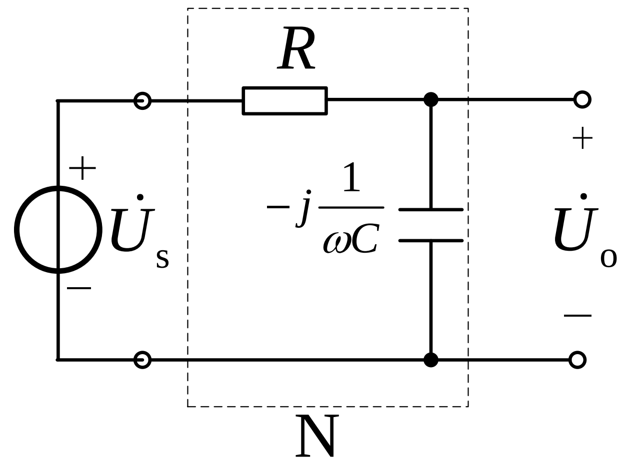
a. 理解频率响应的意义，会计算电路的频率响应；

b. 理解谐振现象及其特点，通过谐振电路的频率响应分析理解滤波的含义。

# 正弦稳态电路的频率响应

**Q1**：下面的正弦稳态电路，参数一定，只改变电源的频率，响应如何变化？

$$\dot{U}_{\mathrm{o}}(\mathrm{j}\omega)=\frac{-j\dfrac{1}{\omega C}}{R-j\dfrac{1}{\omega C}}\times\dot{U}_{\mathrm{s}}(\mathrm{j}\omega)$$




**Q2**：找出描述响应随频率变化的方法？

$$\frac{\dot{U}_{\mathrm{o}}(\mathrm{j}\omega)}{\dot{U}_{\mathrm{s}}(\mathrm{j}\omega)}=\frac{1}{1+j\omega RC}$$

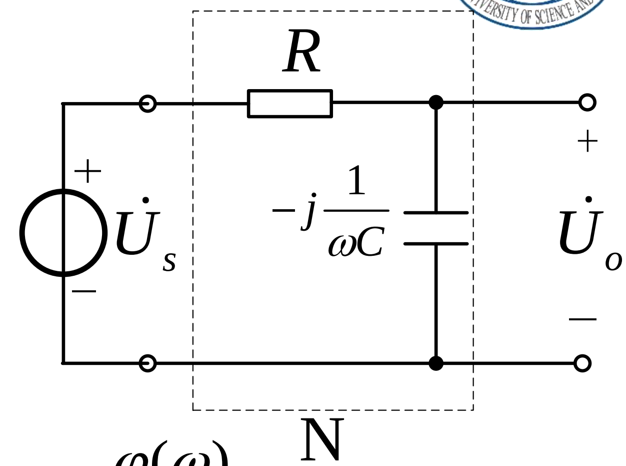
**Q3**：研究响应随频率变化的特点有何意义？

$$u_{\mathrm{s}}=U_{\mathrm{dc}}+\sum_{k}\sqrt{2}U_{k}\cos(k\omega t+\phi_{k}) \quad\Longrightarrow\quad u_{\mathrm{o}}=?$$

# 14.1 网络函数与频率响应

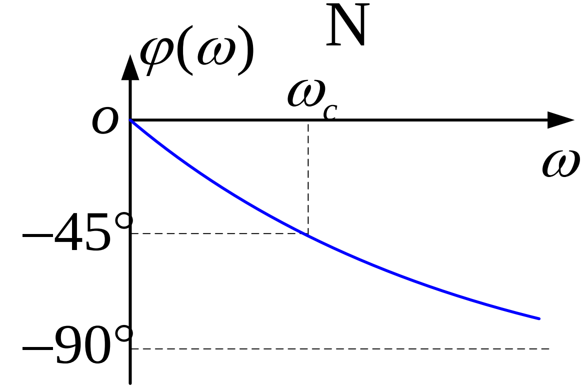
## 1. 网络函数/传递函数

$$H(\mathrm{j}\omega)=\frac{\text{响应}}{\text{激励}}=\frac{\dot{R}(\mathrm{j}\omega)}{\dot{E}(\mathrm{j}\omega)}$$




$$H(j\omega)=\frac{\dot{U}_{\mathrm{o}}(\mathrm{j}\omega)}{\dot{U}_{\mathrm{s}}(\mathrm{j}\omega)}=\frac{-j\dfrac{1}{\omega C}}{R-j\dfrac{1}{\omega C}}=\frac{1}{1+j\omega RC}$$

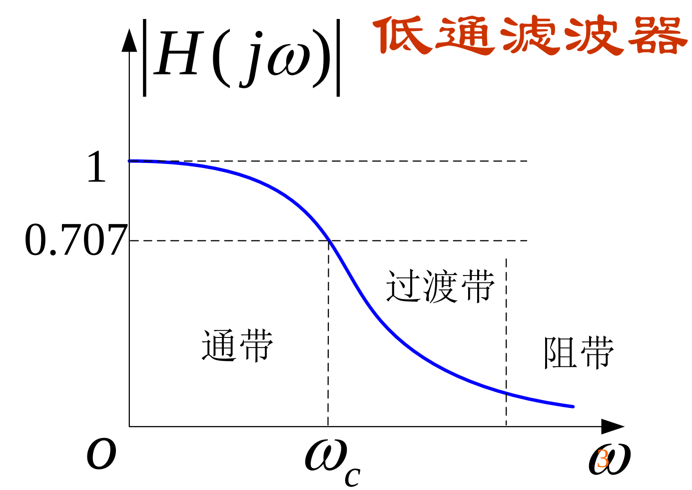
## 2. 频率响应




频率响应： 正弦稳态响应
随激励频率的变化规律。

$$H(j\omega)=|H(j\omega)|\angle\varphi(j\omega)$$

幅频响应　相频响应




$$H(j\omega)=(1+\omega^2R^2C^2)^{-\frac{1}{2}}\angle-\arctan\omega RC$$

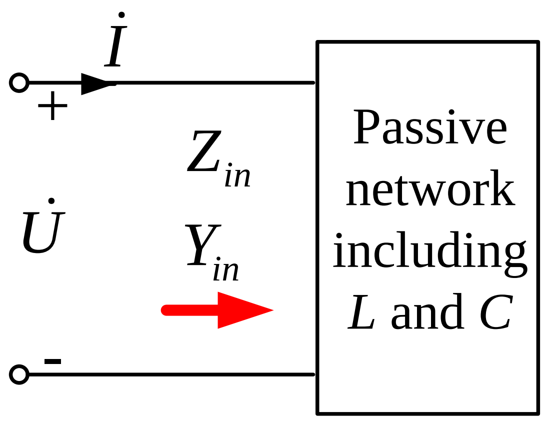
$$H(\mathrm{j}0)=1\angle0° \qquad H(\mathrm{j}\infty)=0\angle-90°$$

# 14.2 谐振电路的频率响应

## 1. 谐振 Resonance

$$Z_{in} = R(\omega) + jX(\omega) \qquad X(\omega) = 0$$

$$Y_{in} = G(\omega) + jB(\omega) \qquad B(\omega) = 0$$




$\dot{U}$ $\dot{I}$ 同相位，端口呈纯阻性。

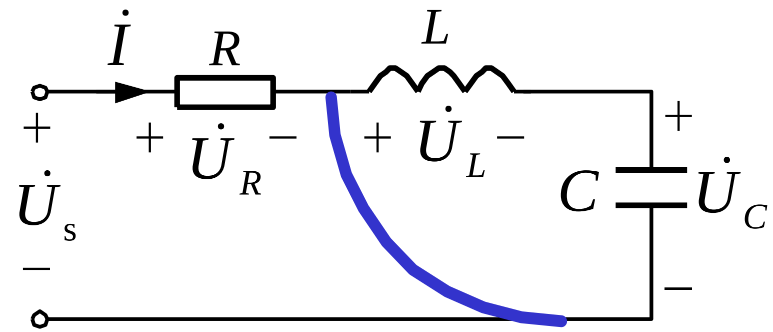
$$P = UI \qquad Q = 0$$

（1）改变激励源频率。

（2）改变网络中$L$ 或 C的值。

# 14.2 RLC串联谐振电路的频率响应

## 2. RLC串联谐振电路及其电气特点




对外相当于短路！

$$Z = R + \mathrm{j}(\omega L - \frac{1}{\omega C})$$

$$\omega = \frac{1}{\sqrt{LC}} = \omega_0$$ 谐振频率

谐振时的电气特点:

(1) $\dot{U}_S$ and $\dot{I}_0$ are in phase.

(2) $|Z(\omega_0)| = R = |Z_{\min}(\omega)|$

(3) $|\dot{I}_0| = \left|\frac{\dot{U}_S}{R}\right| = |\dot{I}_{\max}(\omega)|$

(4) $\dot{U}_{R0} = \dot{U}_S$

(5) $\dot{U}_{L0} = \mathrm{j}\omega_0 L\dot{I}_0 = \mathrm{j}\frac{\omega_0 L}{R}\dot{U}_S$

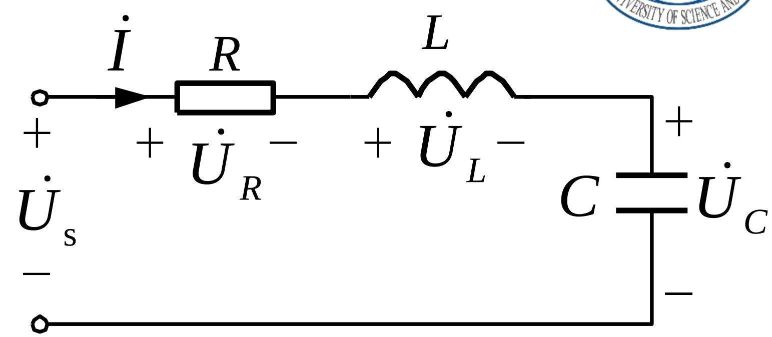
$$\dot{U}_{C0} = -\mathrm{j}\frac{1}{\omega_0 C}\dot{I}_0 = -\mathrm{j}\frac{1}{\omega_0 CR}\dot{U}_S$$

$$U_{L0} = U_{C0} = QU_S$$

$Q$——品质因子

# 14.2 RLC串联谐振电路的频率响应

## 2. RLC串联谐振电路 及其电气特点




### (5) 谐振下的能量关系

**系统中的储能:**

$$w_0 = w_{L0} + w_{C0} = \frac{1}{2} L i_0^{\,2} + \frac{1}{2} C u_{C0}^{\,2}$$

$$= L I_0^{\,2} (\cos\omega_0 t)^2 + C\left(\frac{I_0}{\omega_0 C}\right)^2 [\cos(\omega_0 t - 90°)]^2 \xrightarrow{C\left(\frac{1}{\omega_0 C}\right)^2 = L} = L I_0^{\,2}$$

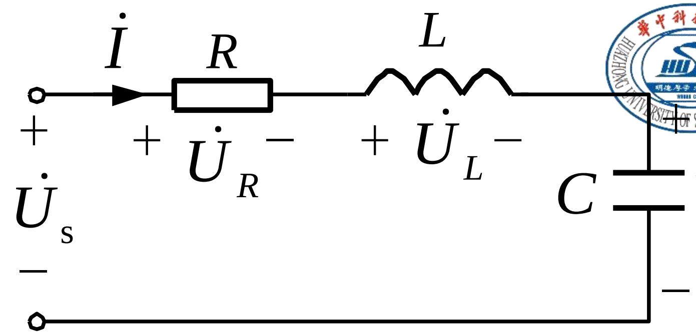
**系统中的耗能(一个周期)**

$$w_{R0} = \int_0^{T_0} p_{R0}\,\mathrm{d}\,t = \int_0^{T_0} i_0^{\,2} R\,\mathrm{d}\,t = I_0^{\,2} R T_0 = I_0^{\,2} R \frac{2\pi}{\omega_0} = 2\pi I_0^{\,2} R \sqrt{LC}$$

$$\boxed{\frac{w_0}{w_{R0}} = \frac{1}{2\pi} \frac{\sqrt{L/C}}{R} = \frac{Q}{2\pi}}$$

# 14.2 谐振电路的频率响应

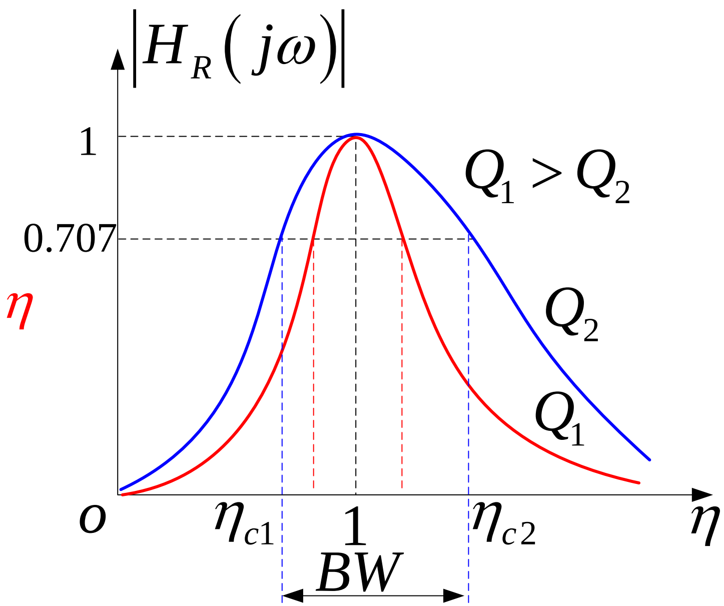
## 3.RLC串联谐振电路的频率响应




$$|H_R(j\omega)| = \left|\frac{\dot{U}_R(j\omega)}{\dot{U}_S(j\omega)}\right| = \frac{R}{\left|R + j(\omega L - \frac{1}{\omega C})\right|}$$

$$= \frac{1}{\sqrt{1 + (\frac{\omega L}{R} - \frac{1}{R\omega C})^2}} = \frac{1}{\sqrt{1 + Q^2(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega})^2}}$$

$$\frac{\omega}{\omega_0} = \eta$$




**半功率点:**

$$P(\omega_{c1,c2}) = \frac{1}{2}P(\omega_0) = \frac{1}{2}\frac{U_S^2}{R} = \frac{(U_S/\sqrt{2})^2}{R}$$

$$|H_R(\eta_{c1,c2})| = \frac{1}{\sqrt{2}} = 0.707$$

$$\left.\begin{aligned} \eta_{c1} &= -\frac{1}{2Q} + \sqrt{(\frac{1}{2Q})^2 + 1} \\ \eta_{c2} &= \frac{1}{2Q} + \sqrt{(\frac{1}{2Q})^2 + 1} \end{aligned}\right\}$$

## 3dB Bandwidth:

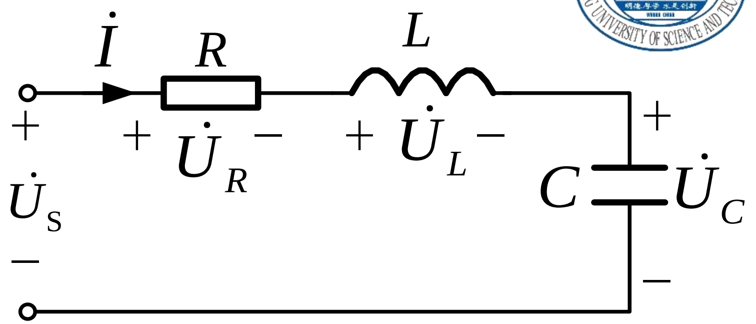
$$BW = (\eta_{c2} - \eta_{c1})\omega_0 = \frac{\omega_0}{Q}$$

$$\sqrt{\omega_{c1}\omega_{c2}} = \sqrt{\eta_{c1}\eta_{c2}}\,\omega_0 = \omega_0$$

$$\omega_{c1,c2} \approx \omega_0 \mp \frac{1}{2}BW \quad (\text{for } Q \geq 10)$$

# 14.2 谐振电路的频率响应

## 3.RLC串联谐振电路的频率响应




$$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{\omega_{c1}\omega_{c2}}$$

$$BW = \omega_{c2} - \omega_{c1} = \frac{R}{L} \qquad BW = \frac{\omega_0}{Q}$$

$$Q = \frac{U_{L0}}{U_S} = \frac{U_{C0}}{U_S} \qquad Q = \frac{X_{L0}}{R} = \frac{X_{C0}}{R} = \frac{\sqrt{L/C}}{R} \qquad Q = 2\pi\frac{w_0}{w_{R0}} \qquad Q = \frac{\omega_0}{BW}$$

$$\omega_{c1,c2} = \mp\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = \mp\frac{\omega_0}{2Q} + \omega_0\sqrt{1 + \left(\frac{1}{2Q}\right)^2}$$

$$\omega_{c1,c2} \approx \omega_0 \mp \frac{BW}{2} \qquad (\text{For } Q \geq 10)$$

# Practice

Design a series *RLC* circuit with *BW*=20 rad/s and $\omega_0 = 1000$ rad/s. (1)Find the circuit's *Q*. (2)If *C*=5μF, find the Value of *L* and *R*. (3)Find the cut-off frequencies.

$$BW = \frac{\omega_0}{Q} \quad \rightarrow Q = \frac{\omega_0}{BW} = \frac{1000}{20} = 50$$

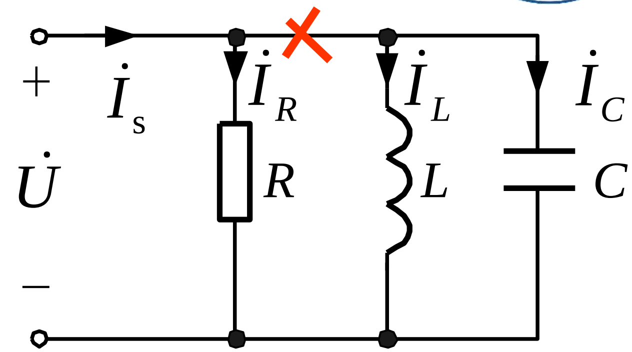
$$\omega_0 = \frac{1}{\sqrt{LC}} = 1000 \qquad L = 200\text{mH}$$

$$Q = \frac{\omega_0 L}{R} \quad \rightarrow R = \frac{\omega_0 L}{Q} = 4\Omega$$

$$\omega_{c1,c2} \approx \omega_0 \mp \frac{BW}{2} = 990\text{rad/s},\ 1010\text{rad/s}$$

# 14.2 谐振电路的频率响应

## 4.RLC并联谐振电路




$$\omega_0 = \frac{1}{\sqrt{LC}}$$

- $\dot{I}_{R0} = \dot{I}_S$
- $\dot{U}_0$ and $\dot{I}_S$ are in phase.
- $|Y(\omega_0)| = G = |Y_{\min}(\omega)|$
- $|\dot{U}_0| = \left|\frac{\dot{I}_S}{G}\right| = |\dot{U}_{\max}(\omega)|$
- $\dot{I}_{L0} = \frac{\dot{U}_0}{\mathrm{j}\omega_0 L} = -\mathrm{j}\frac{R}{\omega_0 L}\dot{I}_S$
- $\dot{I}_{C0} = \mathrm{j}\omega_0 C\dot{U}_0 = \mathrm{j}\omega_0 CR\dot{I}_S$
- $I_{L0} = I_{C0} = QI_S$

$$\omega_{c1,c2} = \mp\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$BW = \frac{1}{RC}$$

串联 $\left\{\begin{array}{c} R \to G \\ L \to C \\ C \to L \\ \dot{U} \to \dot{I} \\ \dot{I} \to \dot{U} \end{array}\right\}$ 并联

# 14.2 谐振电路的频率响应

## 5.其他谐振电路

**与串联或并联谐振电路等效！**

$$Y=\frac{1}{R_1+\mathrm{j}\omega L_1}+\mathrm{j}\omega C_2$$

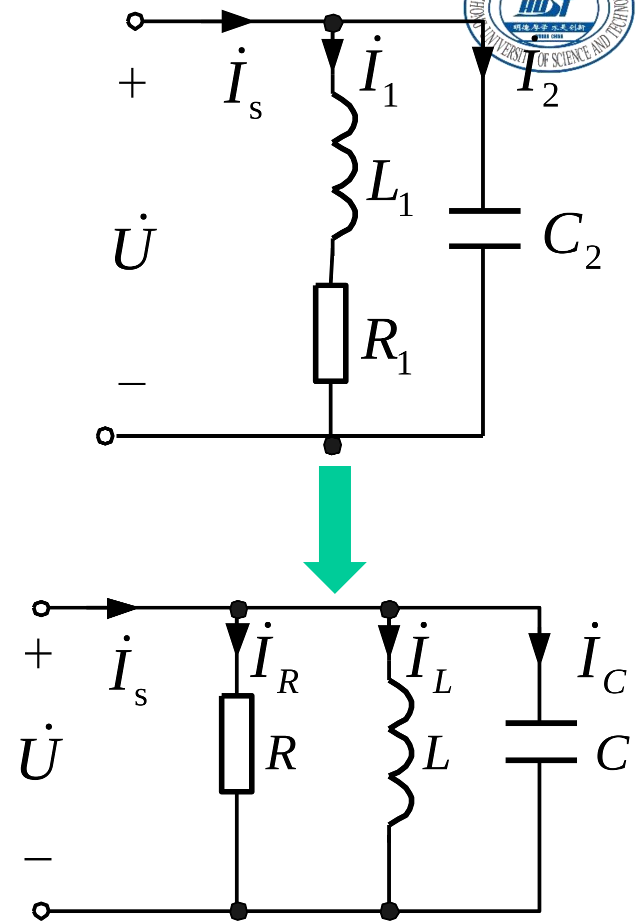
$$=\frac{R_1}{R_1^2+(\omega L_1)^2}+\mathrm{j}[\omega C_2-\frac{\omega L_1}{R_1^2+(\omega L_1)^2}]$$

$$=G+\mathrm{j}(\omega C-\frac{1}{\omega L})$$

$$\omega_0 C_2=\frac{\omega_0 L_1}{R_1^2+(\omega_0 L_1)^2}$$

$$\omega_0=\frac{1}{\sqrt{L_1C_2}}\sqrt{1-\frac{C_2R_1^2}{L_1}}\quad (L_1>C_2R_1^2)$$

$$Q=\frac{B_{L0}}{G}=\frac{\omega_0 L_1}{R_1}=\sqrt{\frac{L_1}{C_2R_1^2}-1}$$




$$I_{20}=QI_s$$

$$I_{10}=\sqrt{I_s^2+(QI_s)^2}$$

$$=I_s\sqrt{1+Q^2}$$

# 作业

- 14.3节：14-9，14-10
- 14.4节：14-14