

第10章

正弦稳态分析

后续章节学习
的基础!!!

10.1 数学基础

10.2 正弦电量

10.3 相量法

10.4 阻抗与导纳

10.5 正弦稳态电路分析方法

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10.1 数学基础

1. 复数的表示形式

$$F = a + jb$$

代数式

($j = \sqrt{-1}$ 为虚数单位)

$$F = |F| e^{j\theta}$$

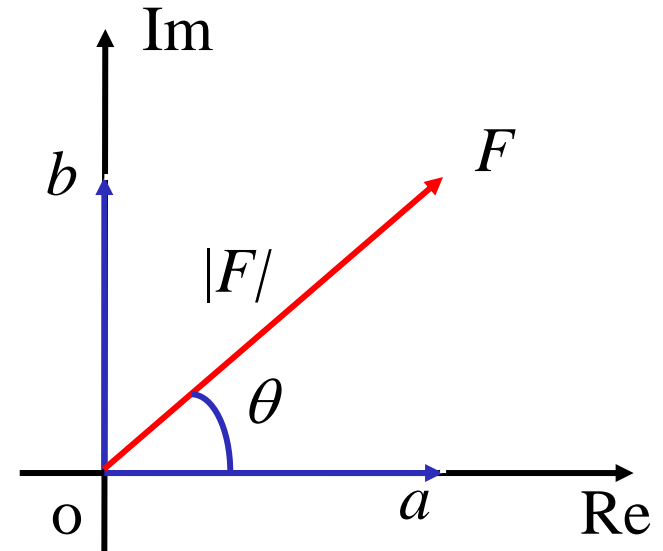
指数式

三角函数式

$$F = |F| e^{j\theta} = |F| (\cos \theta + j \sin \theta) = a + jb$$

$$F = |F| e^{j\theta} = |F| \angle \theta$$

极坐标式



几种表示法的关系：

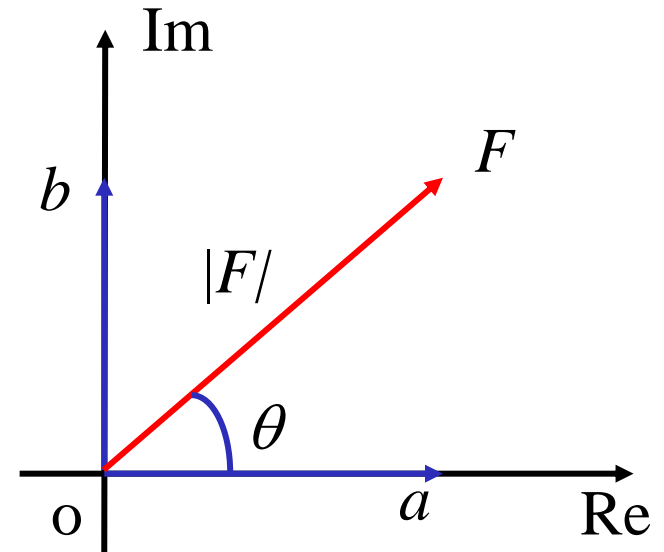
$$F = a + jb$$

$$F = |F|e^{j\theta} = |F| \angle \theta$$

$$\begin{cases} |F| = \sqrt{a^2 + b^2} \\ \theta = \arctan \frac{b}{a} \end{cases}$$

或

$$\begin{cases} a = |F| \cos \theta \\ b = |F| \sin \theta \end{cases}$$

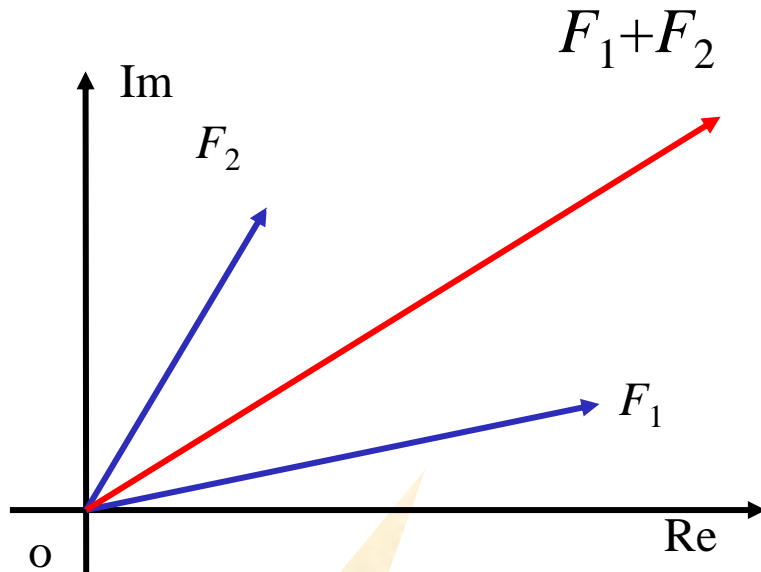


2. 复数运算

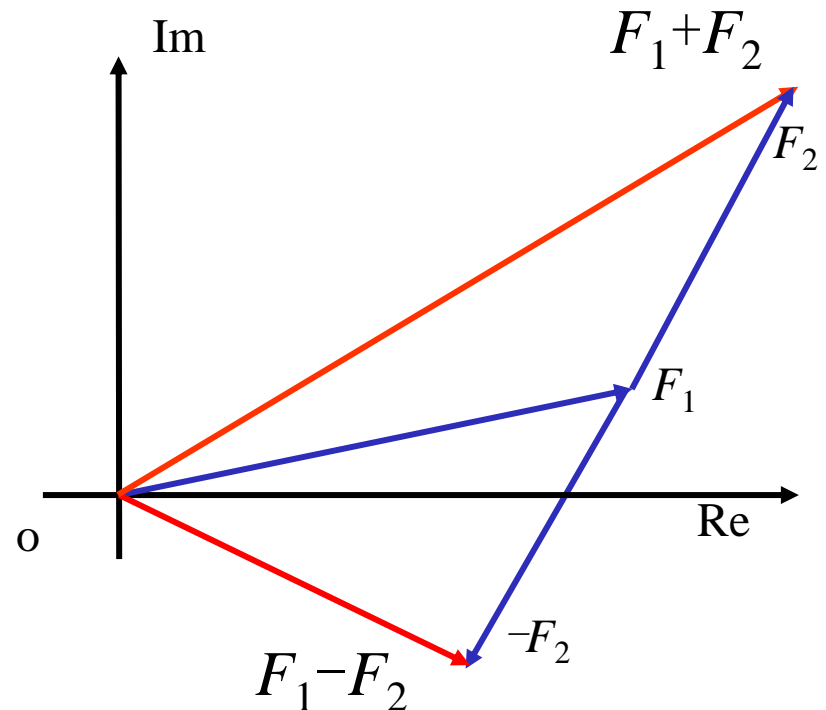
A. 加减运算——采用代数式

若 $F_1 = a_1 + jb_1$, $F_2 = a_2 + jb_2$

则 $F_1 \pm F_2 = (a_1 \pm a_2) + j(b_1 \pm b_2)$



图解法



B. 乘除运算——采用极坐标式

若 $F_1 = |F_1| \angle \theta_1, F_2 = |F_2| \angle \theta_2$

则 $F_1 \cdot F_2 = |F_1| e^{j\theta_1} \cdot |F_2| e^{j\theta_2} = |F_1| |F_2| e^{j(\theta_1 + \theta_2)}$
 $= |F_1| |F_2| \angle \theta_1 + \theta_2$ 模相乘
角相加

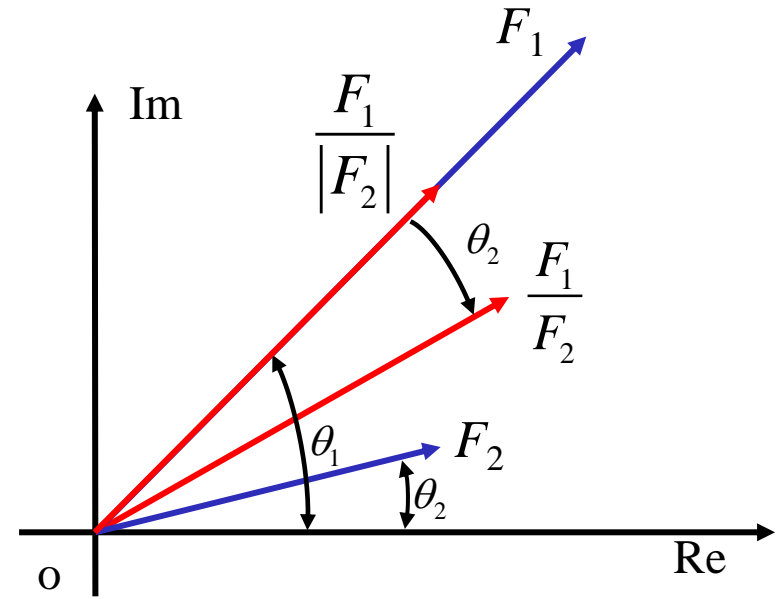
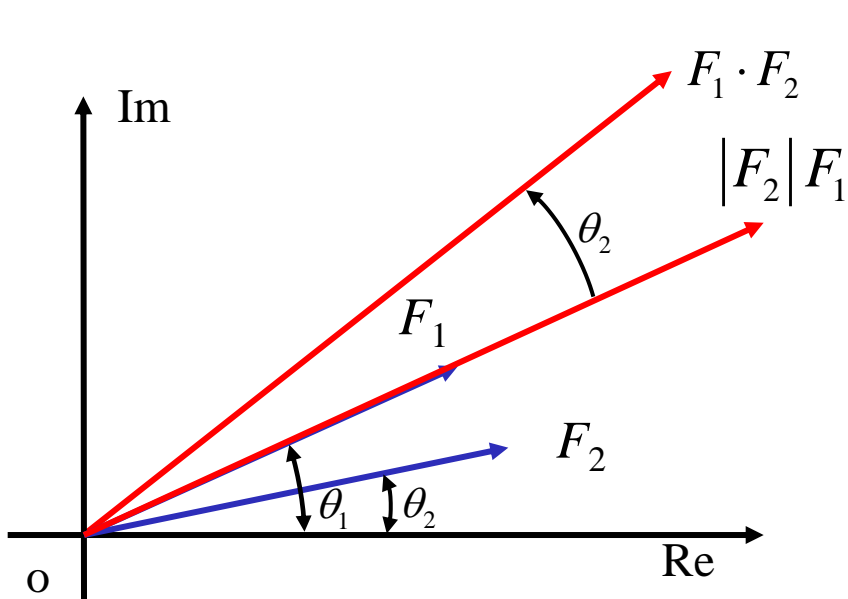
$$\frac{F_1}{F_2} = \frac{|F_1| \angle \theta_1}{|F_2| \angle \theta_2} = \frac{|F_1| e^{j\theta_1}}{|F_2| e^{j\theta_2}} = \frac{|F_1|}{|F_2|} e^{j(\theta_1 - \theta_2)}$$
$$= \frac{|F_1|}{|F_2|} \angle \theta_1 - \theta_2$$

模相除
角相减

$$F_1 \cdot F_2 = |F_1| e^{j\theta_1} \cdot |F_2| e^{j\theta_2}$$

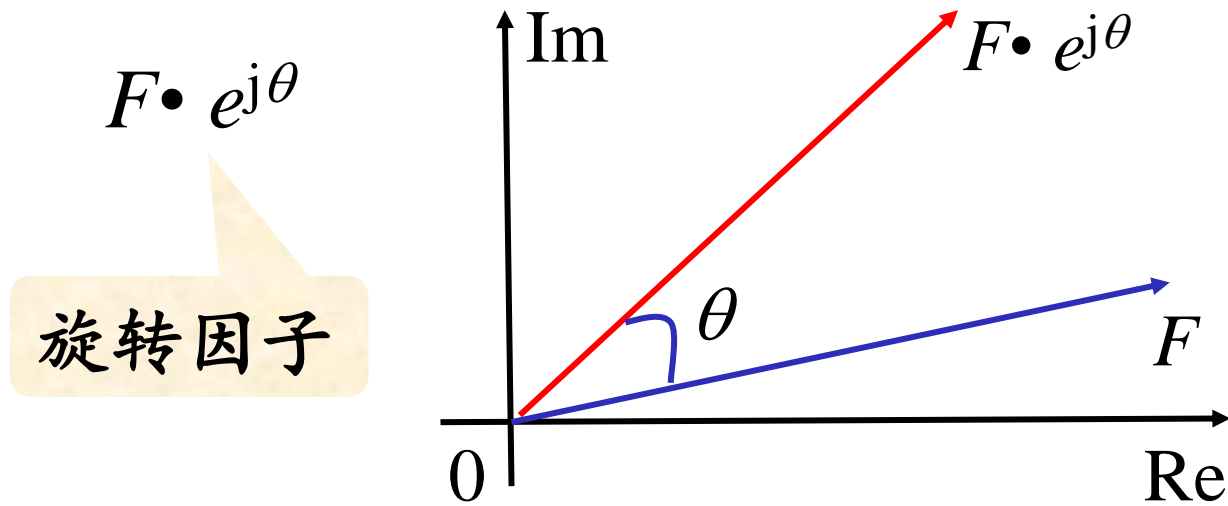
$$= |F_1| |F_2| \angle \theta_1 + \theta_2$$

$$\frac{F_1}{F_2} = \frac{|F_1| e^{j\theta_1}}{|F_2| e^{j\theta_2}} = \frac{|F_1|}{|F_2|} \angle \theta_1 - \theta_2$$



C. 旋转因子

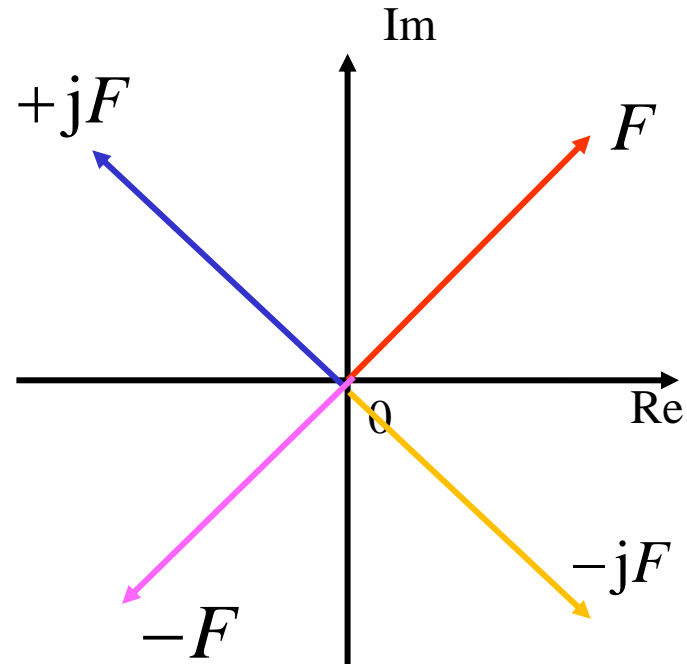
$$e^{j\theta} = \cos\theta + j\sin\theta = 1 \angle \theta$$



特殊旋转因子

$$\theta = \frac{\pi}{2},$$

$$e^{j\frac{\pi}{2}} = \cos \frac{\pi}{2} + j \sin \frac{\pi}{2} = +j$$



$$\theta = -\frac{\pi}{2}, \quad e^{j-\frac{\pi}{2}} = \cos(-\frac{\pi}{2}) + j \sin(-\frac{\pi}{2}) = -j$$

$$\theta = \pm\pi, \quad e^{j\pm\pi} = \cos(\pm\pi) + j \sin(\pm\pi) = -1$$

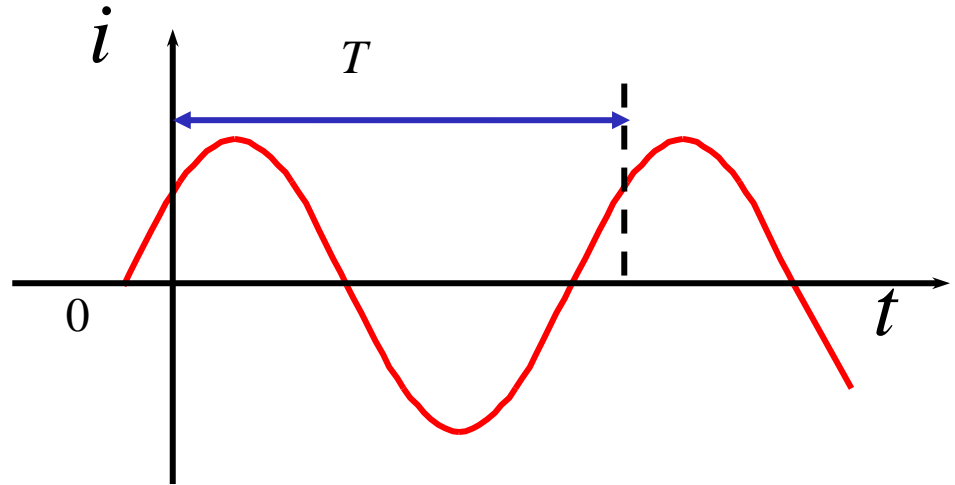
注意：+j、-j、-1 都可以看成旋转因子

10.2 正弦量

1. 正弦量

● 瞬时值表达式

$$i(t) = I_m \cos(\omega t + \psi)$$



正弦量为周期函数： $f(t) = f(t + kT)$

$$f = \frac{1}{T}$$

● 周期 T 和频率 f

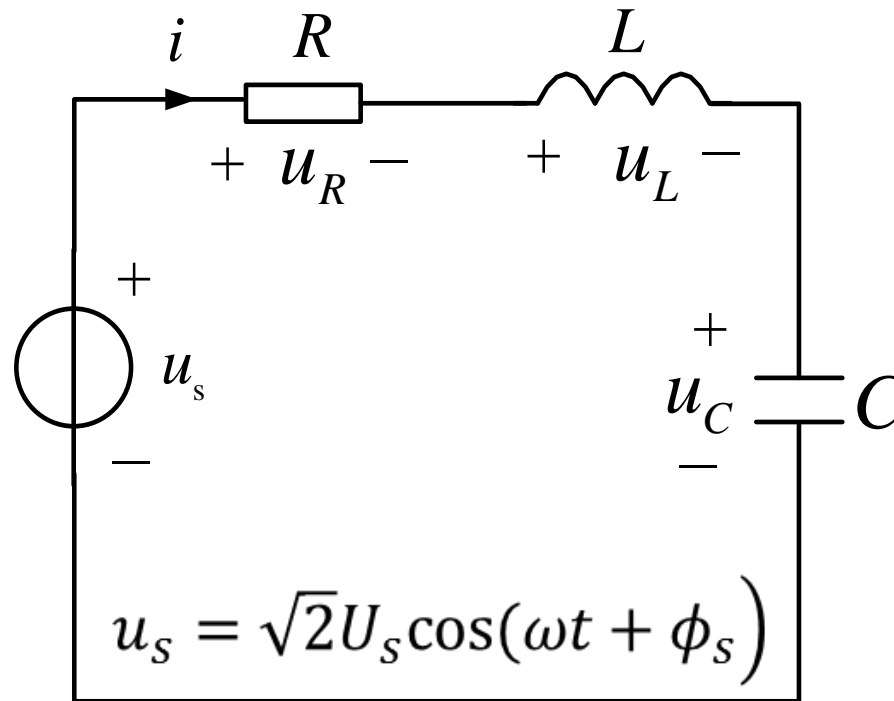
{ 周期 T ：重复变化一次所需的时间。
频率 f ：每秒重复变化的次数。

单位：秒s

单位：赫(兹)Hz

● 正弦电流电路

激励和响应均为**同频率正弦量**的**线性电路**（正弦稳态电路）称为正弦电路或交流电路。



$$LC \frac{d^2 u_C}{dt^2} + RC \frac{du_C}{dt} + u_C = \sqrt{2}U_s \cos(\omega t + \psi_s)$$

●研究正弦电路的意义

✓ 正弦稳态电路在电力系统和电子技术领域占有十分重要的地位。

优点

- 正弦函数是周期函数，其加、减、求导、积分运算后仍是同频率的正弦函数；
- 正弦信号容易产生、传送和使用。

✓ 正弦信号是一种基本信号，任何非正弦周期信号可以分解为按正弦规律变化的分量。

$$f(t) = \sum_{k=1}^n A_k \cos(k\omega t + \theta_k)$$

2. 正弦量的三要素 $i(t) = I_m \cos(\omega t + \psi)$

(1) 幅值 (振幅、最大值) I_m

→ 反映正弦量变化幅度的大小。

(2) 角频率 ω

→ 相位变化的速度，反映正弦量变化快慢。

$$\omega = 2\pi f = \frac{2\pi}{T} \quad \text{单位: rad/s, 弧度/秒}$$

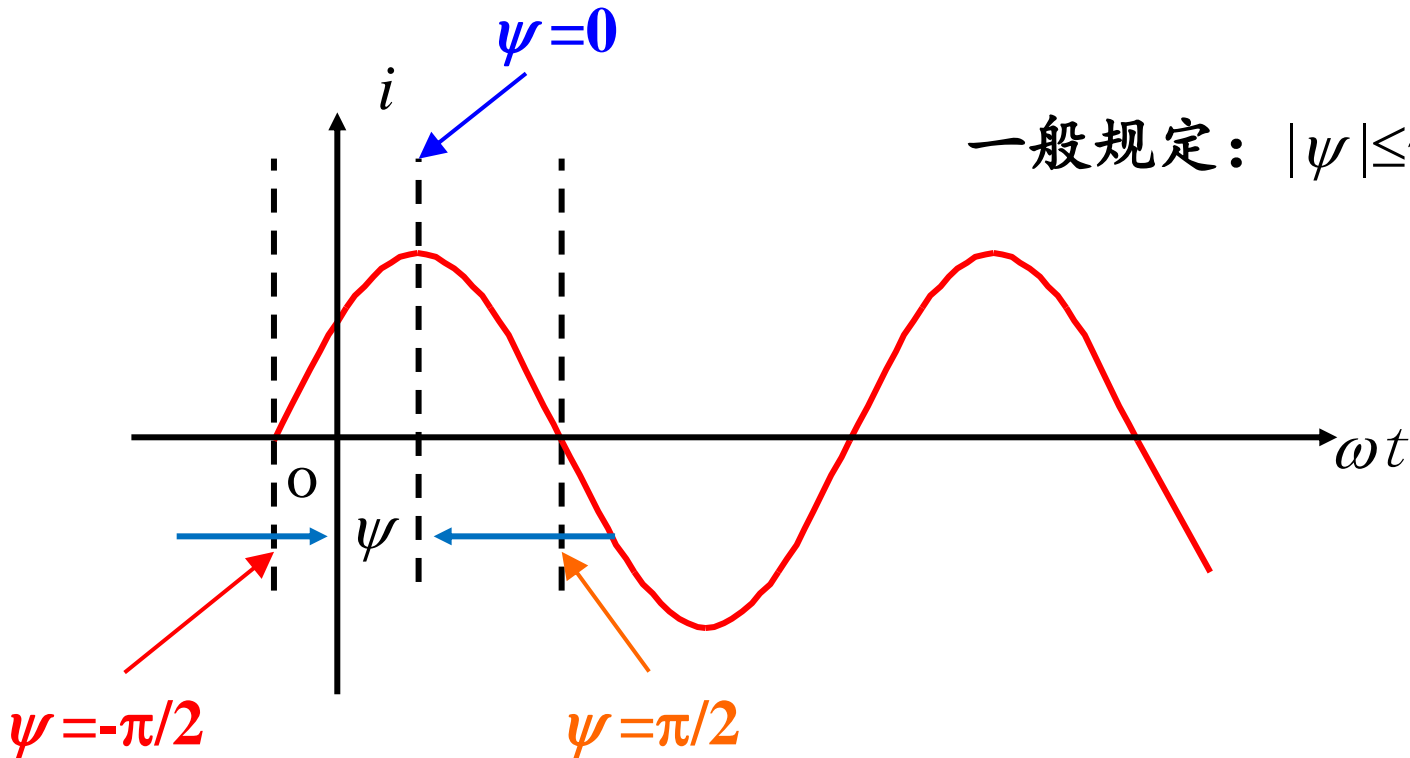
(3) 初相位 ψ

→ 反映正弦量的计时起点，常用角度表示。

注意：同一个正弦量，计时起点不同，初相位不同。

$$i(t)=I_m\cos(\omega t+\psi)$$

一般规定： $|\psi|\leq\pi$ 。



例 已知正弦电流波形如图, $\omega=10^3\text{rad/s}$,

1. 写出 $i(t)$ 表达式; 2. 求最大值发生的时间 t_1

解
$$i(t) = 100 \cos(10^3 t + \psi)$$

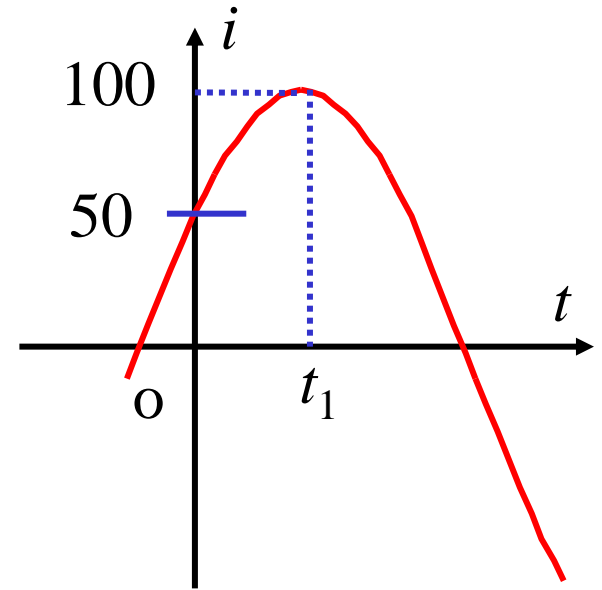
$$t = 0 \rightarrow 50 = 100 \cos \psi$$

$$\rightarrow \psi = \pm \pi/3 \quad \rightarrow \psi = -\frac{\pi}{3}$$

由于最大值发生在计时起点右侧

$$i(t) = 100 \cos(10^3 t - \frac{\pi}{3})$$

$$\text{当 } 10^3 t_1 = \pi/3 \text{ 时有最大值} \quad \rightarrow \quad t_1 = \frac{\pi/3}{10^3} = 1.047\text{ms}$$

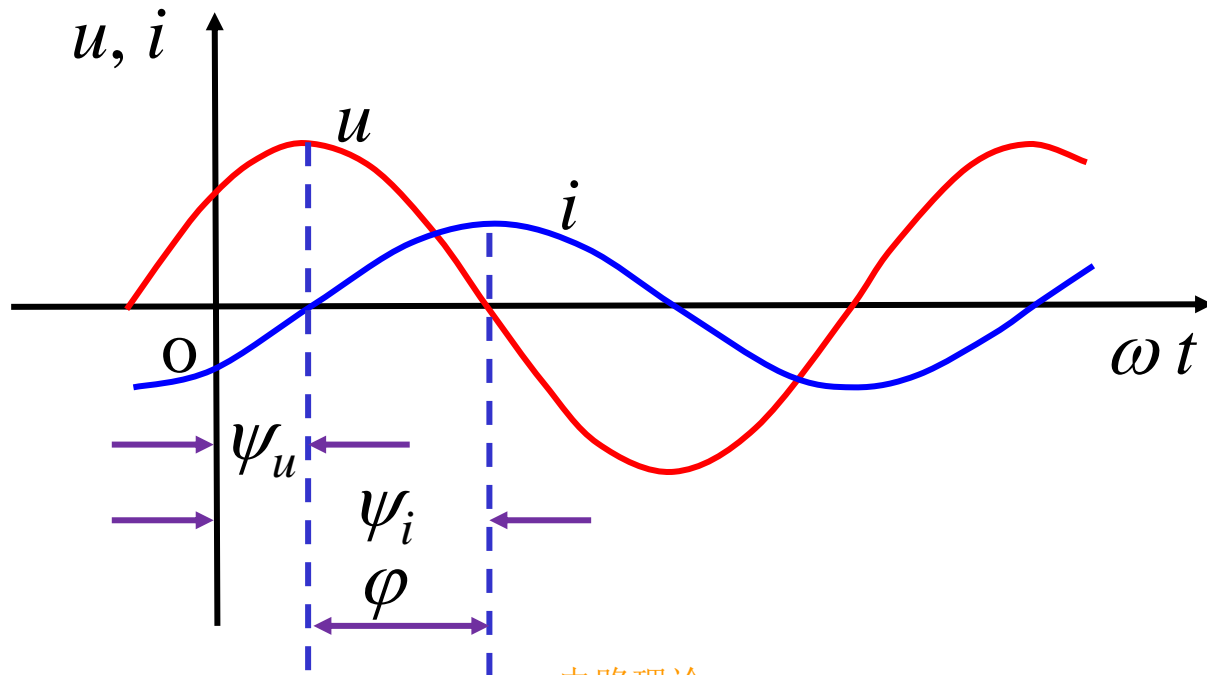


3. 同频率正弦量的相位差

设 $u(t)=U_m\cos(\omega t+\psi_u)$, $i(t)=I_m\cos(\omega t+\psi_i)$

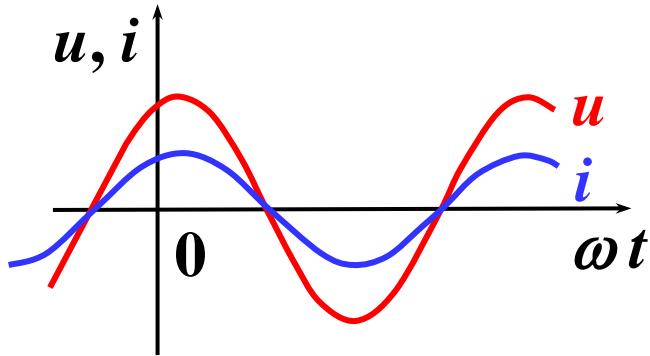
相位差: $\varphi = (\omega t+\psi_u)-(\omega t+\psi_i)=\psi_u-\psi_i$

- $\varphi < 0$, i 超前 u 相位 φ 角, 或 u 滞后 i 相位 φ 角;
- $\varphi > 0$, u 超前 i 相位 φ 角, 或 i 滞后 u 相位 φ 角;

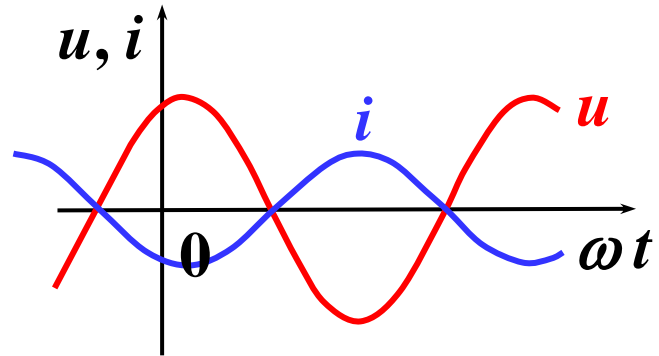


特殊相位关系:

$\varphi = 0$, 同相:



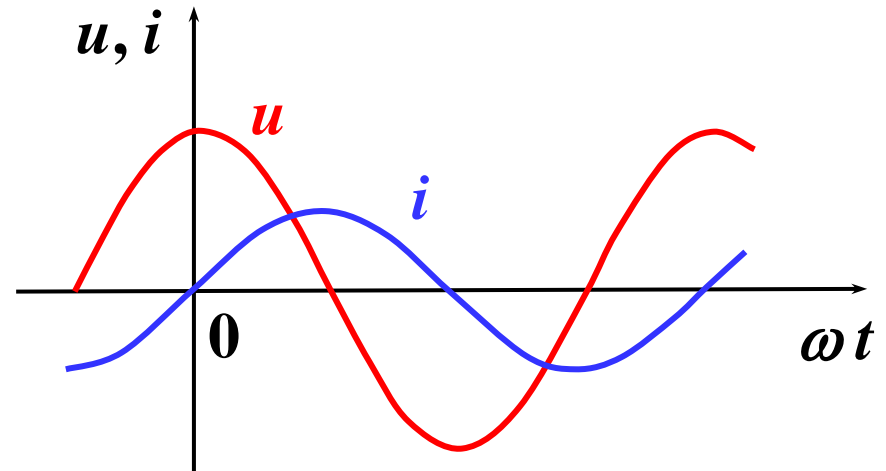
$\varphi = \pm \pi$ ($\pm 180^\circ$), 反相:



$\varphi = 90^\circ$

u 领先 i 90°

或 i 落后 u 90°



规定: $|\varphi| \leq \pi$ (180°)

计算下列两正弦量的相位差。

(1) $i_1(t) = 10 \cos(100\pi t + 3\pi/4)$

$i_2(t) = 10 \cos(100\pi t - \pi/2)$

解: $\varphi = 3\pi/4 - (-\pi/2) = 5\pi/4 > 0$

→ $\varphi = 5\pi/4 - 2\pi = -3\pi/4$

(2) $i_1(t) = 10 \cos(100\pi t + 30^\circ)$

$i_2(t) = 10 \sin(100\pi t - 15^\circ)$

解: $i_2(t) = 10 \cos(100\pi t - 105^\circ)$

→ $\varphi = 30^\circ - (-105^\circ) = 135^\circ$

(3) $u_1(t) = 10 \cos(100\pi t + 30^\circ)$

$u_2(t) = 10 \cos(200\pi t + 45^\circ)$

解: 不能比较相位差

(4) $i_1(t) = 5 \cos(100\pi t - 30^\circ)$

$i_2(t) = -3 \cos(100\pi t + 30^\circ)$

解: $i_2(t) = 3 \cos(100\pi t - 150^\circ)$

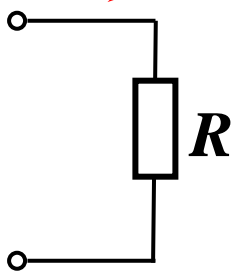
→ $\varphi = -30^\circ - (-150^\circ) = 120^\circ$

结论 两个正弦量进行相位比较时应满足同频率、同函数、同符号，且在主值范围比较。

4. 有效值(effective value)

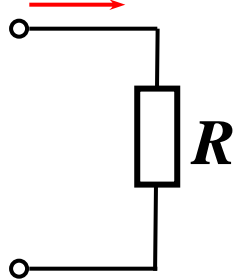
物理含义

$i(t)$ 周期电流



$$W_1 = \int_0^T i^2(t) R dt$$

I 直流电流



$$W_2 = I^2 R T$$

$$I^2 R T = \int_0^T i^2(t) R dt$$

$$I = \sqrt{\frac{1}{T} \int_0^T i^2(t) dt}$$

定义：将周期性电压或电流在一个周期内产生的效应转换为相同效应下的直流电压或电流，则该直流电压或电流成为周期电压或电流的有效值（也称均方根值）

$$I \stackrel{\text{def}}{=} \sqrt{\frac{1}{T} \int_0^T i^2(t) dt}$$

$$U \stackrel{\text{def}}{=} \sqrt{\frac{1}{T} \int_0^T u^2(t) dt}$$

正弦电流、电压的有效值

设 $i(t) = I_m \sin(\omega t + \psi)$

$$I \stackrel{\text{def}}{=} \sqrt{\frac{1}{T} \int_0^T i^2(t) dt}$$

$$I = \sqrt{\frac{1}{T} \int_0^T I_m^2 \sin^2(\omega t + \psi) dt}$$

$$\therefore \int_0^T \sin^2(\omega t + \psi) dt = \int_0^T \frac{1 - \cos 2(\omega t + \psi)}{2} dt = \frac{1}{2} t \Big|_0^T = \frac{1}{2} T$$

$$\therefore I = \sqrt{\frac{1}{T} I_m^2 \cdot \frac{T}{2}} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

$$I_m = \sqrt{2} I$$

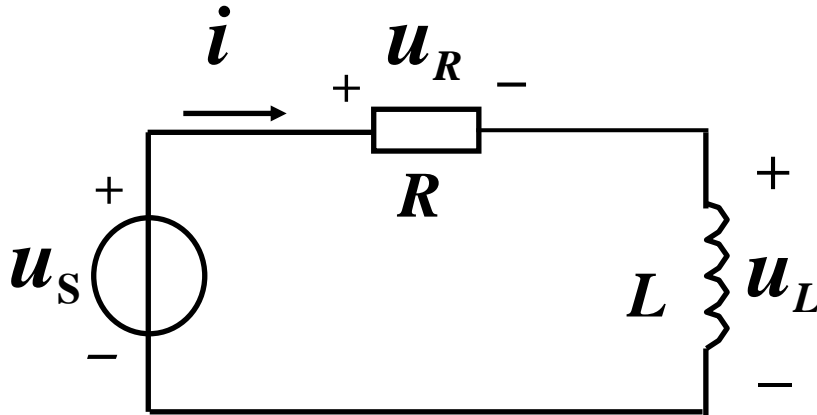
注意: 只适用正弦量

$$i(t) = I_m \sin(\omega t + \psi) = \sqrt{2} I \sin(\omega t + \psi)$$

- 交流电压表、电流表的标尺刻度是有效值；交流电气设备铭牌上的电压、电流是有效值。
- 但绝缘水平、耐压值指的是最大值。

10.3 相量法

1. 问题的提出



$$u_S = U_m \sin(\omega t + \psi_u) \varepsilon(t)$$

求： $i(t)$, $u_L(t)$, $u_R(t)$

$$Ri + L \frac{di}{dt} = U_m \sin(\omega t + \psi_u)$$

稳态解

$$i = A \sin(\omega t + B) + Ce^{-\alpha t}$$

$$i = A \sin(\omega t + B)$$

$$Ri + L \frac{di}{dt} = U_m \sin(\omega t + \psi_u)$$

$$i = A \sin(\omega t + B)$$



$$RA \sin(\omega t + B) + LA\omega \cos(\omega t + B) = U_m \sin(\omega t + \Psi_u)$$



$$A\sqrt{R^2 + (\omega L)^2} \left(\frac{R}{\sqrt{R^2 + (\omega L)^2}} \sin(\omega t + B) + \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}} \cos(\omega t + B) \right) = U_m \sin(\omega t + \Psi_u)$$



$$A\sqrt{R^2 + (\omega L)^2} \sin\left(\omega t + B + \arctan \frac{\omega L}{R}\right) = U_m \sin(\omega t + \Psi_u)$$

$$\left\{ \begin{array}{l} A\sqrt{R^2 + (\omega L)^2} = U_m \quad \longrightarrow \quad A = \frac{U_m}{\sqrt{R^2 + (\omega L)^2}} = I_m \\ B + \arctan\left(\frac{\omega L}{R}\right) = \Psi_u \quad \longrightarrow \quad B = \Psi_u - \arctan\left(\frac{\omega L}{R}\right) = \Psi_u - \varphi \end{array} \right.$$

$$i(t) = \frac{U_m}{\sqrt{R^2 + (\omega L)^2}} \sin\left(\omega t + \Psi_u - \arctan\left(\frac{\omega L}{R}\right)\right)$$

$$u_L(t) = L \frac{di(t)}{dt} = \frac{L\omega U_m}{\sqrt{R^2 + (\omega L)^2}} \sin\left(\omega t + \Psi_u - \arctan\left(\frac{\omega L}{R}\right) + 90^\circ\right)$$

$$u_R(t) = Ri(t) = u_S - u_L(t) = \frac{RU_m}{\sqrt{R^2 + (\omega L)^2}} \sin\left(\omega t + \Psi_u - \arctan\left(\frac{\omega L}{R}\right)\right)$$

所有支路电压电流均以相同频率变化!!

接下来……

$$i(t) = I_m \cos(\omega t + \psi)$$

所有支路电压电流均以相同频率变化!!

➤ 角频率(ω) 可以不考虑

➤ 幅值 (I_m)

➤ 初相角(ψ)

复数

用什么可以同时表示幅值和相位?

KCL、KVL、元件特性如何得到简化?

微分方程的求解如何得到简化?

$$Ri + L \frac{di}{dt} = U_m \sin(\omega t + \psi_u)$$

2. 正弦量的相量表示

构造一个复函数

无物理意义

$$F(t) = \sqrt{2}Ie^{j(\omega t + \Psi)}$$

$$= \sqrt{2}I\cos(\omega t + \Psi) + j\sqrt{2}I\sin(\omega t + \Psi)$$

对 $F(t)$ 取实部 $\text{Re}[F(t)] = \sqrt{2}I\cos(\omega t + \Psi) = i(t)$

是一个**正弦量**
有物理意义

结论：任意一个正弦时间函数都有唯一
与其对应的复数函数。

$$i = \sqrt{2}I\cos(\omega t + \Psi) \leftrightarrow F(t) = \sqrt{2}Ie^{j(\omega t + \Psi)}$$

$F(t)$ 还可以写成 **复常数**

$$F(t) = \sqrt{2} I e^{j\psi} e^{j\omega t} = \sqrt{2} \dot{I} e^{j\omega t}$$

正弦量对应的相量

$F(t)$ 包含了三要素: I 、 Ψ 、 ω ,

复常数(相量)包含了两个要素: I , Ψ 。

$$i(t) = \sqrt{2} I \cos(\omega t + \Psi) \Leftrightarrow \dot{I} = I \angle \Psi$$

时域



相量域

一一对应

注意

{ 相量的**模**表示正弦量的**有效值**
相量的**幅角**表示正弦量的**初相位**

同样可以建立正弦电压与相量的对应关系：

$$u(t) = \sqrt{2}U \cos(\omega t + \theta) \Leftrightarrow \dot{U} = U \angle \theta$$

例1 已知 $i = 141.4 \cos(314t + 30^\circ) \text{ A}$

$$u = 311.1 \cos(314t - 60^\circ) \text{ V}$$

试用相量表示 i, u 。

$$\dot{I} = 100 \angle 30^\circ \text{ A}, \quad \dot{U} = 220 \angle -60^\circ \text{ V}$$

例2 已知 $\dot{I} = 50 \angle 15^\circ \text{ A}$, $f = 50 \text{ Hz}$.

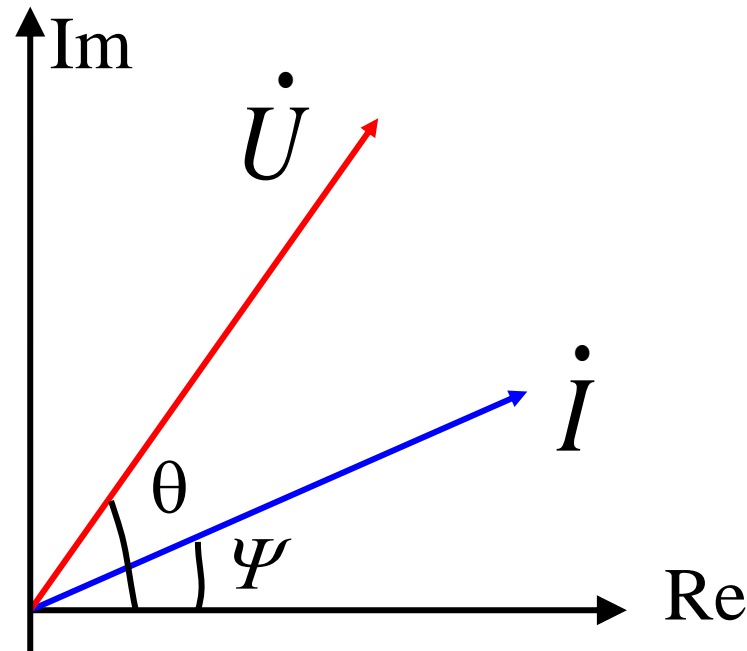
试写出电流的瞬时值表达式。

$$i = 50\sqrt{2} \cos(314t + 15^\circ) \text{ A}$$

3. 相量图 → 在复平面上用向量表示相量的图

$$i(t) = \sqrt{2}I\cos(\omega t + \Psi) \rightarrow \dot{I} = I \angle \Psi$$

$$u(t) = \sqrt{2}U\cos(\omega t + \theta) \rightarrow \dot{U} = U \angle \theta$$



4. 正弦量运算的相量方法

$$\boxed{\times k} \quad ku = \sqrt{2}kU \cos(\omega t + \theta)$$

$$\boxed{ku} \longleftrightarrow \boxed{k\dot{U}}$$

$$\boxed{\pm}$$

$$u_1 = \sqrt{2}U_1 \cos(\omega t + \theta_1) \quad \dot{U}_1 = U_1 e^{j\theta_1} = U_1 \angle \theta_1$$

$$u_2 = \sqrt{2}U_2 \cos(\omega t + \theta_2) \quad \dot{U}_2 = U_2 e^{j\theta_2} = U_2 \angle \theta_2$$

$$\begin{aligned} u_1 \pm u_2 &= \operatorname{Re}[\sqrt{2}U_1 e^{j(\omega t + \theta_1)}] \pm \operatorname{Re}[\sqrt{2}U_2 e^{j(\omega t + \theta_2)}] \\ &= \operatorname{Re}[\sqrt{2} \cdot (U_1 e^{j\theta_1} \pm U_2 e^{j\theta_2}) \cdot e^{j\omega t}] \\ &= \operatorname{Re}[\sqrt{2} \cdot U e^{j\theta} \cdot e^{j\omega t}] \end{aligned}$$

$$\boxed{u = u_1 \pm u_2} \longleftrightarrow \boxed{\dot{U} = \dot{U}_1 \pm \dot{U}_2}$$

$$\begin{array}{ccc} i_1 & \pm & i_2 = i_3 \\ \updownarrow & & \updownarrow \quad \updownarrow \\ I_1 & \pm & I_2 = I_3 \end{array}$$

例 $u_1(t) = 6\sqrt{2}\cos(314t + 30^\circ) \text{ V}$
 $u_2(t) = 4\sqrt{2}\cos(314t + 60^\circ) \text{ V}$ $\rightarrow \left\{ \begin{array}{l} \dot{U}_1 = 6\angle 30^\circ \text{ V} \\ \dot{U}_2 = 4\angle 60^\circ \text{ V} \end{array} \right.$

$$\dot{U} = \dot{U}_1 + \dot{U}_2 = 6\angle 30^\circ + 4\angle 60^\circ$$

$$= 5.19 + j3 + 2 + j3.46 = 7.19 + j6.46$$

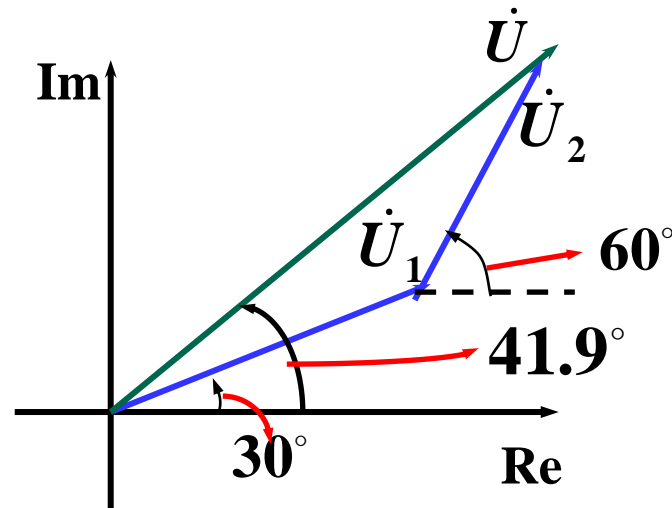
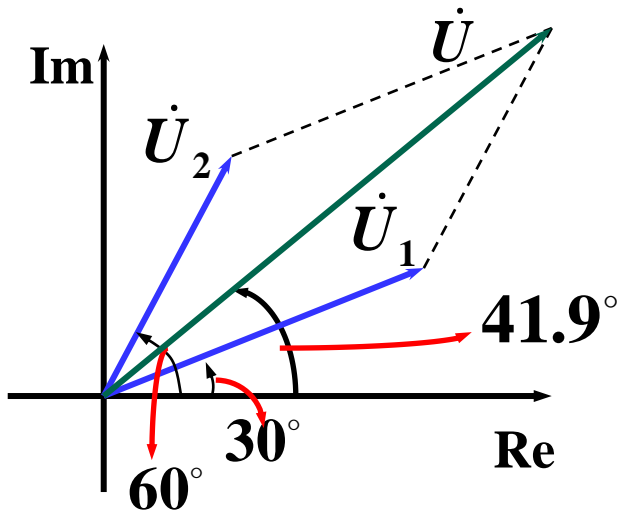
$$= 9.64\angle 41.9^\circ \text{ V}$$

$$\therefore u(t) = u_1(t) + u_2(t) = 9.64\sqrt{2}\cos(314t + 41.9^\circ) \text{ V}$$

借助相量图计算

$$\dot{U}_1 = 6\angle 30^\circ \text{ V}$$

$$\dot{U}_2 = 4\angle 60^\circ \text{ V}$$



同频正弦量的加、减运算可借助相量图进行。相量图在正弦稳态分析中有重要作用，尤其适用于定性分析。

$$\frac{d}{dt}$$

$$u = \sqrt{2}U \cos(\omega t + \theta) \quad \dot{U} = Ue^{j\theta} = U \angle \theta$$

$$u_d = \frac{du}{dt} = \frac{d}{dt} \{ \text{Re}[\sqrt{2}Ue^{j(\omega t + \theta)}] \}$$

$$= \text{Re} \left\{ \frac{d}{dt} [\sqrt{2}Ue^{j(\omega t + \theta)}] \right\} = \text{Re}[\sqrt{2}Ue^{j\theta} \cdot j\omega \cdot e^{j\omega t}]$$

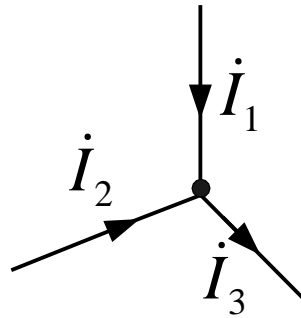
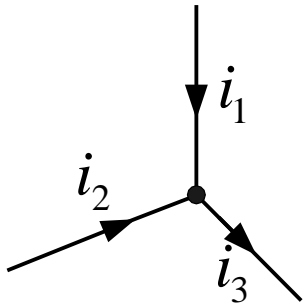
$$= \text{Re}[\sqrt{2}\dot{U} \cdot j\omega \cdot e^{j\omega t}]$$

$$u_d = \frac{du}{dt} \longleftrightarrow \dot{U}_d = j\omega \dot{U}$$

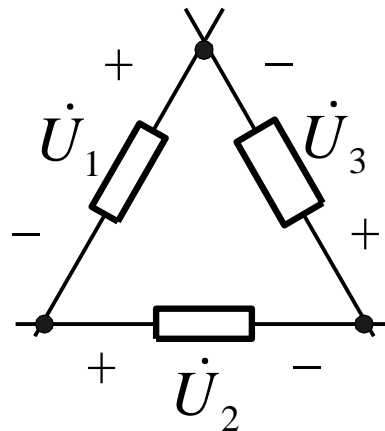
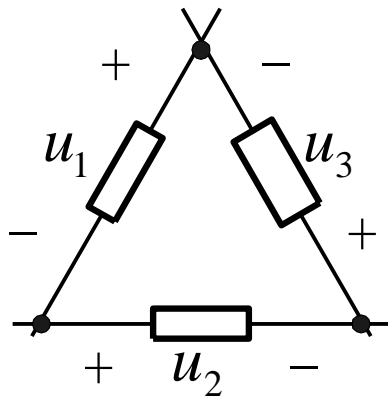
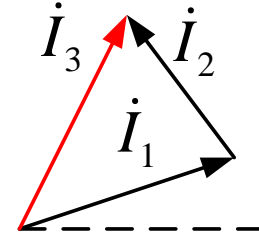
$$\int dt$$

$$u_i = \int u dt \longleftrightarrow \dot{U}_i = \frac{\dot{U}}{j\omega}$$

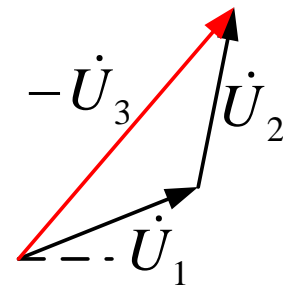
KCL & KVL的相量形式



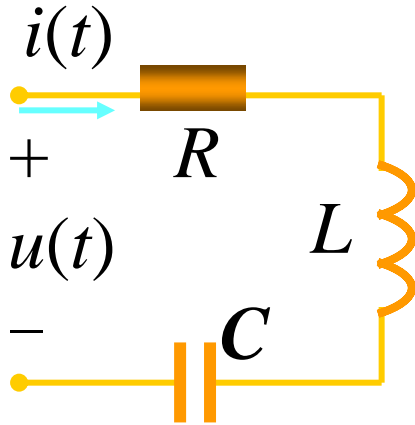
$$\dot{I}_1 + \dot{I}_2 - \dot{I}_3 = 0$$



$$\dot{U}_1 + \dot{U}_2 + \dot{U}_3 = 0$$



例



$$i(t) = \sqrt{2} I \cos(\omega t + \psi_i)$$

$$u(t) = Ri + L \frac{di}{dt} + \frac{1}{C} \int i dt$$

用相量运算：

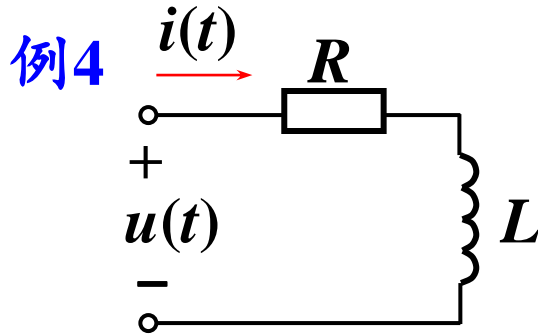
$$\dot{U} = R\dot{I} + j\omega L\dot{I} + \frac{\dot{I}}{j\omega C}$$

相量法的优点

- ✓ 把时域问题变为复数问题；
- ✓ 把微积分方程的运算变为复数方程运算；
- ✓ 可以把直流电路的分析方法直接用于交流电路。

5. 相量法的应用

求解正弦电流电路的**稳态解**(微分方程的特解)。



$$u(t) = U_m \sin(\omega t + \psi_u)$$

解: $u(t) = Ri(t) + L \frac{di(t)}{dt}$ 一阶常系数
线性微分方程

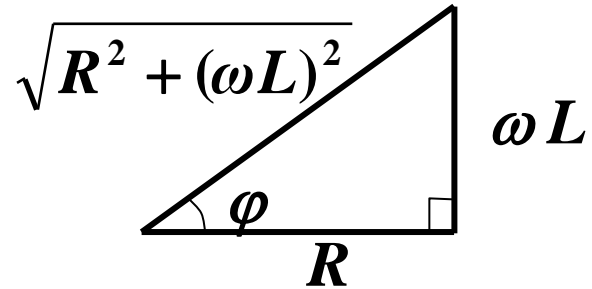
$$\left\{ \begin{array}{l} \text{自由分量(齐次方程通解): } Ae^{-(R/L)t} \\ \text{强制分量(特解): } I_m \sin(\omega t + \psi_i) \end{array} \right.$$

$$\begin{aligned} U_m \sin(\omega t + \psi_u) &= RI_m \sin(\omega t + \psi_i) + \omega LI_m \cos(\omega t + \psi_i) \\ &= \sqrt{(RI_m)^2 + (\omega LI_m)^2} \sin(\omega t + \psi_i + \varphi) \end{aligned}$$

$$U_m = \sqrt{(RI_m)^2 + (\omega LI_m)^2} \Rightarrow I_m = \frac{U_m}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\psi_u = \psi_i + \varphi$$

$$\varphi = \arctan \frac{\omega L}{R}$$

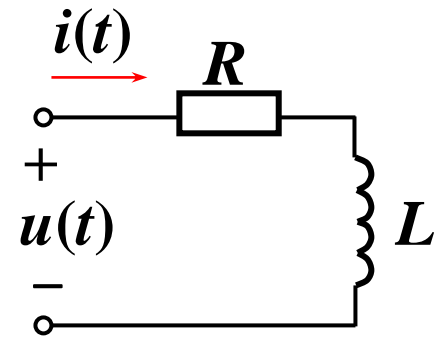


$$i = \frac{\sqrt{2}U}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t + \psi_u - \arctan \frac{\omega L}{R})$$

用相量法求：

$$u(t) = Ri(t) + L \frac{di(t)}{dt}$$

取相量 $\dot{U} = R\dot{I} + j\omega L\dot{I}$



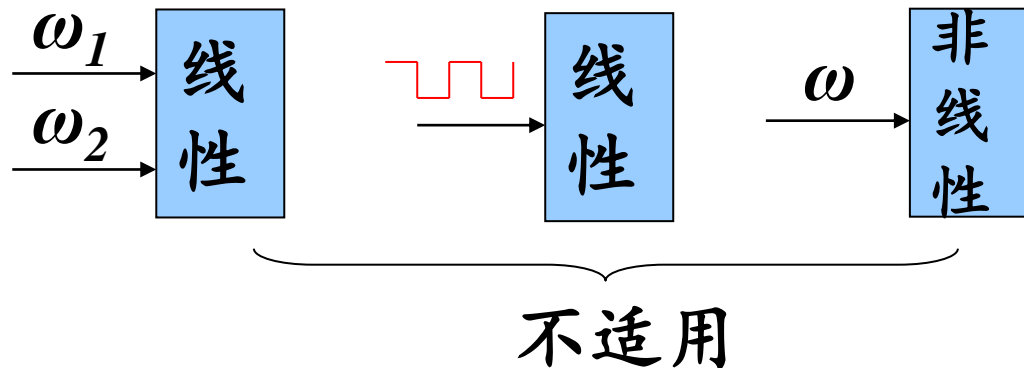
$$\dot{i} = \frac{\dot{U}}{R + j\omega L} = \frac{U \angle \psi_u}{\sqrt{R^2 + \omega^2 L^2} \angle \arctan \frac{\omega L}{R}} = \frac{U}{\sqrt{R^2 + \omega^2 L^2}} \angle (\psi_u - \arctan \frac{\omega L}{R})$$

$$i = \frac{\sqrt{2}U}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t + \psi_u - \arctan \frac{\omega L}{R})$$

小 结

- ① 正弦量 \longleftrightarrow 相量
时域 相量域
正弦波形图 \longleftrightarrow 相量图

- ② 相量法只适用于激励为同频 正弦量的线性时不变电路



- ③ 相量法可以用来分析正弦稳态电路

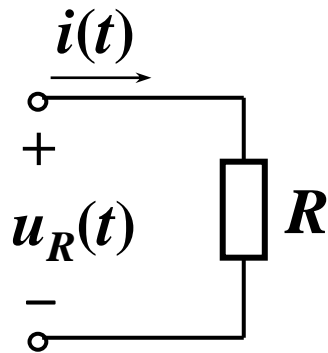
10.4 阻抗与导纳

一、元件特性的相量形式

1. 电阻

已知 $i(t) = \sqrt{2}I \sin(\omega t + \psi)$

则 $u_R(t) = Ri(t) = \sqrt{2}RI \sin(\omega t + \psi)$



相量形式:

$$\dot{I} = I \angle \psi$$

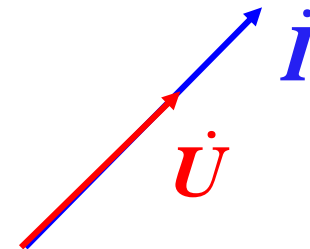
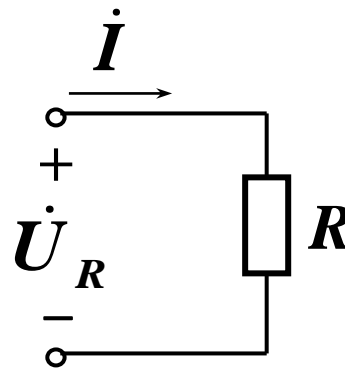
有效值关系: $U_R = RI$

$$\dot{U}_R = RI \angle \psi$$

相位关系: u, i 同相

相量关系

$$\dot{U}_R = R \dot{I}$$



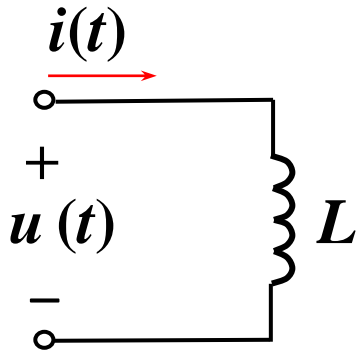
相量图

相量模型

电路理论

2. 电感

时域



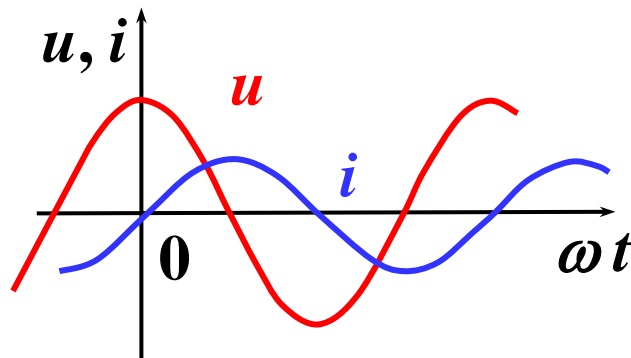
时域模型

$$i(t) = \sqrt{2}I \sin \omega t$$

$$u(t) = L \frac{di(t)}{dt}$$

$$= \sqrt{2}\omega L I \cos \omega t$$

$$= \sqrt{2}\omega L I \sin(\omega t + 90^\circ)$$



波形图

相量域

$$\dot{I} = I \angle 0^\circ$$

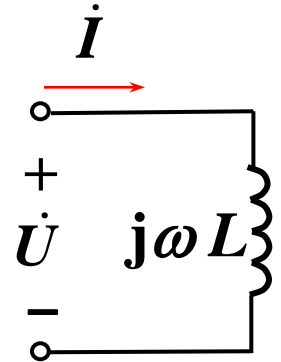
$$\dot{U} = j\omega L \dot{I}$$

有效值关系

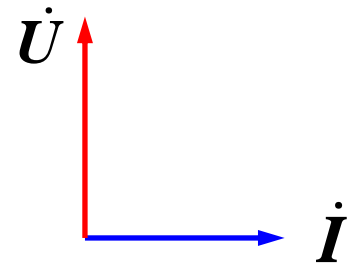
$$U = \omega L I$$

相位关系

u 超前 i 90°



相量模型



相量图

$$U = \omega L I$$

感抗(inductive reactance): $X_L = U/I = \omega L = 2\pi f L$, 单位: Ω

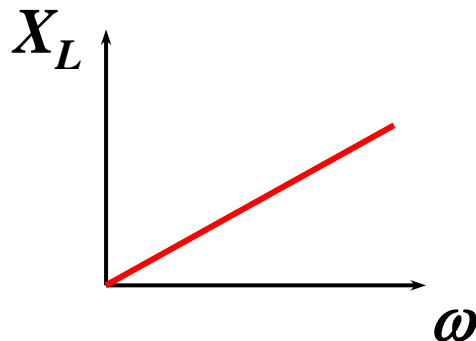
错误的写法 $\omega L \neq \frac{u}{i}$ $\omega L \neq \frac{\dot{U}}{\dot{I}}$

感抗的物理意义:

(1) 表示限制电流的能力;

(2) 感抗和频率成正比。

通直流, 阻交流



$\omega = 0$ (直流), $X_L = 0$, 短路;

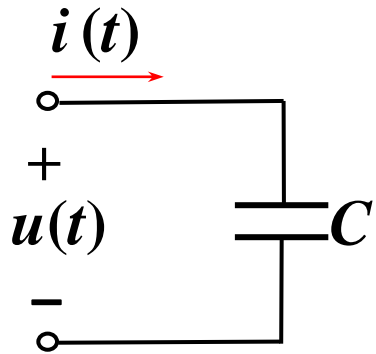
$\omega \rightarrow \infty$, $X_L \rightarrow \infty$, 开路;

(3) 由于感抗的存在使电流的相位落后电压。

感纳(inductive susceptance): $B_L = 1/X_L = 1/\omega L$, 单位: S

3. 电容

时域



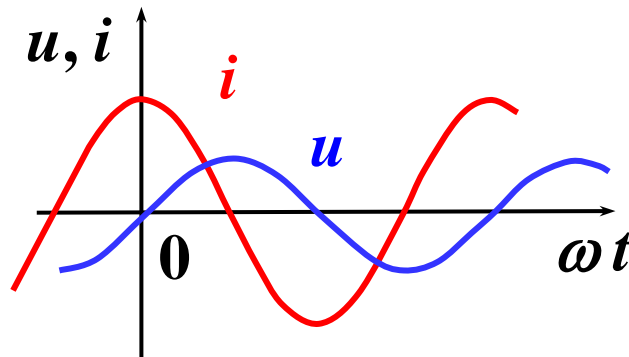
时域模型

$$u(t) = \sqrt{2}U \sin \omega t$$

$$i(t) = C \frac{du(t)}{dt}$$

$$= \sqrt{2}\omega C U \cos \omega t$$

$$= \sqrt{2}\omega C U \sin(\omega t + 90^\circ)$$



波形图

相量域

$$\dot{U} = U \angle 0^\circ$$

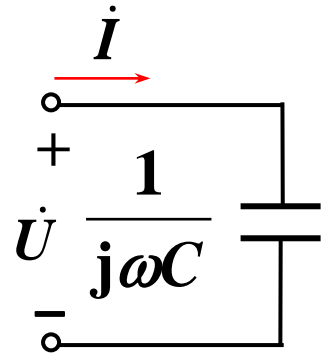
$$\dot{I} = j\omega C \dot{U}$$

有效值关系

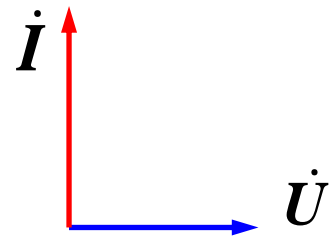
$$I = \omega C U$$

相位关系

i 超前 u 90°



相量模型



相量图

$$I = \omega C U$$

$$\frac{U}{I} = \frac{1}{\omega C}$$

$$X_c = \frac{1}{\omega C}$$

容抗 (*capacitive reactance*)

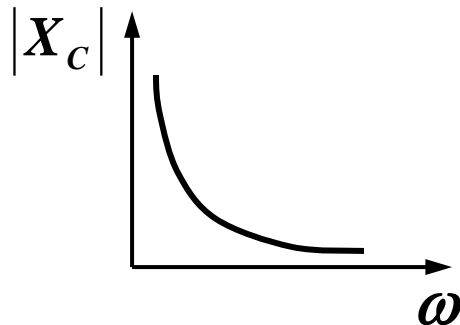
错误的写法

$$\frac{1}{\omega C} \neq \frac{u}{i}$$

$$\frac{1}{\omega C} \neq \frac{\dot{U}}{\dot{I}}$$

容抗的物理意义：

- (1) 表示限制电流的能力；
- (2) 容抗的绝对值和频率成反比。

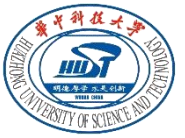


$\omega = 0$ (直流), $|X_c| \rightarrow \infty$, 隔直作用；

$\omega \rightarrow \infty$, $X_c \rightarrow 0$, 旁路作用；

- (3) 由于容抗的存在使电流领先电压。

容纳 (*capacitive susceptance*): $B_c = 1/X_c = \omega C$, 单位: S



电路元件的相量关系：

$$u = Ri \quad \Rightarrow \quad \dot{U} = R\dot{I}$$

$$u = L \frac{di}{dt} \quad \Rightarrow \quad \dot{U} = j\omega L \dot{I}$$

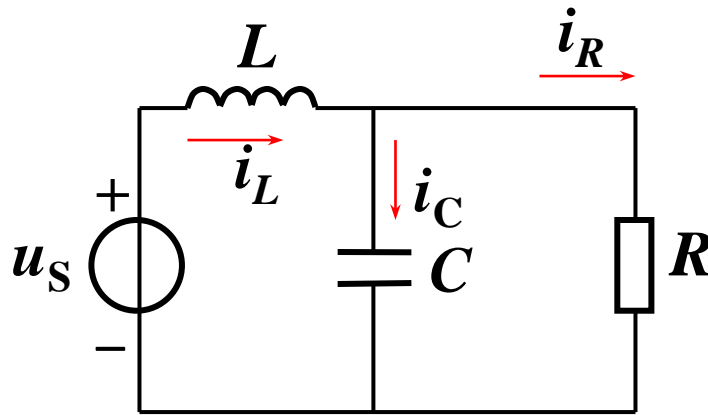
$$u = \frac{1}{C} \int i dt \quad \Rightarrow \quad \dot{U} = \frac{1}{j\omega C} \dot{I}$$

基尔霍夫定律的相量形式：

$$\sum i(t) = 0 \quad \Rightarrow \quad \sum \dot{I} = 0$$

$$\sum u(t) = 0 \quad \Rightarrow \quad \sum \dot{U} = 0$$

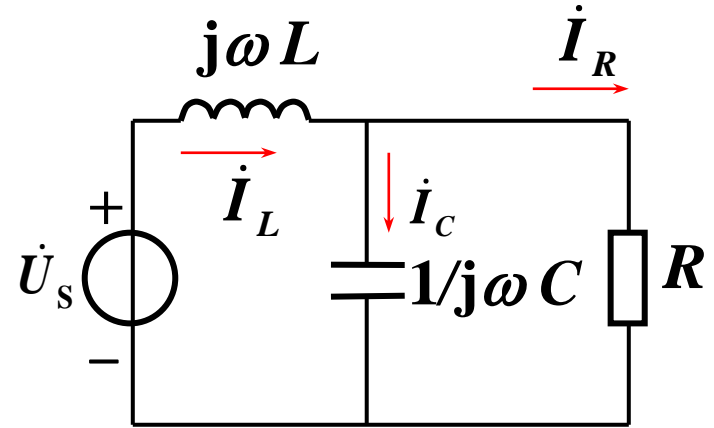
5. 电路的相量模型与相量法



时域电路

$$\begin{cases} i_L = i_C + i_R \\ L \frac{di_L}{dt} + \frac{1}{C} \int i_C dt = u_s \\ Ri_R = \frac{1}{C} \int i_C dt \end{cases}$$

时域列写微分方程



相量模型

$$\begin{cases} \dot{I}_L = \dot{I}_C + \dot{I}_R \\ j\omega L \dot{I}_L + \frac{1}{j\omega C} \dot{I}_C = \dot{U}_s \\ R \dot{I}_R = \frac{1}{j\omega C} \dot{I}_C \end{cases}$$

相量形式代数方程

A. 画
相量
模型

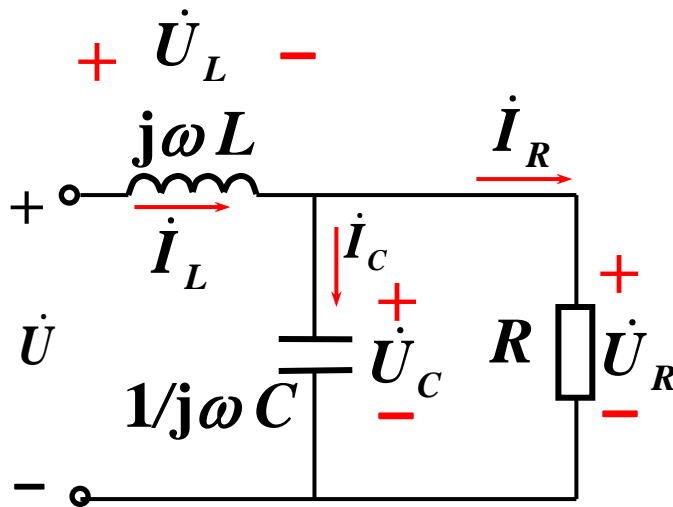
B. 列
代数
方程

D. 变换正弦量

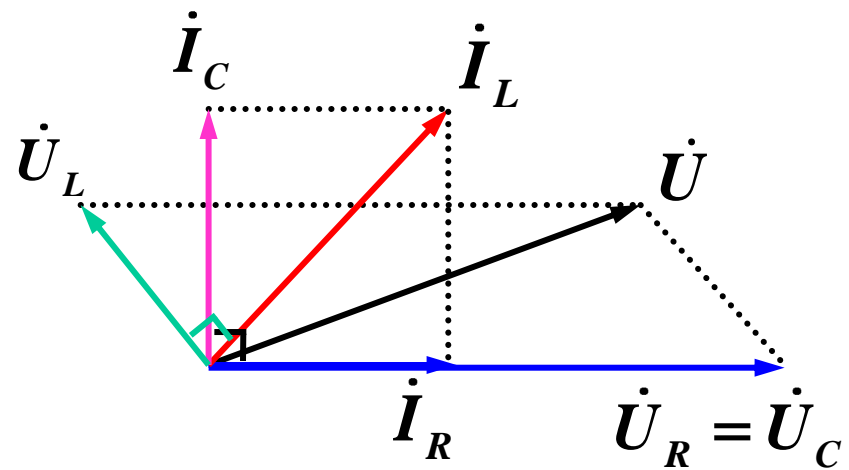
C. 解代数方程

6. 相量图(phasor diagram)

- (1) 同频率的正弦量才能表示在同一个相量图中；
- (2) 选定一个参考相量(设初相位为零)； **非常关键!!!**
- (3) 根据相位关系确定其他相量。



选 \dot{U}_R 为参考相量



例 已知 $i(t) = 5\sqrt{2} \cos(10^6 t + 15^\circ)$, 求: $u_s(t)$



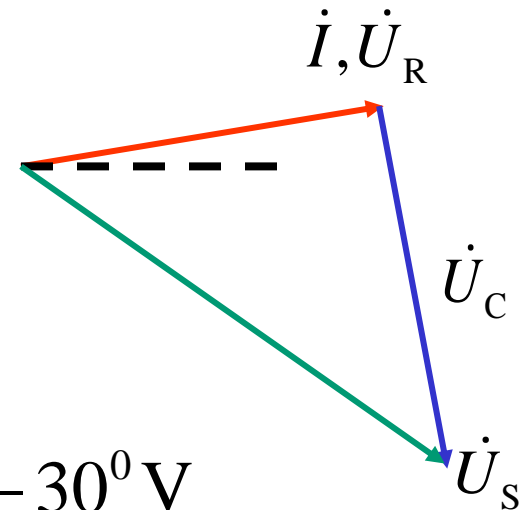
解

$$\dot{I} = 5\angle 15^\circ$$

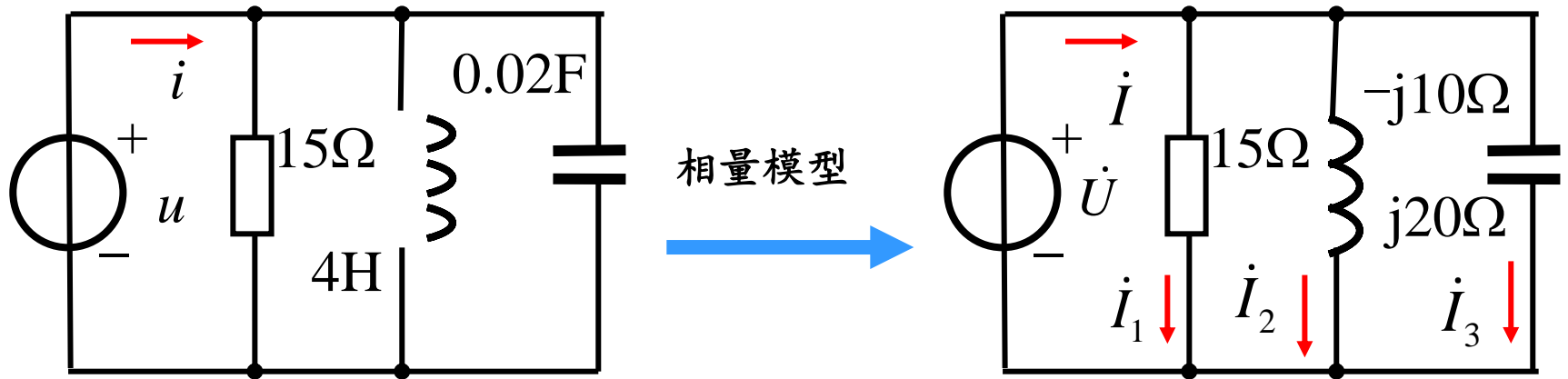
$$-jX_C = -j \frac{1}{10^6 \times 0.2 \times 10^{-6}} = -j5\Omega$$

$$\dot{U}_S = \dot{U}_R + \dot{U}_C = 5\angle 15^\circ (5 - j5)$$

$$= 5\angle 15^\circ \times 5\sqrt{2}\angle -45^\circ = 25\sqrt{2}\angle -30^\circ \text{ V}$$



例 已知 $u(t) = 120\sqrt{2} \cos(5t)$, 求: $i(t)$

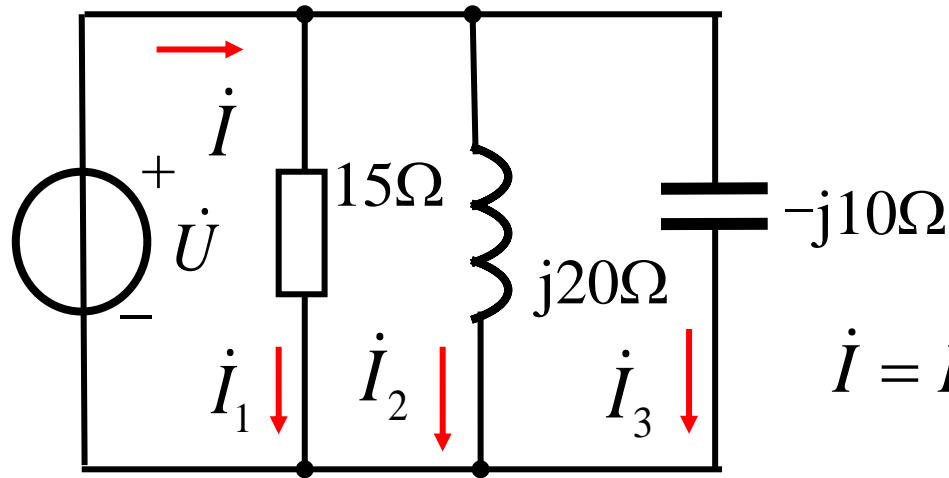


解

$$\dot{U} = 120\angle 0^\circ$$

$$jX_L = j4 \times 5 = j20\Omega$$

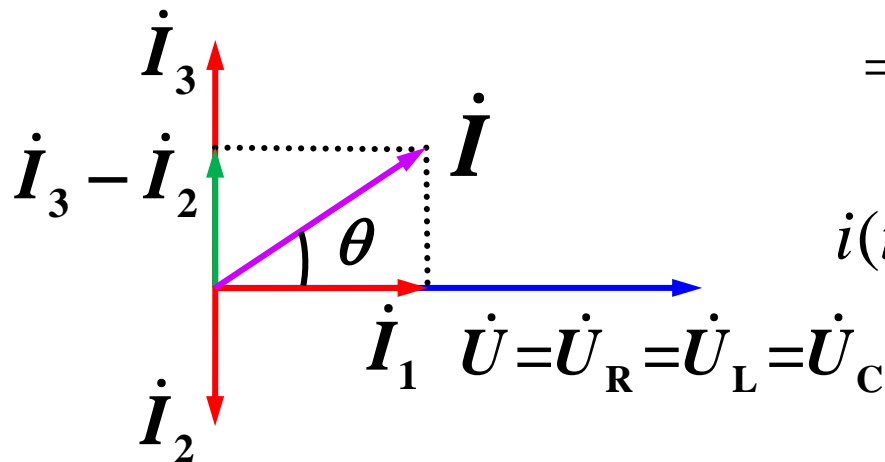
$$-jX_C = -j \frac{1}{5 \times 0.02} = -j10\Omega$$



$$\dot{I} = \dot{I}_R + \dot{I}_L + \dot{I}_C = \frac{\dot{U}}{R} + \frac{\dot{U}}{jX_L} + \frac{\dot{U}}{-jX_C}$$

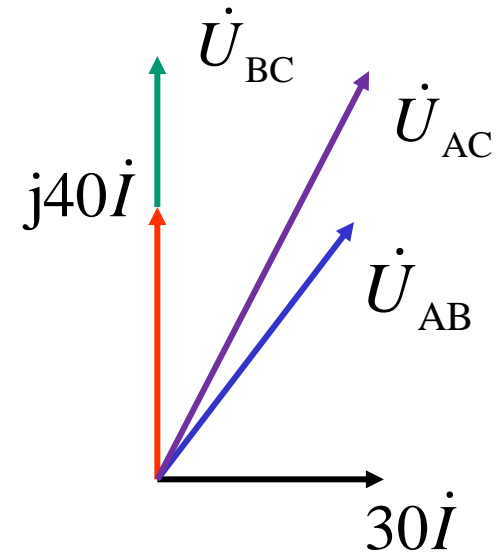
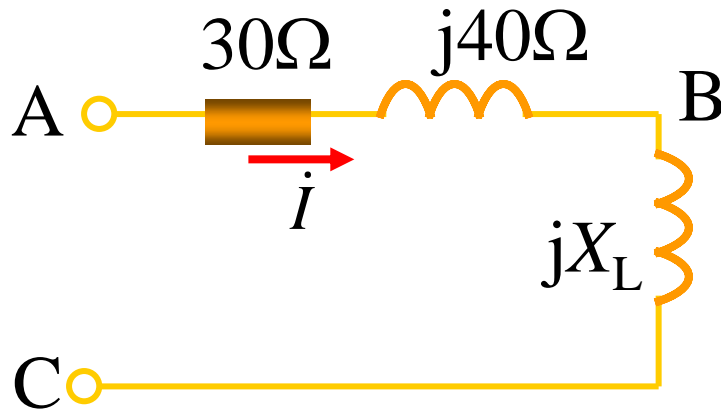
$$= 120 \left(\frac{1}{15} + \frac{1}{j20} - \frac{1}{j10} \right)$$

$$= 8 - j6 + j12 = 8 + j6 = 10 \angle 36.9^\circ \text{ A}$$



$$i(t) = 10\sqrt{2} \cos(5t + 36.9^\circ) \text{ A}$$

例 已知 $U_{AB} = 50V$, $U_{AC} = 78V$, 求: $U_{BC} = ?$



解

$$U_{AB} = \sqrt{(30I)^2 + (40I)^2} = 50I$$

→ $I = 1A, \quad U_R = 30V, \quad U_L = 40V$

$$U_{AC} = 78 = \sqrt{(30)^2 + (40 + U_{BC})^2}$$

→ $U_{BC} = \sqrt{(78)^2 - (30)^2} - 40 = 32V$

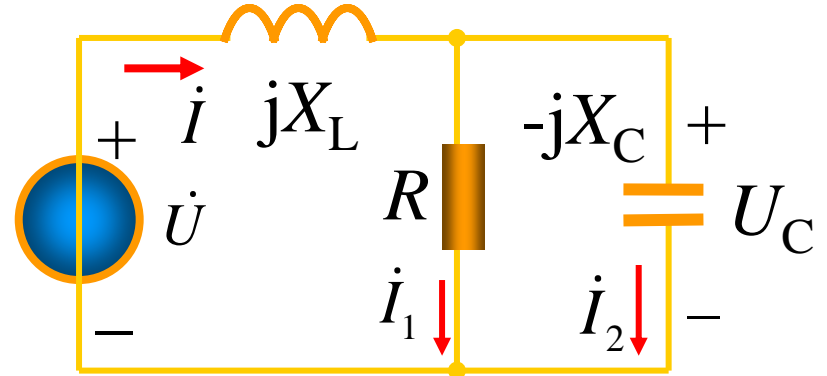
例 图示电路 $I_1=I_2=5\text{A}$, $U=50\text{V}$, 总电压与总电流同相位, 求 I 、 R 、 X_C 、 X_L 。

解法1

设 $\dot{U}_C = U_C \angle 0^\circ$

→ $\dot{I}_1 = 5 \angle 0^\circ$, $\dot{I}_2 = j5$

$$\dot{I} = 5 + j5 = 5\sqrt{2} \angle 45^\circ$$



$$\dot{U} = 50 \angle 45^\circ = (5 + j5) \times jX_L + 5R = \frac{50}{\sqrt{2}} (1 + j)$$

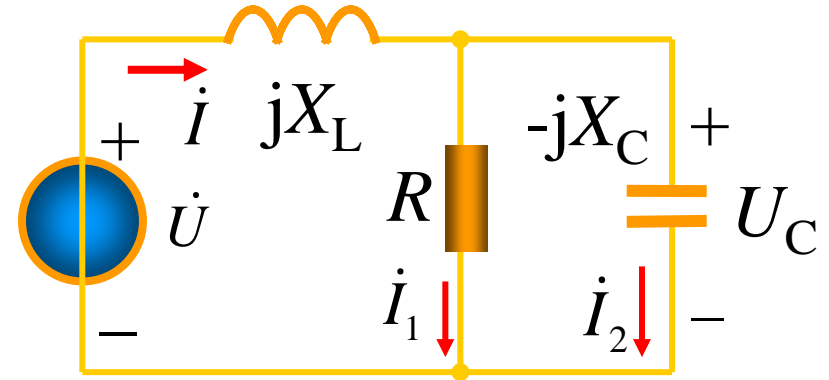
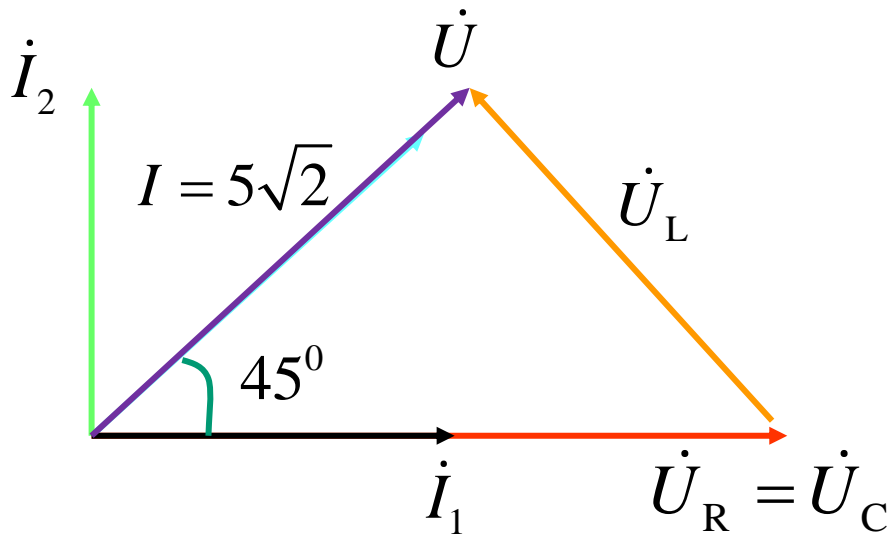
令等式两边实部等于实部, 虚部等于虚部

$$5X_L = 50/\sqrt{2} \Rightarrow X_L = 5\sqrt{2}$$

$$5R = \frac{50}{\sqrt{2}} + 5 \times 5\sqrt{2} = 50\sqrt{2} \Rightarrow R = |X_C| = 10\sqrt{2} \Omega$$

例 图示电路 $I_1 = I_2 = 5\text{A}$, $U = 50\text{V}$, 总电压与总电流同相位, 求 I 、 R 、 X_C 、 X_L 。

解法2 画相量图计算



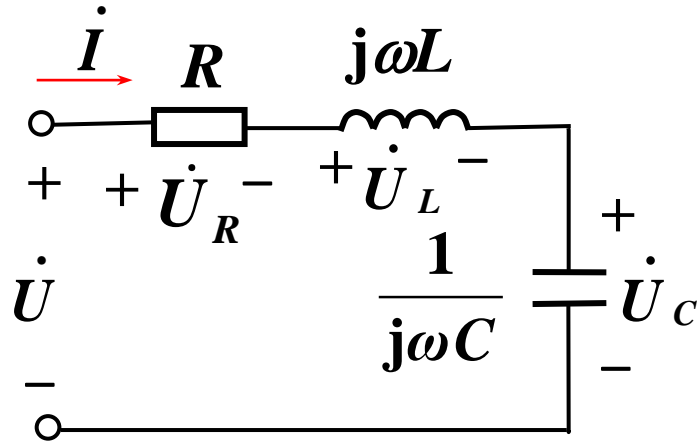
$$U = U_L = 50\text{V}$$

$$X_L = \frac{50}{5\sqrt{2}} = 5\sqrt{2}\Omega$$

$$|X_C| = R = \frac{50\sqrt{2}}{5} = 10\sqrt{2}\Omega$$

10.4 复阻抗和复导纳

1. 复阻抗(complex impedance)



复阻抗 $Z = \frac{\dot{U}}{\dot{I}} = \frac{\dot{U}_R + \dot{U}_L + \dot{U}_C}{\dot{I}}$

$$= R + j\omega L + \frac{1}{j\omega C}$$

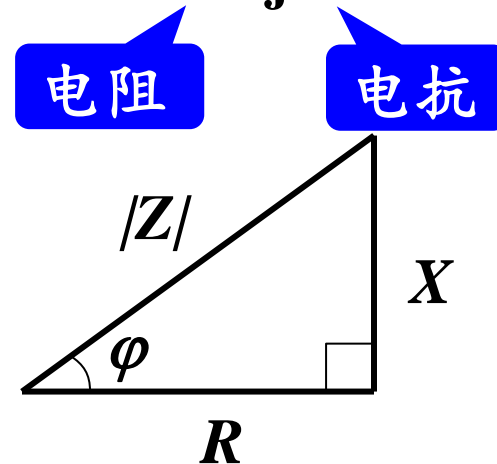
$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$= R + j(X_L - X_C)$$

$$= R + jX$$

$$Z = R + jX = |Z| \angle \varphi$$

$$\left\{ \begin{array}{l} |Z| = \frac{U}{I} \quad \text{阻抗模} \quad \text{单位: } \Omega \\ \varphi = \psi_u - \psi_i \quad \text{阻抗角} \end{array} \right.$$



阻抗三角形

具体分析一下 RLC 串联电路：

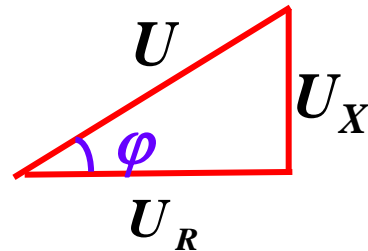
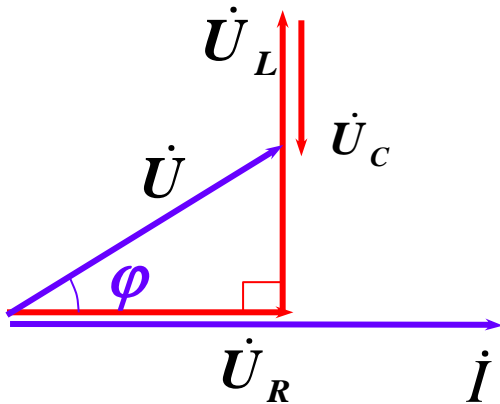
$$Z = R + j(\omega L - 1/\omega C) = |Z| \angle \varphi$$

$\omega L > 1/\omega C$, $X > 0$, $\varphi > 0$, 电压领先电流, 电路呈感性;

$\omega L < 1/\omega C$, $X < 0$, $\varphi < 0$, 电压落后电流, 电路呈容性;

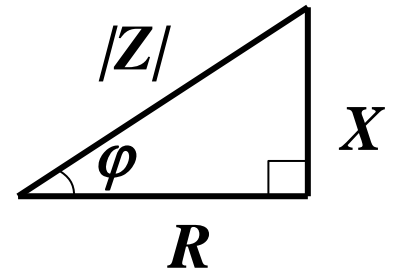
$\omega L = 1/\omega C$, $X = 0$, $\varphi = 0$, 电压与电流同相, 电路呈电阻性。

画相量图：选电流为参考向量($\omega L > 1/\omega C$)



电压三角形

$$U = \sqrt{U_R^2 + U_X^2}$$



阻抗三角形

例 已知: $R=15\Omega$, $L=0.3\text{mH}$, $C=0.2\mu\text{F}$,

$$u = 5\sqrt{2}\cos(\omega t + 60^\circ), f = 3 \times 10^4 \text{ Hz}$$

求 i , u_R , u_L , u_C .

解

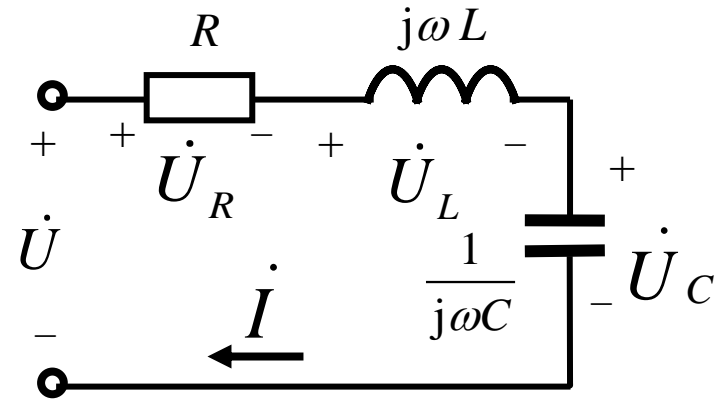
画出相量模型

$$\dot{U} = 5\angle 60^\circ \text{ V}$$

$$\begin{aligned} j\omega L &= j2\pi \times 3 \times 10^4 \times 0.3 \times 10^{-3} \\ &= j56.5 \Omega \end{aligned}$$

$$-j\frac{1}{\omega C} = -j\frac{1}{2\pi \times 3 \times 10^4 \times 0.2 \times 10^{-6}} = -j26.5\Omega$$

$$\begin{aligned} Z &= R + j\omega L - j\frac{1}{\omega C} = 15 + j56.5 - j26.5 \\ &= 33.54\angle 63.4^\circ \Omega \end{aligned}$$



$$\dot{I} = \frac{\dot{U}}{Z} = \frac{5\angle 60^\circ}{33.54\angle 63.4^\circ} = 0.149\angle -3.4^\circ \text{ A}$$

$$\dot{U}_R = R\dot{I} = 15 \times 0.149\angle -3.4^\circ = 2.235\angle -3.4^\circ \text{ V}$$

$$\dot{U}_L = j\omega L\dot{I} = 56.5\angle 90^\circ \times 0.149\angle -3.4^\circ = 8.42\angle 86.4^\circ \text{ V}$$

$$\dot{U}_C = -j\frac{1}{\omega C}\dot{I} = 26.5\angle -90^\circ \times 0.149\angle -3.4^\circ = 3.95\angle -93.4^\circ \text{ V}$$

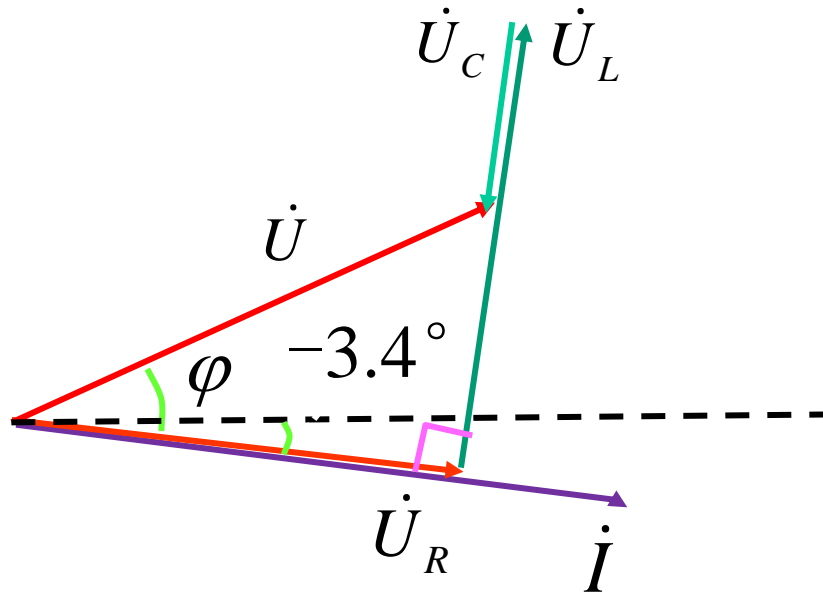
则 $i = 0.149\sqrt{2}\cos(\omega t - 3.4^\circ) \text{ A}$

$$u_R = 2.235\sqrt{2}\cos(\omega t - 3.4^\circ) \text{ V}$$

$$u_L = 8.42\sqrt{2}\cos(\omega t + 86.6^\circ) \text{ V}$$

$$u_C = 3.95\sqrt{2}\cos(\omega t - 93.4^\circ) \text{ V}$$

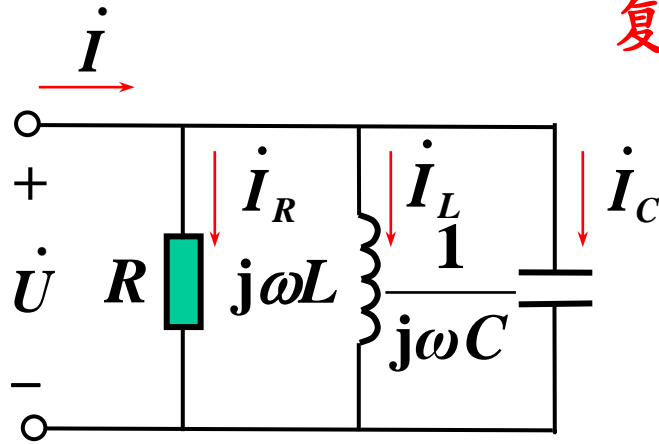
相量图



注意

$U_L = 8.42 > U = 5$, 分电压大于总电压。

2. 复导纳(admittance)



复导纳 $Y = \frac{\dot{I}}{\dot{U}} = \frac{\dot{I}_R + \dot{I}_L + \dot{I}_C}{\dot{U}}$

$$= \frac{1}{R} + \frac{1}{j\omega L} + \frac{1}{1/j\omega C}$$

$$= G - j\frac{1}{\omega L} + j\omega C$$

$$= G + j(B_C - B_L)$$

$$= G + jB$$

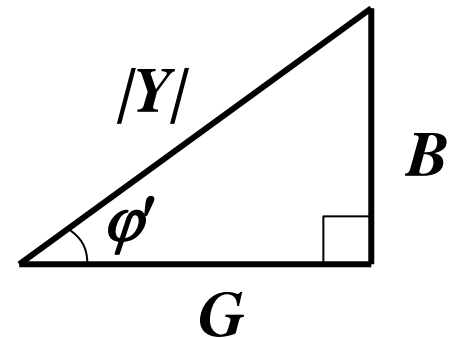
$$Y = \frac{\dot{I}}{\dot{U}} = G + jB = |Y| \angle \varphi'$$

电导

电纳

$$\begin{cases} |Y| = \frac{I}{U} & \text{导纳的模} \\ \varphi' = \psi_i - \psi_u & \text{导纳角} \end{cases}$$

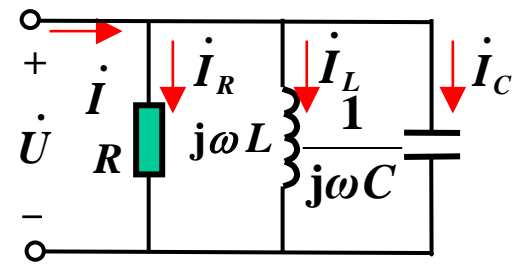
单位: S



导纳三角形

具体分析一下 RLC 并联电路

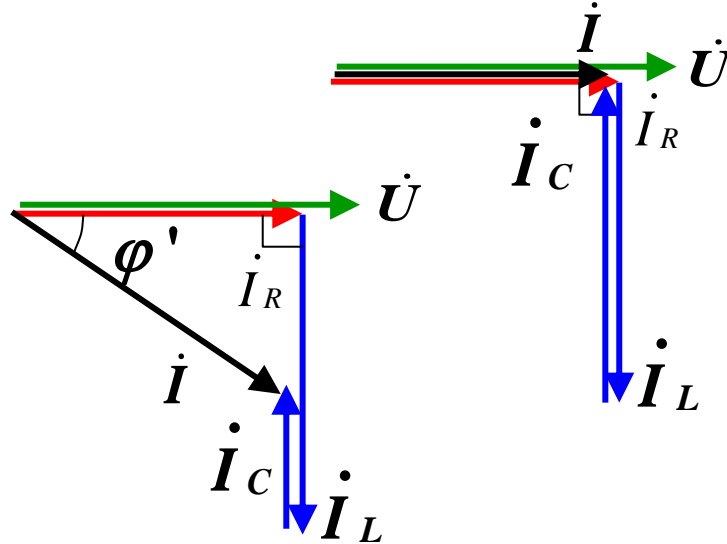
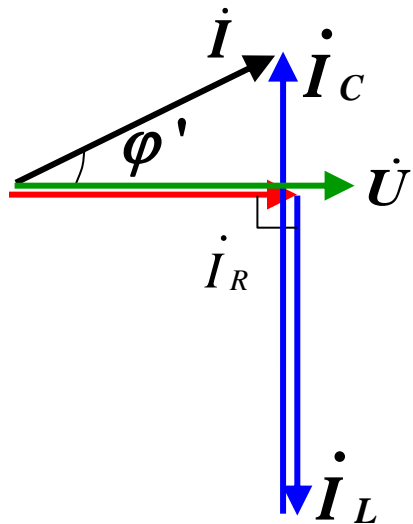
$$Y = G + j(\omega C - 1/\omega L) = |Y| \angle j'$$



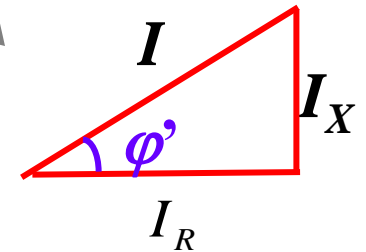
$\omega C > 1/\omega L$, $B > 0$, $j' > 0$, 电压落后电流, 电路呈容性;

$\omega C < 1/\omega L$, $B < 0$, $j' < 0$, 电压领先电流, 电路呈感性;

$\omega C = 1/\omega L$, $B = 0$, $j' = 0$, 电压与电流同相, 电路呈阻性。

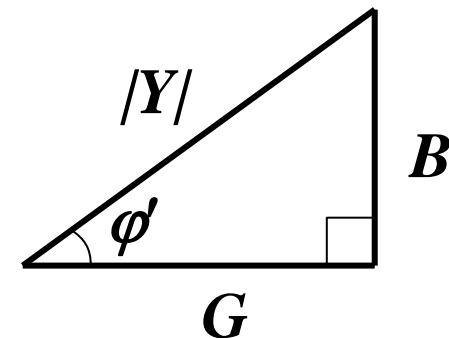


电流三角形

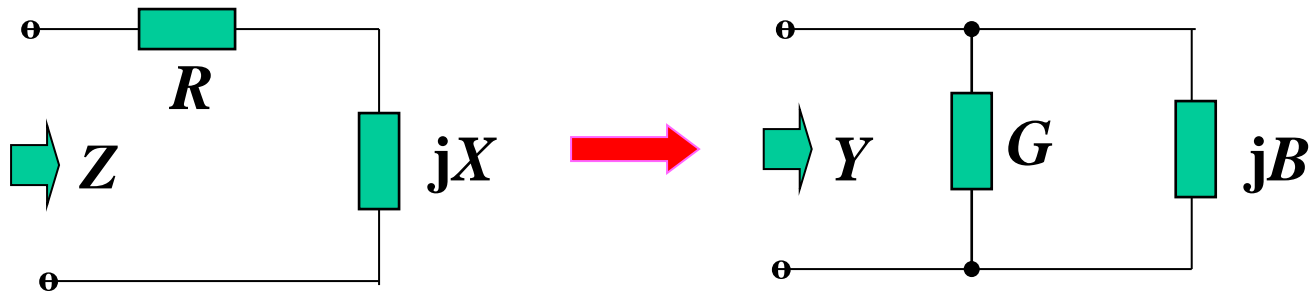


相似三角形

导纳三角形



3. 复阻抗和复导纳的等效变换



$$Z = R + jX = |Z| \angle \varphi \quad \Rightarrow \quad Y = G + jB = |Y| \angle \varphi'$$

$$Y = \frac{1}{Z} = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2} = G + jB$$

$$Y = \frac{1}{Z}$$

$$\therefore G = \frac{R}{R^2 + X^2}, \quad B = \frac{-X}{R^2 + X^2}$$

$$|Y| = \frac{1}{|Z|}, \quad \varphi' = -\varphi$$

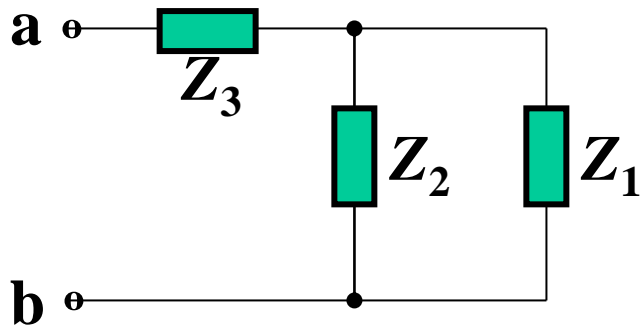
一般情况 $G \neq 1/R$ $B \neq 1/X$

4. 阻抗串、并联

串联: $Z = \sum Z_k$, $\dot{U}_k = \frac{Z_k}{\sum Z_k} \dot{U}$

并联: $Y = \sum Y_k$, $\dot{I}_k = \frac{Y_k}{\sum Y_k} \dot{I}$

例 已知 $Z_1=10+j6.28\Omega$
 $Z_2=20-j31.9\Omega$
 $Z_3=15+j15.7\Omega$
 求 Z_{ab} 。



$$Z_{ab} = Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2} = Z_3 + Z$$

$$Z = \frac{(10 + j6.28)(20 - j31.9)}{10 + j6.28 + 20 - j31.9}$$

$$= \frac{11.81 \angle 32.13^\circ \times 37.65 \angle -57.61^\circ}{39.45 \angle -40.5^\circ}$$

$$= 10.89 + j2.86\Omega$$

$$\begin{aligned} \therefore Z_{ab} &= Z_3 + Z = 15 + j15.7 + 10.89 + j2.86 \\ &= 25.89 + j18.56 = 31.9 \angle 35.6^\circ \Omega \end{aligned}$$

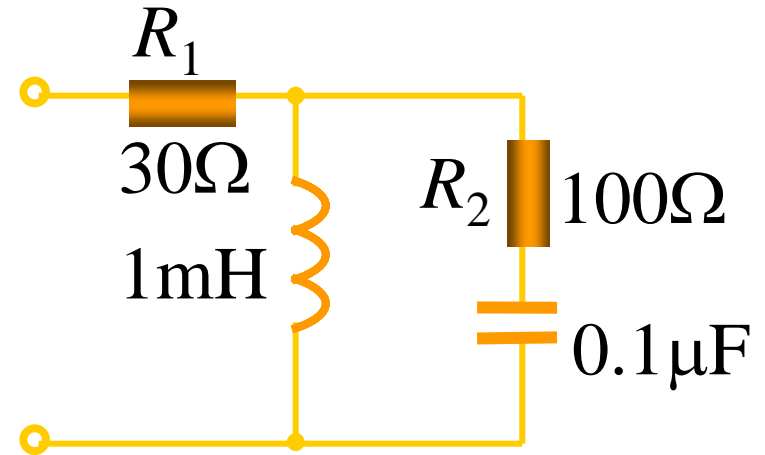
例 求图示电路的等效阻抗, $\omega = 10^5 \text{ rad/s}$ 。

解 感抗和容抗为:

$$X_L = \omega L = 10^5 \times 1 \times 10^{-3} = 100 \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{10^5 \times 0.1 \times 10^{-6}} = 100 \Omega$$

$$\begin{aligned} Z &= R_1 + \frac{jX_L(R_2 - jX_C)}{jX_L + R_2 - jX_C} = 30 + \frac{j100 \times (100 - j100)}{100} \\ &= 130 + j100 \Omega \end{aligned}$$

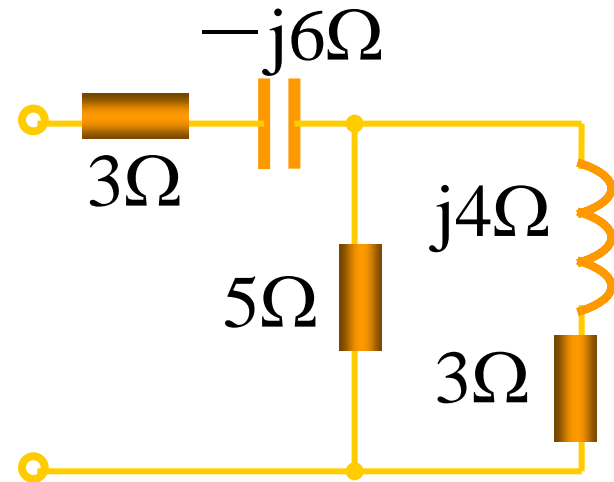


例 图示电路对外呈现感性还是容性？

解

等效阻抗为：

$$\begin{aligned} Z &= 3 - j6 + \frac{5(3 + j4)}{5 + (3 + j4)} \\ &= 3 - j6 + \frac{25 \angle 53.1^\circ}{8 + j4} = 5.5 - j4.75 \Omega \end{aligned}$$



电路对外呈现容性

10.5 正弦稳态电路的分析

电阻电路与正弦电流电路的分析比较：

电阻电路：

$$\left\{ \begin{array}{l} \text{KCL: } \sum i = 0 \\ \text{KVL: } \sum u = 0 \\ u = Ri \quad \text{或} \quad i = Gu \end{array} \right.$$

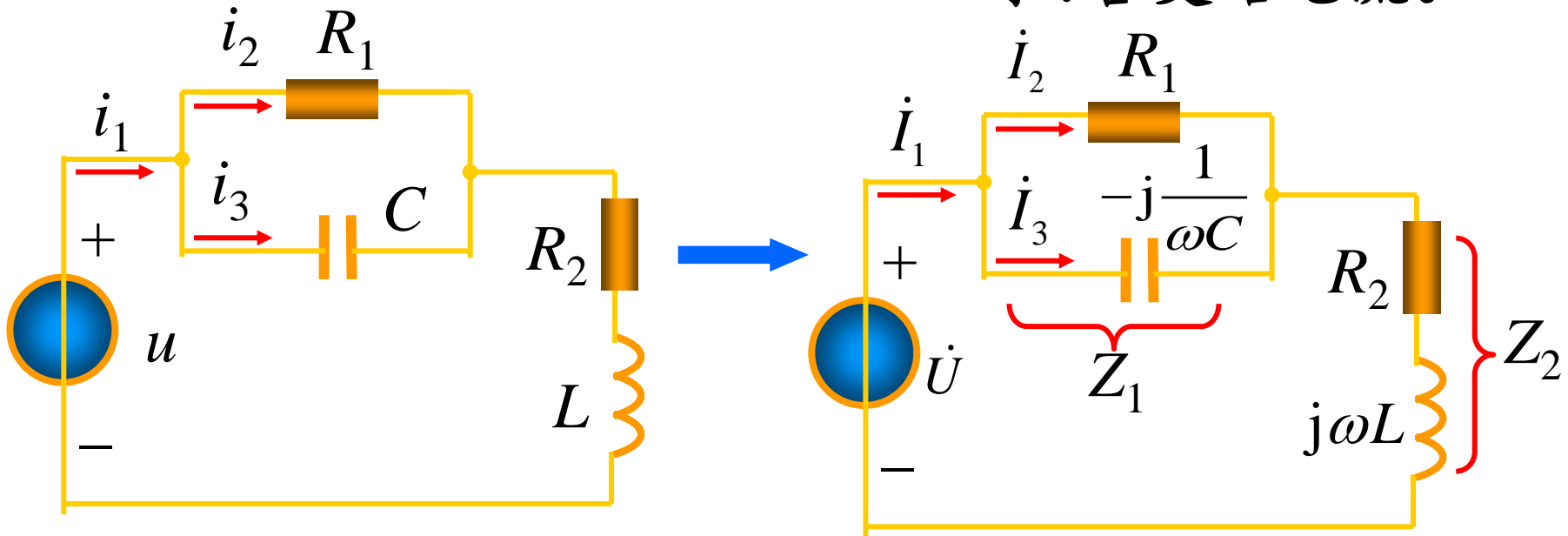
正弦电路相量分析：

$$\left\{ \begin{array}{l} \text{KCL: } \sum \dot{I} = 0 \\ \text{KVL: } \sum \dot{U} = 0 \\ \dot{U} = Z \dot{I} \quad \text{或} \quad \dot{I} = Y \dot{U} \end{array} \right.$$

结论

1. 引入相量法，电阻电路和正弦电流电路依据的电路定律是相似的。
2. 引入电路的相量模型，把列写时域微分方程转为直接列写相量形式的代数方程。
3. 引入阻抗以后，可将电阻电路中讨论的所有网络定理和分析方法都推广应用于正弦稳态的相量分析中。直流 ($f=0$) 是一个特例。

例1 已知: $R_1 = 1000\Omega$, $R_2 = 10\Omega$, $L = 500\text{mH}$, $C = 10\mu\text{F}$,
 $U = 100\text{V}$, $\omega = 314\text{rad/s}$, 求: 各支路电流。



解

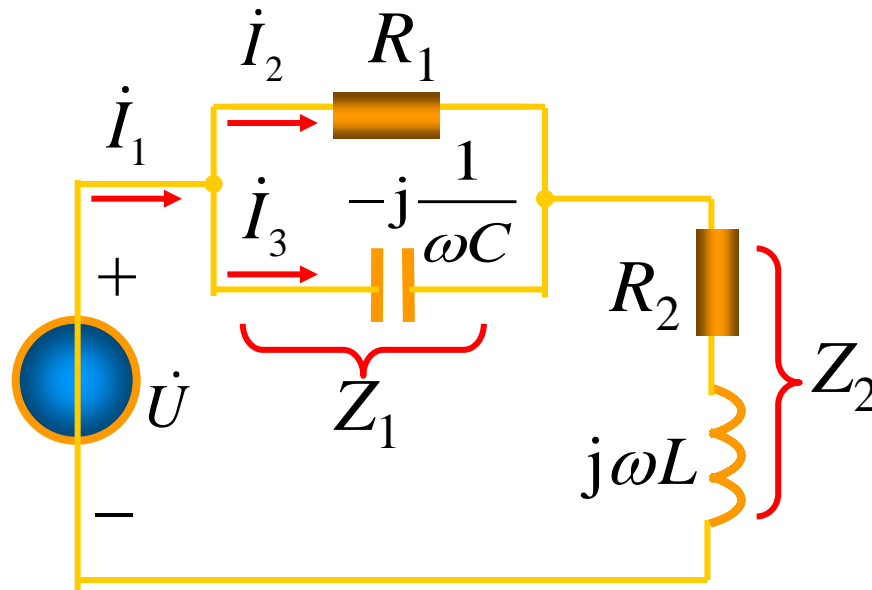
画出电路的相量模型

$$Z_1 = \frac{R_1(-j\frac{1}{\omega C})}{R_1 - j\frac{1}{\omega C}} = \frac{1000 \times (-j318.47)}{1000 - j318.47} = \frac{318.47 \times 10^3 \angle -90^\circ}{1049.5 \angle -17.7^\circ}$$

$$Z_1 = 303.45 \angle -72.3^\circ = 92.11 - j289.13 \, \Omega$$

$$Z_2 = R_2 + j\omega L = 10 + j157 \, \Omega$$

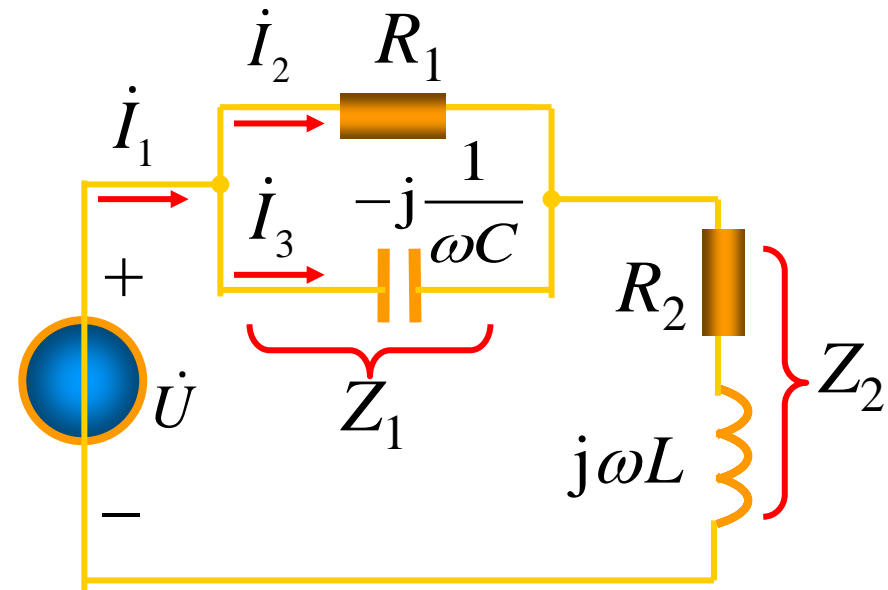
$$\begin{aligned} Z &= Z_1 + Z_2 = 92.11 - j289.13 + 10 + j157 \\ &= 102.11 - j132.13 = 166.99 \angle -52.3^\circ \, \Omega \end{aligned}$$



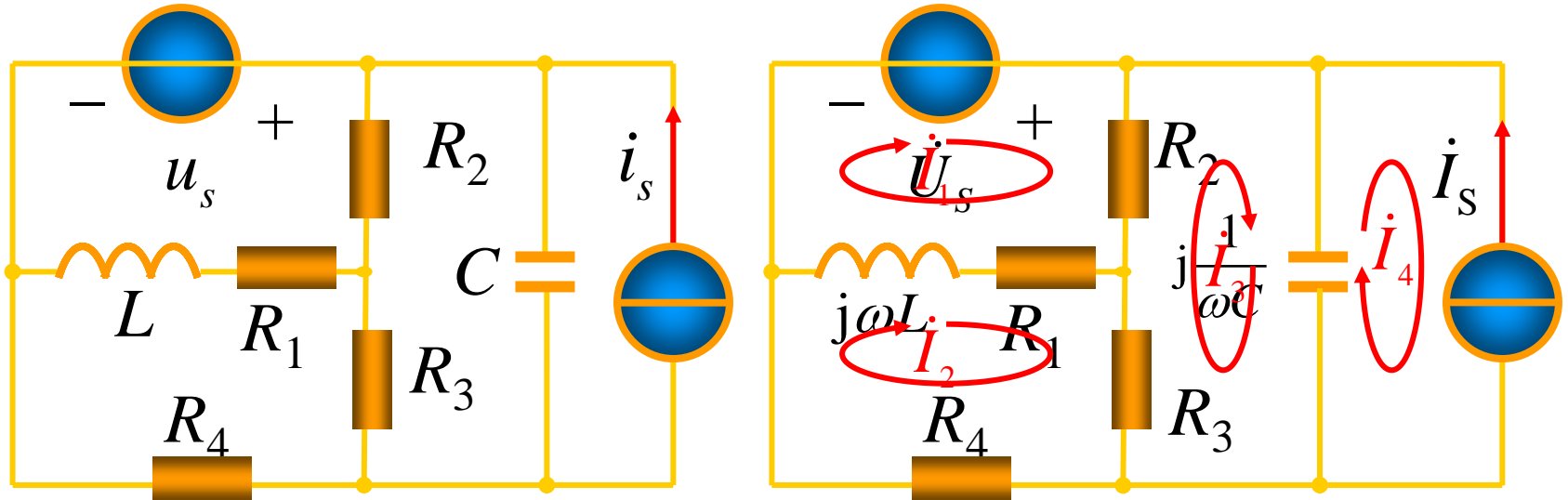
$$\dot{I}_1 = \frac{\dot{U}}{Z} = \frac{100 \angle 0^\circ}{166.99 \angle -52.3^\circ} = 0.6 \angle 52.3^\circ \text{ A}$$

$$\begin{aligned} \dot{I}_2 &= \frac{-j \frac{1}{\omega C}}{R_1 - j \frac{1}{\omega C}} \dot{I}_1 = \frac{-j318.47}{1049.5 \angle -17.7^\circ} \times 0.6 \angle 52.3^\circ \\ &= 0.181 \angle -20^\circ \text{ A} \end{aligned}$$

$$\begin{aligned} \dot{I}_3 &= \frac{R_1}{R_1 - j \frac{1}{\omega C}} \dot{I}_1 \\ &= \frac{1000}{1049.5 \angle -17.7^\circ} \times 0.6 \angle 52.3^\circ = 0.57 \angle 70^\circ \text{ A} \end{aligned}$$



例2 列写电路的回路电流方程和结点电压方程



解

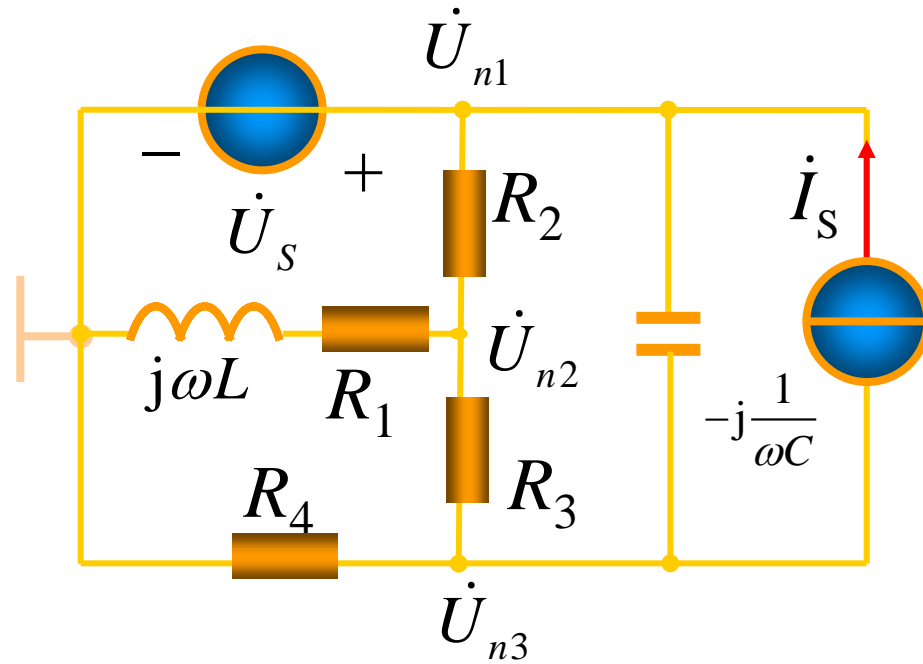
回路方程

$$(R_1 + R_2 + j\omega L)\dot{I}_1 - (R_1 + j\omega L)\dot{I}_2 - R_2\dot{I}_3 = \dot{U}_s$$

$$(R_1 + R_3 + R_4 + j\omega L)\dot{I}_2 - (R_1 + j\omega L)\dot{I}_1 - R_3\dot{I}_3 = 0$$

$$(R_2 + R_3 + \frac{1}{j\omega C})\dot{I}_3 - R_2\dot{I}_1 - R_3\dot{I}_2 + j\frac{1}{\omega C}\dot{I}_4 = 0$$

$$\dot{I}_4 = -\dot{I}_s$$

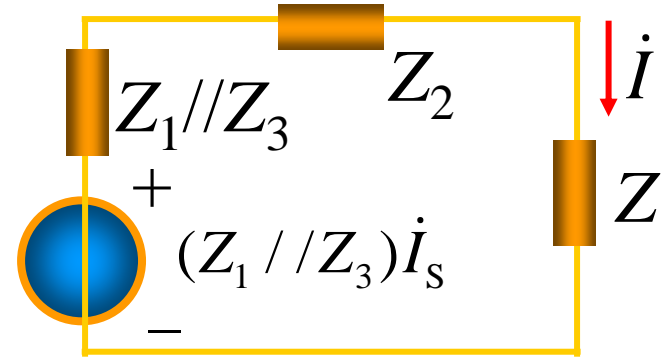
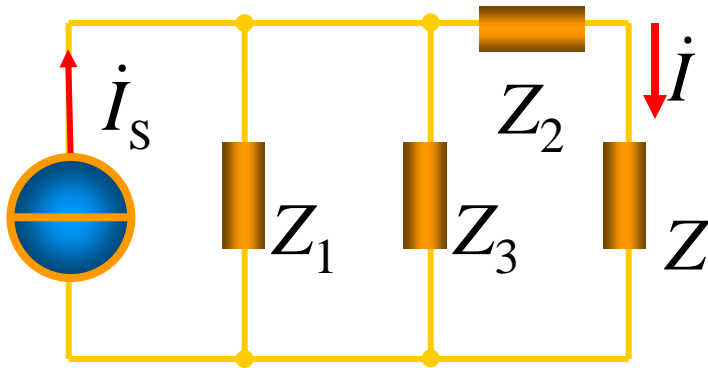


结点方程

$$\left\{ \begin{array}{l} \dot{U}_{n1} = \dot{U}_S \\ \left(\frac{1}{R_1 + j\omega L} + \frac{1}{R_2} + \frac{1}{R_3} \right) \dot{U}_{n2} - \frac{1}{R_2} \dot{U}_{n1} - \frac{1}{R_3} \dot{U}_{n3} = 0 \\ \left(\frac{1}{R_3} + \frac{1}{R_4} + j\omega C \right) \dot{U}_{n3} - \frac{1}{R_3} \dot{U}_{n2} - j\omega C \dot{U}_{n1} = -\dot{I}_S \end{array} \right.$$

例3

已知： $\dot{I}_s = 4\angle 90^\circ \text{ A}$, $Z_1 = Z_2 = -j30 \ \Omega$,
 $Z_3 = 30 \ \Omega$, $Z = 45 \ \Omega$, 求电流 \dot{I} .



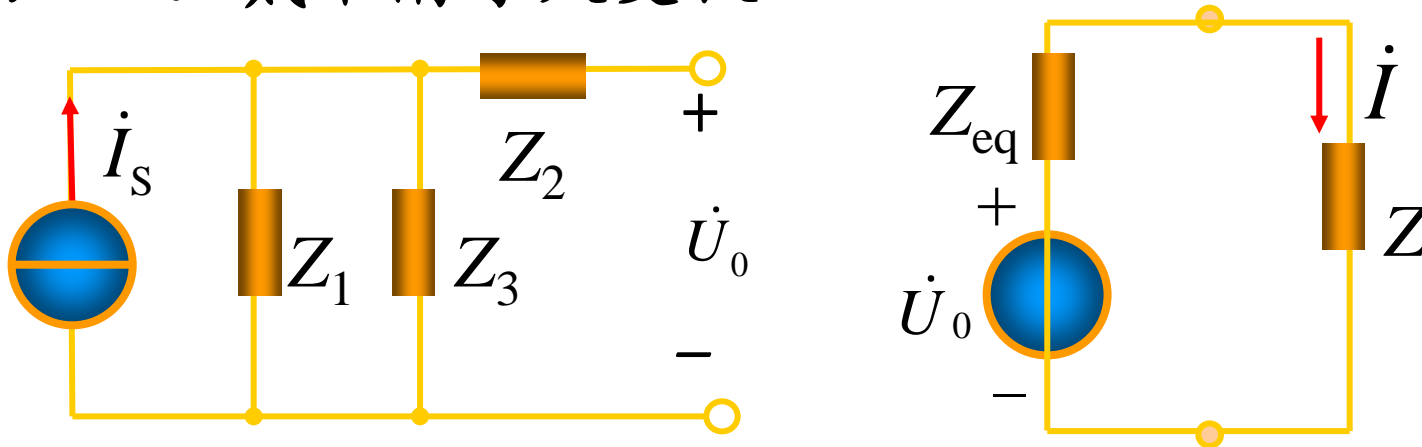
解

方法1：电源变换

$$Z_1 // Z_3 = \frac{30(-j30)}{30 - j30} = 15 - j15 \ \Omega$$

$$\begin{aligned} \dot{I} &= \frac{\dot{I}_s (Z_1 // Z_3)}{Z_1 // Z_3 + Z_2 + Z} = \frac{j4(15 - j15)}{15 - j15 - j30 + 45} \\ &= \frac{5.657\angle 45^\circ}{5\angle -36.9^\circ} = 1.13\angle 81.9^\circ \text{ A} \end{aligned}$$

方法2：戴维南等效变换

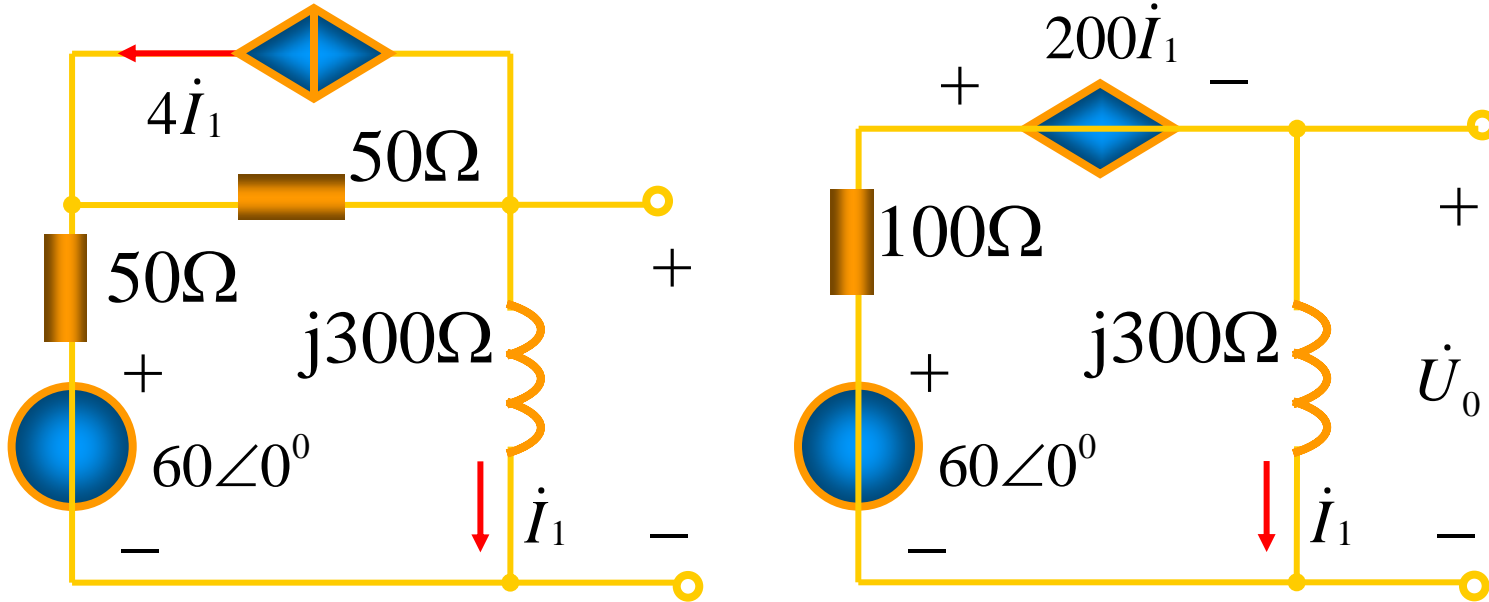


求开路电压： $\dot{U}_0 = \dot{I}_s (Z_1 // Z_3) = 84.86 \angle 45^\circ \text{ V}$

求等效电阻： $Z_{eq} = Z_1 // Z_3 + Z_2 = 15 - j45 \Omega$

$$\dot{I} = \frac{\dot{U}_0}{Z_0 + Z} = \frac{84.86 \angle 45^\circ}{15 - j45 + 45} = 1.13 \angle 81.9^\circ \text{ A}$$

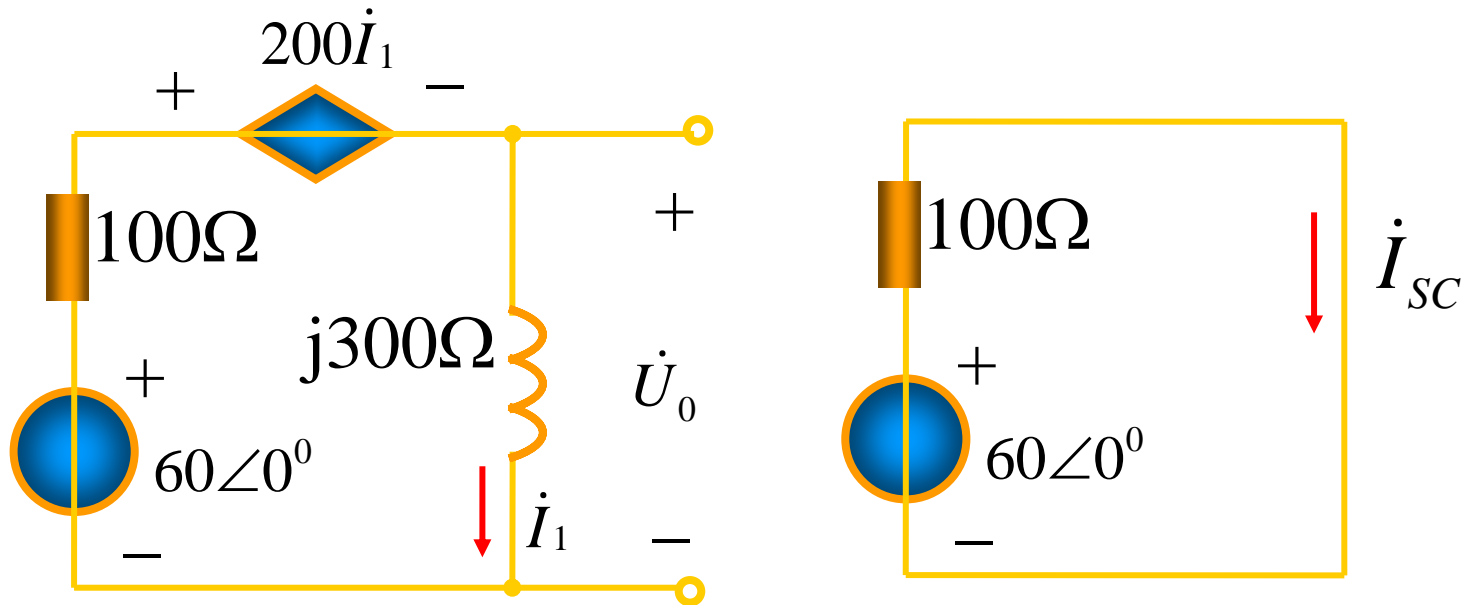
例4 求图示电路的戴维南等效电路。



解 求开路电压：

$$\dot{U}_o = -200\dot{I}_1 - 100\dot{I}_1 + 60 = -300\dot{I}_1 + 60 = -300 \frac{\dot{U}_o}{j300} + 60$$

$$\rightarrow \dot{U}_o = \frac{60}{1-j} = 30\sqrt{2}\angle 45^\circ \text{ V}$$



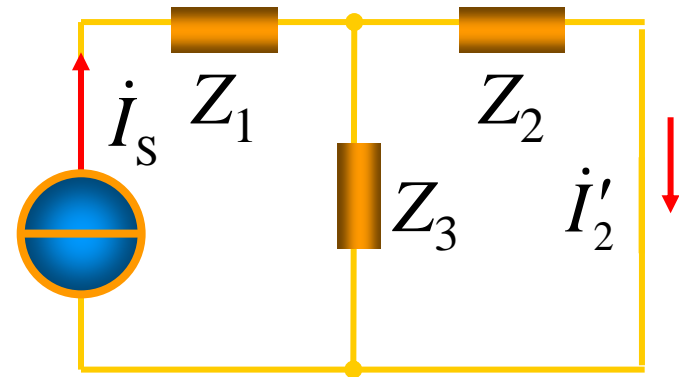
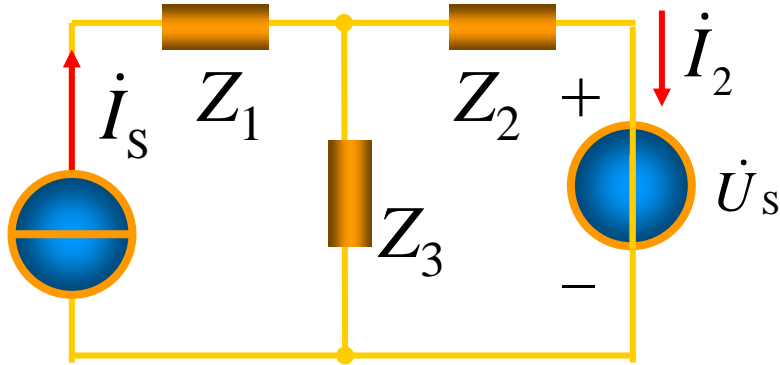
求短路电流：

$$\dot{I}_{sc} = 60/100 = 0.6\angle 0^\circ \text{ A}$$

→
$$Z_{eq} = \frac{\dot{U}_0}{\dot{I}_{sc}} = \frac{30\sqrt{2}\angle 45^\circ}{0.6} = 50\sqrt{2}\angle 45^\circ \Omega$$

例5 用叠加定理计算电流 i_2 已知: $\dot{U}_s = 100\angle 45^\circ \text{ V}$

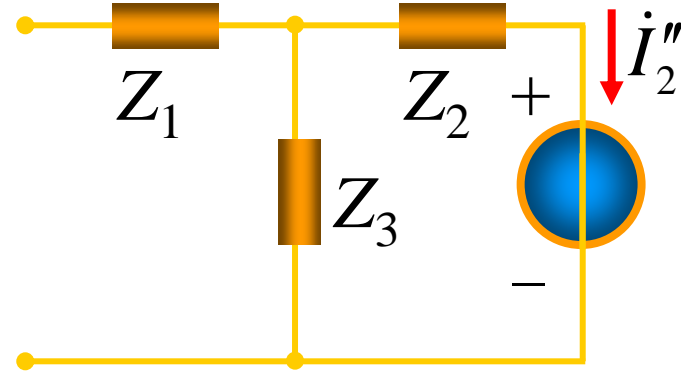
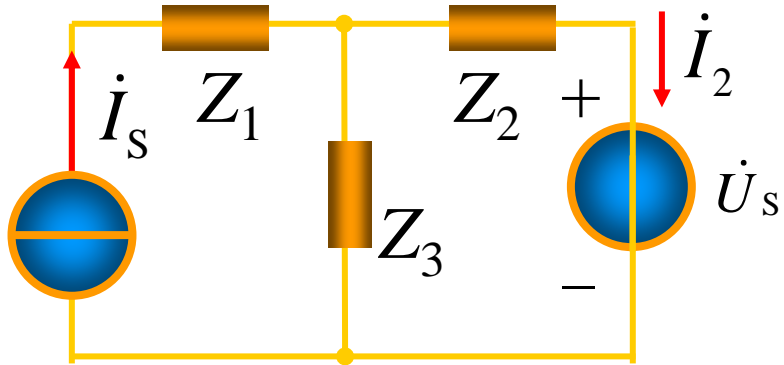
$$\dot{I}_s = 4\angle 0^\circ \text{ A}, Z_1 = Z_3 = 50\angle 30^\circ \Omega, Z_2 = 50\angle -30^\circ \Omega .$$



解

(1) \dot{I}_s 单独作用(\dot{U}_s 置零):

$$\begin{aligned} i'_2 &= \dot{I}_s \frac{Z_3}{Z_2 + Z_3} = 4\angle 0^\circ \times \frac{50\angle 30^\circ}{50\angle -30^\circ + 50\angle 30^\circ} \\ &= \frac{200\angle 30^\circ}{50\sqrt{3}} = 2.31\angle 30^\circ \text{ A} \end{aligned}$$



(2) \dot{U}_s 单独作用(\dot{I}_s 置零):

$$\dot{I}_2'' = -\frac{\dot{U}_s}{Z_2 + Z_3} = \frac{-100\angle 45^\circ}{50\sqrt{3}} = 1.155\angle -135^\circ \text{ A}$$

$$\dot{I}_2 = \dot{I}_2' + \dot{I}_2'' = 2.31\angle 30^\circ + 1.155\angle -135^\circ \text{ A}$$

例6 已知平衡电桥 $Z_1=R_1$, $Z_2=R_2$, $Z_3=R_3+j\omega L_3$ 。
求: $Z_x=R_x+j\omega L_x$ 。

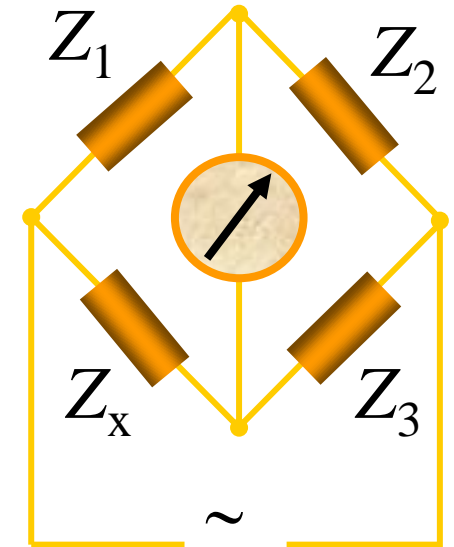
解 平衡条件: $Z_1 Z_3 = Z_2 Z_x$ 得:

$$|Z_1| \angle \varphi_1 \cdot |Z_3| \angle \varphi_3 = |Z_2| \angle \varphi_2 \cdot |Z_x| \angle \varphi_x$$

$$\rightarrow \begin{cases} |Z_1| |Z_3| = |Z_2| |Z_x| \\ \varphi_1 + \varphi_3 = \varphi_2 + \varphi_x \end{cases}$$

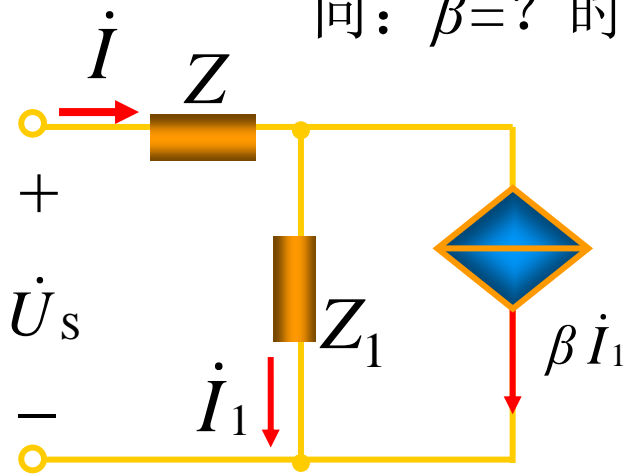
$$R_1(R_3+j\omega L_3)=R_2(R_x+j\omega L_x)$$

$$\therefore R_x=R_1 R_3 / R_2, \quad L_x=L_3 R_1 / R_2$$



例7 已知： $Z=10+j50\Omega$, $Z_1=400+j1000\Omega$ 。

问： $\beta=?$ 时， \dot{I}_1 与 \dot{U}_s 相位相差 90° ？



解

$$\dot{U}_s = Z\dot{I} + Z_1\dot{I}_1 = Z(1 + \beta)\dot{I}_1 + Z_1\dot{I}_1$$

$$\frac{\dot{U}_s}{\dot{I}_1} = (1 + \beta)Z + Z_1 = 410 + 10\beta + j(50 + 50\beta + 1000)$$

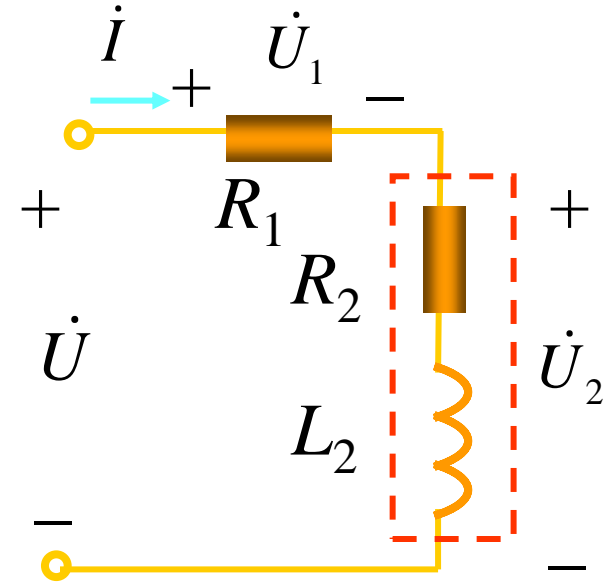
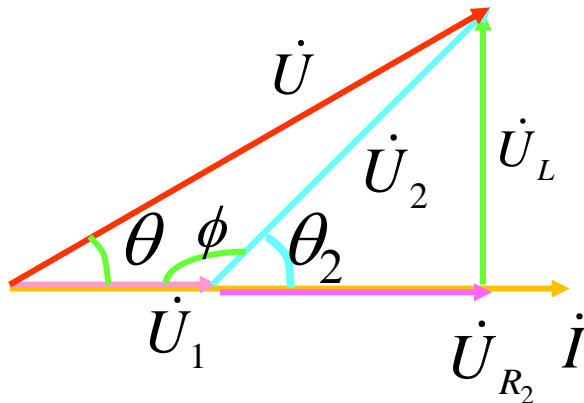
得 $410 + 10\beta = 0$, $\beta = -41$

$$\frac{\dot{U}_s}{\dot{I}_1} = -j1000 \quad \text{电流领先电压 } 90^\circ$$

例8 已知： $U=115\text{V}$, $U_1=55.4\text{V}$, $U_2=80\text{V}$, $R_1=32\Omega$,
 $f=50\text{Hz}$ 。求：线圈的电阻 R_2 和电感 L_2 。

解 方法一、画相量图分析。

$$\dot{U} = \dot{U}_1 + \dot{U}_2 = \dot{U}_1 + \dot{U}_{R_2} + \dot{U}_L$$



$$U^2 = U_1^2 + U_2^2 - 2U_1U_2 \cos \phi$$

$$\cos \phi = -0.4237 \quad \therefore \phi = 115.1^\circ$$

$$\theta_2 = 180^\circ - \phi = 64.9^\circ$$

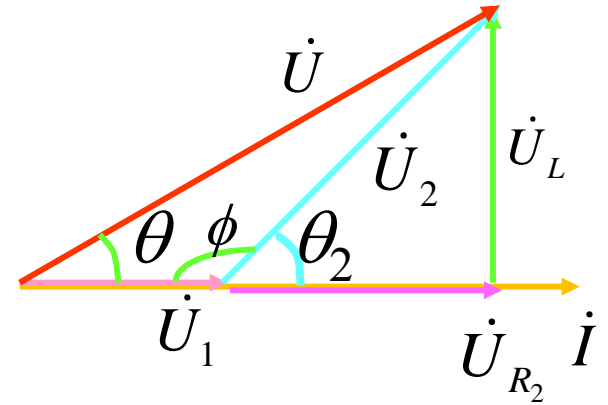
$$I = U_1 / R_1 = 55.4 / 32 = 1.73\text{A}$$

$$|Z_2| = U_2 / I = 80 / 1.73 = 46.2\Omega$$

$$R_2 = |Z_2| \cos \theta_2 = 19.6\Omega$$

$$X_2 = |Z_2| \sin \theta_2 = 41.8\Omega$$

$$L = X_2 / (2\pi f) = 0.133\text{H}$$



已知： $U=115\text{V}$, $U_1=55.4\text{V}$, $U_2=80\text{V}$, $R_1=32\Omega$,
 $f=50\text{Hz}$ 。 求：线圈的电阻 R_2 和电感 L_2 。

方法二

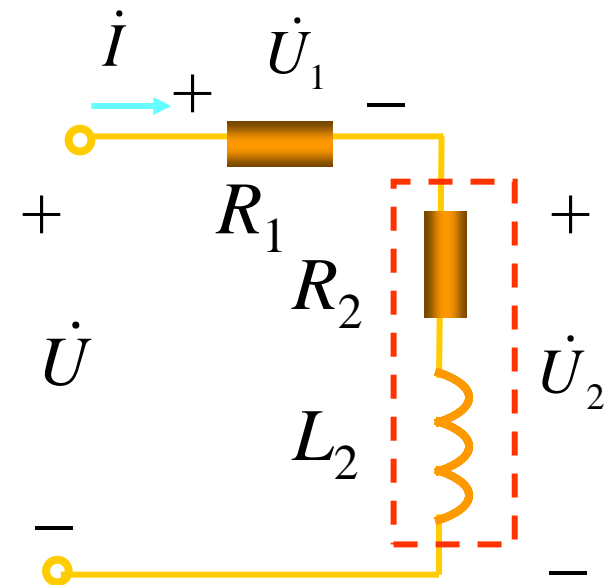
$$\dot{U} = \dot{U}_1 + \dot{U}_2 = 55.4\angle 0^\circ + 80\angle \phi = 115\angle \theta$$

$$\left\{ \begin{array}{l} 55.4 + 80 \cos \phi = 115 \cos \theta \\ 80 \sin \phi = 115 \sin \theta \end{array} \right.$$

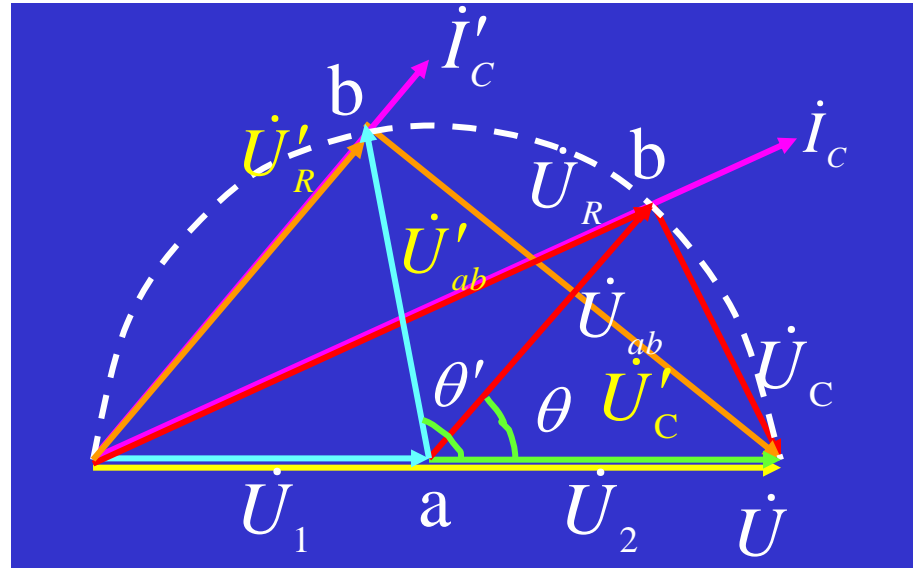
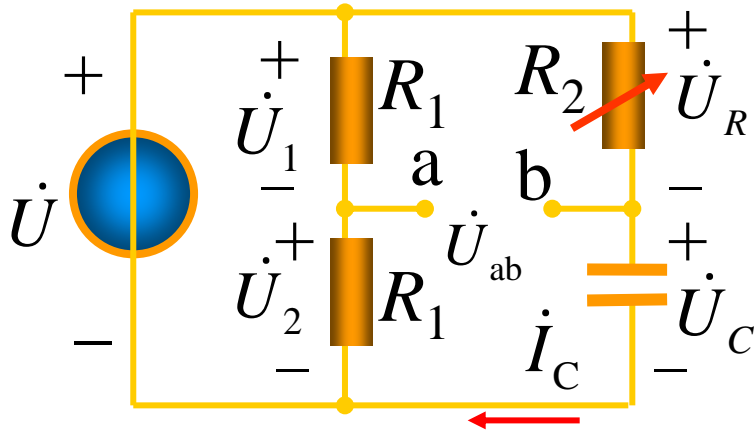
$$\cos \phi = 0.424$$

$$\phi = 64.93^\circ$$

其余步骤同解法一。



例9 移相桥电路。当 R_2 由 $0 \rightarrow \infty$ 时， \dot{U}_{ab} 如何变化？



解 用相量图分析

$$\dot{U} = \dot{U}_1 + \dot{U}_2, \quad \dot{U}_1 = \dot{U}_2 = \frac{\dot{U}}{2}$$

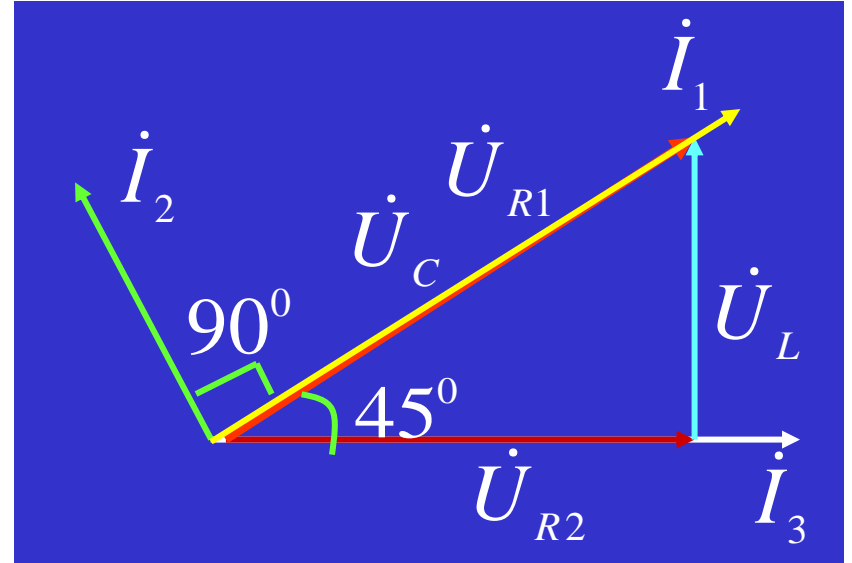
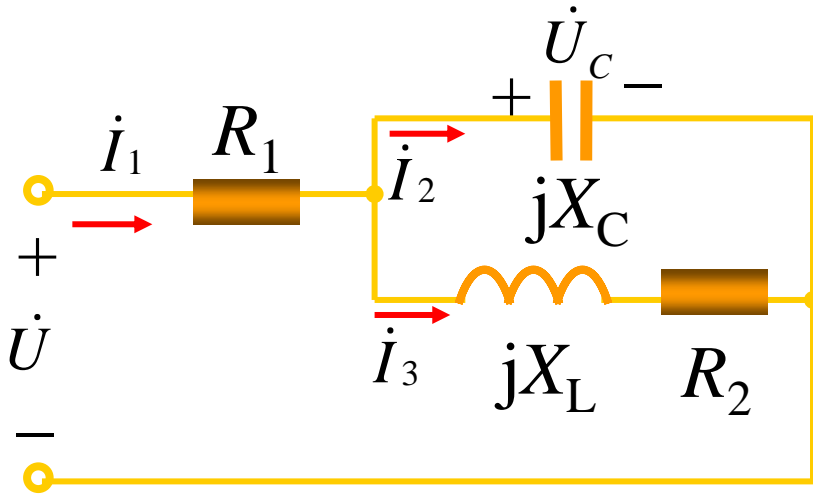
当 $R_2=0$, $\theta=180^\circ$;

$$\dot{U} = \dot{U}_R + \dot{U}_C \quad \dot{U}_{ab} = \dot{U}_R - \dot{U}_1$$

当 $R_2 \rightarrow \infty$, $\theta=0^\circ$ 。

由相量图可知，当 R_2 改变， $U_{ab}=0.5U$ 不变，相位改变 θ 为移相角，移相范围为 $180^\circ \rightarrow 0^\circ$

例10 图示电路, $I_2 = 10\text{A}$ 、 $I_3 = 10\sqrt{2}\text{A}$ 、 $U = 200\text{V}$ 、 $R_1 = 5\Omega$ 、 $R_2 = X_L$, 求: I_1 、 X_C 、 X_L 、 R_2 。



解 $\dot{I}_1 = \dot{I}_2 + \dot{I}_3 = 10\angle 135^\circ + 10\sqrt{2} = 10\angle 45^\circ \Rightarrow I_1 = 10\text{A}$

$$\dot{U} = \dot{U}_{R1} + \dot{U}_C \Rightarrow 200 = 5 \times 10 + U_C \Rightarrow U_C = 150\text{V}$$

$$\dot{U}_C = \dot{U}_{R2} + \dot{U}_L \Rightarrow U_C = \sqrt{2U_{R2}^2} \Rightarrow U_{R2} = U_L = 75\sqrt{2}$$

$$X_C = \frac{150}{10} = 15\Omega \quad R_2 = X_L = \frac{75\sqrt{2}}{10\sqrt{2}} = 7.5\Omega$$

本章小结

➤ 正弦电量：三要素 $i(t) = I_m \cos(\omega t + \Psi)$

有效值 $I_m = \sqrt{2}I$

➤ 相量法： $i(t) = \sqrt{2}I \cos(\omega t + \Psi) \Leftrightarrow \dot{I} = I \angle \Psi$

✓ 把时域问题变为复数问题；

✓ 把微积分方程的运算变为复数方程运算；

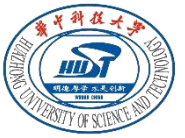
➤ 阻抗与导纳：

相量图法

$$\text{元件} \begin{cases} \dot{U} = R\dot{I} \\ \dot{U} = j\omega L\dot{I} \\ \dot{U} = \frac{1}{j\omega C}\dot{I} \end{cases} \xrightarrow{\text{复阻抗}} \dot{U} = Z\dot{I}$$

$$\text{电路} \begin{cases} \sum \dot{I} = 0 \\ \sum \dot{U} = 0 \end{cases}$$

分析电阻电路的方法可以直接用于分析相量模型



作业

- 10.3节：10-13
- 10.4节：10-34
- 10.5节：10-41（只要求用戴维南定理）
- 10.6节：10-51
- 综合：10-53