Chapter 13

含磁耦合的电路

13. 1 耦合电感

Coupled inductors

13. 2 含耦合电感电路的分析

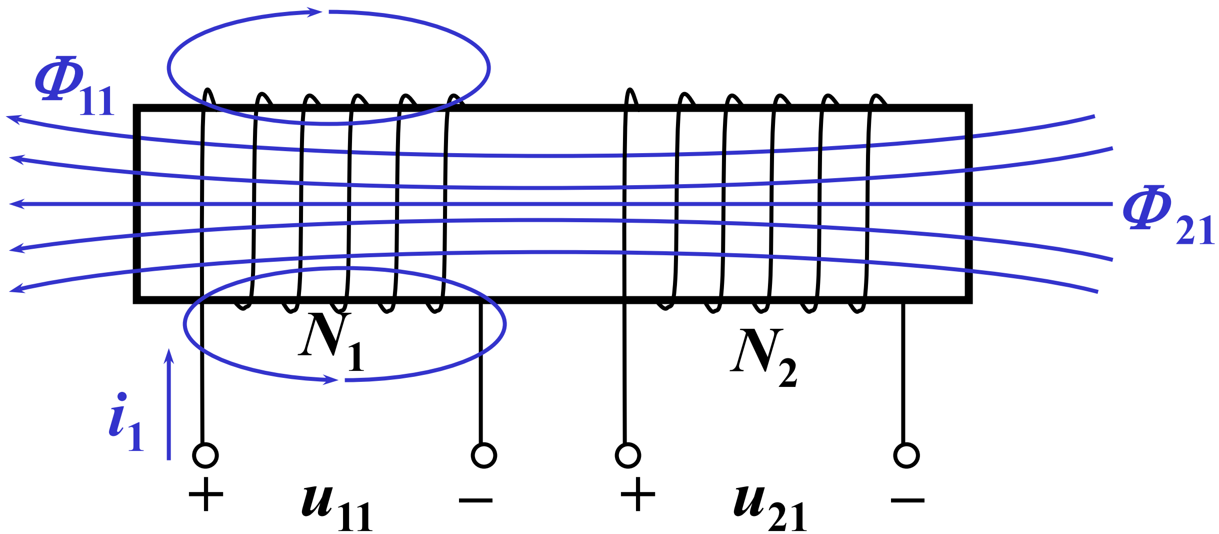
Analysis of coupled circuits

13. 3 变压器

Transformers

13.1 耦合电感

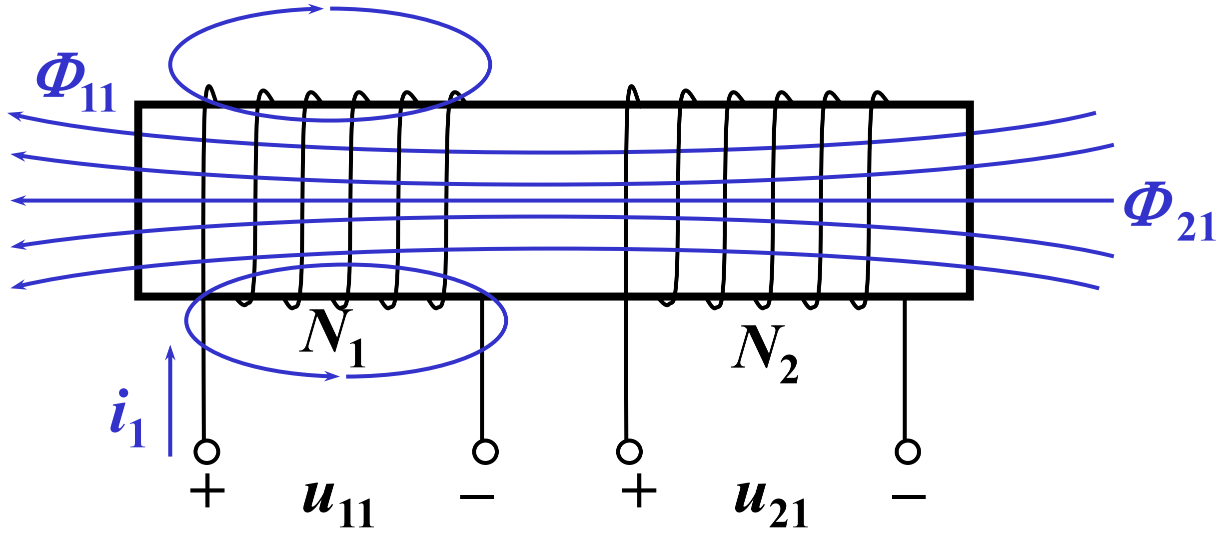
一、互感（mutual inductance）和互感电压（mutual voltage）



当线圈1中通入电流i_1时

$\Psi_{11}=N_1\Phi_{11}$

载流回路1中的电流 i → 磁感应强度 B → 磁通 Φ_{11} → 磁链 Ψ_{11}

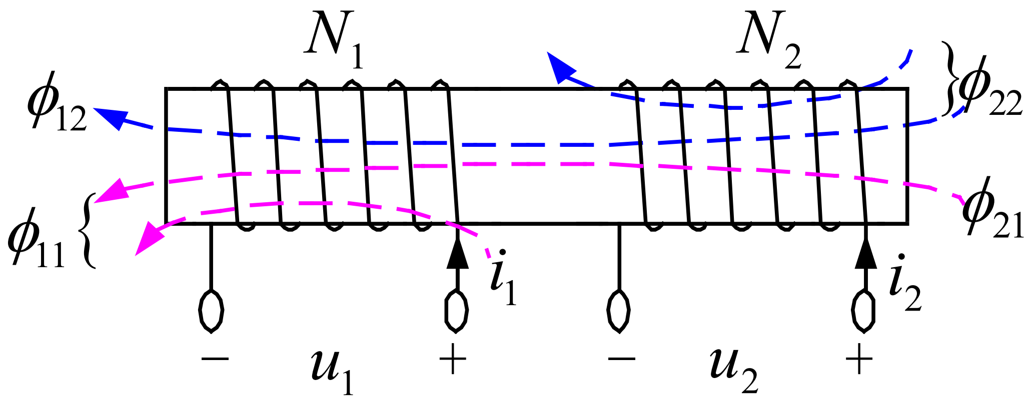


由电磁感应定律（**Farady′s law**）和楞次定律（**Lenz′s law**）可得

$$u_{11}=\frac{\mathrm{d}\Psi_{11}}{\mathrm{d}t}=N_1\frac{\mathrm{d}\Phi_{11}}{\mathrm{d}t}\qquad\text{—自感电压}$$

$$u_{21}=\frac{\mathrm{d}\Psi_{21}}{\mathrm{d}t}=N_2\frac{\mathrm{d}\Phi_{21}}{\mathrm{d}t}\qquad\text{—互感电压}$$

当两个线圈同时通以电流时



$$\begin{cases} \phi_1 = \phi_{11} + \phi_{12} \\ \phi_2 = \phi_{21} + \phi_{22} \end{cases}$$

$$\begin{cases} \psi_1 = \psi_{11} + \psi_{12} \\ \psi_2 = \psi_{21} + \psi_{22} \end{cases}$$

$$\boxed{\begin{cases} u_1 = \dfrac{d\psi_1}{dt} \\ u_2 = \dfrac{d\psi_2}{dt} \end{cases}}$$

$$\begin{cases} \psi_1 = N_1\phi_1 = N_1\phi_{11} + N_1\phi_{12} \\ \psi_2 = N_2\phi_2 = N_2\phi_{21} + N_2\phi_{22} \end{cases}$$

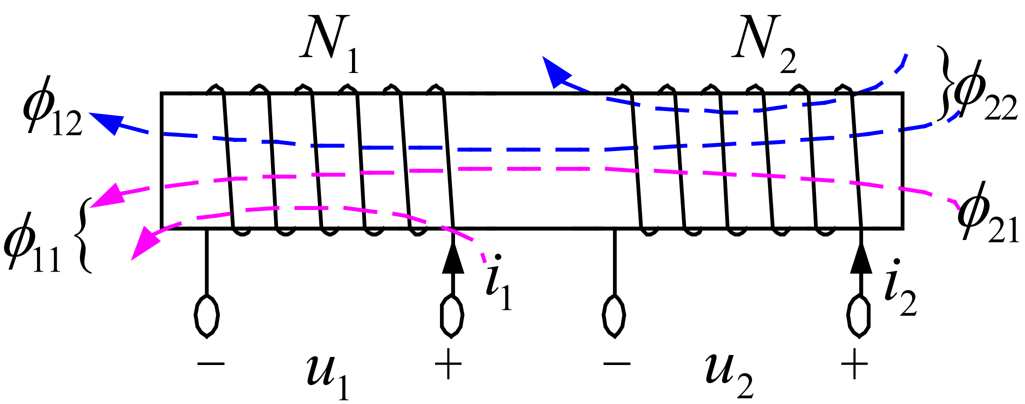
$$\begin{cases} u_1 = u_{11} + u_{12} = L_1 \dfrac{\mathrm{d}i_1}{\mathrm{d}t} + M \dfrac{\mathrm{d}i_2}{\mathrm{d}t} \\ u_2 = u_{21} + u_{22} = M \dfrac{\mathrm{d}i_1}{\mathrm{d}t} + L_2 \dfrac{\mathrm{d}i_2}{\mathrm{d}t} \end{cases}$$

$$\begin{cases} \dot{U}_1 = \mathrm{j}\omega L_1 \dot{I}_1 + \mathrm{j}\omega M \dot{I}_2 \\ \dot{U}_2 = \mathrm{j}\omega M \dot{I}_1 + \mathrm{j}\omega L_2 \dot{I}_2 \end{cases}$$

二、耦合系数（coupling coefficient）k

k 表示两个线圈磁耦合（magnetic coupling）的紧密程度。

$$k \overset{\text{def}}{=} \frac{M}{\sqrt{L_1 L_2}}$$



可以证明，$k \le 1$

全耦合时：$\Phi_{11} = \Phi_{21}$，$\Phi_{22} = \Phi_{12}$

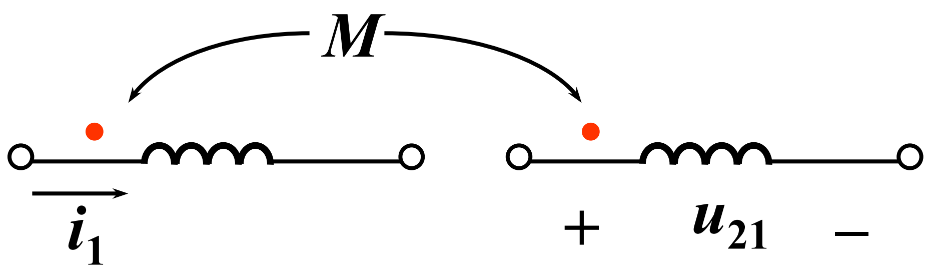
$$\because \quad L_1 = \frac{N_1 \Phi_{11}}{i_1}, \quad L_2 = \frac{N_2 \Phi_{22}}{i_2}$$

$$M_{21} = \frac{N_2 \Phi_{21}}{i_1}, \quad M_{12} = \frac{N_1 \Phi_{12}}{i_2}$$

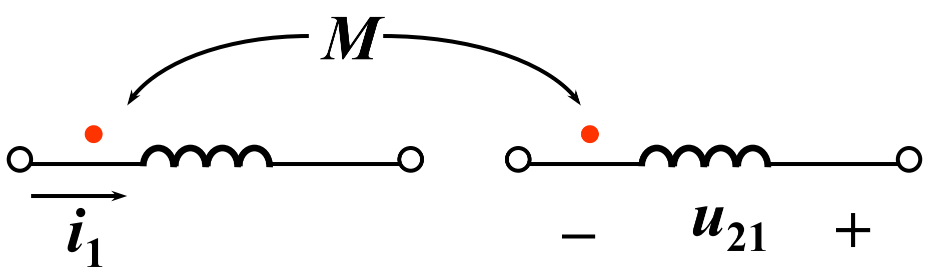
$$\therefore \quad M_{12} M_{21} = L_1 L_2, \quad M^2 = L_1 L_2, \quad k = 1$$

三、互感线圈的同名端（ Dot convention ）

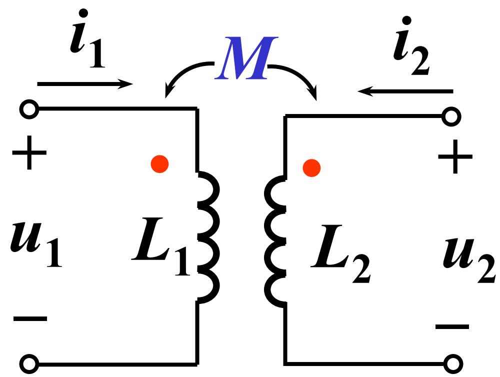
当两个电流分别从这两个端点流入或流出，两个电感的磁链耦合互相加强，为加强型耦合。

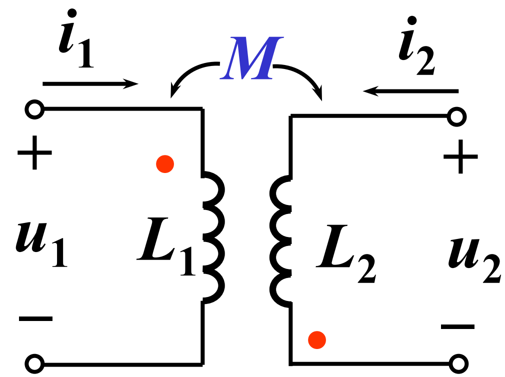


$$u_{21} = M\frac{\mathrm{d}i_1}{\mathrm{d}t}$$



$$u_{21} = -M\frac{\mathrm{d}i_1}{\mathrm{d}t}$$



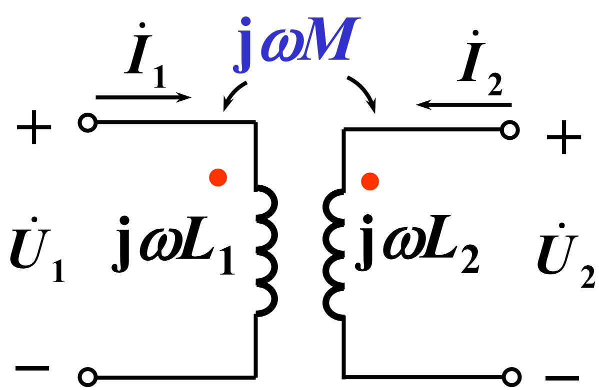


时域形式

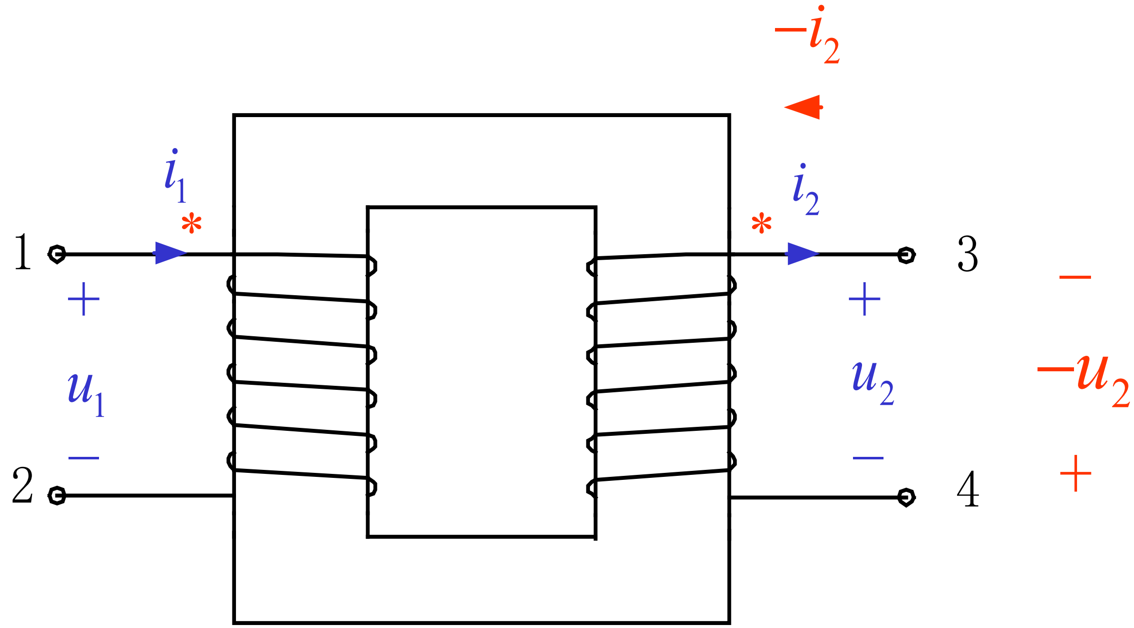
$$\begin{cases} u_1 = L_1 \dfrac{\mathrm{d}i_1}{\mathrm{d}t} + M \dfrac{\mathrm{d}i_2}{\mathrm{d}t} \\ u_2 = M \dfrac{\mathrm{d}i_1}{\mathrm{d}t} + L_2 \dfrac{\mathrm{d}i_2}{\mathrm{d}t} \end{cases}$$

$$\begin{cases} u_1 = L_1 \dfrac{\mathrm{d}i_1}{\mathrm{d}t} - M \dfrac{\mathrm{d}i_2}{\mathrm{d}t} \\ u_2 = -M \dfrac{\mathrm{d}i_1}{\mathrm{d}t} + L_2 \dfrac{\mathrm{d}i_2}{\mathrm{d}t} \end{cases}$$

在正弦交流电路中，其相量形式的电路模型和方程分别为



$$\begin{cases} \dot{U}_1 = \mathrm{j}\omega L_1 \dot{I}_1 + \mathrm{j}\omega M \dot{I}_2 \\ \dot{U}_2 = \mathrm{j}\omega M \dot{I}_1 + \mathrm{j}\omega L_2 \dot{I}_2 \end{cases}$$

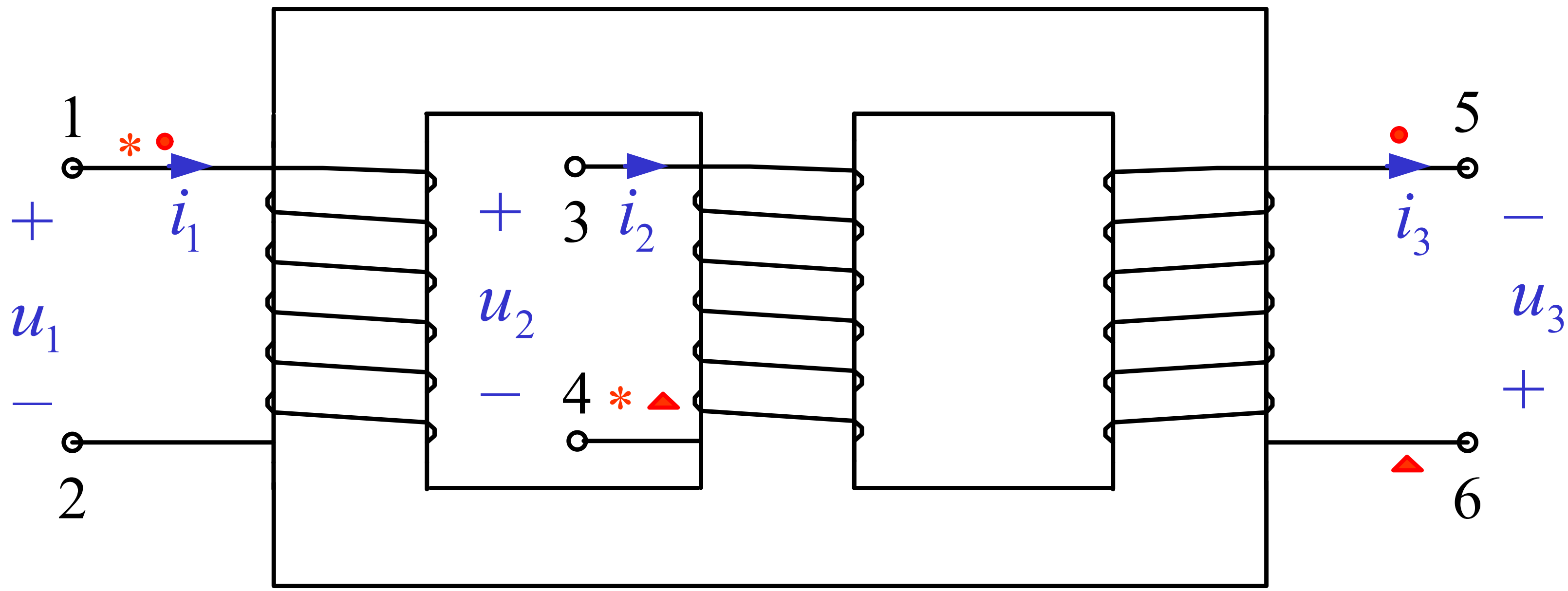


$$u_1 = L_1 \frac{di_1}{dt} + M \frac{d(-i_2)}{dt}$$

$$u_2 = +M \frac{di_1}{dt} + L_2 \frac{d(-i_2)}{dt}$$

$$u_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$-u_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$



$$u_1 = L_1\frac{di_1}{dt} - M_{12}\frac{di_2}{dt} - M_{13}\frac{di_3}{dt}$$

$$u_2 = L_2\frac{di_2}{dt} - M_{12}\frac{di_1}{dt} - M_{23}\frac{di_3}{dt}$$

$$u_3 = L_3\frac{di_3}{dt} - M_{13}\frac{di_1}{dt} - M_{23}\frac{di_2}{dt}$$

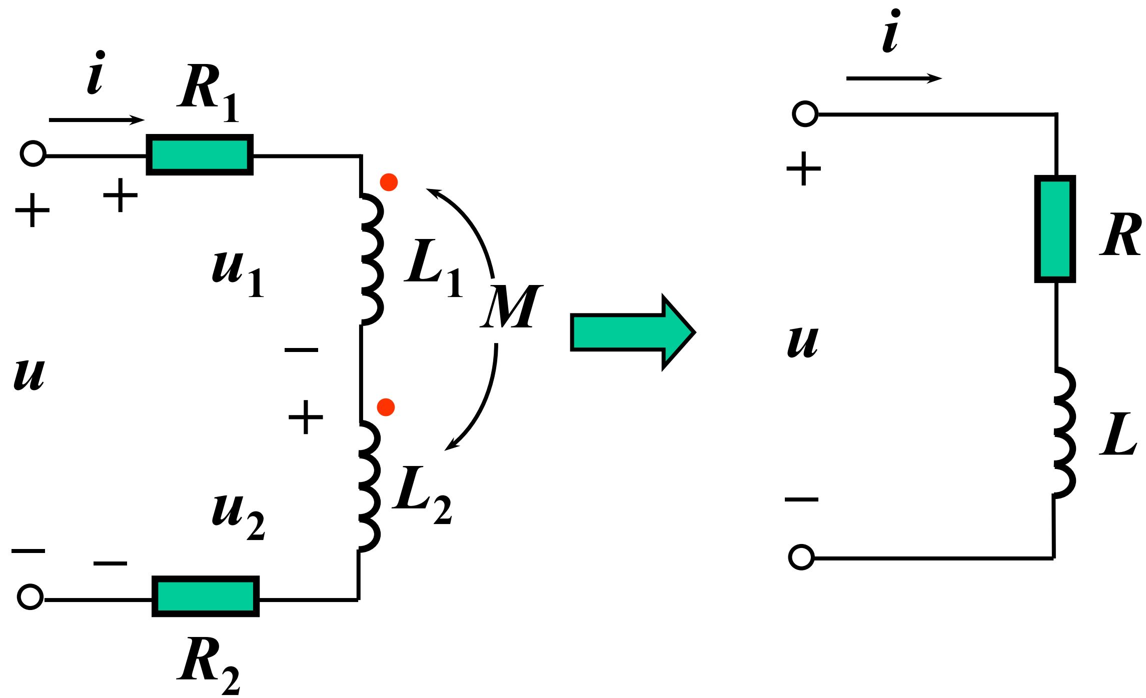
$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} L_1 & -M_{12} & -M_{13} \\ -M_{12} & L_2 & -M_{23} \\ -M_{13} & -M_{23} & L_3 \end{bmatrix} \begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \\ \frac{di_3}{dt} \end{bmatrix}$$

13.2 互感线圈的串并联

一、互感线圈的串联

1. 同名端顺串

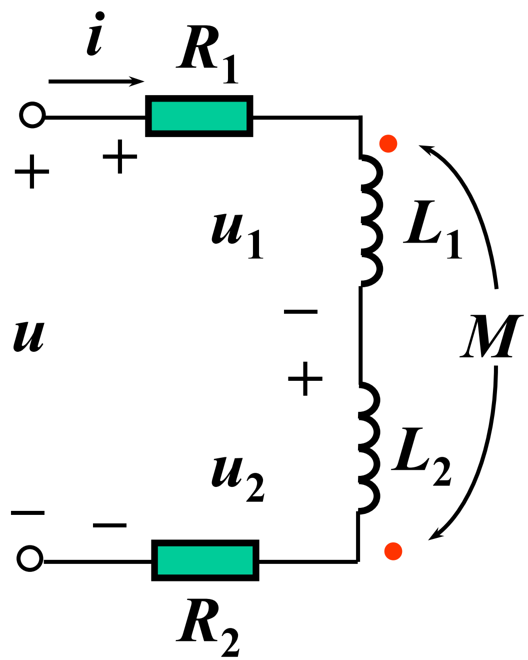
时域



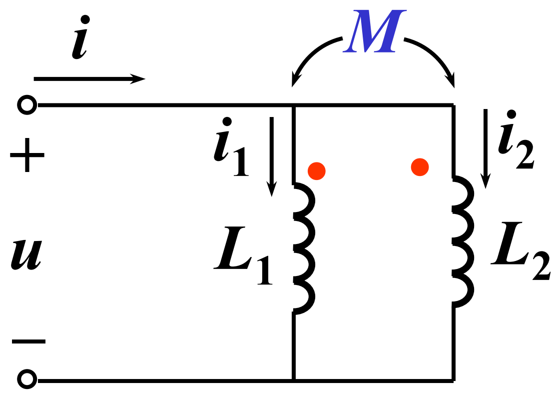
$$u = R_1 i + L_1\frac{\mathrm{d}i}{\mathrm{d}t} + M\frac{\mathrm{d}i}{\mathrm{d}t} + L_2\frac{\mathrm{d}i}{\mathrm{d}t} + M\frac{\mathrm{d}i}{\mathrm{d}t} + R_2 i$$

$$= (R_1 + R_2)i + (L_1 + L_2 + 2M)\frac{\mathrm{d}i}{\mathrm{d}t} = Ri + L\frac{\mathrm{d}i}{\mathrm{d}t}$$

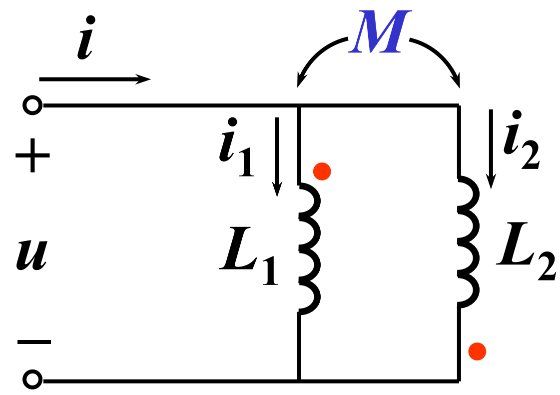
其中 $R = R_1 + R_2$ ， $L = L_1 + L_2 + 2M$



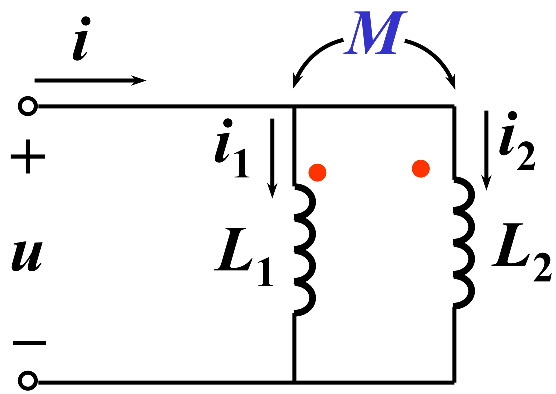
i
R1
+
+
u1
L1
M
u
−
+
L2
u2
−
−
R2



i
M
+
i1
i2
u
L1
L2
−



i
M
+
i1
i2
u
L1
L2
−



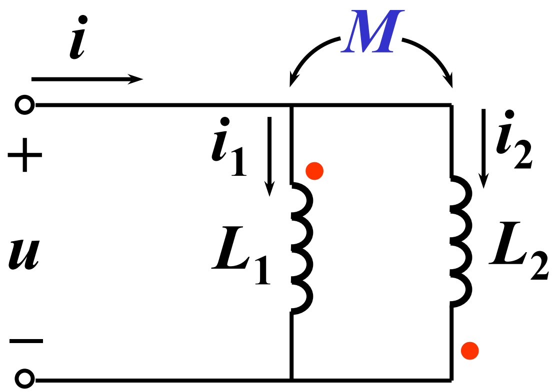
$$\begin{cases} u = L_1 \dfrac{\mathrm{d}i_1}{\mathrm{d}t} + M \dfrac{\mathrm{d}i_2}{\mathrm{d}t} \\ u = L_2 \dfrac{\mathrm{d}i_2}{\mathrm{d}t} + M \dfrac{\mathrm{d}i_1}{\mathrm{d}t} \\ i = i_1 + i_2 \end{cases}$$

解得u, i的关系

$$u = \frac{(L_1 L_2 - M^2)}{L_1 + L_2 - 2M} \frac{\mathrm{d}i}{\mathrm{d}t} \quad \Rightarrow \quad L_{eq} = \frac{(L_1 L_2 - M^2)}{L_1 + L_2 - 2M} \geq 0$$

故 $M \leq \sqrt{L_1 L_2}$

互感小于两元件自感的几何平均值。



$$\begin{cases} u = L_1 \dfrac{\mathrm{d}i_1}{\mathrm{d}t} - M \dfrac{\mathrm{d}i_2}{\mathrm{d}t} \\ u = L_2 \dfrac{\mathrm{d}i_2}{\mathrm{d}t} - M \dfrac{\mathrm{d}i_1}{\mathrm{d}t} \\ i = i_1 + i_2 \end{cases}$$

解得u, i的关系

$$u = \frac{(L_1 L_2 - M^2)}{L_1 + L_2 + 2M} \frac{\mathrm{d}i}{\mathrm{d}t}$$

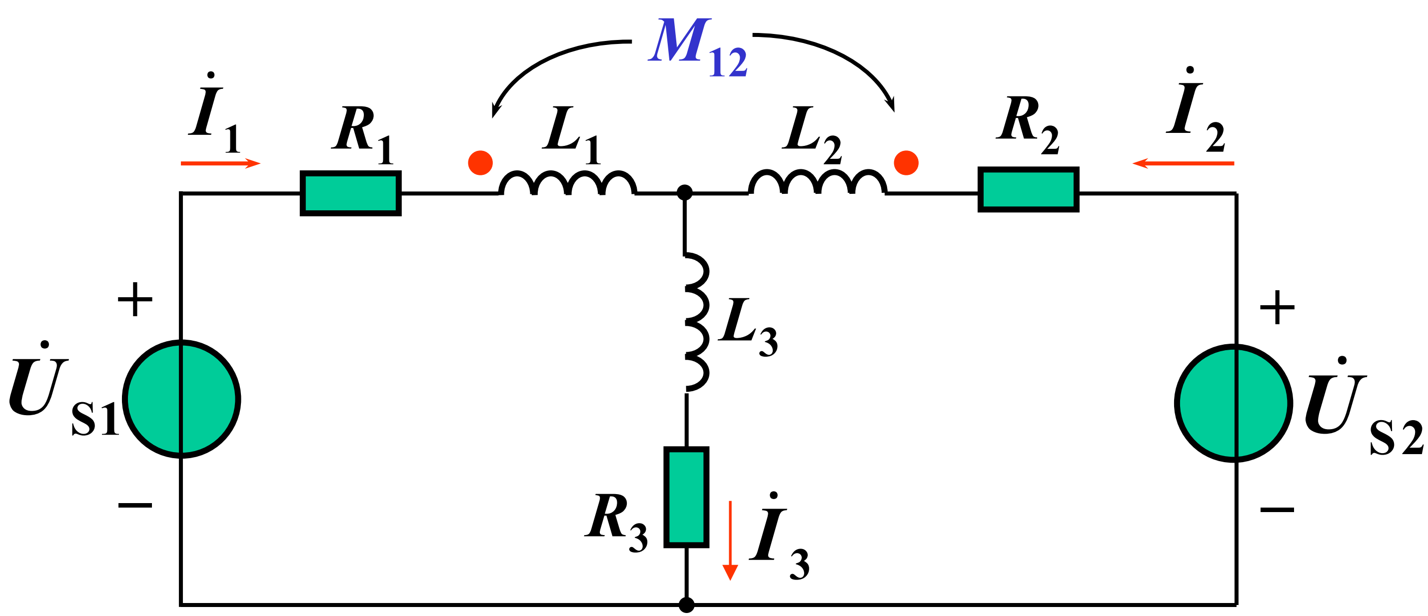
$$L_{eq} = \frac{(L_1 L_2 - M^2)}{L_1 + L_2 + 2M} \geq 0$$

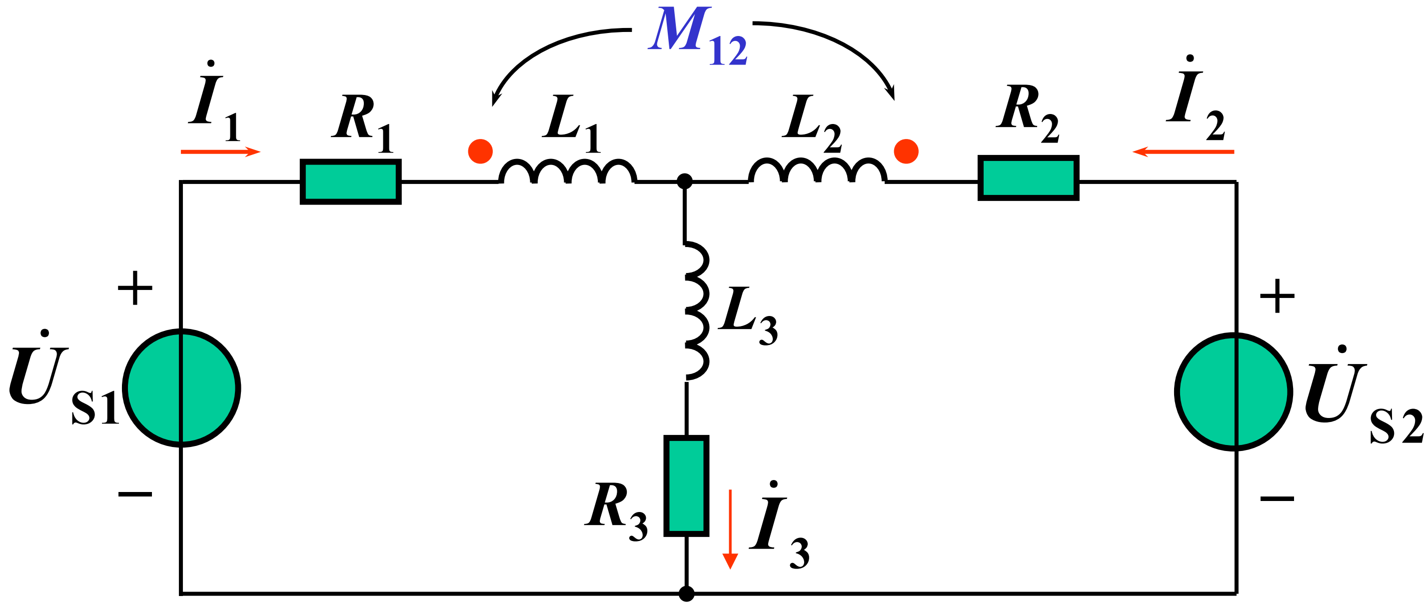
13.3 含耦合电感电路分析

有互感的电路的计算仍属正弦稳态分析，前面介绍的相量分析的的方法均适用。只需注意互感线圈上的电压除自感电压外，还应包含互感电压。

1.网孔分析法

例 1 列写下图电路的方程。





网孔分析法：

$$\begin{cases} R_1\dot{I}_1 + \mathrm{j}\omega L_1\dot{I}_1 + \mathrm{j}\omega M\dot{I}_2 + \mathrm{j}\omega L_3\dot{I}_3 + R_3\dot{I}_3 = \dot{U}_{S1} \\ R_2\dot{I}_2 + \mathrm{j}\omega L_2\dot{I}_2 + \mathrm{j}\omega M\dot{I}_1 + \mathrm{j}\omega L_3\dot{I}_3 + R_3\dot{I}_3 = \dot{U}_{S2} \\ \dot{I}_3 = \dot{I}_1 + \dot{I}_2 \end{cases}$$

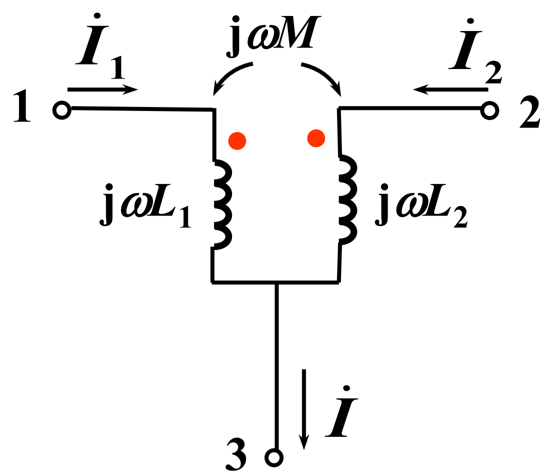
注意：线圈上互感电压的表示式及正负号。

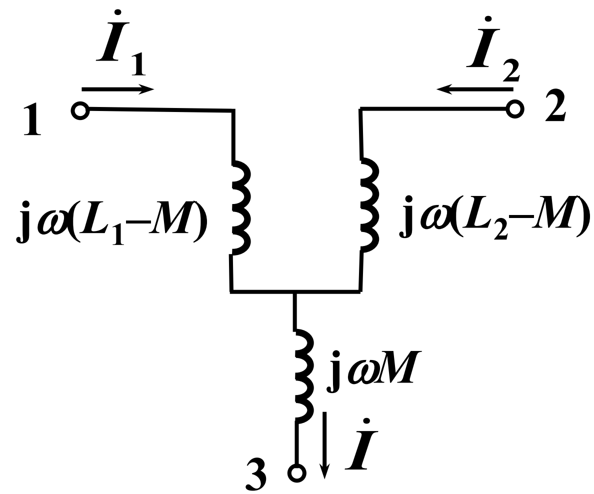
含互感的电路，直接用节点法列写方程不方便。

2.互感的去耦等效（两电感有公共端）

当耦合的两个线圈有一个公共端时，可以等效为非耦合的三个电感，称为**去耦等效电路**

（a） 两个线圈的同名端接在公共端



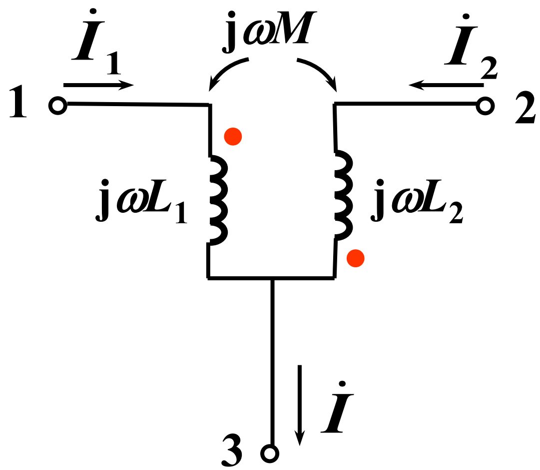


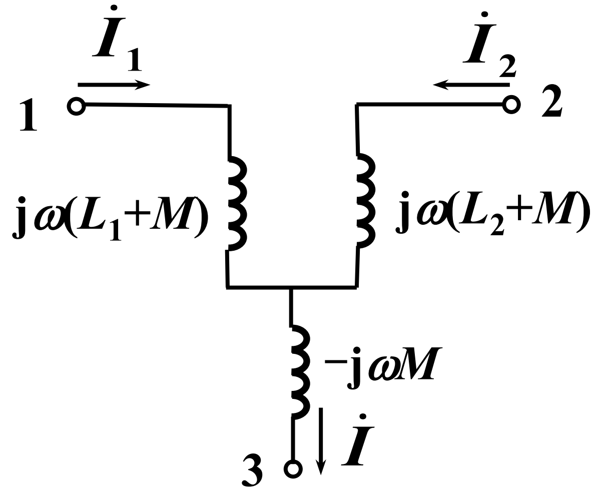
$$\begin{cases} \dot{U}_{13} = \mathrm{j}\omega L_1 \dot{I}_1 + \mathrm{j}\omega M \dot{I}_2 \\ \dot{U}_{23} = \mathrm{j}\omega L_2 \dot{I}_2 + \mathrm{j}\omega M \dot{I}_1 \\ \dot{I} = \dot{I}_1 + \dot{I}_2 \end{cases}$$

整理得

$$\begin{cases} \dot{U}_{13} = \mathrm{j}\omega (L_1 - M) \dot{I}_1 + \mathrm{j}\omega M \dot{I} \\ \dot{U}_{23} = \mathrm{j}\omega (L_2 - M) \dot{I}_2 + \mathrm{j}\omega M \dot{I} \\ \dot{I} = \dot{I}_1 + \dot{I}_2 \end{cases}$$

（b）两个线圈的异名端接在公共端



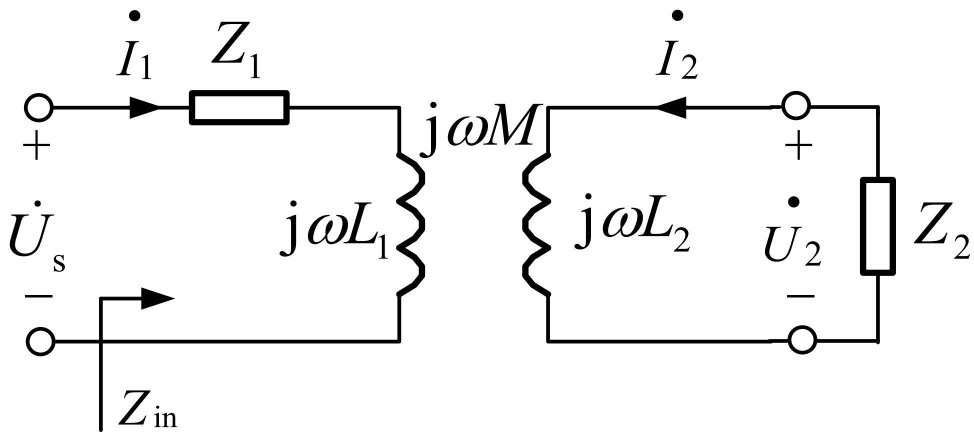


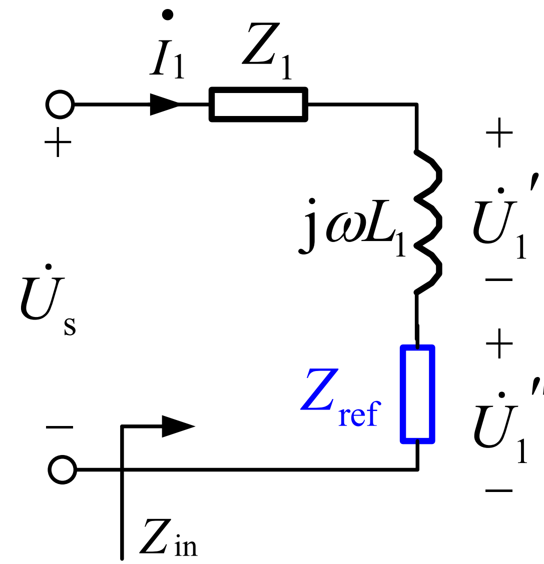
$$\begin{cases} \dot{U}_{13} = \mathrm{j}\omega L_1\dot{I}_1 - \mathrm{j}\omega M\dot{I}_2 \\ \dot{U}_{23} = \mathrm{j}\omega L_2\dot{I}_2 - \mathrm{j}\omega M\dot{I}_1 \\ \dot{I} = \dot{I}_1 + \dot{I}_2 \end{cases}$$

整理得

$$\begin{cases} \dot{U}_{13} = \mathrm{j}\omega(L_1 + M)\dot{I}_1 - \mathrm{j}\omega M\dot{I} \\ \dot{U}_{23} = \mathrm{j}\omega(L_2 + M)\dot{I}_2 - \mathrm{j}\omega M\dot{I} \\ \dot{I} = \dot{I}_1 + \dot{I}_2 \end{cases}$$

3. 映射阻抗





负载回路对电源回路的影响可用Z_{ref}表示，称Z_{ref}为负载回路在电源回路的映射阻抗。

$$Z_{in} = \frac{\dot{U}_S}{\dot{I}_1} = Z_1 + \frac{j\omega L_1 \dot{I}_1 \pm j\omega M \dot{I}_2}{\dot{I}_1} = (Z_1 + j\omega L_1) + (\pm j\omega M)\frac{\dot{I}_2}{\dot{I}_1}$$

$$\dot{U}_2 = j\omega L_2 \dot{I}_2 \pm j\omega M \dot{I}_1 = -Z_2 \dot{I}_2$$

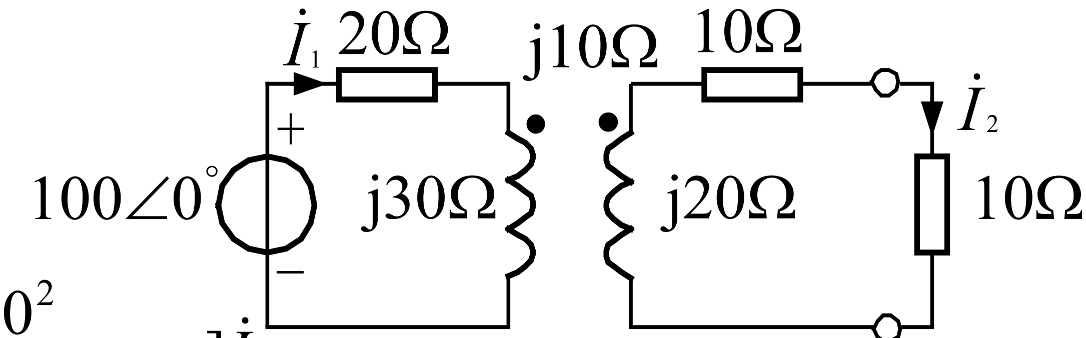
$$\frac{\dot{I}_2}{\dot{I}_1} = -\frac{(\pm j\omega M)}{Z_2 + j\omega L_2}$$

$$Z_{in} = (Z_1 + j\omega L_1) + \frac{(\omega M)^2}{Z_2 + j\omega L_2}$$

$$= Z_{11} + \frac{(\omega M)^2}{Z_{22}}$$

$$= Z_{11} + Z_{ref}$$

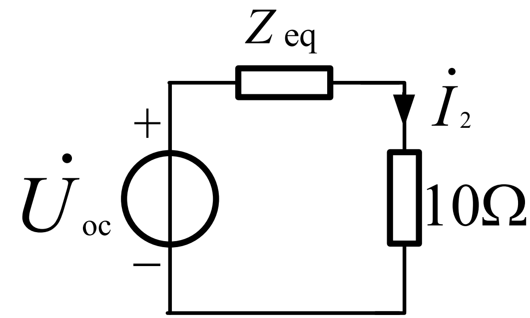
3. 映射阻抗



$$\dot{U}_s = [Z_{11} + \frac{(\omega M)^2}{Z_{22}}]\dot{I}_1$$

$$100\angle 0° = [(20 + j30) + \frac{10^2}{(10 + 10 + j20)}]\dot{I}_1$$

$$10\dot{I}_2 + 10\dot{I}_2 + (j20\dot{I}_2 - j10\dot{I}_1) = 0$$



如何先求 $\dot{I}_2$?

$$\dot{U}_{oc} = j\omega M\dot{I}_1 = j\omega M\frac{\dot{U}_s}{Z_{11}} = j10 \times \frac{100\angle 0°}{20 + j30}$$

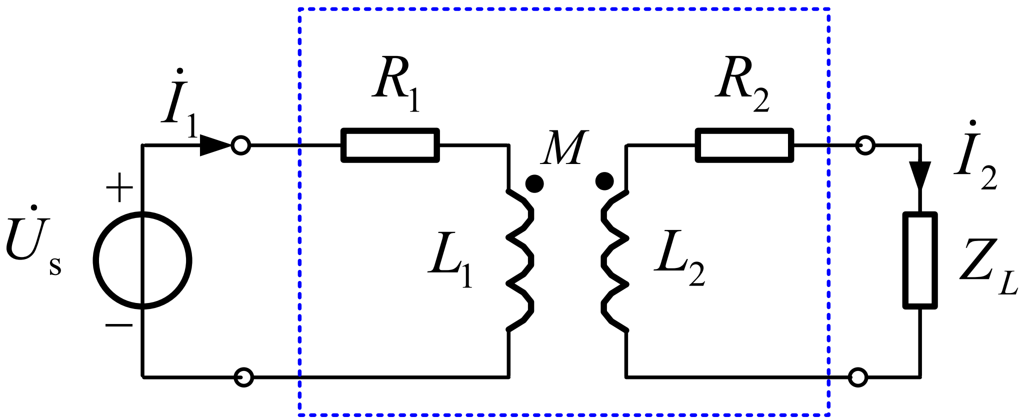
$$Z_{eq} = (10 + j20) + \frac{10^2}{20 + j30}$$

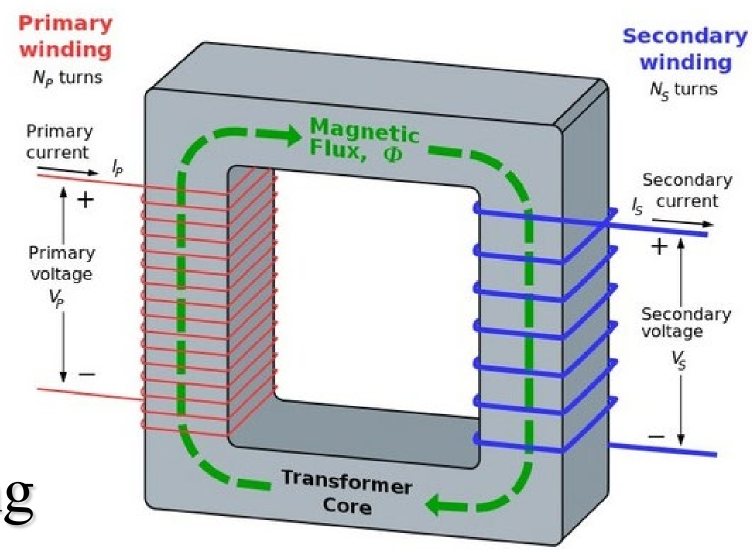
$$\dot{I}_2 = \frac{\dot{U}_{oc}}{10 + Z_{eq}}$$

$$20\dot{I}_1 + j30\dot{I}_1 - j10\dot{I}_2 = 100\angle 0°$$

13.4 变压器 Transformers

1. 变压器模型



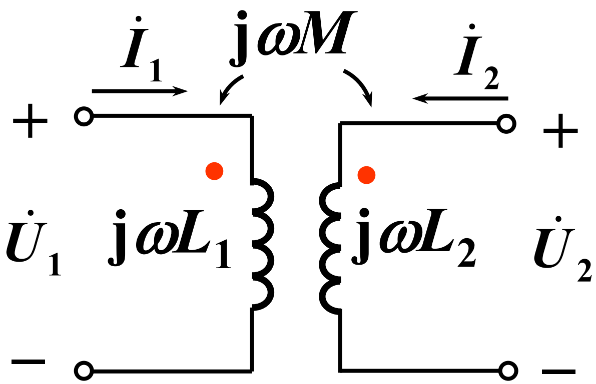


Primary winding 初级(原方)

secondary winding 次级(副方)

- 交流变压、变流
- 电隔离
- 传送功率
- 阻抗匹配

2. 全耦合变压器



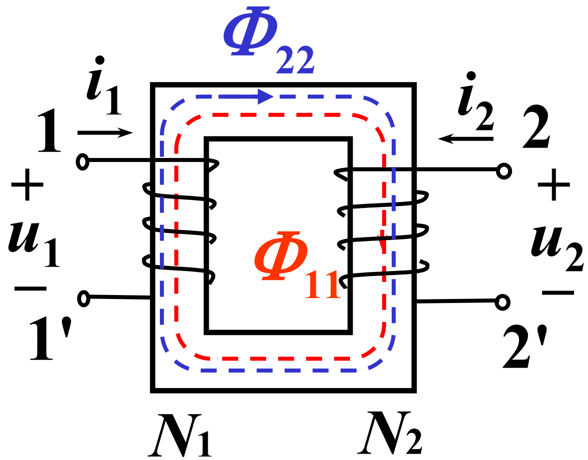
$$\begin{cases} \dot{U}_1 = \mathrm{j}\omega L_1 \dot{I}_1 + \mathrm{j}\omega M \dot{I}_2 \\ \dot{U}_2 = \mathrm{j}\omega L_2 \dot{I}_2 + \mathrm{j}\omega M \dot{I}_1 \end{cases}$$

全耦合时 $M = \sqrt{L_1 L_2}$ ， $k = 1$

$$\dot{I}_1 = \frac{\dot{U}_2 - \mathrm{j}\omega L_2 \dot{I}_2}{\mathrm{j}\omega M}$$

$$\dot{U}_1 = \frac{L_1}{M}(\dot{U}_2 - \mathrm{j}\omega L_2 \dot{I}_2) + \mathrm{j}\omega M \dot{I}_2 = \frac{L_1}{M}\dot{U}_2$$

$$\frac{\dot{U}_1}{\dot{U}_2} = \sqrt{\frac{L_1}{L_2}}$$



$$\Phi = \Phi_{11} + \Phi_{22}$$

$$u_1 = N_1 \frac{\mathrm{d}\Phi}{\mathrm{d}t} \quad , \quad u_2 = N_2 \frac{\mathrm{d}\Phi}{\mathrm{d}t}$$

$$\frac{u_1}{u_2} = \frac{N_1}{N_2} = n$$

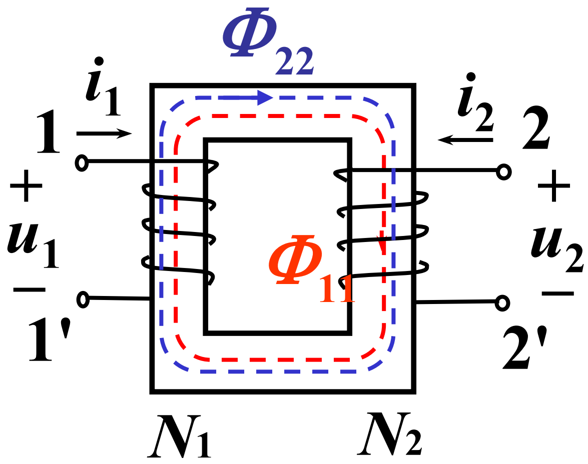
则

$$\frac{u_1}{u_2} = \frac{N_1}{N_2} = n = \frac{L_1}{M} = \frac{M}{L_2} = \sqrt{\frac{L_1}{L_2}}$$

全耦合变压器的电压、电流关系

$$\begin{cases} \dot{U}_1 = n\dot{U}_2 \\ \dot{I}_1 = \dfrac{\dot{U}_1 - \mathrm{j}\omega M \dot{I}_2}{\mathrm{j}\omega L_1} = \dfrac{\dot{U}_1}{\mathrm{j}\omega L_1} - \dfrac{\mathrm{j}\omega M}{\mathrm{j}\omega L_1}\dot{I}_2 = \dfrac{\dot{U}_1}{\mathrm{j}\omega L_1} - \dfrac{1}{n}\dot{I}_2 \end{cases}$$

3. 理想变压器

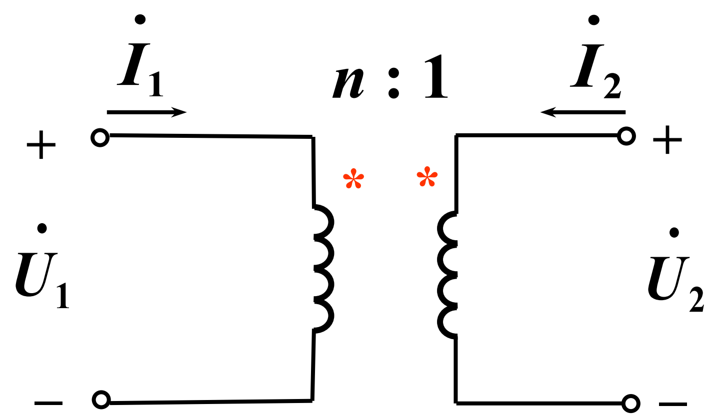


全耦合变压器 $\begin{cases} \dot{U}_1 = n\dot{U}_2 \\ \dot{I}_1 = \dfrac{\dot{U}_1}{\mathrm{j}\omega L_1} - \dfrac{1}{n}\dot{I}_2 \end{cases}$

当 L_1 , M, $L_2 \to \infty$，L_1/L_2 比值不变 (磁导率 $\mu \to \infty$) ，则有

$$\begin{cases} \dot{U}_1 = n\dot{U}_2 \\ \dot{I}_1 = -\dfrac{1}{n}\dot{I}_2 \end{cases}$$

理想变压器的元件特性

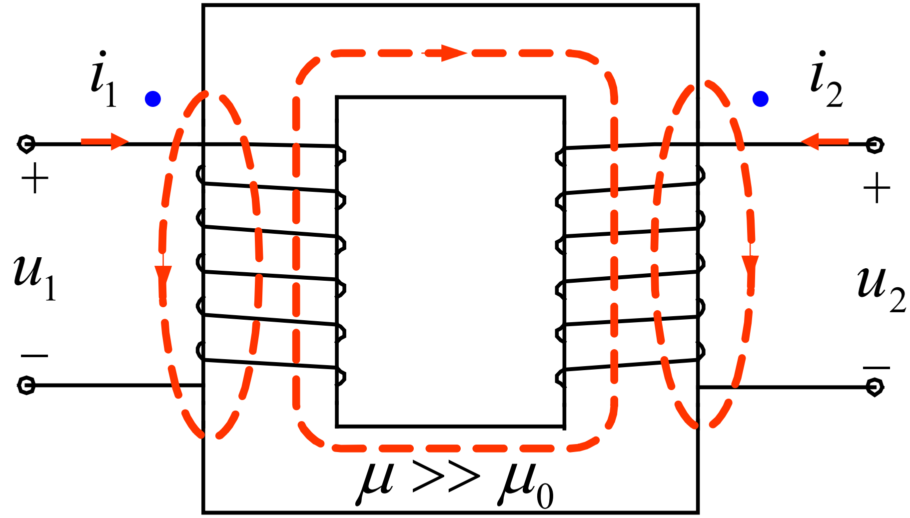


理想变压器的电路模型

$$\boxed{\begin{array}{l} k=1 \\ \mu \to \infty \\ R_1 = 0 = R_2 \end{array}}$$

$L_1, L_2, M \to \infty$

没有损耗



$H=0$

$$u_1 = \frac{d\psi_1}{dt} = \frac{d(N_1\phi)}{dt} = N_1 \frac{d\phi}{dt}$$

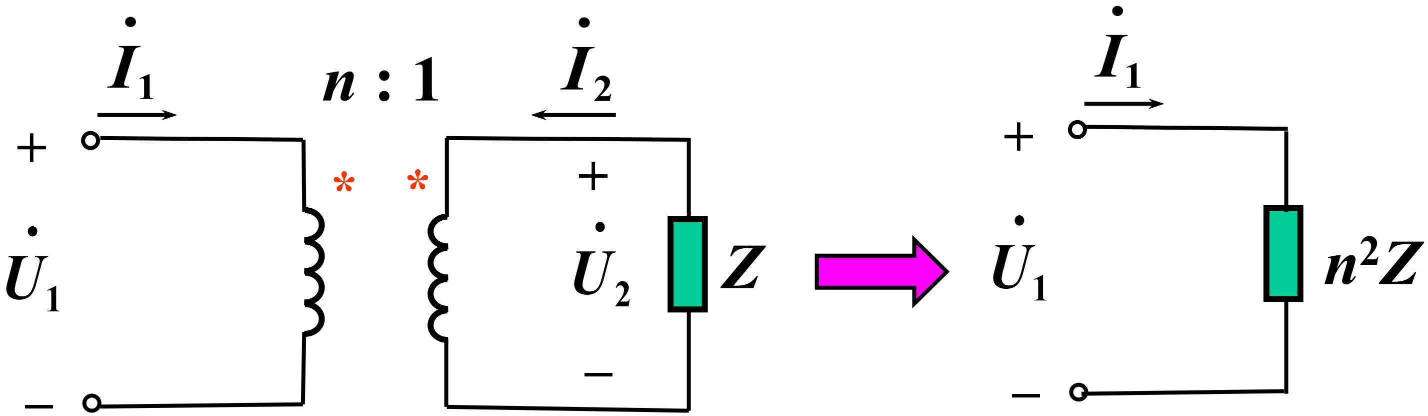
$$u_2 = \frac{d\psi_2}{dt} = \frac{d(N_2\phi)}{dt} = N_2 \frac{d\phi}{dt}$$

$$\oint \vec{H} \cdot d\vec{l} = \sum i = N_1 i_1 + N_2 i_2 = 0$$

$$\boxed{\frac{u_1}{u_2} = \frac{N_1}{N_2} = n} \qquad \boxed{i_1 = -\frac{N_2}{N_1} i_2 = -\frac{1}{n} i_2} \qquad \boxed{u_1 i_1 + u_2 i_2 = 0}$$

理想变压器的性质：

(a) 阻抗变换性质

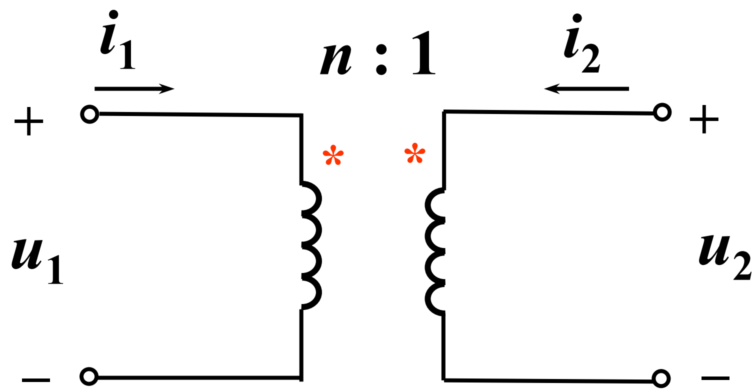


$$\frac{\dot{U}_1}{\dot{I}_1} = \frac{n\dot{U}_2}{-1/n\dot{I}_2} = n^2\left(-\frac{\dot{U}_2}{\dot{I}_2}\right) = n^2 Z$$

(b) 功率传输

理想变压器的特性方程为代数关系，因此无记忆作用。

$$\begin{cases} u_1 = nu_2 \\ i_1 = -\dfrac{1}{n} i_2 \end{cases}$$



$$p = u_1 i_1 + u_2 i_2 = u_1 i_1 + \frac{1}{n} u_1 \times (-n i_1) = 0$$

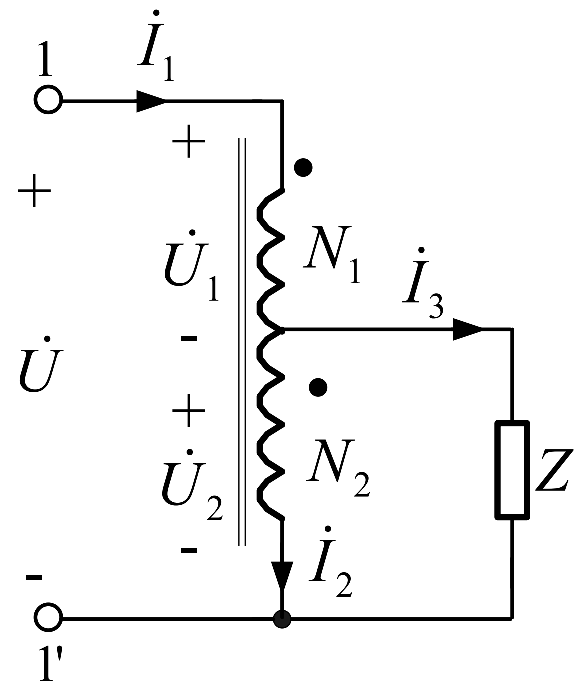
由此可以看出，理想变压器既不储能，也不耗能，在电路中只起传递信号和能量的作用。

4. 自耦变压器

自耦变压器是闭合铁心上只有一个线圈，从线圈中间接出一个抽头，线圈的一部分为一次绕组（或二次绕组），线圈的全部为二次绕组（或一次绕组）。

$$\frac{\dot{U}_1}{\dot{U}_2}=\frac{N_1}{N_2} \longrightarrow \frac{\dot{U}}{\dot{U}_2}=\frac{N_1+N_2}{N_2}$$

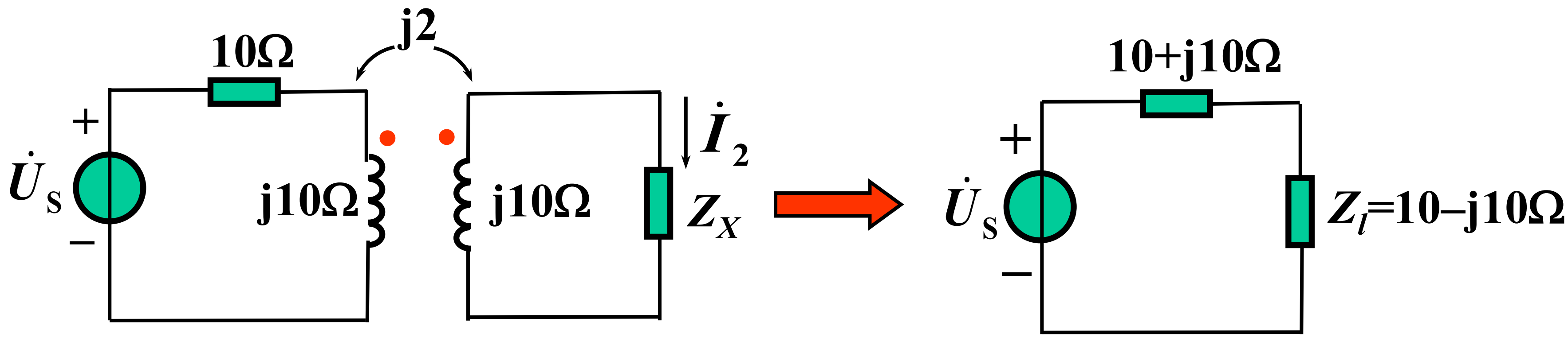
$$\frac{\dot{I}_1}{\dot{I}_2}=-\frac{N_2}{N_1} \longrightarrow \frac{\dot{I}_1}{\dot{I}_3}=\frac{\dot{I}_1}{\dot{I}_1-\dot{I}_2}=\frac{N_2}{N_1+N_2}$$



推导要点：能量守恒或者安培环路定理。

例1 已知 U_S=20 V，原边引入阻抗 Z_l=10－j10Ω。

求：Z_X，并求负载获得的有功功率。



原边等效电路

解 $$Z_l=\frac{\omega^2 M^2}{Z_{22}}=\frac{4}{Z_X+\mathrm{j}10}$$

$$\therefore Z_X=0.2+\mathrm{j}9.8\Omega$$

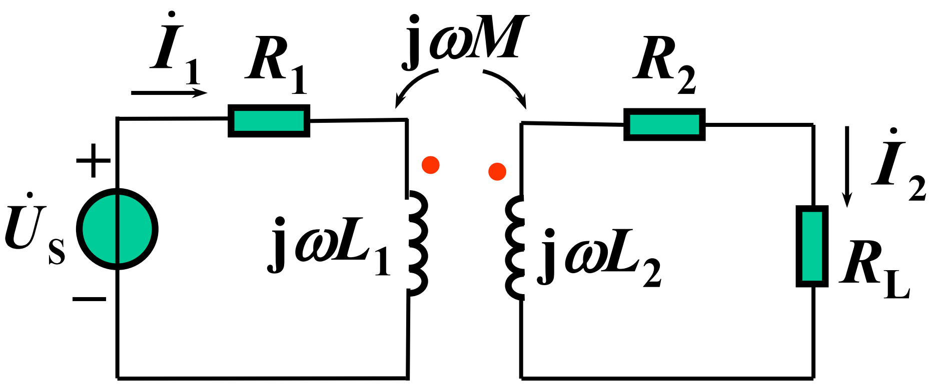
此时负载获得的功率 $P=P_{R引}=(\frac{20}{10+10})^2 R_l=10\text{ W}$

本例实际是最佳匹配状态

$$Z_l=Z_{11}^*,\qquad P=\frac{U_S^2}{4R}=10\text{ W}$$

变压器是否消耗功率？

例2 已知L_1=3.6H，L_2=0.06H，M=0.465H，R_1=20Ω，R_2=0.08Ω，

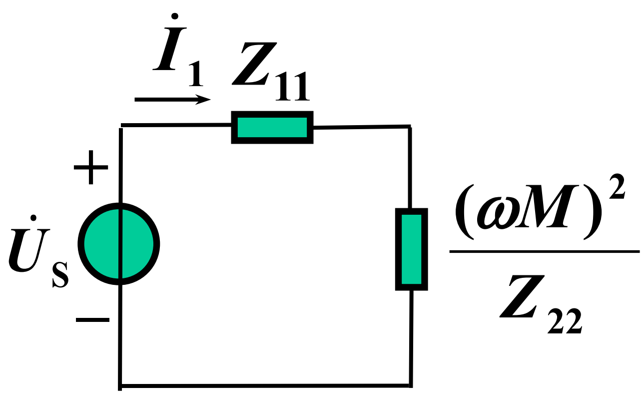


R_L=42Ω，ω=314rad/s，

$\dot{U}_s = 115\angle 0°\mathrm{V}$。

求：$\dot{I}_1$，$\dot{I}_2$。

解 原边等效电路



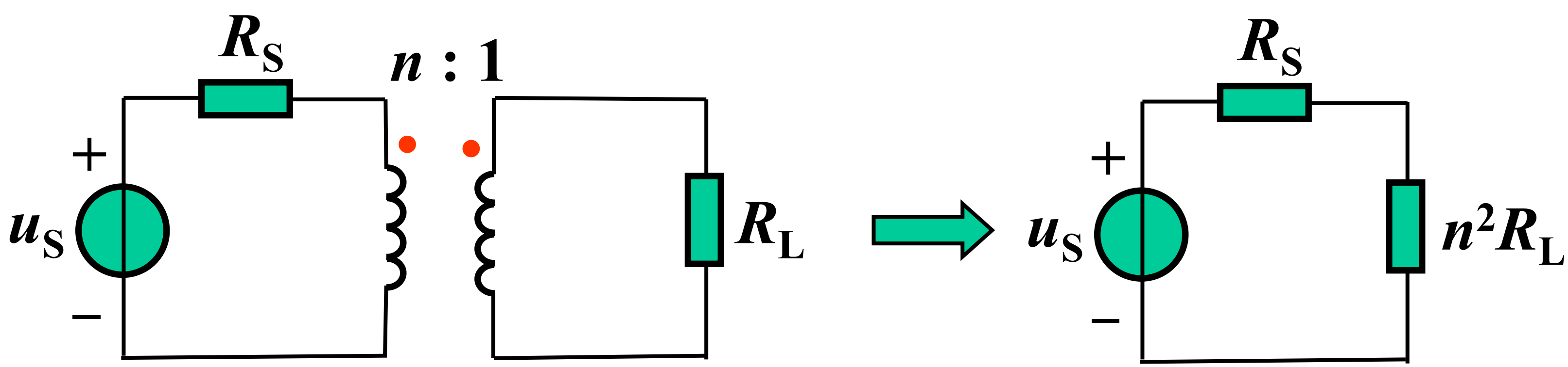
$$Z_{11} = R_1 + \mathrm{j}\omega L_1 = 20 + \mathrm{j}1131\Omega$$

$$Z_{22} = R_2 + R_L + \mathrm{j}\omega L_2 = 42.08 + \mathrm{j}18.85\,\Omega$$

$$Z_l = \frac{X_M^2}{Z_{22}} = 464\angle -24.1^{\circ}\ \ \Omega$$

$$\dot{I}_1 = \frac{\dot{U}_S}{Z_{11} + Z_l} = 0.111\angle -64.9^{\circ}\ \ \mathrm{A} \qquad \dot{I}_2 = \frac{\mathrm{j}\omega M \dot{I}_1}{Z_{22}} = 0.351\angle\ 1^{\circ}\ \mathrm{A}$$

例3 已知电源内阻R_S=1kΩ，负载电阻R_L=10Ω。为使R_L上获得最大功率，求理想变压器的变比n。

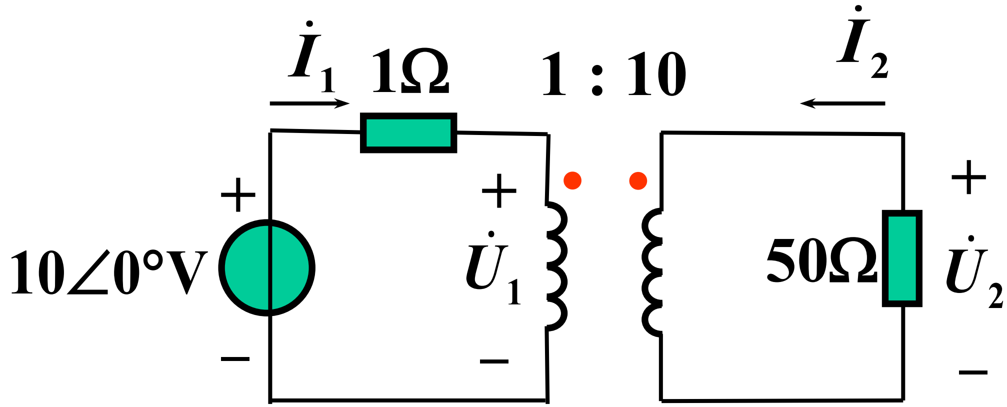


解 当 $n^2R_L=R_S$时匹配，即

$$10n^2=1000$$

$$\therefore\ n^2=100，\quad n=10.$$

例4 已知如图求 $\dot{U}_2$。

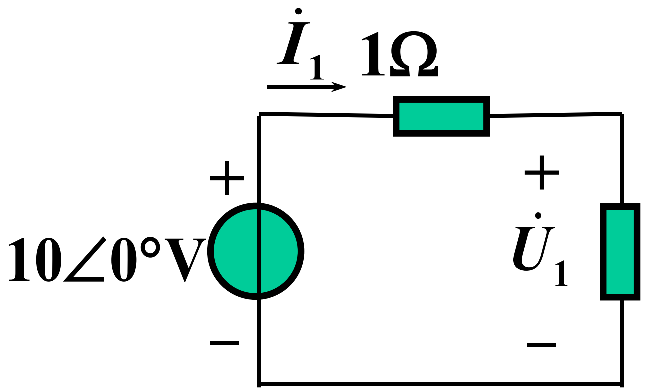


方法1 列方程

$$\begin{cases} 1\times \dot{I}_1+\dot{U}_1=10\angle 0^\circ \\ 50\dot{I}_2+\dot{U}_2=0 \\ \dot{U}_1=\dfrac{1}{10}\dot{U}_2 \\ \dot{I}_1=-10\dot{I}_2 \end{cases}$$

解得 $\Rightarrow$ $\dot{U}_2=33.33\angle 0^\circ\text{V}$

方法2　阻抗变换



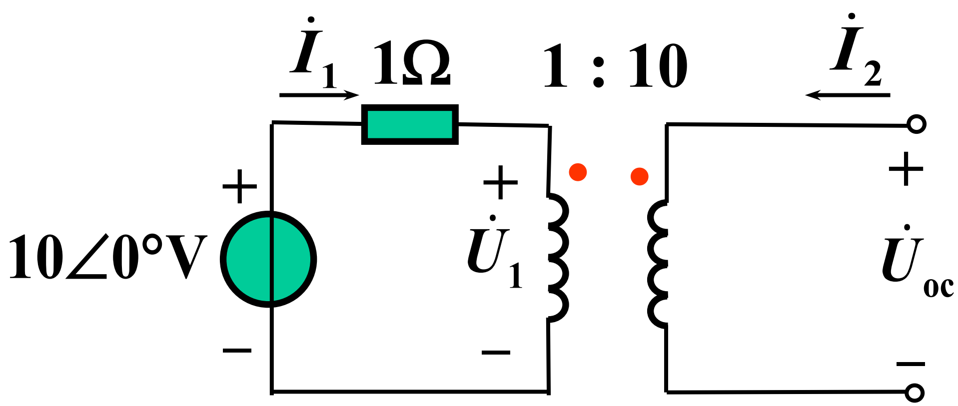
$$\left(\frac{1}{10}\right)^2 \times 50 = \frac{1}{2}\Omega$$

$$\dot{U}_1 = \frac{10\angle 0^\circ}{1+1/2} \times \frac{1}{2} = \frac{10}{3}\angle 0^\circ \text{V}$$

$$\dot{U}_2 = n\dot{U}_1 = 10\,\dot{U}_1$$

$$= 33.33\angle 0^\circ \text{V}$$

方法3　戴维南等效



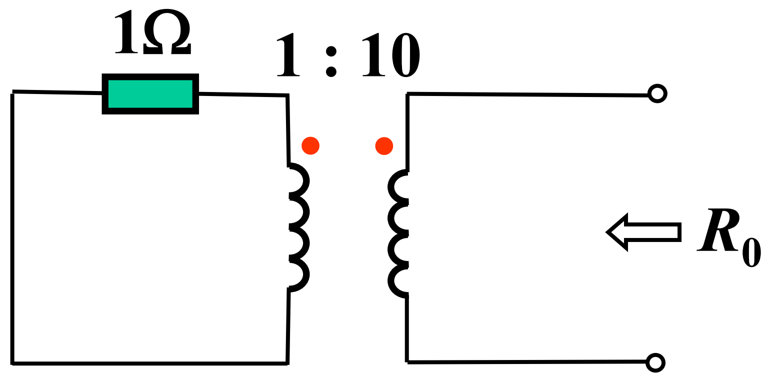
（1）求$\dot{U}_{oc}$

$$\because \dot{I}_2 = 0, \quad \therefore \dot{I}_1 = 0$$

$$\dot{U}_{oc} = 10\dot{U}_1 = 10\dot{U}_S$$

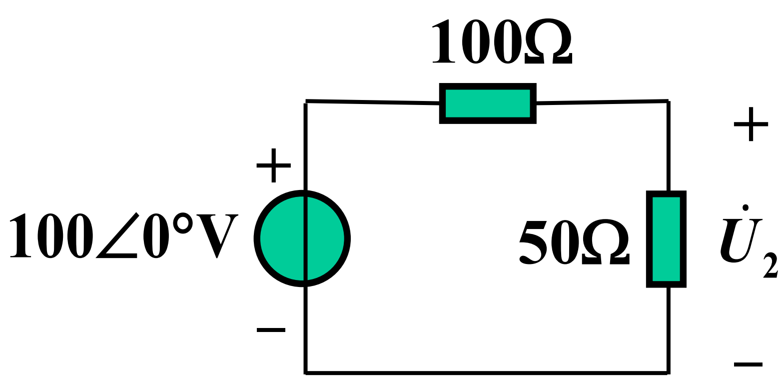
$$= 100\angle 0^\circ \text{V}$$

（2） 求R_0



$R_0=10^2\times1=100\Omega$

戴维南等效电路



$$\dot{U}_2=\frac{100\angle0°}{100+50}\times50=33.33\angle0°\text{V}$$

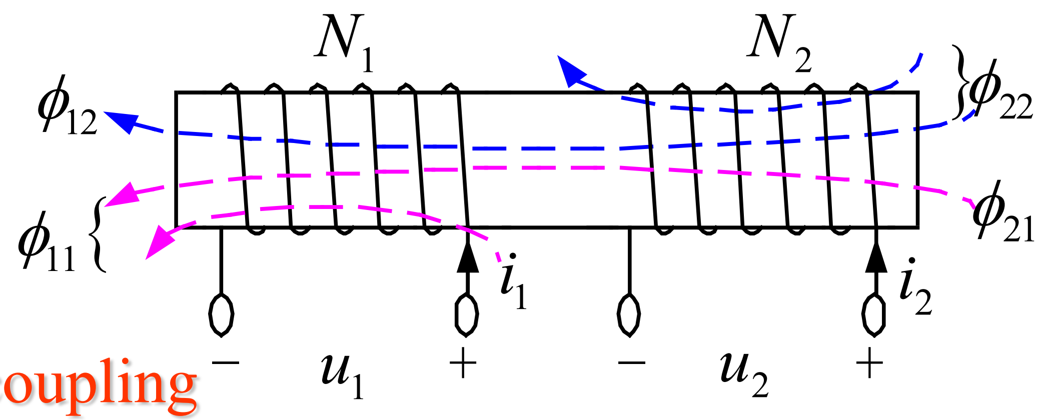
作业

- 13.2节：13-6
- 13.3节：13-9
- 13.4节：13-15
- 13.5节：13-20

3. 自感和互感　Self-inductance and mutual inductance

L_1、L_2 ——Self-inductance

M ——Mutual inductance



4. 耦合系数　Coefficient of coupling

$$k = \frac{M}{\sqrt{L_1 L_2}} \quad (0 \le k \le 1)$$

$$k = \frac{M}{\sqrt{L_1 L_2}} = \sqrt{\frac{M_{12} M_{21}}{L_1 L_2}} = \sqrt{\frac{\frac{\psi_{12}}{i_2} \cdot \frac{\psi_{21}}{i_1}}{\frac{\psi_{11}}{i_1} \cdot \frac{\psi_{22}}{i_2}}}$$

$$= \sqrt{\frac{N_1 \phi_{12} \cdot N_2 \phi_{21}}{N_1 \phi_{11} \cdot N_2 \phi_{22}}} = \sqrt{\frac{\phi_{12} \cdot \phi_{21}}{\phi_{11} \cdot \phi_{22}}} \le 1$$

$$\begin{cases} \psi_1 = \psi_{11} + \psi_{12} \\ \psi_2 = \psi_{21} + \psi_{22} \end{cases}$$

$$= L_1 i_1 + M_{12} i_2$$

$$= M_{21} i_1 + L_2 i_2$$

$$\begin{cases} \psi_1 = N_1 \phi_{11} + N_1 \phi_{12} \\ \psi_2 = N_2 \phi_{21} + N_2 \phi_{22} \end{cases}$$