Chapter 16　二端口网络
Two-port Networks

16.1 二端口网络的特性　Characteristics of two-ports

16.2 二端口网络的参数　Parameters of two-ports

16.3 参数之间的关系　Relationships between parameters

16.4 二端口网络的等效电路　Equivalent circuits

16.5 二端口网络的相互连接　Interconnections of two-ports

目标：

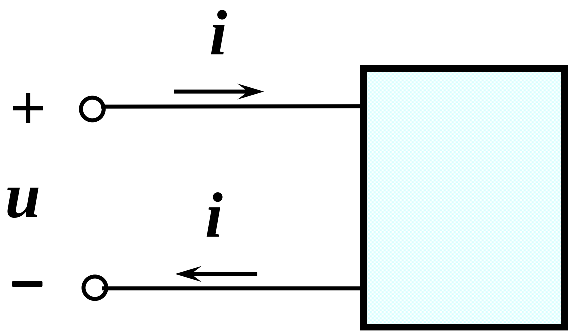
a.　理解二端口网络特性的描述方法；

b.　计算、测量二端口网络的任何参数，并可相互转换；

c.　计算带负载二端口网络的端口电量；

d.　分析相互连接的二端口网络。

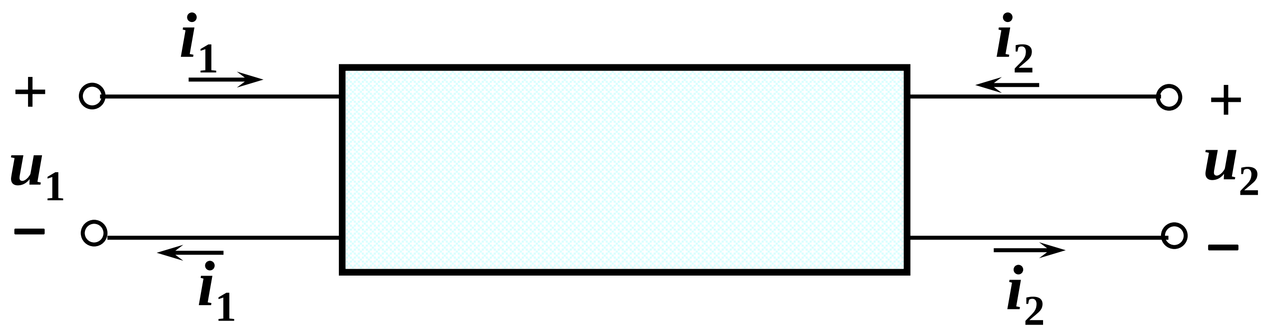
一、 端口 （port）



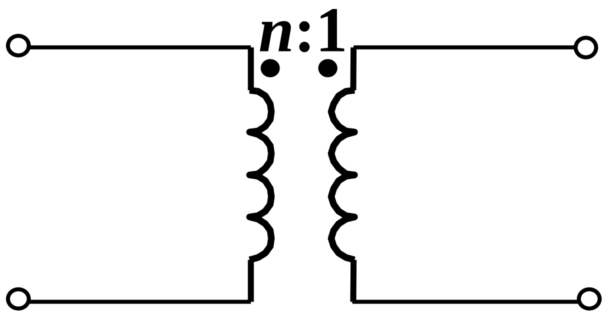
端口由一对端钮构成，且满足从一个端钮流入的电流等于从另一个端钮流出的电流。

二、二端口（two-port）

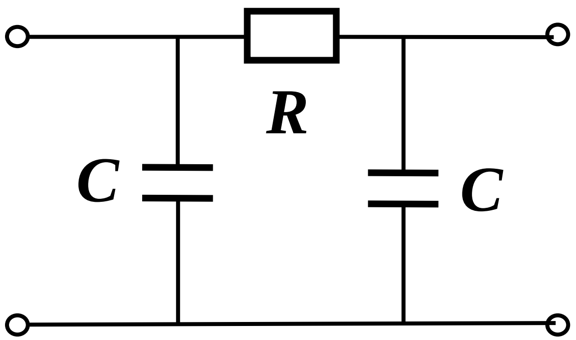
当一个电路与外部电路通过两个端口连接时称此电路为二端口。



典型二端口网络



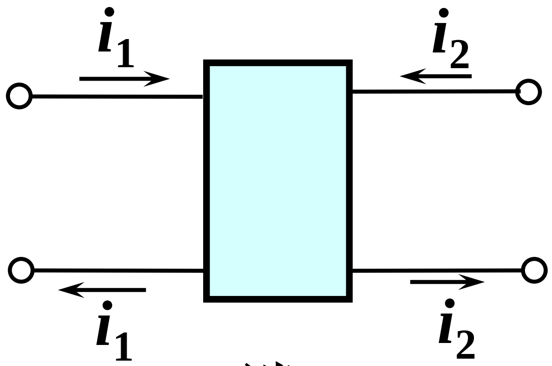
理想变压器



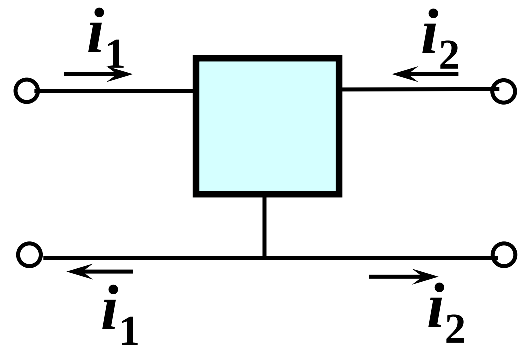
滤波器电路

- ✓ 二端口网络大量出现在通信、控制系统和电力系统中，信号（或能量）从一个端口输入，另一个端口输出。
- ✓ 很多情况下，我们只关心输入、输出端口的电压、电流，而并不关心二端口网络内部的情况，因此可以把二端口网络是为“黑盒子”来分析。

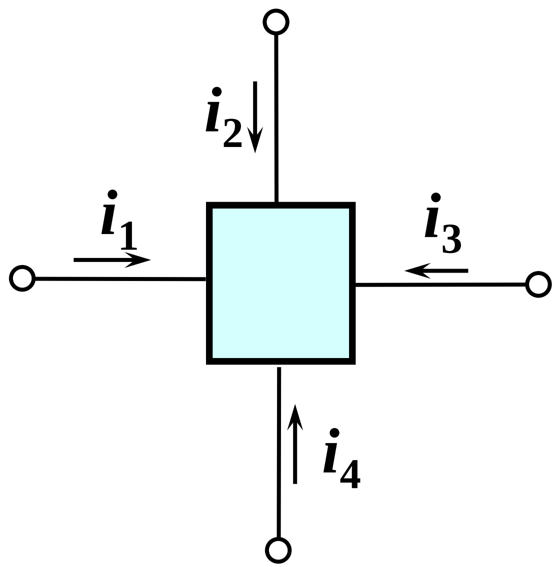
三、二端口与四端网络



二端口

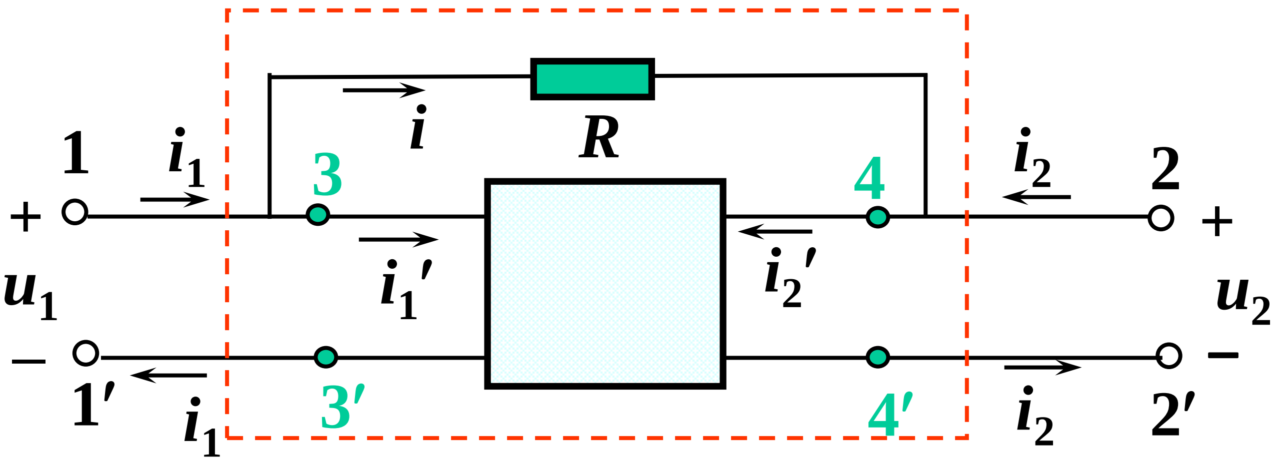


具有公共端的二端口



四端网络

例



1−1′， 2−2′ 是二端口。

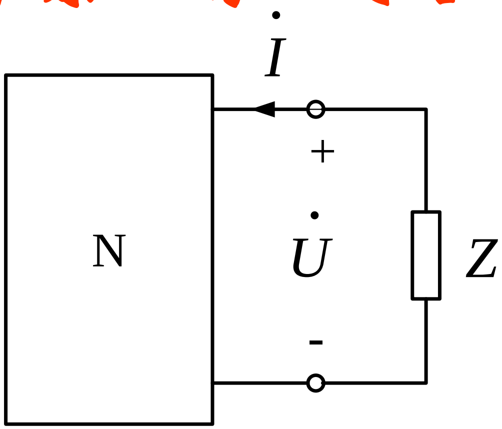
3−3′， 4−4′ 不是二端口，是四端网络。

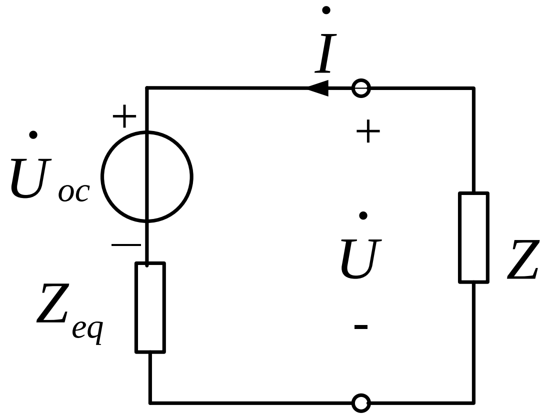
因为

$$\left.\begin{array}{l} i_1' = i_1 - i \neq i_1 \\ i_2' = i_2 + i \neq i_2 \end{array}\right\}$$ 不满足端口条件

16.2 二端口的端口特性方程

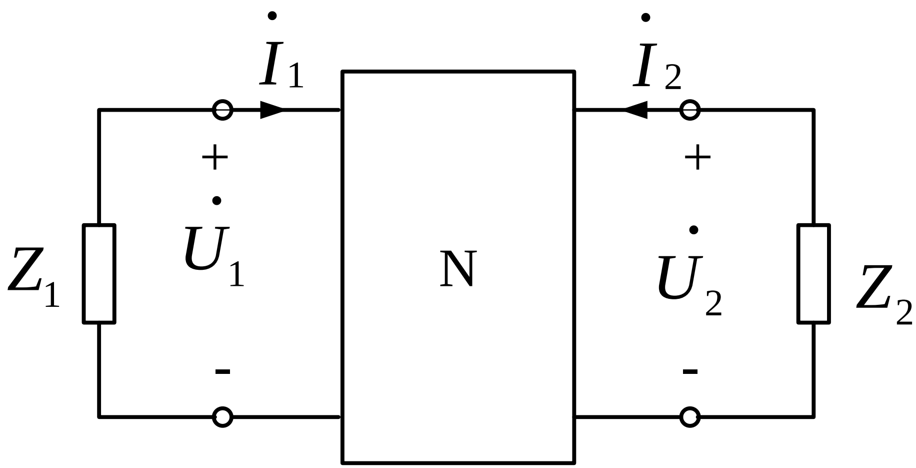
含源一端口网络





$$\begin{cases} \dot{U} = \dot{U}_{oc} + Z_{eq}\dot{I} \\ \dot{U} = -Z\dot{I} \end{cases}$$

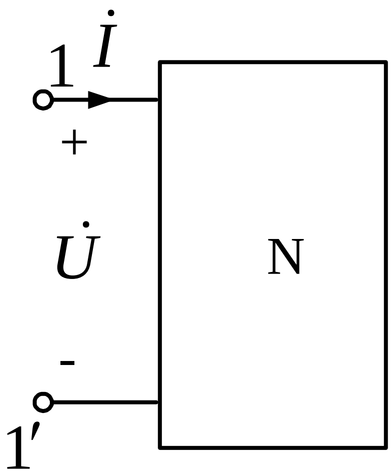
含源二端口网络



$$\begin{cases} ? \\ ? \\ \dot{U}_1 = -Z_1\dot{I}_1 \\ \dot{U}_2 = -Z_2\dot{I}_2 \end{cases}$$

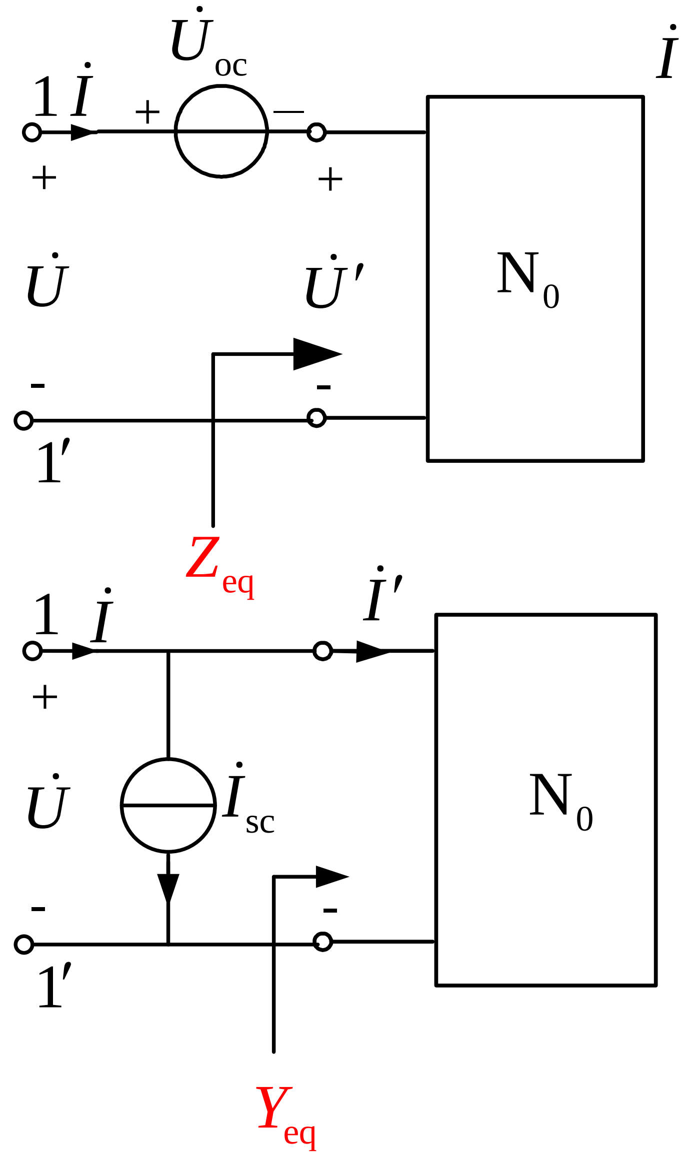
关于4个端口变量的两个方程

含源一端口网络



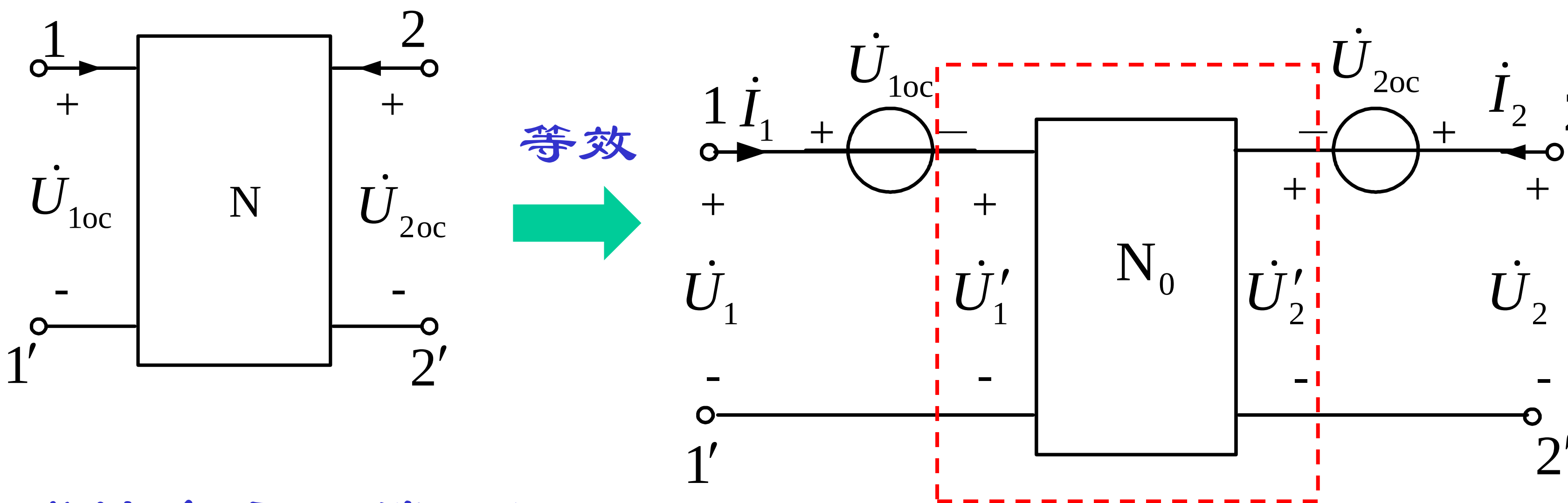
$$\dot{U} = \dot{U}_{oc} + Z_{eq}\dot{I}$$

$$\dot{I} = Y_{eq}\dot{U} + \dot{I}_{sc}$$

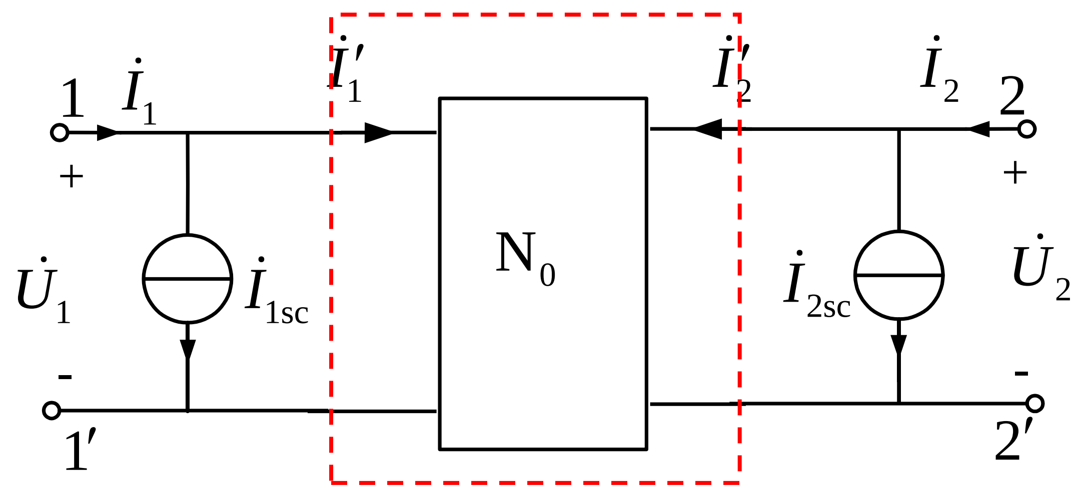


含源一端口网络可以等效为电压源与将原一端口网络内部独立电源置零后所得的无源一端口网络串联

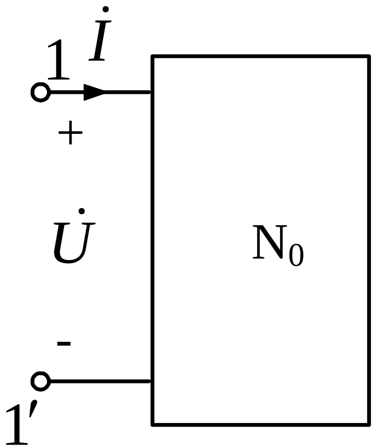
含源二端口网络



- ✓ 线性含源二端口网络，可等效为电压源与不含源二端口网络的连接



- ✓ 不含独立源的二端口网络称为**松弛二端口网络**



无源一端口网络

$$\dot{U} = Z\dot{I}$$

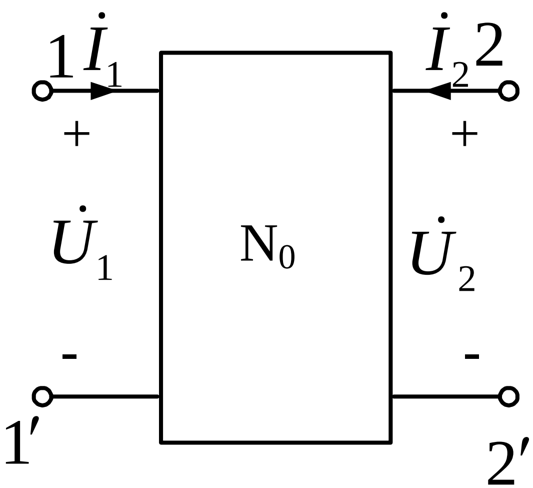
$$\dot{I} = Y\dot{U}$$

$$\begin{cases} \dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 \\ \dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2 \end{cases}$$

阻抗参数方程

$$\begin{cases} \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 \end{cases}$$

导纳参数方程



无源二端口网络

$$\begin{cases} \dot{U}_1 = h_{11}\dot{I}_1 + h_{12}\dot{U}_2 \\ \dot{I}_2 = h_{21}\dot{I}_1 + h_{22}\dot{U}_2 \end{cases}$$

$$\begin{cases} \dot{I}_1 = g_{11}\dot{U}_1 + g_{12}\dot{I}_2 \\ \dot{U}_2 = g_{21}\dot{U}_1 + g_{22}\dot{I}_2 \end{cases}$$

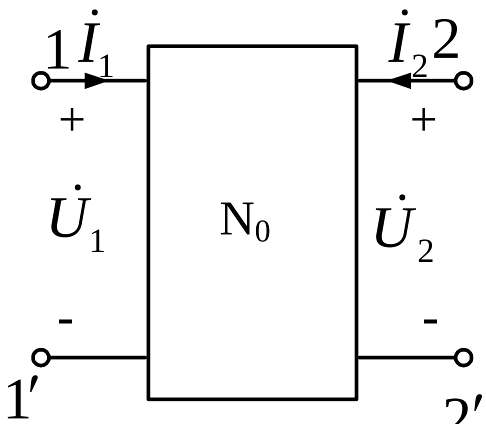
混和参数方程

$$\begin{cases} \dot{U}_1 = A\dot{U}_2 + B(-\dot{I}_2) \\ \dot{I}_1 = C\dot{U}_2 + D(-\dot{I}_2) \end{cases}$$

$$\begin{cases} \dot{U}_2 = A'\dot{U}_1 + B'(-\dot{I}_1) \\ \dot{I}_2 = C'\dot{U}_1 + D'(-\dot{I}_1) \end{cases}$$

传输参数方程

16.2 二端口网络的参数　Parameters of two-ports



$$\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}=\begin{bmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{bmatrix}\begin{bmatrix}\dot{I}_1\\ \dot{I}_2\end{bmatrix}=\boldsymbol{Z}\begin{bmatrix}\dot{I}_1\\ \dot{I}_2\end{bmatrix}$$

$$\begin{bmatrix}\dot{I}_1\\ \dot{I}_2\end{bmatrix}=\begin{bmatrix}Y_{11} & Y_{12}\\ Y_{21} & Y_{22}\end{bmatrix}\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}=\boldsymbol{Y}\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}$$

$\boldsymbol{ZY}=\boldsymbol{1}$

$$\begin{bmatrix}\dot{U}_1\\ \dot{I}_2\end{bmatrix}=\begin{bmatrix}h_{11} & h_{12}\\ h_{21} & h_{22}\end{bmatrix}\begin{bmatrix}\dot{I}_1\\ \dot{U}_2\end{bmatrix}=\boldsymbol{H}\begin{bmatrix}\dot{I}_1\\ \dot{U}_2\end{bmatrix}$$

$$\begin{bmatrix}\dot{I}_1\\ \dot{U}_2\end{bmatrix}=\begin{bmatrix}g_{11} & g_{12}\\ g_{21} & g_{22}\end{bmatrix}\begin{bmatrix}\dot{U}_1\\ \dot{I}_2\end{bmatrix}=\boldsymbol{G}\begin{bmatrix}\dot{U}_1\\ \dot{I}_2\end{bmatrix}$$

$\boldsymbol{HG}=\boldsymbol{1}$

$$\begin{bmatrix}\dot{U}_1\\ \dot{I}_1\end{bmatrix}=\begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}\dot{U}_2\\ -\dot{I}_2\end{bmatrix}=\boldsymbol{T}\begin{bmatrix}\dot{U}_2\\ -\dot{I}_2\end{bmatrix}$$

$$\begin{bmatrix}\dot{U}_2\\ \dot{I}_2\end{bmatrix}=\begin{bmatrix}A' & B'\\ C' & D'\end{bmatrix}\begin{bmatrix}\dot{U}_1\\ -\dot{I}_1\end{bmatrix}=\boldsymbol{T}'\begin{bmatrix}\dot{U}_1\\ -\dot{I}_1\end{bmatrix}$$

$$\begin{bmatrix}A & -B\\ C & -D\end{bmatrix}\begin{bmatrix}A' & -B'\\ C' & -D'\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

参数是什么？——参数的物理含义

参数如何获得？——计算参数、测量参数

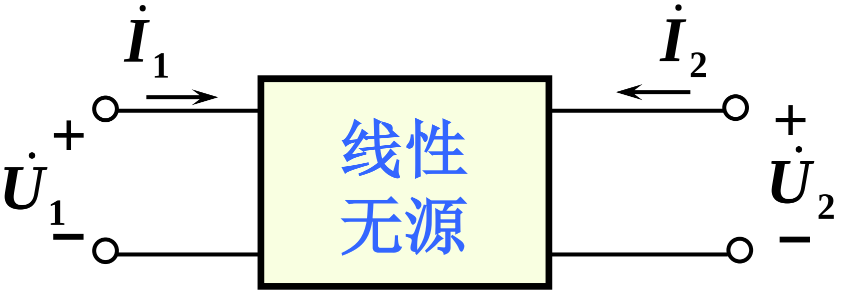
参数如何应用？——计算端口变量、获得等效电路

16.3 二端口的参数和方程

一、 Z 参数（阻抗参数admittance parameters）方程

其矩阵形式为

$$\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}=\begin{bmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{bmatrix}\begin{bmatrix}\dot{I}_1\\ \dot{I}_2\end{bmatrix}$$



$$Z=\begin{bmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{bmatrix}$$

称为Z参数矩阵。

Z参数的实验测定

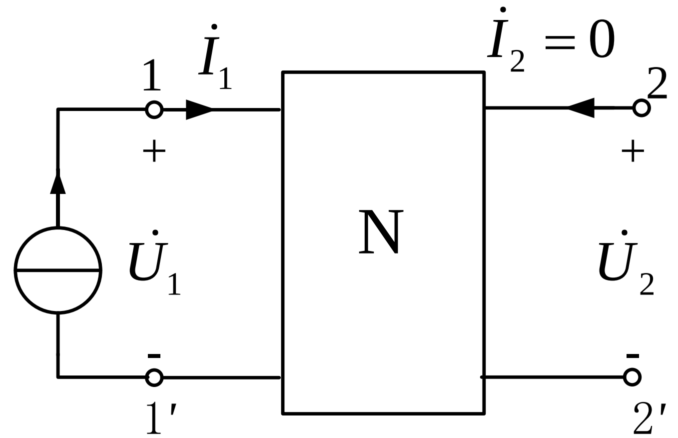
$$\dot{U}_1=Z_{11}\dot{I}_1+Z_{12}\dot{I}_2$$
$$\dot{U}_2=Z_{21}\dot{I}_1+Z_{22}\dot{I}_2$$

用端口开路实验测Z参数。

$$\dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2$$

$$\dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2$$

实验电路图

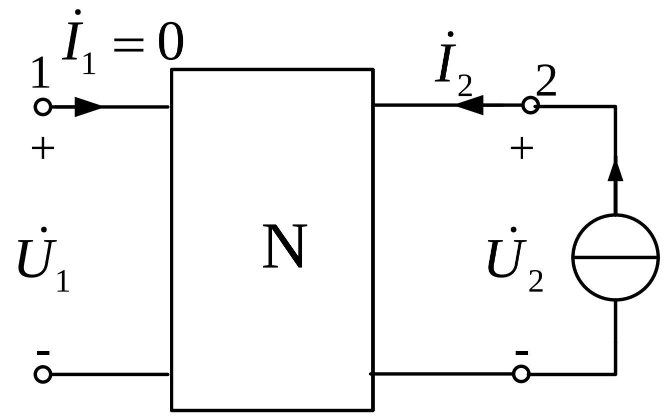


$$Z_{11} = \left.\frac{\dot{U}_1}{\dot{I}_1}\right|_{\dot{I}_2=0}$$

2−2′ 开路
1−1′入端阻抗

$$Z_{21} = \left.\frac{\dot{U}_2}{\dot{I}_1}\right|_{\dot{I}_2=0}$$

2−2′ 开路
转移阻抗



$$Z_{12} = \left.\frac{\dot{U}_1}{\dot{I}_2}\right|_{\dot{I}_1=0}$$

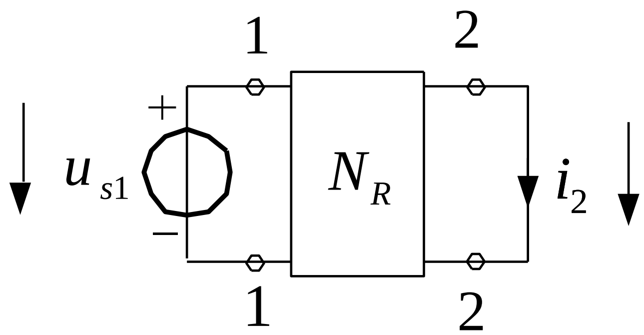
1−1′ 开路
转移阻抗

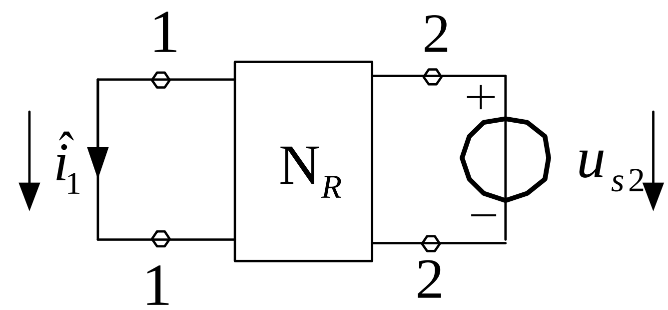
$$Z_{22} = \left.\frac{\dot{U}_2}{\dot{I}_2}\right|_{\dot{I}_1=0}$$

1−1′ 开路
2−2′入端阻抗

Z参数又称开路阻抗（open impedance）参数

第一种形式的证明



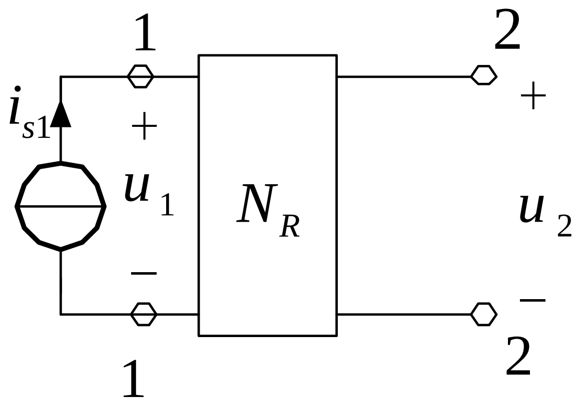


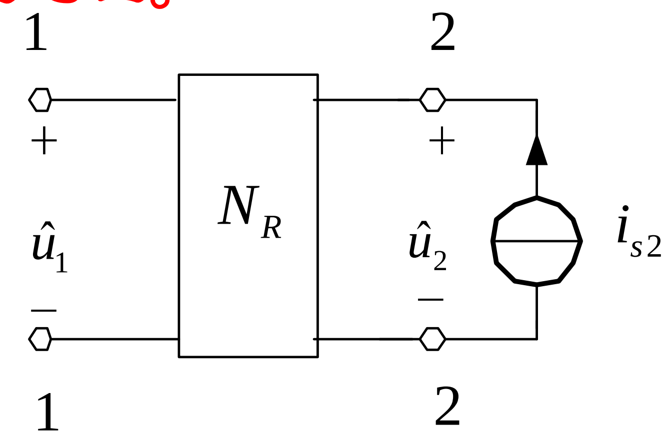
N$_R$——Consists of linear resistors only—— reciprocity network

$$\left.\begin{aligned} u_{s1}\hat{i}_1 + 0 + \sum_{k=3}^{b} u_k \hat{i}_k &= 0 \\ 0 + u_{s2} i_2 + \sum_{k=3}^{b} \hat{u}_k i_k &= 0 \end{aligned}\right\} \quad \hat{u}_k i_k = R_k \hat{i}_k i_k = u_k \hat{i}_k \longrightarrow \boxed{\frac{u_{s1}}{i_2} = \frac{u_{s2}}{\hat{i}_1}}$$

两电路的激励与响应之比相等

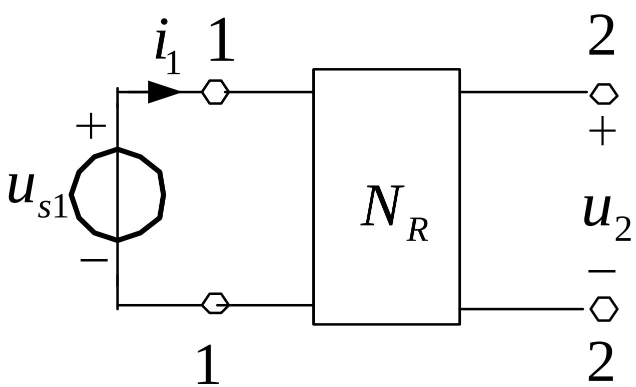
第二种形式： 激励是电流源，响应是电压。

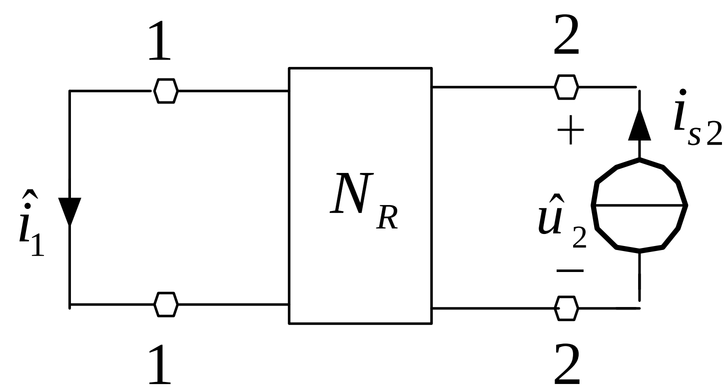




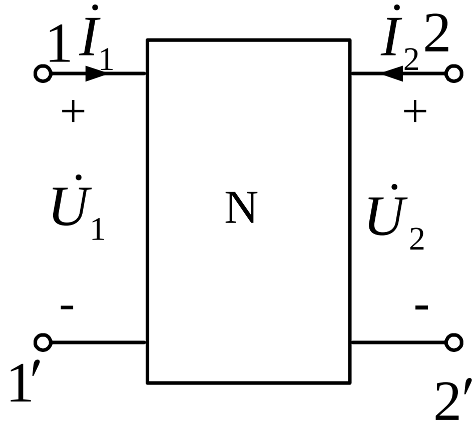
$$\boxed{\frac{i_{s1}}{u_2}=\frac{i_{s2}}{\hat{u}_1}}$$

第三种形式：电路1激励是电压源，响应是电压；电路2激励是电流源，响应是电流



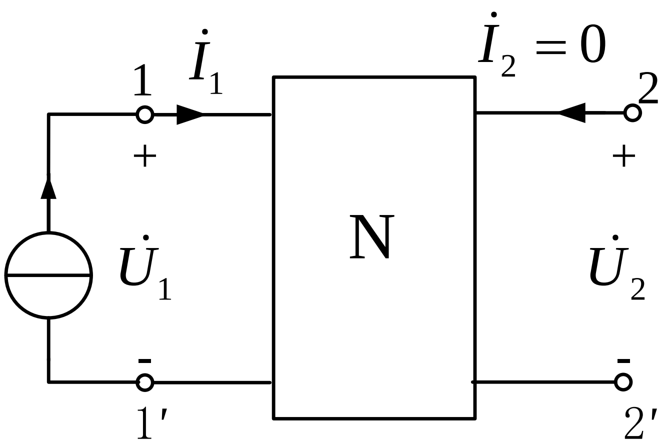


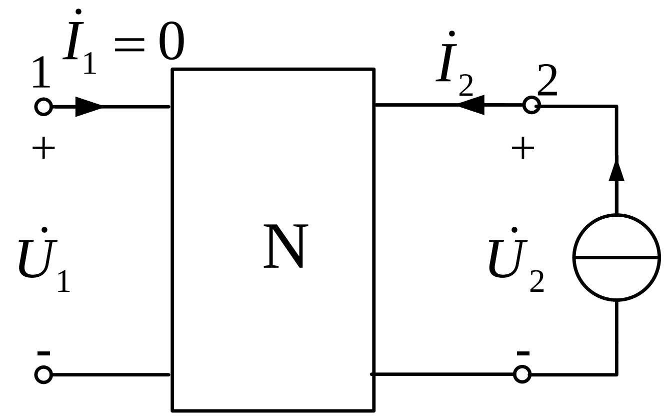
$$\boxed{\frac{u_{s1}}{u_2}=\frac{i_{s2}}{\hat{i}_1}}$$



$$Z_{11}=\left.\frac{\dot{U}_1}{\dot{I}_1}\right|_{\dot{I}_2=0} \qquad Z_{22}=\left.\frac{\dot{U}_2}{\dot{I}_2}\right|_{\dot{I}_1=0}$$

$$Z_{21}=\left.\frac{\dot{U}_2}{\dot{I}_1}\right|_{\dot{I}_2=0} \qquad Z_{12}=\left.\frac{\dot{U}_1}{\dot{I}_2}\right|_{\dot{I}_1=0}$$



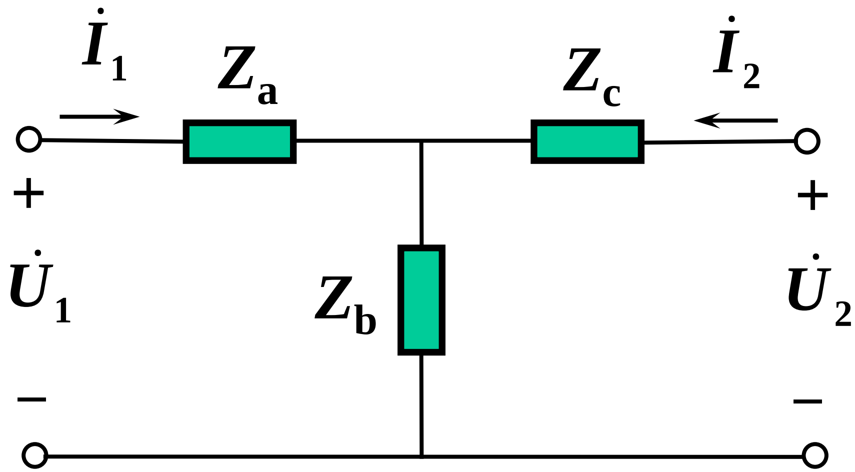


互易二端口？ → $Z_{12}=Z_{21}$

对称二端口？ → $Z_{12}=Z_{21}$, $Z_{11}=Z_{22}$

例1 求所示电路的Z 参数

$$\dot U_1 = Z_{11}\dot I_1 + Z_{12}\dot I_2$$
$$\dot U_2 = Z_{21}\dot I_1 + Z_{22}\dot I_2$$



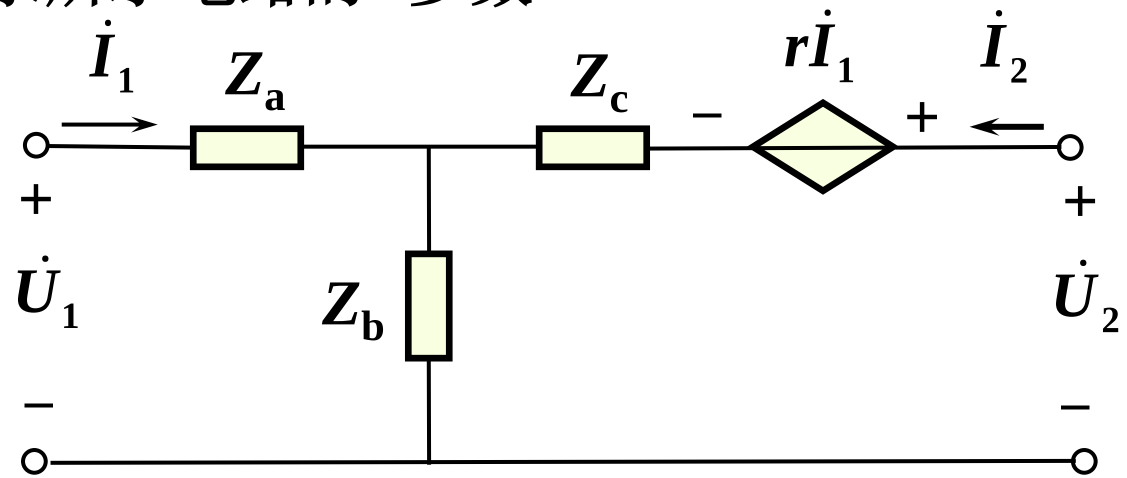
由实验测定得参数

$$Z_{11} = \frac{\dot U_1}{\dot I_1}\bigg|_{\dot I_2=0} = Z_a + Z_b \qquad Z_{12} = \frac{\dot U_1}{\dot I_2}\bigg|_{\dot I_1=0} = Z_b$$

$$Z_{21} = \frac{\dot U_2}{\dot I_1}\bigg|_{\dot I_2=0} = Z_b \qquad Z_{22} = \frac{\dot U_2}{\dot I_2}\bigg|_{\dot I_1=0} = Z_b + Z_c$$

互易二端口，当 $Z_a=Z_c$ 时为对称二端口。

例2　求所示电路的Z参数



直接列端口电压、电流方程

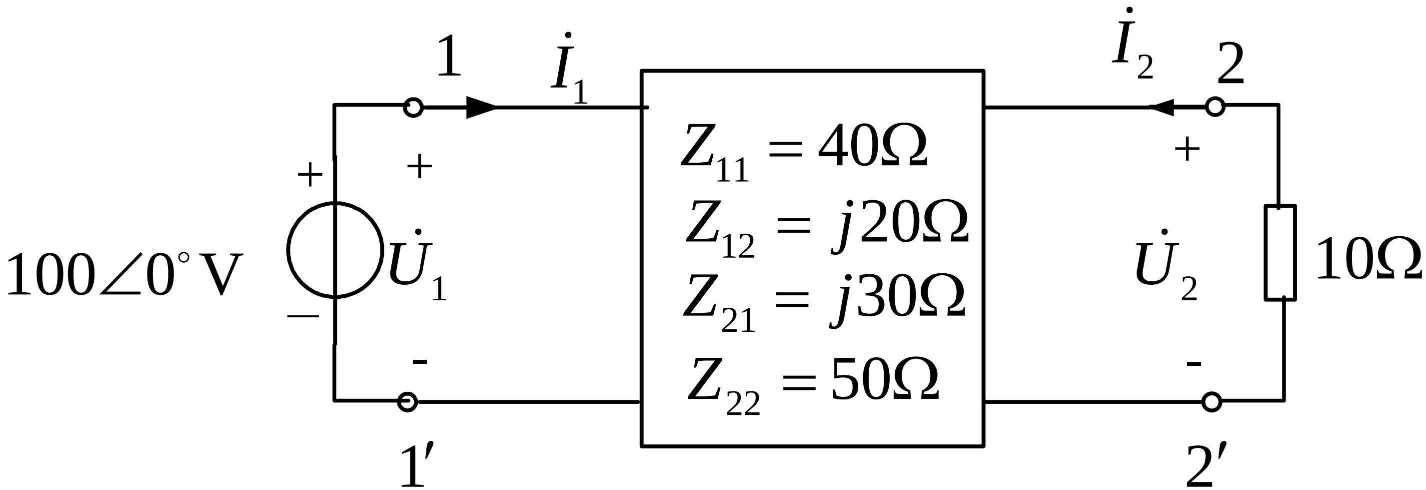
$$\dot{U}_1 = Z_a\dot{I}_1 + Z_b(\dot{I}_1 + \dot{I}_2)$$

$$\dot{U}_2 = r\dot{I}_1 + Z_c\dot{I}_2 + Z_b(\dot{I}_1 + \dot{I}_2)$$

$$\mathbf{Z} = \begin{bmatrix} Z_a + Z_b & Z_b \\ r + Z_b & Z_b + Z_c \end{bmatrix}$$

4个独立参数

例3 计算端口电流



VCR of the branches:

$$\dot{U}_1 = 100\angle 0^\circ$$
$$\dot{U}_2 = -10\dot{I}_2$$

Equations of the two-ports:

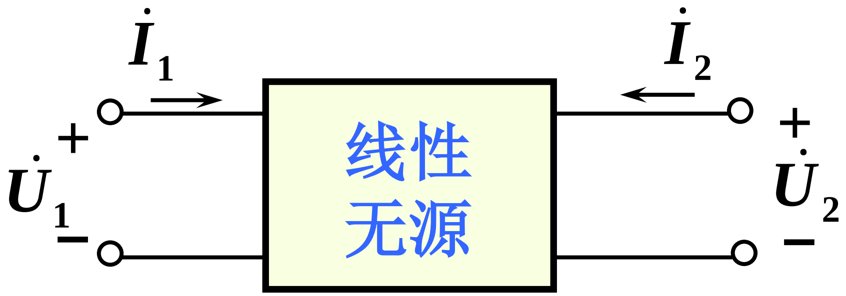
$$\dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2$$
$$\dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2$$

已知参数，确定端口变量

$$\begin{cases} 100\angle 0^\circ = 40\dot{I}_1 + j20\dot{I}_2 \\ -10\dot{I}_2 = j30\dot{I}_1 + 50\dot{I}_2 \end{cases}$$

$$\dot{I}_1 = 2\angle 0^\circ A$$
$$\dot{I}_2 = 1\angle -90^\circ A$$

二、 Y 参数（导纳参数admittance parameters）方程



矩阵形式为

$$\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix}$$

令 $Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$ 称为Y 参数矩阵。

Y 参数方程

$$\begin{cases} \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 \end{cases}$$

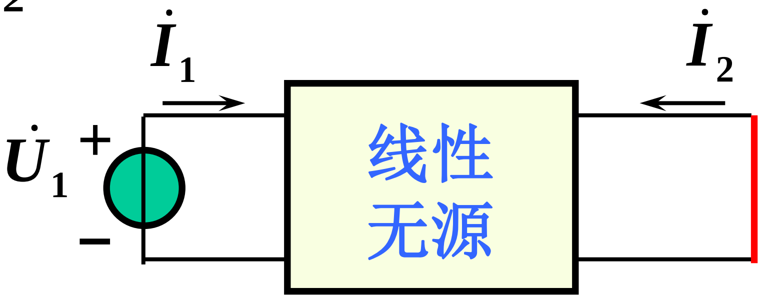
端口电流 $\dot{I}_1$和$\dot{I}_2$可视为$\dot{U}_1$和$\dot{U}_2$共同作用产生。

Y参数的实验测定

$$\dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2$$

$$\dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2$$

实验电路图



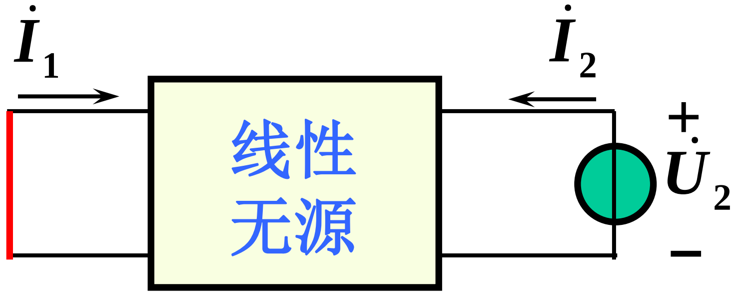
2−2′ 短路

$$Y_{11} = \left.\frac{\dot{I}_1}{\dot{U}_1}\right|_{\dot{U}_2=0}$$

2−2′ 短路
1−1′入端导纳

$$Y_{21} = \left.\frac{\dot{I}_2}{\dot{U}_1}\right|_{\dot{U}_2=0}$$

2−2′ 短路
转移导纳



1−1′ 短路

$$Y_{12} = \frac{\dot{I}_1}{\dot{U}_2}\bigg|_{\dot{U}_1=0}$$

1−1′ 短路
转移导纳

$$Y_{22} = \frac{\dot{I}_2}{\dot{U}_2}\bigg|_{\dot{U}_1=0}$$

1−1′ 短路
2−2′ 入端导纳

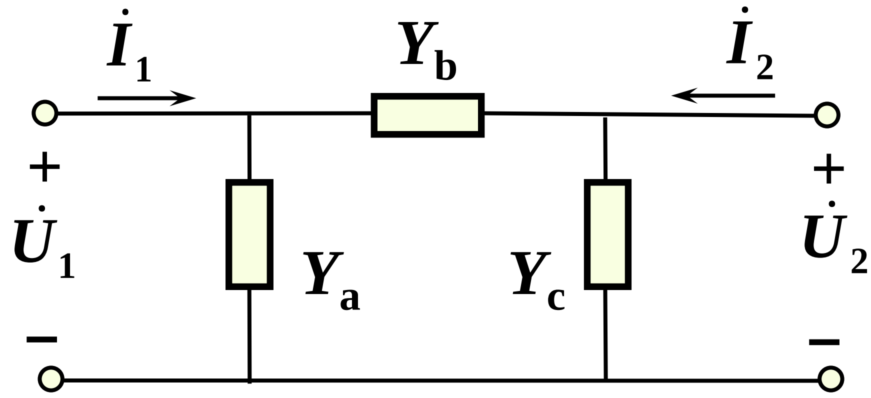
Y **参数也称为短路导纳（short admittance）参数。**

若二端口网络内部无受控源，电路满足互易定理，则

互易二端口有 $Y_{12}= Y_{21}$ ，只有三个参数是独立的。

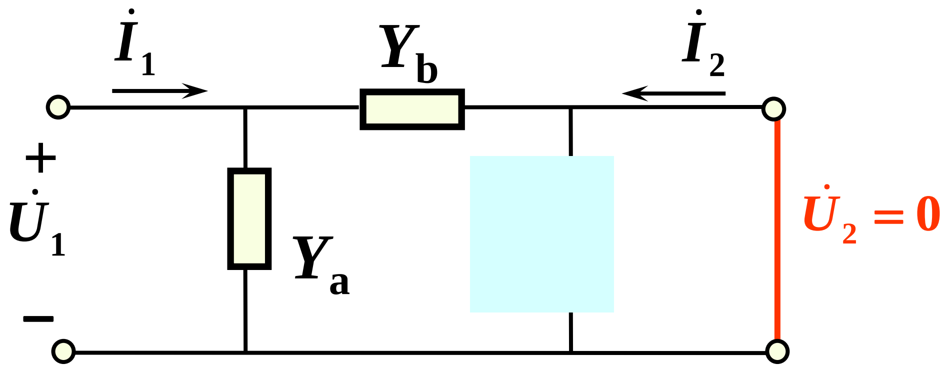
若二端口网络对称，则 $Y_{12}= Y_{21}$， $Y_{11}= Y_{22}$

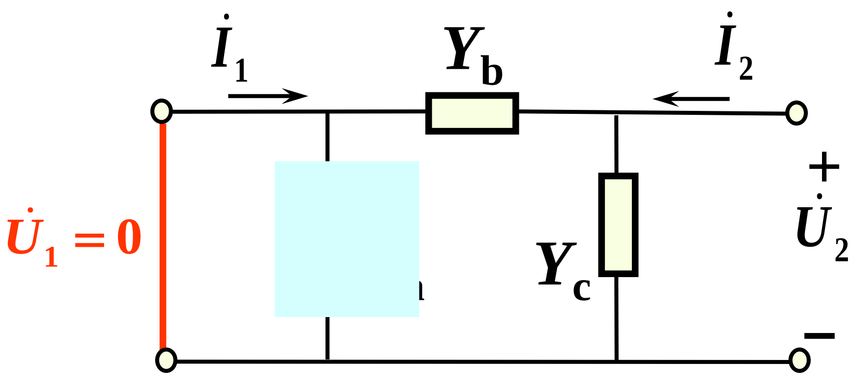
例1　求图示二端口的Y参数。



解

$$\begin{cases} \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 \end{cases}$$





$$Y_{11} = \left.\frac{\dot{I}_1}{\dot{U}_1}\right|_{\dot{U}_2=0} = Y_a + Y_b$$

$$Y_{21} = \left.\frac{\dot{I}_2}{\dot{U}_1}\right|_{\dot{U}_2=0} = -Y_b$$

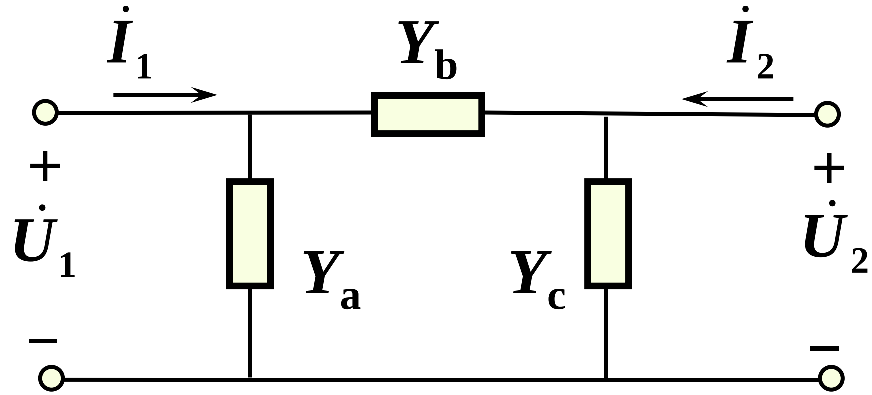
$$Y_{12} = \left.\frac{\dot{I}_1}{\dot{U}_2}\right|_{\dot{U}_1=0} = -Y_b$$

$$Y_{22} = \left.\frac{\dot{I}_2}{\dot{U}_2}\right|_{\dot{U}_2=0} = Y_b + Y_c$$

$$Y_{12} = Y_{21} = -Y_b$$

互易二端口

$$\boldsymbol{Y}=\begin{bmatrix} Y_a+Y_b & -Y_b \\ -Y_b & Y_b+Y_c \end{bmatrix}$$



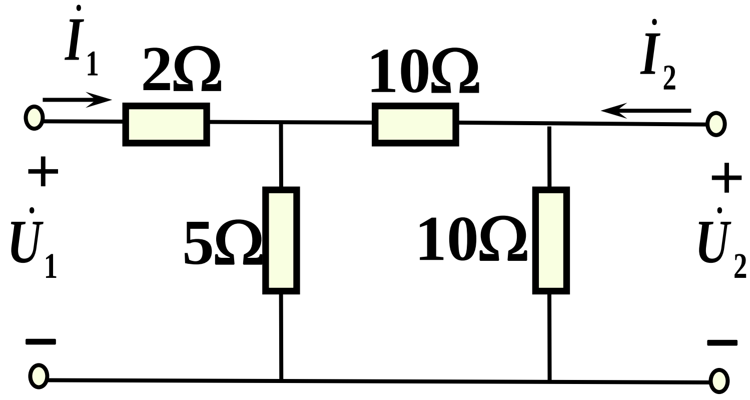
若 $Y_a=Y_c$，则$Y_{11}=Y_{22}$。

有 $Y_{12}=Y_{21}$ 且$Y_{11}=Y_{22}$称为对称二端口。

对称二端口只有两个参数是独立的。

对称二端口是指两个端口电气特性上对称。电路结构左右对称的，端口电气特性对称；电路结构不对称的二端口，其电气特性也可能是对称的。这样的二端口也是对称二端口。

例



电路结构左右不对称

思路1:

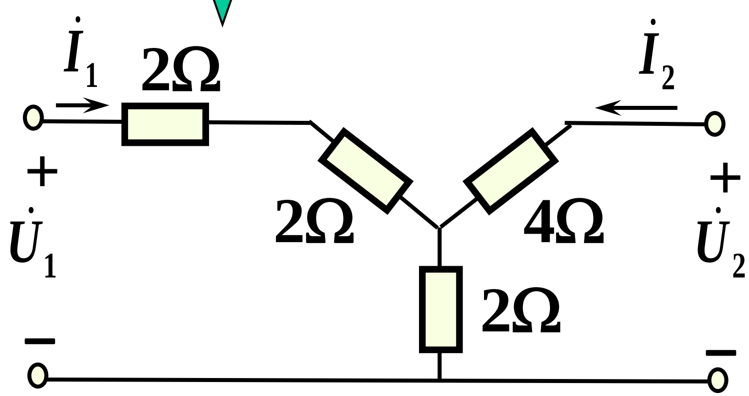
电阻网络，互易 $Y_{12} = Y_{21}$

$$Y_{11} = \frac{1}{2+5//10} = \frac{3}{16}\text{S}$$

$$Y_{22} = \frac{1}{10//(10+2//5)} = \frac{3}{16}\text{S}$$

思路2:

Y–Δ 等效变换

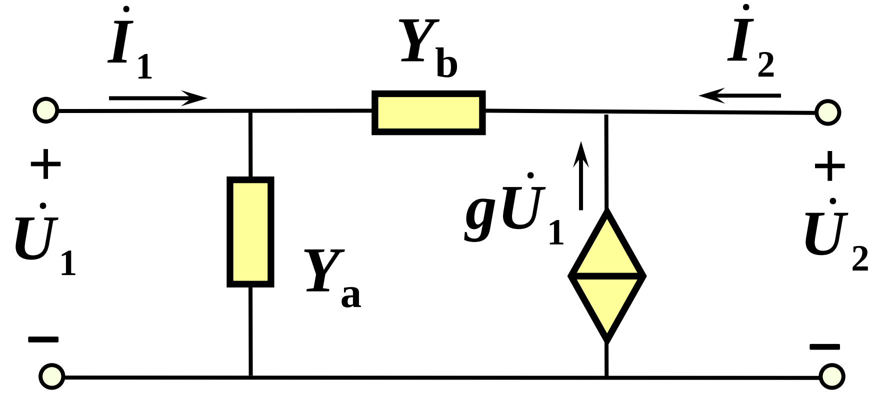


电路结构左右对称

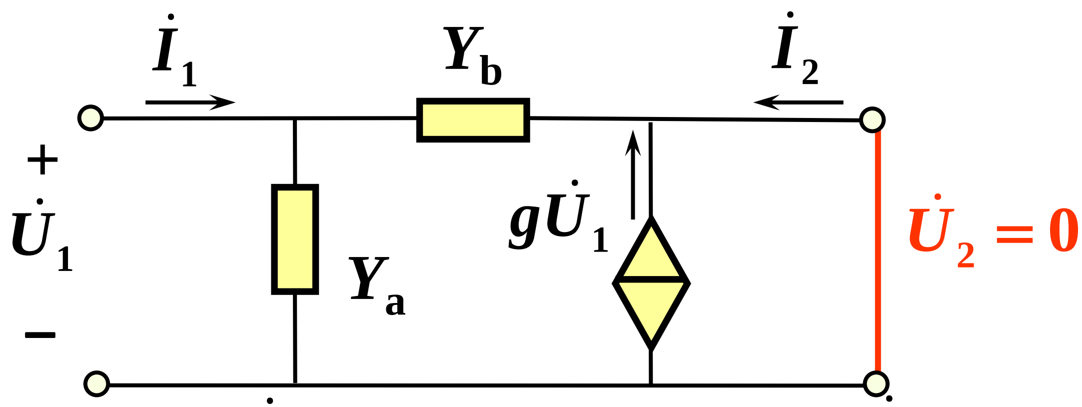
对称二端口（电气对称）

例2 求所示电路的Y参数。

$$\begin{cases}\dot I_1 = Y_{11}\dot U_1 + Y_{12}\dot U_2 \\ \dot I_2 = Y_{21}\dot U_1 + Y_{22}\dot U_2\end{cases}$$

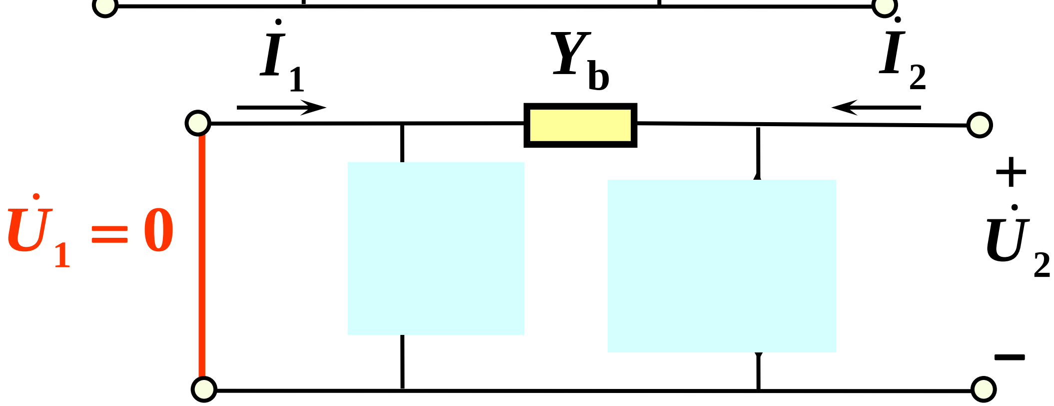


解法一 由实验测定得参数



$$Y_{11}=\frac{\dot I_1}{\dot U_1}\bigg|_{\dot U_2=0}=Y_a+Y_b$$

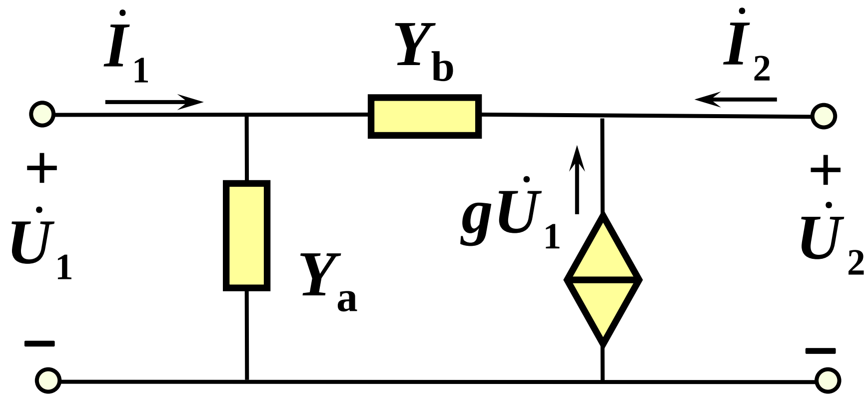
$$Y_{21}=\frac{\dot I_2}{\dot U_1}\bigg|_{\dot U_2=0}=-Y_b-g$$



$$Y_{12}=\frac{\dot I_1}{\dot U_2}\bigg|_{\dot U_1=0}=-Y_b$$

$$Y_{22}=\frac{\dot I_2}{\dot U_2}\bigg|_{\dot U_1=0}=Y_b$$

解法二　直接列写端口电压电流方程，得参数



$$\dot{I}_1 = Y_a\dot{U}_1 + Y_b(\dot{U}_1 - \dot{U}_2)$$
$$\dot{I}_2 = Y_b(\dot{U}_2 - \dot{U}_1) - g\dot{U}_1$$

$$\Rightarrow$$

$$\dot{I}_1 = (Y_a + Y_b)\dot{U}_1 - Y_b\dot{U}_2$$
$$\dot{I}_2 = (-g - Y_b)\dot{U}_1 + Y_b\dot{U}_2$$

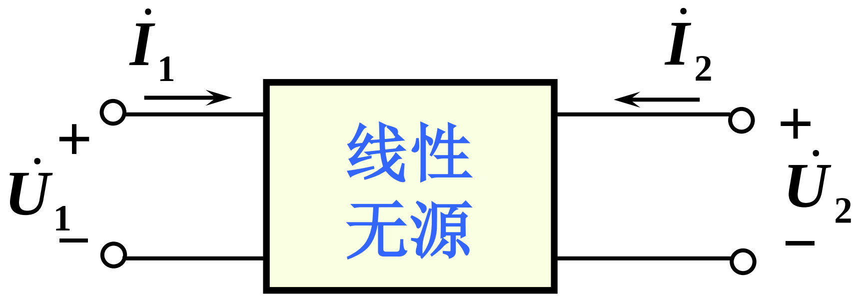
$$\mathbf{Y} = \begin{bmatrix} Y_a + Y_b & -Y_b \\ -g - Y_b & Y_b \end{bmatrix}$$

非互易二端口网络（网络内部有受控源）四个独立参数。

三、H 参数（混合参数）和方程（hybrid parameters）

H 参数方程

$$\dot{U}_1 = H_{11}\dot{I}_1 + H_{12}\dot{U}_2$$
$$\dot{I}_2 = H_{21}\dot{I}_1 + H_{22}\dot{U}_2$$



矩阵形式

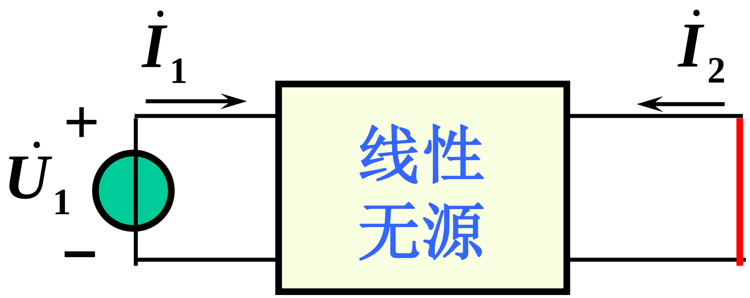
$$\begin{bmatrix} \dot{U}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{U}_2 \end{bmatrix}$$

H 参数的实验测定

$$\dot{U}_1 = H_{11}\dot{I}_1 + H_{12}\dot{U}_2$$

$$\dot{I}_2 = H_{21}\dot{I}_1 + H_{22}\dot{U}_2$$

$$\left.\begin{aligned} H_{11} &= \left.\frac{\dot{U}_1}{\dot{I}_1}\right|_{\dot{U}_2=0} \\ H_{21} &= \left.\frac{\dot{I}_2}{\dot{I}_1}\right|_{\dot{U}_2=0} \end{aligned}\right\} \text{短路参数} \qquad \left.\begin{aligned} H_{12} &= \left.\frac{\dot{U}_1}{\dot{U}_2}\right|_{\dot{I}_1=0} \\ H_{22} &= \left.\frac{\dot{I}_2}{\dot{U}_2}\right|_{\dot{I}_1=0} \end{aligned}\right\} \text{开路参数}$$



2−2′ 短路

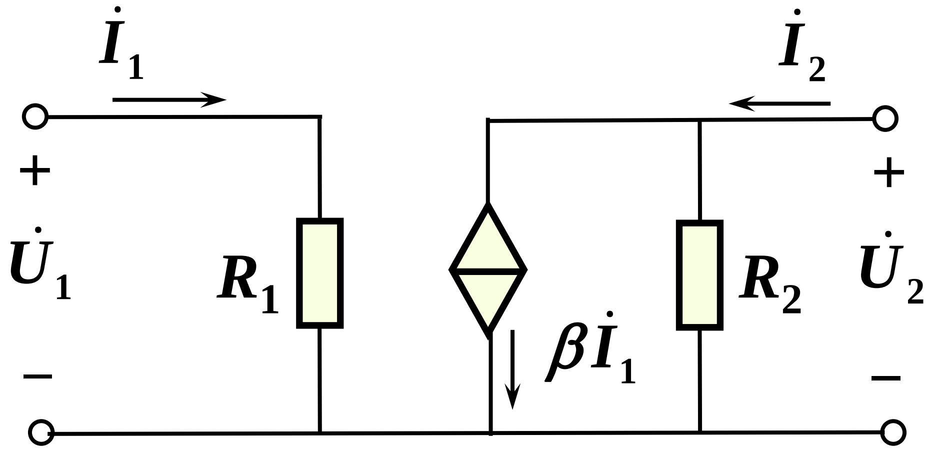
1−1′ 开路

互易二端口 $H_{12} = -H_{21}$

对称二端口 $H_{11}H_{22} - H_{12}H_{21} = 1$

$$\dot{U}_1 = H_{11}\dot{I}_1 + H_{12}\dot{U}_2$$
$$\dot{I}_2 = H_{21}\dot{I}_1 + H_{22}\dot{U}_2$$

例　求所示电路的H参数

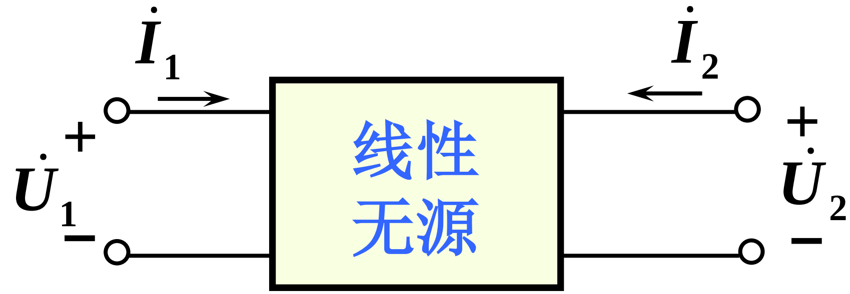


端口电压、电流方程

$$\dot{U}_1 = R_1\dot{I}_1$$
$$\dot{I}_2 = \beta\,\dot{I}_1 + \frac{1}{R_2}\dot{U}_2$$

$$H = \begin{bmatrix} R_1 & 0 \\ \beta & 1/R_2 \end{bmatrix}$$

四、*T* 参数 （传输参数）方程
（transmission parameters）



其矩阵形式

$$\begin{bmatrix}\dot{U}_1\\ \dot{I}_1\end{bmatrix}=\begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}\dot{U}_2\\ -\dot{I}_2\end{bmatrix}$$

（注意负号）

$$T=\begin{bmatrix}A & B\\ C & D\end{bmatrix}$$

称为*T* 参数矩阵。

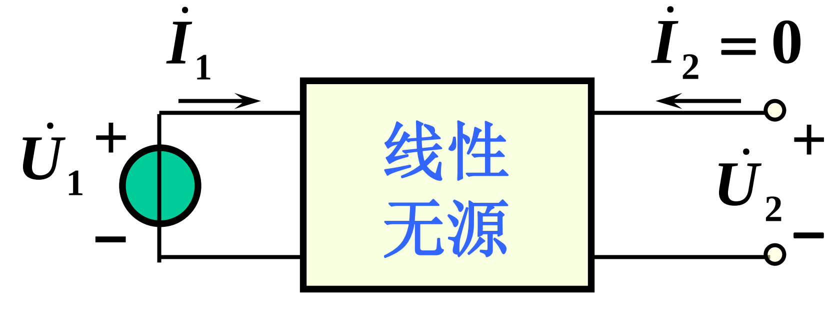
T 参数方程

$$\dot{U}_1 = A\dot{U}_2 - B\dot{I}_2$$
$$\dot{I}_1 = C\dot{U}_2 - D\dot{I}_2$$

T参数能方便的描述信号或能量从一个端口向另一端口的传输特性。

T 参数的实验测定

$$\dot{U}_1 = A\dot{U}_2 - B\dot{I}_2$$
$$\dot{I}_1 = C\dot{U}_2 - D\dot{I}_2$$

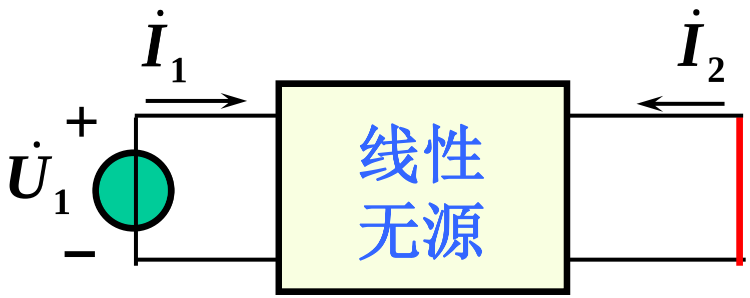


2−2′ 开路

$$A = \left.\frac{\dot{U}_1}{\dot{U}_2}\right|_{\dot{I}_2=0}$$

$$C = \left.\frac{\dot{I}_1}{\dot{U}_2}\right|_{\dot{I}_2=0}$$

开路参数



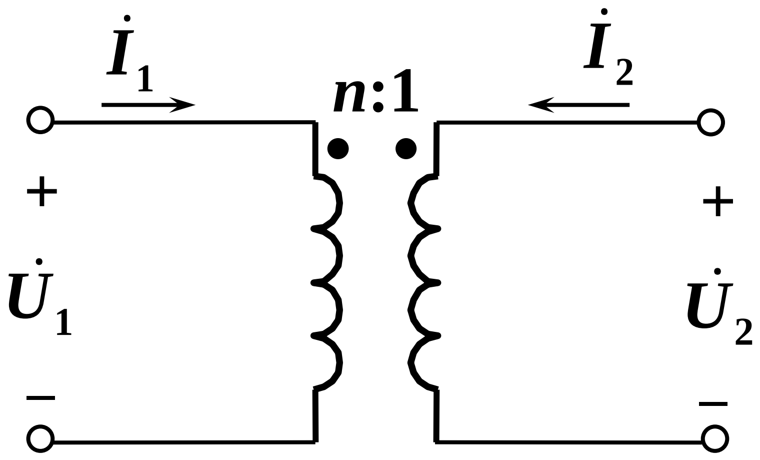
2−2′ 短路

$$B = \left.\frac{\dot{U}_1}{-\dot{I}_2}\right|_{\dot{U}_2=0}$$

$$D = \left.\frac{\dot{I}_1}{-\dot{I}_2}\right|_{\dot{U}_2=0}$$

短路参数

例1 求所示电路的T参数。



理想变压器

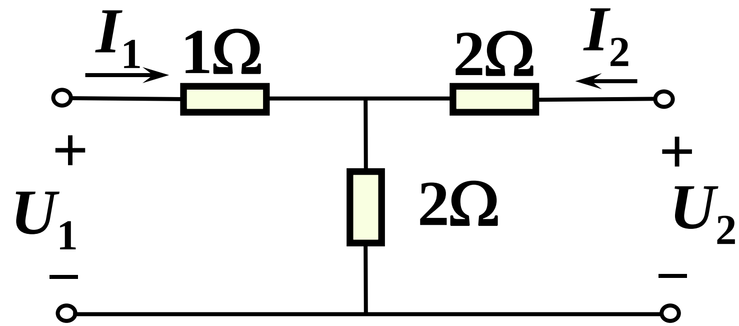
$$\dot{U}_1 = n\dot{U}_2$$

$$\dot{I}_1 = -\frac{1}{n}\dot{I}_2$$

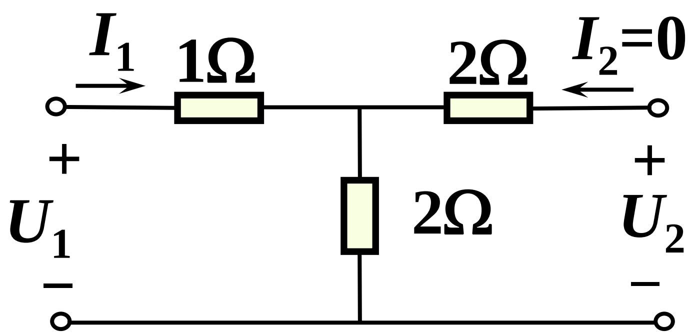
即 $$\begin{bmatrix} \dot{U}_1 \\ \dot{I}_1 \end{bmatrix} = \begin{bmatrix} n & 0 \\ 0 & \frac{1}{n} \end{bmatrix} \begin{bmatrix} \dot{U}_2 \\ -\dot{I}_2 \end{bmatrix}$$

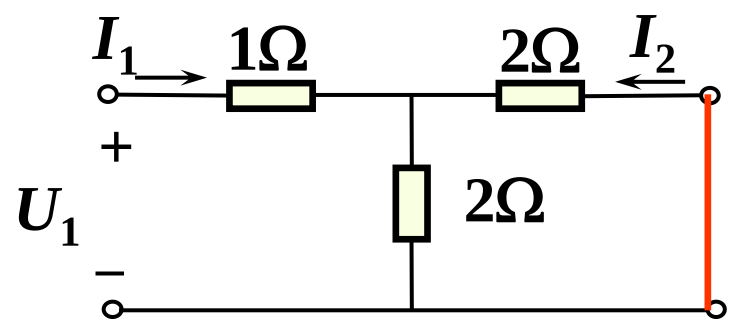
则 $$T = \begin{bmatrix} n & 0 \\ 0 & \frac{1}{n} \end{bmatrix}$$

例2　求图示电路的T 参数。



由实验测定得参数





$$T_{11}=\frac{U_1}{U_2}\bigg|_{I_2=0}=\frac{1+2}{2}=1.5$$

$$T_{12}=\frac{U_1}{-I_2}\bigg|_{U_2=0}=\frac{I_1[1+(2/\!/2)]}{0.5I_1}=4\ \Omega$$

$$T_{21}=\frac{I_1}{U_2}\bigg|_{I_2=0}=0.5\ \text{S}$$

$$T_{22}=\frac{I_1}{-I_2}\bigg|_{U_2=0}=\frac{I_1}{0.5I_1}=2$$

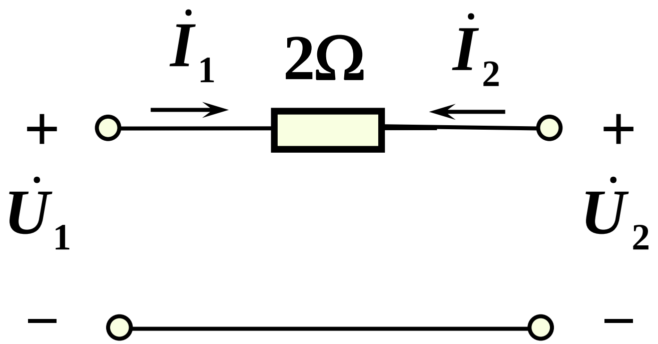
小结

（1）六套参数，还有逆传输参数 和逆混合参数。

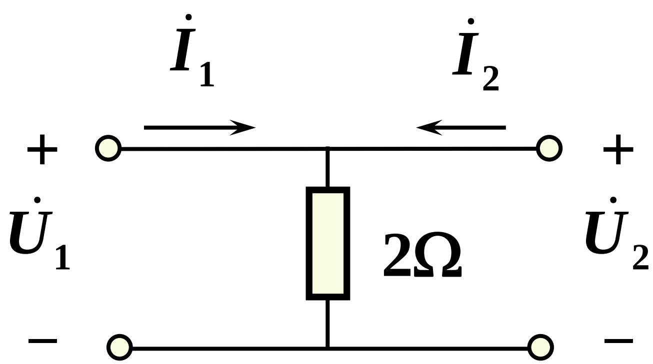
（2）为什么用这么多参数表示？

（a）为描述电路方便，测量方便。

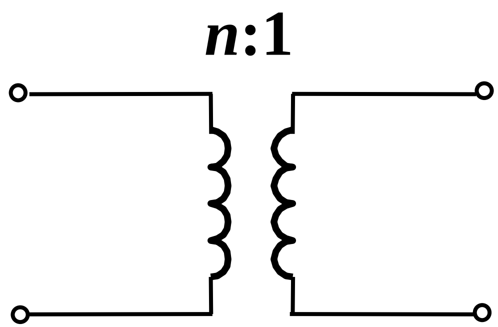
（b）有些电路只存在某几种参数。



$$Y = \begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 0.5 \end{bmatrix} \mathrm{s}$$ ，Z参数 不存在。



$$Z = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \Omega$$ ，Y 参数不存在。



存在T参数，H参数。

Z , Y 均不存在。

（3）可用不同的参数来表示以不同的方式联接的二端口。

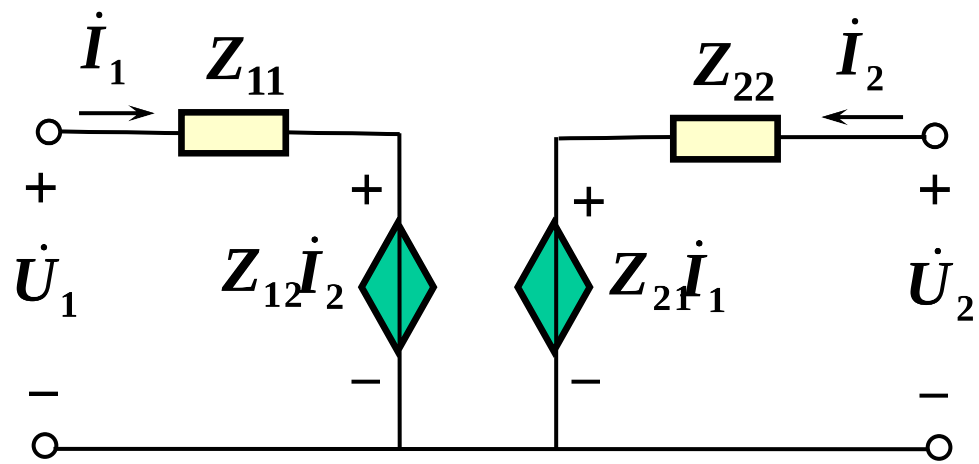
（4）线性无源二端口为互易二端口网络

16.4 二端口的电路模型

两个二端口等效是指对外电路而言，端口的电压、电流关系相同。

一、由Z参数方程作等效电路

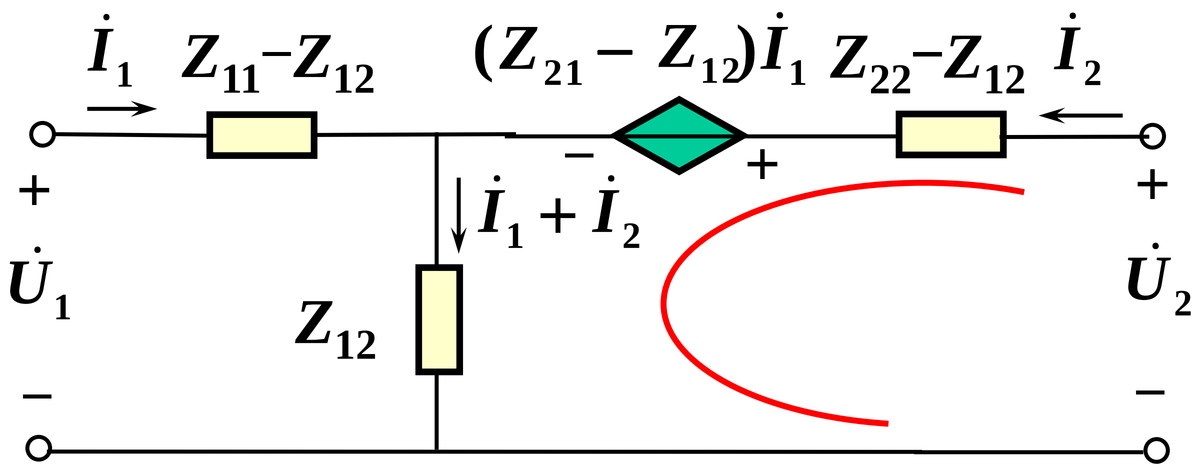
$$\dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2$$
$$\dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2$$



改写为

$$\dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 + Z_{12}\dot{I}_1 - Z_{12}\dot{I}_1$$

$$\dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2 + Z_{12}\dot{I}_1 - Z_{12}\dot{I}_1 + Z_{12}\dot{I}_2 - Z_{12}\dot{I}_2$$



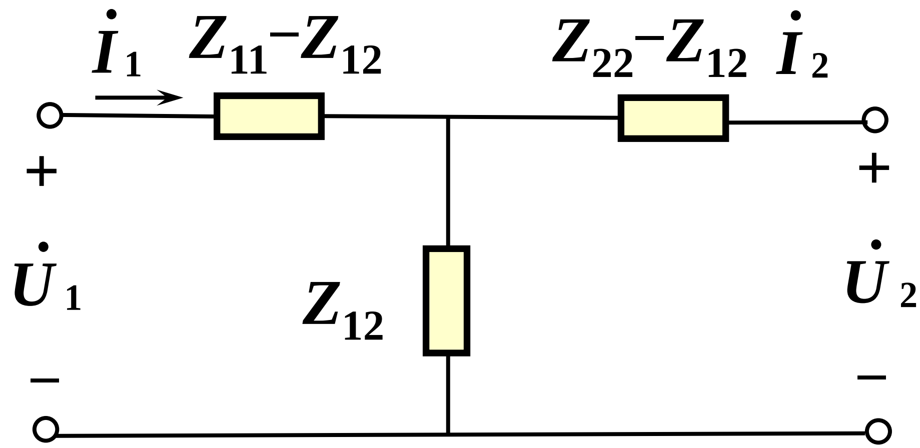
同一个参数方程，可以作出结构不同的等效电路。

表明等效电路不唯一。

上述电路模型称为二端口网络的T形电路模型。

对于互易二端口

$Z_{12}=Z_{21}$

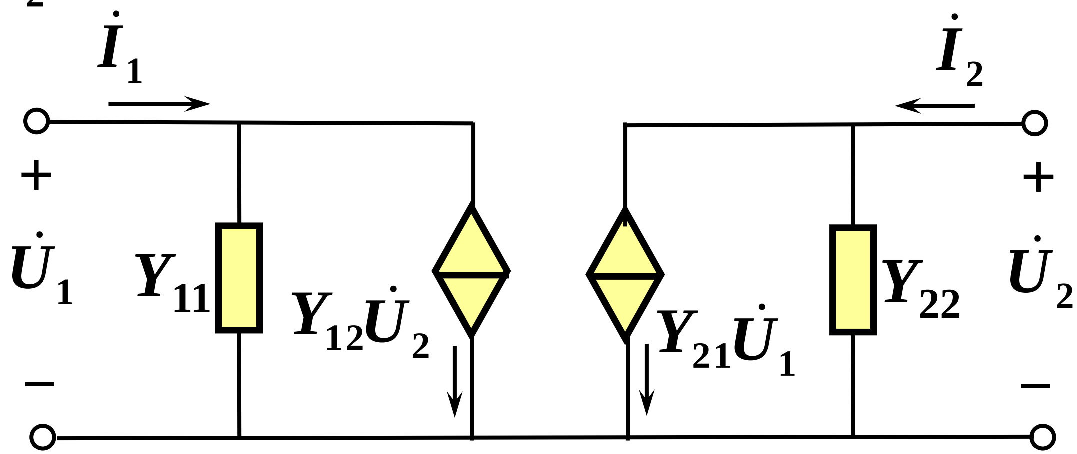


互易二端口等效电路

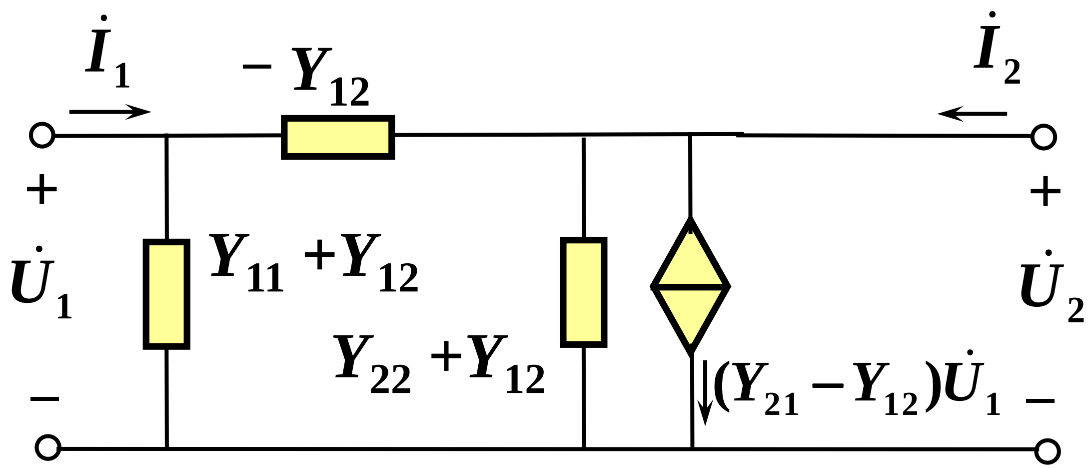
若二端口是对称的（$Z_{12}=Z_{21}$，$Z_{11}=Z_{22}$），则等效电路结构也对称。

二、由Y参数方程作等效电路

$$\begin{cases} \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 \end{cases}$$

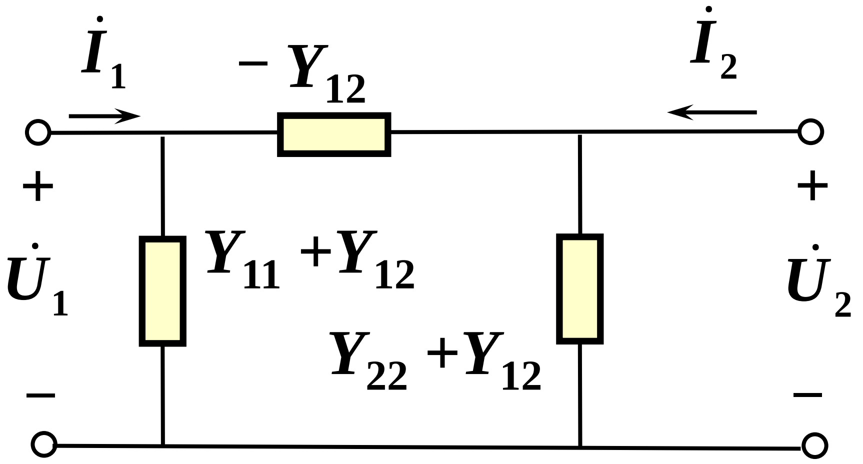


另一种形式



互易二端口

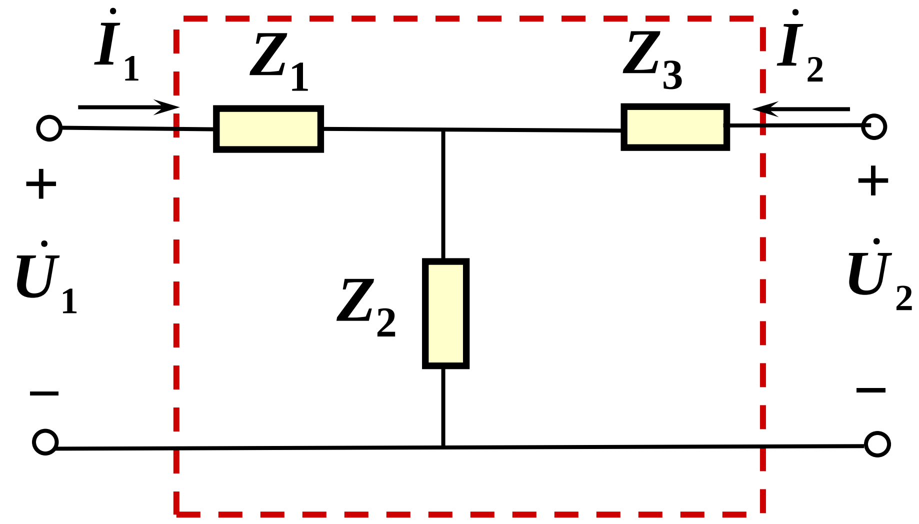
$Y_{12}=Y_{21}$



若二端口是对称的（$Y_{12}=Y_{21}$，$Y_{11}=Y_{22}$），则等效电路结构也对称。

上述电路模型称为二端口网络的Π形电路模型。

例 互易网络的等效电路如图所示，求等效电路的T参数。



开路电压比

$$T_{11}=\frac{\dot{U}_1}{\dot{U}_2}\bigg|_{\dot{I}_2=0}=\frac{Z_1+Z_2}{Z_2}$$

开路转移导纳

$$T_{21}=\frac{\dot{I}_1}{\dot{U}_2}\bigg|_{\dot{I}_2=0}=\frac{1}{Z_2}$$

短路电流比

$$T_{22}=\frac{\dot{I}_1}{-\dot{I}_2}\bigg|_{\dot{U}_2=0}=\frac{Z_3+Z_2}{Z_2}$$

可求得等效电路元件的参数

$$Z_2=1/T_{21}$$

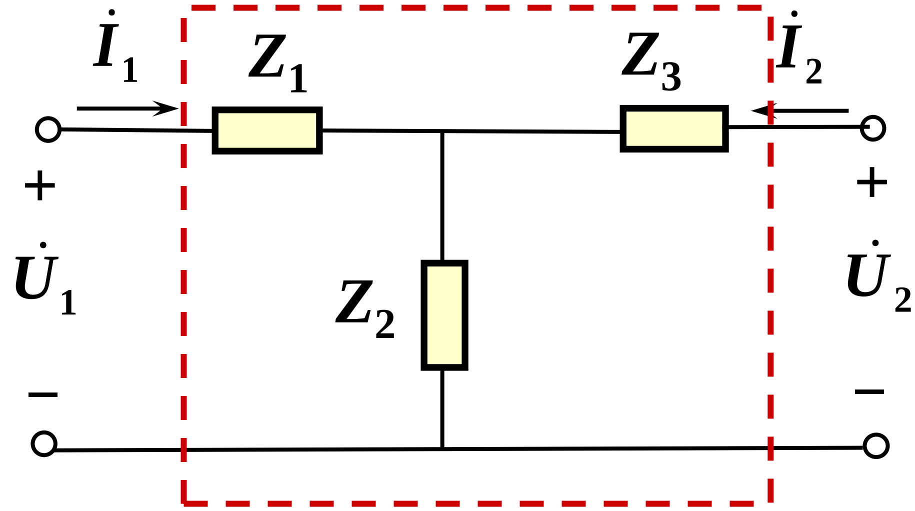
$$Z_1=(T_{11}-1)/T_{21}$$

$$Z_3=(T_{22}-1)/T_{21}$$

也可通过列端口电压、电流关系得到参数方程

$$\dot{U}_1 = Z_1\dot{I}_1 - Z_3\dot{I}_2 + \dot{U}_2$$

$$\dot{I}_1 = \frac{\dot{U}_2 - Z_3\dot{I}_2}{Z_2} - \dot{I}_2$$



将 $\dot{I}_1$ 代入第一式并经整理，可得

$$\dot{U}_1 = \underbrace{(1+\frac{Z_1}{Z_2})}_{T_{11}}\dot{U}_2 - (Z_1 + Z_3 + \frac{Z_1 Z_3}{Z_2})\dot{I}_2$$

$$\dot{I}_1 = \underbrace{\frac{1}{Z_2}}_{T_{21}}\dot{U}_2 - \underbrace{(1+\frac{Z_3}{Z_2})}_{T_{22}}\dot{I}_2$$

也可求得

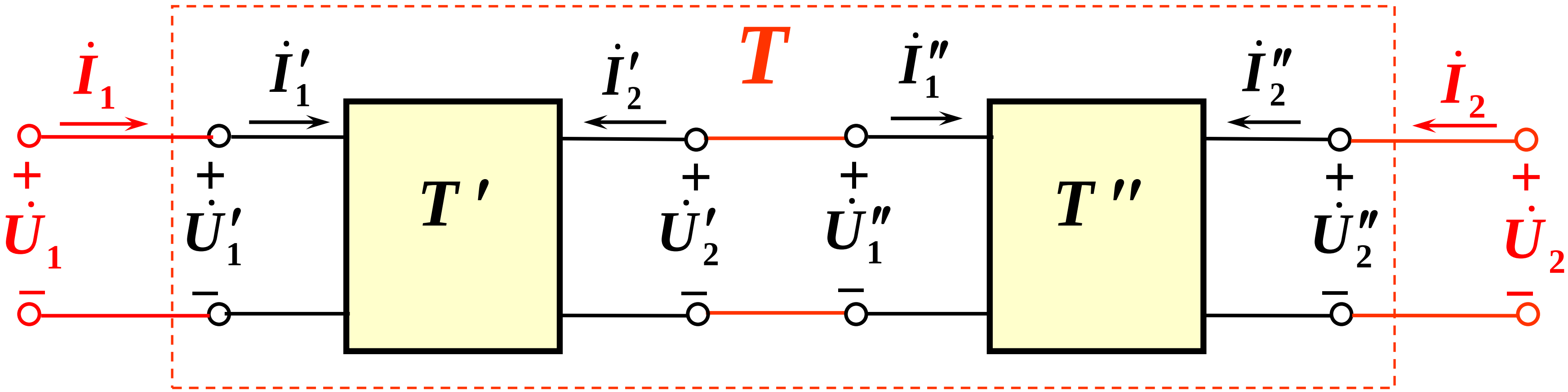
$Z_2 = 1 / T_{21}$

$Z_1 = (T_{11} - 1) / T_{21}$

$Z_3 = (T_{22} - 1) / T_{21}$

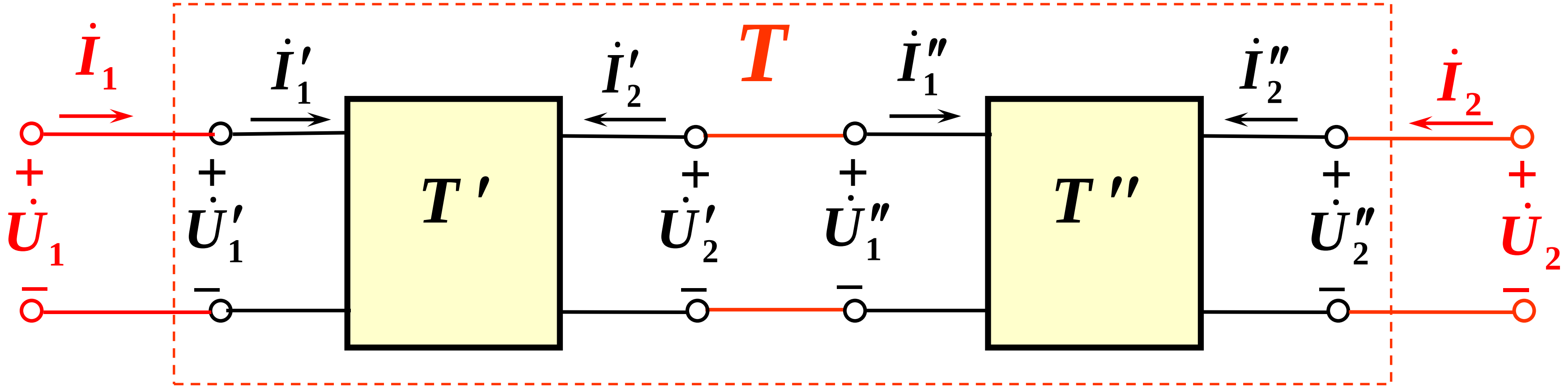
16.5 二端口的联接

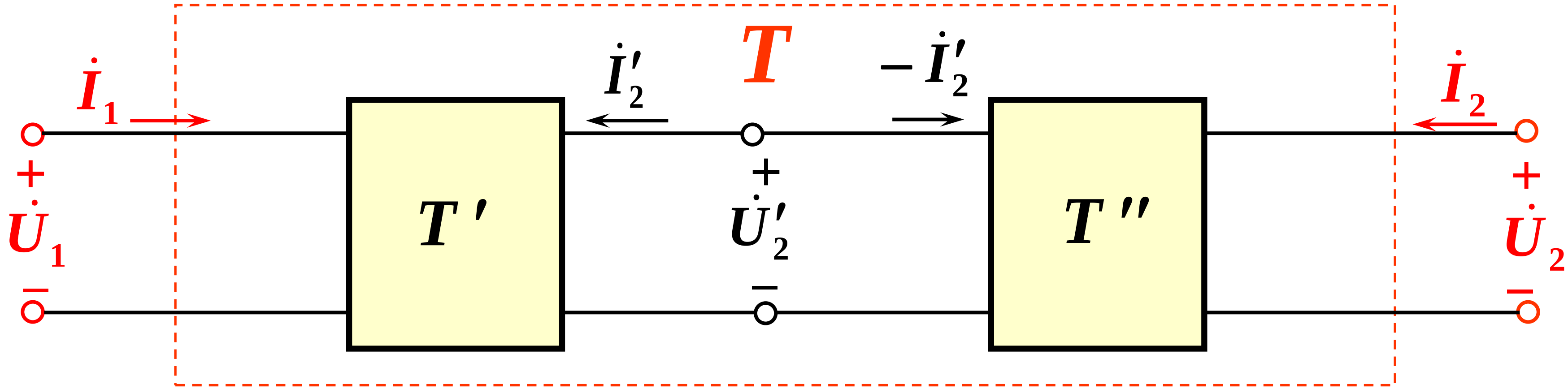
一、级联（cascade connection）（链联）



设 $[T']=\begin{bmatrix} T'_{11} & T'_{12} \\ T'_{21} & T'_{22} \end{bmatrix}$ $[T'']=\begin{bmatrix} T''_{11} & T''_{12} \\ T''_{21} & T''_{22} \end{bmatrix}$

即 $\begin{bmatrix} \dot U_1' \\ \dot I_1' \end{bmatrix}=\begin{bmatrix} T'_{11} & T'_{12} \\ T'_{21} & T'_{22} \end{bmatrix}\begin{bmatrix} \dot U_2' \\ -\dot I_2' \end{bmatrix}$ $\begin{bmatrix} \dot U_1'' \\ \dot I_1'' \end{bmatrix}=\begin{bmatrix} T''_{11} & T''_{12} \\ T''_{21} & T''_{22} \end{bmatrix}\begin{bmatrix} \dot U_2'' \\ -\dot I_2'' \end{bmatrix}$





得

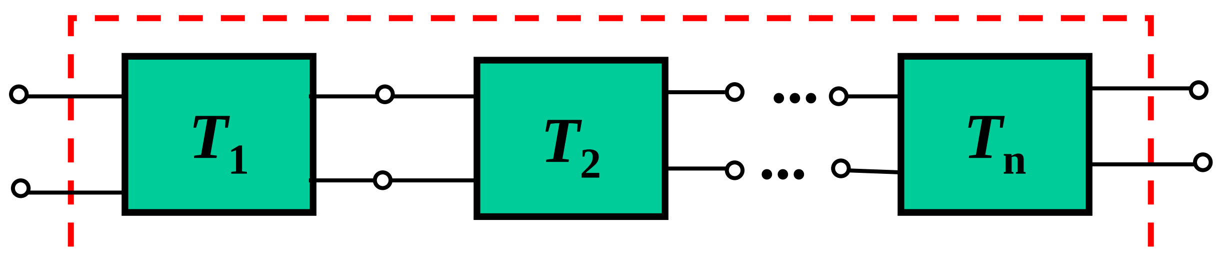
$$\begin{bmatrix}\dot U_1\\ \dot I_1\end{bmatrix}=\begin{bmatrix}T_{11}' & T_{12}'\\ T_{21}' & T_{22}'\end{bmatrix}\begin{bmatrix}\dot U_2'\\ -\dot I_2'\end{bmatrix}=\begin{bmatrix}T_{11}' & T_{12}'\\ T_{21}' & T_{22}'\end{bmatrix}\begin{bmatrix}T_{11}'' & T_{12}''\\ T_{21}'' & T_{22}''\end{bmatrix}\begin{bmatrix}\dot U_2\\ -\dot I_2\end{bmatrix}$$

$$\begin{bmatrix} \dot{U}_1 \\ \dot{I}_1 \end{bmatrix} = \begin{bmatrix} T'_{11} & T'_{12} \\ T'_{21} & T'_{22} \end{bmatrix} \begin{bmatrix} T''_{11} & T''_{12} \\ T''_{21} & T''_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_2 \\ -\dot{I}_2 \end{bmatrix}$$

得 $\boldsymbol{T = T' \, T''}$

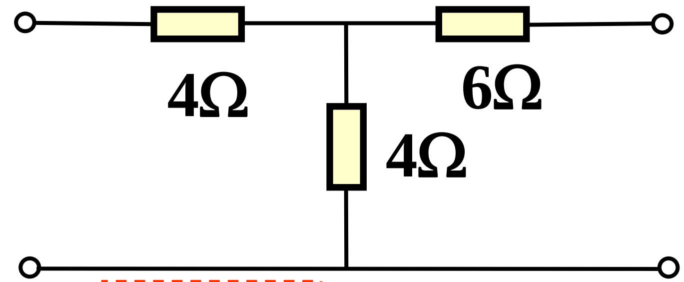
结论

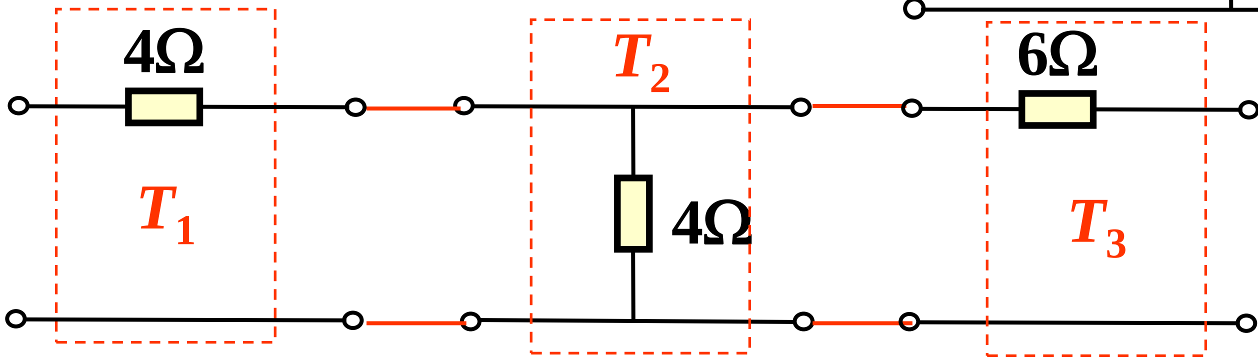
级联后所得复合二端口（**composite two-port**）T参数矩阵等于级联的二端口T 参数矩阵相乘。上述结论可推广到n个二端口级联的关系。



$T=[T_1][T_2]$ …. $[T_n]$

例　求图示电路的T参数。





易求出

$$T_1=\begin{bmatrix}1 & 4\Omega\\ 0 & 1\end{bmatrix}\qquad T_2=\begin{bmatrix}1 & 0\\ 0.25\,\mathrm{S} & 1\end{bmatrix}\qquad T_3=\begin{bmatrix}1 & 6\Omega\\ 0 & 1\end{bmatrix}$$

得

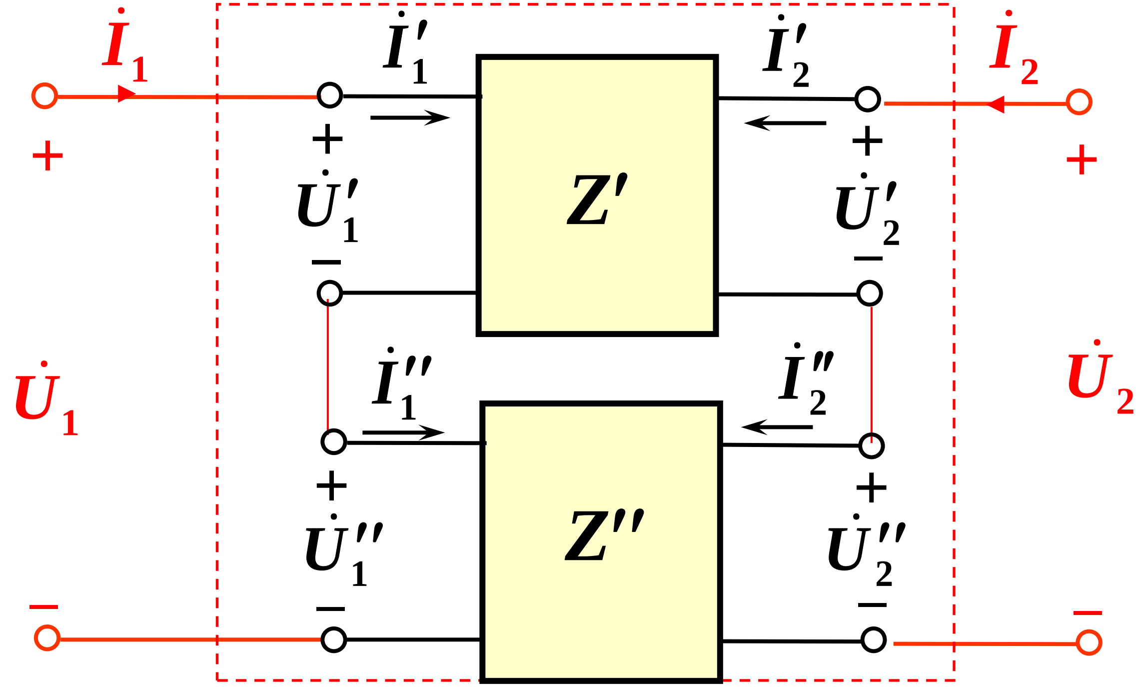
$$[T]=[T_1][T_2][T_3]=\begin{bmatrix}1 & 4\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0.25 & 1\end{bmatrix}\begin{bmatrix}1 & 6\\ 0 & 1\end{bmatrix}=\begin{bmatrix}2 & 16\Omega\\ 0.25\,\mathrm{S} & 2.5\end{bmatrix}$$

二、串联

输入端口串联

输出端口串联

采用Z 参数



$$\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}=\begin{bmatrix}\dot{U}_1'\\ \dot{U}_2'\end{bmatrix}+\begin{bmatrix}\dot{U}_1''\\ \dot{U}_2''\end{bmatrix}=[Z']\begin{bmatrix}\dot{I}_1'\\ \dot{I}_2'\end{bmatrix}+[Z'']\begin{bmatrix}\dot{I}_1''\\ \dot{I}_2''\end{bmatrix}$$

串联电流相等 $$\begin{bmatrix}\dot{I}_1\\ \dot{I}_2\end{bmatrix}=\begin{bmatrix}\dot{I}_1'\\ \dot{I}_2'\end{bmatrix}=\begin{bmatrix}\dot{I}_1''\\ \dot{I}_2''\end{bmatrix}$$

则 $$\mathbf{Z}=\mathbf{Z}'+\mathbf{Z}''$$

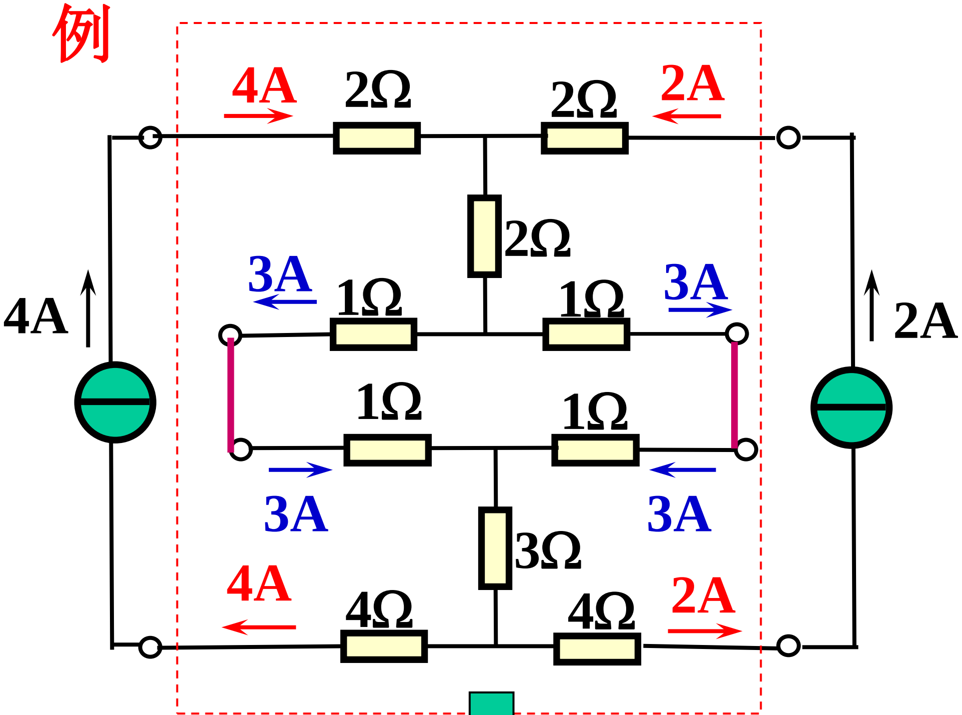
即 $$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}=\begin{bmatrix} Z'_{11} & Z'_{12} \\ Z'_{21} & Z'_{22} \end{bmatrix}+\begin{bmatrix} Z''_{11} & Z''_{12} \\ Z''_{21} & Z''_{22} \end{bmatrix}$$

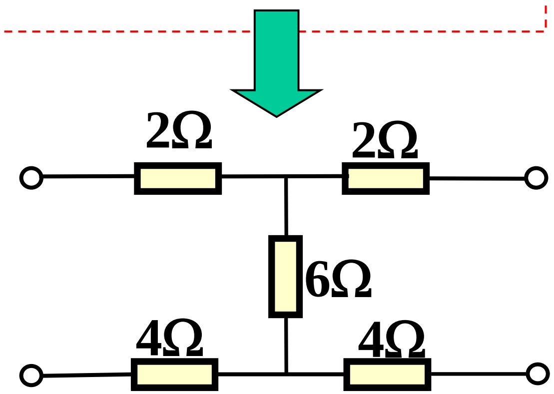
结论

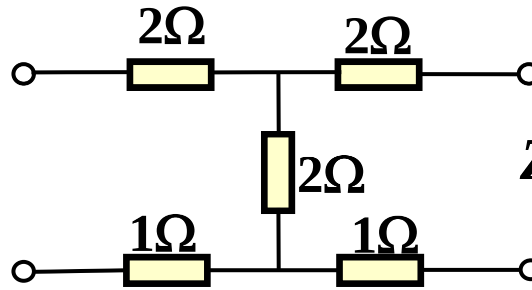
串联后复合二端口Z 参数矩阵等于原二端口Z 参数矩阵相加。可推广到 ***n*端口串联**。

二端口串联也要注意串联后原端口条件是否满足，即是否是正规连接，只有在正规连接的条件下，上述结论才能成立。

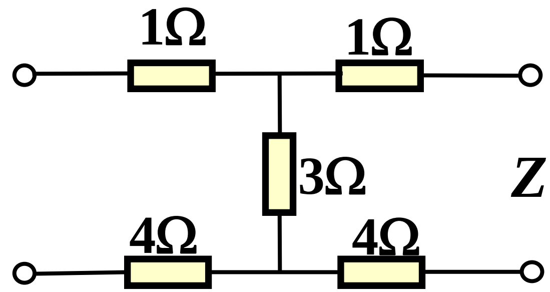
例





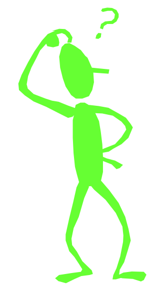


$$Z' = \begin{bmatrix} 5 & 2 \\ 2 & 5 \end{bmatrix}$$



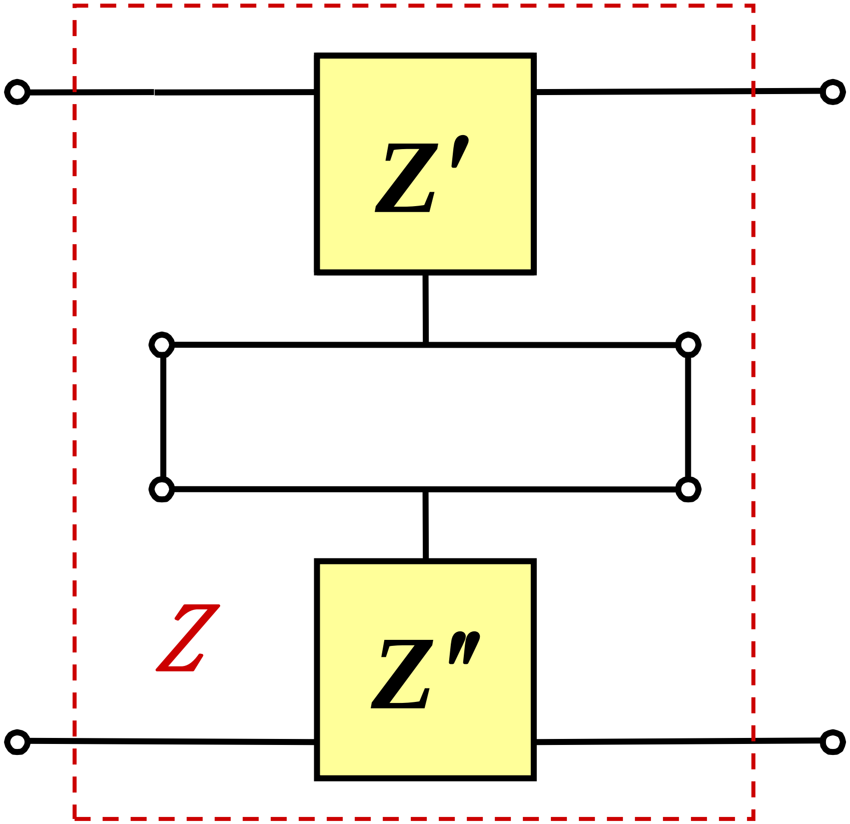
$$Z'' = \begin{bmatrix} 8 & 3 \\ 3 & 8 \end{bmatrix}$$

$$Z = \begin{bmatrix} 12 & 6 \\ 6 & 12 \end{bmatrix} \neq Z' + Z''$$



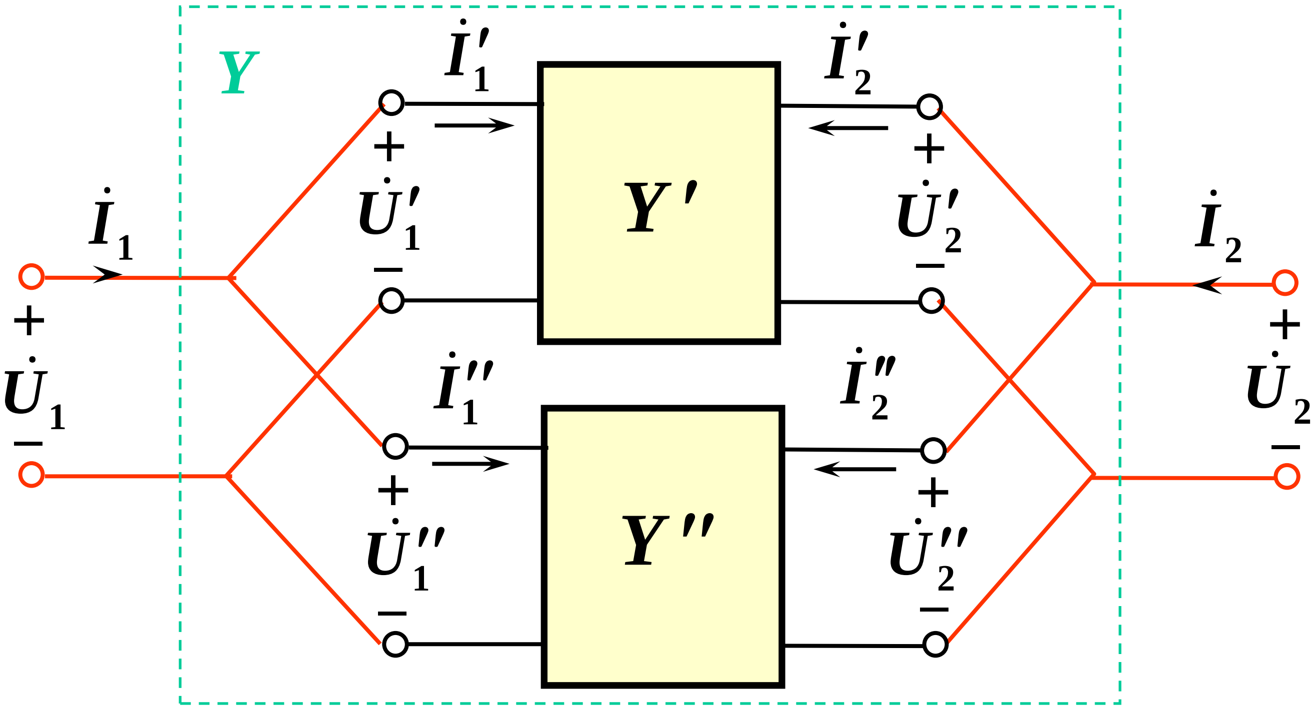
端口条件破坏，不正规连接!

具有公共端的二端口，将公共端串联时将不会破坏端口条件。

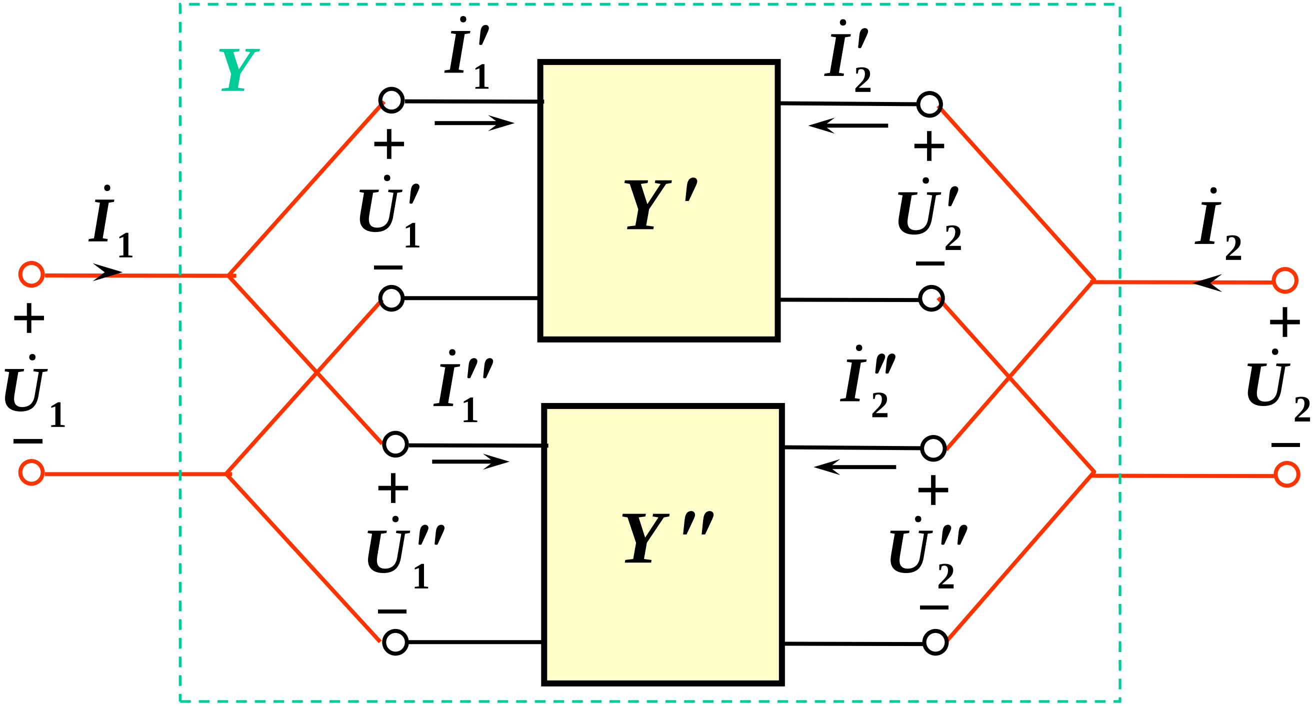


$$Z = Z' + Z''$$

三、并联：输入端口并联，输出端口并联



$$\begin{bmatrix} \dot{I}'_1 \\ \dot{I}'_2 \end{bmatrix} = \begin{bmatrix} Y'_{11} & Y'_{12} \\ Y'_{21} & Y'_{22} \end{bmatrix} \begin{bmatrix} \dot{U}'_1 \\ \dot{U}'_2 \end{bmatrix} \qquad \begin{bmatrix} \dot{I}''_1 \\ \dot{I}''_2 \end{bmatrix} = \begin{bmatrix} Y''_{11} & Y''_{12} \\ Y''_{21} & Y''_{22} \end{bmatrix} \begin{bmatrix} \dot{U}''_1 \\ \dot{U}''_2 \end{bmatrix}$$



并联后

$$\begin{bmatrix}\dot{I}_1\\ \dot{I}_2\end{bmatrix}=\begin{bmatrix}\dot{I}'_1\\ \dot{I}'_2\end{bmatrix}+\begin{bmatrix}\dot{I}''_1\\ \dot{I}''_2\end{bmatrix}=\begin{bmatrix}Y'_{11} & Y'_{12}\\ Y'_{21} & Y'_{22}\end{bmatrix}\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}+\begin{bmatrix}Y''_{11} & Y''_{12}\\ Y''_{21} & Y''_{22}\end{bmatrix}\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}$$

$$\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} Y'_{11}+Y''_{11} & Y'_{12}+Y''_{12} \\ Y'_{11}+Y''_{11} & Y'_{22}+Y''_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix} = [Y'+Y''] \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix}$$

可得　　$Y = Y' + Y''$

结论

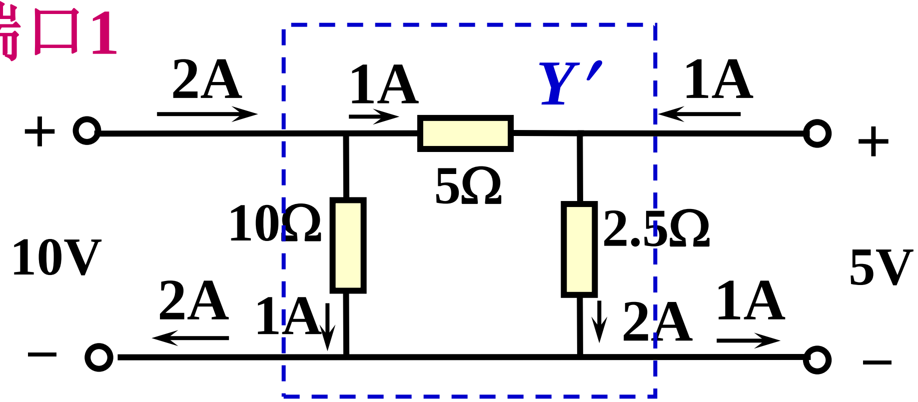
二端口并联所得复合二端口的Y参数矩阵等于两个二端口Y参数矩阵相加。

注意

两个二端口并联时，其端口条件可能被破坏，此时上述关系式就不成立。

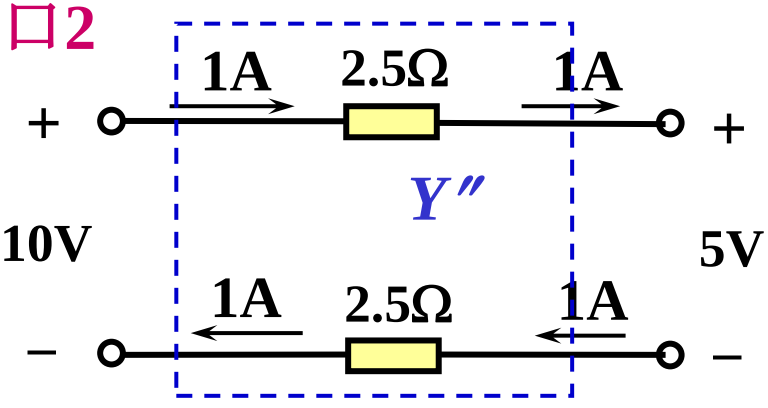
例

二端口1



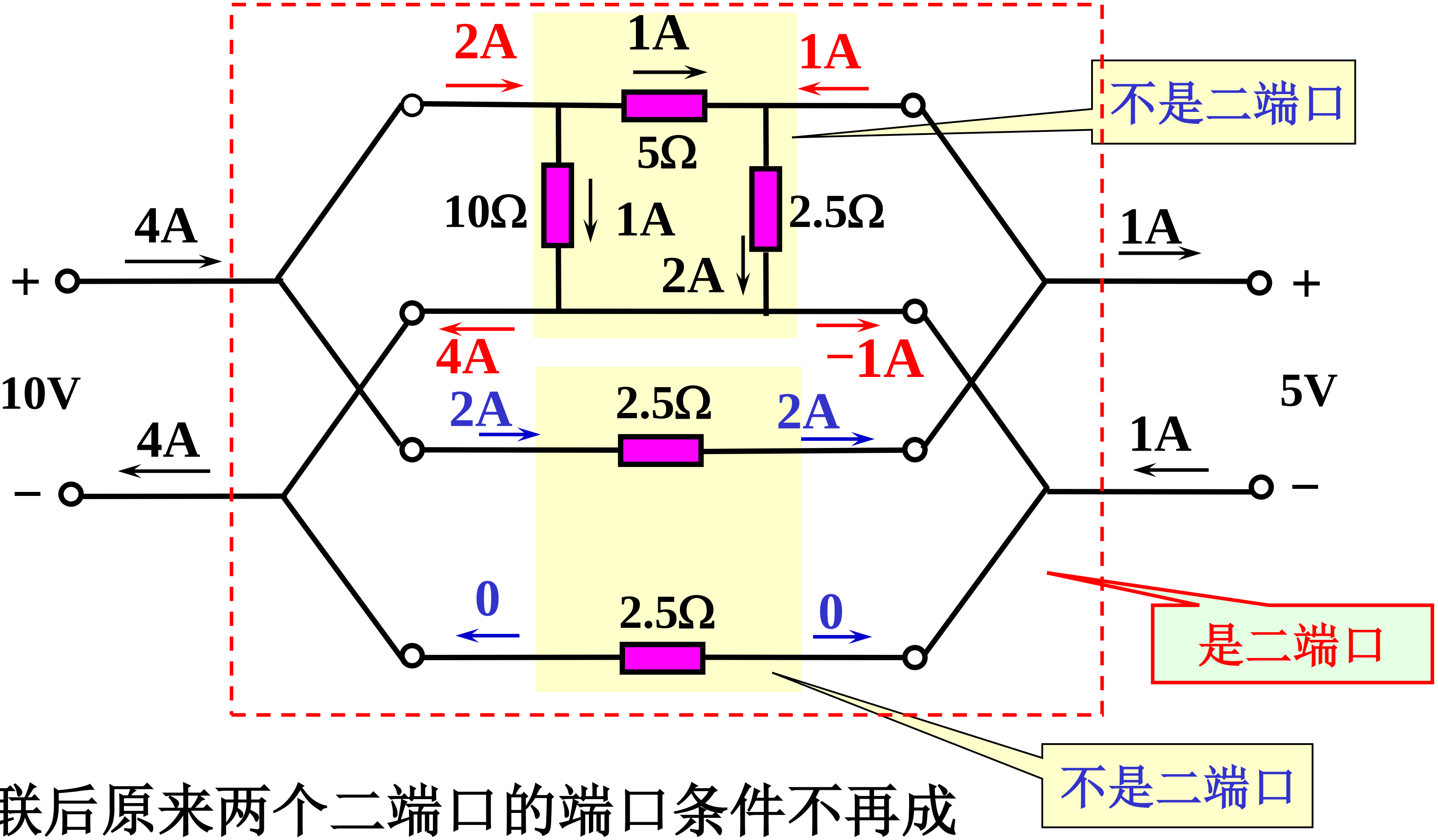
短路导纳参数Y'

二端口2



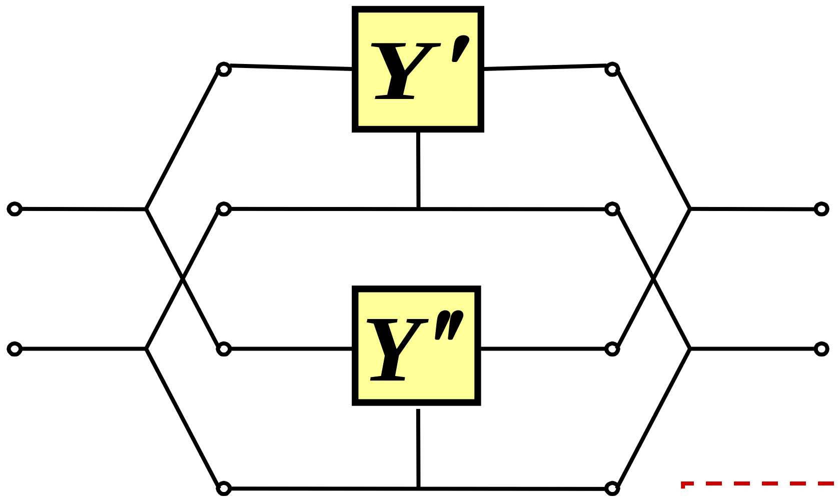
短路导纳参数Y''

二端口1和二端口2并联

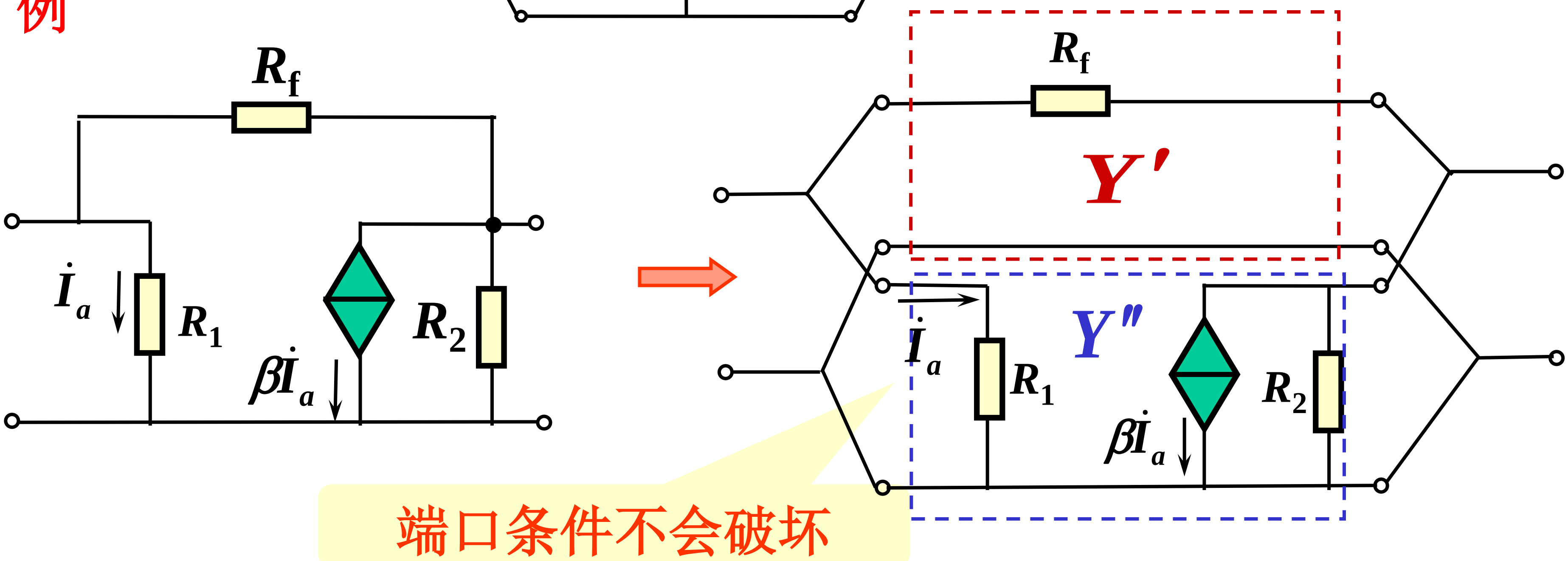


并联后原来两个二端口的端口条件不再成立，称为不正规连接，则并联后的Y参数不能用原来的Y参数相加得到，即 $Y \neq Y' + Y''$

具有公共端的二端口，将公共端并在一起将不会破坏端口条件。



例



$$[Y'] = \begin{bmatrix} \dfrac{1}{R_f} & -\dfrac{1}{R_f} \\ -\dfrac{1}{R_f} & \dfrac{1}{R_f} \end{bmatrix}$$

$$[Y''] = \begin{bmatrix} \dfrac{1}{R_1} & 0 \\ \dfrac{\beta}{R_1} & \dfrac{1}{R_2} \end{bmatrix}$$

$$[Y] = [Y'] + [Y''] = \begin{bmatrix} \dfrac{R_1 + R_f}{R_1 R_f} & -\dfrac{1}{R_f} \\ \dfrac{\beta R_f - R_1}{R_1 R_f} & \dfrac{R_2 + R_f}{R_2 R_f} \end{bmatrix}$$

作业

- 16.2节：16-2
- 16.3节：16-11，16-16
- 16.4节：16-26
- 16.5节：16-30
- 综合：16-38