Chapter 16
二端口网络

16.1 二端口概述

16.2 二端口网络的特性

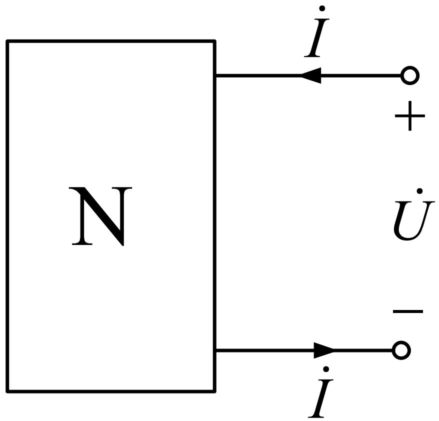
16.3 二端口网络的参数

16.4 二端口网络的电路模型

16.5 二端口网络的相互连接

16.1 二端口概述

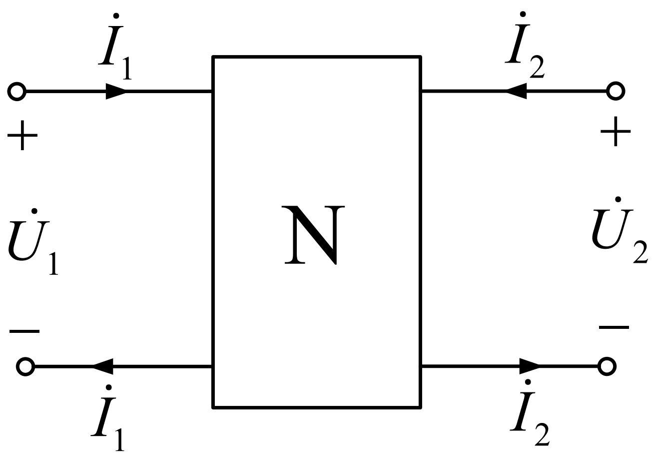
1. 端口

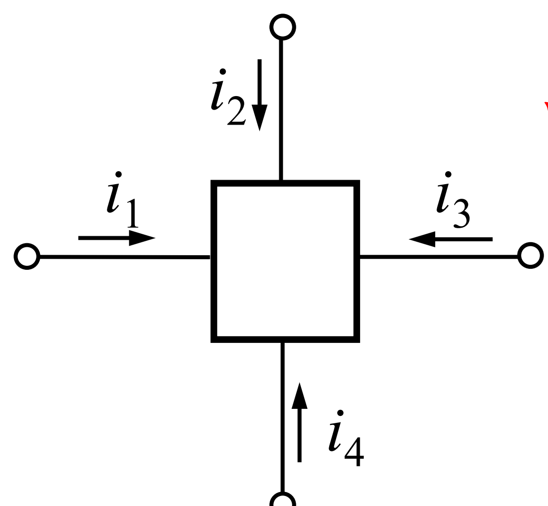


由一对端钮构成，且满足从一个端钮流入的电流等于从另一个端钮流出的电流。

2. 二端口网络

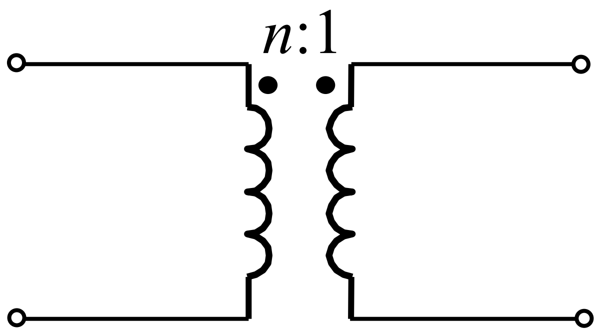
具有两个端口和外部相连的网络



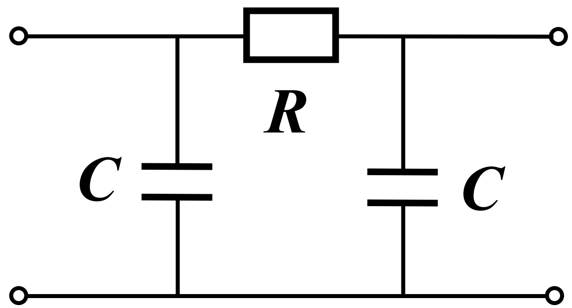


四端网络

典型二端口网络



理想变压器



滤波器电路

✓ 二端口网络大量出现在通信、控制系统和电力系统中，信号（或能量）从一个端口输入，另一个端口输出。

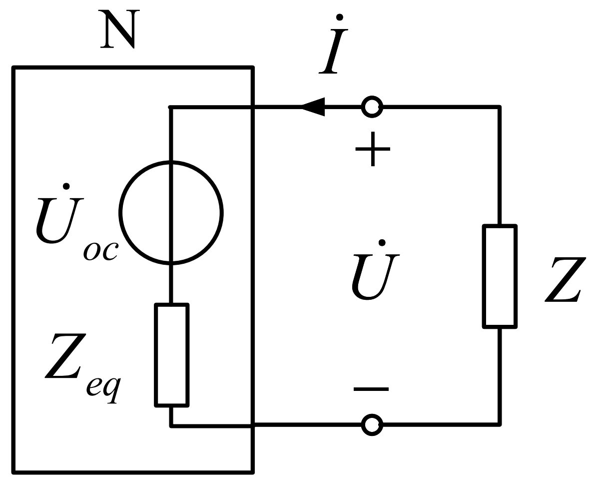
✓ 很多情况下，我们只关心输入、输出端口的电压、电流，而并不关心二端口网络内部的情况，因此可以把二端口网络是为“黑盒子”来分析。

✓ 无需弄清网络内部的元件连接关系，只需了解二端口网络的端口特性即可。

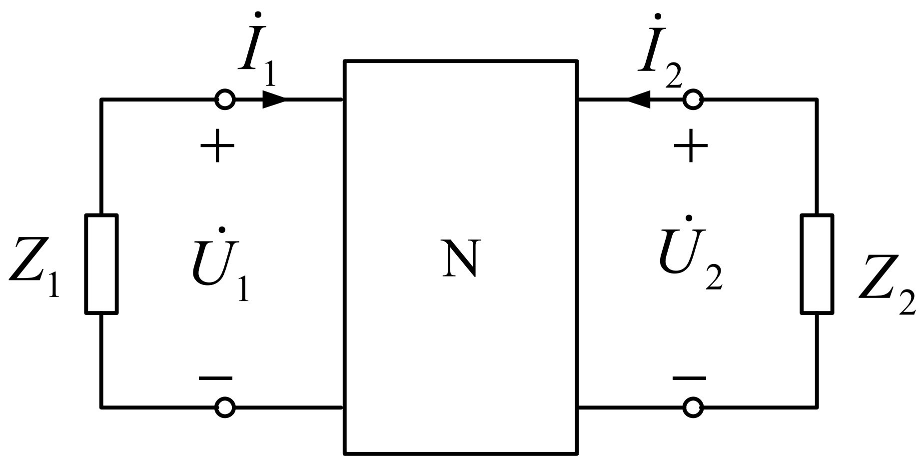
16.2 二端口网络的特性

1. 一端口网络

$$\begin{cases} \dot{U} = \dot{U}_{oc} + Z_{eq}\dot{I} \\ \dot{U} = Z\left(-\dot{I}\right) \end{cases}$$



2. 二端口网络 求解4个端口变量需要列写4个方程



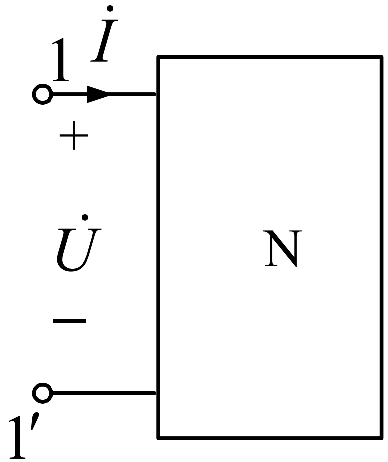
$$\begin{cases} ? \\ ? \\ \dot{U}_1 = -Z_1\dot{I}_1 \\ \dot{U}_2 = -Z_2\dot{I}_2 \end{cases}$$

端口特性方程该如何列写呢?

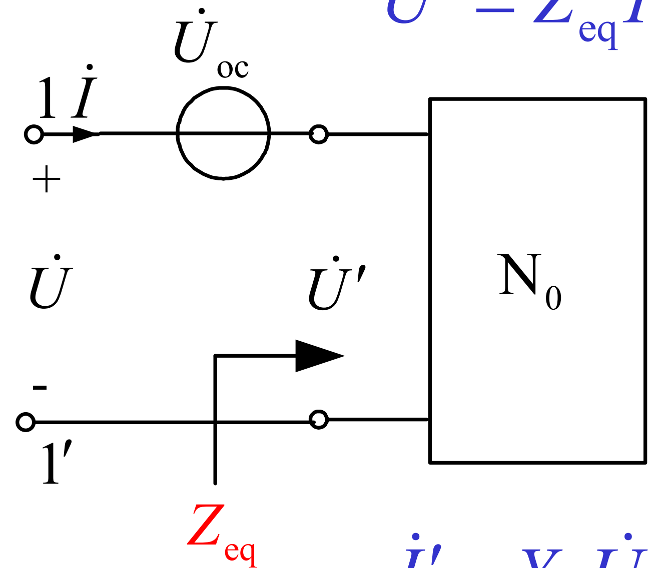
3. 含源网络

(1) 含源一端口网络

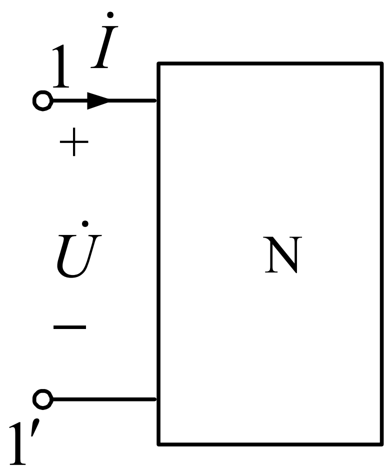
$$\dot{U}' = Z_{eq}\dot{I}$$



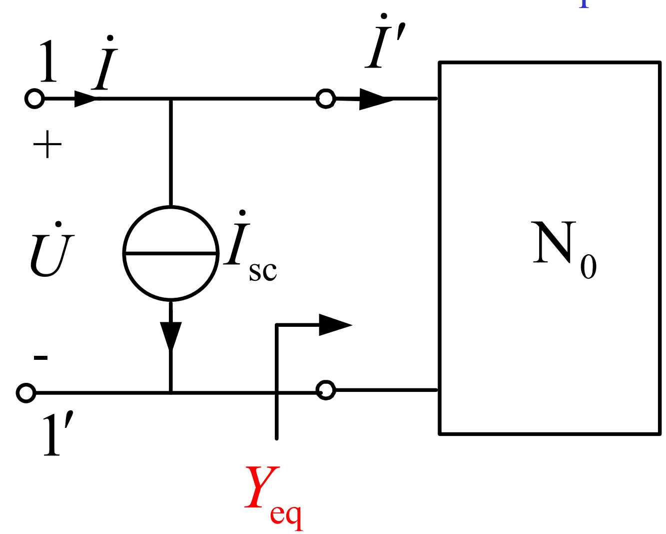
$$\dot{U} = \dot{U}_{oc} + Z_{eq}\dot{I}$$



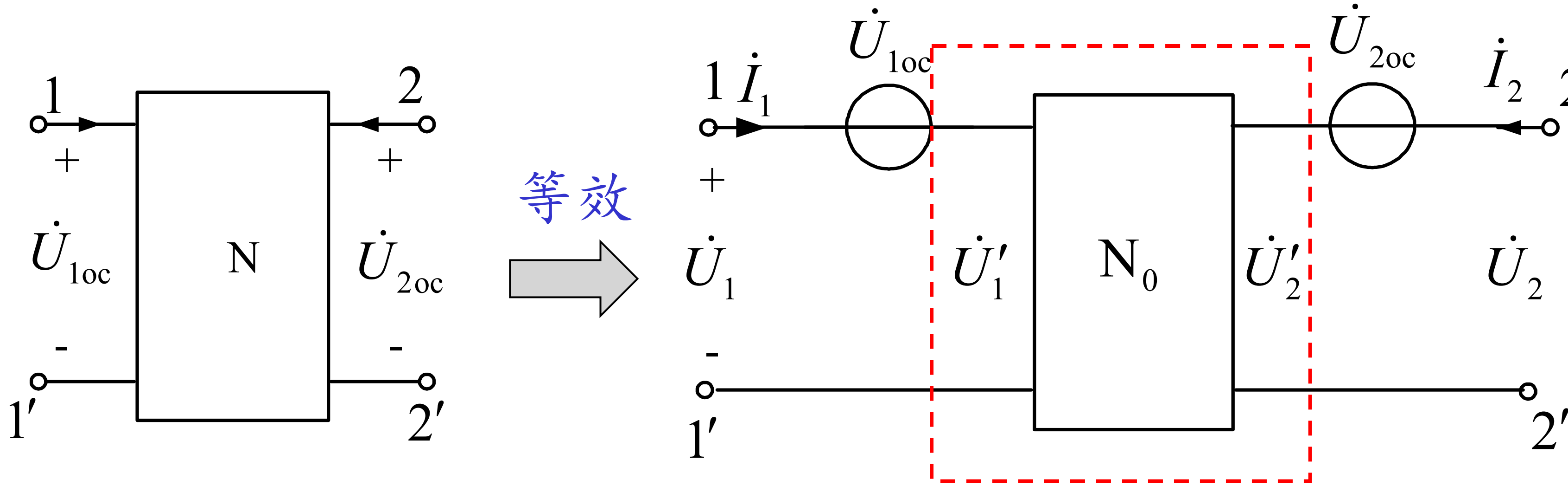
$$\dot{I}' = Y_{eq}\dot{U}$$



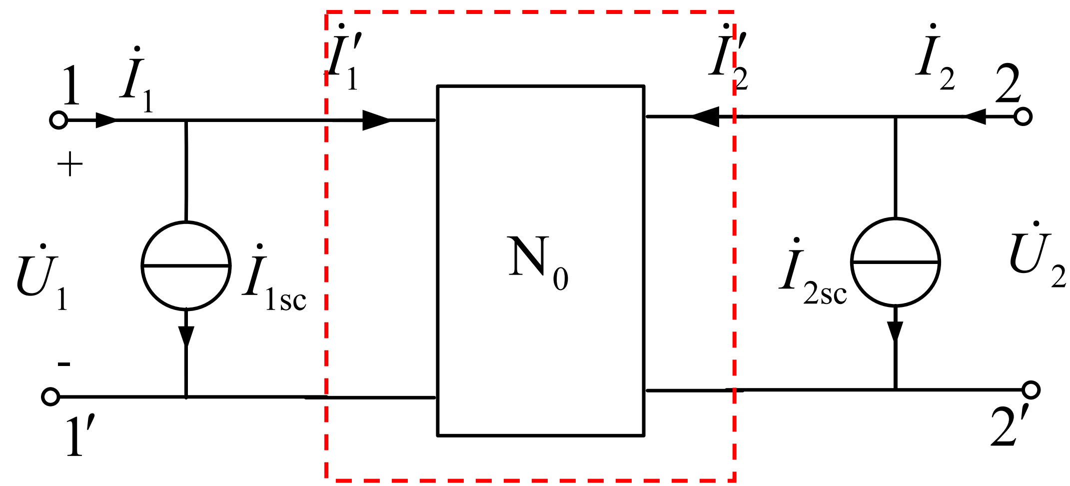
$$\dot{I} = Y_{eq}\dot{U} + \dot{I}_{sc}$$



（2）含源二端口网络 研究松弛二端口网络的端口特性



- ✓ 线性含源二端口网络，可等效为电压/电流源与不含源二端口网络的连接



- ✓ 不含独立源的二端口网络称为**松弛二端口网络**

六组松弛二端口网络的端口特性方程

两个端口
四个变量

阻抗参数方程(Z)

$$\begin{cases}\dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 \\ \dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2\end{cases}$$

导纳参数方程(Y)

$$\begin{cases}\dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2\end{cases}$$

同一类型变量
位于不同端口

混合参数方程(H和G)

$$\begin{cases}\dot{U}_1 = h_{11}\dot{I}_1 + h_{12}\dot{U}_2 \\ \dot{I}_2 = h_{21}\dot{I}_1 + h_{22}\dot{U}_2\end{cases}$$

$$\begin{cases}\dot{I}_1 = g_{11}\dot{U}_1 + g_{12}\dot{I}_2 \\ \dot{U}_2 = g_{21}\dot{U}_1 + g_{22}\dot{I}_2\end{cases}$$

不同类型变量
位于不同端口

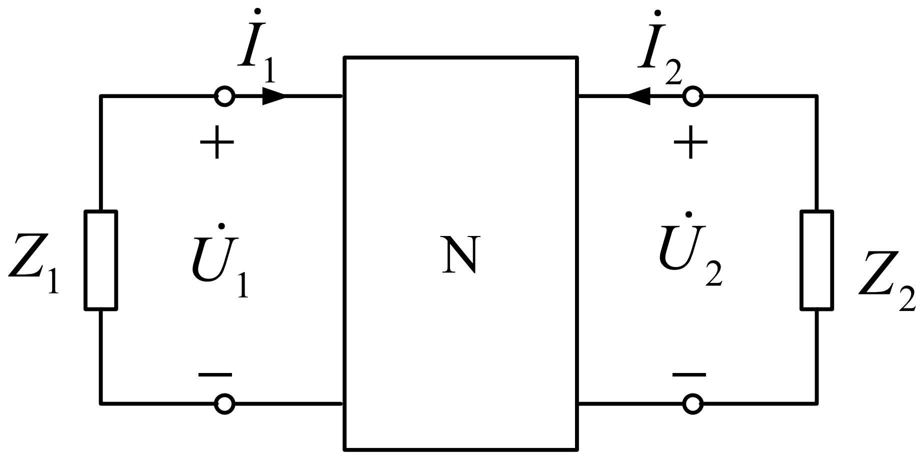
传输参数方程（T和T'）

$$\begin{cases}\dot{U}_1 = A\dot{U}_2 + B\left(-\dot{I}_2\right) \\ \dot{I}_1 = C\dot{U}_2 + D\left(-\dot{I}_2\right)\end{cases}$$

$$\begin{cases}\dot{U}_2 = A'\dot{U}_1 + B'\left(-\dot{I}_1\right) \\ \dot{I}_2 = C'\dot{U}_1 + D'\left(-\dot{I}_1\right)\end{cases}$$

不同类型变量
位于同一端口

16.3 二端口网络的参数



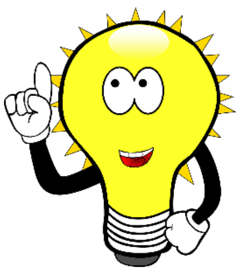
四个端口参数

$\dot{U}_1$　$\dot{I}_2$　$\dot{I}_1$　$\dot{U}_2$

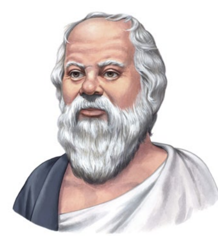
六组特性方程

Z参数、Y参数、H参数、

G参数、T参数、T'参数



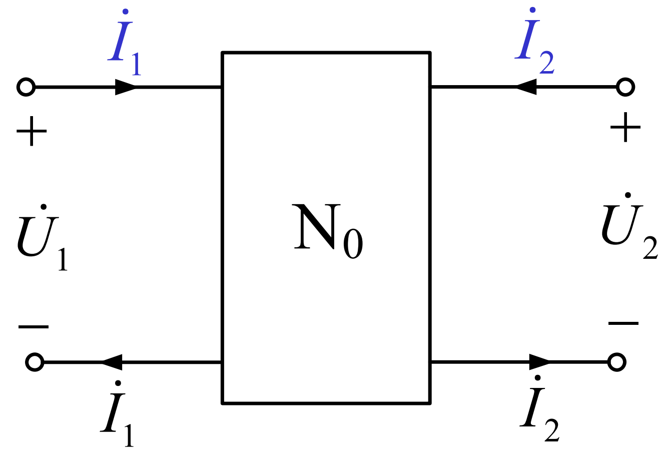
灵魂三问？？？



我是谁？	我从哪里来？	我要到哪里去？
参数是什么？ 各种参数的物理含义	参数如何获得？ 参数计算、测量方法	参数如何应用？ 计算端口变量、获得等效电路

1. 阻抗参数方程（Z参数）

$$\begin{cases} \dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 \\ \dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2 \end{cases}$$



矩阵形式：

$$\begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} \qquad Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

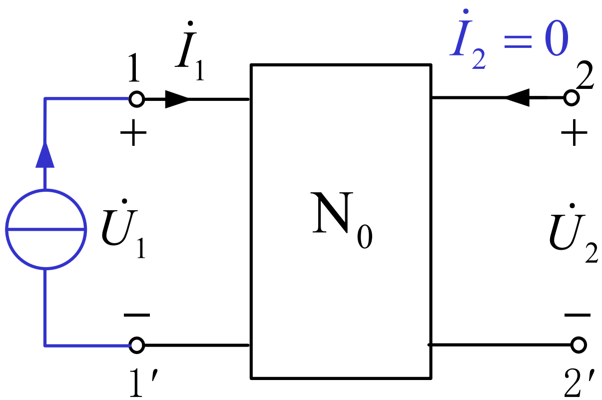
Q：如何测量Z参数？

用端口开路实验测Z参数

$$\begin{cases} \dot{I}_2 = 0 \Rightarrow \text{计算} Z_{11}、Z_{21} \\ \dot{I}_1 = 0 \Rightarrow \text{计算} Z_{12}、Z_{22} \end{cases}$$

阻抗参数测量 $\begin{cases}\dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 \\ \dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2\end{cases}$

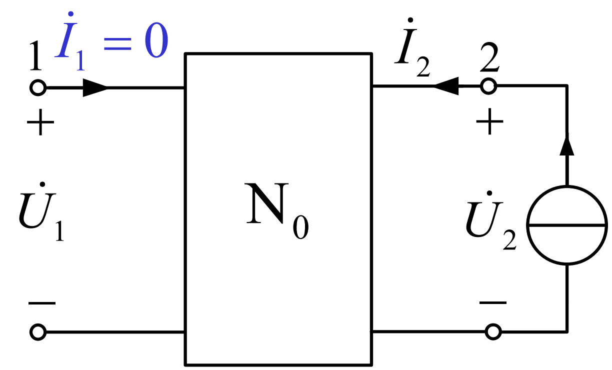
2−2′ 端口开路：



$$Z_{11} = \left.\frac{\dot{U}_1}{\dot{I}_1}\right|_{\dot{I}_2=0}$$ 输入阻抗

$$Z_{21} = \left.\frac{\dot{U}_2}{\dot{I}_1}\right|_{\dot{I}_2=0}$$ 转移阻抗

1−1′ 端口开路：



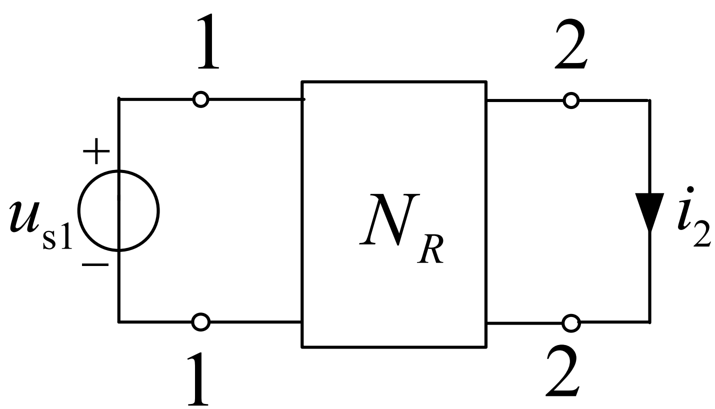
$$Z_{12} = \left.\frac{\dot{U}_1}{\dot{I}_2}\right|_{\dot{I}_1=0}$$ 转移阻抗

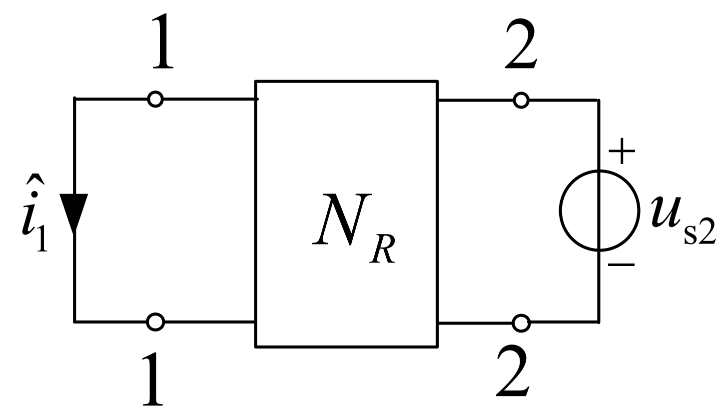
$$Z_{22} = \left.\frac{\dot{U}_2}{\dot{I}_2}\right|_{\dot{I}_1=0}$$ 输入阻抗

Q：两个转移阻抗值是否相等？ 互易定理

互易定理（第四章内容回顾！）

第一种形式：激励是电压源，响应是电流。





N_R只包含线性电阻

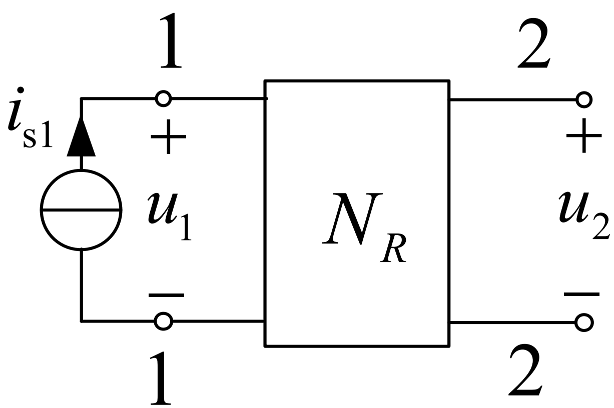
$$\left.\begin{aligned} u_{s1}\hat{i}_1 + 0 + \sum_{k=3}^{b} u_k \hat{i}_k = 0 \\ 0 + u_{s2} i_2 + \sum_{k=3}^{b} \hat{u}_k i_k = 0 \end{aligned}\right\} \xrightarrow{\hat{u}_k i_k = R_k \hat{i}_k i_k = u_k \hat{i}_k} \boxed{\frac{i_2}{u_{s1}} = \frac{\hat{i}_1}{u_{s2}}}$$

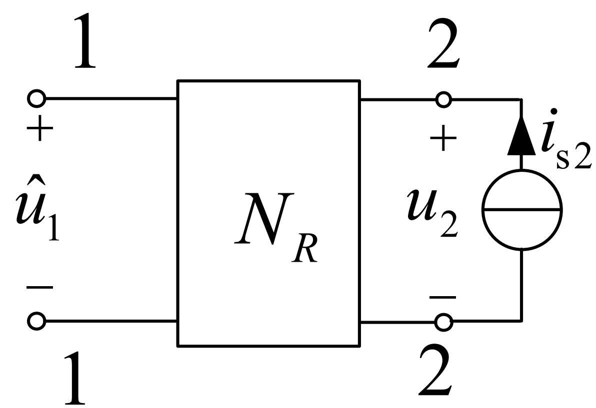
两电路的响应与激励之比相等

返回

互易定理（第四章内容回顾！）

第二种形式：激励是电流源，响应是电压。

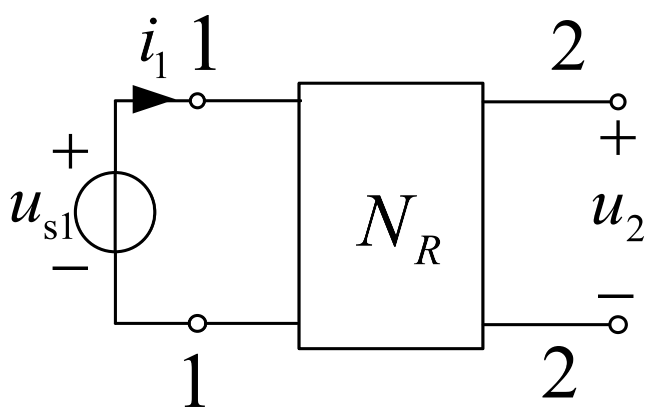


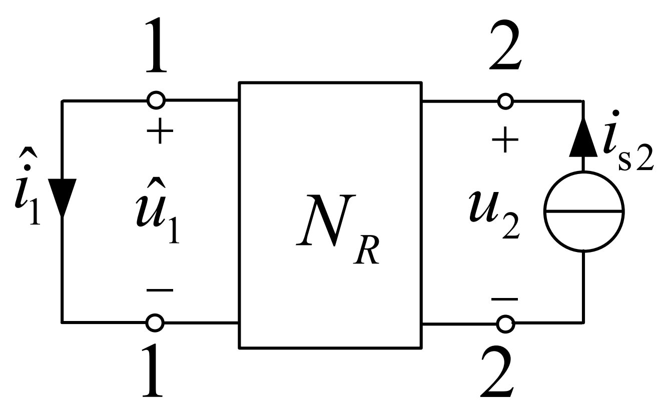


$$\frac{u_2}{i_{s1}} = \frac{\hat{u}_1}{i_{s2}}$$

第三种形式：电路1激励是电压源，响应是电压；

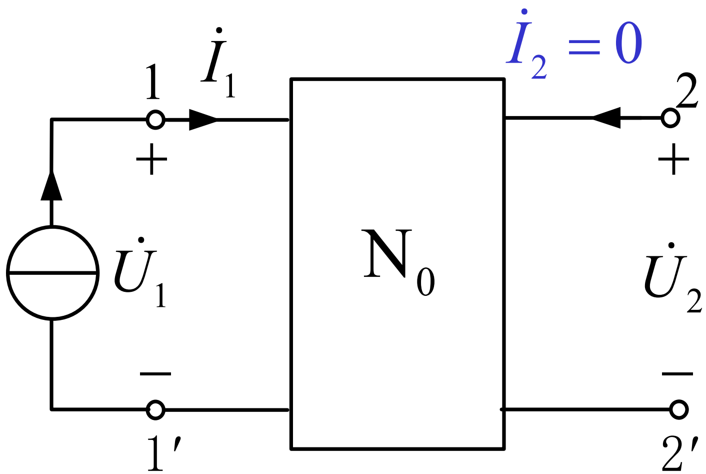
电路2激励是电流源，响应是电流。

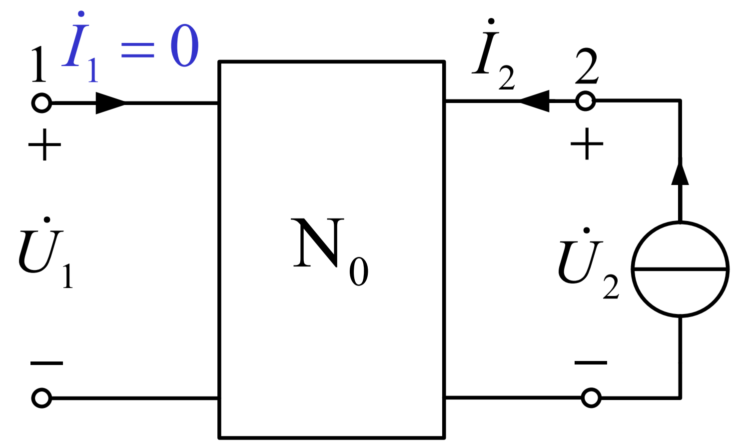




$$\frac{u_2}{u_{s1}} = \frac{\hat{i}_1}{i_{s2}}$$

返回





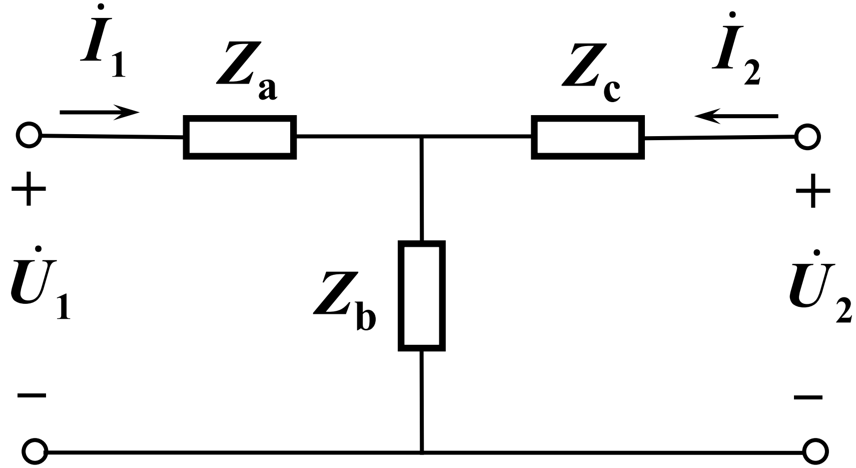
$$Z_{11} = \left.\frac{\dot{U}_1}{\dot{I}_1}\right|_{\dot{I}_2=0} \qquad Z_{21} = \left.\frac{\dot{U}_2}{\dot{I}_1}\right|_{\dot{I}_2=0} \qquad Z_{12} = \left.\frac{\dot{U}_1}{\dot{I}_2}\right|_{\dot{I}_1=0} \qquad Z_{22} = \left.\frac{\dot{U}_2}{\dot{I}_2}\right|_{\dot{I}_1=0}$$

若二端口网络内部无受控源，电路满足互易定理2

互易二端口？ ⇒ $Z_{12} = Z_{21}$

对称二端口？ ⇒ $Z_{12} = Z_{21}$，$Z_{11} = Z_{22}$

例 求所示电路的Z参数。



$$\dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2$$
$$\dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2$$

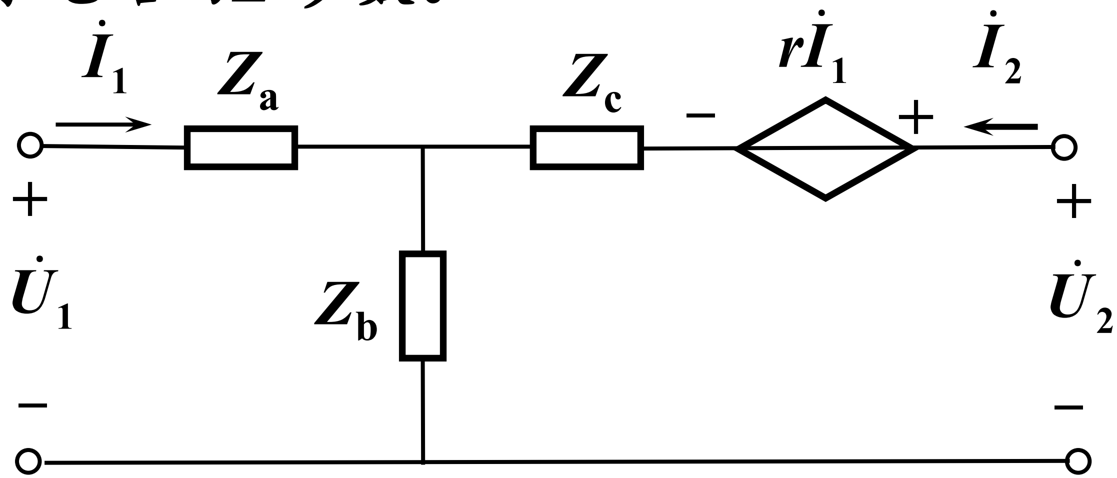
解 由实验测量法得到参数

$$Z_{11} = \frac{\dot{U}_1}{\dot{I}_1}\bigg|_{\dot{I}_2=0} = Z_a + Z_b \qquad Z_{12} = \frac{\dot{U}_1}{\dot{I}_2}\bigg|_{\dot{I}_1=0} = Z_b$$

$$Z_{21} = \frac{\dot{U}_2}{\dot{I}_1}\bigg|_{\dot{I}_2=0} = Z_b \qquad Z_{22} = \frac{\dot{U}_2}{\dot{I}_2}\bigg|_{\dot{I}_1=0} = Z_b + Z_c$$

互易二端口，且当 $Z_a=Z_c$ 时为对称二端口。

例 求所示电路的Z参数。



解

方法一：实验测量法

方法二：列写网孔方程

$$\dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2$$

$$\dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2$$

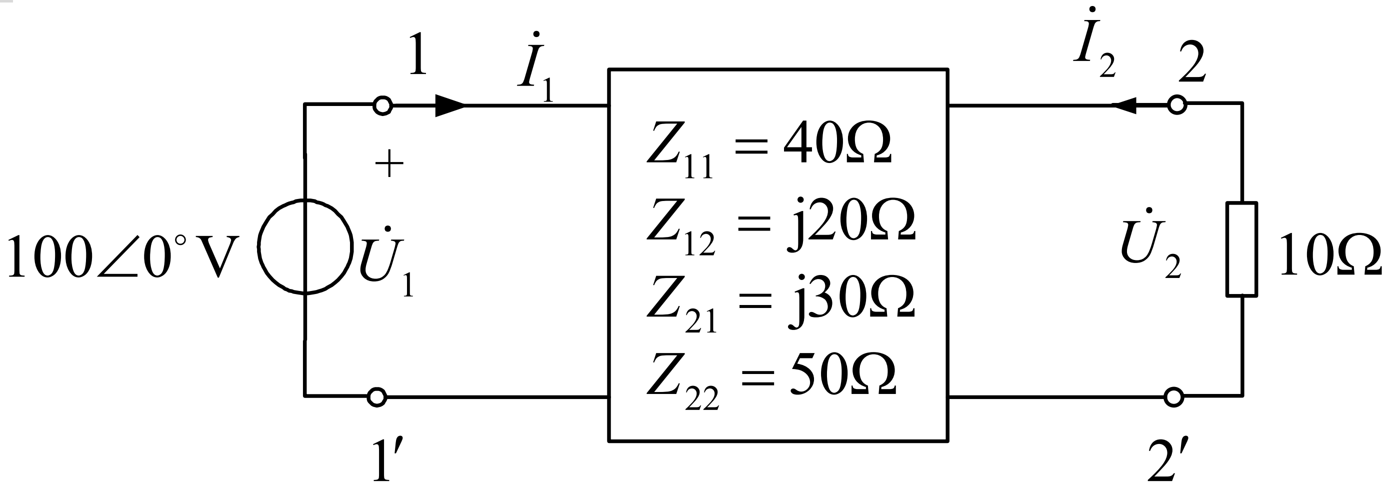
$$\dot{U}_1 = Z_a\dot{I}_1 + Z_b(\dot{I}_1 + \dot{I}_2)$$

$$\dot{U}_2 = r\dot{I}_1 + Z_c\dot{I}_2 + Z_b(\dot{I}_1 + \dot{I}_2)$$

$$Z = \begin{bmatrix} Z_a + Z_b & Z_b \\ r + Z_b & Z_b + Z_c \end{bmatrix}$$

4个独立参数

例 计算端口电流



解 已知参数，确定端口变量

端口电压、电流方程

$$\begin{cases}\dot{U}_1=100\angle 0^\circ \\ \dot{U}_2=-10\dot{I}_2 \\ \dot{U}_1=Z_{11}\dot{I}_1+Z_{12}\dot{I}_2 \\ \dot{U}_2=Z_{21}\dot{I}_1+Z_{22}\dot{I}_2\end{cases}$$

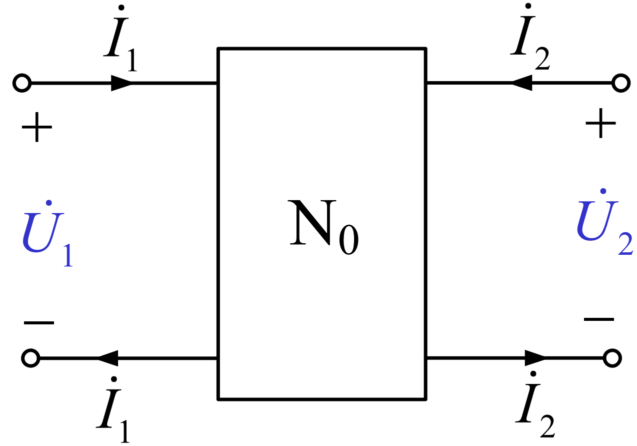
端口支路方程

端口特性方程

$$\begin{cases}100\angle 0^\circ=40\dot{I}_1+j20\dot{I}_2 \\ -10\dot{I}_2=j30\dot{I}_1+50\dot{I}_2\end{cases} \Rightarrow \begin{aligned}\dot{I}_1&=2\angle 0^\circ\ \text{A} \\ \dot{I}_2&=1\angle -90^\circ\ \text{A}\end{aligned}$$

2. 导纳参数方程（Y参数）

$$\begin{cases} \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 \end{cases}$$



矩阵形式：

$$\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix}$$

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$\begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} \Rightarrow \boxed{YZ = 1}$$

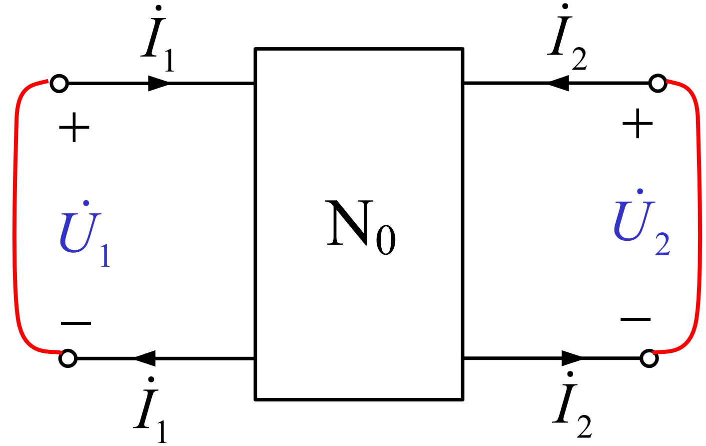
同一网络的Z参数、Y参数矩阵互逆

导纳参数测量

2−2′ 端口短路： $\dot{U}_2=0$

$$Y_{11}=\left.\frac{\dot{I}_1}{\dot{U}_1}\right|_{\dot{U}_2=0}$$ 输入导纳

$$Y_{21}=\left.\frac{\dot{I}_2}{\dot{U}_1}\right|_{\dot{U}_2=0}$$ 转移导纳



$$\begin{cases}\dot{I}_1=Y_{11}\dot{U}_1+Y_{12}\dot{U}_2\\ \dot{I}_2=Y_{21}\dot{U}_1+Y_{22}\dot{U}_2\end{cases}$$

1−1′ 端口短路： $\dot{U}_1=0$

$$Y_{12}=\left.\frac{\dot{I}_1}{\dot{U}_2}\right|_{\dot{U}_1=0}$$ 转移导纳

$$Y_{22}=\left.\frac{\dot{I}_2}{\dot{U}_2}\right|_{\dot{U}_1=0}$$ 输入导纳

互易二端口： $\boxed{\text{互易定理1}}$

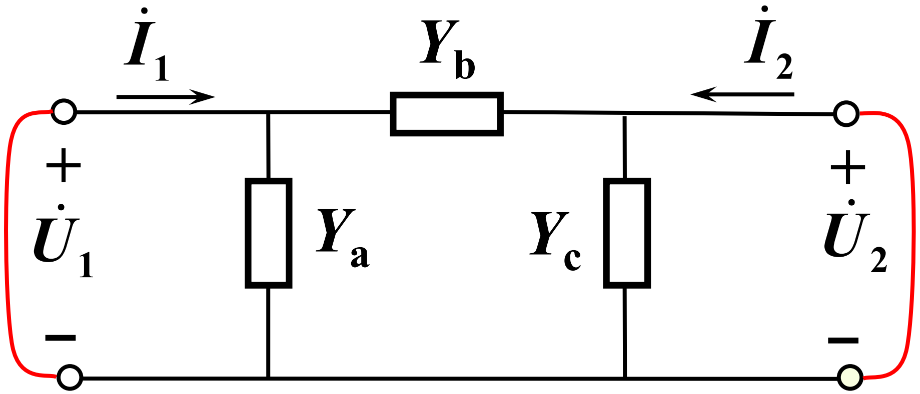
$$Y_{12}=Y_{21}$$

对称二端口：

$$Y_{12}=Y_{21}\quad Y_{11}=Y_{22}$$

实验测量法是确定任何参数的通用方法！

例 求图示二端口的Y参数。



$$\begin{cases} \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 \end{cases}$$

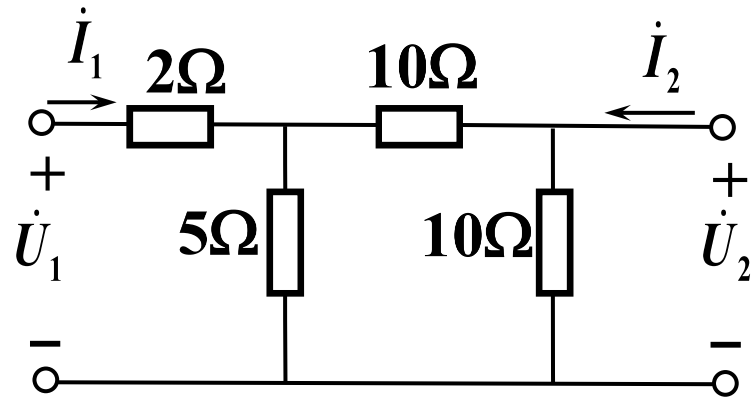
解 由实验测量法得到参数

$$Y_{11} = \frac{\dot{I}_1}{\dot{U}_1}\bigg|_{\dot{U}_2=0} = Y_a + Y_b \qquad Y_{12} = \frac{\dot{I}_1}{\dot{U}_2}\bigg|_{\dot{U}_1=0} = -Y_b$$

$$Y_{21} = \frac{\dot{I}_2}{\dot{U}_1}\bigg|_{\dot{U}_2=0} = -Y_b \qquad Y_{22} = \frac{\dot{I}_2}{\dot{U}_2}\bigg|_{\dot{U}_2=0} = Y_b + Y_c$$

互易二端口，且当Y_a=Y_c时为对称二端口。

例 求图示二端口的Y参数。



解 电阻网络，互易

Y–Δ 等效变换

$$Y_{12}=Y_{21}=-\frac{1}{16}\ \mathrm{S}$$

$$Y_{11}=\frac{1}{2+5//10}=\frac{3}{16}\mathrm{S}$$

$$Y_{22}=\frac{1}{10//(10+2//5)}=\frac{3}{16}\mathrm{S}$$

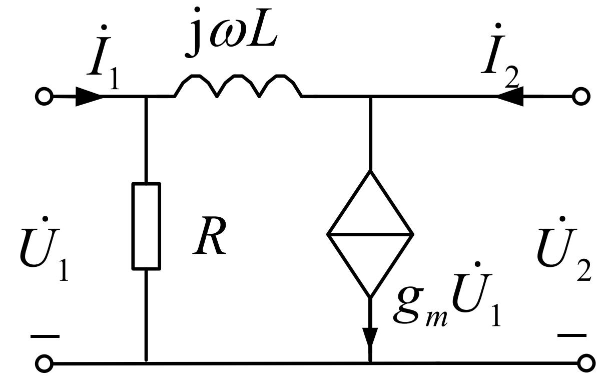
对称二端口网络

$$Y_{11}=Y_{22}$$

$$Y_{12}=Y_{21}$$

对称二端口是指两个端口电气特性上对称。电路结构左右对称的，端口电气特性对称；电路结构不对称的二端口，其电气特性也可能是对称的。

例 求图示二端口的Y参数。



解 方法一：实验测量法

方法二：列写结点方程

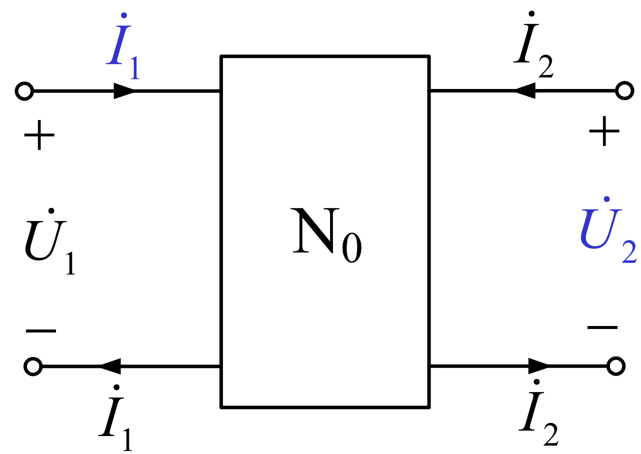
$$\begin{cases}\left(\dfrac{1}{R}+\dfrac{1}{\mathrm{j}\omega L}\right)\dot{U}_1-\dfrac{1}{\mathrm{j}\omega L}\dot{U}_2=\dot{I}_1\\ -\dfrac{1}{\mathrm{j}\omega L}\dot{U}_1+\dfrac{1}{\mathrm{j}\omega L}\dot{U}_2=\dot{I}_2-g_m\dot{U}_1\end{cases}$$

$$\begin{cases}\dot{I}_1=Y_{11}\dot{U}_1+Y_{12}\dot{U}_2\\ \dot{I}_2=Y_{21}\dot{U}_1+Y_{22}\dot{U}_2\end{cases}$$

$$\begin{cases}\dot{I}_1=\left(\dfrac{1}{R}+\dfrac{1}{\mathrm{j}\omega L}\right)\dot{U}_1-\dfrac{1}{\mathrm{j}\omega L}\dot{U}_2\\ \dot{I}_2=\left(g_m-\dfrac{1}{\mathrm{j}\omega L}\right)\dot{U}_1+\dfrac{1}{\mathrm{j}\omega L}\dot{U}_2\end{cases}$$

$$Y=\begin{bmatrix}\dfrac{1}{R}+\dfrac{1}{\mathrm{j}\omega L} & -\dfrac{1}{\mathrm{j}\omega L}\\ g_m-\dfrac{1}{\mathrm{j}\omega L} & \dfrac{1}{\mathrm{j}\omega L}\end{bmatrix}$$

3. 混合参数方程（H参数和G参数）



H参数 $\begin{cases} \dot{U}_1 = h_{11}\dot{I}_1 + h_{12}\dot{U}_2 \\ \dot{I}_2 = h_{21}\dot{I}_1 + h_{22}\dot{U}_2 \end{cases}$

G参数 $\begin{cases} \dot{I}_1 = g_{11}\dot{U}_1 + g_{12}\dot{I}_2 \\ \dot{U}_2 = g_{21}\dot{U}_1 + g_{22}\dot{I}_2 \end{cases}$

矩阵形式： $\boxed{HG = 1}$

$$\begin{bmatrix} \dot{U}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{U}_2 \end{bmatrix} \qquad \begin{bmatrix} \dot{I}_1 \\ \dot{U}_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ \dot{I}_2 \end{bmatrix}$$

混合参数测量（H参数）

2−2′ 端口短路： $\dot{U}_2 = 0$

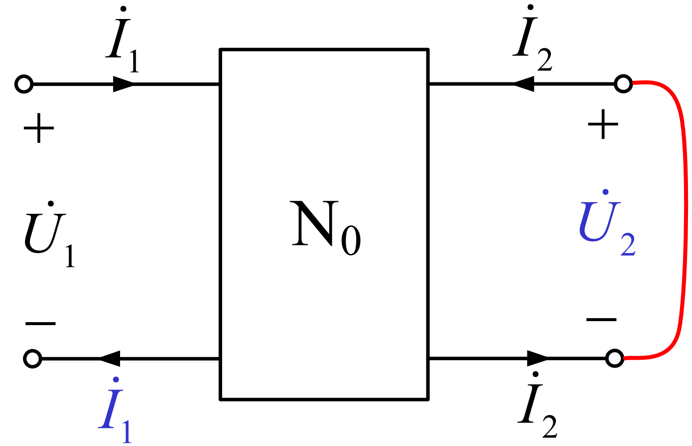
$$h_{11} = \left.\frac{\dot{U}_1}{\dot{I}_1}\right|_{\dot{U}_2=0} \quad \text{输入阻抗}$$

$$h_{21} = \left.\frac{\dot{I}_2}{\dot{I}_1}\right|_{\dot{U}_2=0} \quad \text{电流增益}$$

1−1′ 端口开路： $\dot{I}_1 = 0$

$$h_{12} = \left.\frac{\dot{U}_1}{\dot{U}_2}\right|_{\dot{I}_1=0} \quad \text{电压增益}$$

$$h_{22} = \left.\frac{\dot{I}_2}{\dot{U}_2}\right|_{\dot{I}_1=0} \quad \text{输入导纳}$$



$$\begin{cases} \dot{U}_1 = h_{11}\dot{I}_1 + h_{12}\dot{U}_2 \\ \dot{I}_2 = h_{21}\dot{I}_1 + h_{22}\dot{U}_2 \end{cases}$$

互易二端口： $\boxed{\text{互易定理3}}$

$$h_{12} = -h_{21}$$

对称二端口：

$$h_{12} = -h_{21}$$

$$h_{11}h_{22} - h_{12}h_{21} = 1$$

证明对称二端口H参数需满足：$h_{12}=-h_{21}\quad h_{11}h_{22}-h_{12}h_{21}=1$

Y参数 $\begin{cases}\dot I_1=Y_{11}\dot U_1+Y_{12}\dot U_2\\ \dot I_2=Y_{21}\dot U_1+Y_{22}\dot U_2\end{cases}$ 对称二端口： $Y_{12}=Y_{21}$ $Y_{11}=Y_{22}$

H参数 $\dot U_1=h_{11}\dot I_1+h_{12}\dot U_2 \Rightarrow \dot I_1=\dfrac{1}{h_{11}}\dot U_1-\dfrac{h_{12}}{h_{11}}\dot U_2$

$$\dot I_2=h_{21}\dot I_1+h_{22}\dot U_2$$

$$\Rightarrow \dot I_2=h_{21}\left(\frac{\dot U_1}{h_{11}}-\frac{h_{12}\dot U_2}{h_{11}}\right)+h_{22}\dot U_2$$

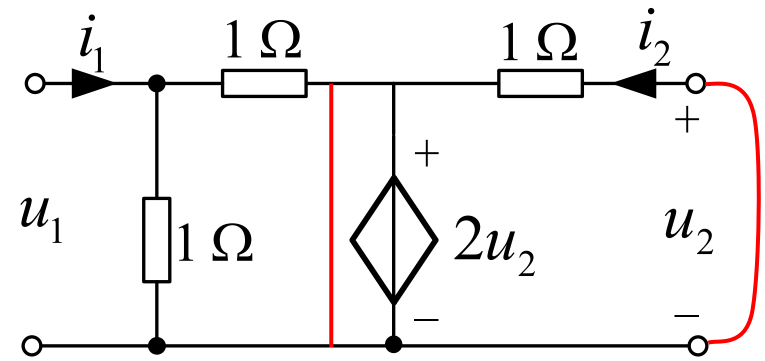
$$=\frac{h_{21}}{h_{11}}\dot U_1+\frac{h_{11}h_{22}-h_{21}h_{12}}{h_{11}}\dot U_2$$

$$Y_{12}=\frac{-h_{12}}{h_{11}}=\frac{h_{21}}{h_{11}}=Y_{21}$$

$$Y_{11}=\frac{1}{h_{11}}=\frac{h_{11}h_{22}-h_{21}h_{12}}{h_{11}}=Y_{22}$$

G参数测量方法以及互易、对称条件类似，课后自学

例 求图示二端口的H参数。



解 含受控源，不满足互易定理

方法一：实验测量法

$2\text{-}2'$ 端口短路： $u_2=0 \quad 2u_2=0$

$$\begin{cases}\dot{U}_1=h_{11}\dot{I}_1+h_{12}\dot{U}_2\\ \dot{I}_2=h_{21}\dot{I}_1+h_{22}\dot{U}_2\end{cases}$$

$$u_1=\frac{1}{2}i_1 \Rightarrow h_{11}=\frac{1}{2}$$

$$i_2=0 \Rightarrow h_{21}=0$$

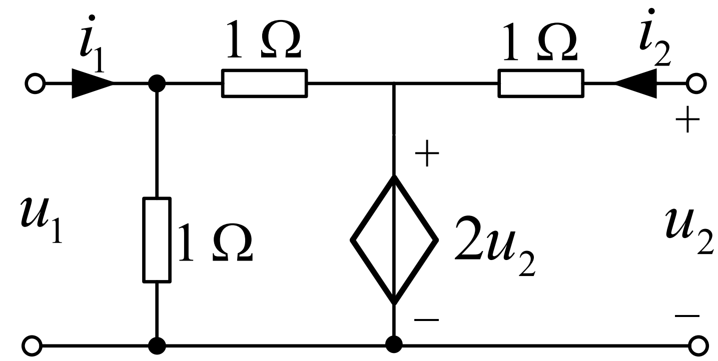
$1\text{-}1'$ 端口开路： $i_1=0$

$$2u_2=u_1+u_1 \Rightarrow h_{12}=1$$

$$u_2=1\cdot i_2+2u_2 \Rightarrow h_{22}=-1$$

$$H=\begin{bmatrix}h_{11} & h_{12}\\ h_{21} & h_{22}\end{bmatrix}=\begin{bmatrix}0.5 & 1\\ 0 & -1\end{bmatrix}$$

例 求图示二端口的H参数。



$$\begin{cases} \dot{U}_1 = h_{11}\dot{I}_1 + h_{12}\dot{U}_2 \\ \dot{I}_2 = h_{21}\dot{I}_1 + h_{22}\dot{U}_2 \end{cases}$$

解 方法二：列写网孔方程

$$u_1 = \left(i_1 - \frac{u_1}{1}\right)\times 1 + 2u_2$$

$$\Rightarrow u_1 = \frac{1}{2}i_1 + u_2$$

$$i_2 = \frac{u_2 - 2u_2}{1} = -u_2$$

$$h_{11} = \frac{1}{2} \qquad h_{12} = 1$$

$$h_{21} = 0 \qquad h_{22} = -1$$

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 0.5 & 1 \\ 0 & -1 \end{bmatrix}$$

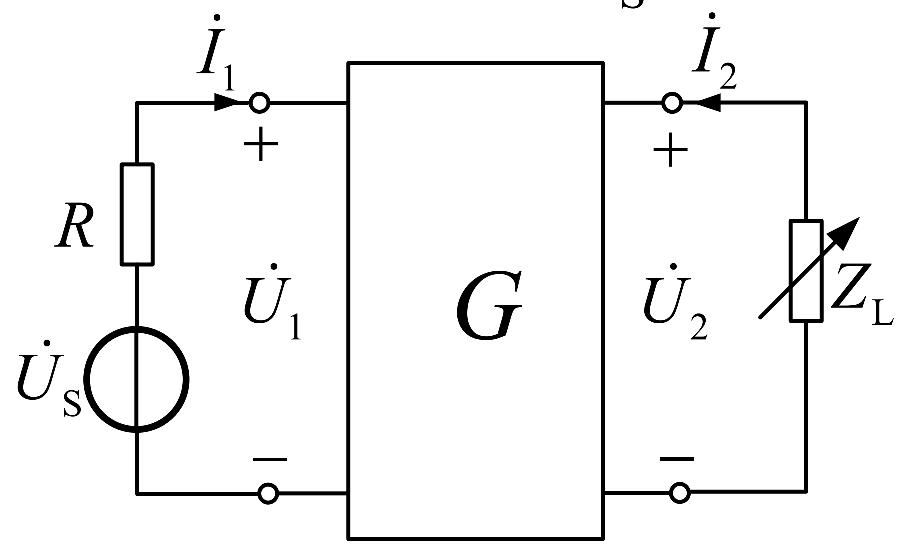
例 已知 $U_S = 42\sqrt{2}\cos(5000t)$ V，$R = 6\ \Omega$， 无源二端口网络G参数为 $G = \begin{bmatrix} \frac{1}{6} - \mathrm{j}\frac{1}{6} & -0.5 + \mathrm{j}0.5 \\ 0.5 - \mathrm{j}0.5 & 1.5 + \mathrm{j}2.5 \end{bmatrix}$， 求负载$Z_L$的最大功率$P_L$及此时电源提供的功率$P_S$。

解

$$\dot{I}_1 = g_{11}\dot{U}_1 + g_{12}\dot{I}_2$$

$$\dot{U}_2 = g_{21}\dot{U}_1 + g_{22}\dot{I}_2$$

$$\dot{U}_1 = 42 - 6\dot{I}_1$$



戴维南等效

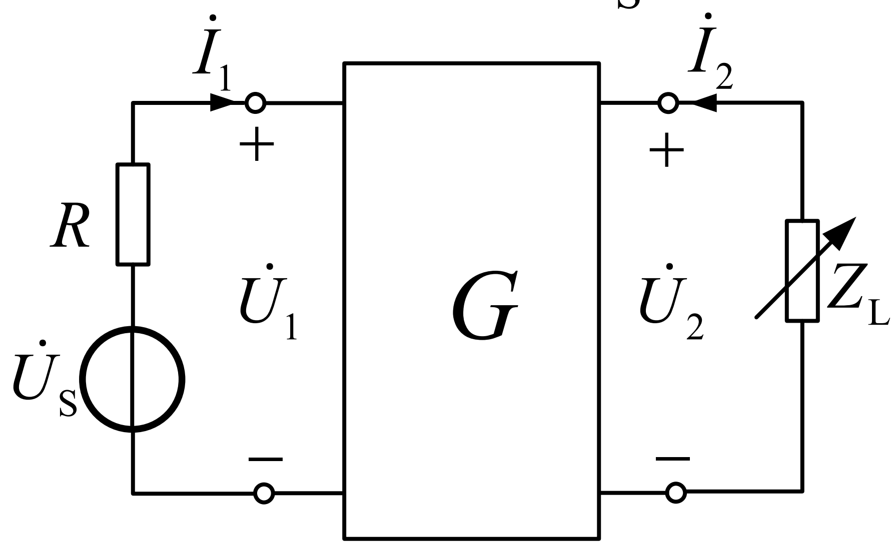
端口2开路：$\dot{I}_2 = 0 \Rightarrow \dot{U}_{2OC}$

$$\dot{U}_2 = \dot{U}_{2OC} + Z_{eq}\dot{I}_{2SC}$$

端口2短路：$\dot{U}_2 = 0 \Rightarrow \dot{I}_{2SC} = \dot{I}_2$

例 已知 $U_S = 42\sqrt{2}\cos(5000t)$ V，$R = 6\ \Omega$，无源二端口网络G参数为 $G = \begin{bmatrix} \frac{1}{6} - \mathrm{j}\frac{1}{6} & -0.5 + \mathrm{j}0.5 \\ 0.5 - \mathrm{j}0.5 & 1.5 + \mathrm{j}2.5 \end{bmatrix}$，求负载$Z_L$的最大功率$P_L$及此时电源提供的功率$P_S$。

解 思路2：戴维南等效



$$\dot{U}_2 = \dot{U}_{2OC} + Z_{eq}\dot{I}_{2SC}$$
$$= \dot{U}_{2OC} + Z_{eq}\dot{I}_2$$

只需找到$\dot{U}_2$和$\dot{I}_2$之间的关系

$$\dot{I}_1 = g_{11}\dot{U}_1 + g_{12}\dot{I}_2$$
$$\dot{U}_2 = g_{21}\dot{U}_1 + g_{22}\dot{I}_2 \rightarrow \dot{U}_2 = \underbrace{42\frac{g_{21}}{1+6g_{11}}}_{\dot{U}_{2OC}} + \underbrace{\frac{g_{22}+6(g_{11}g_{22}-g_{12}g_{21})}{1+6g_{11}}}_{Z_{eq}}\dot{I}_2$$
$$\dot{U}_1 = 42 - 6\dot{I}_1$$

$$\dot{U}_{2\mathrm{OC}}=42\frac{g_{21}}{1+6g_{11}}=13.28\angle-18.43^\circ\ \mathrm{V}$$

$$Z_{\mathrm{eq}}=\frac{g_{22}+6\left(g_{11}g_{22}-g_{12}g_{21}\right)}{1+6g_{11}}=2.1+\mathrm{j}1.3\ \Omega$$

(1) 最大功率P_{L}

$$Z_{\mathrm{L}}=2.1-\mathrm{j}1.3\ \Omega \qquad \rightarrow P_{\mathrm{L}}=\frac{U_{\mathrm{OC}}^{2}}{4\mathrm{Re}\left(Z_{\mathrm{L}}\right)}=21\ \mathrm{W}$$

(2) 电源功率P_{S}

$$\dot{I}_2=-\frac{\dot{U}_{2\mathrm{OC}}}{Z_{\mathrm{eq}}+Z_{\mathrm{L}}}=3.16\angle161.57^\circ\ \mathrm{A}$$

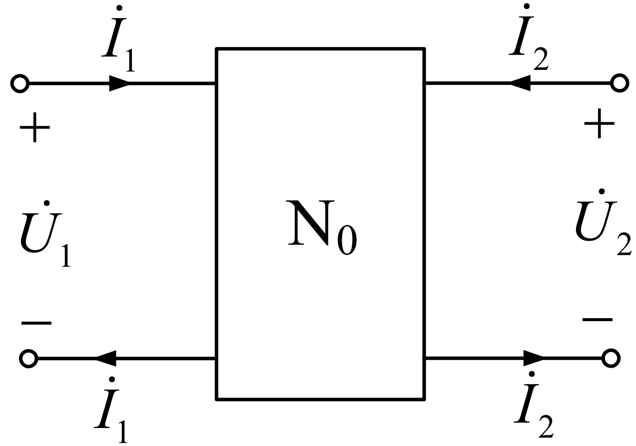
$$\begin{aligned}\dot{I}_1&=g_{11}\dot{U}_1+g_{12}\dot{I}_2\\ \dot{U}_1&=42-6\dot{I}_1\end{aligned} \quad \rightarrow \dot{I}_1=\frac{g_{11}\dot{U}_1+g_{12}\dot{I}_2}{1+6g_{11}}=5.38\angle-21.8^\circ\ \mathrm{A}$$

$$\rightarrow P=42\times5.38\times\cos\left(21.8^\circ\right)=210\ \mathrm{W}$$

4. 传输参数方程（T参数和T'参数）

T参数
$$\begin{cases}\dot U_1 = A\dot U_2 + B\left(-\dot I_2\right)\\ \dot I_1 = C\dot U_2 + D\left(-\dot I_2\right)\end{cases}$$

T'参数
$$\begin{cases}\dot U_2 = A'\dot U_1 + B'\left(-\dot I_1\right)\\ \dot I_2 = C'\dot U_1 + D'\left(-\dot I_1\right)\end{cases}$$



注意负号

矩阵形式：

$$\begin{bmatrix}\dot U_1\\ \dot I_1\end{bmatrix}=\begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}\dot U_2\\ -\dot I_2\end{bmatrix} \qquad \begin{bmatrix}\dot U_2\\ \dot I_2\end{bmatrix}=\begin{bmatrix}A' & B'\\ C' & D'\end{bmatrix}\begin{bmatrix}\dot U_1\\ -\dot I_1\end{bmatrix}$$

T参数能方便的描述信号或能量从一个端口向另一端口的传输特性。

$$\begin{bmatrix}\dot U_1\\ \dot I_1\end{bmatrix}=\begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}\dot U_2\\ -\dot I_2\end{bmatrix}\qquad \begin{bmatrix}\dot U_2\\ \dot I_2\end{bmatrix}=\begin{bmatrix}A' & B'\\ C' & D'\end{bmatrix}\begin{bmatrix}\dot U_1\\ -\dot I_1\end{bmatrix}$$

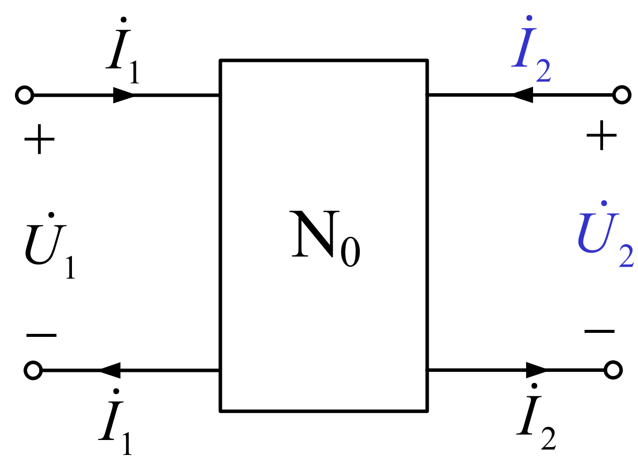
$$\boxed{\begin{bmatrix}A & -B\\ C & -D\end{bmatrix}\begin{bmatrix}A' & -B'\\ C' & -D'\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}$$

传输参数测量（T参数）

2−2′ 端口开路： $\dot I_2 = 0$

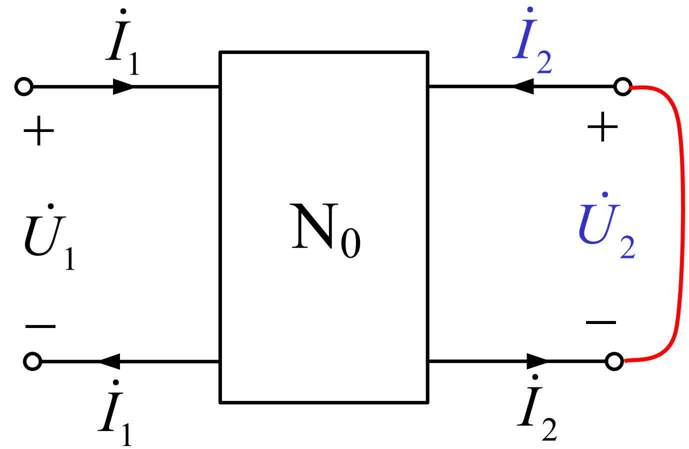
$$A=\left.\frac{\dot U_1}{\dot U_2}\right|_{\dot I_2=0}\qquad A^{-1}\text{电压增益}$$

$$C=\left.\frac{\dot I_1}{\dot U_2}\right|_{\dot I_2=0}\qquad C^{-1}\text{转移阻抗}$$



传输参数测量（T参数）

2−2′ 端口短路： $\dot{U}_2 = 0$



$$D = \left.\frac{\dot{I}_1}{-\dot{I}_2}\right|_{\dot{U}_2=0} \quad -D^{-1}\text{电流增益}$$

$$B = \left.\frac{\dot{U}_1}{-\dot{I}_2}\right|_{\dot{U}_2=0} \quad -B^{-1}\text{转移导纳}$$

互易二端口：

$$AD - BC = 1$$

如何证明？

对称二端口：

$$AD - BC = 1 \qquad A = D$$

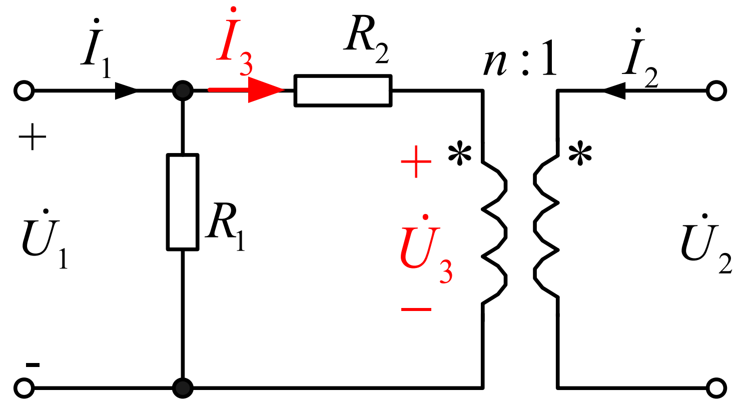
T '参数的测量方法课后自学

例 求图示二端口的T参数。

解 方法一：实验测量法

2−2′ 端口开路： $\dot{I}_2=0$

$$\dot{I}_3=0 \quad \dot{U}_2=\frac{1}{n}\dot{U}_3 \quad \dot{U}_3=\dot{U}_1$$



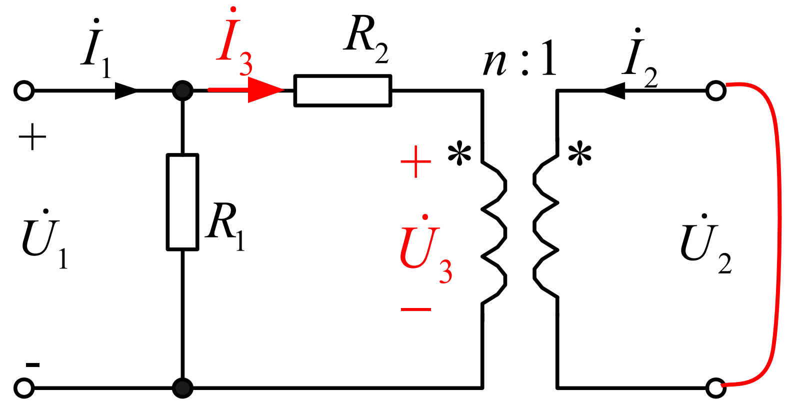
$$\dot{U}_2=\frac{1}{n}\dot{U}_3=\frac{1}{n}\dot{U}_1$$

$$\begin{cases}\dot{U}_1=A\dot{U}_2+B\left(-\dot{I}_2\right)\\ \dot{I}_1=C\dot{U}_2+D\left(-\dot{I}_2\right)\end{cases}$$

$$\dot{I}_1=\frac{\dot{U}_1}{R_1}=\frac{n}{R_1}\dot{U}_2$$

$$A=\left.\frac{\dot{U}_1}{\dot{U}_2}\right|_{\dot{I}_2=0}=n \qquad C=\left.\frac{\dot{I}_1}{\dot{U}_2}\right|_{\dot{I}_2=0}=\frac{\dfrac{\dot{U}_1}{R_1}}{\dfrac{1}{n}\dot{U}_1}=\frac{n}{R_1}$$

例 求图示二端口的T参数。



解 方法一：实验测量法

2−2′ 端口短路： $\dot{U}_2=0$

$$\dot{U}_3=0 \quad \dot{I}_3=\frac{\dot{U}_1}{R_2}$$

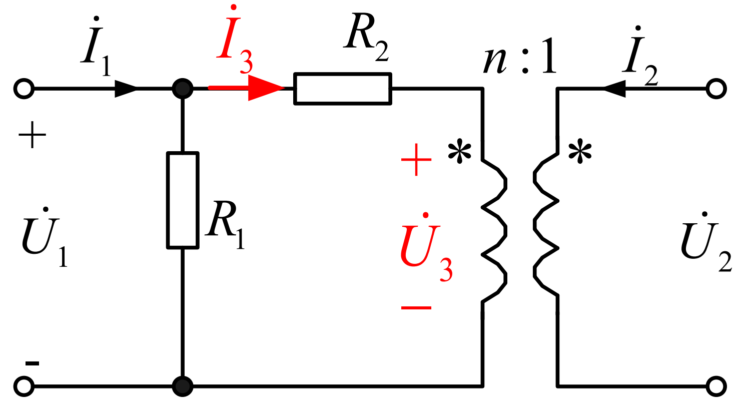
$$\dot{I}_1=\frac{\dot{U}_1}{R_1}+\dot{I}_3=(\frac{1}{R_1}+\frac{1}{R_2})\dot{U}_1$$

$$\dot{I}_2=-n\dot{I}_3=-\frac{n}{R_2}\dot{U}_1$$

$$\begin{cases}\dot{U}_1=A\dot{U}_2+B\left(-\dot{I}_2\right)\\ \dot{I}_1=C\dot{U}_2+D\left(-\dot{I}_2\right)\end{cases}$$

$$B=\left.\frac{\dot{U}_1}{-\dot{I}_2}\right|_{\dot{U}_2=0}=\frac{\dot{U}_1}{\frac{n}{R_2}\dot{U}_1}=\frac{R_2}{n} \qquad D=\left.\frac{\dot{I}_1}{-\dot{I}_2}\right|_{\dot{U}_2=0}=\frac{(\frac{1}{R_1}+\frac{1}{R_2})\dot{U}_1}{\frac{n}{R_2}\dot{U}_1}=\frac{R_1+R_2}{nR_1}$$

例 求图示二端口的T参数。



解 方法二：列写端口特性方程

由理想变压器特性：
$$\dot{I}_3 = -\frac{1}{n}\dot{I}_2$$
$$\dot{U}_3 = n\dot{U}_2$$

$$\begin{cases} \dot{U}_1 = A\dot{U}_2 + B\left(-\dot{I}_2\right) \\ \dot{I}_1 = C\dot{U}_2 + D\left(-\dot{I}_2\right) \end{cases}$$

$$\dot{U}_1 = \dot{I}_3 R_2 + \dot{U}_3 = n\dot{U}_2 - \frac{R_2}{n}\dot{I}_2$$

$$\dot{I}_1 = \frac{\dot{U}_1}{R_1} + \dot{I}_3 = \frac{1}{R_1}(n\dot{U}_2 - \frac{R_2}{n}\dot{I}_2) - \frac{1}{n}\dot{I}_2$$

$$\begin{cases} \dot{U}_1 = n\dot{U}_2 + \frac{R_2}{n}(-\dot{I}_2) \\ \dot{I}_1 = \frac{n}{R_1}\dot{U}_2 + \left(\frac{R_2}{nR_1} + \frac{1}{n}\right)\left(-\dot{I}_2\right) \end{cases}$$

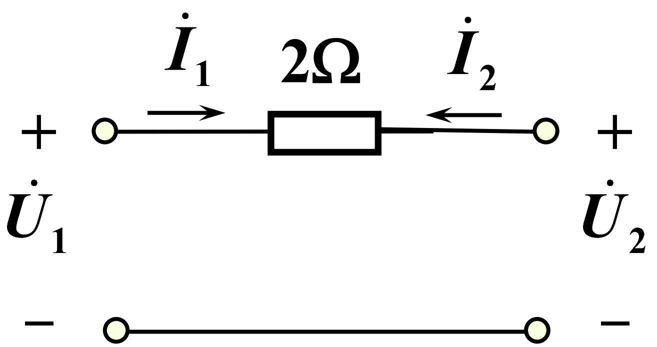
$$A = n \qquad B = \frac{R_2}{n}$$

$$C = \frac{n}{R_1} \qquad D = \frac{R_1 + R_2}{nR_1}$$

小结

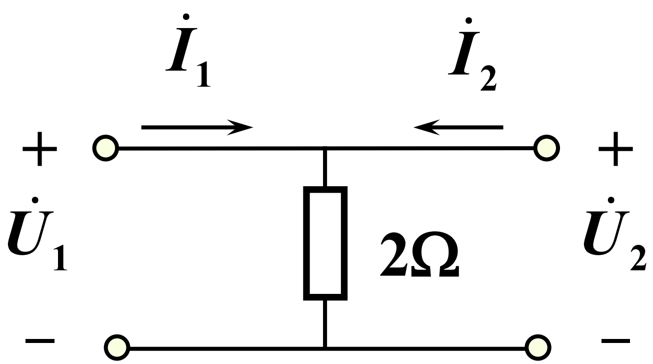
（1）为什么用这么多参数表示二端口网络特性？

- 为描述电路方便，测量方便；
- 有些电路只存在某几种参数。



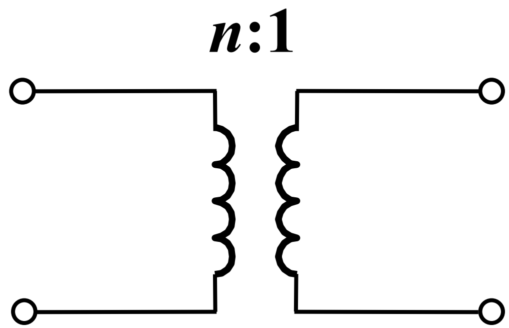
$$Y=\begin{bmatrix}0.5 & -0.5\\ -0.5 & 0.5\end{bmatrix}\mathrm{s}$$

*Z*参数不存在



$$Z=\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}\Omega$$

*Y*参数不存在



存在T参数，H参数。

Z，Y均不存在。

（2）线性无源二端口为互易二端口网络；

（3）含有受控源的电路有四个独立参数；

（4）不同参数之间可以进行互换。 数学方程变换

H参数 $\begin{cases} \dot{U}_1 = h_{11}\dot{I}_1 + h_{12}\dot{U}_2 \\ \dot{I}_2 = h_{21}\dot{I}_1 + h_{22}\dot{U}_2 \end{cases}$

$$\Longrightarrow \dot{I}_1 = \frac{1}{h_{11}}\dot{U}_1 - \frac{h_{12}}{h_{11}}\dot{U}_2$$

$$\dot{I}_2 = h_{21}\left(\frac{\dot{U}_1}{h_{11}} - \frac{h_{12}\dot{U}_2}{h_{11}}\right) + h_{22}\dot{U}_2$$

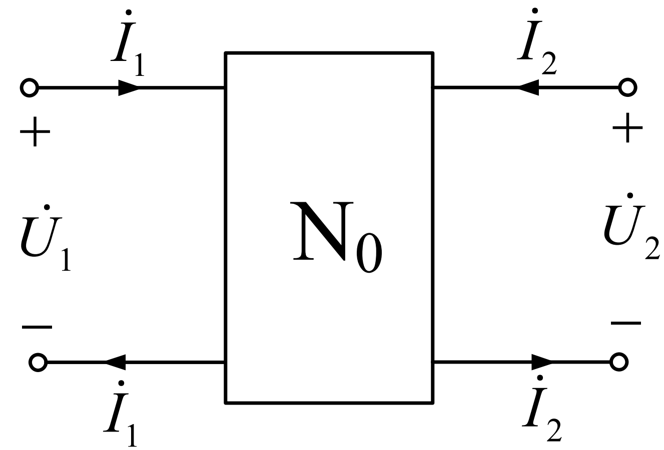
$$= \frac{h_{21}}{h_{11}}\dot{U}_1 + \frac{h_{11}h_{22} - h_{21}h_{12}}{h_{11}}\dot{U}_2$$

Y参数 $\begin{cases} \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 \end{cases}$

互易二端口网络、对称二端口网络参数特点

	互易二端口	对称二端口	
Z	$Z_{12}=Z_{21}$	$Z_{11}=Z_{22}$	$Z_{12}=Z_{21}$
Y	$Y_{12}=Y_{21}$	$Y_{11}=Y_{22}$	$Y_{12}=Y_{21}$
H	$h_{12}=-h_{21}$	$h_{11}h_{22}-h_{12}h_{21}=1$	$h_{12}=-h_{21}$
G	$g_{12}=-g_{21}$	$g_{11}g_{22}-g_{12}g_{21}=1$	$g_{12}=-g_{21}$
T	$AD-BC=1$	$A=D$	$AD-BC=1$
T'	$A'D'-B'C'=1$	$A'=D'$	$A'D'-B'C'=1$

例 已知：端口2开路时，U_1=10mV，I_1=10μA，U_2=-40V；端口2短路时，U_1=24mV，I_1=20μA，I_2=1mA。计算网络*H*参数。



解

$$\begin{cases} \dot{U}_1 = h_{11}\dot{I}_1 + h_{12}\dot{U}_2 \\ \dot{I}_2 = h_{21}\dot{I}_1 + h_{22}\dot{U}_2 \end{cases}$$

端口2短路时：

$$h_{11} = \frac{\dot{U}_1}{\dot{I}_1}\bigg|_{\dot{U}_2=0} = \frac{24\text{mV}}{20\mu\text{A}} = 1.2\ \text{k}\Omega$$

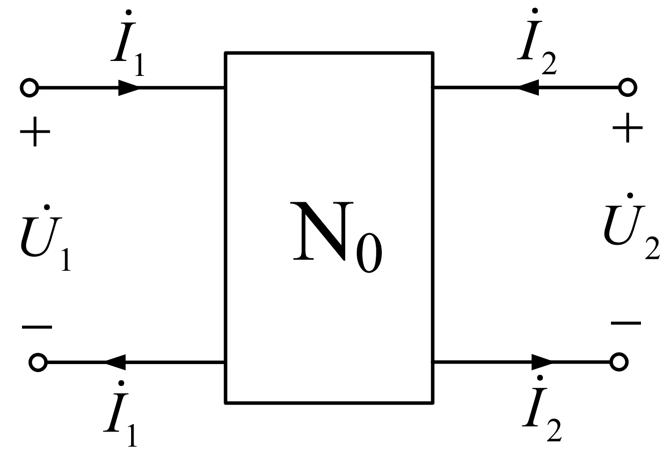
$$h_{21} = \frac{\dot{I}_2}{\dot{I}_1}\bigg|_{\dot{U}_2=0} = \frac{1\text{mA}}{20\mu\text{A}} = 50$$

端口1开路时：

$$h_{12} = \frac{\dot{U}_1}{\dot{U}_2}\bigg|_{\dot{I}_1=0} = ?$$

$$h_{22} = \frac{\dot{I}_2}{\dot{U}_2}\bigg|_{\dot{I}_1=0} = ?$$

例 已知：端口2开路时，U_1=10mV，I_1=10μA，U_2=-40V；端口2短路时，U_1=24mV，I_1=20μA，I_2=1mA。计算网络*H*参数。



解 端口2开路 ⇨ $\dot{I}_2 = 0$

端口2短路 ⇨ $\dot{U}_2 = 0$

T参数方程 $\begin{cases} \dot{U}_1 = A\dot{U}_2 + B\left(-\dot{I}_2\right) \\ \dot{I}_1 = C\dot{U}_2 + D\left(-\dot{I}_2\right) \end{cases}$

$$A = \frac{\dot{U}_1}{\dot{U}_2} = \frac{10\text{mV}}{-40\text{V}} = -2.5\times10^4 \qquad B = \frac{\dot{U}_1}{-\dot{I}_2} = \frac{24\text{mV}}{-1\text{mA}} = -24\ \Omega$$

$$C = \frac{\dot{I}_1}{\dot{U}_2} = \frac{10\mu\text{A}}{-40\text{V}} = -2.5\times10^7\ \text{S} \qquad D = \frac{\dot{I}_1}{-\dot{I}_2} = \frac{20\mu\text{A}}{-1\text{mA}} = -0.02$$

$$A=-2.5\times10^{4}\qquad C=-2.5\times10^{7}\ \mathrm{S}$$

$$B=-24\ \Omega\qquad D=-0.02$$

T参数方程

$$\begin{cases}\dot{U}_1=A\dot{U}_2+B\left(-\dot{I}_2\right)\\ \dot{I}_1\ =C\dot{U}_2+D\left(-\dot{I}_2\right)\end{cases}$$

H参数方程

$$\begin{cases}\dot{U}_1=h_{11}\dot{I}_1+h_{12}\dot{U}_2\\ \dot{I}_2=h_{21}\dot{I}_1+h_{22}\dot{U}_2\end{cases}$$

$$\dot{I}_1\ =C\dot{U}_2+D\left(-\dot{I}_2\right)\ \Rightarrow\ \dot{I}_2=-\frac{1}{D}\dot{I}_1+\frac{C}{D}\dot{U}_2$$

$$h_{21}=-\frac{1}{D}=50\qquad h_{22}=\frac{C}{D}=12.5\ \mu\mathrm{S}$$

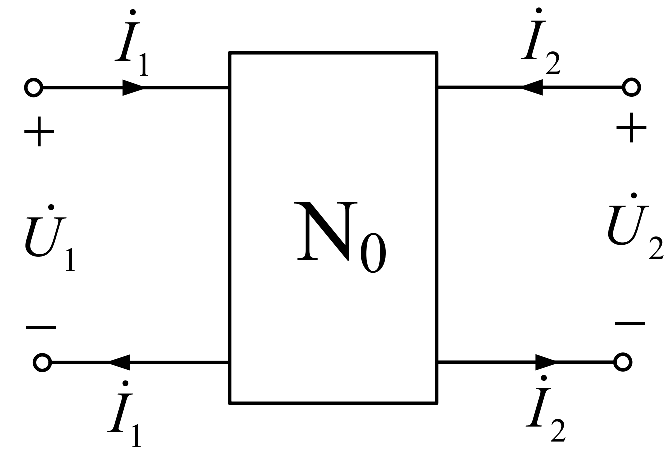
$$\Downarrow$$

$$\dot{U}_1=A\dot{U}_2+B\left(\frac{1}{D}\dot{I}_1-\frac{C}{D}\dot{U}_2\right)\ \Rightarrow$$

$$h_{11}=\frac{B}{D}=1.2\ \mathrm{k}\Omega$$

$$h_{12}=\frac{AD-BC}{D}=5\times10^{-5}$$

例 已知：端口2开路时，U_1=10mV，I_1=10μA，U_2=-40V；端口2短路时，U_1=24mV，I_1=20μA，I_2=1mA。计算网络*H*参数。



解 方法二：

$$\begin{cases} \dot{U}_1 = h_{11}\dot{I}_1 + h_{12}\dot{U}_2 \\ \dot{I}_2 = h_{21}\dot{I}_1 + h_{22}\dot{U}_2 \end{cases}$$

端口2短路时：

$$h_{11} = \left.\frac{\dot{U}_1}{\dot{I}_1}\right|_{\dot{U}_2=0} = \frac{24\text{mV}}{20\mu\text{A}} = 1.2\ \text{k}\Omega$$

$$h_{21} = \left.\frac{\dot{I}_2}{\dot{I}_1}\right|_{\dot{U}_2=0} = \frac{1\text{mA}}{20\mu\text{A}} = 50$$

端口2开路时：$\dot{I}_2 = 0$

$$h_{22} = \frac{-h_{21}\dot{I}_1}{\dot{U}_2} = 12.5\ \mu\text{S}$$

$$h_{12} = \frac{\dot{U}_1 - h_{11}\dot{I}_1}{\dot{U}_2} = 5\times 10^{-5}$$

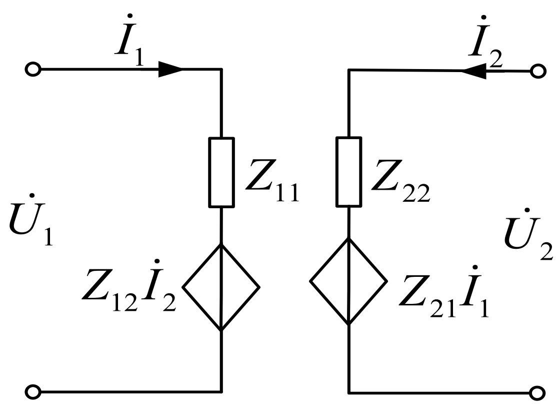
无论端口电量如何变化，网络参数不变

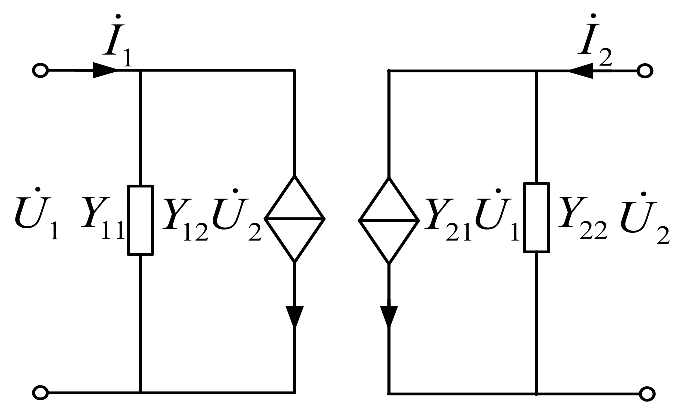
16.4二端口网络的电路模型

1. 等效二端口网络

两个二端口等效是指**对外电路而言，端口的电压、电流关系相同。**

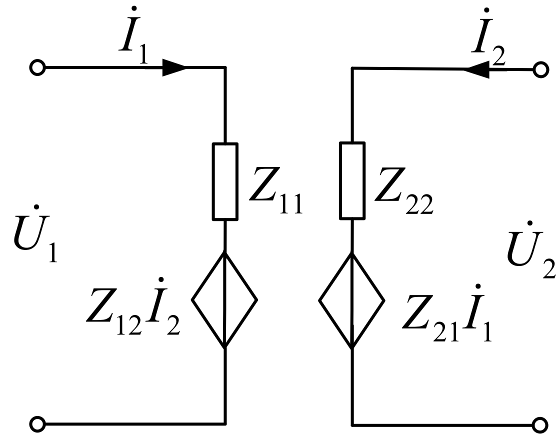
- 可以用最简单的二端口网络来代替复杂的网络；
- 等效二端口网络电路模型并不唯一；
- 最简单的二端口网络应包含4个独立的电路元件。



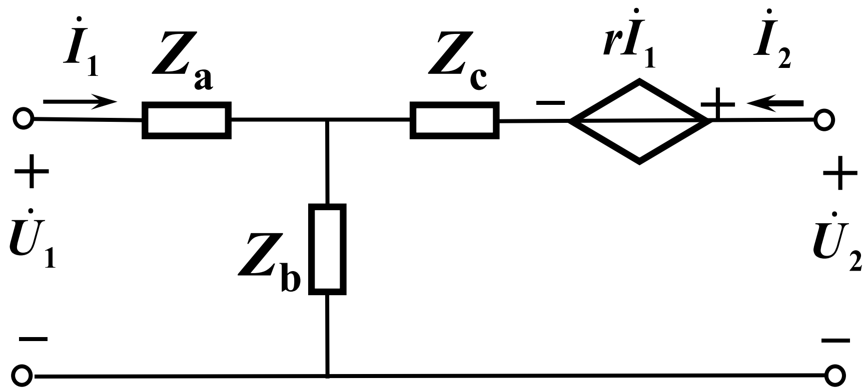


2. 由Z参数方程做等效电路 ⟹ T形电路模型

$$\begin{cases} \dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 \\ \dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2 \end{cases}$$



回顾

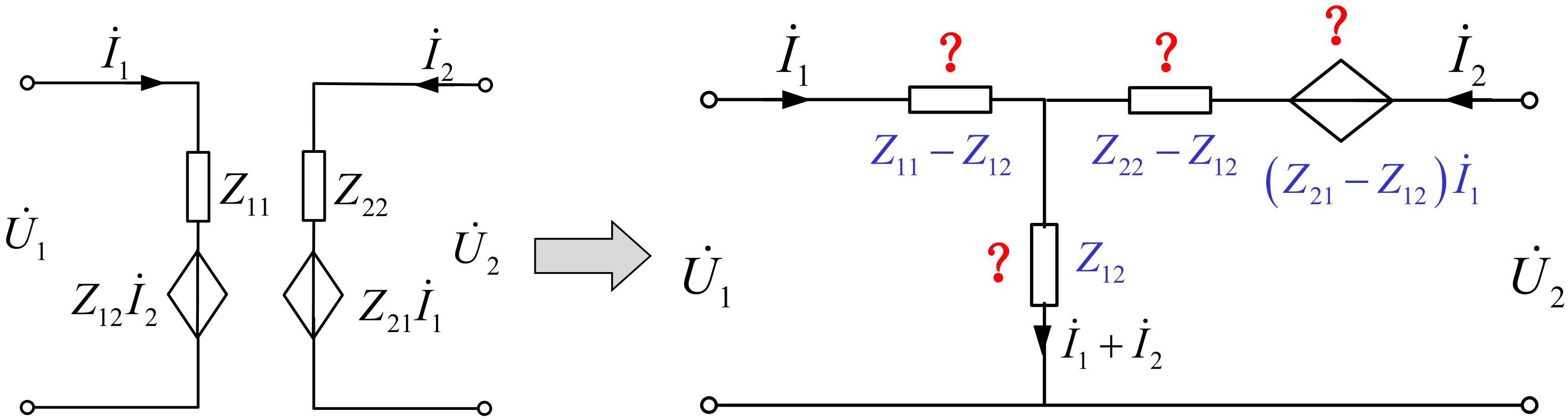


方法二：列写网孔方程

$$\dot{U}_1 = Z_a\dot{I}_1 + Z_b(\dot{I}_1 + \dot{I}_2)$$

$$\dot{U}_2 = r\dot{I}_1 + Z_c\dot{I}_2 + Z_b(\dot{I}_1 + \dot{I}_2)$$

2. 由Z参数方程做等效电路　T形电路模型



$$\begin{cases}\dot U_1 = Z_{11}\dot I_1 + Z_{12}\dot I_2 \\ \dot U_2 = Z_{21}\dot I_1 + Z_{22}\dot I_2\end{cases} \Rightarrow \dot U_1 = Z_{11}\dot I_1 + Z_{12}\dot I_2 + Z_{12}\dot I_1 - Z_{12}\dot I_1$$

$$= (Z_{11}-Z_{12})\dot I_1 + Z_{12}(\dot I_1+\dot I_2)$$

$$\dot U_2 = Z_{21}\dot I_1 + Z_{22}\dot I_2 + Z_{12}(\dot I_1+\dot I_2) - Z_{12}(\dot I_1+\dot I_2)$$

$$= (Z_{21}-Z_{12})\dot I_1 + (Z_{22}-Z_{12})\dot I_2 + Z_{12}(\dot I_1+\dot I_2)$$

记住推导过程，勿死记硬背公式

3. 由Y参数方程做等效电路　π形电路模型

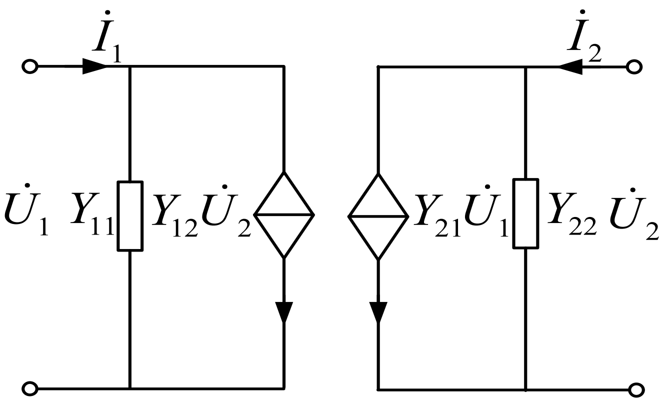
$$\begin{cases}\dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2\end{cases}$$

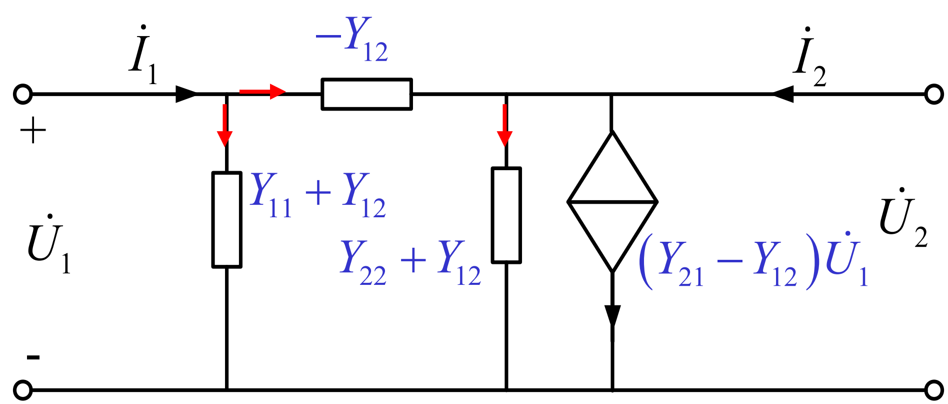
$$\Rightarrow \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 - Y_{12}\dot{U}_1 + Y_{12}\dot{U}_1$$ 结点法

$$= (Y_{11} + Y_{12})\dot{U}_1 + (-Y_{12})(\dot{U}_1 - \dot{U}_2)$$

$$\Rightarrow \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 + (-Y_{12})(\dot{U}_1 - \dot{U}_2) - (-Y_{12})(\dot{U}_1 - \dot{U}_2)$$

$$= (Y_{21} - Y_{12})\dot{U}_1 + (Y_{22} + Y_{12})\dot{U}_2 + (-Y_{12})(\dot{U}_1 - \dot{U}_2)$$





通过已知参数，确定二端口网络等效电路模型

例 已知二端口参数矩阵为$Y=\begin{bmatrix}5 & -2\\ 0 & 3\end{bmatrix}$S，$Z=\begin{bmatrix}\frac{60}{9} & \frac{40}{9}\\ \frac{40}{9} & \frac{100}{9}\end{bmatrix}\Omega$

求它们的π形等效电路。

解 (1) $\because Y_{21}-Y_{12}\neq 0$

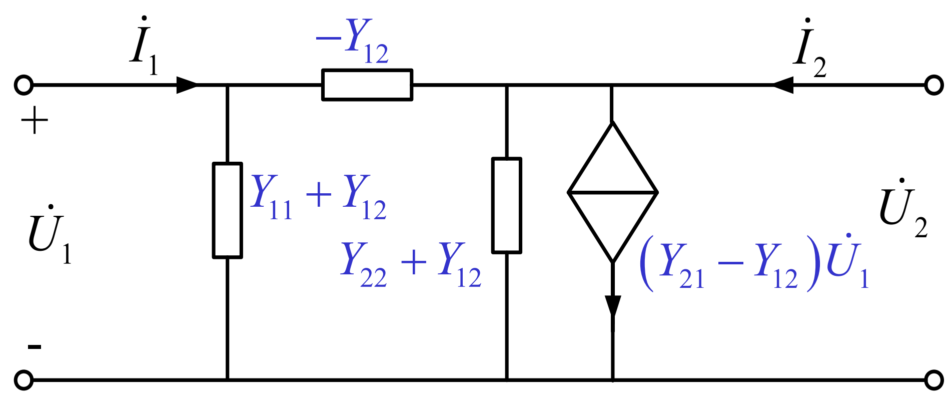
$\therefore$ 非互易二端口网络，含受控源

$$Y_{11}+Y_{12}=3\text{ S}$$

$$-Y_{12}=2\text{ S}$$

$$Y_{22}+Y_{12}=1\text{ S}$$

$$(Y_{21}-Y_{12})\dot{U}_1=2\dot{U}_1$$



例 已知二端口参数矩阵为 $Y=\begin{bmatrix}5 & -2\\ 0 & 3\end{bmatrix}$ S，$Z=\begin{bmatrix}\frac{60}{9} & \frac{40}{9}\\ \frac{40}{9} & \frac{100}{9}\end{bmatrix}\Omega$

求它们的π形等效电路。

解 （2）Z参数转换为Y参数矩阵

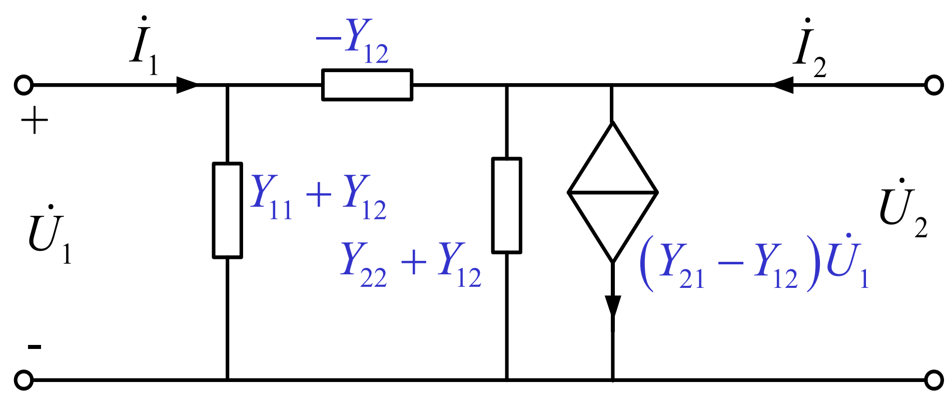
$$YZ=1 \qquad Y=Z^{-1}=\begin{bmatrix}0.2045 & -0.0818\\ -0.0818 & 0.1227\end{bmatrix}\text{S}$$

$$Y_{11}+Y_{12}=0.1227\ \text{S}$$

$$-Y_{12}=0.0818\ \text{S}$$

$$Y_{22}+Y_{12}=0.0409\ \text{S}$$

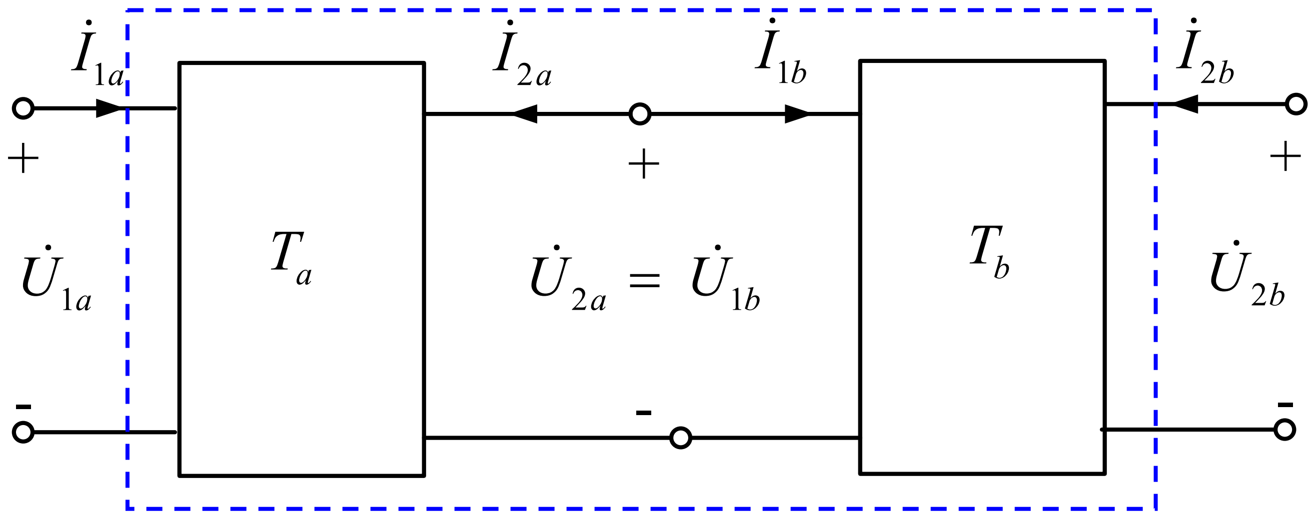
$$Y_{21}-Y_{12}=0$$



二端口网络等效电路模型不唯一，也可等效为T形电路模型

16.5 二端口网络的相互连接

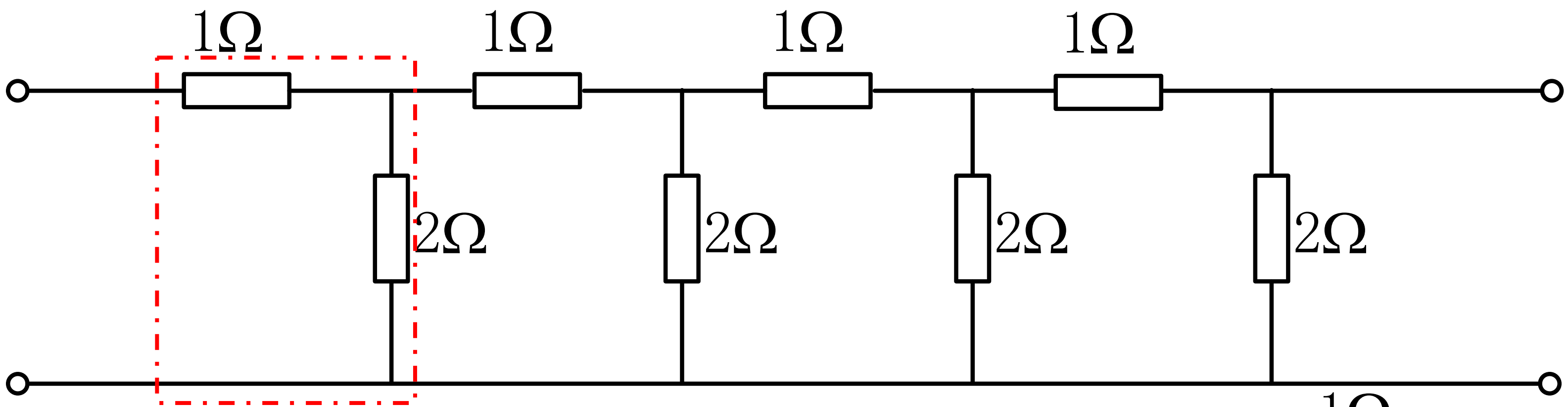
1. 级联 广泛用于电子信号滤波和传输中

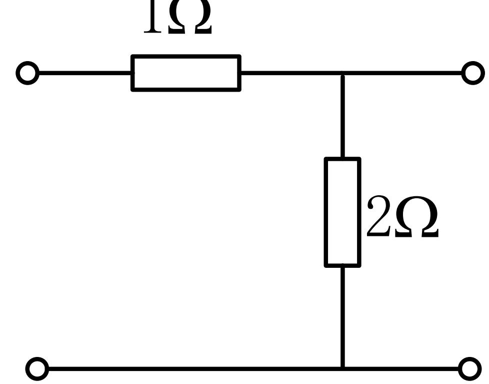


$$\begin{bmatrix} \dot{U}_{1a} \\ \dot{I}_{1a} \end{bmatrix} = \boldsymbol{T}_a \begin{bmatrix} \dot{U}_{2a} \\ -\dot{I}_{2a} \end{bmatrix} = \boldsymbol{T}_a \begin{bmatrix} \dot{U}_{1b} \\ \dot{I}_{1b} \end{bmatrix} = \boldsymbol{T}_a \times \boldsymbol{T}_b \begin{bmatrix} \dot{U}_{2b} \\ -\dot{I}_{2b} \end{bmatrix}$$

$$\boldsymbol{T} = \boldsymbol{T}_a \times \boldsymbol{T}_b$$

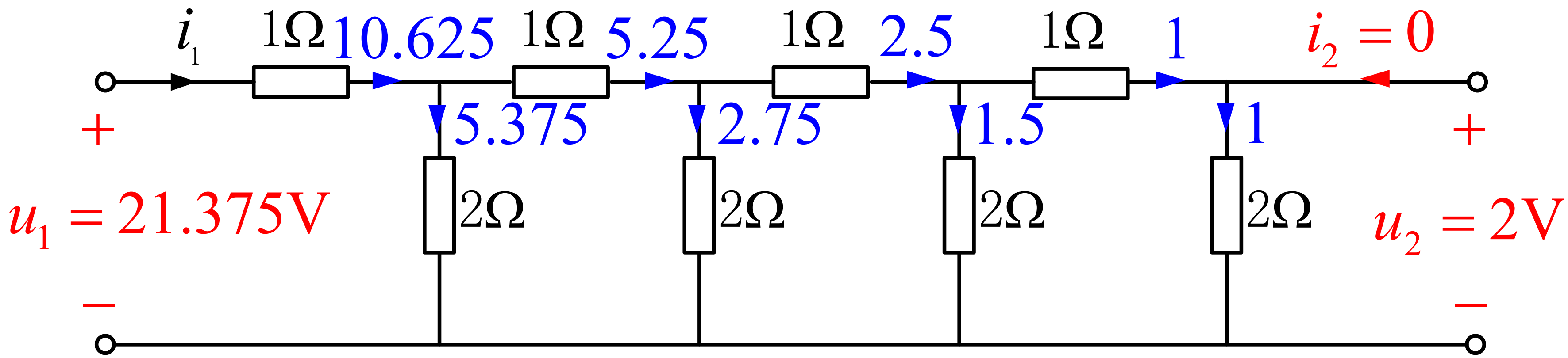
例 求T参数矩阵。



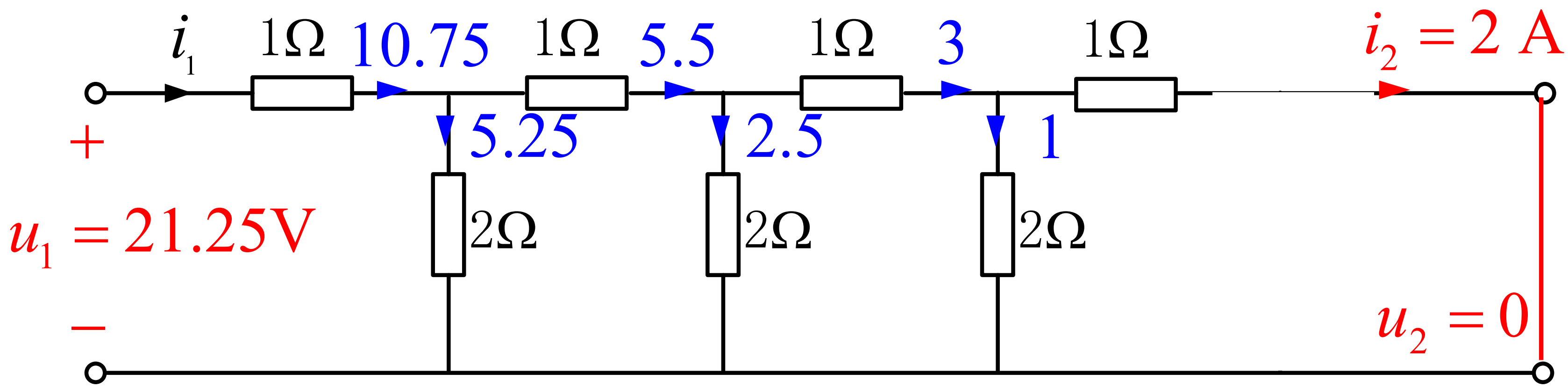


解 $T_1 = \begin{bmatrix} 1.5 & 1 \\ 0.5 & 1 \end{bmatrix}$ $\begin{cases} \dot{U}_1 = A\dot{U}_2 + B\left(-\dot{I}_2\right) \\ \dot{I}_1 = C\dot{U}_2 + D\left(-\dot{I}_2\right) \end{cases}$

$$T = T_1^4 = (T_1^2)^2 = \begin{bmatrix} 10.6875 & 10.625 \\ 5.3125 & 5.375 \end{bmatrix}$$



$$A=\left.\frac{u_1}{u_2}\right|_{i_2=0}=\frac{21.375}{2}=10.6875 \qquad C=\left.\frac{i_1}{u_2}\right|_{i_2=0}=\frac{10.625}{2}=5.3125\ \text{S}$$



$$B=\left.\frac{u_1}{i_2}\right|_{u_2=0}=\frac{21.25}{2}=10.625\ \Omega \qquad D=\left.\frac{i_1}{i_2}\right|_{u_2=0}=\frac{10.75}{2}=5.375$$

作业

- 16.2节：16-2
- 16.3节：16-11，16-16
- 16.4节：16-26
- 16.5节：16-30
- 综合：16-38