

## Problem 1

Consider an aircraft moving in a plane with constant speed (the magnitude of the velocity vector) and turning with a constant angular rate. Such a model is often used by air traffic control tracking algorithms to describe aircraft executing coordinated turns. Given 2D inertial position variables  $\xi(t)$ , east position, and  $\eta(t)$ , north position. The equations of motion are

$$\begin{aligned}\ddot{\xi} &= -\Omega\dot{\eta} \\ \ddot{\eta} &= \Omega\dot{\xi}\end{aligned}$$

where  $\Omega$  is the constant angular rate, such that  $\Omega > 0$  implies a counterclockwise turn. Using the state representation  $x(t) = [\xi, \dot{\xi}, \eta, \dot{\eta}]^T$ , it can be shown that

$$e^{A\Delta t} = \begin{bmatrix} 1 & \frac{\sin(\Omega\Delta t)}{\Omega} & 0 & -\frac{1-\cos(\Omega\Delta t)}{\Omega} \\ 0 & \cos(\Omega\Delta t) & 0 & -\sin(\Omega\Delta t) \\ 0 & \frac{1-\cos(\Omega\Delta t)}{\Omega} & 1 & \frac{\sin(\Omega\Delta t)}{\Omega} \\ 0 & \sin(\Omega\Delta t) & 0 & \cos(\Omega\Delta t) \end{bmatrix}$$

Where  $A$  is the CT LTI state matrix for the system.