ASEN 5044 Statistical Estimation for Dynamical Systems
Fall 2018
Homework 1
Out: Thursday 09/06/2018 (posted on Canvas)
Due: Thursday 09/13/2018 (via Canvas - no credit for illegible submissions)

Show all your work and explain your reasoning.

1. Compute determinants for the following matrices **by hand** (i.e. do not use Matlab, Mathematica, etc.), and state whether or not each is invertible.

(a) $A = \begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 8 & 9 & 7 \end{bmatrix}$

(b) $A = \begin{bmatrix} 11 & 26 & 0 & 0 \\ 64 & 57 & 0 & 10 \\ 83 & 91 & 1 & 71 \\ 54 & 23 & 0 & 71 \end{bmatrix}$

(c) $A = \begin{bmatrix} -34 & 16 & 17 & \pi \\ 22 & 10 & -11 & e \\ -64 & 31 & 32 & \sqrt{2} \\ 76 & 37 & -38 & 42 \end{bmatrix}$ (hint: this is the easiest one)

(d) $A = \begin{bmatrix} 1 & 1 & 2 & 3 & 5 \\ 0 & 8 & 13 & 21 & 34 \\ 0 & 0 & 55 & 89 & 144 \\ 0 & 0 & 0 & 233 & 377 \\ 0 & 0 & 0 & 0 & 610 \end{bmatrix}$

2. Prove each of the following statements (stick to solid mathematical facts and reasoning; eschew numerical or hand-wavy arguments):

(a) If a and b are non-zero $n \times 1$ vectors, then matrix ab^T has rank $= 1$.

(b) $\text{tr}(AB) = \text{tr}(BA)$ if A is an $m \times n$ matrix and B is $n \times m$ (hint: consider expressing A and B in terms of stacked row or column vectors)

(c) If A is invertible, then $|A^{-1}| = \frac{1}{|A|}$ (hint: note that $|AB| = |BA| = |B||A|$ if A and B are any two compatible square matrices).

3. Consider the equations of motion for the coupled 2 mass/3 spring system, like the one discussed in lecture,

$$m_1 \ddot{q}_1 = -k_1 q_1 - k_2(q_1 - q_2) - u_1$$
$$m_2 \ddot{q}_2 = -k_2(q_2 - q_1) - k_3 q_2 + u_1 + u_2,$$

where q_1 is the displacement of mass m_1, q_2 is the displacement of mass m_2, the k's are spring constants, and the u's are external forces. Find a set of A,B,C,D matrices for the state vector definition,

$$x = [q_1 - q_2, \dot{q}_1 - \dot{q}_2, q_1 + q_2, \dot{q}_1 + \dot{q}_2]^T,$$

and for observations $y = [q_1, q_2]^T$ and inputs $u = [u_1, u_2]^T$. For this problem, you may use Matlab or other software to assist with any required intermediate *numerical* matrix calculations (i.e. no symbolic tools allowed).

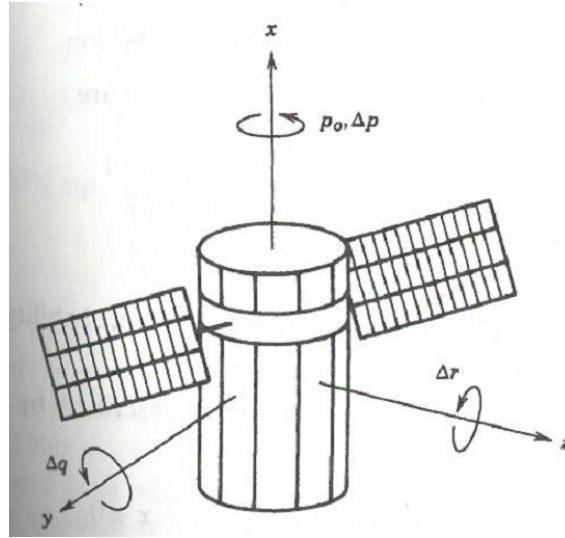**4.** The linearized equations of motion for an orbiting satellite spinning with nominal angular rate p_o about the x axis are

Figure 1: Spinning satellite variables.

$$\Delta \dot{p} = \frac{M_x}{I_x}$$
$$\Delta \dot{q} = \frac{p_o(I_x - I_z)\Delta r + M_y}{I_y}$$
$$\Delta \dot{r} = \frac{p_o(I_y - I_x)\Delta q + M_z}{I_z}$$

where I_x, I_y, and I_z are the moments of inertia about the roll, pitch, and yaw axes; M_x, M_y and M_z are the corresponding input (control and disturbance) torques; and Δp, Δq and Δr are perturbations in rolling, pitching, and yawing rates from the linearization point.

(a) Using the state vector $x = [\Delta p, \Delta q, \Delta r]^T$, input vector $u = [M_x, M_y, M_z]^T$, and output vector $y = x$, put this system into state space form (i.e. find the A, B, C, D parameter matrices).

(b) Use Matlab's `expm` function to compute the state transition matrix for this system, assuming that $I_y = 750$ kg m^3, $I_z = 1000$ kg m^3, $I_x = 500$ kg m^3, $p_o = 20$ rad/s, and $\Delta t = 0.1$ s.

(c) Use the state transition matrix to compute and plot the state time history for 5 s, assuming zero inputs and assuming initial states $\Delta q(0) = 0.1$ rad/s, and $\Delta p(0) = \Delta r(0) = 0$. Be sure to label each of the state plots carefully. What can you say about the behavior of this system in terms of stability, i.e. does it want to stay near the conditions used to linearize the equations of motion?

5. Do the following end of chapter problems from the Simon textbook:

(a) Problem 1.5

(b) Problem 1.8

Advanced Questions *PhD students in the class MUST answer ALL questions below in addition to regular homework questions above* – *non-PhD students are welcome to try any of these for extra credit (only given if all regular problems turned in on time as well). In either case, Submit your responses for these questions with rest of your homework, but make sure these are clearly labeled and start on separate pages* – *indicate on the top of the front page of your assignment if you answered these questions (as a PhD student, or for extra credit) so they can be spotted, graded and recorded more easily.*

AQ1. Explain why matrix element (4,4) in problem 1c belongs in the last column (hint: Google it, if you must...).

AQ2. Using the infinite series definition of the matrix exponential, find the analytical expression for the state transition matrix (STM) of the spinning satellite system in problem 4, assuming a generic value for Δt and using the values for I_x, I_y, I_z, and p_o given in that problem. Your answer should be a function of Δt. Note: do *not* use a symbolic calculation tool (e.g. Matlab's symbolic toolbox, Maple, Mathematica, etc.) – you must derive the STM entirely by hand using the series definition of the matrix exponential, and show your work. *Hint:* it may help to simplify certain terms by defining intermediate variables and by recalling some basic Taylor series expansions for certain trigonometric functions. It's also wise to verify that your answer agrees with your results from #4.