Statistical Estimation	**Midterm 1**
ASEN 5044 Fall 2018	**Due Date: Oct 11, 2018**
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Problem 1

Inverted pendulum with equations of motion:

$$(M+m)\ddot{z} - ml\ddot{\theta}\cos\theta + ml\dot{\theta}^2\sin\theta = P$$
$$l\ddot{\theta} - g\sin\theta = \ddot{z}\cos\theta$$

Part (a)

The system's state equations can be expressed as follows:

$$\dot{x} = \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{P - g\sin x_3\cos x_3 - mlx_4^2\sin x_3}{M + m\sin^2 x_3}\cos x_3 \\ x_4 \\ \frac{P\cos x_3 + (M+m)g\sin x_3 + mlx_4^2\sin x_3\cos x_3}{Ml + ml\sin^2 x_3}\end{bmatrix}$$

To demonstrate the system is in equilibrium at $\dot{z} = 0$, $\theta = 0$, $\dot{\theta} = 0$, and $P(t) = 0$ we note first that at the given conditions the equations of motion become

$$(M+m)\ddot{z} - ml\ddot{\theta} = 0$$
$$l\ddot{\theta} = \ddot{z}$$

If we plug the second equation back into the first we get

$$(M+m)\ddot{z} - m\ddot{z} = M\ddot{z} = 0$$

Because we know M is not equal to zero, this means $\ddot{z}$ must be equal to zero. Additionally, because $\ddot{z} = l\ddot{\theta}$ and $l \neq 0$ we can also conclude that $\ddot{\theta} = 0$. This means $\dot{x} = 0$ under the given conditions and the system is therefore in equilibrium.

Part (b)

Part (c)

Part (d)

Part (e)

Part (f)

Part (g)

Problem 2

Part (a)

Part (b)

Part (c)

Part (d)

Problem 3

Part (a)

Part (b)

Part (c)

Problem 4

Part (a)

Part (b)

Problem AQ1

Part (a)

Part (b)

Part (c)

Part (d)