ASEN 5044 Statistical Estimation for Dynamical Systems
Fall 2018
Homework 2

Out: Thursday 09/13/2018 (posted on Canvas)

Due: Thursday 09/20/2018 (Canvas - **no credit for illegible submissions**)

Show all your work and explain your reasoning.

1. Consider the equations of motion for a unit mass subjected to an inverse square law force field, e.g. a satellite orbiting a planet,

$$\ddot{r} = r\dot{\theta}^2 - \frac{k}{r^2} + u_1(t) \tag{1}$$

$$\ddot{\theta} = -\frac{2\dot{\theta}\dot{r}}{r} + \frac{1}{r}u_2(t) \tag{2}$$

where r represents the radius from the center of the force field, θ gives the angle with respect to a reference direction in the orbital plane, k is a constant, and u_1 and u_2 represent radial and tangential thrusts, respectively. It is easily shown that for the initial conditions $r(0) = r_0$, $\theta(0) = 0$, $\dot{r}(0) = 0$, and $\dot{\theta}(0) = \omega_0$ with nominal thrusts $u_1(t) = 0$ and $u_2(t) = 0$ for all $t \geq 0$, the equations of motion have as a solution the circular orbit given by

$$r(t) = r_0 = \text{constant} \tag{3}$$

$$\dot{\theta}(t) = \omega_0 = \text{constant} = \sqrt{\frac{k}{r_0^3}}, \tag{4}$$

$$\theta(t) = \omega_0 t + \text{constant} \tag{5}$$

(a) Pick a state vector for this system, and express the original nonlinear ODEs in 'standard' nonlinear state space form.

(b) Linearize this system's nominal equations of motion about the nominal solution $r(t) = r_0$, $\dot{r}(0) = 0$, $\theta(t) = \omega_0 t + \text{constant}$ and $\dot{\theta}(t) = \omega_0$ with $u_1(t) = 0$ and $u_2(t) = 0$. Find (A, B, C, D) matrices for output $y(t) = [r(t), \theta(t)]^T$ for the linearized system of equations about the nominal solution.

(c) Convert the continuous time (A, B, C, D) matrices you found from part (b) into discrete time (F, G, H, M) matrices, using a discretization step size of $\Delta t = 10s$ and setting $k = 398600\ km^3/s^2$ and $r_0 = 6678\ km$.

(d) Interpret the results for the STM in part (c), i.e. what is the physical meaning of each column vector that makes up F?

2. Simon, Problem 1.13 (for part (b): use only the first and last expressions in eq. 1.71, i.e. you can skip the Laplace transform technique). Note: there is an error in the problem statement: the equation for p should be $p = p_0 + \dot{p}_0 t + \frac{1}{2}\ddot{p}_0 t^2$. The problem statement for part (a) should also be clarified as asking for the state space equation that describes the system whose solution is given by this formula for p.

3. Simon, Problem 1.14.

4. Simon, Problem 1.19. For part (d), use Matlab's `ode45` command (or a similar command in whatever language you are using) to simulate the full nonlinear dynamics and the linearized model – see the `doc ode45` file for more details and examples of how to use this function correctly if you are using Matlab (note that you will need to express both the nonlinear and linearized dynamics in 'standard' state space form). Also note: for part d of this problem, the book errata says there should be an absolute value in the input $u(t)$, so that it should be $u(t) = u_0(t) + \Delta u \cdot \mathrm{abs}(\cos(t))$ – you need this to obtain physical results. Also assume that the initial conditions to be used in the simulation should be the nominal initial conditions, with $x_{3,0} = 1000$ kg.

Advanced Questions ***PhD students in the class MUST answer ALL questions below in addition to regular homework questions above*** – *non-PhD students are welcome to try any of these for extra credit (only given if all regular problems turned in on time as well). In either case, Submit your responses for these questions with rest of your homework, but make sure these are clearly labeled and start on separate pages* – ***indicate on the top of the front page of your assignment if you answered these questions (as a PhD student, or for extra credit) so they can be spotted, graded and recorded more easily***.

AQ1. Simon, Problem 1.6.

AQ2. Simon, Problem 1.11. Note: there is a mistake in the problem statement: the expression to be proven should read $|\exp(At)| = \exp(\mathrm{tr}(A)t)$ (also stated in the posted errata for the Simon book).