Statistical Estimation	**Midterm 1**
ASEN 5044 Fall 2018	**Due Date: Oct 11, 2018**
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Problem 1

Inverted pendulum with equations of motion:

$$(M+m)\ddot{z} - ml\ddot{\theta}\cos\theta + ml\dot{\theta}^2\sin\theta = P$$
$$l\ddot{\theta} - g\sin\theta = \ddot{z}\cos\theta$$

Part (a)

The system's state equations can be expressed as follows:

$$\dot{x} = \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{P - g\sin x_3\cos x_3 - mlx_4^2\sin x_3}{M+m\sin^2 x_3}\cos x_3 \\ x_4 \\ \frac{P\cos x_3 + (M+m)g\sin x_3 + mlx_4^2\sin x_3\cos x_3}{Ml+ml\sin^2 x_3}\end{bmatrix}$$

To demonstrate the system is in equilibrium at $\dot{z}=0,\ \theta=0,\ \dot{\theta}=0$, and $P(t)=0$ we note first that at the given conditions the equations of motion become

$$(M+m)\ddot{z} - ml\ddot{\theta} = 0$$
$$l\ddot{\theta} = \ddot{z}$$

If we plug the second equation back into the first we get

$$(M+m)\ddot{z} - m\ddot{z} = M\ddot{z} = 0$$

Because we know M is not equal to zero, this means $\ddot{z}$ must be equal to zero. Additionally, because $\ddot{z} = l\ddot{\theta}$ and $l \neq 0$ we can also conclude that $\ddot{\theta} = 0$. This means $\dot{x} = 0$ under the given conditions and the system is therefore in equilibrium.

Part (b)

Part (c)

Part (d)

Part (e)

Part (f)

Part (g)

Problem 2

Two 6-sided dice rolls with R_1 and R_2 denoting the outcome of the first and second die, respectively.

Part (a)

$P(R_1) = P(R_2) = \frac{1}{6}$ for all R_1 and R_2. Because the outcomes R_1 and R_2 are independent, $P(R_1, R_2) = P(R_1) * P(R_2)$. So

$$P(R_1, R_2) = \frac{1}{36}, \ \forall R_1, R_2$$

Part (b)

The joint probabilities for X and Y are shown in table 2 below

Table 1: Joint Probabilities

X	Y=1	Y=2	Y=3	Y=4	Y=5	Y=6
1	1/36	2/36	2/36	2/36	2/36	2/36
2	0	1/36	2/36	2/36	2/36	2/36
3	0	0	1/36	2/36	2/36	2/36
4	0	0	0	1/36	2/36	2/36
5	0	0	0	0	1/36	2/36
6	0	0	0	0	0	1/36

Part (c)

The marginal probabilities of X obtained from the sum $\sum_y P(X = x, Y = y)$ and the marginal probabilities of Y obtained from the sum $\sum_x P(X = x, Y = y)$ are shown below in table **??**.

Table 2: Marginal Probabilities

	X	Y
1	11/36	1/36
2	9/36	3/36
3	7/36	5/36
4	5/36	7/36
5	3/36	9/36
6	1/36	11/36

Part (d)

X and Y are not independent. Two variables are considered independent if the realization of one variable does not affect the probability of the other. This is not the case for X and Y. By the definitions of X and Y, the value of Y cannot be less than the value of X, since the maximum of R_1 and R_2 cannot be less than the minimum. So the $P(Y = 3|X = 5) = 0$, while $P(Y = 3|X = 1) > 0$.

Problem 3

Part (a)

Part (b)

Part (c)

Problem 4

Part (a)

Part (b)

Problem AQ1

Part (a)

Part (b)

Part (c)

Part (d)