Statistical Estimation	**Homework 5**
ASEN 5044 Fall 2018	**Due Date: October 18, 2018**
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Problem 1

Consider two zero-mean uncorrelated random variables W and V with standard deviations σ_w σ_v, respectively. What is the standard deviation of the random variable $X = W + V$?

Problem 2

Consider two scalar RVs X and Y.

Part a

Prove that if X and Y are independent their correlation coefficient $\rho = 0$.

Part b

Find an example of two RVs that are not independent but have a correlation coefficient of zero.

Part c

Prove that if Y is a linear function of X then $P = \pm 1$.

Problem 3

Consider the following function

$$f_{XY} = \begin{cases} ae^{-2x}e^{-3y} & x > 0,\ y > 0 \\ 0 & \text{otherwise} \end{cases}$$

Part a

Find the value of a so that $f_{XY}(x, y)$ is a valid joint probability density function.

Part b

Calculate $\bar{x}$ and $\bar{y}$.

Part c

Calculate $E(X^2)$, $E(Y^2)$, and $E(XY)$.

Part d

Calculate the autocorrelation matrix of the random vector $[X\ Y]^T$.

Part e

Calculate the variance σ_x^2 and σ_y^2 and the covariance C_{XY}.

Part f

Calculate the autocovariance matrix of the random vector $[X\ Y]^T$.

Part g

Calculate the correlation coefficient between X and Y.

Problem 4

Prove the following two results used in lectire to derive the theoretical expectations for the Gaussian sampling experiment where $x \sim \mathcal{N}(\bar{x}, \sigma_x^2)$, $e \sim \mathcal{N}(0, \sigma_e^2)$, and $y = cx + d$.

Part a

$\mathrm{cov}(X, Y) = E[(x - \bar{x})(y - \bar{y})] = E[XY] - \bar{x}\bar{y}$

Part b

$\mathrm{var}(Y) = E[(y - \bar{y})^2] = c^2\sigma_x^2 + d^2\sigma_e^2$

Problem 5

Consider two continuous random variables x and y, where $y = \ln(x)$ and $x > 0$. Derive analytical closed-form expressions for each of the following:

Part a

$p(y)$ if $p(x) = \mathcal{U}[a, b]$ (i.e. if x has a uniform pdf for $0 < a \leq x \leq b$)

Part b

$p(y)$ if $p(\frac{1}{x}) = \mathcal{U}[c, d]$ (i.e. if $\frac{1}{x}$ has a uniform pdf $0 < c \leq \frac{1}{x} \leq d$)

Part c

$p(x)$ if $p(y) = \mathcal{U}[l, m]$ (i.e. if y has a uniform pdf for $l \leq y \leq m$)

Part d

$p(x)$ if $p(y) = \mathcal{N}(\mu_y, \sigma_y^2)$ (i.e. if y has a Gaussian pdf with mean μ_y and variance σ_y^2)