MBLT

not MBL! MBL Transition.

Wan take 2023/12/4

Berry's conjecture

在与混沌经典系统相对应的通用基础上，量子系统的高能本征态系数是独立的高斯变量，类似于相应随机矩阵集合中特征状态的分布

Deutsch将随机矩阵理论联系起来，表明用随机矩阵扰动哈密顿量会导致热化。

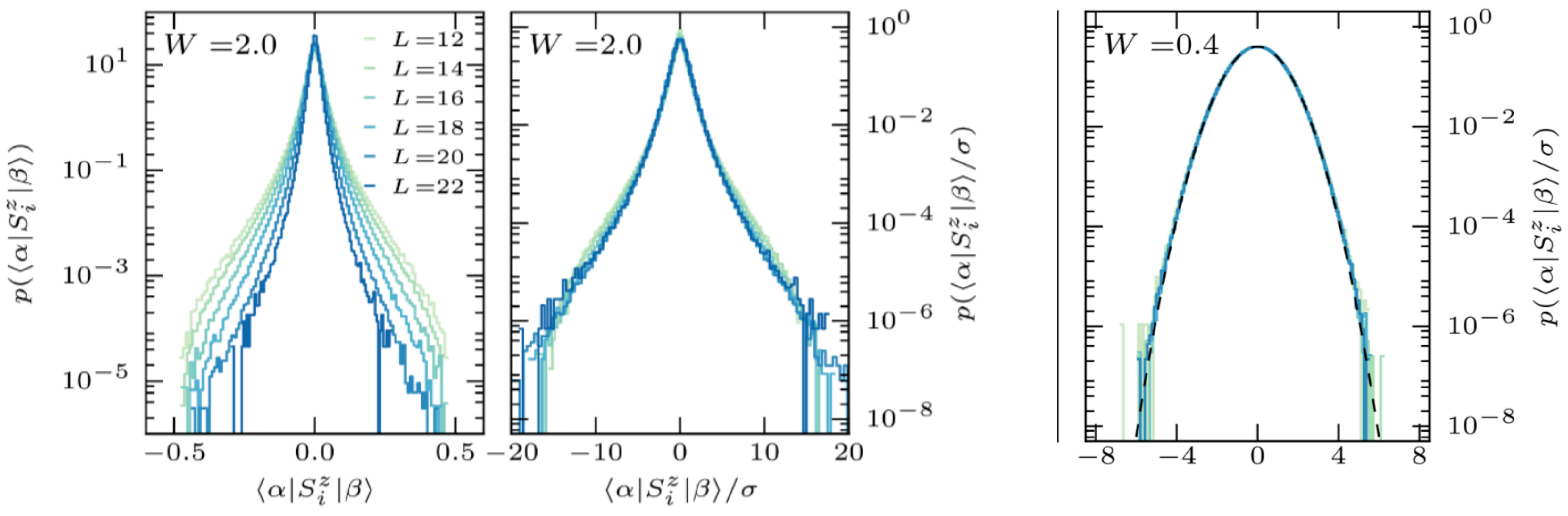
$$\langle\alpha|\hat{O}|\beta\rangle = \overline{O}(E)\delta_{\alpha\beta} + e^{-S(E)/2} f(E,\omega) R_{\alpha\beta},$$

The first, diagonal term in the ETH ansatz is equal to the microcanonical expectation value of the corresponding observable, thus, representing a static thermodynamic quantity.

The exponential decay with system size of the second term, as well as the validity of the Gaussian distribution of the noise $R_{\alpha\beta}$, was subsequently verified for a number of generic quantum systems

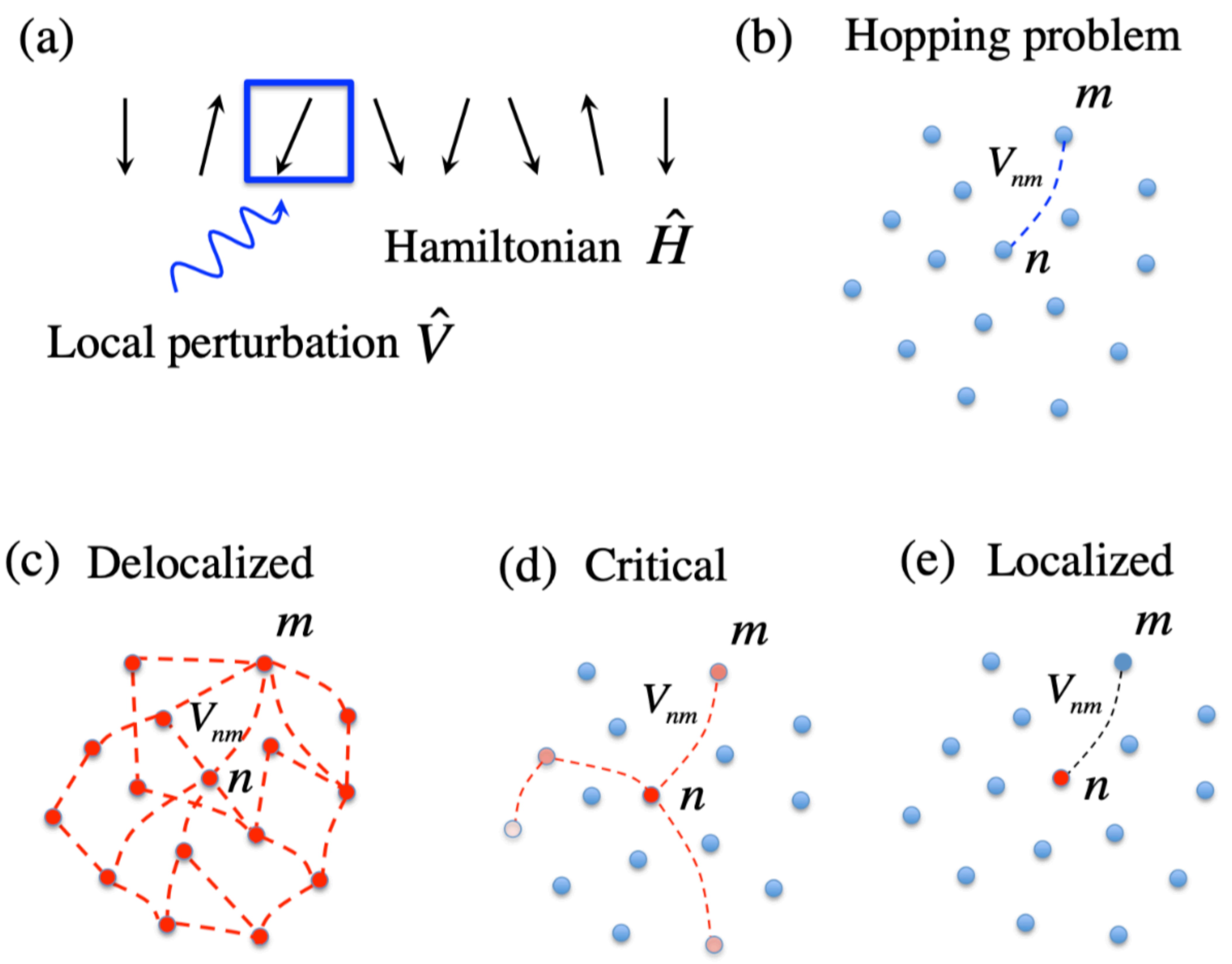
PhysRevLett.117.170404

在弱无序状态下，非对角矩阵元素的分布是高斯分布，此时动力学大致是扩散的，而在较强无序状态下，当系统变得亚扩散时，非对角矩阵元素的分布就变成了强非高斯分布。



$$V_{nm} = e^{-S(E,L)/2} f(E_n, E_m) R_{nm},$$

Phys. Rev. X **5**, *041047*



(a) 系统受到局部物理算子$\hat{V}$的扰动，该算子混合了未扰动哈密顿量$\hat{H}$的本征态，从而产生了(b)
(b) 所示的有效跳变问题：网格的顶点对应于未扰动的本征态，算子$\hat{V}$的矩阵元素决定了hopping幅度。

底图说明了对哈密顿进行局部扰动的可能结果：

(c) 在遍历阶段，所有特征状态都混合在一起
(e) 在 MBL 阶段，扰动只混合了有限数量的状态
(d) 在临界机制中，本征态预计是multifractal

(a) The system is perturbed by a local physical operator $\hat{V}$ that mixes the eigenstates of the unperturbed Hamiltonian $\hat{H}$, leading to an effective hopping problem shown in the panel (b). Vertices of the lattice correspond to the unperturbed eigenstates, and matrix elements of the operator $\hat{V}$ determine the hopping amplitudes. Bottom panels illustrate the possible outcomes of a local perturbation applied to the Hamiltonian: all the eigenstates are mixed in the ergodic phase (panel c), whereas in the MBL phase the perturbation admixes a finite number of states (panel e). In the critical regime, the eigenstates are expected to be multifractal

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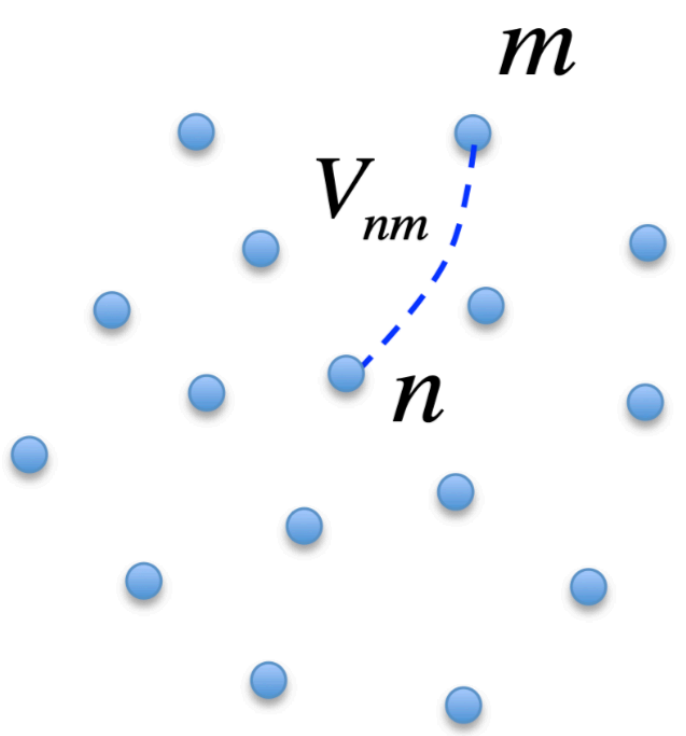
The problem of finding the spectrum and eigenstates of $H + V$ can be viewed as a hopping problem on a lattice

Hopping problem



$H + V$

hopping amplitudes V_{nm} between sites n, m

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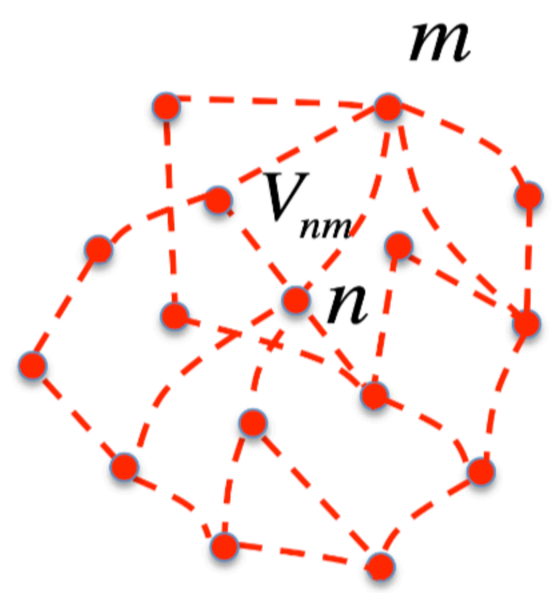
an unperturbed Hamiltonian H and eigenstates |n>,

Sites represent eigenstates of H $\boxed{H|n\rangle \;=\; E_n|n\rangle}$

on-site energies $\boxed{E_n' = E_n + V_{nn}}$

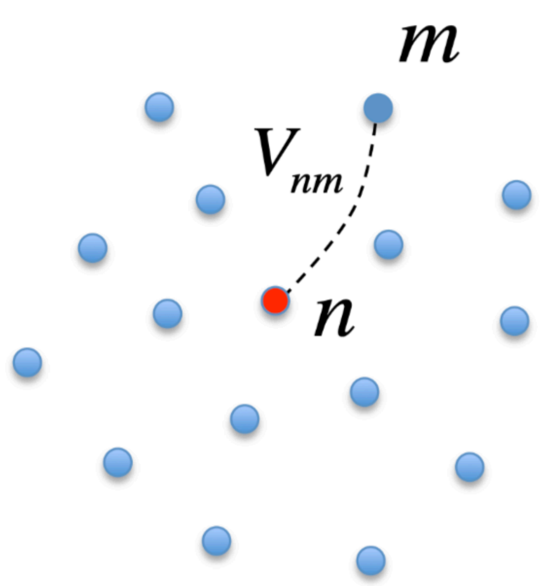
all the information is encoded in eigenenergies E_n'
nand in the offdiagonal matrix elements V_{nm}.

(c) Delocalized



- the perturbation strongly admixes a given eigenstate with only a finite number of other eigenstates

(e) Localized



In the MBL phase, the perturbation V can only affect the degrees of freedom within a localization length ξ away from its support.

在MBL相中，扰动 V 只能影响其支持区域内一个局域长度 ξ 范围内的自由度

To characterize this hybridization, introduce the parameter

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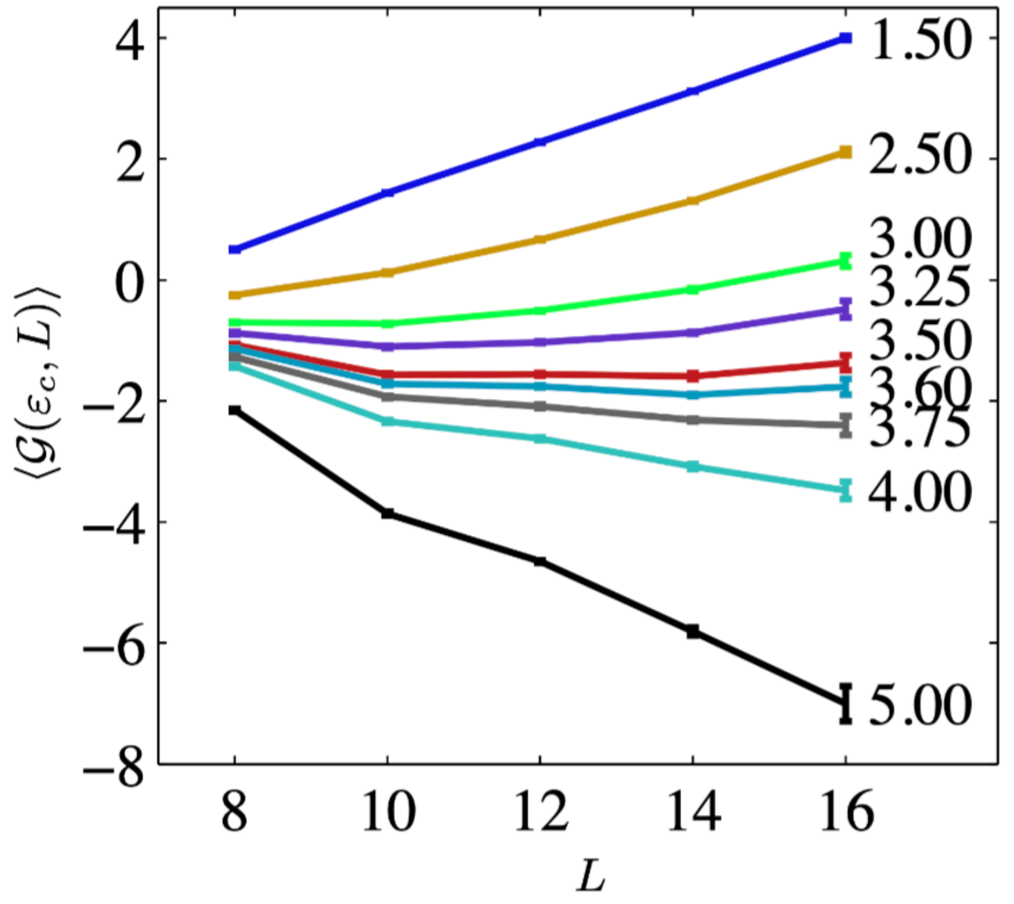
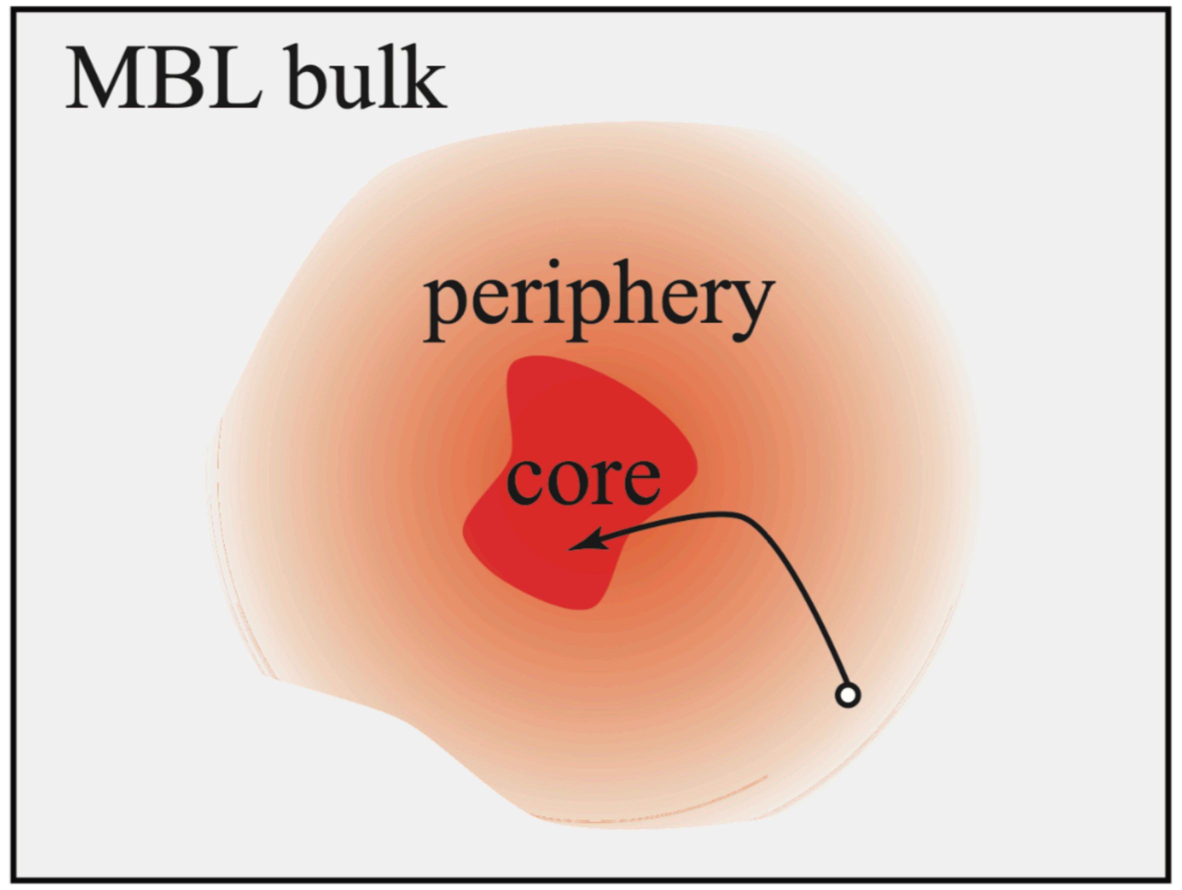


$$\boxed{\mathcal{G}(\varepsilon, L) = \ln \frac{|V_{n,n+1}|}{E'_{n+1} - E'_n},}$$

Critical: = 1

the ergodic phase the ETH implies that a local perturbation mixes the energy levels very strongly, and this requires $\boxed{\mathcal{G}(L) \propto +L.}$

in the MBL phase at L >> ξ the eigenstates close in energy typically have very different spatial structure due to their different values local integrals of motion



PHYSICAL REVIEW B **96**, 104201 (2017)

Thouless energy and multifractality across the many-body localization transition

Maksym Serbyn,[1,2] Z. Papić,[3] and Dmitry A. Abanin[4]

[1]*Department of Physics, University of California, Berkeley, California 94720, USA*

[2]*Institute of Science and Technology (IST) Austria, Am Campus 1, 3400 Klosterneuburg, Austria*

[3]*School of Physics and Astronomy, University of Leeds, Leeds, LS2 9JT, United Kingdom*

[4]*Department of Theoretical Physics, University of Geneva, 24 quai Ernest-Ansermet, 1211 Geneva, Switzerland*

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Thermal and many-body localized phases are separated by a dynamical phase transition of a new kind. We analyze the distribution of off-diagonal matrix elements of local operators across this transition in two different models of disordered spin chains. We show that the behavior of matrix elements can be used to characterize the breakdown of thermalization and to extract the many-body Thouless energy. We find that upon increasing the disorder strength the system enters a critical region around the many-body localization transition. The properties of the system in this region are: (i) the Thouless energy becomes smaller than the level spacing, (ii) the matrix elements show critical dependence on the energy difference, and (iii) the matrix elements, viewed as amplitudes of a fictitious wave function, exhibit strong multifractality. This critical region decreases with the system size, which we interpret as evidence for a diverging correlation length at the many-body localization transition. Our findings show that the correlation length becomes larger than the accessible system sizes in a broad range of disorder strength values and shed light on the critical behavior near the many-body localization transition.

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XXZ：1D random-field spin-1/2 Heisenberg model

$$H = \frac{1}{2}\sum_{i=1}^{L} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_{i+1} + \sum_{i=1}^{L} h_i \sigma_i^z,$$

random field $h_i \in [-W; W]$.

Transverse field Ising model with a disordered longitudinal field

$$H_{\mathrm{Ising}} = \sum_{i=1}^{L} g\Gamma\sigma_i^x + (h + g\sqrt{1-\Gamma^2}G_j)\sigma_i^z + \sum_{i=1}^{L} \sigma_i^z\sigma_{i+1}^z,$$

disorder strength $\Gamma \in [0,1]$,

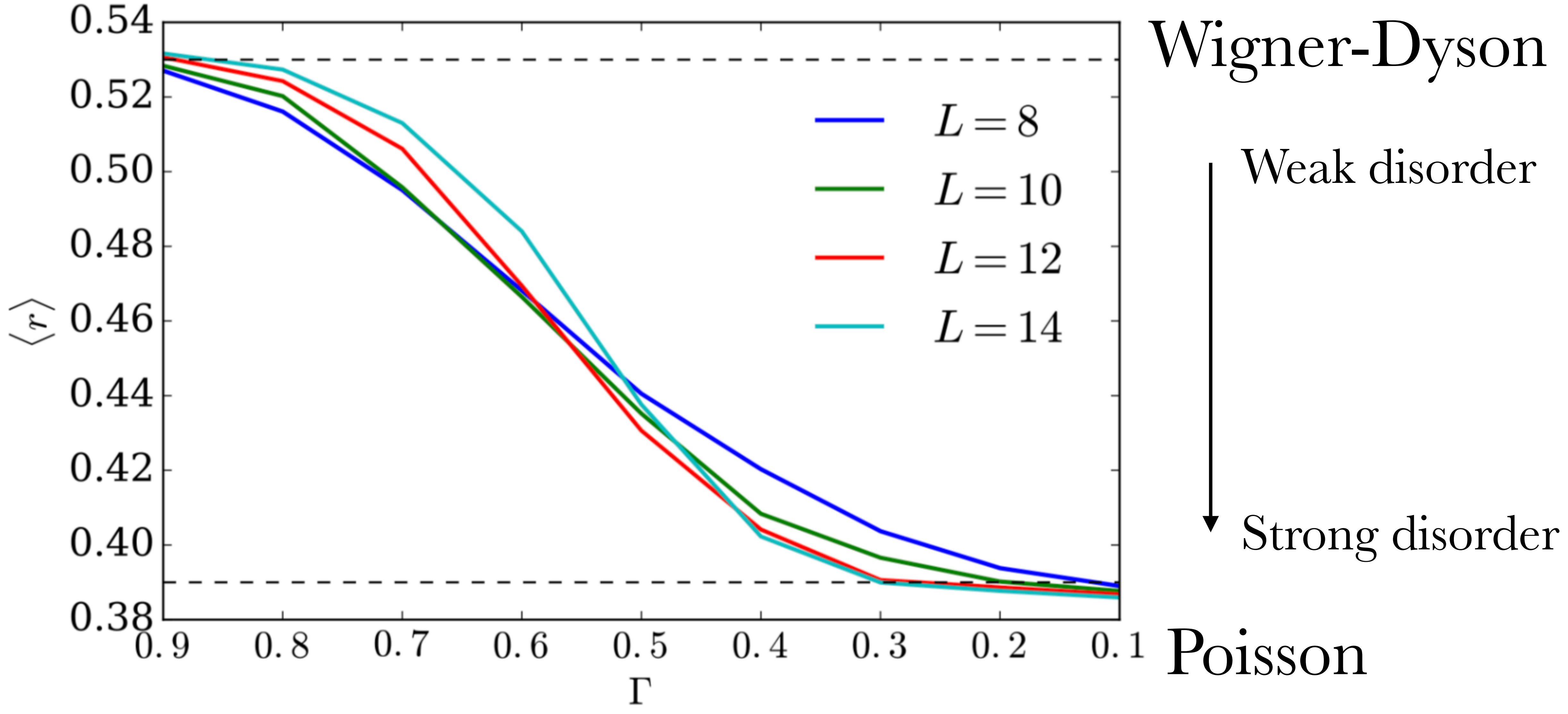
$$\Gamma = 1, H = \sum_{i}^{L} g\sigma_i^x + (h+0)\sigma_i^z + \dots$$

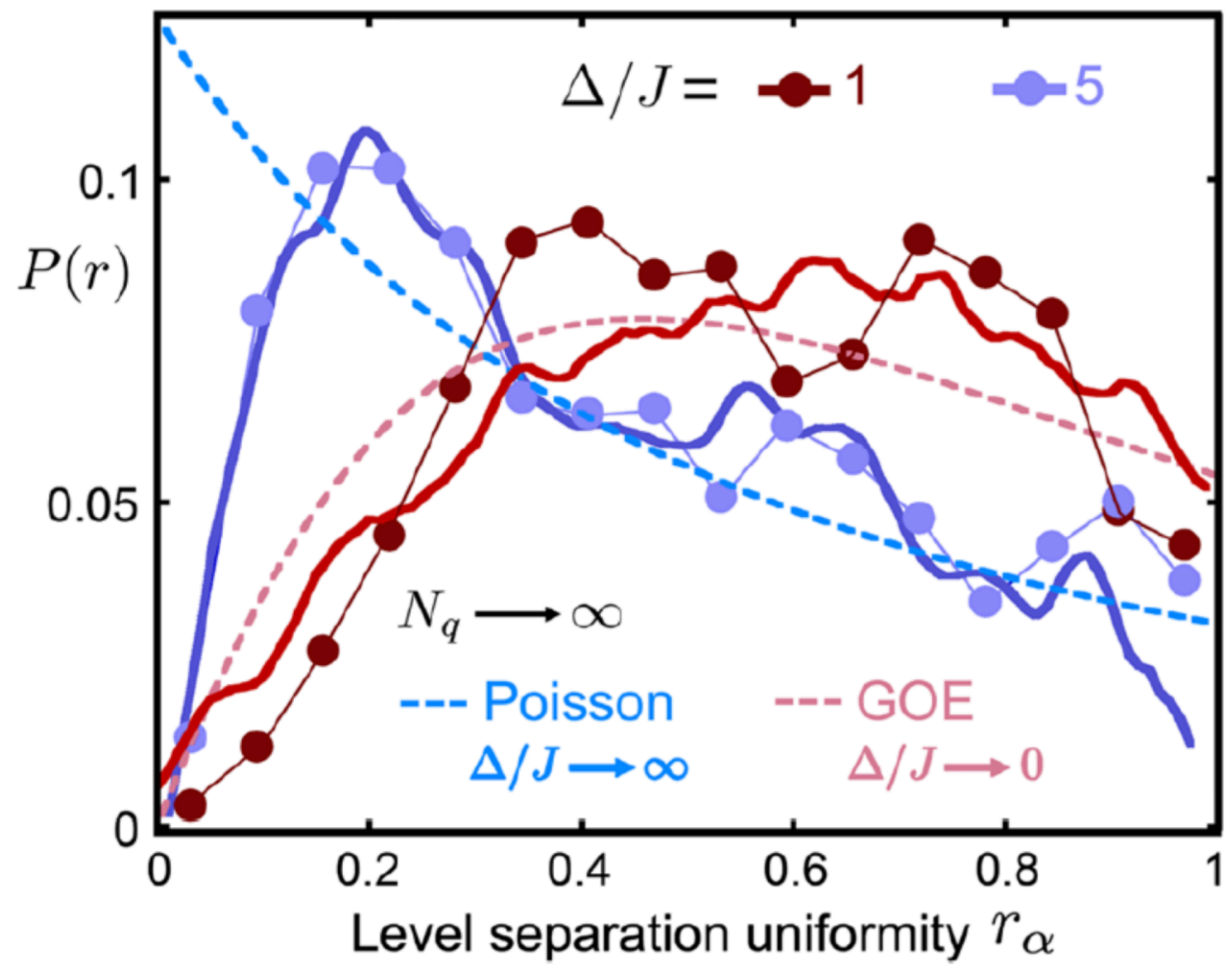
$$\Gamma = 0, H = 0 + (h + gG_j)\sigma_i^z$$

coupling constants $g = 0.9, h = 0.8$.

$\Gamma \uparrow, \rightarrow h\sigma_i^z$，纵场无序减弱，expectly $MBL \rightarrow$;

$\Gamma \downarrow, \rightarrow (h + gG_j)\sigma_i^z$，纵场无序增强，expectly $\rightarrow MBL$



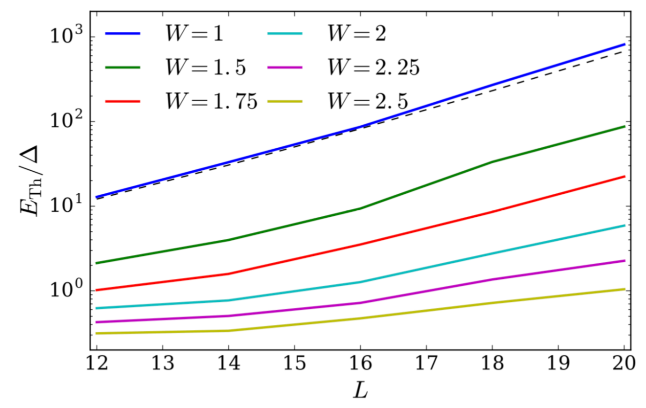


$$\boxed{\langle r \rangle = \left\langle \frac{\min(\delta_n, \delta_{n+1})}{\max(\delta_n, \delta_{n+1})} \right\rangle.}$$

level spacings $\delta_n = E_{n+1} - E_n$, analyze the distribution of off-diagonal matrix elements of local operators $\hat{O} = \sigma_1^z$

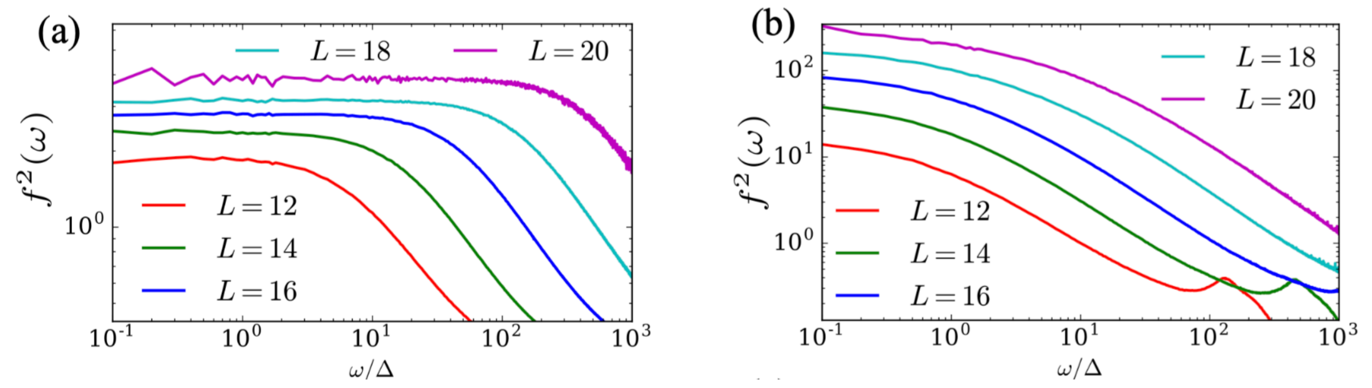
behavior of matrix elements can be used to characterize the breakdown of thermalization and to extract the many-body Thouless energy.

<u>increasing the disorder strength</u>



(i) the Thouless energy becomes smaller than the level spacing

critical region decreases with the system size



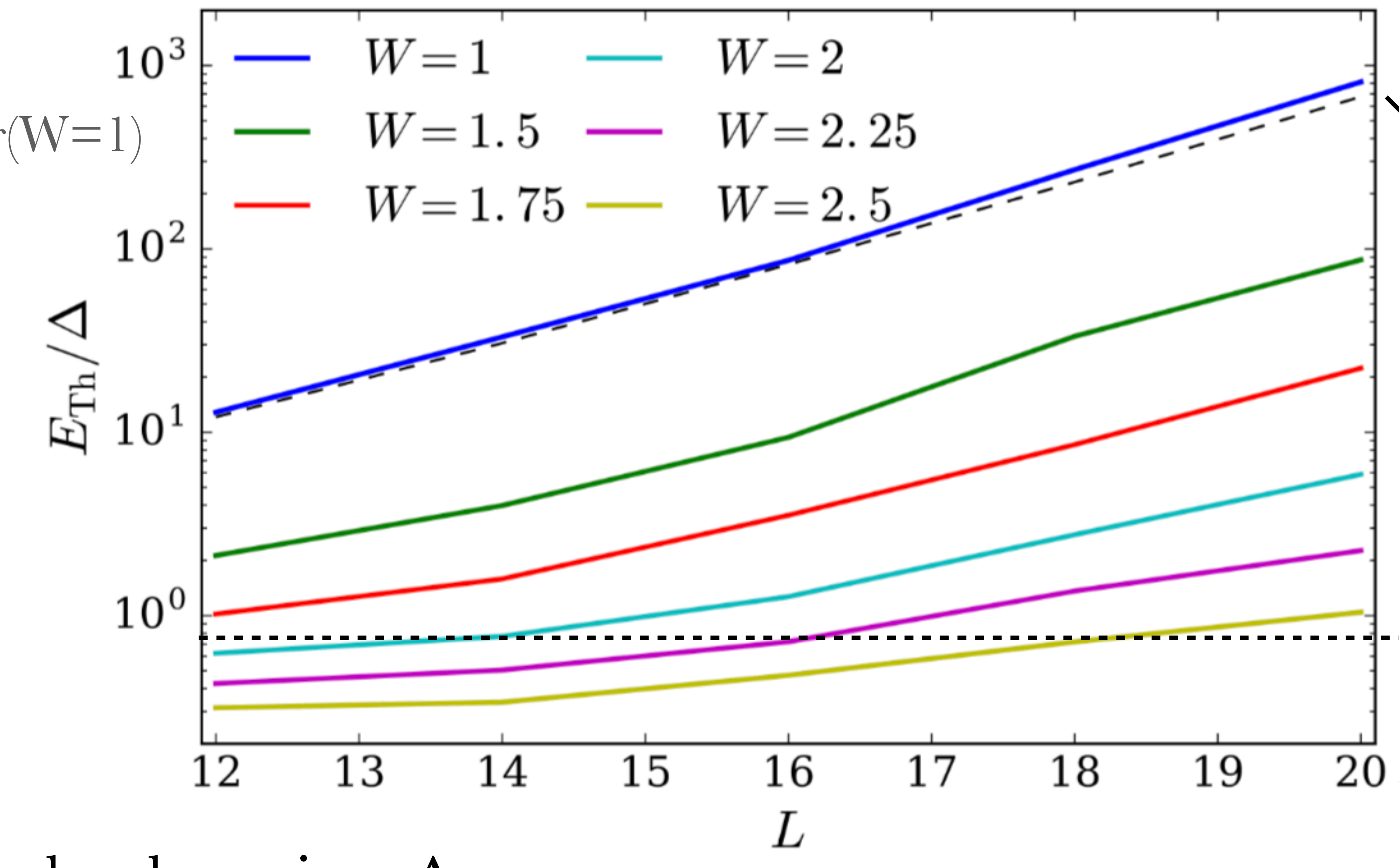
(ii) the matrix elements show critical dependence on the energy difference

(iii) the matrix elements, viewed as amplitudes of a fictitious wave function, exhibit strong multifractality.

the Thouless energy becomes smaller than the level spacing with increasing the disorder strength

Thouless energy for the XXZ model at various disorders

Weak disorder(W=1)



dashed line

the diffusive ($\gamma = 1/2$) scaling of $E \propto L^{-1/\gamma}$

$$\boxed{E_{\rm Th}/\Delta < 1}$$

Strong disorder

Many-body level spacing Δ

$$\Delta \approx \frac{\text{无序强度}W \times \text{系统大小}\sqrt{L}}{\text{希尔伯特空间维度}}$$

随着W的增大，随着L增大的效应越来越弱

Ansatz

$$f^2(\omega) \propto \begin{cases} \text{const}, & \omega \leqslant E_{\rm Th}, \\ 1/\omega^{\phi}, & E_{\rm Th} \ll \omega \ll J_{\rm loc}, \end{cases}$$

diffusion $\gamma = 1/2, f^2(\omega) \propto 1/\omega^{1/2}$

subdiffusion $\gamma < 1/2, f^2(\omega) \propto 1/\omega^{1/\phi}$

$$f^2(\omega) = \frac{f^2(0)}{1 + (\omega/E_{\rm Th})^{\phi}}.$$

$$E_{\rm Th} \propto L^{-1/\gamma}, \quad \phi = 1 - \gamma.$$

In critical region:

$$E_{\rm Th} \propto e^{-\kappa L} \text{ with } \kappa < \ln 2.$$

RG $\boxed{L \sim \ln t \text{ in this region.}}$

Thouless Energy & MBL

Weak disorder

- smallness of E_{Th}
- exponential scaling

$$\boxed{E_{\text{Th}} \propto L^{-1/\gamma}, \quad \phi = 1 - \gamma.}$$

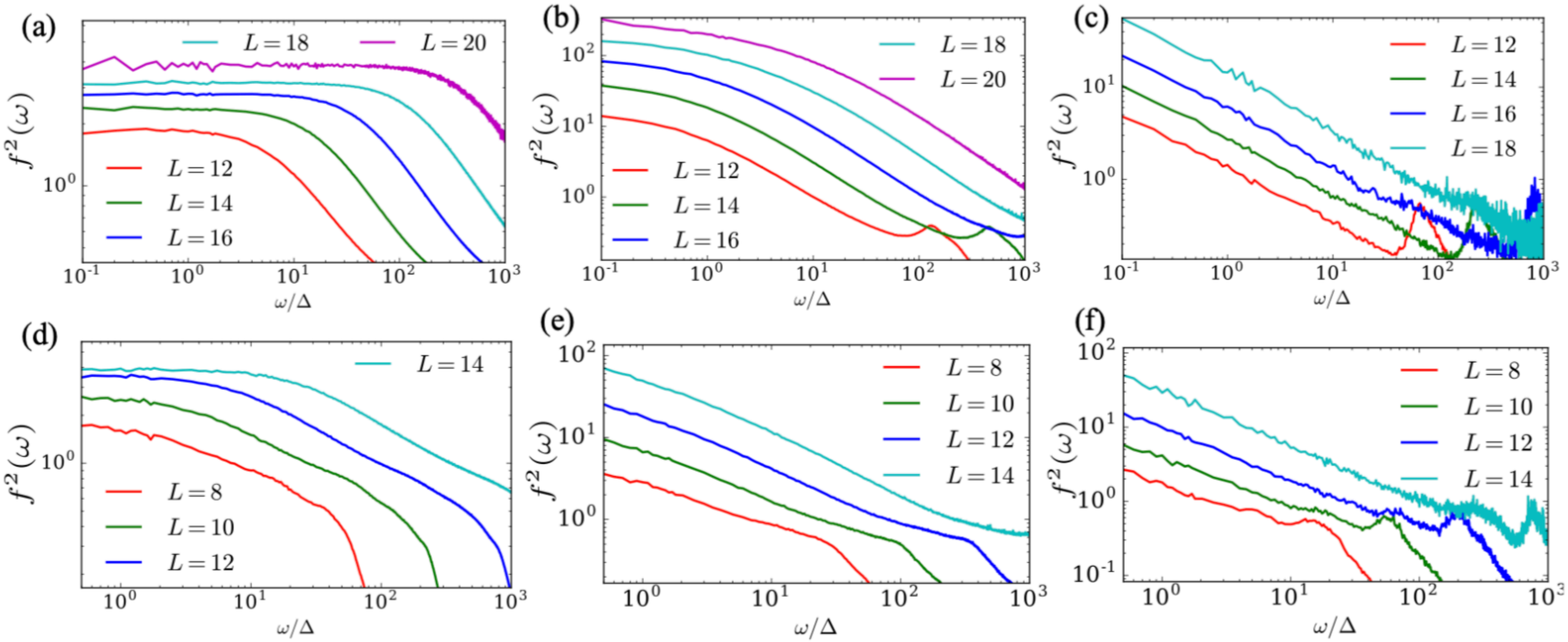
overlap of two different eigenfunctions.

the Thouless energy can be extracted from the overlap of different wave functions at the same point but at different energy

in order to be interpreted as an inverse relaxation timescale of local observables, the Thouless energy must exceed the many-body level spacing.

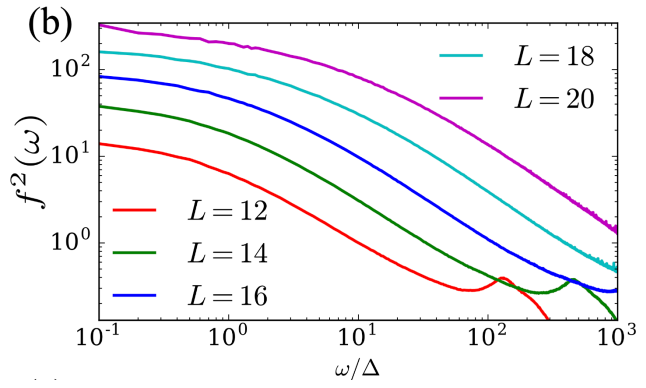
$\boxed{E_{\text{Th}}/\Delta < 1}$ gives a probability that two nearby eigenstates are strongly mixed with each other

W = 1 W = 2 In the MBL phase W = 5

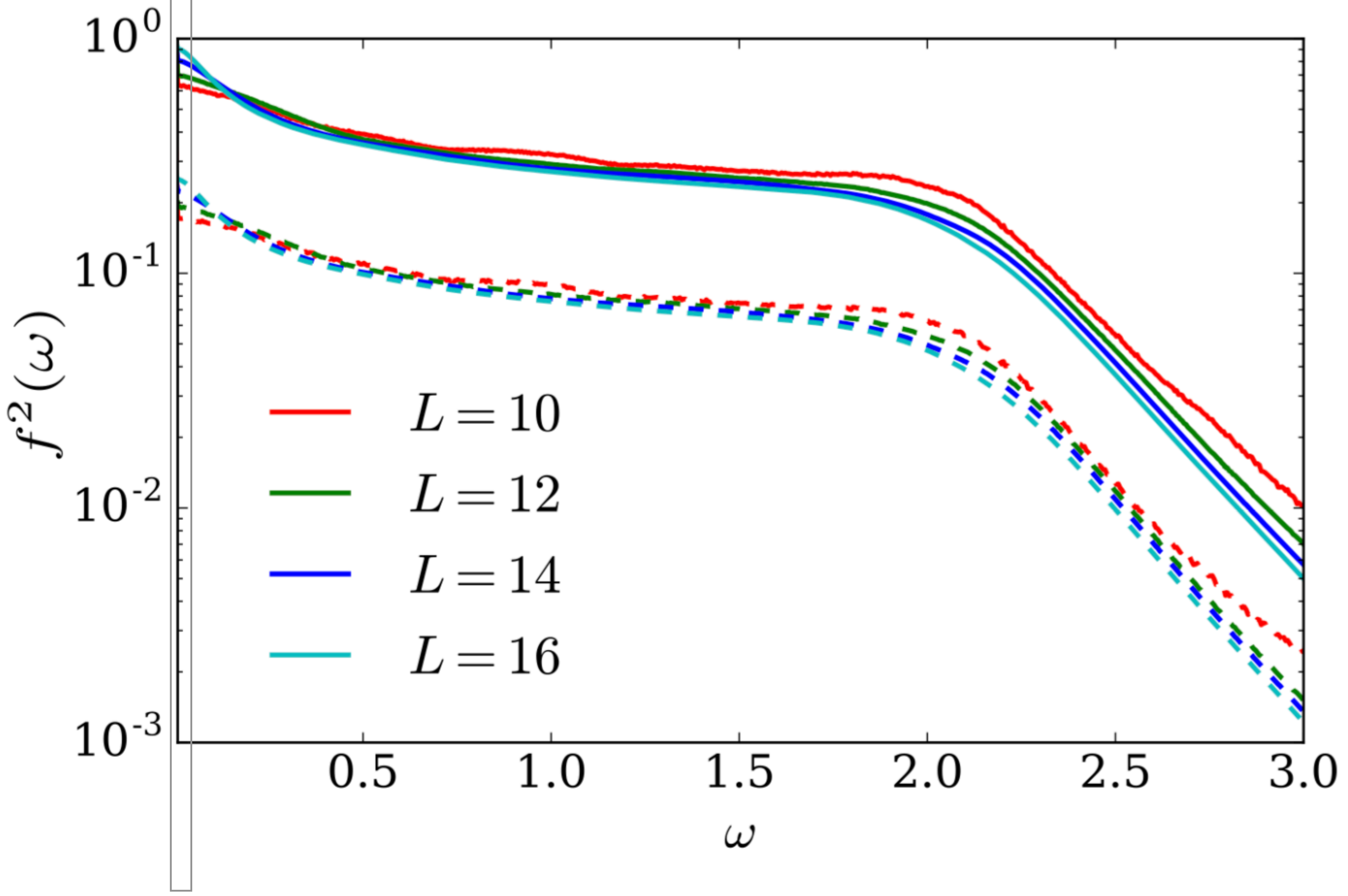


$$f^2(\omega) \propto \begin{cases} \text{const}, & \omega \leqslant E_{\text{Th}}, \\ 1/\omega^{\phi}, & E_{\text{Th}} \ll \omega \ll J_{\text{loc}}, \end{cases}$$

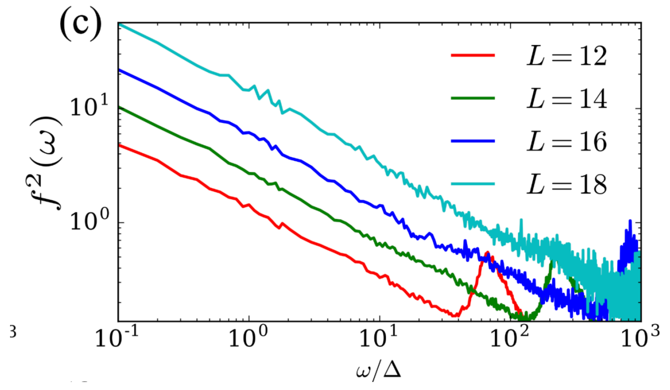
$$E_{\text{Th}} \propto L^{-1/\gamma}, \quad \phi = 1 - \gamma.$$



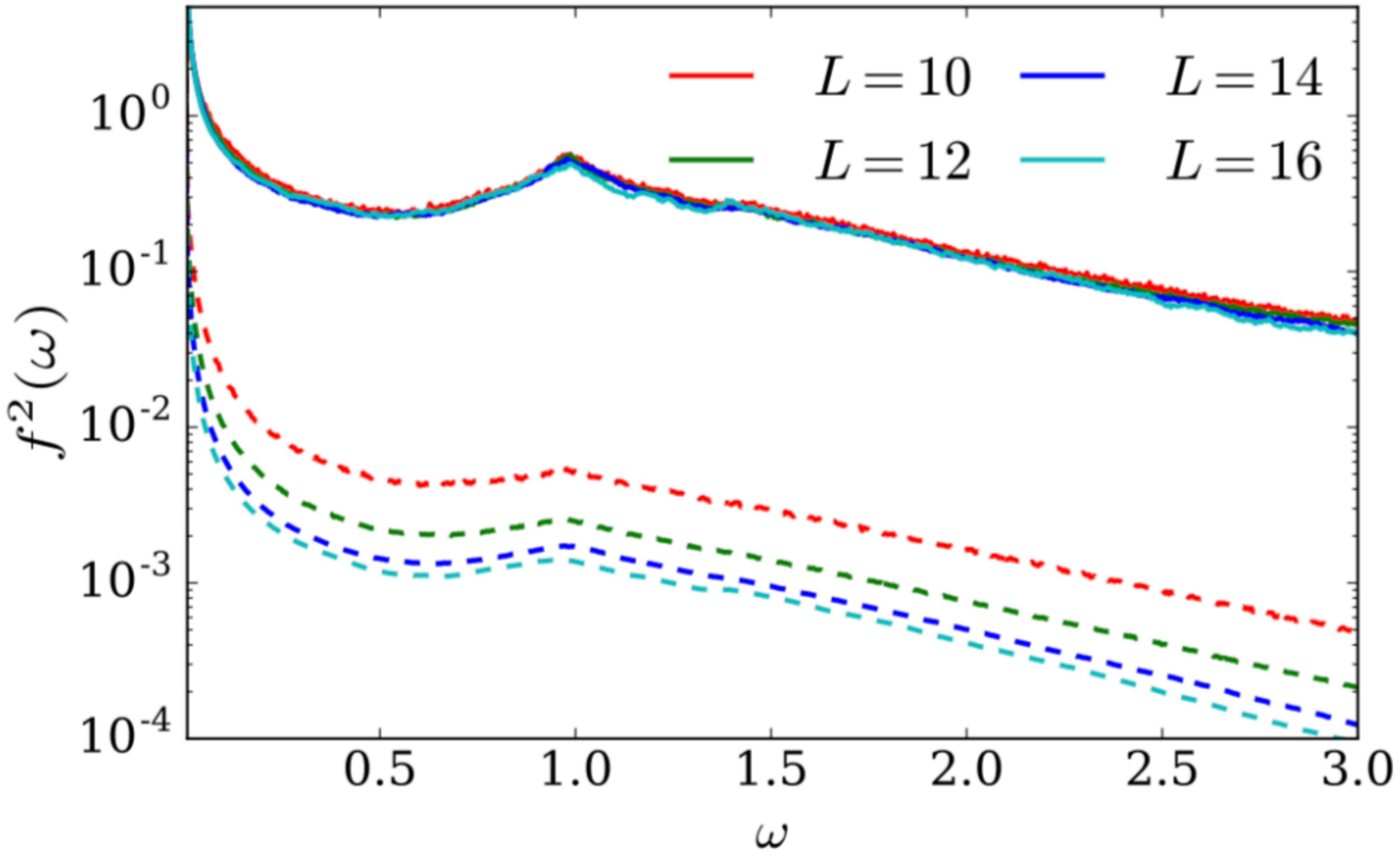
weak disorder W = 0.5



E_{Th}



strong disorder W = 2.75



$$[f^2(\omega)]_{\mathrm{typ}} = \exp(\langle \ln f^2(\omega) \rangle),$$

$L = 10, E_{Th} \propto L^{-1/\gamma} \rightarrow,$

$$E_{\mathrm{Th}} \propto 10^{-1/(1/3)} = 10^{-3},$$
$$\text{when } \gamma = 1/2, E_{\mathrm{Th}} \propto 10^{-2}$$

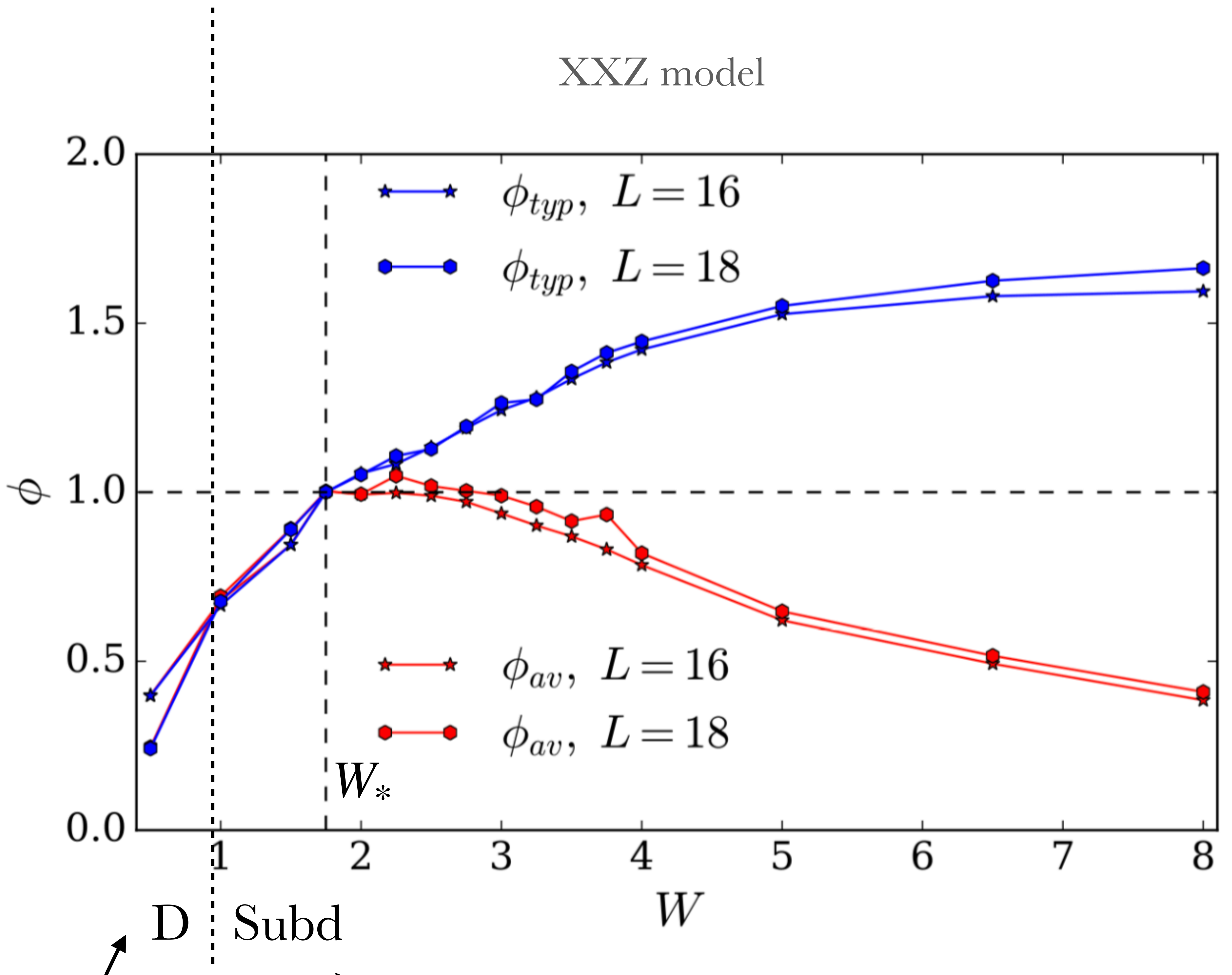
$$\omega <= E_{\mathrm{Th}}, f^2(\omega) \propto \text{const}$$
$$E_{\mathrm{Th}} << \omega << J_{\mathrm{loc}}, f^2(\omega) \propto 1/\omega^{\phi}$$

$\gamma = 1/2$:

$$L = 10, \quad 12 \quad 14 \quad 16$$
$$E_{Th} \propto 0.01, 0.0069, 0.0051, 0.0039$$

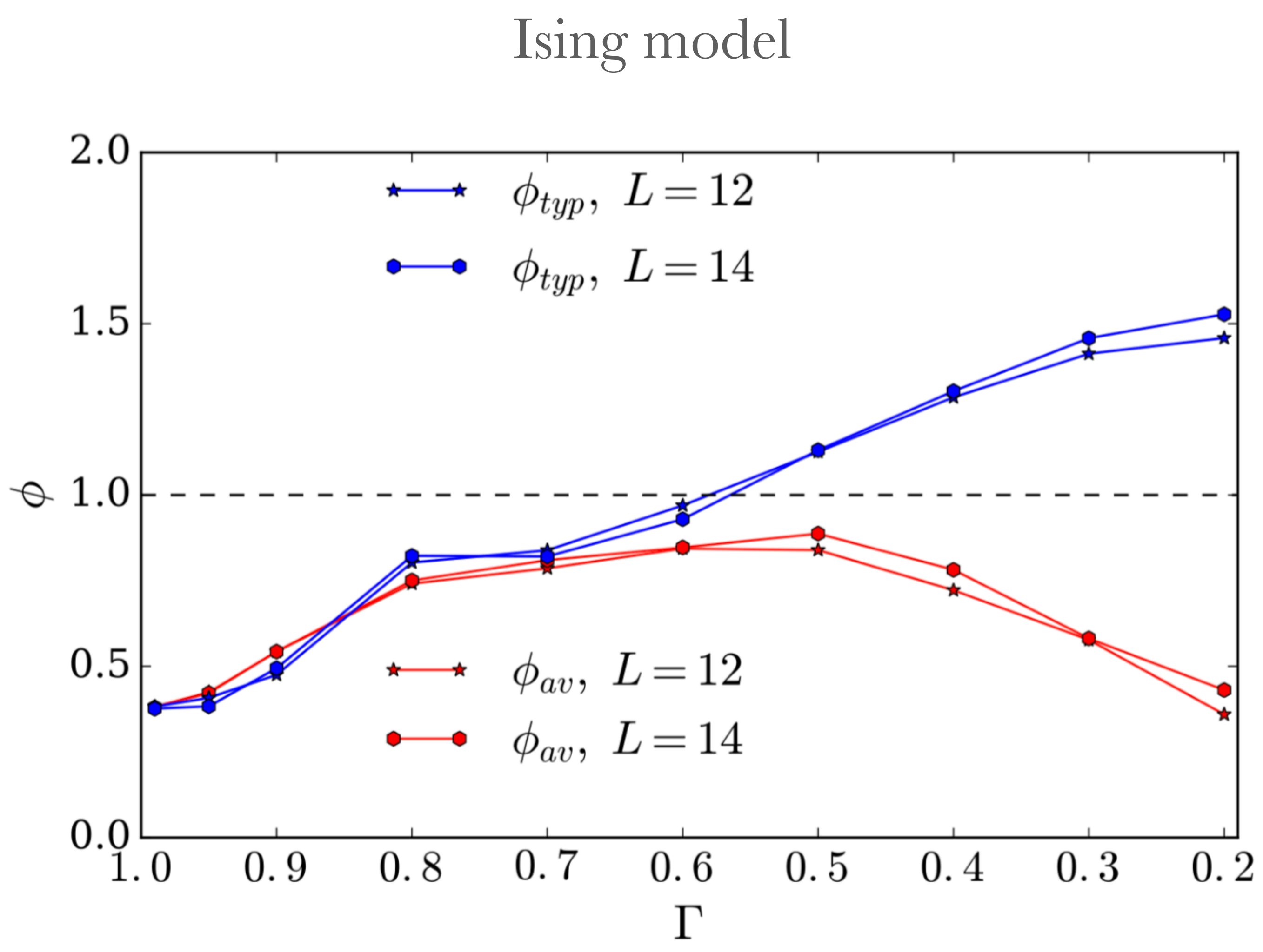
$\gamma = 1/3$:

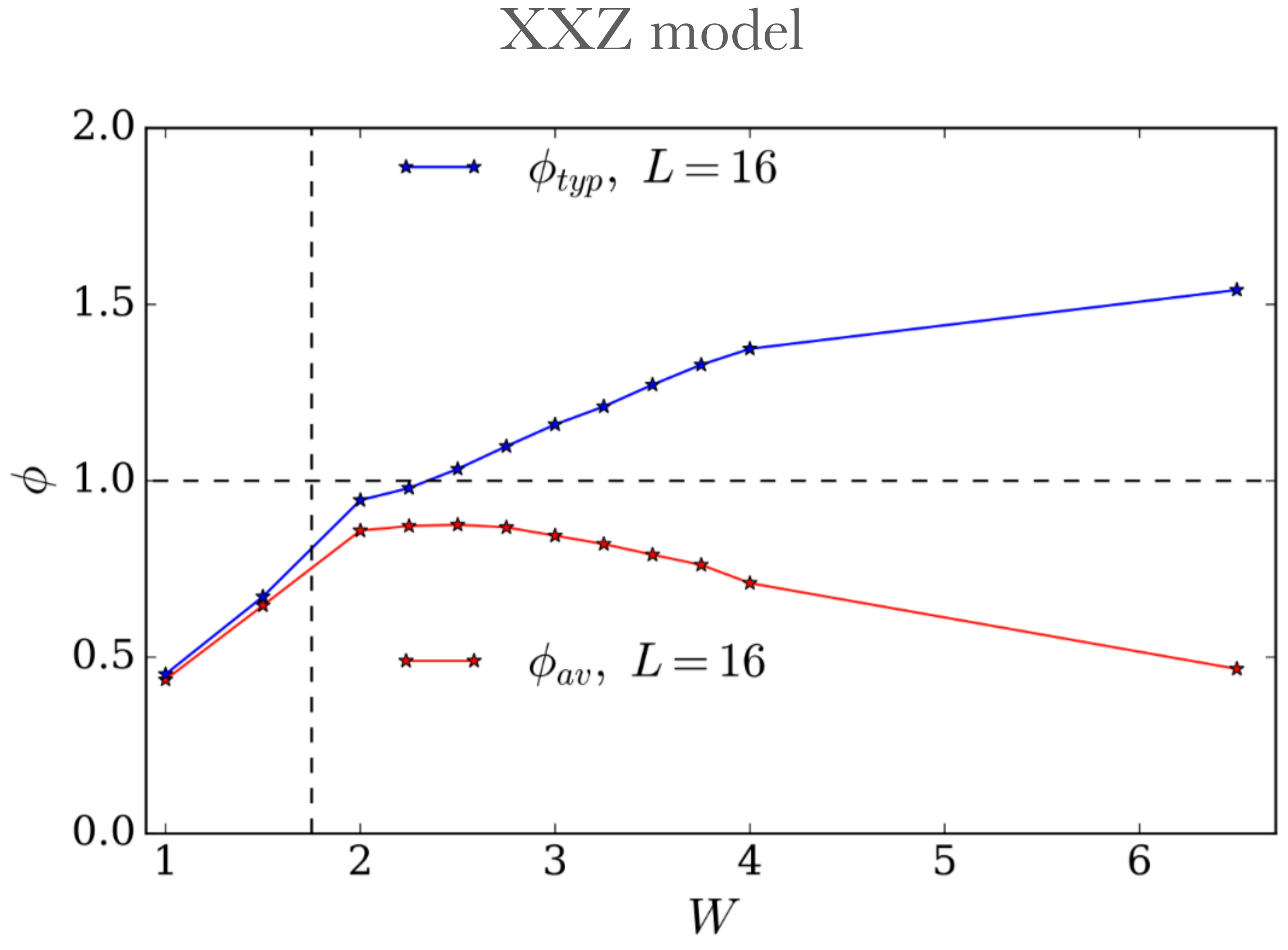
$$L = 10, \quad 12 \quad 14 \quad 16$$
$$E_{Th} \propto 0.001, 0.00057, 0.00036, 0.00024$$



$$f^2(\omega) \propto \begin{cases} \text{const}, & \omega \leqslant E_{\mathrm{Th}}, \\ 1/\omega^{\phi}, & E_{\mathrm{Th}} \ll \omega \ll J_{\mathrm{loc}}, \end{cases}$$

$$[f^2(\omega)]_{\mathrm{typ}} = \exp(\langle \ln f^2(\omega) \rangle),$$





<u>for strong enough disorder</u> $\quad W > W_*$

$$1/2 < \phi = 1 - \gamma < 1$$
$$\rightarrow 0 < \gamma < 1/2$$

- $f^2(\omega)$ is not smooth
- similar breakdown
- relation of ϕ to the exponent γ may no longer hold

diffusion $\gamma = 1/2, f^2(\omega) \propto 1/\omega^{1/2}$

subdiffusion $\gamma < 1/2, f^2(\omega) \propto 1/\omega^{1/\phi}$

exponent γ & MBL

Weak disorder

$$\phi \ ? \ \gamma \longleftarrow \phi = 1 - \gamma \quad \text{or} \quad \gamma = 1 - \phi$$

Power-law decay of spectral function

$$\boxed{f^2(\omega) \propto \begin{cases} \text{const}, & \omega \leqslant E_{\text{Th}}, \\ 1/\omega^{\phi}, & E_{\text{Th}} \ll \omega \ll J_{\text{loc}}, \end{cases}}$$

Reason of

$$[f^2(\omega)]_{typ} = \exp(\langle \ln f^2(\omega) \rangle) \neq f^2(\omega)_{av} = \langle \exp(\ln f^2(\omega)) \rangle$$

the log-averaged spectral function is dominated by the most probable matrix elements

the average function is dominated by rare resonances that give matrix elements of order one

Phys. Rev. B ***92****, 104202*

Still don’t know

participation ratios of the wave function with the Hilbert space dimension

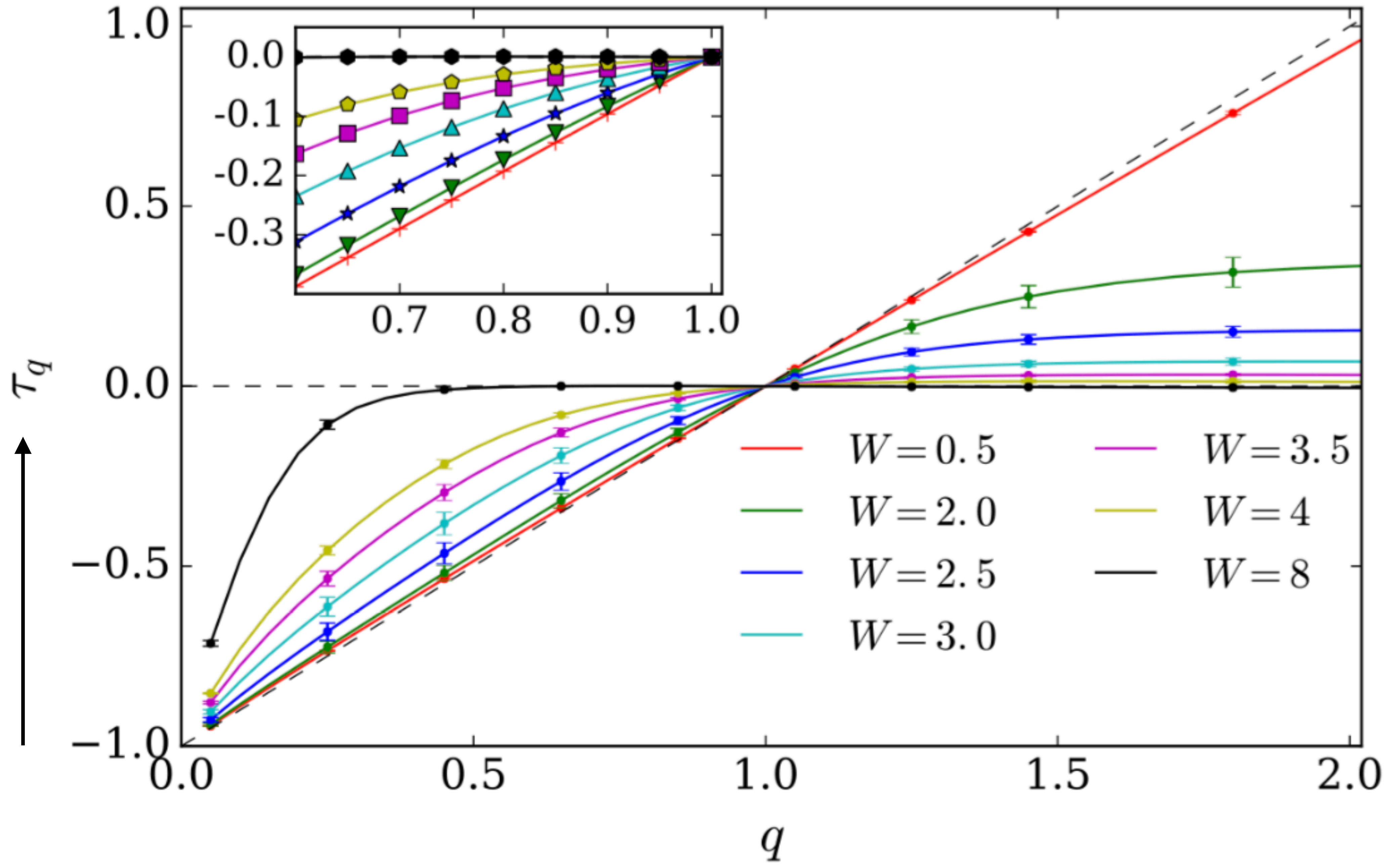
the scaling dimension τ_q & W

$$\boxed{P_q = \sum_\beta \langle |O_{\alpha\beta}|^{2q} \rangle \propto \frac{1}{\mathcal{D}^{\tau_q}},}$$

dashed line

$\tau_q = q - 1$

strong disorder



strong disorder

$$P_q = \sum_\beta \langle \; O_{\alpha\beta} \;^{2q} \rangle \propto \frac{1}{D^{\tau_q}}$$

$$P_q = \sum_\beta \langle \; \psi_\alpha(\beta)^{2q} \; \rangle$$

If the distribution of matrix elements is very narrow

$$\psi(\beta) \;^2 \propto \frac{1}{D}$$

$$P_q \propto \frac{1}{D^q} \to \tau_q = q?$$

Critical $\quad \psi(\beta) \;^2 \propto \dfrac{1}{D^c} \quad \tau_q \to const \quad \longleftarrow \quad$ Weak disorder

$$\tau_0 = -1, \tau_1 = 0$$
$$\to \tau_q = q - 1$$

Participation ratio

*A. De Luca and A. Scardicchio 2013 EPL **101** 37003*

We introduce the inverse participation ratios as the moments

$$\mathrm{IPR}_q = \sum_E |\langle E|\psi_0\rangle|^{2q}. \qquad (2)$$

where the sum runs over the full set of eigenstates $|E\rangle$. The long time average of the survival probability (the average removes some finite size effects like quasi-periodicity etc.) can be expressed as

$$\overline{P} \equiv \lim_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau dt\, |G(t)|^2 = \mathrm{IPR}_2\,. \qquad (3)$$

Here $(\mathrm{IPR}_2)^{-1}$ is therefore a measure of the portion of explored Hilbert space during the quantum dynamics and it is usually dubbed *participation ratio* (PR). Analogously, higher order IPR_q's describe finer details of the dynamics.

Eigenstate characterization – Localized wave-functions The most striking characterization of the localized phase is perhaps that the eigenfunctions of the Hamiltonian H resemble those of the infinite disorder case: they are localized around some localization center $j_{\mathrm{loc}} \in \mathbb{Z}^d$, and are decaying exponentially away from it with a localization length ζ_{loc}:

$$\psi_{\mathrm{loc}}(j) \sim e^{-|j-j_{\mathrm{loc}}|/\zeta_{\mathrm{loc}}}\,. \qquad (15)$$

Note that, similarly as in the infinite disorder case but in contrast with the free case, these eigenfunctions are true normalizable eigenfunctions of H even in the $L\to\infty$ limit. A useful characterization of the localized character of an eigenfunction ψ is to introduce its participation ratio

$$I_q := \sum_j |\psi(j)|^{2q}\,. \qquad (16)$$

Taking L finite, one expects the scaling $I_q \sim L^{d(1-q)}$ for a delocalized eigenfunction and $I_q \sim 1$ for a localized wave-function. This quantities are also useful to quantify more complex behavior: fractals wave-functions are also observed exactly at the transition between the localized and delocalized phase and a non-trivial fractal dimension $0 < D(q) < d$ (multifractality) is introduced as $I_q \sim L^{D(q)(1-q)}$.

function in Eq. (13),

$$P_q = \sum_\beta \langle |\psi_\alpha(\beta)|^{2q}\rangle, \qquad (14)$$

where the brackets denote averaging over disorder and eigenstates α from a narrow band around the energy $E = 0$. The scaling of P_q with the Hilbert space dimension $\mathcal{D}$ is given by

$$\boxed{P_q = \sum_\beta \langle |O_{\alpha\beta}|^{2q}\rangle \propto \frac{1}{\mathcal{D}^{\tau_q}},} \qquad (15)$$

which defines the scaling dimension τ_q. We note that in contrast to ALT, where participation ratios can be in principle probed in tunneling experiments, the relation of the above P_q to physical properties is not clear.

ALT:

Inverse participation ratio: a)Local $\quad I \sim 1$

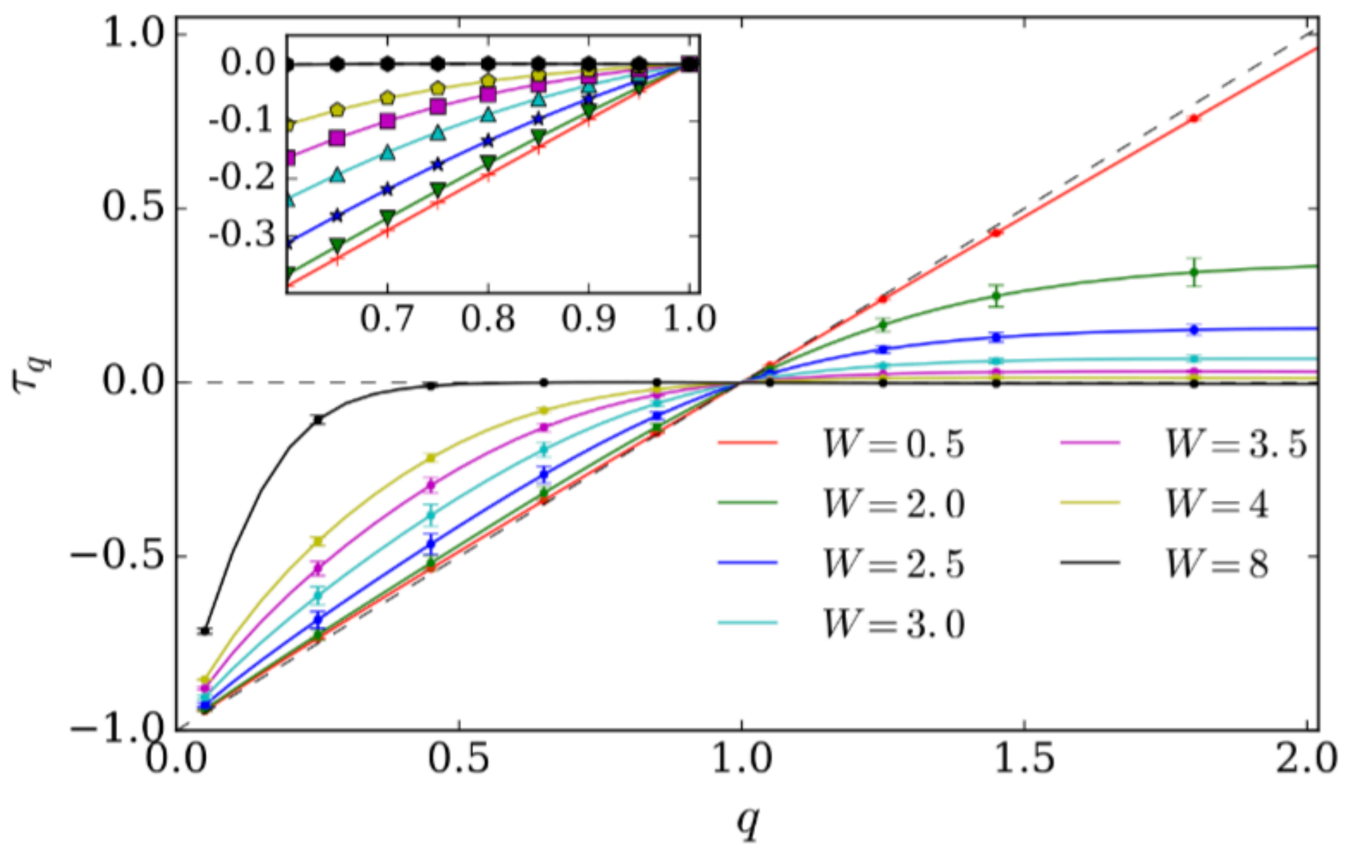
b)delocal $\quad I \sim L^{D(q)(1-q)}$

MBLT:

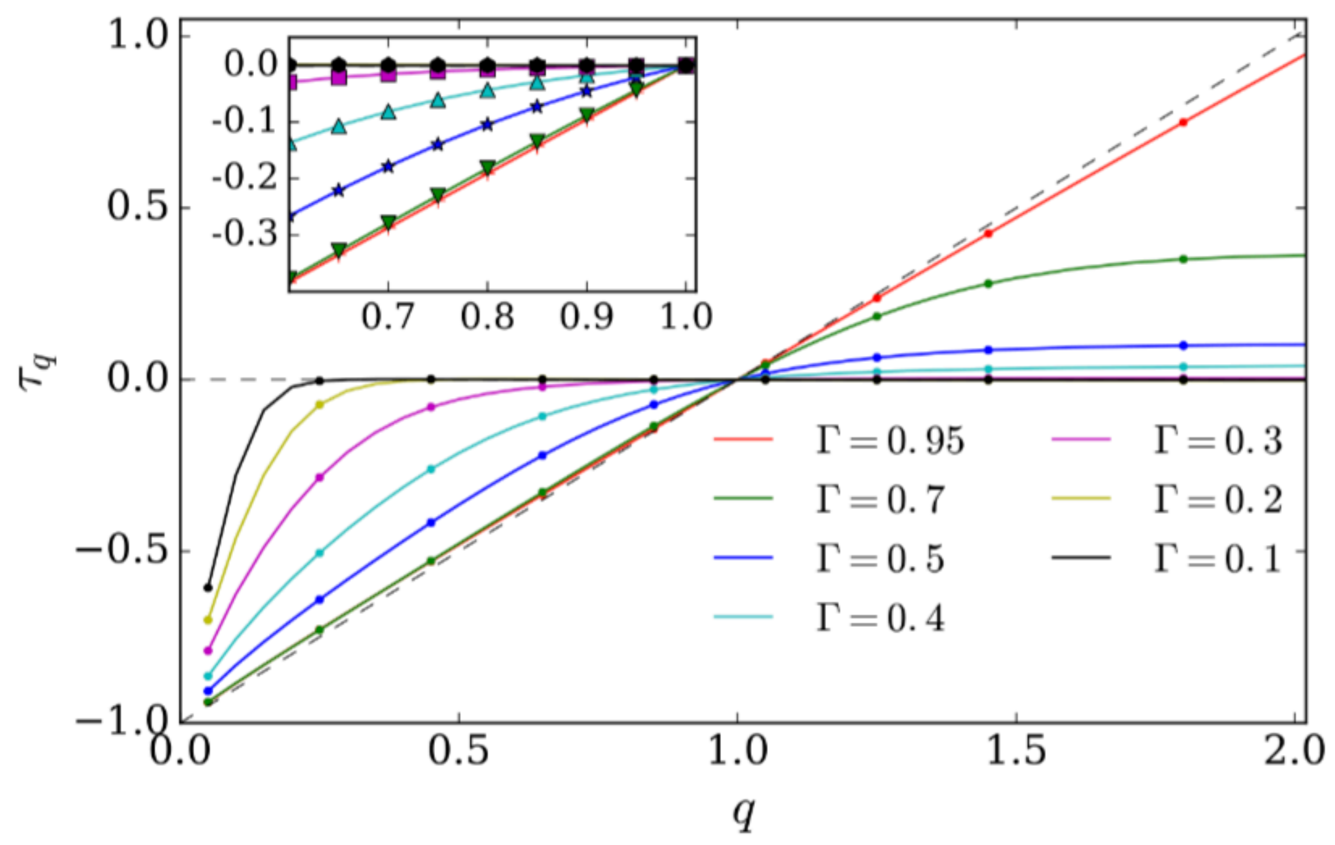
Participation ratio: A) local $\quad$ High P

B) delocal $\quad$ Low P

XXZ model

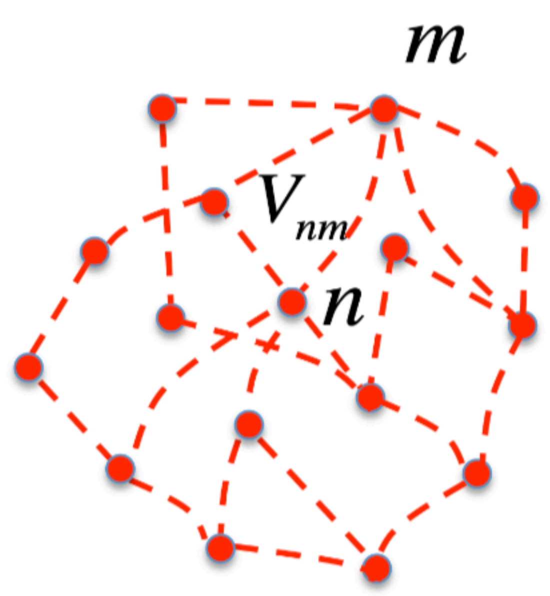


Ising model

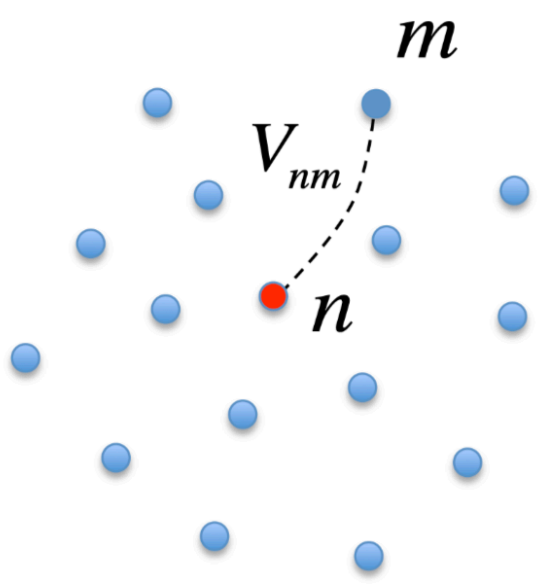


$\boxed{0.4 > \Gamma_c > 0.3}$

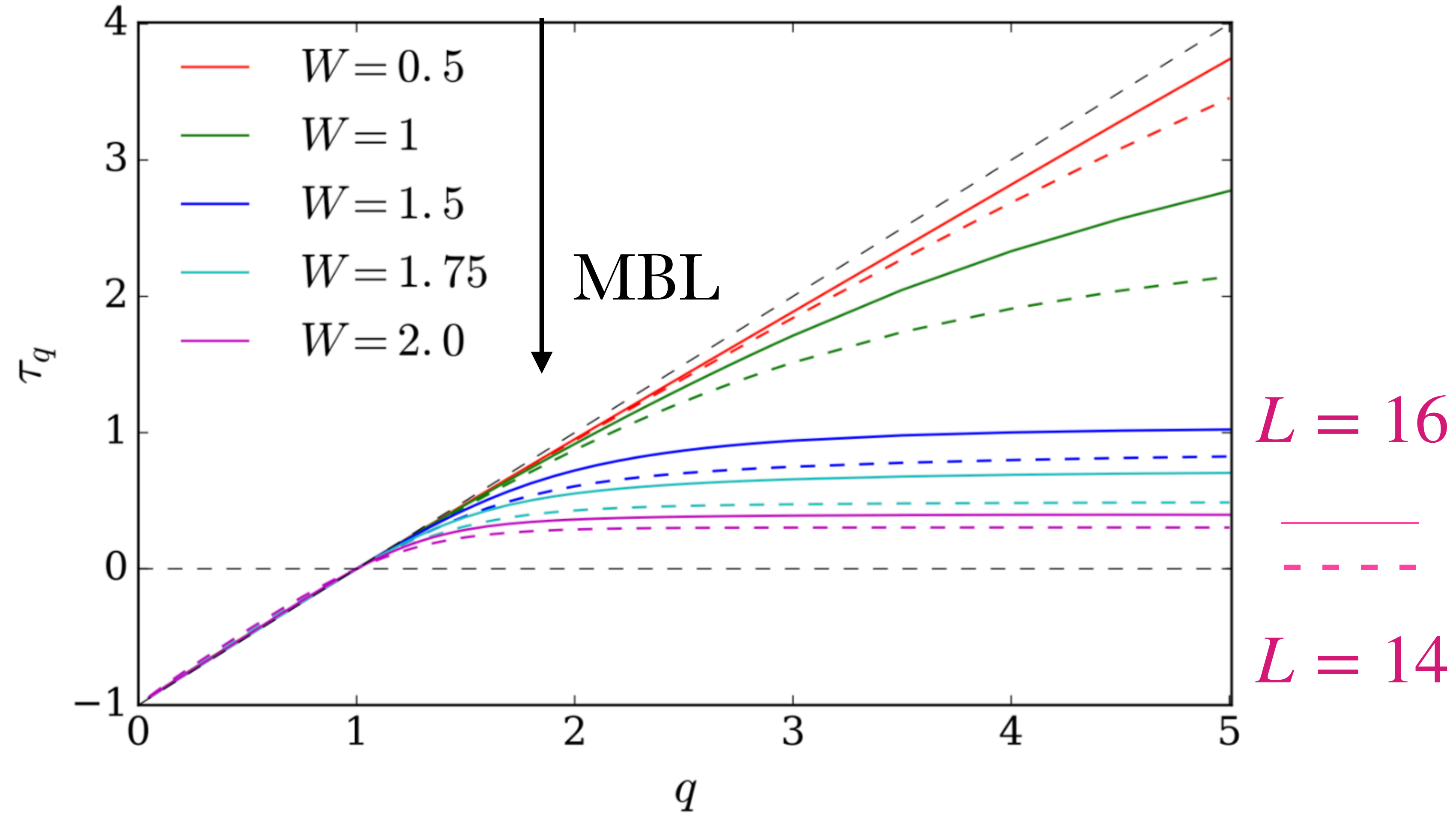
(c) Delocalized



(e) Localized

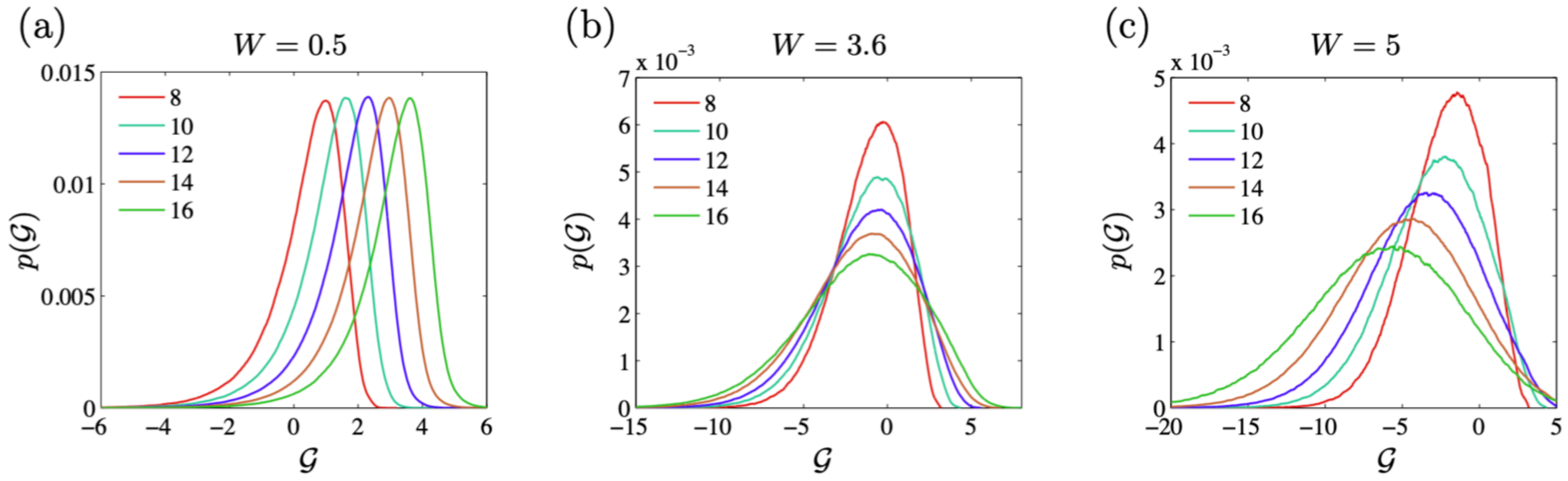


the scaling dimension τ_q & L



- exponentially suppressed in the system size

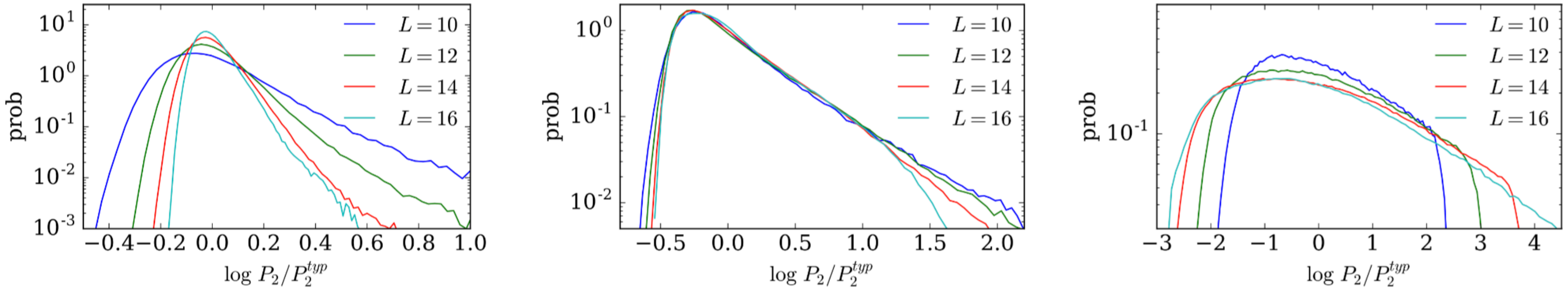
Distribution of G across the MBL transition displays qualitatively different scaling with system size of the one-dimensional random-field XXZ model



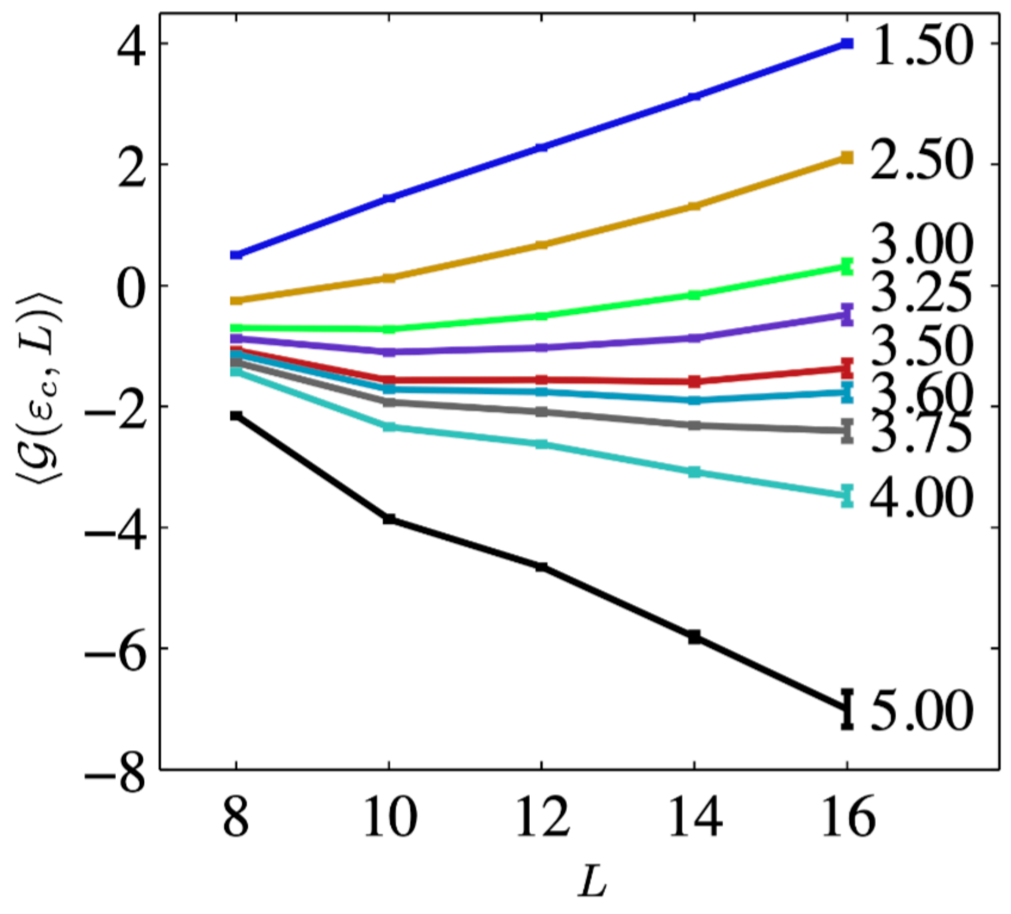
$$\boxed{\mathcal{G}(\varepsilon, L) = \ln \frac{|V_{n,n+1}|}{E'_{n+1} - E'_n},}$$

Phys. Rev. X ***5****, 041047*

Distribution of participation ratios P_2 on the ergodic side of the MBLT in the XXZ model.



W = 0.5 W = 1 W = 2



$$\boxed{p(P_2/P_2^{\text{typ}}) \propto 1/P_2^{1+x_2}}$$

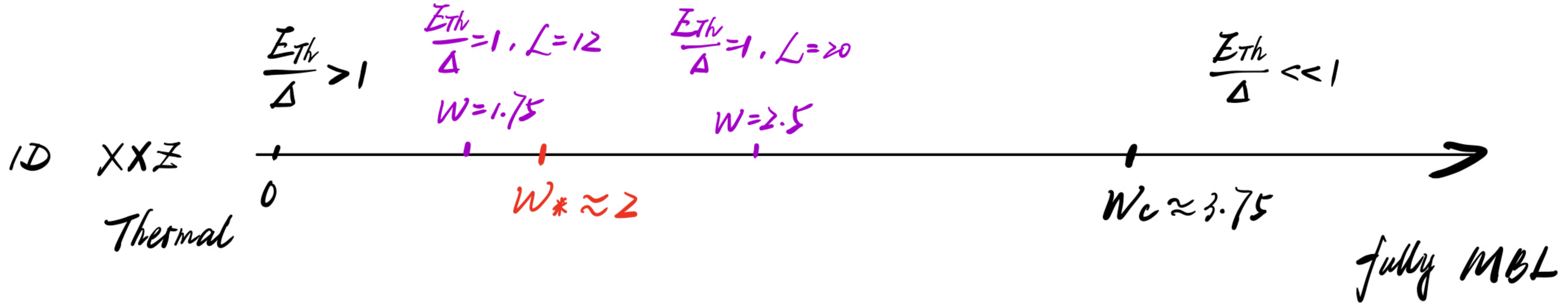
?

L 越大展宽越大

In summary

introduce a local operator and explore off-diagonal matrix elements of it

Thouless energy



scaling: weak disorder $\xrightarrow[\gamma \downarrow]{W \uparrow}$ strong disorder ($W \geq W_* \approx 2$)

$E_{Th} \propto L^{-1/\gamma}$

$\gamma = \frac{1}{2}$ D

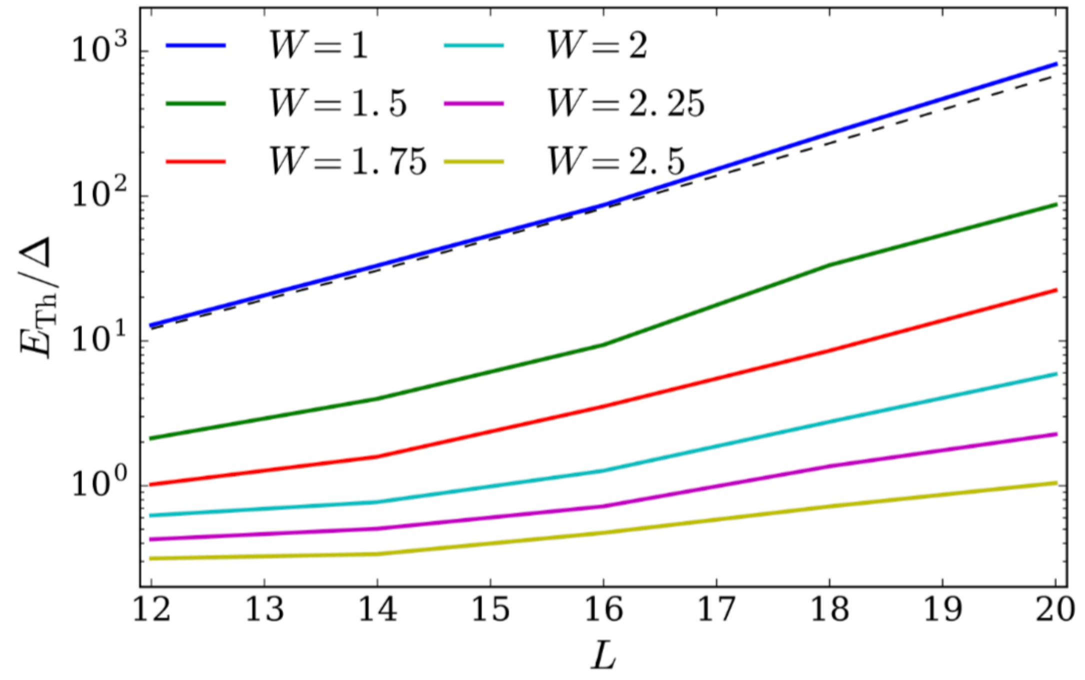
$\gamma < \frac{1}{2}$ SubD

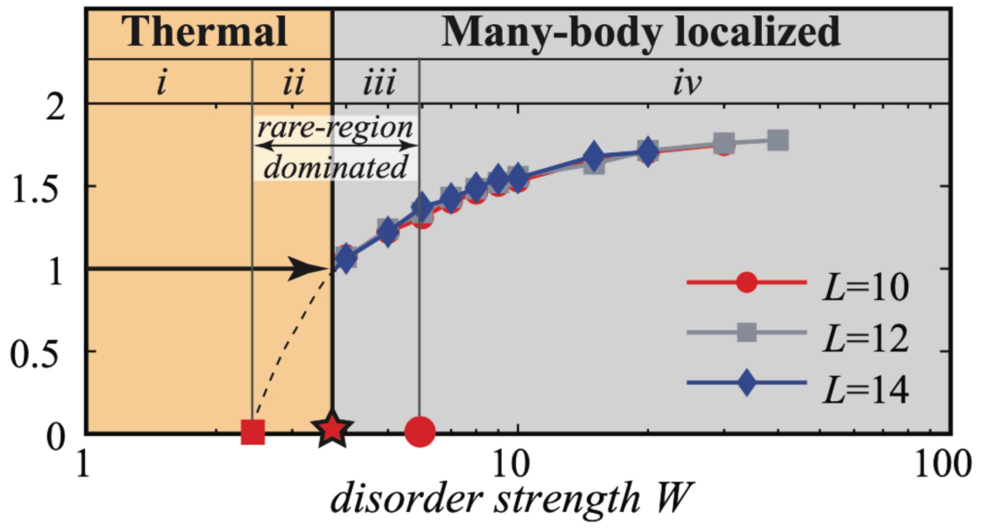
power-law decay

very small $\gamma \Rightarrow E \propto e^{-\kappa L}$

$\kappa \approx \ln 2$

exponentially decays





(i) the diffusive thermal phase;
(ii) the subdiffusive thermal phase

Phys. Rev. B 92, 104202

Spectral Function

$$f^2(\omega) = \begin{cases} const, & \omega \le E_{Th} \\ \frac{1}{\omega^{\phi}}, & E_{Th} \ll \omega \ll J \end{cases}$$

$\frac{E_{Th}}{\Delta} \ll 1$

1D XXZ

Thermal 0 — $W_* \approx 2$ — $W_c \approx 3.75$ → fully MBL

$W_* \approx 2$ → 平台消失

$E_{Th} \sim \Delta$

$E_{Th} \propto L^{-1/\nu}$

$$\phi = 1-\gamma \begin{cases} \gamma = \frac{1}{2}, D \\ \gamma < \frac{1}{2}, SubD \end{cases}$$

$f^2(\omega) \propto const$ 平台

$+$

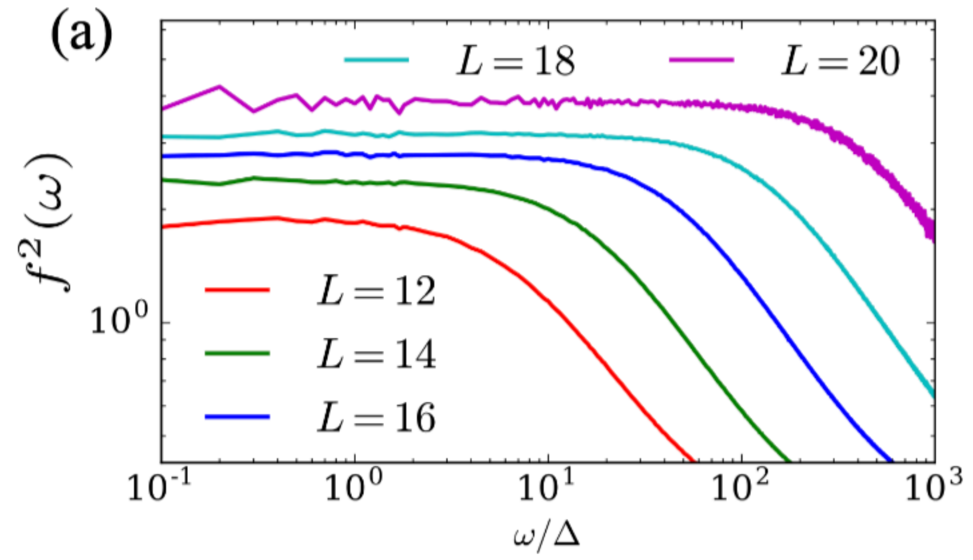
$f^2(\omega) \propto \frac{1}{\omega^{\phi}}$ power-law decay

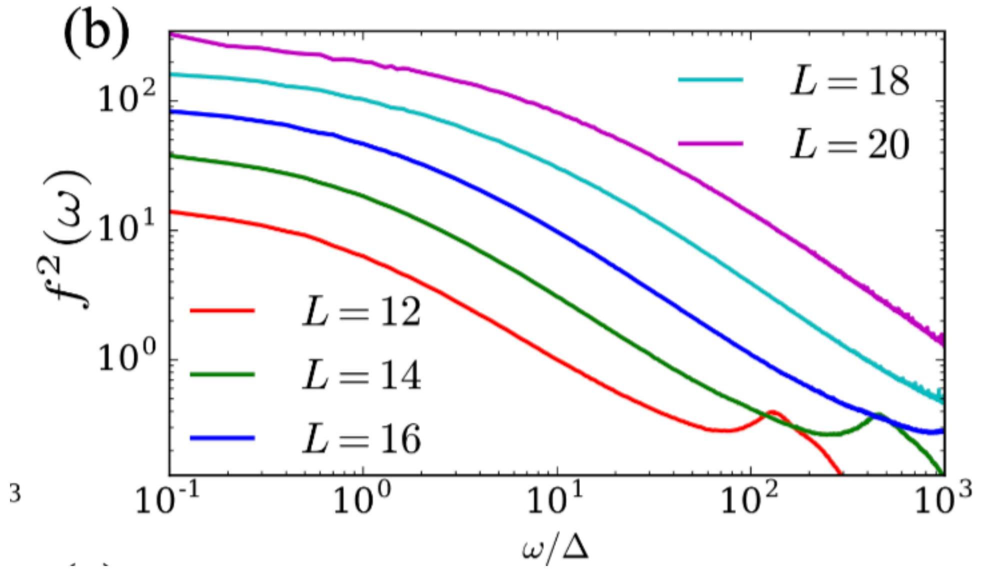
$\phi \neq 1-\gamma$

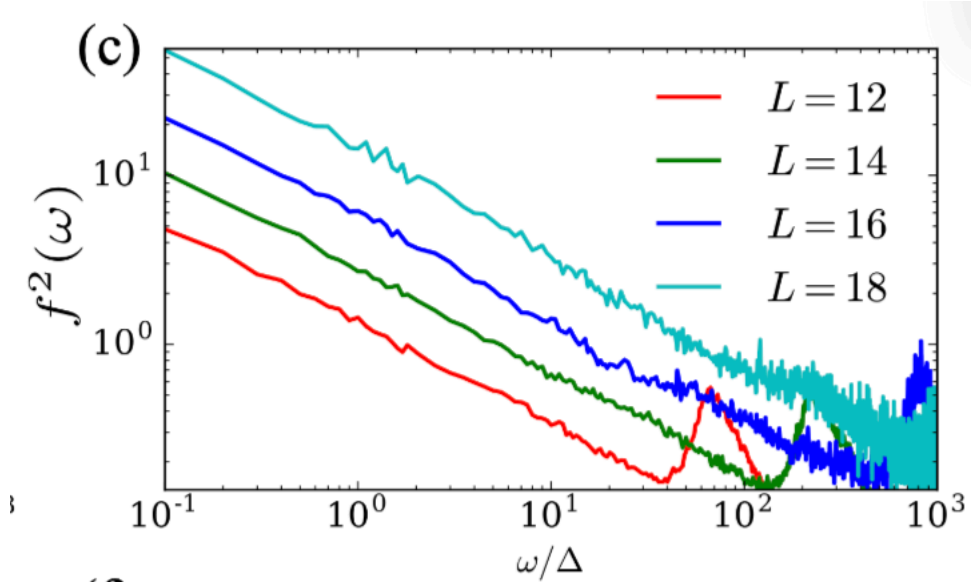
$\phi_{typ} \neq \phi_{av} \Leftarrow f^2(\omega)_{typ} = e^{\langle \ln f^2(\omega) \rangle}$

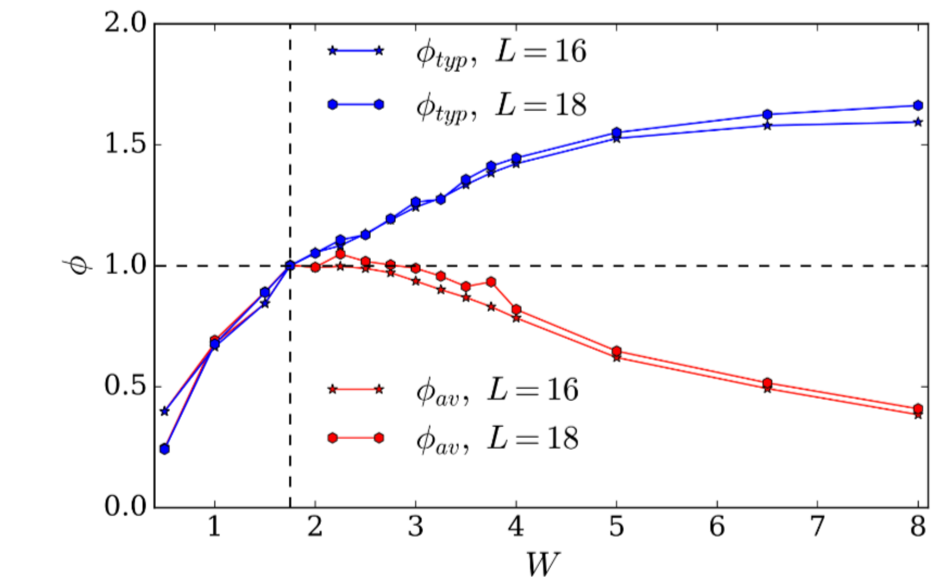
$f^2(\omega) \propto \frac{1}{\omega^{\phi}}$ power-law decay

not smooth









MULTIFRACTAL ANALYSIS

1D XXZ

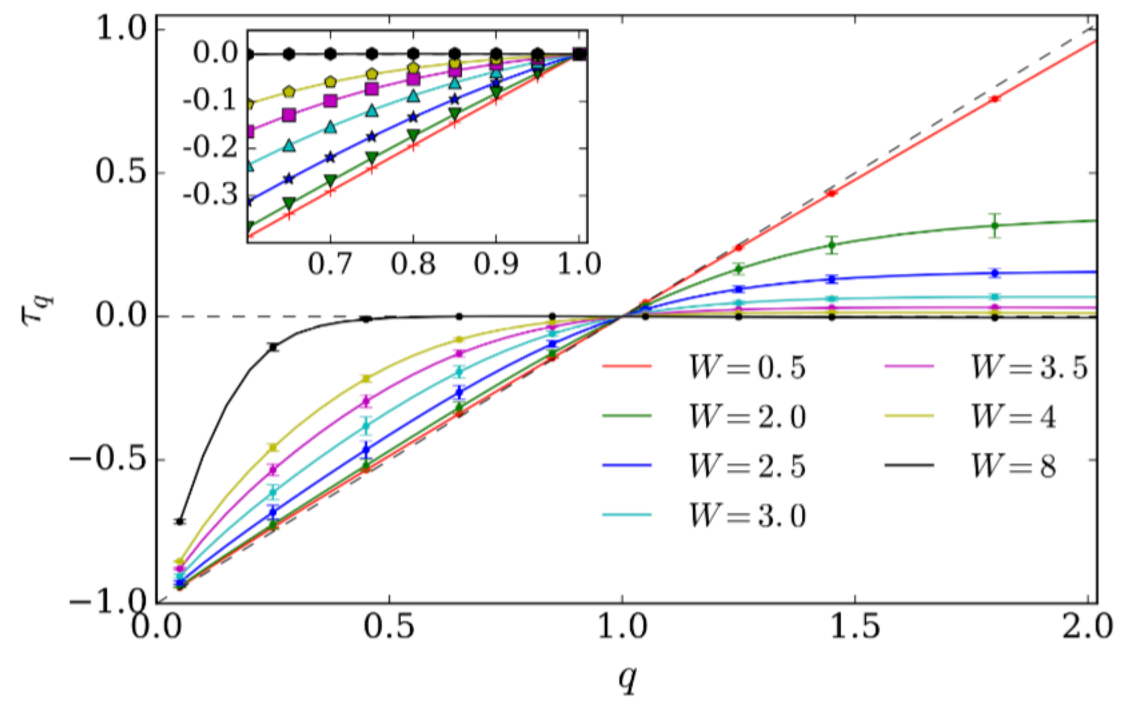
$\tau_q = q-1$ $W=0.5$ — $\tau_q = 0$ $W=4$

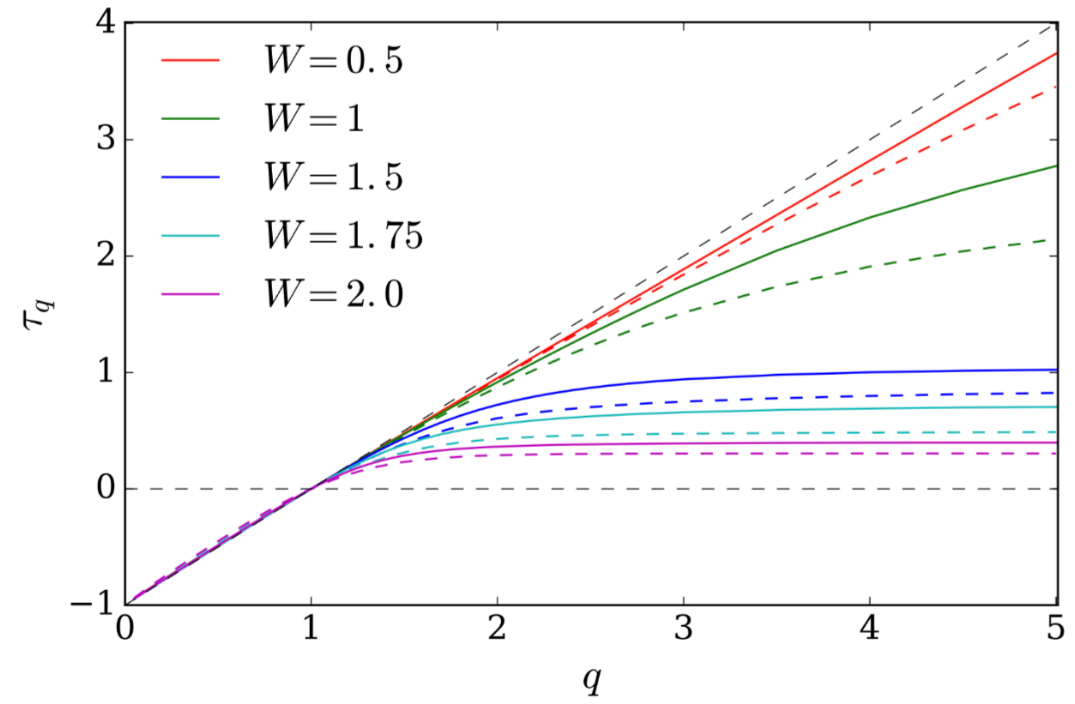
0 Thermal — $W_* \approx 2$ — $W_c \approx 3.75$ — fully MBL

$q \geq 1$ $\tau_q = q-1$ 偏离 $\tau_q \to$ const

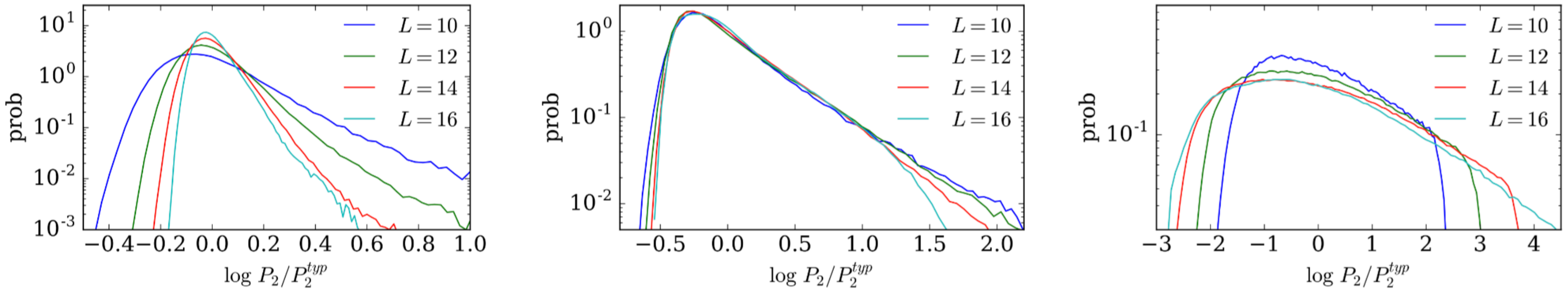
$q \leq 1$ $\tau_q = q-1$ 偏离 exponentially decays

τ_q对L的依赖非常不明显





$$P_q = \sum_\beta \langle |O_{\alpha\beta}|^{2q} \rangle \propto \frac{1}{\mathcal{D}^{\tau_q}},$$



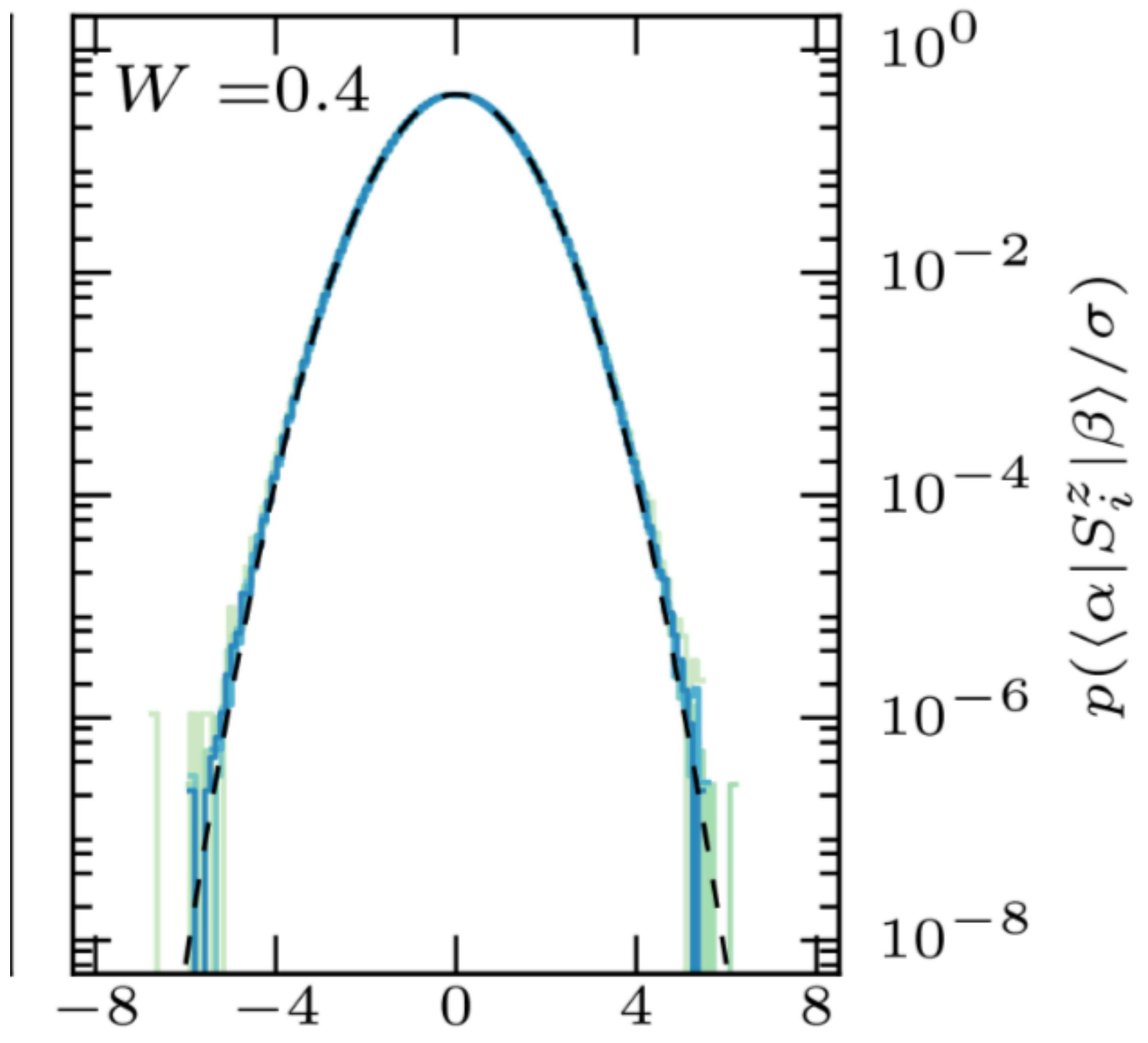
eigenstate thermalization hypothesis (ETH) anzatz

$$\langle\alpha|\hat{O}|\beta\rangle = \overline{O}(E)\delta_{\alpha\beta} + e^{-S(E)/2} f(E,\omega) R_{\alpha\beta},$$

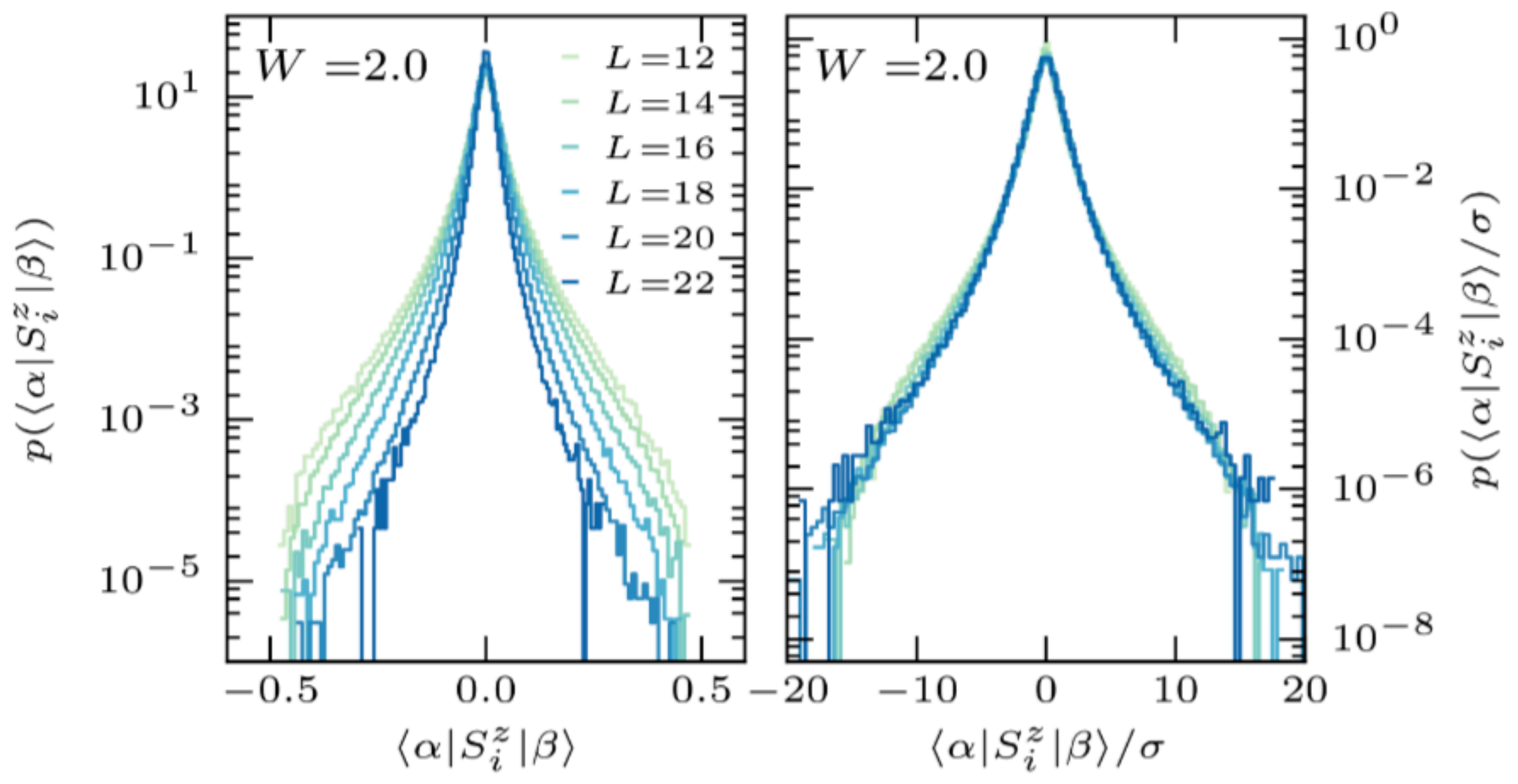
$$O_{\alpha\beta} \propto e^{-Ls(E)/2} L^{(1-\gamma)/(2\gamma)} R_{\alpha\beta}, \quad |E_\alpha - E_\beta| < L^{-1/\gamma},$$

$R_{\alpha\beta}$ the normal distribution with zero mean and unit variance of the random term

the probability distribution of the **off-diagonal** elements computed for different disorder strengths and system sizes, renormalized by their standard deviation σ（标准差）



- 虚线是高斯分布
- 弱无序中频谱非对角算符元素符合高斯分布



- 较强的无序下，非对角元素非高斯分布

Closer to the MBL transition, the shape of the distribution is clearly non-Gaussian

there is a class of ergodic systems,
anomalous (nondiffusive) relaxation to equilibrium,
while still satisfying a modified ETH ansatz,

the off-diagonal elements in (ETH) anzatz include a power law correction to their scaling with the system size.

在依旧满足ETH的情况下，有一类遍历系统，非对角元素随系统尺寸的缩放中包含一个幂律校正

$$\boxed{|f(E_\alpha,\omega)|^2 \propto \int_{-\infty}^{\infty} dt|t|^{-\gamma} e^{i\omega t} \propto |\omega|^{-(1-\gamma)}.}$$

the off-diagonal elements should scale with system size as

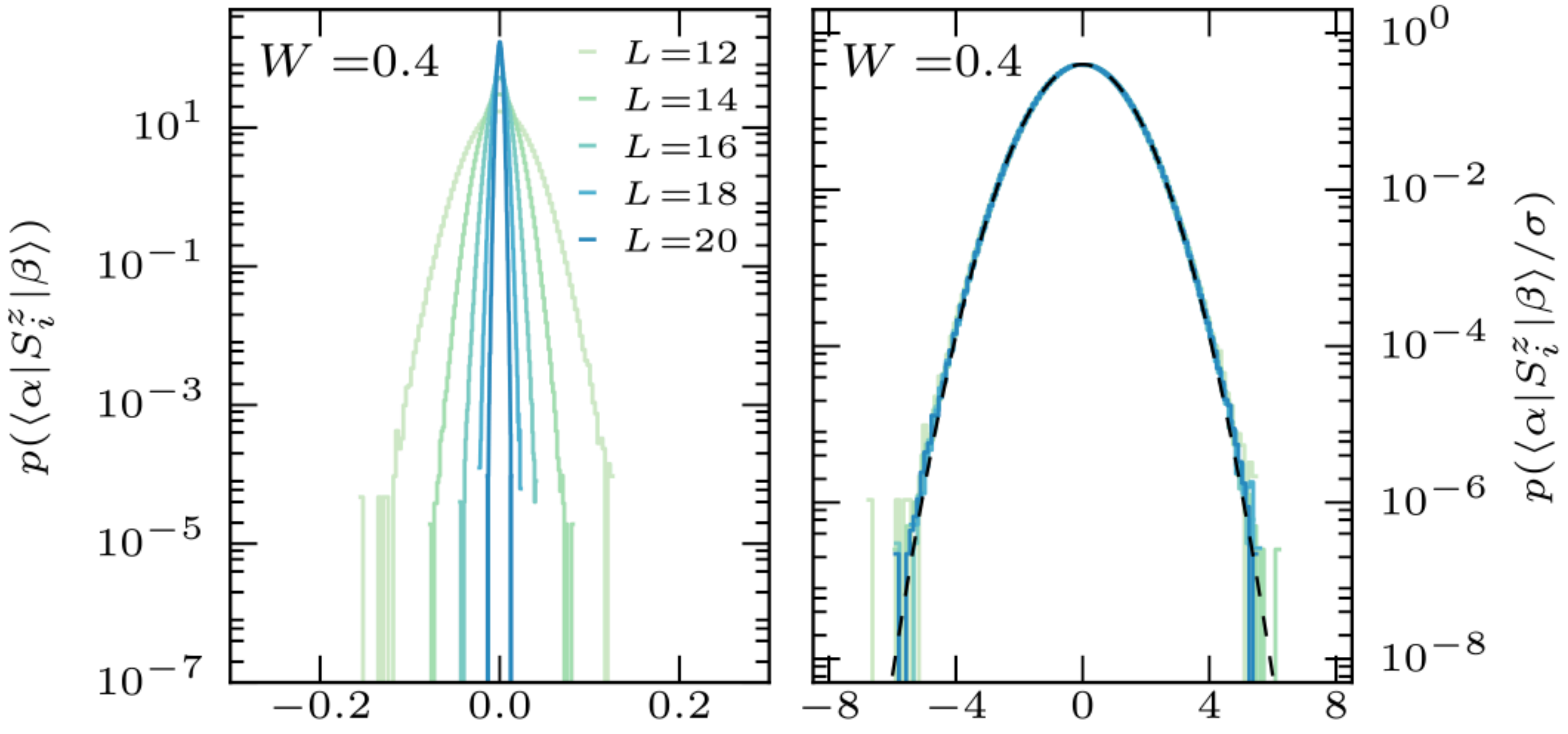
$$\boxed{O_{\alpha\beta} \propto e^{-Ls(E)/2} L^{(1-\gamma)/(2\gamma)} R_{\alpha\beta}, \quad |E_\alpha - E_\beta| < L^{-1/\gamma},}$$

For a finite system of size L, saturation will occur after time $t_c \approx L^{1/\gamma}$

the power-law dependence will be cut off for frequencies, $\omega < t_c \approx L^{-1/\gamma}$

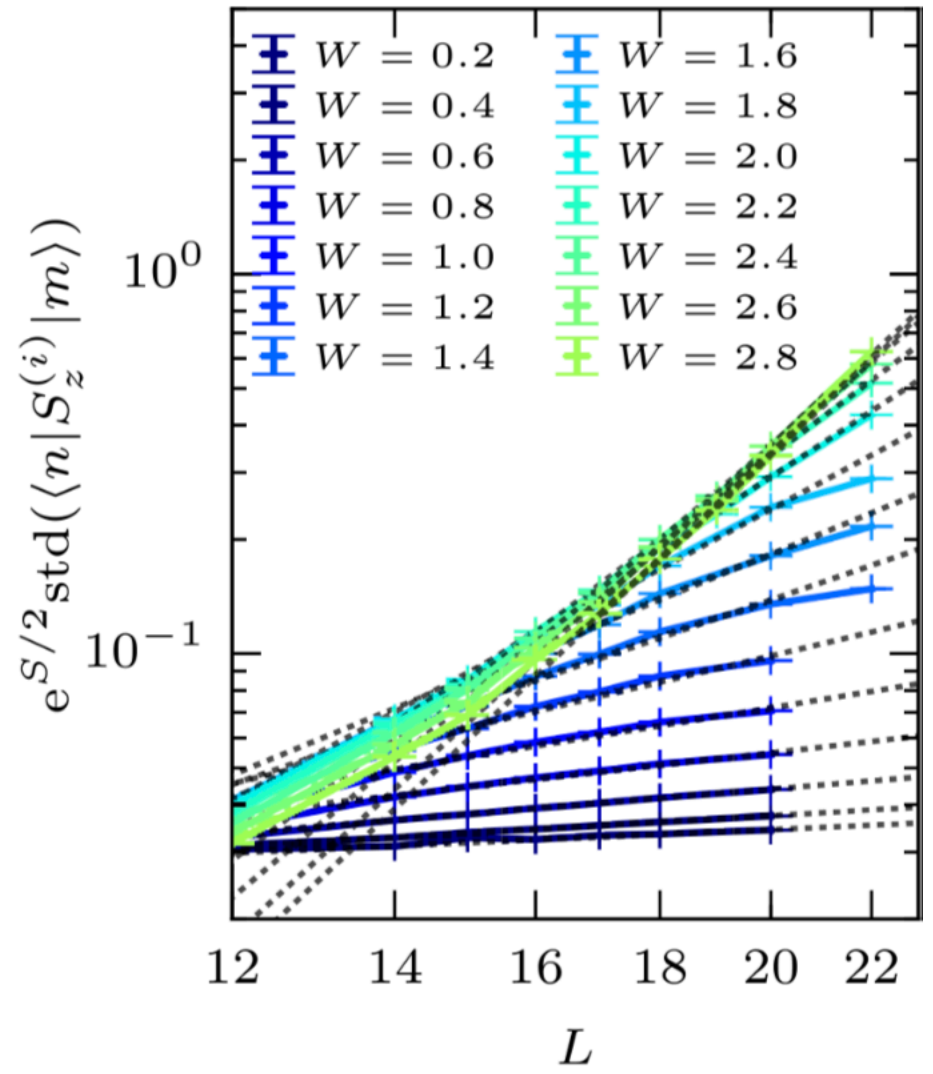
and $f(E_\alpha, \omega)$ ^{2} will become structureless

$$\boxed{|f(E_\alpha,\omega)|^2 \approx t_c^{1-\gamma} = L^{(1-\gamma)/\gamma}, \qquad \omega < L^{-1/\gamma}.}$$



右图数据塌缩，体现高斯行为

左图可以看到 depends on L 行为



approaching the MBL transition, visible deviations from power law behavior appear, signaling the violation of the scaling

不符合虚线，对数关系

$$O_{\alpha\beta} \propto e^{-Ls(E)/2} L^{(1-\gamma)/(2\gamma)} R_{\alpha\beta}, \quad |E_\alpha - E_\beta| < L^{-1/\gamma},$$

随着更靠近MBL侧，幂律衰减逐渐被破坏

In ergodic systems, the eigenstates are similar to random vectors in the Hilbert space (modulo global conservation laws, e.g. energy); therefore, the eigenstates have an extensive, volume-law entanglement entropy.

在遍历性系统中，本征态类似于希尔伯特空间中的随机向量（考虑到全局守恒律，例如能量），因此，这些本征态具有广泛的、遵循体积定律的纠缠熵。

entanglement spreads ballistically and grows linearly in time

纠缠以弹道方式传播并随时间线性增长

the eigenstates of MBL systems can be obtained from product states by quasi-local unitary transformations, and have low entanglement that obeys the “area law”

相反，多体局域化（MBL）系统的本征态可以通过准局部幺正变换从乘积态获得，并且具有较低的纠缠，遵循“面积定律”

the spreading is logarithmic in time(the dynamics of entanglement)

传播是对数时间的

关于这篇文献，我还想知道哪些问题?

correlation length ξ(W) & MBL

如果存在一个相关长度 $\xi(W)$ 它依赖于无序强度并在MBLT($W = W_c$)处发散，那么即使是在小尺寸($L <= \xi(W)$)、小无序($W < W_c$)，也可能表现类似MBLT(MBL transition)行为

study systems of size L > ξ(W) to see the delocalized behavior

$$
\boxed{V_{nm} = e^{-S(E,L)/2} f(E_n, E_m) R_{nm},}
$$

fractal properties depend on the choice of the basis

even trivial product state may acquire “fractal”statistics of the wave function amplitudes in the improperly chosen basis