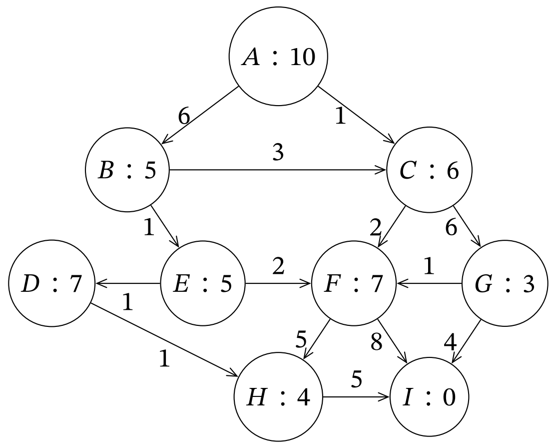
Assignment3 – Search

Problem 3.1 (Search Algorithms)
Consider the following directed graph:



Every node is labeled with $n : h(n)$ where n is the identifier of the node and $h(n)$ is the heuristic for estimating the cost from n to a goal node. Every edge is labeled with its actual cost.

1. Assume that I is the goal node. Argue whether or not the heuristic is admissible.

Solution: It is not admissible: The cost from D to the goal is $1 + 5 = 6 < 7 = h(D)$, and a heuristic must not overestimate that cost.

Now assume you have already expanded the node A. List the **next** 4 **nodes (i.e., excluding** A**)** that will be expanded using the respective algorithm. If there is a tie, break it using alphabetical order.

As presented in the notes, the same state may be inserted into the fringe multiple times if it is encountered along multiple paths. The second insertion may happen *before* or *after* the first one has been expanded.

- before: Here both nodes are in the fringe at the same time, and a practical implementation can easily remove duplicates from the fringe as follows:
  - DFS: keep the later node
  - BFS: keep the earlier node
  - UCS, A^*: keep the node with the lower cost
  - greedy: keep either node (they're identical)

  This can speed up search and possibly avoid some cycles.

- after: Here the fringe does not contain the same state twice, and an implementation would have to perform a more sophisticated analysis to check if a state has been expanded previously. This would avoid all redundant duplicate expansion and cycles. However, in the case of UCS and A^*, if the second occurrence of the state has a lower cost, the state would still have to be expanded a second time.

In the sequel, we assume that the before-check happens but not the after-check.

2. depth-first search

   Solution: B, C, F, H

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3. breadth-first search

   Solution: B, C, E, F (Without the before-check, we would get B, C, C, E.)

   ---

4. uniform-cost search

   Solution: C, F, B, E

   ---

5. greedy-search

   Solution: B, E, C, G

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6. A^*-search

   Solution: C, F, G, B

Problem 3.2 (Formally Modeling a Search Problem)

Consider the Towers of Hanoi for 7 disks initially stacked on peg A.

Is this problem deterministic? Is it fully observable?

Formally model it as a search problem in the sense of the mathematical definition from the slides. Explain how your mathematical definition models the problem.

Note that the formal model only defines the problem — we are not looking for solutions here.

Note that modeling the problem corresponds to defining it in a programming language, except that we use mathematics instead of a programming language. Then explaining the model corresponds to documenting your implementation.

Solution: We need to give $(S, \mathcal{A}, T, I, G)$.

Because the problem is deterministic, we know $\#(T(a, s)) \leq 1$. Because the problem is fully observable, we know $\#(I) = 1$.

Let $D = \{1, 2, 3, 4, 5, 6, 7\}$ (the set of disks) and $P = \{A, B, C\}$ (the set of pegs). We put:

- $S = D \to P$, i.e., a state s is a function from disks to pegs.

  Explanation: In state s, the value $s(d)$ is the peg that disk d is on. Because disks must always be ordered by size, we do not have to explicitly store the order in which the disks sit on the pegs.

- $\mathcal{A} = \{(A,B), (B,A), (A,C), (C,A), (B,C), (C,B)\}$, i.e., an action a is a pair of different pegs.

  Explanation: (p,q) represents the action of moving the top disk of peg p to peg q.

- For $s \in S$ and $p \in P$, we abbreviate as $top(s, p)$ the smallest $d \in D$ such that $s(d) = p$.

  Explanation: $top(s, p)$ is the top (smallest) disk on peg p in state s.

  Then $T : \mathcal{A} \times S \to \mathcal{P}(S)$ is defined as follows:

  - If $top(s, q) > top(s, p)$, we put $T((p, q), s) = \{s'\}$ where $s' : D \to P$ is given by
    - $s'(d) = q$ if $d = top(s, p)$
    - $s'(d) = s(d)$ for all other values of d
  - otherwise, $T((p, q)), s) = \emptyset$

  Explanation: In state s, if the top disk of q is bigger than the top disk of p, the action (p,q) is applicable, and the successor states s' of s after applying (p,q) is the same as s except that the top (smallest) disk on peg p is now on peg q.

- $I = \{i\}$ where the state i is given by $i(d) = A$ for all $d \in D$.

  Explanation: Initially, all disks are on peg A.

- $G = \{g\}$ where the state g is given by $g(d) = B$ for all $d \in D$.

  Explanation: There is only one goal state, described by all disks being on peg B.

Note that there are many different correct solutions to this problem. In particular, you can use different definitions for S (i.e., model the state space differently), in which case everything else in the model will be different, too. Often a good model for the state space can be recognized by how straightforward it is to define the rest of the model formal.

Even if you used a different state space, a good self-study exercise is to check that the above (a) is indeed a search problem and (b) correctly models the Towers of Hanoi. Continuing the above analogy to programming languages, (a) corresponds to compiling/type-checking your implementation and (b) to checking that your implementation is correct.

Problem 3.3 (Heuristic Searches)

Consider the graph of Romanian cities with edges labeled with costs $c(m,n)$ of going from m to n. $c(m,n)$ is always bigger than the straight-line distance from m to n. $c(m,n)$ is infinite if there is no edge.

Our search algorithm keeps:

- a list E of expanded nodes n together with the cost $g(n)$ of the cheapest path to n found so far,
- a fringe F containing the unexpanded neighbors of expanded nodes.

We want to find a cheap path from Lugoj to Bucharest. Initially, E is empty, and F contains only Lugoj. We terminate if E contains Bucharest.

Expansion of a node n in F moves it from F to E and adds to F every neighbor of n that is not already in E or F. We obtain $g(n)$ by minimizing $g(e)+c(e,n)$ over expanded nodes e.

As a heuristic $h(n)$, we use the straight-line distance from n to Bucharest as given by the table in the lectures.

Explain how the following algorithms choose which node to expand next:

1. greedy search with heuristic h

   Solution: The search expands the node n that minimizes the function $h(n)$.

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2. A* search with path cost g and heuristic h

   Solution: The search expands the node n that minimizes the function $g(n)+h(n)$

   ---

3. Explain what h^* is here and why h is admissible.

   Solution: $h^*(n)$ is the cost of the shortest path from n to Bucharest (which we do not know unless we expand all nodes). Because $c(m,n)$ is always bigger than the straight-line distance, every path is longer than the straight-line distance between its end points. Thus $h(n) \leq h^*(n)$.

   ---

4. For each search, give the order in which nodes are expanded.
(You only have to give the nodes to get the full score. But to get partial credit in case you're wrong, you may want to include for each step all nodes in the fringe and their cost.)

Solution: Lugoj (244), Mehadia (241), Drobeta (242), Craiova (160), Pitesti (100), Bucharest (0)
astarSearch-search: Lugoj (0+244), Mehadia (70+241), Drobeta ((70+75)+242), Craiova (70+75+120)+160), Timisoara (111+329), Pitesti ((70+75+120+138)+100), Bucharest ((70+75+120+138+101)+0)

Problem 3.4 (Heuristics)

Consider heuristic search with heuristic h.

1. Briefly explain what is the same and what is different between A^* search and greedy search regarding the decision which node to expand next.

Solution: Both choose the node that minimizes a certain function. As that function, A^* uses the sum of path cost and heuristic whereas greedy only uses the heuristic.

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2. Is the constant function $h(n) = 0$ an admissible heuristic for A^* search?

Solution: Yes. (But it's a useless one.)

Problem 3.5 (Tree Search in ProLog)

Implement the following tree search algorithms in Prolog:

1. BFS
2. DFS
3. Iterative Deepening (with step size 1)

Remarks:

- In the lectures, we talked about *expanding* nodes. That is relevant in many AI applications where the tree is not built yet (and maybe even too big to hold in memory), such as game trees in move-based games or decision trees of agents interacting with an environment. In those cases, when visiting a node, we have to expand it, i.e., compute what its children are.

In this problem, we work with smaller trees where the search algorithm receives the fully expanded tree as input. The algorithm must still visit every node and perform some operation on it — the search algorithm determines in which order the nodes are visited.

In our case, the operation will be to *write out the label* of the node.

- In the lectures, we worked with goal nodes, where the search stops when a goal node is found. Here we do something simpler: we *visit all the nodes and operate on each one* without using a goal state. (Having a goal state is then just the special case where the operation is to test the node and possibly stop.)

Concretely, your submission **must** be a single Prolog file that extends the following implementation:

```
% tree(V,TS) represents a tree.
% V must be a string - the label/value/data V of the root node
% TS must be a list of trees - the children/subtrees of the root node
% In particular, a leaf node is a tree with the empty list of children
istree(tree(V,TS)) :- string(V), istreelist(TS).

% istreelist(TS) holds if TS is a list of trees.
% Note that the children are a list not a set, i.e., they are ordered.
istreelist([]).
istreelist([T|TS]) :- istree(T), istreelist(TS).

% The following predicates define search algorithms that take a tree T
% and visit every node each time writing out the line D:L where
% * D is the depth (starting at 0 for the root)
% * L is the label

% dfs(T) visits every node in depth-first order
dfs(T) :- ???
% bfs(T) visits every node in beadth-first order
bfs(T) :- ???
%itd(T):- visits every node in iterative deepening order
itd(T) :- ???
```

Here "must" means you can define any number of additional predicates. But the predicates specified above must exist and must have that arity and must work correctly on any input `T` that satisfies `istree(T)`. "Working correctly" means the predicates must write out exactly what is specified, e.g.,

```
0:A
1:B
```

for the depth-first search of the tree `tree("A",[tree("B",[])])`.

Solution:

```
% initialize with depth 0
dfs(T) :- dfsD(T,0).
```

```
% write out depth and value V of the current node,
% then search all children with depth D+1
dfsD(tree(V,TS), D) :- write(D), write(":"), writeln(V),
                     Di is D+1, dfsAll(TS,Di).

% calls dfsD on all trees in a list
dfsAll([],_).
dfsAll([T|TS],D) :- dfsD(T,D), dfsAll(TS,D).

% initialize with the fringe containing T at depth 0
bfs(T) :- bfsFringe([(0,T)]).

% empty fringe - done
bfsFringe([]).
% take the first pair (D,T) in the fringe, write out D and the
% value V of T, append children TS of T paired with depth D+1
% to the *end* of F, and recurse
bfsFringe([(D,tree(V,TS))|F]) :- write(D), write(":"), writeln(V),
   Di is D+1, pair(Di,TS, DTS), append(F,DTS,F2), bfsFringe(F2).

% pair(D,L,DL) takes value D and list L and pairs every element
% in L with D, returning DL
pair(_,[],[]).
pair(D,[H|T],[(D,H)|DT]) :- pair(D,T,DT).

% initialize with cutoff 0
itd(T) :- itdUntilDone(T,0),!.

% calls dfsUpTo with cutoff C and initial depth
itdUntilDone(T,C) :- dfsUpTo(T,0,C,Done), increaseCutOffIfNotDone(T,C,Done).
% depending on the value of Done, terminate or increase the cutoff.
increaseCutOffIfNotDone(_,_,Done) :- Done=1.
increaseCutOffIfNotDone(T,C,Done) :- Done=0, Ci is C+1, itdUntilDone(T,Ci).

% dfsUpTo(T,D,U,Done) is like dfs(T,D) except that
% * we stop at cutoff depth U
% * we return Done (0 or 1) if there were no more nodes to explore

% cutoff depth reach, more nodes left
dfsUpTo(_, D, U, Done) :- D > U, Done is 0.
% write data, recurse into all children with depth D+1
dfsUpTo(tree(V,TS), D, U, Done) :- write(D), write(":"), writeln(V),
   Di is D+1, dfsUpToAll(TS,Di,U, Done).

% dfsUpToAll(TS,D,U,Done) calls dfsUpTo(T,D,U,_) on all elements of TS;
% it returns 1 if all children did
dfsUpToAll([],_,_,Done) :- Done is 1.
dfsUpToAll([T|TS],D,U,Done) :- dfsUpTo(T,D,U,DoneT),
   dfsUpToAll(TS,D,U,DoneTS), Done is DoneT*DoneTS.
```