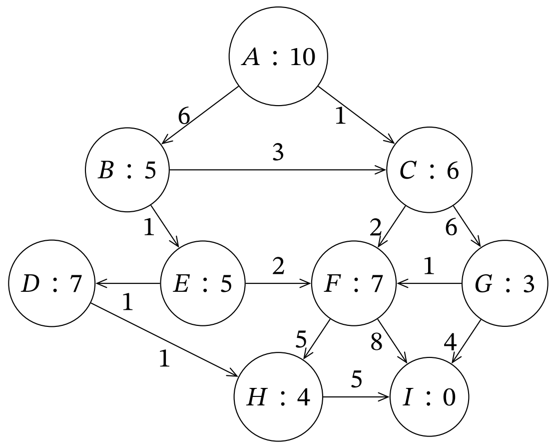
# Assignment3 – Search

**Problem 3.1 (Search Algorithms)**

Consider the following *directed graph*:




Every *node* is *labeled* with $n : h(n)$ where $n$ is the identifier of the *node* and $h(n)$ is the *heuristic* for estimating the *cost* from $n$ to a *goal node*. Every *edge* is *labeled* with its actual *cost*.

1. Assume that $I$ is the *goal node*. Argue whether or not the *heuristic* is *admissible*.

Now assume you have already *expanded* the *node* $A$. List the **next** 4 ***nodes*** **(i.e., excluding** $A$**)** that will be *expanded* using the respective *algorithm*. If there is a tie, break it using *alphabetical order*.

2. *depth-first search*
3. *breadth-first search*
4. *uniform-cost search*
5. *greedy-search*
6. $A^*$*-search*

**Problem 3.2 (Formally Modeling a Search Problem)**

Consider the Towers of Hanoi for 7 disks initially stacked on peg A.

Is this problem *deterministic*? Is it *fully observable*?

*Formally model* it as a *search problem* in the sense of the *mathematical definition* from the slides. *Explain* how your *mathematical definition* models the problem.

Note that the *formal model* only defines the problem — we are not looking for solutions here.

Note that modeling the problem corresponds to defining it in a *programming language*, except that we use *mathematics* instead of a *programming language*. Then *explaining* the model corresponds to documenting your *implementation*.

**Problem 3.3 (Heuristic Searches)**

Consider the *graph* of Romanian cities with *edges labeled* with costs $c(m, n)$ of going from $m$ to $n$. $c(m, n)$ is always bigger than the *straight-line distance* from $m$ to $n$. $c(m, n)$ is *infinite* if there is no *edge*.

Our *search algorithm* keeps:

- a *list* $E$ of *expanded nodes* $n$ together with the *cost* $g(n)$ of the cheapest *path* to $n$ found so far,
- a *fringe* $F$ containing the *unexpanded neighbors* of *expanded nodes*.

We want to find a cheap *path* from Lugoj to Bucharest. Initially, $E$ is *empty*, and $F$ *contains* only Lugoj. We *terminate* if $E$ *contains* Bucharest.

*Expansion* of a *node* $n$ in $F$ moves it from $F$ to $E$ and adds to $F$ every *neighbor* of $n$ that is not already in $E$ or $F$. We obtain $g(n)$ by *minimizing* $g(e) + c(e, n)$ over *expanded nodes* $e$.

As a *heuristic* $h(n)$, we use the *straight-line distance* from $n$ to Bucharest as given by the table in the *lectures*.

*Explain* how the following *algorithms* choose which *node* to *expand* next:

1. *greedy search* with *heuristic* $h$
2. A* *search* with *path cost* $g$ and *heuristic* $h$
3. *Explain* what $h^*$ is here and why $h$ is *admissible*.
4. For each *search*, give the *order* in which *nodes* are *expanded*.
   (You only have to give the *nodes* to get the full score. But to get partial credit in case you're wrong, you may want to include for each step all *nodes* in the *fringe* and their *cost*.)

**Problem 3.4 (Heuristics)**

Consider *heuristic search* with *heuristic* $h$.

1. Briefly *explain* what is the same and what is different between $A^*$ *search* and *greedy search* regarding the decision which *node* to *expand* next.
2. Is the *constant function* $h(n) = 0$ an *admissible heuristic* for $A^*$ *search*?

**Problem 3.5 (Tree Search in ProLog)**

*Implement* the following *tree search algorithms* in *Prolog*:

1. *BFS*
2. *DFS*
3. *Iterative Deepening* (with *step size* 1)

Remarks:

- In the *lectures*, we talked about expanding *nodes*. That is relevant in many AI *applications* where the *tree* is not built yet (and maybe even too big to hold in *memory*), such as *game trees* in *move*-based *games* or *decision trees* of *agents interacting* with an *environment*. In those cases, when visiting a *node*, we have to *expand* it, i.e., *compute* what its *children* are.

  In this problem, we work with smaller *trees* where the *search algorithm* receives the fully *expanded tree* as *input*. The *algorithm* must still visit every *node* and perform some operation on it — the *search algorithm* determines in which *order* the *nodes* are visited.

  In our case, the operation will be to *write out the* label of the *node*.

- In the *lectures*, we worked with *goal nodes*, where the *search* stops when a *goal node* is found. Here we do something simpler: we *visit all the* nodes *and operate on each one* without using a *goal state*. (Having a *goal state* is then just the special case where the operation is to *test* the *node* and possibly stop.)

*Concretely*, your submission **must** be a single *Prolog file* that extends the following *implementation*:

```
% tree(V,TS) represents a tree.
% V must be a string - the label/value/data V of the root node
% TS must be a list of trees - the children/subtrees of the root node
% In particular, a leaf node is a tree with the empty list of children
istree(tree(V,TS)) :- string(V), istreelist(TS).

% istreelist(TS) holds if TS is a list of trees.
% Note that the children are a list not a set, i.e., they are ordered.
istreelist([]).
istreelist([T|TS]) :- istree(T), istreelist(TS).

% The following predicates define search algorithms that take a tree T
% and visit every node each time writing out the line D:L where
% * D is the depth (starting at 0 for the root)
% * L is the label

% dfs(T) visits every node in depth-first order
dfs(T) :- ???
% bfs(T) visits every node in beadth-first order
bfs(T) :- ???
%itd(T):- visits every node in iterative deepening order
itd(T) :- ???
```

Here "must" *means* you can define any *number* of additional *predicates*. But the *predicates* specified above must exist and must have that *arity* and must work correctly on any *input* `T` that satisfies `istree(T)`. "Working correctly" *means* the *predicates* must write out exactly what is specified, e.g.,

```
0:A
1:B
```

for the *depth-first search* of the *tree* `tree("A",[tree("B",[])])`.