Assignment7 – Calculi

Problem 7.1 (FOL-Signatures)

1. Represent the syntax of integers using operations for 0 ,1, $x+y$, $-x$, and $x < y$ as a FOL signature. (FOL and PLNQ signatures are the same.)

Now consider the signature given by

- $\Sigma_0^f = \{a, b\}$
- $\Sigma_1^f = \{f, g\}$
- $\Sigma_2^f = \{h\}$
- $\Sigma_0^p = \{p\}$
- $\Sigma_1^p = \{q\}$
- $\Sigma_2^p = \{r\}$
- all other sets empty

2. Give a term over this signature that uses all function symbols.
3. Give a formula over this signature that uses all function and predicate symbols.

Problem 7.2 (FOL Models)

1. Consider the syntax of integers using the symbols `zero`, `one`, `plus`, `minus` (unary), and `less`. Give the model that assigns the usual interpretations in the integers.

Now consider the signature given by

- $\Sigma_0^f = \{a\}$
- $\Sigma_1^f = \{f\}$
- $\Sigma_2^f = \{h\}$
- $\Sigma_0^p = \{p\}$
- $\Sigma_2^p = \{r\}$
- all other sets empty

2. Give a model for it.
3. Give a formula over this signature that uses all function and predicate symbols and is satisfied by your model.

Problem 7.3 (Natural Deduction)

Prove the following formula using the propositional Natural Deduction calculus.

$$(A \vee B) \wedge (A \Rightarrow C) \wedge (B \Rightarrow C) \Rightarrow C$$

Problem 7.4 (Proving in Tableau Calculus)

We use the *propositional variables* P, Q, and R and define *formulae* A, B, and C by

$$A = Q \wedge (Q \Rightarrow R)$$

$$B = P \Rightarrow A$$

$$C = P \Rightarrow R$$

Prove the *formula* $B \Rightarrow C$ using the *propositional tableau calculus* $\mathcal{T}_0$.

Problem 7.5 (Logical Systems)

Fix a set V of propositional variables. We define a logical system $\langle L, K, \models \rangle$. (Note: This logical system is different from the ones in the lecture and only used here as an exercise.)

- L is the powerset of V, i.e., a formula is a set of propositional variables.
- K is the set of functions $V \to \{F, T\}$.
- For $A \in L$ and $M \in K$, $M \models A$ holds if $M(p) = T$ for all $p \in A$.

1. Try to give examples of formulas that are
   1. satisfiable
   2. falsifiable
   3. unsatisfiable
   4. valid
2. Give a sound and complete calculus for this logical system.
3. Consider the relation $H \vdash A$ holding if $A = \bigcup_{h \in H} h$. Check if $\vdash$ is a derivation relation.