Assignment10 – Knowledge Representation

Problem 10.1 (Modeling in Description Logic)

Consider the following situation:

- Some beings are persons, some are animals.
- Persons and animals may like other persons or animals.
- Alice is a person, and she likes the animal Bubbles.

1. Model this situation as a semantic network. Explain the different kinds of nodes and edges occurring in your network.

Solution: Kinds of nodes:

- represent concepts: being, person, animal
- represent individuals: Alice, Bubbles

Kinds of edges:

- assert that one concept is a subconcept of another (is-a): person-being, animal-being
- assert that one individual is an instance of a concept (inst): Alice-person, Bubbles-animal
- represent a relation between two concepts: an edge labeled 'like' relation person-animal (These edges can often be omitted.)
- assert that two individuals are in a relation: an edge labeled 'like' Alice-Bubbles

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2. Model the same situation in first-order logic and compare the results.

Solution: Nodes (by kind):

- For each concept-node c, we need a unary predicate symbol: $\mathtt{being} \in \Sigma_1^p$, $\mathtt{person} \in \Sigma_1^p$, $\mathtt{animal} \in \Sigma_1^p$.
- For each individual-node i, we need a nullary function symbol: $\mathtt{Alice} \in \Sigma_0^f$, $\mathtt{Bubbles} \in \Sigma_0^f$.

Edges (by kind):

- For each subconcept-edge (is-a) from c to d, we need an axiom $\forall x.c(x) \Rightarrow d(x)$. Here: $\forall x.\mathtt{person}(x) \Rightarrow \mathtt{being}(x)$ and $\forall x.\mathtt{animal}(x) \Rightarrow \mathtt{being}(x)$.
- For each instance-edge (inst) from i to c, we need an axiom $c(i)$. Here: $\mathtt{person}(\mathtt{Alice})$ and $\mathtt{person}(\mathtt{Bubbles})$.

- For each relation-representing-edge r from c to d, we need a binary predicate symbol. Here: $like \in \Sigma_2^p$. To represent the subject and object concepts, we can add axioms $\forall x, y.r(x, y) \Rightarrow c(x)$ and $\forall x, y.r(x, y) \Rightarrow d(y)$. Here: $\forall x, y.\texttt{like}(x, y) \Rightarrow \texttt{person}(x)$ and $\forall x, y.\texttt{like}(x, y) \Rightarrow \texttt{animal}(y)$.
- For each relation-asserting-edge for r from i to j, we need axiom $r(i, j)$. Here: $\texttt{like}(\texttt{Alice}, \texttt{Bubbles})$.

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3. Explain the difference between inst and is-a edges.

Solution: inst edges are between an individual and a concept. They correspond to $\in$ math.
is-a edges are between two concepts. They correspond to $\subseteq$ in math.

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4. Explain the difference between having a relation edge between two concepts vs. asserting a relation between two individuals.

Solution: The former models that a relation r exists between individuals of the respective concepts. That corresponds to a FOL-predicate symbol in Σ_2^p. For each pair of individuals the relation may be true or false.
The latter asserts that that relation holds for two individuals. For example, an r-edge from i to j corresponds to the FOL-axiom $r(i, j)$.

Problem 10.2 (ALC)

Consider the following description logic signature

- concept symbols: i (for instructor), s (for student), c (for course), p (for program)
- role symbol m (for is-member-of) used for
  - instructors giving a course
  - students taking a course
  - students being enrolled in a degree program
  - courses being part of a degree program

We use an extension of $\mathcal{ALC}$, in which there are dual roles: there is a role m^{-1} that captures the relation has-as-member, e.g., $MK\ m\ AI$ iff $AI\ m^{-1}\ MK$.

1. For the signature above, give a concept axiom that captures that instructors can only be members of courses.

Solution: $i \sqsubseteq \forall m.c$

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2. Give a concept axiom for the above signature that captures: courses that are taken by a student, must be given by an instructor.

Solution: $c \sqcap \exists m^{-1}.s \sqsubseteq \exists m^{-1}.i$

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3. Calculate the translation to first-order logic of $s \sqsubseteq \forall m.\exists m.p$.

Solution: $\forall x.s(x) \Rightarrow (\forall y.m(x,y) \Rightarrow (\exists z.m(y,z) \wedge p(z)))$

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4. Given a first-order model $\langle \mathcal{D}, \mathcal{I} \rangle$, define an appropriate case of the interpretation mapping for the formula $\forall r^{-1}.C$.

Solution: $[\![\forall r^{-1}.C]\!] = \{u \in D \mid \text{ for } v \in \mathcal{D}, \text{ if } (v,u) \in [\![r]\!], \text{ then } v \in [\![C]\!]\}$

Problem 10.3 (ALC Semantics)

Consider the $\mathcal{ALC}$ concepts $\forall R.(C \sqcap D)$ and $\forall R.C \sqcap \forall R.D$.

1. By applying the semantics of $\mathcal{ALC}$, show that the two are equivalent.

Solution: We have:

$$[\![\forall R.(C \sqcap D)]\!]$$
$$= \{x \in \mathcal{D} \mid \text{ for all } y \in \mathcal{D}, \text{ if } (x,y) \in [\![R]\!], \text{ then } y \in [\![C \sqcap D]\!]\}$$
$$= \{x \in \mathcal{D} \mid \text{ for all } y \in \mathcal{D}, \text{ if } (x,y) \in [\![R]\!], \text{ then } y \in ([\![C]\!] \cap [\![D]\!])\}$$

$$[\![\forall R.C \sqcap \forall R.D]\!]$$
$$= [\![\forall R.C]\!] \cap [\![\forall R.D]\!]$$
$$= \{x \in \mathcal{D} \mid \text{ for all } y \in \mathcal{D}, \text{ if } (x,y) \in [\![R]\!], \text{ then } y \in ([\![C]\!] \cap Q)\}$$

where

$$Q = \{x \in \mathcal{D} \mid \text{ for all } y \in \mathcal{D}, \text{ if } (x,y) \in [\![R]\!], \text{ then } y \in [\![D]\!]\}$$

Now to prove that sets are equal, consider an $x \in \mathcal{D}$ and see that both conditions are equivalent to

$$\text{for all } y \in \mathcal{D}, \text{ if } (x,y) \in [\![R]\!], \text{ then } y \in [\![C]\!] \text{ and } y \in [\![D]\!]$$

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2. Translate both formulas to first-order logic and state which FOL formula we would need to prove (e.g., with the ND calculus) to show that the two are equivalent.

Solution: The translation yields

$$C_1(x) = \forall y.R(x, y) \Rightarrow C(y) \wedge D(y)$$

$$C_2(x) = (\forall y.R(x, y) \Rightarrow C(y)) \wedge (\forall y.R(x, y) \Rightarrow D(y))$$

We need to show

$$\forall x.C_1(x) \Leftrightarrow C_2(x)$$

Problem 10.4 (ALC TBox)

Consider $\mathcal{ALC}$ with the following

- primitive concepts: woman, man
- roles: has_child, has_parent, has_sibling, has_spouse

Give an $\mathcal{ALC}$ TBox that defines the concepts person, parent, mother, father, grandmother, aunt, uncle, sister, brother, onlychild, cousin, nephew, niece, fatherinlaw, motherinlaw.

Solution: $\mathsf{person} = \mathsf{man} \sqcup \mathsf{woman}$

$\mathsf{parent} = \mathsf{person}\exists\mathsf{has\_child}.\mathsf{person}$
$\mathsf{mother} = \mathsf{womanparent}$
$\mathsf{father} = \mathsf{manparent}$
$\mathsf{grandmother} = \mathsf{woman}\exists\mathsf{has\_child}.\mathsf{parent}$
$\mathsf{aunt} = \mathsf{woman}\exists\mathsf{has\_sibling}.\mathsf{parent}$
$\mathsf{uncle} = \mathsf{man}\exists\mathsf{has\_sibling}.\mathsf{parent}$
$\mathsf{sister} = \mathsf{woman}\exists\mathsf{has\_sibling}.\mathsf{person}$
$\mathsf{brother} = \mathsf{man}\exists\mathsf{has\_sibling}.\mathsf{person}$
$\mathsf{onlychild} = \mathsf{person}\overline{\mathsf{brother} \sqcup \mathsf{sister}}$
$\mathsf{cousin} = \mathsf{person}\exists\mathsf{has\_parent}.\exists\mathsf{has\_sibling}.\mathsf{parent}$
$\mathsf{nephew} = \mathsf{man}\exists\mathsf{has\_parent}.\exists\mathsf{has\_sibling}.\mathsf{person}$
$\mathsf{niece} = \mathsf{woman}\exists\mathsf{has\_parent}.\exists\mathsf{has\_sibling}.\mathsf{person}$
$\mathsf{fatherinlaw} = \mathsf{man}\exists\mathsf{has\_child}.\exists\mathsf{has\_spouse}.\mathsf{person}$
$\mathsf{motherinlaw} = \mathsf{woman}\exists\mathsf{has\_child}.\exists\mathsf{has\_spouse}.\mathsf{person}$
