# Assignment9 – First-Order Logic

**Problem 9.1 (Predicate Logic)**

Consider the following *first-order signature* $\Sigma$:

- *binary function constant* $f$
- *unary predicate constant* $p$

1. Give a *closed formula* over $\Sigma$ that uses all its constants.
2. Give a *first-order model* for $\Sigma$
3. Consider the *formula* $A = p(x) \wedge p(y)$ and assume a *first-order model* with *universe* $\mathbb{N}$ and $\mathcal{I}(p) = \{0\}$.
   Give an *assignment* $\alpha$ such that $\mathcal{I}_\alpha(A) = \mathsf{T}$.
4. Prove or refute the following statement: A *first-order model* that *satisfies* $\forall x.p(x)$, *satisfies* all *formulae*.

**Problem 9.2 (Unification)**

Let $S_1 \in \Sigma_2^f$, $S_2 \in \Sigma_3^f$, $f \in \Sigma_1^f$, $g \in \Sigma_2^f$, $c \in \Sigma_0^f$

Decide whether (and how or why not) the following pairs of terms are unifiable.

1. $S_1(g(f(x), g(x, y)), y)$ and $S_1(g(z, v), f(w))$
2. $S_2(g(f(x), g(x, u)), f(y), z)$ and $S_2(g(g(g(u, v), f(w)), f(c)), f(g(u, v)), f(c))$

**Problem 9.3 (Natural Deduction)**

Let $R \in \Sigma_2^p$, $P \in \Sigma_1^p$, $c \in \Sigma_0^f$.

Prove the following formula in Natural Deduction:

$$(\forall X.\forall Y.R(Y, X) \Rightarrow P(Y)) \wedge (\exists Y.R(c, Y)) \Rightarrow P(c)$$

**Problem 9.4 (First-Order Tableaux)**

Prove or refute the following formula using the first-order *free variable tableaux calculus*. We have $P, Q \in \Sigma_1^p$.

$$(\forall X.P(X) \Rightarrow Q(X)) \Rightarrow (((\forall X.P(X)) \Rightarrow (\forall X.Q(X))))$$

**Problem 9.5 (First-Order Resolution)**

Let $P, Q \in \Sigma_1^p$, $R \in \Sigma_2^p$, $c, d \in \Sigma_0^f$. Prove the following *formula* using the *first-order resolution calculus* $\mathcal{R}_1$.

$$\exists X.\forall Y.\exists W.\exists Z.\neg((R(Z, Y) \vee \neg P(Z)) \wedge (\neg Q(d) \vee P(c)) \wedge (Q(d) \vee \neg P(c)) \wedge (\neg R(Z, Y) \vee \neg P(W) \vee \neg Q(X)) \wedge P(c))$$

*Hint:* Note, that the formula is already (close to) a negated CNF, so if you spend any significant amount of time transforming the formula, you are most likely doing something wrong.