Assignment6 – Propositional Logic

Problem 6.1 (PL Concepts)

Which of the following statements are true? In each case, give an informal argument why it is true or a counter-example.

1. Every satisfiable formula is valid.
2. Every valid formula is satisfiable.
3. If A is satisfiable, then $\neg A$ is unsatisfiable.
4. If $A \vDash B$, then $A \wedge C \vDash B \wedge C$.
5. Every admissible inference rule is derivable.
6. If $\vdash$ is sound for $\vDash$ and $\{A, B\} \vdash C$, then C is satisfiable if A and B are.

Problem 6.2 (Propositional Logic in Prolog)

We implement propositional logic in Prolog.
We use the following Prolog terms to represent Prolog formulas

- lists of strings for signatures (each element being the name of a propositional variables)
- `var(s)` for a propositional variable named `s`, which is a string,
- `neg(F)` for negation,
- `disj(F,G)` for disjunction,
- `conj(F,G)` for conjunction,
- `impl(F,G)` for implication.

1. Implement a Prolog predicate `isForm(S,F)` that checks if `F` is well-formed formula relative to signature `S`.
   Examples:

```
?- isForm(["a","b"],neg(var("a"))).
True
```

```
?- isForm(["a","b"],neg(var("c"))).
False
```

```
?- isForm(["a","b"],conj(var("a"),impl(var("b")))).
False
```

2. Implement a Prolog predicate `simplify(F,G)` that replaces all disjunctions and implications with conjunction and negation.
   Examples:

```
?- simplify(disj(var("a"),var("b")), X).
X = neg(conj(neg(var("a")),neg(var("b")))).
```

Note that there is more than one possible simplification of a term, so your results may be different (but should be logically equivalent).

3. Implement a predicate `eval(P,F,V)` that evaluates a formula under assignment `P`. Here `P` is a list of terms `assign(s,v)` where `s` is the name of a propositional variable and `v` is a truth value (either 1 or 0). You can assume that `P` provides exactly one assignment for every propositional variable in `F`.
   Example:

```
?- eval([assign("a",1),assign("b",0)], conj(var("a"), var("b")), V).
V = 0.

?- eval([assign("a",1),assign("b",1)], conj(var("a"), var("b")), V).
V = 1.
```

Problem 6.3 (PL Semantics)

We work with a propositional logic signature declaring variables A and B. For each of the fomulae blow we use a fixed but arbitrary assignment φ for the propositional variables.

For each of the two formulas F, apply the definition of the interpretation $\mathcal{I}_\varphi(F)$ step-by-step to obtain the semantic condition that F holds under φ. Afterwards determine if F is valid or not by one of the following:

- argue why $\mathcal{I}_\varphi(F)$ is true, which means F is valid because it holds for an arbitrary φ,
- give an assignment φ that makes $\mathcal{I}_\varphi(F)$ false

1. $A \Rightarrow (B \Rightarrow A)$
2. $A \wedge B \Rightarrow A \wedge C$