Assignment4 – Adversarial Search

Problem 4.1 (Games for Adversarial Search)

Consider the following properties a game can have.

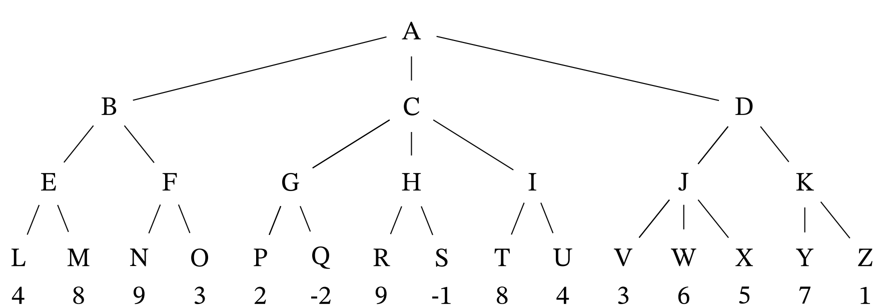
- A 2 players alternating moves
- I discrete state space
- C players have complete information about state
- F finite number of move options per state
- E deterministic successor states n
- T games guaranteed to terminate
- U terminal state has zero-sum utility.

For each of the following games, state whether the game violates the property; fill in the corresponding letters (no spaces) into the box or write "none".

1. 2-player poker (until one player is bankrupted): `CET`
2. Backgammon: `ET`
3. Wrestling (one 5 minute round): `AIF`
4. Connect Four: `none`
5. Rock-Paper-Scissors (with a repeat to break ties): `AT`
6. Meta-Game (player 1's first move is to choose a game that satisfies all properties, subsequent moves play that game): `F`

Problem 4.2 (Game Tree)

Consider the following game tree. Assume it is the maximizing player's turn to move. The values at the leaves are the evaluation function values of the states at each of those nodes.



1. Compute the minimax game value of nodes A, B, C, and D.

Solution: B = 8, C = 2, D = 6, A = 8

2. Max would select move $\boxed{B}$
3. List the nodes that the alpha-beta algorithm would prune (i.e., not visit). Assume children of a node are visited left-to-right.

Solution: O, H (and R and S), I (and T and U), K (and Y and Z)

4. What reordering of the evaluation function values at the bottom would prune as many branches as possible?

Solution:

Problem 4.3 (Minimax Applicability)

Consider the following game:

- Initially, 2 players have 10 tokens each, and there is an empty bag of tokens in the center.
- The players take turns either putting an odd number of tokens into the bag or taking a non-zero even number of tokens from the bag.
- A player loses if they have no more tokens.

Explain why minimax can/cannot be used to find a perfect strategy for this game.

Solution: It cannot be used. The game is not guaranteed to be finite, e.g., the move sequence put 1, put 1, take 2 could repeat forever.

Problem 4.4 (Minimax Search in ProLog)

Consider the following game:

1. There is a pile of n matches in the middle.
2. Two players alternate taking away 1, 2, or 3 matches.
3. The winner is whoever takes the last match.

Solve this game (for all values of n) by implementing the minimax algorithm in Prolog. Specifically, implement exactly the following

- a Prolog predicate `value(S,P)` that holds if player `P` wins from initial state `S`,
- where the Prolog constructor `state(N,P)` represents the game state with `N` remaining matches and player `P` going next,
- where we represent players `P` using 1 for the starting player and −1 for the opponent.

Note: A partial solution will be explained in the tutorials, especially the use of `\+` for negation-as-failure and `!` for cut.

Solution:

```
% Game state: number N of remaining matches and current player P=1 or P=-1

% possible moves in state(N,P) yielding successor state T
successor(state(N,P),T) :- N>0, N2 is N-1, P2 is -P, T=state(N2,P2).
successor(state(N,P),T) :- N>1, N2 is N-2, P2 is -P, T=state(N2,P2).
successor(state(N,P),T) :- N>2, N2 is N-3, P2 is -P, T=state(N2,P2).

% membership in a list
contains([H|T],A) :- not(H=A), contains(T,A).
contains([A|_],A).

% find list Ts of succesor states of S using accumulator Acc
successors(S, Acc, Ts) :- successor(S,T), \+ contains(Acc,T), !,
                                        successors(S, [T|Acc], Ts).
successors(_, Acc, Acc).

% shown until here in the tutorials

% minvalue(Ss,Sofar,V) holds if V is the minimum value of list of states Ss
% Sofar is accumulator for minimum value seen so far

% end of list - return accumulator
minvalue([],Sofar,Sofar).
% next state has smaller value, replace accumulator and continue with rest
minvalue([S|Ss],Sofar,V) :- value(S,V1), V1<Sofar, minvalue(Ss,V1,V).
% next state has non-smaller value, keep accumulator and continue with rest
minvalue([S|Ss],Sofar,V) :- value(S,V1), V1>=Sofar, minvalue(Ss,Sofar,V).

% like minvalue
maxvalue([],Sofar,Sofar).
maxvalue([S|Ss],Sofar,V) :- value(S,V1), V1>=Sofar, maxvalue(Ss,V1,V).
maxvalue([S|Ss],Sofar,V) :- value(S,V1), V1<Sofar, maxvalue(Ss,Sofar,V).

% value(S,V) holds if state S has winner V (1 or -1)

% our turn (P=1): choose successor with maximal value
% we lose if no possible move (Ts=[], accumulator initialized to -1)
value(S,V) :- state(_,P)=S, P = 1, successors(S,[],Ts), maxvalue(Ts,-1,V).

% opponent's turn (P=-1): choose successor with minimal value
% we win if no possible move (Ts=[], accumulator initialized to 1)
value(S,V) :- state(_,P)=S, P = -1, successors(S,[],Ts), minvalue(Ts,1,V).
```