Prof. Alexandra Nilles

Western Washington University

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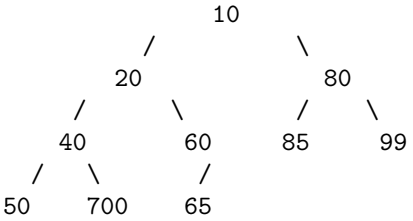
Announcements

- Mid-term is underway!
- If in DAC testing center, message me your scheduled time
- HW 4 due this Friday
- Delayed office hours today

Heaps

- Definition: a complete binary tree with vertical ordering
- vertical ordering: for every element X with parent P, P <= X
- Complete: every "level" is full except last level, which is filled left to right. Guarantees height goes like O(log N).
- "Min heap": minimum element is always the root, parents always smaller than children
- "Max heap": reverse ordering, max value at root

Add element to heaps



1. Add 15: right child of 60
2. Restore heap ordering: new element is shifted up tree to proper place
   - "percolate" / "bubbling" / "sifting"
   - swap with parent

Remove element from heap

- Case: remove root
- replace with rightmost leaf (to maintain completeness)
  - but this breaks our ordering property!
- Solution: bubble *down*: swap with smaller child (why?)
- log(n) bubbles up or down for adding / removing

Array representation

- Since we know we have a *complete* tree, with careful indexing we can implement a heap in an array
- Let index of root = 1
- For any node n at index i:
  - index of `n.left` = 2i
  - index of `n.right` = 2i+1
  - Parent index of n?
    - floor of i/2
- If array runs out of space, we can copy data into larger array

Heap Sort

- Basic idea: add each element to heap, then remove top element N times
- "Selection sort with the right data structure"
- In-place solution: maintain max-heap in array, build sorted array from back to front of array
- Algorithm performance? O(n lg n)
- Unstable, in-place
- Good bound on worst-case scenarios, makes it well suited for real-time aapplications
- Not easily parallelizable

Priority Queue

- "Abstract Data Type" usually implemented with heaps
- ADT vs data structure?
  - ADT is more formal, explicitly a mathematical model, defined by semantics: in, out, and invariants / guarantees
  - data structure refers to a particular implementation