



# Continuous Wavelet Transformation for Spectroscopy

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University of Minnesota

ASCEND summer training 2024



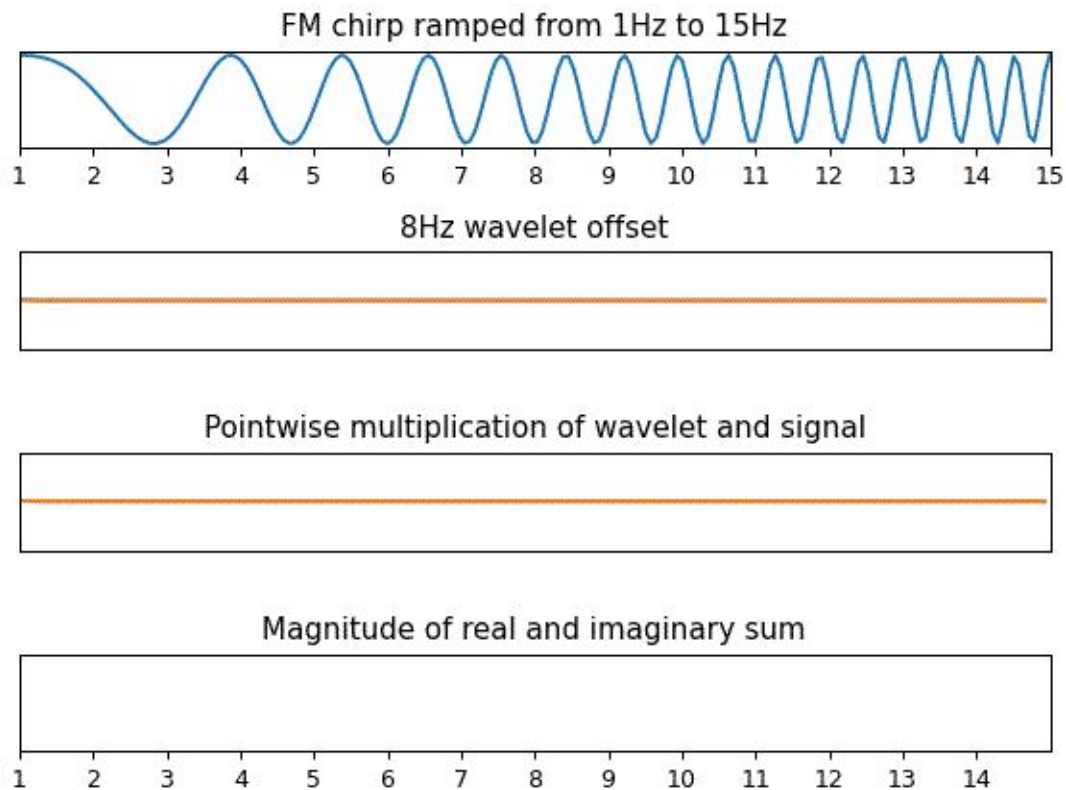
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# Continuous Wavelet Transformation

- CWT is a powerful tool for analyzing signals that vary over time (e.g., spectrum).
- It provides an overcomplete representation of a signal ( $t$ ) by letting the translation ( $b$ ) and scale ( $a$ ) parameter of the wavelets ( $f$ ) vary **continuously**.

$$\text{CWT}(a, b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} f(t) \psi^* \left( \frac{t-b}{a} \right) dt$$

# Continuous Wavelet Transformation

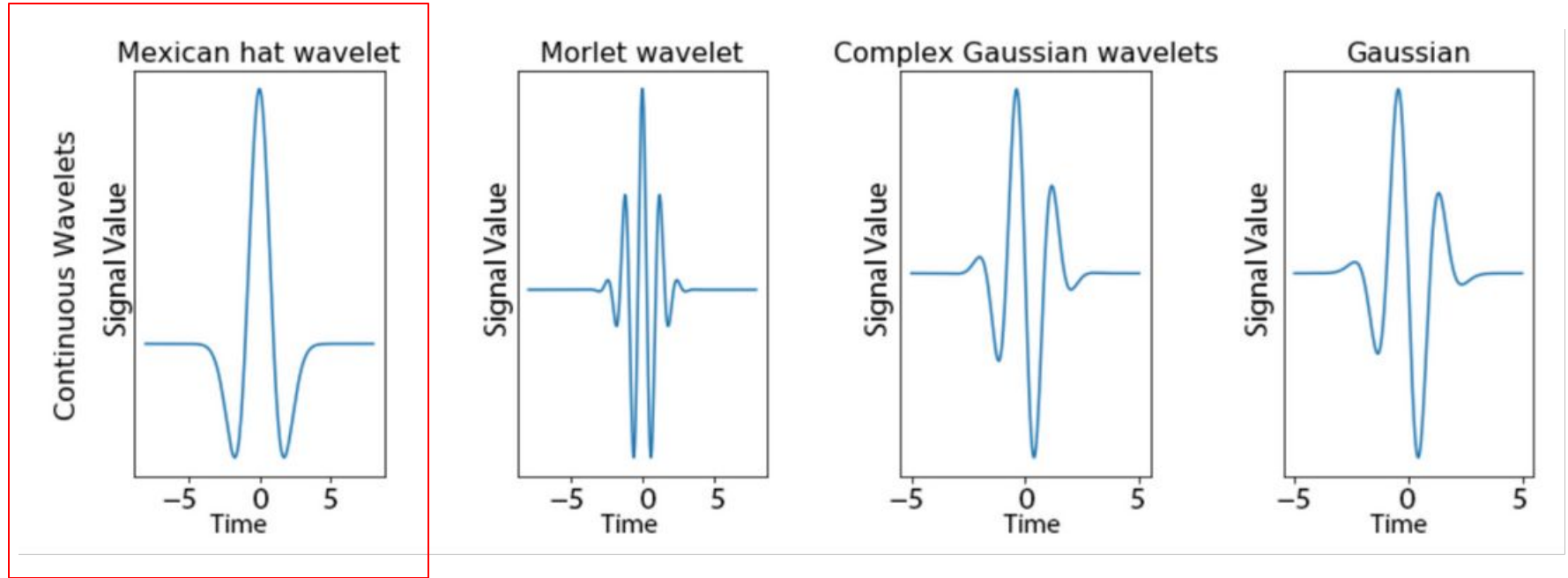


# Continuous Wavelet Transformation

Important elements of CWT:

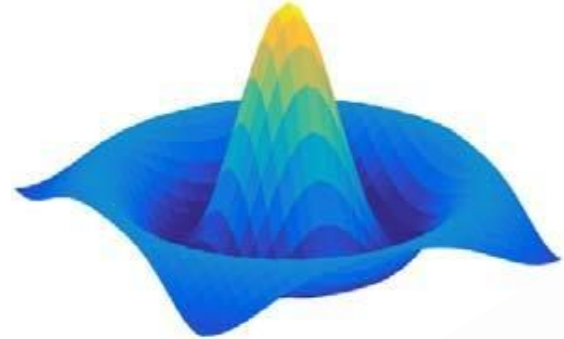
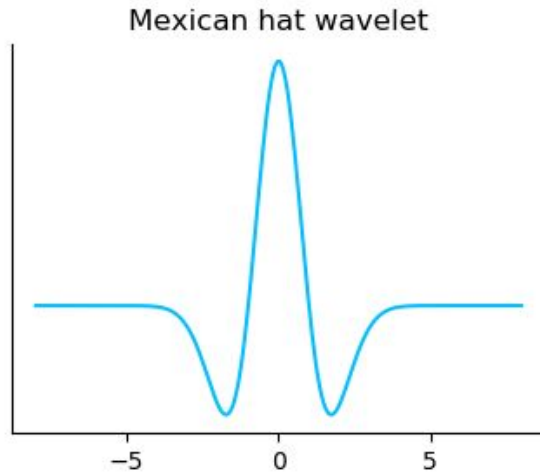
- Wavelet function (mother wavelet)
- Scale factor
- Translation (how the wavelet moves through the signal)

# Wavelet function

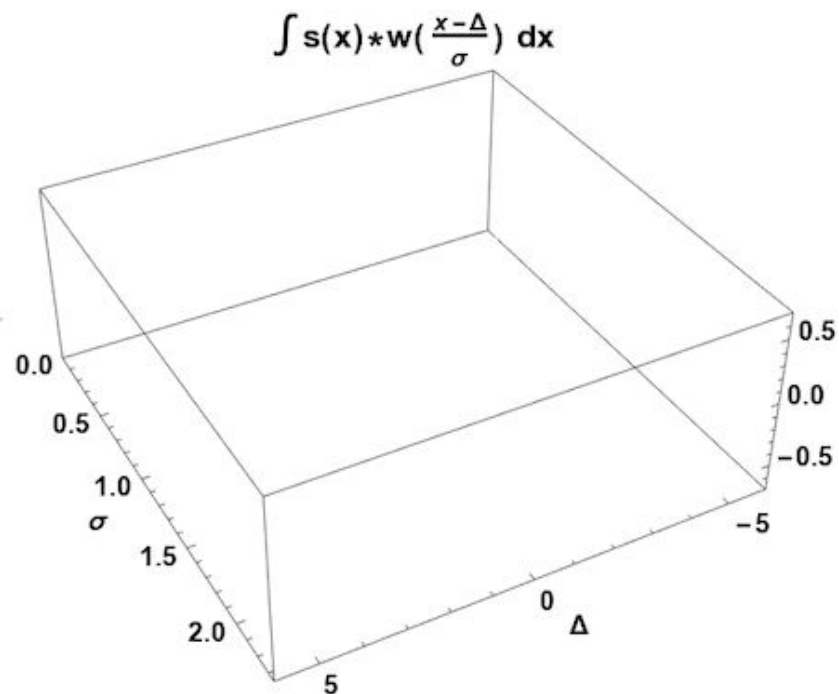
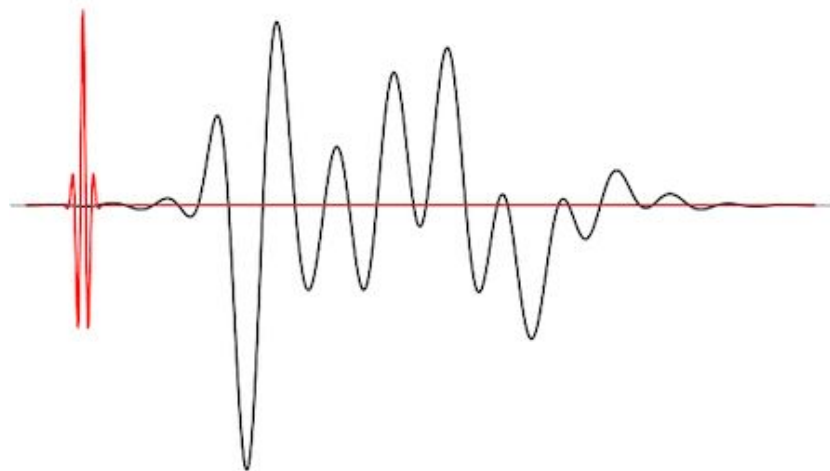


# Wavelet function

Mexican hat wavelet is also known as second derivative gaussian or Ricker wavelet.



# Scale factor



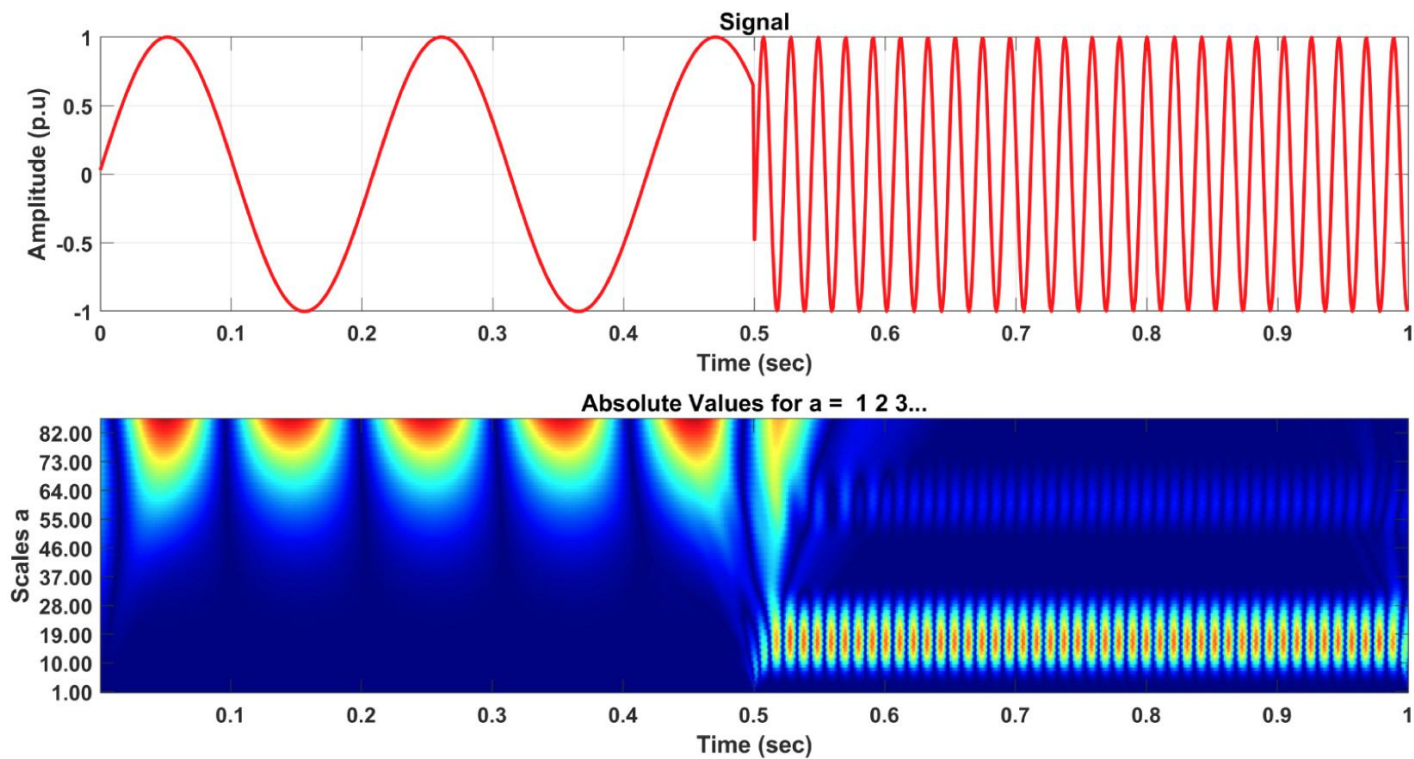
# Scale factor

Principles associated with the scales:

- It must be non-zero
- It must be applied on regular time-series (regularly spaced intervals)
- It can not be smaller than the spaced intervals



# Scalogram



# CWT and spectroscopy

Some papers applied CWT:

- Rivard et al. 2002



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Remote Sensing of Environment 112 (2008) 2850–2862

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of  
Environment

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[www.elsevier.com/locate/rse](http://www.elsevier.com/locate/rse)

Continuous wavelets for the improved use of spectral libraries and  
hyperspectral data

B. Rivard<sup>\*</sup>, J. Feng, A. Gallie, A. Sanchez-Azofeifa

# CWT and spectroscopy

Some papers applied CWT:

- Rivard et al. 2002
- Chen et al. 2011
- Chen et al. 2012
- Chen et al. 2014



Journal of Plant Physiology  
Volume 169, Issue 12, 15 August 2012, Pages 1134-1142



Predicting leaf gravimetric water content from foliar reflectance across a range of plant species using continuous wavelet analysis



Remote Sensing of Environment  
Volume 115, Issue 2, 15 February 2011, Pages 659-670



Spectroscopic determination of leaf water content using continuous wavelet analysis



ISPRS Journal of Photogrammetry and Remote Sensing  
Volume 87, January 2014, Pages 28-38



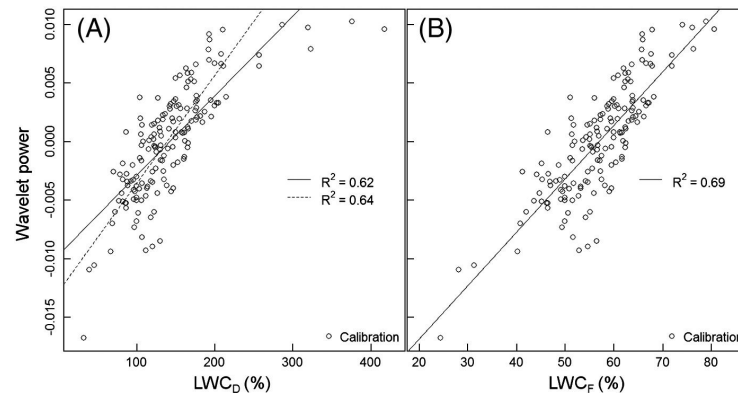
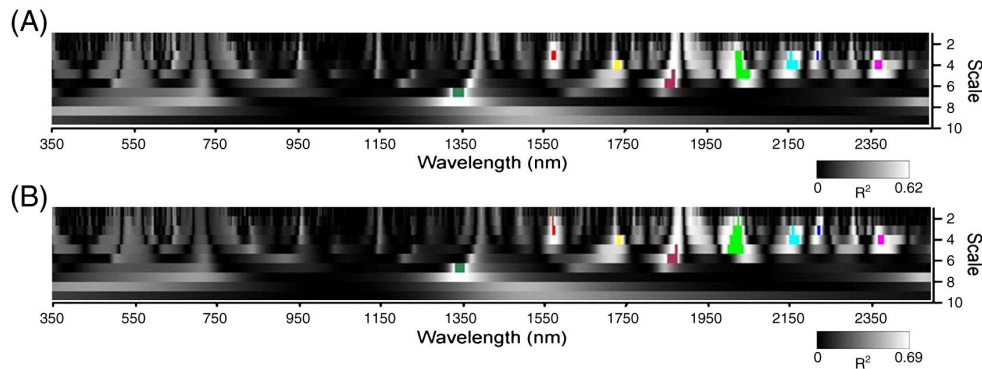
Deriving leaf mass per area (LMA) from foliar reflectance across a variety of plant species using continuous wavelet analysis

Tao Cheng<sup>a</sup> , Benoit Rivard<sup>b</sup>, Arturo G. Sánchez-Azofeifa<sup>b, c</sup>, Jean-Baptiste Féret<sup>d</sup>, Stéphane Jacquemoud<sup>e</sup>, Susan L. Ustin<sup>a</sup>

# CWT and spectroscopy

Some papers applied CWT:

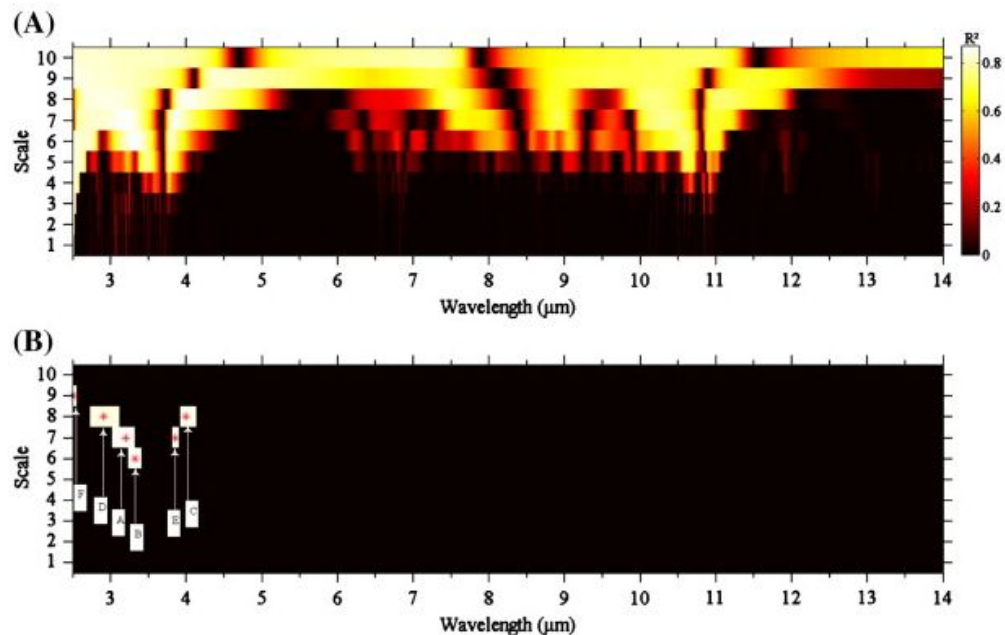
- Rivard et al. 2002
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# CWT and spectroscopy

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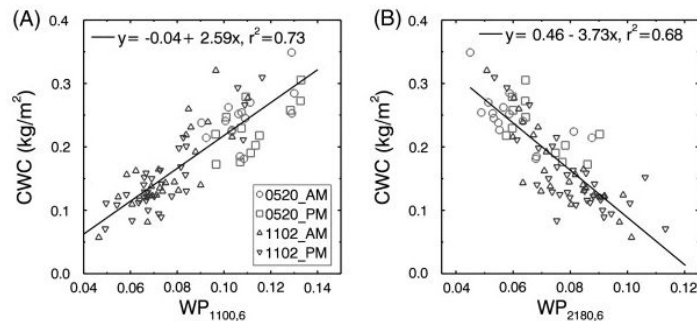
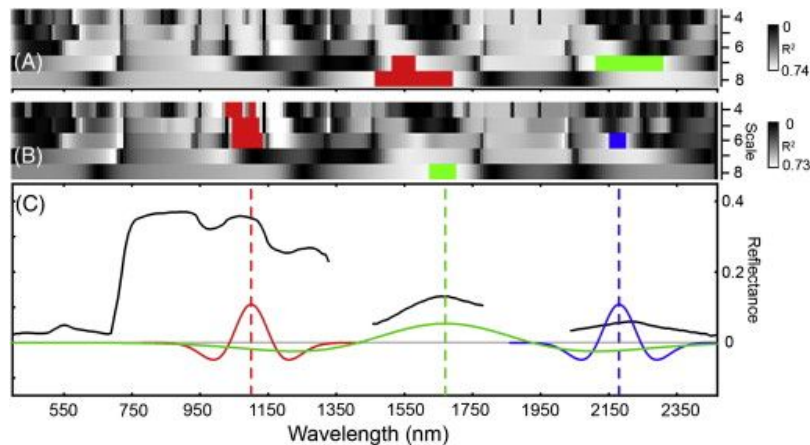
- Rivard et al. 2002
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- Chen et al. 2014
- Ullah et al. 2012



# CWT and spectroscopy

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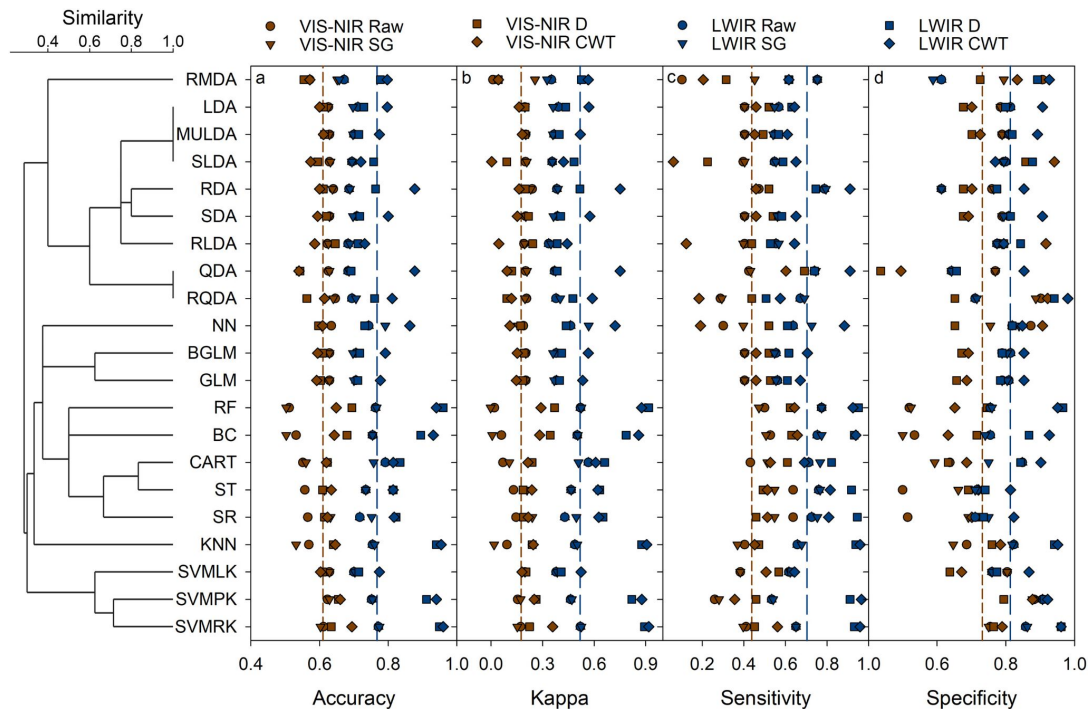
- Rivard et al. 2002
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- Chen et al. 2014
- Ullah et al. 2012
- **Cheng et al. 2014**



# CWT and spectroscopy

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- Rivard et al. 2002
- Chen et al. 2011
- Chen et al. 2012
- Chen et al. 2014
- Ullah et al. 2012
- Cheng et al. 2014
- **Harrison et al. 2018**
- **Guzmán et al. 2018**

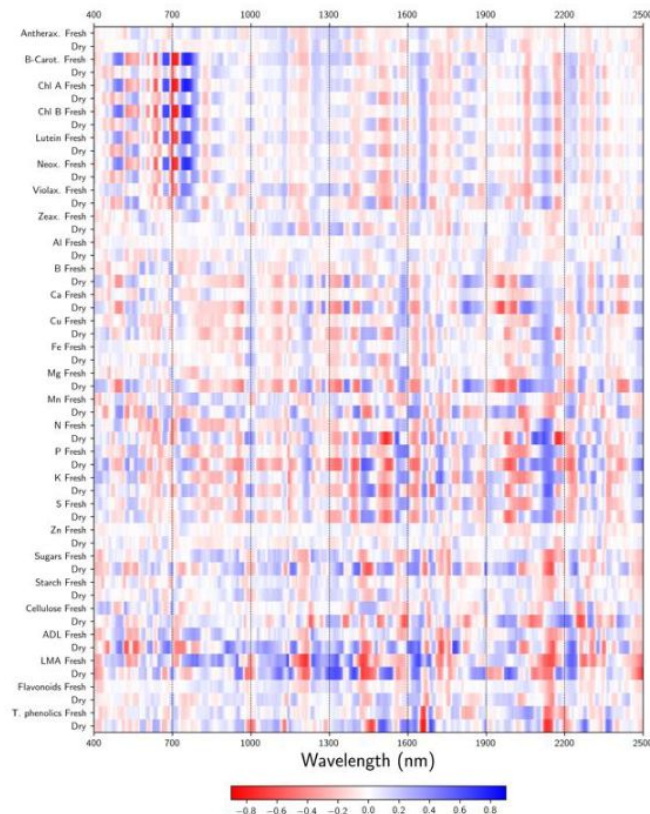




# CWT and spectroscopy

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- Chen et al. 2014
- Ullah et al. 2012
- Cheng et al. 2014
- Harrison et al. 2018
- Guzmán et al. 2018
- **Adams Chlu's dissertation 2020**



**Figure 1.7** Correlation matrix showing mean wavelet correlations across all scales for each trait and spectral measurement type.



# CWT and spectroscopy

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- Chen et al. 2012
- Chen et al. 2014
- Ullah et al. 2012
- Cheng et al. 2014
- Harrison et al. 2018
- Guzmán et al. 2018
- Adams Chlu's dissertation 2020
- **Guzmán and Sanchez-Azofeifa 2021**



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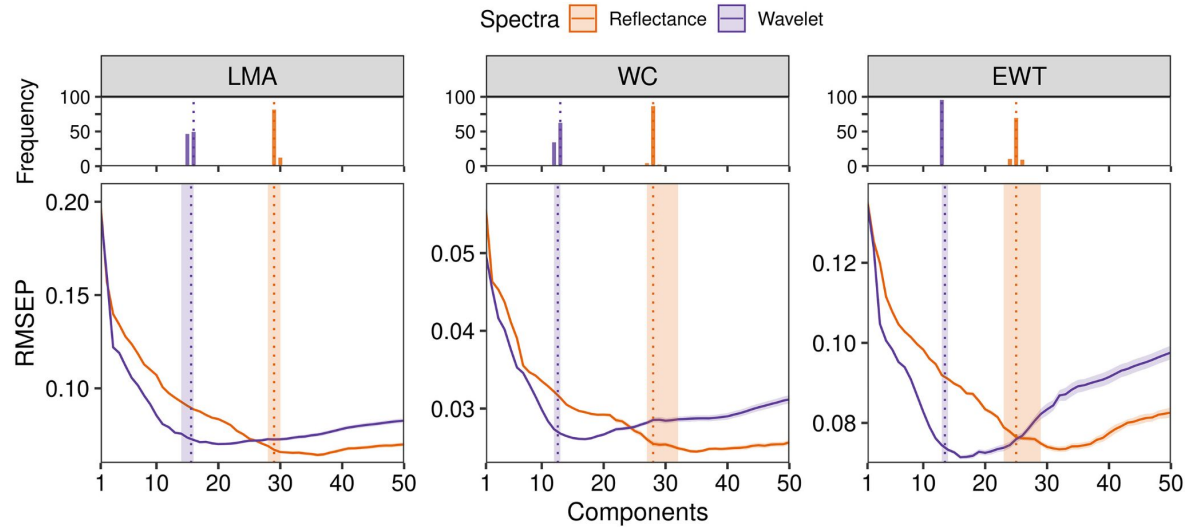
Prediction of leaf traits of lianas and trees via the integration of wavelet spectra in the visible-near infrared and thermal infrared domains

J. Antonio Guzmán Q.  , G. Arturo Sanchez-Azofeifa  

# CWT and spectroscopy

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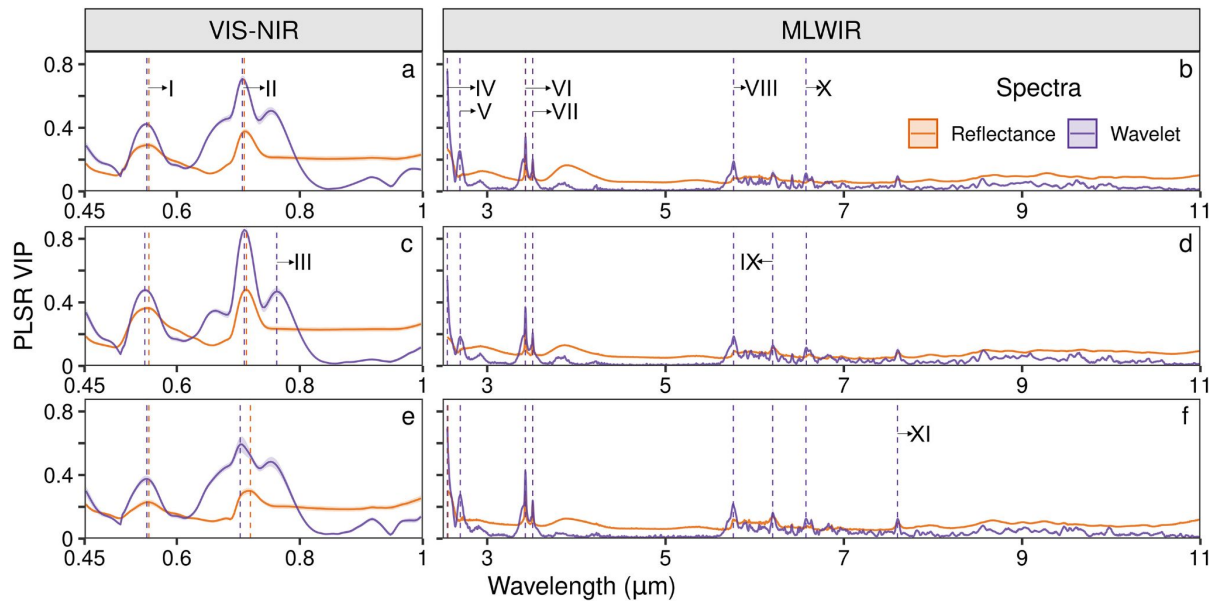
- Guzmán and Sanchez-Azofeifa 2021



# CWT and spectroscopy

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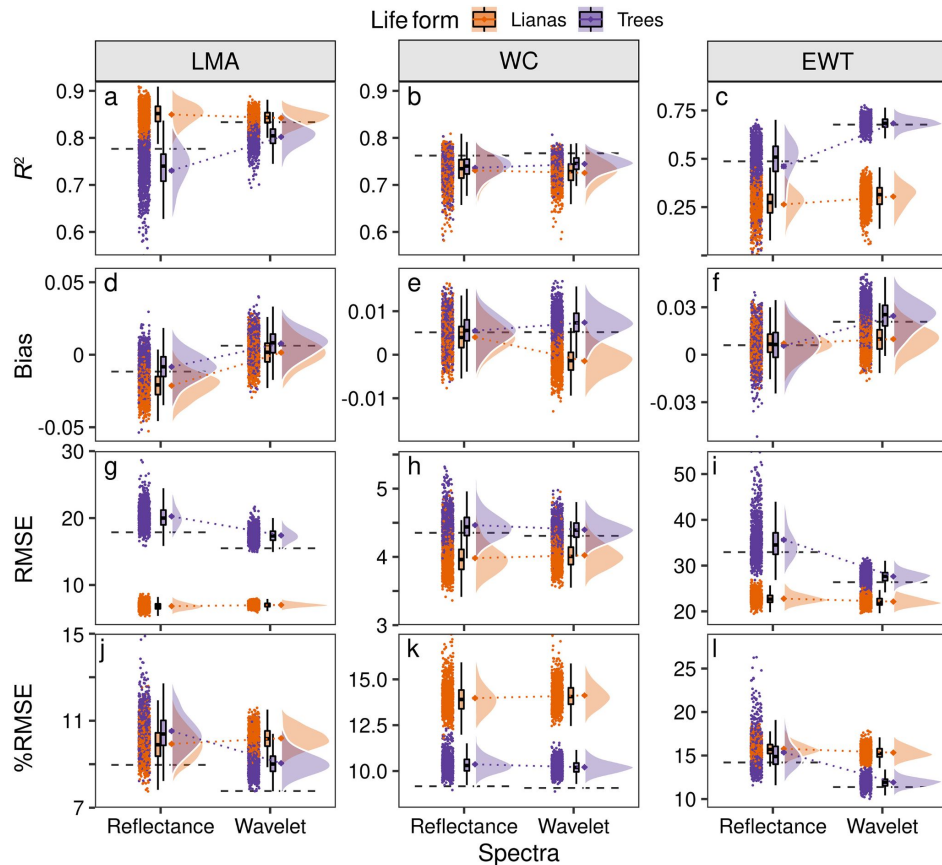
- Guzmán and Sanchez-Azofeifa 2021



# CWT and spectroscopy

Some papers applied CWT:

- Guzmán and Sanchez-Azofeifa 2021



# Let's play with wavelets

The goal:

- Apply wavelet spectra to evaluate its properties
- Evaluate the integration of wavelet spectra with PLSR to predict leaf traits

<https://github.com/ASCEND-BII/wavelets-training>

## wavelets-training

A repository for the use training on "Continuous Wavelet Transformation for Spectroscopy" at ASCEND summer training 2024

### Requirements

Users will need to have R ( $\geq 4.0.0$ ) and RStudio installed in their computers. Users will also have to clone through Github or download this repository in their local computer. Then, install the following libraries:

```
install.packages("data.table")
install.packages("CWT")
install.packages("ccrmm")
install.packages("pls")
install.packages("plsVarSel")
install.packages("parallel")
install.packages("ggplot2")
install.packages("viridis")
```

