Continuous Wavelet Transformation for Spectroscopy

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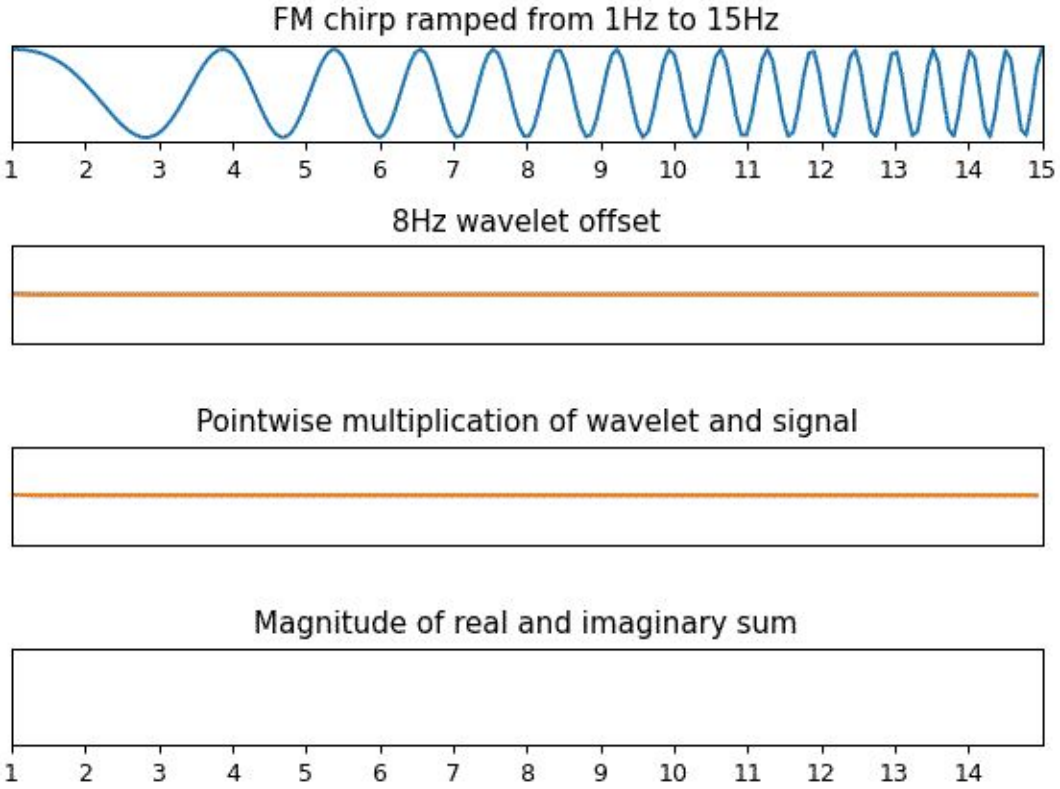
Continuous Wavelet Transformation

- CWT is a powerful tool for analyzing signals that vary over time (e.g., spectrum).

- It provides an overcomplete representation of a signal (*t*) by letting the translation (*b*) and scale (*a*) parameter of the wavelets (*f*) vary **continuously**.

$$\mathrm{CWT}(a,b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} f(t)\psi^* \left(\frac{t-b}{a}\right) dt$$

Continuous Wavelet Transformation



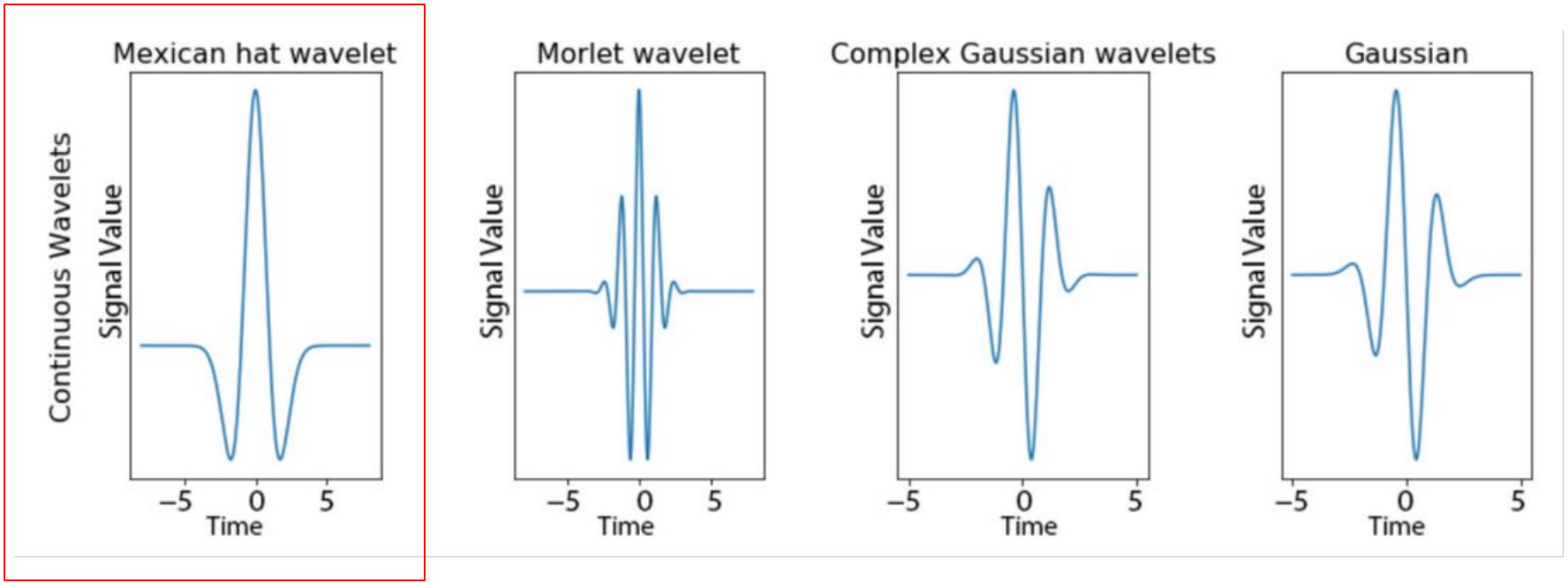
Barrett (2022) - peterbbryan.medium.com

Continuous Wavelet Transformation

Important elements of CWT:

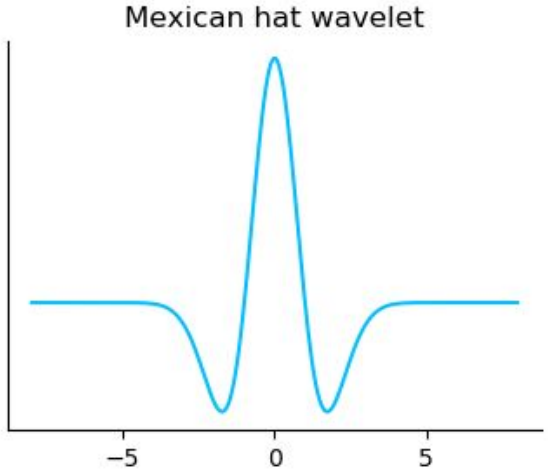
- Wavelet function (mother wavelet)
- Scale factor
- Translation (how the wavelet moves though the signal)

Wavelet function

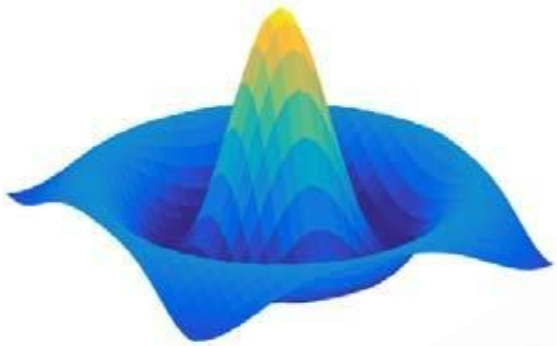


Li et al. 2021; https://doi.org/10.3390/s21175801

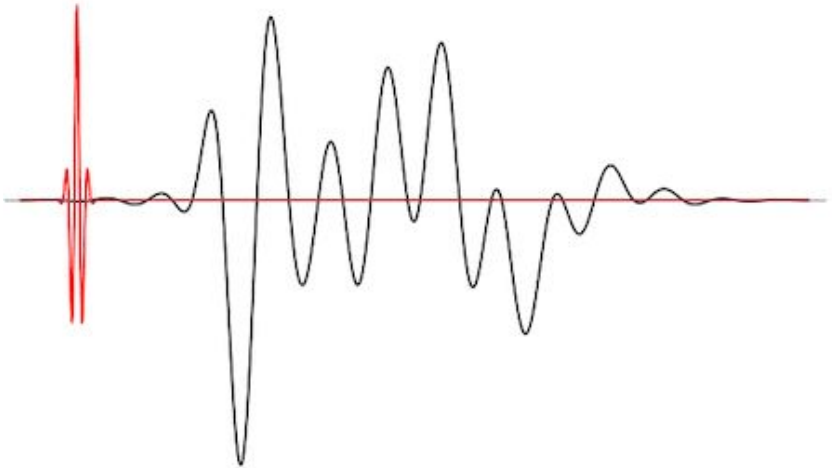
Wavelet function

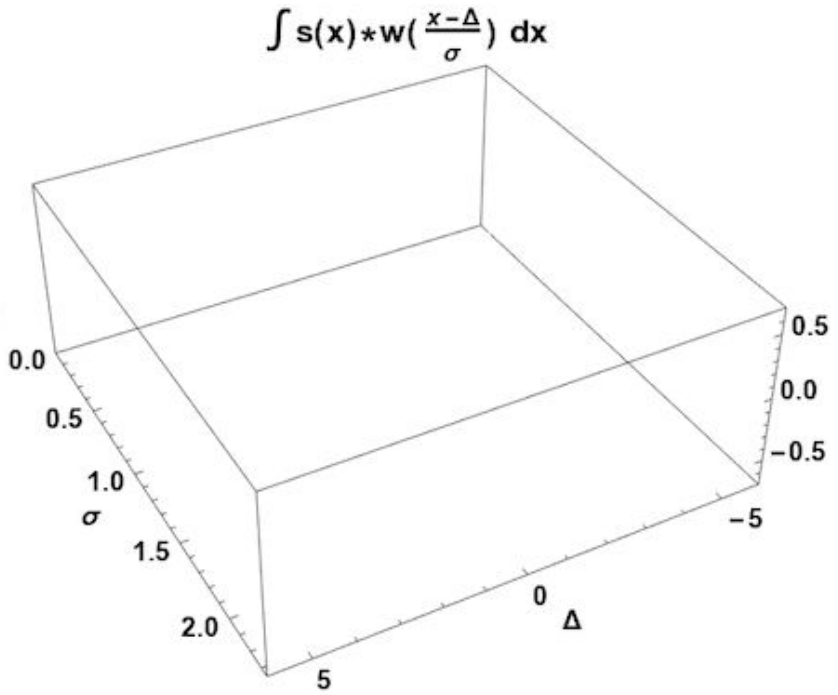


Mexican hat wavelet is also known as second derivative gaussian or Ricker wavelet.



Scale factor



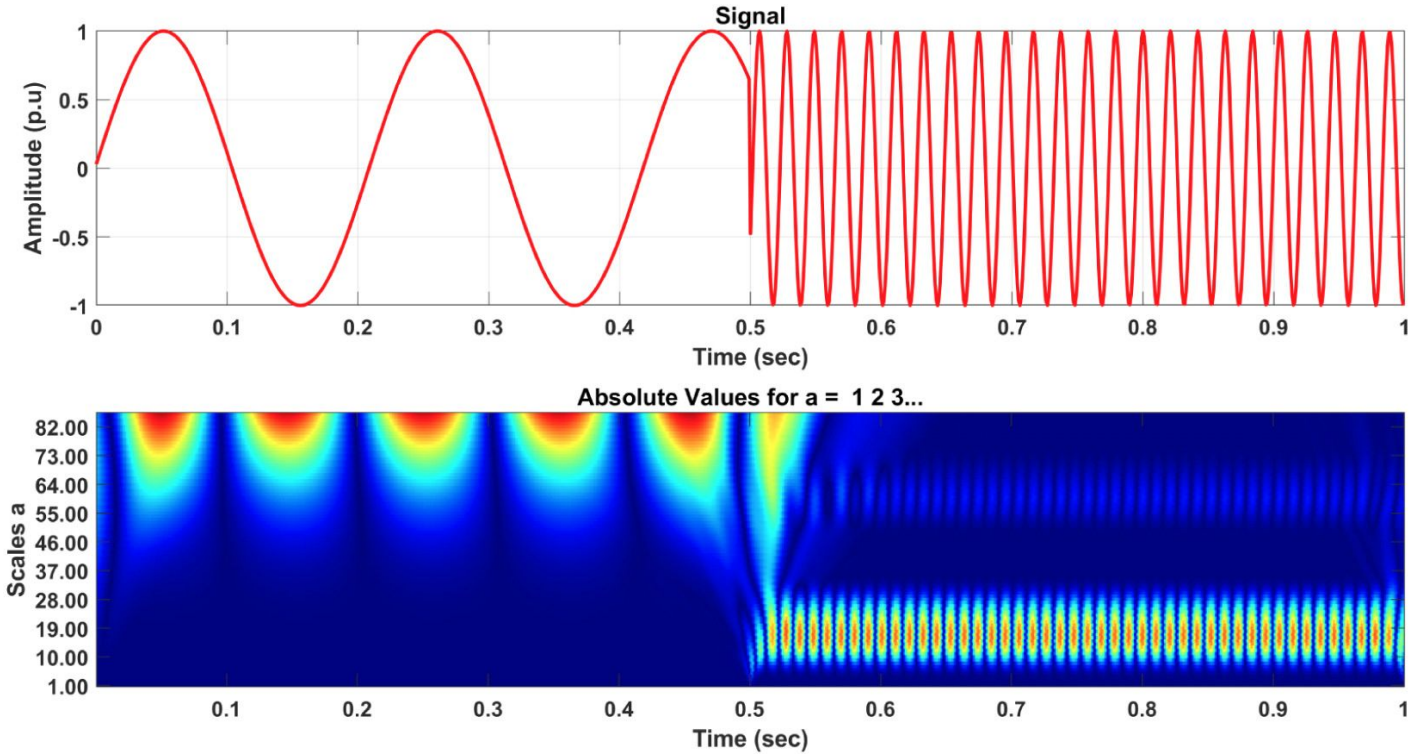


Scale factor

Principles associated with the scales:

- It must be non-zero
- It must be applied on regular time-series (regularly spaced intervals)
- It can not be smaller than the spaced intervals

Scalogram

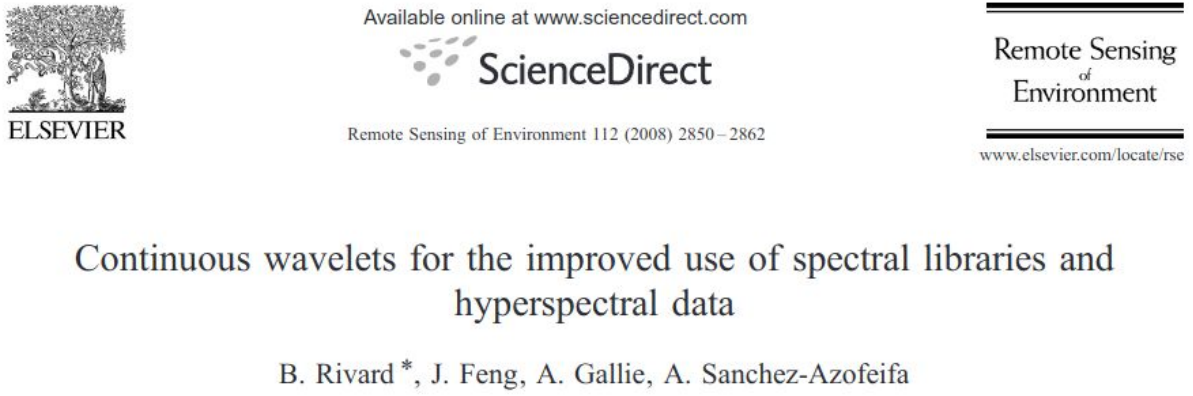


Salles and Ribeiro (2023); https://doi.org/10.1016/j.epsr.2022.108834

CWT and spectroscopy

Some papers applied CWT:

- **Rivard et al. 2002**



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Remote Sensing of Environment 112 (2008) 2850–2862

Remote Sensing of Environment

www.elsevier.com/locate/rse

Continuous wavelets for the improved use of spectral libraries and hyperspectral data

B. Rivard *, J. Feng, A. Gallie, A. Sanchez-Azofeifa

CWT and spectroscopy

Some papers applied CWT:

- Rivard et al. 2002
- **Chen et al. 2011**
- **Chen et al. 2012**
- **Chen et al. 2014**

ELSEVIER

Remote Sensing of Environment
Volume 115, Issue 2, 15 February 2011, Pages 659-670

Spectroscopic determination of leaf water content using continuous wavelet analysis

ELSEVIER

Journal of Plant Physiology
Volume 169, Issue 12, 15 August 2012, Pages 1134-1142

Predicting leaf gravimetric water content from foliar reflectance across a range of plant species using continuous wavelet analysis

ELSEVIER

ISPRS Journal of Photogrammetry and Remote Sensing
Volume 87, January 2014, Pages 28-38

Deriving leaf mass per area (LMA) from foliar reflectance across a variety of plant species using continuous wavelet analysis

Tao Cheng [a], Benoit Rivard [b], Arturo G. Sánchez-Azofeifa [b c], Jean-Baptiste Féret [d], Stéphane Jacquemoud [e], Susan L. Ustin [a]

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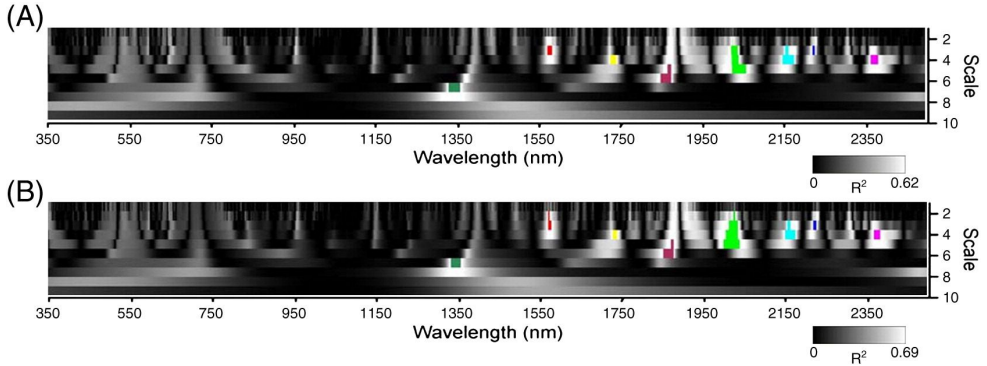

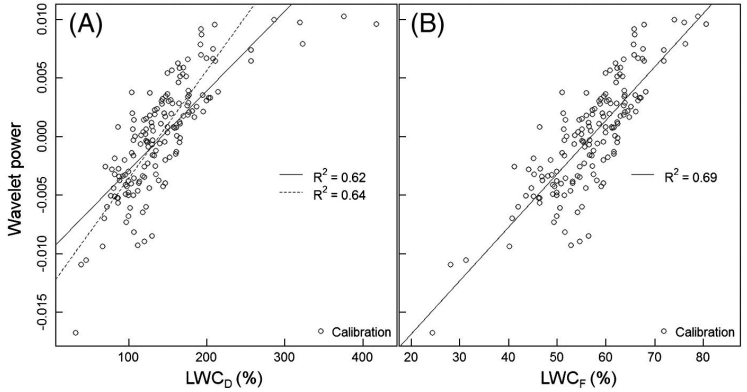

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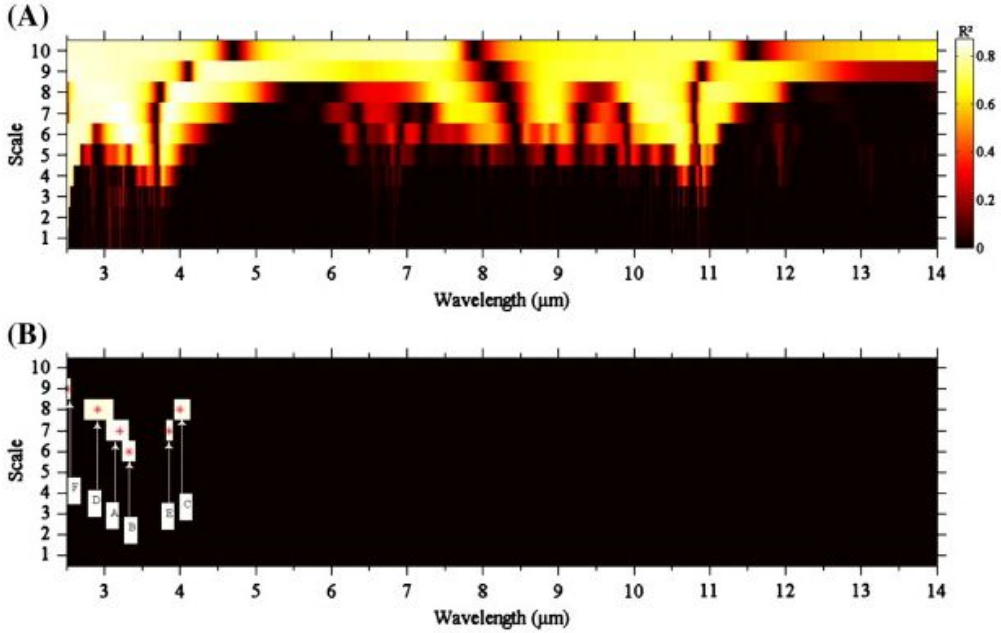
- **Ullah et al. 2012**



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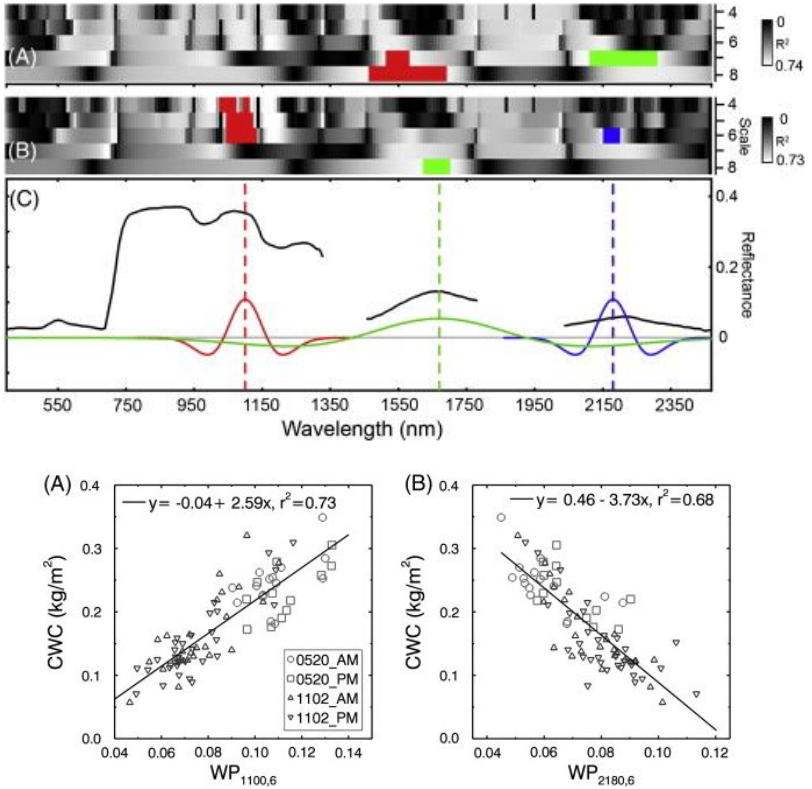
- Ullah et al. 2012
- **Cheng et al. 2014**



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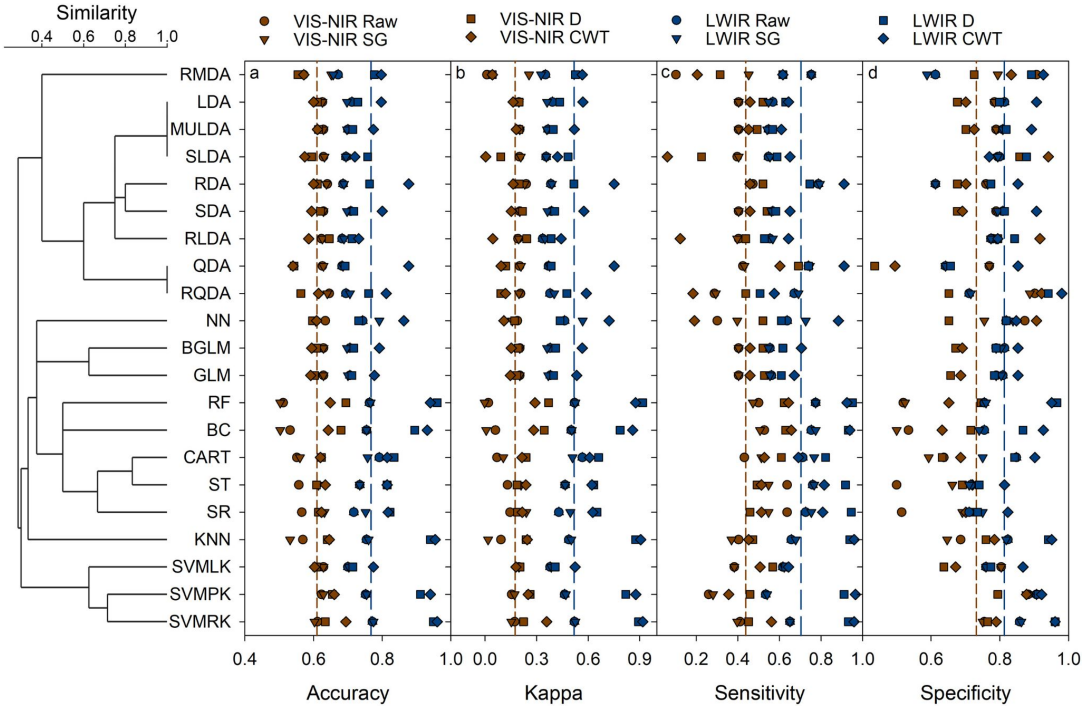
- Cheng et al. 2014
- **Harrison et al. 2018**
- **Guzmán et al. 2018**



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- Harrison et al. 2018
- Guzmán et al. 2018
- **Adams Chlu's dissertation 2020**

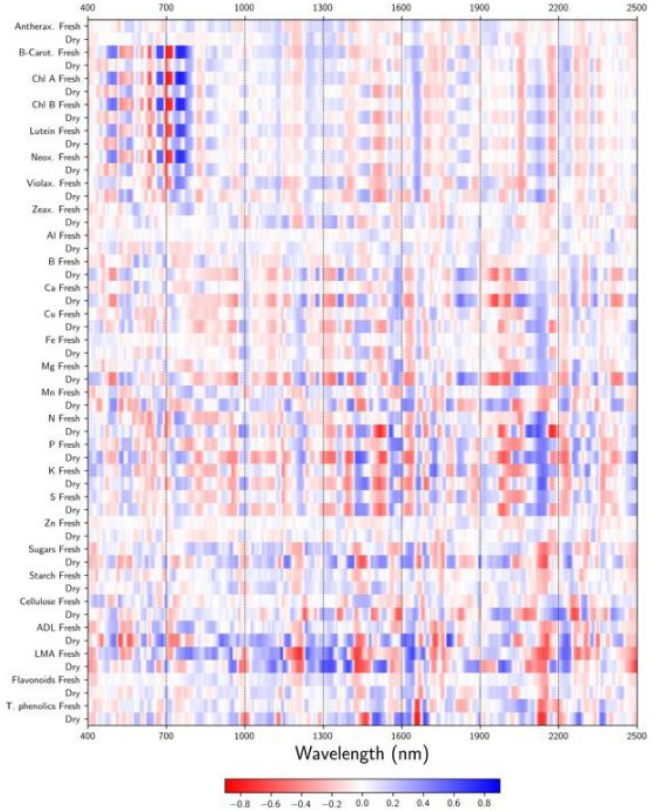

Figure 1.7 Correlation matrix showing mean wavelet correlations across all scales for each trait and spectral measurement type.

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- Guzmán et al. 2018
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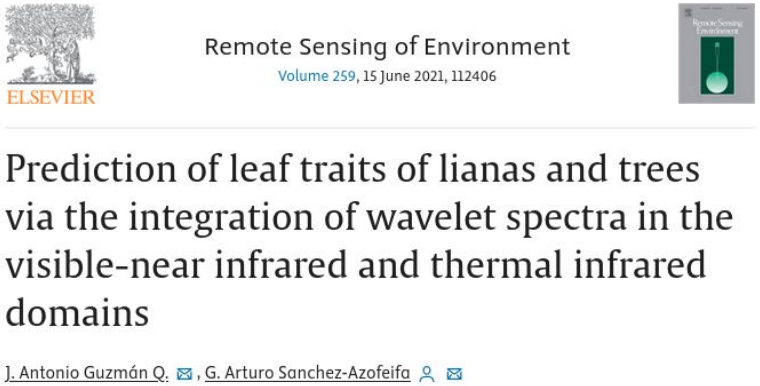
- **Guzmán and Sanchez-Azofeifa 2021**



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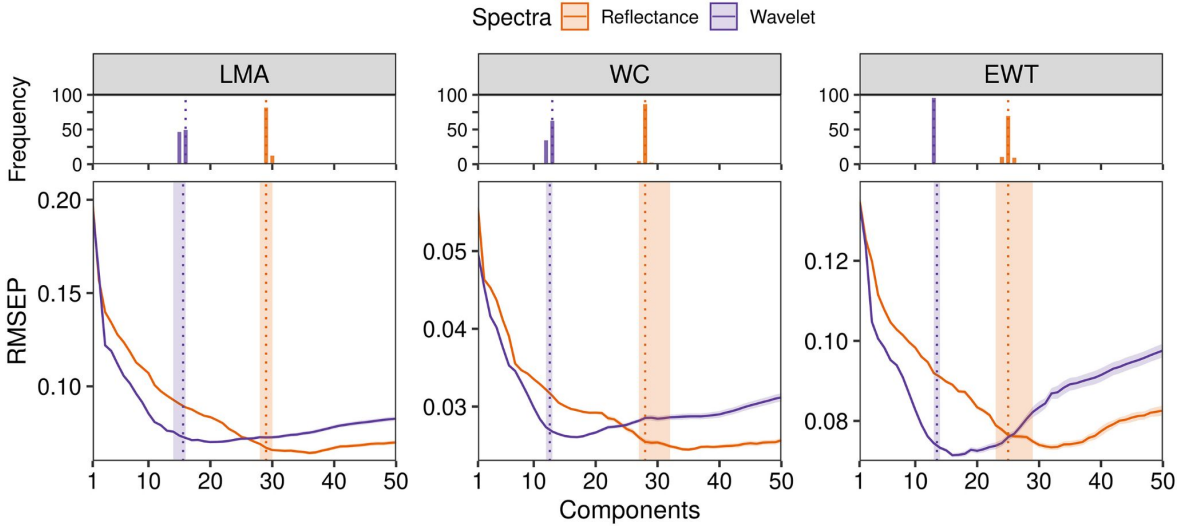

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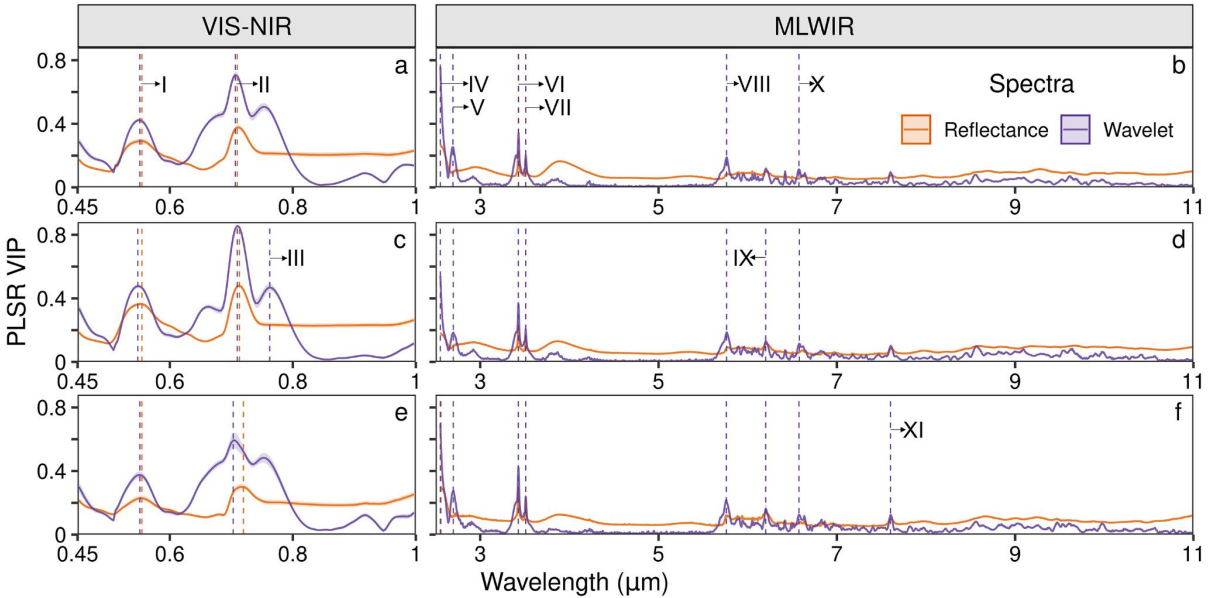

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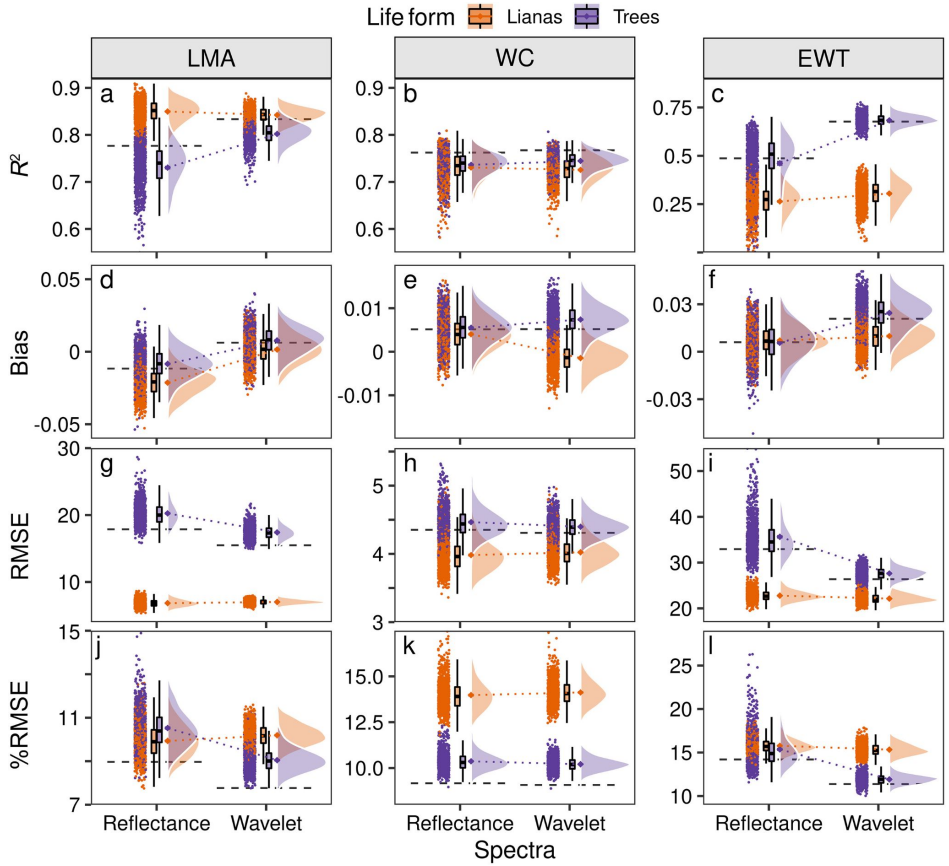

Let’s play with wavelets

The goal:

- Apply wavelet spectra to evaluate its properties
- Evaluate the integration of wavelet spectra with PLSR to predict leaf traits