Optimal Behaviour of Regulated Firms in Solar Renewable Energy Certificate (SREC) Markets

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Solar Renewable Energy Certificate (SREC) markets

- Conceptually similar to cap-and-trade
- Regulator sets floor on amount of energy generated from renewable sources, distributes certificates for each MWh generated
- Certificates offset against this floor at the end of a compliance period (energy year)
- Multiple consecutive, non-overlapping periods linked together by *banking*

What is the problem?

- How should firms behave in these systems?

Previous related work

- Stochastic equilibrium models for spot price of carbon and its properties: Seifert, Uhrig-Homburg, and Wagner (2008), Hitzemann and Uhrig-Homburg (2014), Carmona, Fehr, and Hinz (2009), Carmona et al (2010)
- Structural models for spot price of carbon: Howison and Schwarz (2012), Carmona, Coulon, and Schwarz (2012)
- Stochastic models for SREC generation and price: Coulon, Khazaei, and Powell (2015)

Where our work differs

- Focused on optimal firm behaviour as opposed to properties of spot price of certificates
- Modelling interaction between agents (not covered in preliminary results)
- SREC vs carbon

Simplest possible setup

- Formulate optimal behaviour as a stochastic control problem
- Consider a single firm that is regulated for a single compliance period which ends at time T
- Need controls, state variables, and performance criteria
- Can formulate in discrete or continuous time

Simplest possible setup cont'd

Some notation

- g_i - generation rate at i
- Γ_i - trading rate at i
- h_i - 'baseline' generation rate at i
- R - SREC requirement
- P - monetary penalty per unit of non-compliance for the period
- b_i - number of SRECs on hand at i
- S_i - SREC spot price at i
- Control variable, Measurable system variable, State variable

Simplest possible setup (continuous time)

Performance criteria for firm:

$$J(g,\Gamma,t,b,S) = \mathbb{E}\Bigg[\underbrace{-\frac{1}{2}\zeta\int_0^T (g_u - h_u)^2 du}_{\text{Cost of Generation}} - \underbrace{\int_0^T \Gamma_u S_u^{g,\Gamma} du}_{\text{Cost of Trading}}$$

$$- \underbrace{\frac{\gamma}{2}\int_0^T \Gamma_u^2 du}_{\text{Trading Speed Penalty}} - \underbrace{P_T(R_T - b_T^{g,\Gamma})_+}_{\text{Noncompliance Penalty}}\Bigg]$$

State variable dynamics:

$$dS_t^{g,\Gamma} = (\mu_F + \eta\Gamma_t - \psi g_t)dt + \sigma_F dWt$$

$$db_t^{g,\Gamma} = (g_t + \Gamma_t)dt$$

Simplest possible setup (continuous time)

Value function:

$$V(t,b,S) = \sup_{g,\Gamma} J(g,\Gamma,t,b,S)$$

Optimal controls in feedback form:

$$g^{\mathrm{OPT}} = \frac{\partial_b V - \psi \partial_S V + \zeta h}{\zeta}$$

$$\Gamma^{\mathrm{OPT}} = \frac{\partial_b V + \eta \partial_S V - S}{\gamma}$$

HJB equation:

$$\partial_t V + \mu_F \partial_s V + \frac{1}{2}\sigma_F^2 \partial_{ss} V - \frac{1}{2}\zeta h^2$$

$$+\frac{(\partial_b V - \psi \partial_s V + \zeta h)^2}{2\zeta} + \frac{(\partial_b V + \eta \partial_S V - S)^2}{2\gamma} = 0$$

$$V(T,b,S) = P(R-b)_+$$

Optimality conditions and FBSDE for controls

Optimality conditions:

$$P\mathbb{P}_t(b_T < R) - \eta\mathbb{E}_t\left[\int_t^T \Gamma_u du\right] - S_t - \Gamma_t\gamma = 0$$

$$P\mathbb{P}_t(b_T < R) + \psi\mathbb{E}_t\left[\int_t^T \Gamma_u du\right] - \zeta(g_t - h_t) = 0$$

FBSDE for controls:

$$d\Gamma_t = \frac{1}{\gamma}(dM_t - \mu_t dt + \psi g_t dt) \qquad \Gamma_T = \frac{1}{\gamma}(P\mathbb{I}_{b_T<R} - S_T)$$

$$dg_t = \frac{1}{\zeta}(dZ_t + \zeta dh_t - \psi\Gamma_t dt) \qquad g_T = \frac{1}{\zeta}(P\mathbb{I}_{b_T<R} + \zeta h_T)$$

where

$$M_t = P\mathbb{P}_t(b_T < R) - \eta\mathbb{E}_t\left[\int_t^T \Gamma_u du\right] \quad Z_t = P\mathbb{P}_t(b_T < R) + \psi\mathbb{E}_t\left[\int_t^T \Gamma_u du\right]$$

Simplest possible setup (discrete time)

Performance criteria for firm:

$$J(g,\Gamma,t,b,S) = \mathbb{E}\Bigg[\underbrace{\frac{1}{2}\zeta\sum_{i=1}^{n}(g_{t_i}-h_{t_i})^2\Delta t}_{\text{Cost of Generation}} + \underbrace{\sum_{i=1}^{n}\Gamma_{t_i}S_{t_i}^{g,\Gamma}\Delta t}_{\text{Cost of Trading}}$$

$$+ \underbrace{\frac{\gamma}{2}\sum_{i=0}^{n}\Gamma_{t_i}^2\Delta t}_{\text{Trading Speed Penalty}} + \underbrace{P_T(R_T - b_T^{g,\Gamma})_+}_{\text{Noncompliance Penalty}}\Bigg]$$

Minimize this with respect to the controls

Simplest possible setup cont'd

State variable dynamics:

$$
\begin{aligned}
\tilde{S}_{t_i}^{g,\Gamma} &= S_{t_{i-1}}^{g,\Gamma} + \Delta t(\mu + \eta \Gamma_{t_{i-1}} - \psi g_{t_{i-1}}) + \sigma \sqrt{\Delta t} Z \\
S_{t_i}^{g,\Gamma} &= \min(\max(\tilde{S}_{t_i}^{g,\Gamma}, 0), P) \\
b_{t_i}^{g,\Gamma} &= b_{t_i}^{g,\Gamma} + \Delta t(g_{t_{i-1}} + \Gamma_{t_{i-1}}) \\
Z &\sim N(0,1)
\end{aligned}
$$

Other models for S can be used.

Applying the Bellman Principle

$$V(t, b, S) = \inf_{g,\Gamma} J(g, \Gamma, t, b, S)$$

Apply the Bellman Principle to cost function to obtain:

$$\begin{aligned} V(t_i, b_{t_i}, S_{t_i}) = \inf_{g_{t_i}, \Gamma_{t_i}} \Big\{ & -\frac{1}{2}\zeta(g_{t_i} - h_{t_i})^2 \Delta t + \Gamma_{t_i} S_{t_i}^{g,\Gamma} \Delta t - \frac{\gamma}{2} \Gamma_{t_i}^2 \Delta t \\ & + \inf_{(g_t)_{t=t_{i+1}}^{t=t_n}, (\Gamma_t)_{t=t_{i+1}}^{t=t_n}} \mathbb{E}[V(t_{i+1}, b_{t_{i+1}}, S_{t_{i+1}})] \Big\} \\ V(T, b_T, S_T) = P(R - b_T)_+ & \end{aligned}$$

Numerically solve this optimization problem through backwards induction.

Numerical implementation - parameter choice

Compliance parameters:

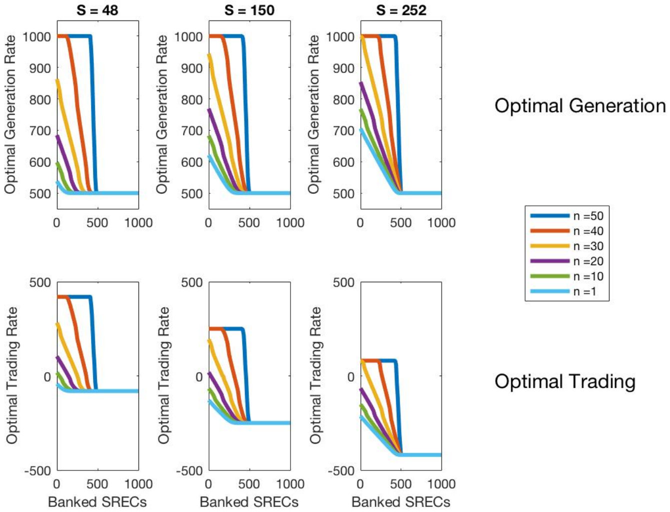
n	T	P	R	h
52	1	300	500	500

Model Parameters:

μ	σ	ψ	η	ζ	γ
0	10	0	0	0.6	0.6

These parameters chosen for illustrative purposes.

Single firm, single period - optimal behaviour



Intuition of optimal behaviour

Three regimes (heuristically)

- Will fail to comply regardless of strategy (within reason)
- Will certainly comply regardless of strategy (within reason)
- Compliance attainable in many ways - room for profit seeking

Intuition of optimal behaviour (with a bit more math)

Consider a firm taking a constant strategy
If we are guaranteed to fail to comply:

$$J(g,\Gamma) = \frac{1}{2}\zeta(g-h)^2\Delta t + S\Gamma\Delta t + \frac{1}{2}\gamma^2\Delta t + P(R-b-(g+\Gamma)\Delta t)$$

$$\implies \frac{\partial J}{\partial g} = \zeta(g-h)\Delta t - P\Delta t \implies g^{\text{OPT}} = \frac{P}{\zeta} + h$$

Similarly,

$$\frac{\partial J}{\partial \Gamma} = (S + \gamma\Gamma - P)\Delta t \implies \Gamma^{\text{OPT}} = \frac{P-S}{\gamma}$$

Can do same if we are guaranteed to comply and get:

$$g^{\text{OPT}} = h \text{ and } \Gamma^{\text{OPT}} = -\frac{S}{\gamma}$$

Takeaways

- Small range where these optimal controls have non-zero derivatives in b
- Around kinks, small changes in banked SRECs result in very different optimal behaviours (and consequently, PnL)
- Trading is much more sensitive to SREC price than generation
- Regulated firms fraught with uncertainty when compliance is in question

Sample path

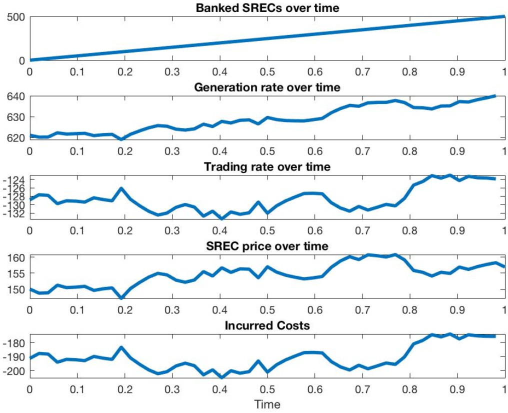


Figure: Sample path for firm with $b_0 = 0$, $S_0 = 150$

Multiple periods

- Little conceptual difference between single compliance period and multiple compliance periods
- Need to make assumptions about linking mechanisms (banking allowed)
- Only additional cost is more computational time
- Omitted from this talk in the interest of time

Parameter sensitivity

- ζ, γ are the most interesting parameters to perturb - will focus on that
- Generally, decreasing ζ, γ corresponds to an expansion of the regime where controls vary with b

Parameter sensitivity (Generation)

For $n = 52, S_n = 150$:

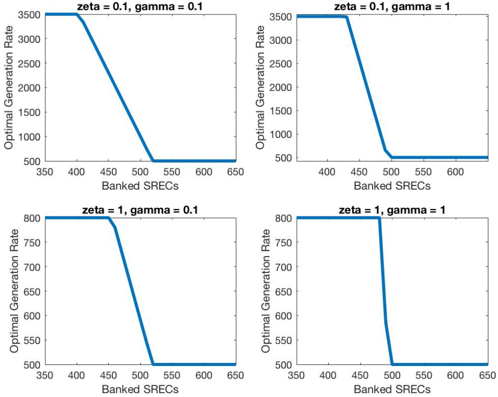


Figure: Optimal generation for differing levels of ζ, γ

Parameter sensitivity (Trading)

For $n = 52, S_n = 150$:

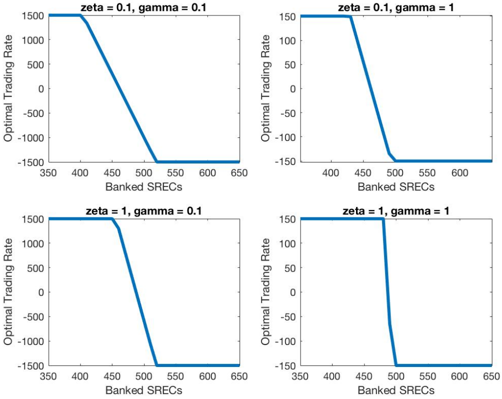


Figure: Optimal trading for differing levels of ζ, γ

Next steps

- Calibration of parameters to real world for increased realism
- More realistic SREC price process
- Multiple agents interacting with one another
- Incorporating partial information