Optimal Behaviour of Regulated Firms in Solar Renewable Energy Certificate (SREC) Markets

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Abstract. SREC markets are a relatively novel market-based system to incentivize the production of energy from renewable means (in particular solar). It requires a floor on the amount of energy each firm must generate from solar power in a given period and penalizes them for any shortfall. SREC contracts may be used to offset their generation. In this work, we formulate a stochastic control problem for generating and trading in SREC markets for a regulated firm and discuss potential takeaways for both a regulated firm under this system and regulatory bodies in charge of designing them.

Key words. stochastic control, SREC, cap and trade, market design

AMS subject classifications. placeholder

1. Introduction.

As the impacts of climate change continue to be felt worldwide, policies to reduce greenhouse gas emissions and promote renewable energy generation will increase in importance. One policy approach which encapsulates many policies is market-based solutions. The most well-known of the policies which fall under this umbrella are carbon cap-and-trade markets.

In these markets, regulators impose a limit on the amount of carbon dioxide (CO_2) that a set of regulated firms can emit during a certain time period (referred to as a compliance period). They also distribute allowances (credits) to these firms in that amount, each allowing for a unit of CO_2 emission, usually one tonne. Firms must offset each of their units of emissions with an allowance, or face a monetary penalty for each allowance they are lacking. These allowances are tradable assets, allowing firms who require more credits than what they were allocated to buy them, and firms who require less to sell them. In this way, cap-and-trade markets attempt to find an efficient way of allocating the costs of CO_2 abatement across the regulated firms.

In practice, these systems regulate multiple consecutive and disjoint compliance periods, which are linked together through mechanisms such as *banking*, where unused allowances in period n can be carried over to period $n+1$. Other linking mechanisms include *borrowing* from future periods (where a firm may reduce its allotment of allowances in period $n+1$ in order to use them in period n) and *withdrawal*, where non-compliance in period n reduces period $n+1$ allowances by the amount of non-compliance (in addition to the monetary penalty mentioned above).

A closely related alternative to these cap-and-trade markets are 'renewable energy certificate' markets (REC markets). A regulator sets a floor on the amount of energy generated from renewable sources for each firm (which is based on a percentage of their total energy generation), and provides certificates for each MWh of energy produced via these means [1].

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[1]Not all generators of renewable energy who participate in REC markets are regulated LSEs, though in this

To ensure compliance, they require that each firm surrender certificates in the amount of this floor at the end of each compliance period, with a monetary penalty paid for each lacking certificate. The certificates are traded assets, allowing regulated LSEs to make a choice about whether to produce electricity from renewable means themselves, or purchase the certificates on the market (or a mix of both). This naturally provokes the question of what the optimal behaviour for these regulated firms are.

REC markets can be used to encourage growth of a particular type of renewable energy. The most notable of these systems are Solar REC markets (SREC markets), which have been implemented in many areas of the northeastern United States[2], and are the focus of this work.

The similarities between SREC markets and carbon cap-and-trade markets are clear. However, there are also some notable differences. One key difference between the SREC market and traditional carbon cap-and-trade markets is the uncertainty in the former market is the supply of certificates (driven by some generation process), while in the latter, the uncertainty is in the demand for allowances (driven by an emissions process). In SREC markets, banking is implemented, but borrowing and withdrawal are not. In a sense, this is the inverse of a cap-and-trade system.

Prior works on SREC markets have primarily been concerned with certificate price formation. In [9], the authors present a stochastic model for SREC generation, calibrated to the New Jersey SREC market, ultimately solving for the certificate price as a function of economy-wide generation capacity and banked SRECs, as well as investigating the role and impact of regulatory parameters on these markets. In [13], a follow-up paper by the same authors, an alternative design scheme for SREC markets is proposed and shown to stabilize SREC prices. The volatility of REC prices has been noted in other works, such as [4] and [12]. The latter focuses on the Swedish-Norwegian electricity certificate market, developing a stochastic model to analyze price dynamics and policy in said market.

While literature in SREC markets is fairly limited, there is a great deal of work that has been done in carbon cap-and-trade markets, particularly in developing stochastic equilibrium models for emissions markets. [10] present a general stochastic framework for firm behaviour leading to the expression of allowance price as a strip of European binary options written on economy-wide emissions. Agents' optimal strategies and properties of allowance prices are also studied by [8] and [17] within a single compliance period setup, with the former also making significant contributions through detailed analyses of potential shortcomings of these markets and their alternatives. [7] also proposes a stochastic equilibrium model to explain allowance price formation, with the fundamental contribution being the development of such a model where abatement (switching from less green to more green fuel sources) costs are stochastic. There has also been significant work on structural models for financial instruments in emissions markets, such as [11] and [6].

As alluded to earlier, a natural question that arises in these systems is how regulated LSEs should behave. Through the use of stochastic control techniques, we characterize the optimal behaviour of these firms through their generation and trading behaviour and discuss potential takeaways from a market design perspective. As such, these results will be of interest to both

work, we largely focus on the decisions faced by regulated firms

[2]The largest and most mature SREC market in North America is the New Jersey SREC Market

regulators and the designers of these markets and the firms regulated by them.

Specifically, we explore a cost minimization problem of a single regulated firm in a single-period SREC market with the goal of understanding their optimal behaviour as a function of their current level of compliance and the market price of SRECs. To do this, we formally pose this as a continuous time stochastic control problem, making use of the powerful theory in this field of mathematics. We are able to produce optimality conditions, as well as gain intuition from the form of the optimal controls in feedback form. In addition, we numerically solve for the optimal controls of the regulated firm for various sets of parameters, including detailed analysis of various scenarios, sample paths, and comparisons of various strategies. We also explore the sensitivity of the optimal controls to the various parameters in the model.

There are some notable differences between our work and many of the referenced papers above. The first and most obvious is that we focus on the SREC market, which is far less studied in comparison to carbon cap-and-trade markets. This work is also focused on the optimal behaviour of firms explicitly, rather than what optimal behaviour implies about the price of SRECs. While most prior works solve a stochastic control problem in order to learn about the behaviour of the allowance prices, we begin with assumed dynamics for the price process of an SREC (which regulated agents exert some control over) and are interested in how the agent should behave given this.

The remainder of this paper is laid out as follows. The next section discusses our model and poses the general optimal behaviour problem in continuous time. In section 3, we present optimality results in a continuous time setting. In section 4, we put forth and numerically solve a dynamic program to characterize the optimal behaviour of a regulated firm. Finally, in section 5, we present the results of the program, including sensitivity analysis and the implications for market design.

2. Model.

2.1. SREC Market Rules.

We assume the following set of rules for the SREC market. These rules are exogenously given and fixed prior to the market beginning. A regulated firm is required to submit R SRECs at time T, representing their production for the compliance period $[0, T]$. A penalty P is imposed for each missing SREC at time T. Firms receive an SREC for each MWh of electricity they produce through solar energy.

In an n-period framework, A firm is required to submit $(R_1, ..., R_n)$ SRECs corresponding to the compliance periods $[0, T_1], ..., [T_{n-1}, T_n]$. For the $[T_{i-1}, T_i]$, firms must pay P_i for each SREC below R_i at T_i. We assume firms may bank leftover SRECs not needed for compliance into the next period, with no expiry on SRECs. This is a simplifying assumption we make - many SREC markets have limitations on how long an SREC can be banked for (in New Jersey's SREC market, the largest and most mature in North America, an SREC can be banked for a maximum of four years). This assumption helps us drastically reduce the dimensionality of the problem. After T_n, all SRECs are forfeited.

2.2. Firm Behaviours.

For now, we restrict ourselves to a world where we only consider the behaviour of a single firm in a single compliance period SREC system. A regulated firm can control their generation rate (SRECs/year) at any given time (g_t) and their trading rate (SRECs/year) at any given time (Γ_t). The processes g_t, Γ_t represent the controls of the firm.

The trading rate can be positive or negative, reflecting that firms can either buy or sell SRECs at the prevailing market rate for SRECs. Firms also incur a trading rate penalty of $\frac{1}{2}\gamma\Gamma_t^2$, $\gamma > 0$. This represents constraints on their trading speed. In general, this could be any function convex in Γ_t.

We assume that a firm has a baseline generation level, represented by $h(t)$ (SRECs/year), which we further assume to be deterministic in time. We could use a method similar to [9] to find an appropriate function for this baseline. In order to deviate from their baseline production, either positively or negatively, a firm must incur costs, represented by $C(g_t, h(t)) := \frac{1}{2}\zeta(g_t - h(t))^2$, which is similar to [1]. This is both differentiable and convex (any choice of C with these properties would be appropriate).

From a mathematical perspective, all objects are defined on the filtered probability space $(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$. We restrict the set of admissible controls to be the set of all progressively measurable (with respect to $\mathbb{F}$) processes g_t, Γ_t such that $\mathbb{E}[\int_0^T g_t^2 dt] < \infty$ and $\mathbb{E}[\int_0^T \Gamma_t^2 dt] < \infty$ for all $t \in [0, T]$. We denote this set by $\mathcal{A}$.

At any time, the firm holds $b_t^{g,\Gamma}$ SRECs, and the price process for SRECs is denoted by $S_t^{g,\Gamma}$.

The dynamics of these are as follows:

$$dF_t = \mu(t)dt + \sigma(t)dBt \tag{2.1}$$

$$dS_t^{g,\Gamma} = dF_t + \eta\Gamma_t dt - \psi g_t dt \tag{2.2}$$

$$db_t^{g,\Gamma} = (g_t + \Gamma_t)dt \tag{2.3}$$

Here, B_t is a Brownian Motion on the filtered probability space $(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$, and μ, σ are deterministic functions. Note that we impose permanent linear impacts in trading and generation, in opposite directions (permanent price impacts are commonly studied in optimal execution problems - see [2], [3]) This is to reflect the fact that buying (selling) the SREC should push the price up (down) and that generating SRECs should push the price downwards. In this sense, a firms behaviour has impact on the rest of the market.

We assume that firms are risk-neutral, and thus, their performance criteria for the single-period problem is as follows:

$$J(t, b, S, g, \Gamma) = \mathbb{E}\left[-\int_0^T C(g_u, h(u))du - \int_0^T \Gamma_u S_u^{g,\Gamma} du - \frac{\gamma}{2}\int_0^T \Gamma_u^2 du - P(R - b_T^{g,\Gamma})_+\right] \tag{2.4}$$

As such, the firm's cost minimization can be expressed as a stochastic control problem:

$$V(t, b, S) = \sup_{g,\Gamma\in\mathcal{A}} J(t, b, S, g, \Gamma) \tag{2.5}$$

In the next section, we leverage the powerful theory of continuous time optimal control in order to gain insight about this problem.

3. Continuous time approach.

3.1. Stochastic Maximum Approach. One approach to solving such a control problem is to use the Stochastic Maximum Principle (Pontryagin Maximum Principle). Two seminal works in establishing this principle are [14] and [15]. We apply Stochastic Maximum Principle to our current problem in a similar fashion to [10]. In doing so, we obtain optimality conditions for the problem, as well as a system of coupled forward backward stochastic differential equations (FBSDEs) that the optimal controls must satisfy. This is summarized in the following proposition.

Proposition 3.1 (Optimality Conditions). *In the setup described in the previous section, for the problem represented by (2.5), we have that the optimal control processes* (g_t, Γ_t) *satisfy the following conditions:*

$$P\mathbb{P}_t(b_T < R) - \eta\mathbb{E}_t\left[\int_t^T \Gamma_u du\right] - S_t - \Gamma_t\gamma = 0 \tag{3.1}$$

$$P\mathbb{P}_t(b_T < R) + \psi\mathbb{E}_t\left[\int_t^T \Gamma_u du\right] - \zeta(g_t - h(t)) = 0 \tag{3.2}$$

for all $t \in [0, T]$

Furthermore, the optimal controls (g_t, Γ_t) *are the solution to the following coupled FBSDE:*

$$d\Gamma_t = \frac{1}{\gamma}(dM_t - \mu(t)dt + \psi g_t dt) \qquad \Gamma_T = \frac{1}{\gamma}(P\mathbb{I}_{b_T<R} - S_T) \tag{3.3}$$

$$dg_t = \frac{1}{\zeta}(dZ_t + \zeta h'(t) - \psi\Gamma_t dt) \qquad g_T = \frac{1}{\zeta}(P\mathbb{I}_{b_T<R} + \zeta h(T)) \tag{3.4}$$

where Z_t,M_t *are both martingales.*

Proof. A strategy (g, Γ) that maximizes the performance criteria (2.4) also will maximize the following Hamiltonian:

$$\mathcal{H}(t, b, S, g, \Gamma, y) = y_b(g + \Gamma) + y_S(\mu(t) + \eta\Gamma - \psi g) - \frac{\zeta}{2}(g - h(t))^2 - S\Gamma - \frac{\gamma}{2}\Gamma^2 + \sigma(t)z_S \tag{3.5}$$

This is clearly concave in the controls and state variables.

The adjoint processes y_b, y_S satisfy the following BSDEs:

$$dy_{b,t} = z_{b,t}dB_t \qquad y_{b,T} = P\mathbb{I}_{b_T<R} \tag{3.6}$$

$$dy_{S,t} = \Gamma_t dt + z_{S,t}dB_t \qquad y_{S,T} = 0 \tag{3.7}$$

This results in a trivial solution to each, as both are linear BSDEs (see [16], Chapter 6). Namely:

$$y_{b,t} = P\mathbb{P}_t(b_T < R) \tag{3.8}$$

$$y_{S,t} = -\mathbb{E}_t\left[\int_t^T \Gamma_u du\right] \tag{3.9}$$

If we differentiate the Hamiltonian with respect to the controls, we obtain the following first order conditions:

$$\frac{\partial \mathcal{H}}{\partial \Gamma} : y_b + \eta y_S - S - \gamma\Gamma = 0 \tag{3.10}$$

$$\frac{\partial \mathcal{H}}{\partial g} : y_b - \psi y_S - \zeta(g - h) = 0 \tag{3.11}$$

Substituting the solutions to the adjoint processes into the above first order conditions, we obtain the required optimality conditions.

$$P\mathbb{P}_t(b_T < R) - \eta\mathbb{E}_t\left[\int_t^T \Gamma_u du\right] - S_t - \Gamma_t\gamma = 0 \tag{3.12}$$

$$P\mathbb{P}_t(b_T < R) + \psi\mathbb{E}_t\left[\int_t^T \Gamma_u du\right] - \zeta(g_t - h(t)) = 0 \tag{3.13}$$

Stochastic Maximum Principle tells us that the solutions to (3.1) and (3.2) are the optimal controls.

We turn these optimality conditions into a coupled FBSDE to solve through the following steps.

$$P\mathbb{P}_t(b_T < R) - \eta\mathbb{E}_t\left[\int_0^T \Gamma_u du\right] + \eta\int_0^t \Gamma_u du - S_t - \Gamma_t\gamma = 0 \qquad \text{(adding and subtracting } \eta \textstyle\int_0^t \Gamma_u du)$$

$$M_t + \eta\int_0^t \Gamma_u du - S_t - \Gamma_t\gamma = 0 \qquad \text{(letting } M_t \text{ be a martingale)}$$

$$M_t + \eta\int_0^t \Gamma_u du - S_t = \Gamma_t\gamma \tag{3.14}$$

Note that M_t is a martingale. Through some simple algebraic manipulation, we arrive at its stochastic differential equation and terminal condition.

$$d\Gamma_t = \frac{1}{\gamma}(dM_t - \mu(t)dt + \psi g_t dt) \tag{3.15}$$

$$\Gamma_T = \frac{1}{\gamma}(P\mathbb{I}_{b_T<R} - S_T) \tag{3.16}$$

We use a similar method to get an FBSDE for g_t and associated terminal condition. In doing so, we obtain:

$$dg_t = \frac{1}{\zeta}(dZ_t + \zeta h'(t) - \psi\Gamma_t dt) \tag{3.17}$$

$$g_T = \frac{1}{\zeta}(P\mathbb{E}_{b_T<R} + \zeta h(T)) \tag{3.18}$$

where Z_t is a martingale.

This forms a coupled FBSDE for which the solution represents the optimal control to our cost minimization problem, as required. ∎

We end this subsection with a few comments regarding the results of this proposition and interpretations of the optimality conditions. We use a similar approach as in [10], where the authors develop optimality conditions in the style of (3.1)-(3.2) in a multi-period carbon cap-and-trade system. The addition of the trading speed penalty in our model setup and the assumed dynamics of SREC prices lead to slightly different optimality conditions. We note that if $\eta = \psi = 0$, (3.2) would reduce to $P\mathbb{P}_t(b_T < R) = \zeta(g_t - h(t))$, which is an equation of the same form as in [10], where the marginal cost of generation is equal to the product of the penalty and probability of non-compliance. Meanwhile, (3.1) would become $P\mathbb{P}_t(b_T < R) - \gamma\Gamma_t = S_t$, suggesting that SREC price is also expressed by the product of the penalty and the probability of non-compliance, modified by the time-t optimal trading of the firm.

In the general case where $\eta > 0, \psi > 0$, we see similar behaviour. In (3.1), we see that SREC price is reflected by the product of the penalty and the probability of non-compliance, modified by the time-t marginal cost of the firm's trading and our expectations of their future trading. That is, low prices would be associated with high current trading and high expected future trading.

In (3.2), the product of penalty and the probability of non-compliance is expressed as the difference between the marginal cost of generation and expected future trading (multiplied by ψ).

One could attempt to solve the FBSDEs (3.3)-(3.4) using Least Square Monte Carlo (LSMC) techniques, however, we will consider other approaches to the general problem of solving (2.5) that do not involve the use of these techniques.

3.2. HJB Approach. We also gain insight from using classical control techniques. We now make the assumption that $\mu(t), \sigma(t)$ are constants, and represent them by μ and σ. We apply the Dynamic Programming Principle to (2.5) and end up with the Dynamic Programming Equation as a necessary condition for our value function to satisfy.

$$\partial_t V + \sup_{g,\Gamma}\{\mathcal{L}^{b,S} + F(t,b,S,g,\Gamma)\} = 0 \tag{3.19}$$

$$V(T,b,S) = G(b) \tag{3.20}$$

where $G(b,S) = P(R-b)_+$ and $F(t,b,S,g,\Gamma) = -\frac{1}{2}\zeta(g-h(t))^2 - S\Gamma - \frac{\gamma}{2}\Gamma^2$

We also have that $\mathcal{L}^{b,S} = (\mu + \eta\Gamma - \psi g)\partial_S V + (g+\Gamma)\partial_b V + \frac{1}{2}\sigma^2\partial_{SS}V$.

By satisfying the first order conditions of the controls with respect to the expression $\sup_{g,\Gamma}\{\mathcal{L}^{b,S} + F(t,b,S,g,\Gamma)\}$, we obtain the optimal controls in feedback form.

$$g^*(t,b,S) = h(t) + \frac{\partial_b V(t,b,S) - \psi\partial_S V(t,b,S)}{\zeta} \tag{3.21}$$

$$\Gamma^*(t,b,S) = \frac{\partial_b V(t,b,S) + \eta\partial_S V(t,b,S) - S}{\gamma} \tag{3.22}$$

There is some natural intuition regarding these optimal controls, even in feedback form. Note that the optimal level of trading has a negative linear relationship with S, the SREC market price. This implies that as S increases, the optimal level of trading decreases. That is, the firm will buy less (or equivalently, sell more).

Similarly, one may interpret g^* as the production of the baseline amount of SRECs in addition to the marginal value gained by the generation of an SREC with respect to its impact on banked SRECs and the SREC market price.

Note also the opposite signs preceding the $\partial_S V$ term. If an incremental change in SREC price increases the value function, the $\partial_S V$ adds to Γ^* and subtracts from g^*, reflecting that the purchase of SRECs on the market will also push the price upwards by η per our assumed SREC price dynamics. The opposite also holds.

Ultimately, substituting these into the HJB equation leads to the PDE:

$$\partial_t V + \mu \partial_s V + \frac{1}{2}\sigma^2 \partial_{ss} V - \frac{1}{2}\zeta h^2 + \frac{(\partial_b V - \psi \partial_s V + \zeta h)^2}{2\zeta} + \frac{(\partial_b V + \eta \partial_S V - S)^2}{2\gamma} = 0$$

$$V(T, b, S) = G(b)$$

This is a semi-linear parabolic PDE that is difficult to solve analytically. One could use finite differences methods in order to numerically solve this PDE and use the equations (3.21)-(3.22) in order to solve for the optimal controls. However, in attempting to do so, we experienced numerical instability unless using a prohibitively large grid. As a result, we consider the alternative approach of numerically solving this problem in discrete time instead.

4. Discrete time version of problem.

We have formulated the cost minimization problem of a single firm regulated in a single-period SREC market through a continuous time optimal control framework in order to leverage the powerful theory of continuous time stochastic control. However, we choose to numerically solve the problem in discrete time for numerical stability reasons. It should be noted, however, that a discrete time setup more closely approximates a real life situation, where a regulated firm chooses their behaviour at fixed time points within a compliance period in order to minimize their costs.

Let n be the number of decision points within a single compliance period (we are still in the single firm, single-period case), which occur at $0 = t_1 < t_2 < ... < t_n < T = t_{n+1}$. We assume that these time points are equally spaced out, with Δt representing the time between them.

As such, instead of g_t, Γ_t representing continuous time processes for generation and trading rate, the controls are now $\{g_{t_i}, \Gamma_{t_i} \forall i \in [n]\}$. Intuitively, at each time point, the regulated firm chooses their trading and generating behaviour over the next interval of length Δt. We adjust our notation in this section such that g, Γ represent the vector whose elements are these controls.

Under the same assumptions as the prior sections, the performance criteria then becomes:

(4.1)
$$J(t,b,S,g,\Gamma) = \mathbb{E}\left[\frac{1}{2}\zeta\sum_{i=1}^{n}(g_{t_i}-h(t_i))^2\Delta t + \sum_{i=1}^{n}\Gamma_{t_i}S_{t_i}^{g,\Gamma}\Delta t + \frac{\gamma}{2}\sum_{i=0}^{n}\Gamma_{t_i}^2\Delta t + P(R-b_T^{g,\Gamma})_+\right]$$

We modify the dynamics of the state variables (b, S) to fit our discrete time setup: The dynamics of these are as follows ($\forall i \in [n]$):

$$S_{t_i}^{g,\Gamma} = \min(\max(S_{t_{i-1}}^{g,\Gamma} + \mu_F\Delta t + \eta\Gamma_{t_{i-1}}\Delta t - \psi g_{t_{i-1}}\Delta t + \sigma_F\sqrt{\Delta t}Z_{t_i}, 0), P) \tag{4.2}$$

$$b_{t_i}^{g,\Gamma} = b_{t_i}^{g,\Gamma} + \Delta t(g_{t_{i-1}} + \Gamma_{t_{i-1}}) \tag{4.3}$$

where $Z_{t_i} \sim N(0,1)$ for all $i \in [n]$.

Note that (4.2)) is the discrete time analogue of (2.2), which is then capped at P and floored at 0[3]. This cap and floor is to ensure that SREC price never falls below 0 and never rises above P. Logically, these cannot occur in a real market.

We aim to optimize (4.1) with respect to g, Γ and determine the value of the position the regulated firm, as well as their optimal behaviour. Mathematically, this is represented by:

$$V(t,b,S) = \inf_{g,\Gamma} J(t,b,S,g,\Gamma) \tag{4.4}$$

We apply the Bellman Principle to (4.1) to obtain:

$$\begin{aligned} V(t_i, b_{t_i}, S_{t_i}) = \inf_{g_{t_i},\Gamma_{t_i}} \Big\{ & -\frac{1}{2}\zeta(g_{t_i}-h(t_i))^2\Delta t + \Gamma_{t_i}S_{t_i}^{g,\Gamma}\Delta t - \frac{\gamma}{2}\Gamma_{t_i}^2\Delta t \\ & + \mathbb{E}[V(t_{i+1}, b_{t_{i+1}}^{g,\Gamma}, S_{t_{i+1}}^{g,\Gamma)}]\Big\} \end{aligned} \tag{4.5}$$

$$V(T, b_T, S_T) = P(R-b_T)_+ \tag{4.6}$$

We numerically solve this optimization problem through backwards induction.

5. Solution Algorithm and Results.

5.1. Parameter Choice and Optimal Behaviour. The solution algorithm to solve (4.5)-(4.6) with state variable dynamics described by (4.2)-(4.3)is as follows:

1. Choose a grid of b and S values for which we wish to solve (4.5)-(4.6). We choose a grid for b from 0 to $2R$ of length 101 (in order to obtain a grid point at exactly R), and a grid for S such that $\Delta S = \sqrt{3\Delta t}\sigma$. As with any numerical solution, the trade-off made in the choice of grid is between the size of the grid (correspondingly, the accuracy of the dynamic program solution) and the run time of the program. We find that the grid points chosen provide an acceptable trade-off between accuracy and run time.

[3]This could also effectively be accomplished by the assumption of GBM-like dynamics (in discrete time) for S

2. Begin at time t_n and minimize (4.5) with respect to g_{t_n}, Γ_{t_n}, for every point in the grid of b and S. This is a convex optimization problem at t_n, as the only costs the firm experiences are deterministic due to the terminal condition.
3. Step backwards in time, minimizing (4.5) with respect to g_{t_i}, Γ_{t_i} at time t_i at each grid point of b and S. In order to minimize, we must calculate $\mathbb{E}[V(t_{i+1}, b_{t_{i+1}}^{g_{t_i},\Gamma_{t_i}}, S_{t_{i+1}}^{g_{t_i},\Gamma_{t_i}})]$ for candidate values of g_{t_i}, Γ_{t_i}. This requires calculation of $b_{t_{i+1}}$ and $S_{t_{i+1}}$[4]. The former is entirely deterministic as a function of g_{t_i}, Γ_{t_i} (see (4.3)).
   For the latter, we simulate 100 instances of $S_{t_{i+1}}$ through the update equation (4.2). As a result, we have 100 pairs of $(b_{t_{i+1}}, S_{t_{i+1}})$, for given values of g_{t_i}, Γ_{t_i}. For each of these 100 pairs, we evaluate the value function $V(t_{i+1}, b_{t_{i+1}}, S_{t_{i+1}})$, which has already been calculated, as we have iterated backwards in time. If necessary, we linearly interpolate between grid points to return this value. We set $\mathbb{E}[V(t_{i+1}, b_{t_{i+1}}^{g_{t_i},\Gamma_{t_i}}, S_{t_{i+1}}^{g_{t_i},\Gamma_{t_i}})]$ to be the mean of these 100 values. This allows us to calculate the RHS of (4.5) at time t_i as a function of (g_{t_i}, Γ_{t_i}), and therefore minimize it with respect to those variables. In all cases, minimization was done through Matlab's 'fminsearch' function.

This yields the value of (4.5) for all values of the grid chosen in step 1, at all values t_i, as well as the optimal choice of generation and trading as a function of t_i, b, and S.

We solve this dynamic programming for the following parameter choices:

Compliance parameters:

n	T	P (\$/ lacking SRECs)	R (SRECs)	$h(t)$ (SREC/y)
50	1	300	500	500

Model Parameters:

μ	σ	ψ	η	ζ	γ
0	10	0	0	0.6	0.6

These parameters are chosen for illustrative purposes; indeed, calibration to a particular firm in these markets is itself a non-trivial problem as it requires proprietary knowledge of their cost functions and baseline production (which will also vary significantly from firm to firm). Rather, we provide broad-level explanations and intuitions regarding the optimal behaviour of a firm in a single-period SREC market with what we feel are reasonable parameters. In Section 5.4, we consider alternative parameter schemes. In particular, we explore how various levels of ζ, γ impact firm behaviour, as well as the case where $\psi \neq 0, \eta \neq 0$ (i.e. the firm has price impact). In this work, we restrict ourselves to $h(t)$ being constant. We study alternative (constant) choices for it in 5.4 and its impact on total generated SRECs by the firm, which is of importance from the perspective of a market designer.

A regulated firm's optimal behaviour (the output to our previously described algorithm) is summarized in Figure 1 across various time-steps (t), SREC price levels (S), and banked SREC levels (b).

The most notable feature of the firm's optimal behaviour are the very distinct regimes in which they exist. For low levels of banked SRECs and high values of t, the firm generates until the marginal cost of producing another SREC exceeds the penalty, and purchases until the marginal cost of purchasing another SREC exceeds the penalty, as the firm is almost assured to fail to comply. This follows the classic microeconomic adage of conducting an

[4]For convenience, we drop the superscripts in this discussion of our algorithm

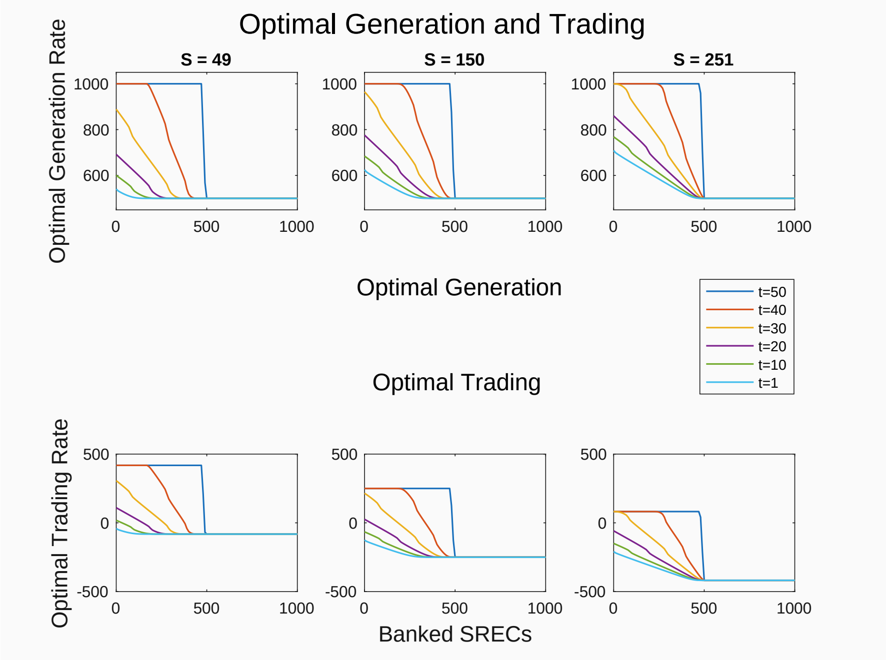

Figure 1. *Optimal firm behaviour as a function of banked SRECs across various time-steps and SREC market prices*

activity until the marginal benefit from the activity equals the marginal cost. In this regime, the marginal benefit of an additional SREC to the firm is P, as each additional SREC lowers their non-compliance obligation by P.

As b increases, the firm reaches a point where the marginal benefit from an additional SREC decreases from P. This occurs as the probability of compliance becomes non-negligible, as additional SRECs over R don't save the firm any money unless sold. This leads to a decrease in optimal generation and optimal trading. Eventually, optimal generation plateaus at the baseline amount, $h(t)$ and optimal trading plateaus where the marginal benefit from trading equals the marginal cost. At this point, the marginal benefit of an additional SREC is zero. More concretely, having an additional SREC does not increase the firm's likelihood of compliance, nor can they sell the additional SREC to make a profit.

We note that if we hold b, S constant, generation and trading are increasing in t. This is natural, as with less time until the end of a compliance period, the firm needs to accumulate more SRECs in order to comply (for the same values of b).

It is also immediately clear that trading is heavily influenced by a change in SREC price, which makes intuitive sense and coincides with the theoretical results from Section 3. As SREC prices increase, the regulated firm chooses to buy less, across all levels of b (banked SRECs). Meanwhile, we see that higher SREC prices also generally imply higher generation, as the firm chooses to generate their own SRECs, either to avoid paying a large price for them in the market, or to sell in the market and capitalize on the high prices (which of these two factors is the larger contributor depends on their value of b).

5.2. Sample Paths. We also model the path of an optimally behaving firm throughout the course of the compliance period, as shown in Figure 2 with initial values $S_0 = 150, b_0 = 0$. Here, we simulate a path for S and at each time-step, the firm behaves optimally (per our solution) given their values of banked SRECs and the SREC price. As a result, we obtain a simulated compliance period across which the firm behaves optimally at all time-steps.

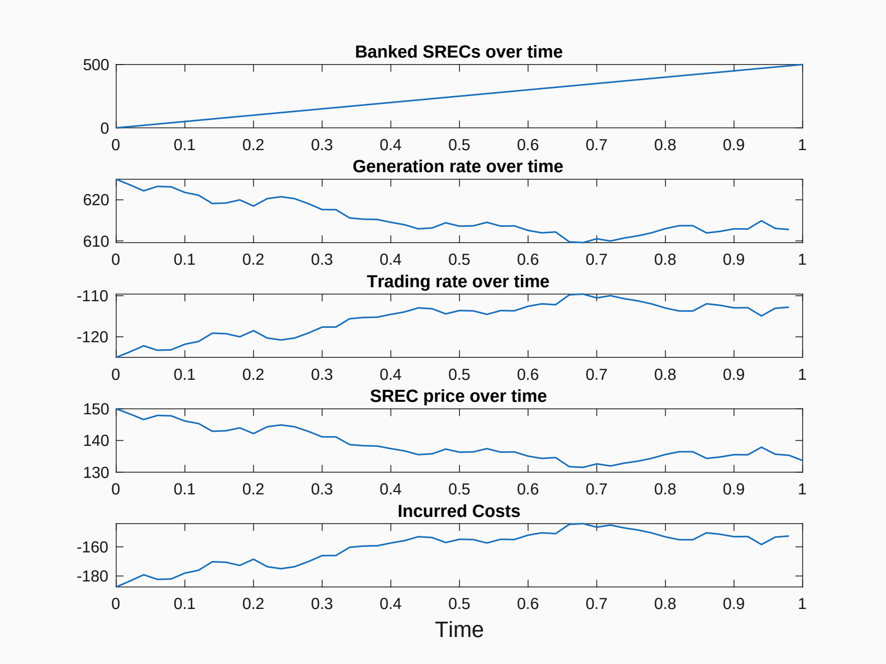

Figure 2. *Optimal firm behaviour with* $S_0 = 150, b_0 = 0$

From Figure 2, we see that the regulated firm acquires SRECs at a consistent rate. However, the generation and trading processes exhibit notable variation. In particular, the inverse relationship between trading rate and SREC price is notable, as is the positive relationship between generation rate and SREC price. Accordingly, we also observe that generation rate and trading rate mirror one another. Note that incurred costs being negative means that the firm is actually making profits (from the sales of SRECs) at each time-step.

If we simulate many paths similar to the above, we also obtain summary statistics and learn about the distribution of various interesting quantities for the firm, such as their final SREC total (b_T), total generated amount ($\int_0^T g_u du$), total traded amount ($\int_0^T \Gamma_u du$), and total costs incurred.

For the same parameter choice as above, and with $S_0 = 150$, we present summary statistics for these quantities, based on 1000 simulated paths of S. Note that we consider total profit below, which is simply the negative of total incurred cost.

It is clear that in this one-period setup, the firm's optimal behaviour results in $b_T = 500$ SRECs in almost every path; there is little to no variation in the firm's final SREC total. This is logical, as there is no advantage to additional SRECs above the requirement in a single-period framework, and any strategy where $b_T < 500$ exposes the firm to the non-compliance

Statistic	Mean	SD	1st Quartile	3rd Quartile	Skewness	Kurtosis
b_T	500.00	7.60e-07	500.00	500.00	3.75	21.98
$\int_0^T g_u du$	624.89	4.58	621.96	628.01	0.02	3.06
$\int_0^T \Gamma_u du$	-124.89	4.58	-128.01	-121.96	-0.02	3.06
Profit	9.38e+03	687.25	8.93e+0.3	9.84e+03	0.17	3.15

Table 1

Summary statistics of various quantities for 1000 sample paths of S with firm behaving optimally

penalty. We would expect this behaviour to change in a multi-period SREC system.

We turn our attention to the other quantities, noting that they exhibit little skewness or excess kurtosis. This is also apparent when looking at the histograms of each of these quantities across these simulations.

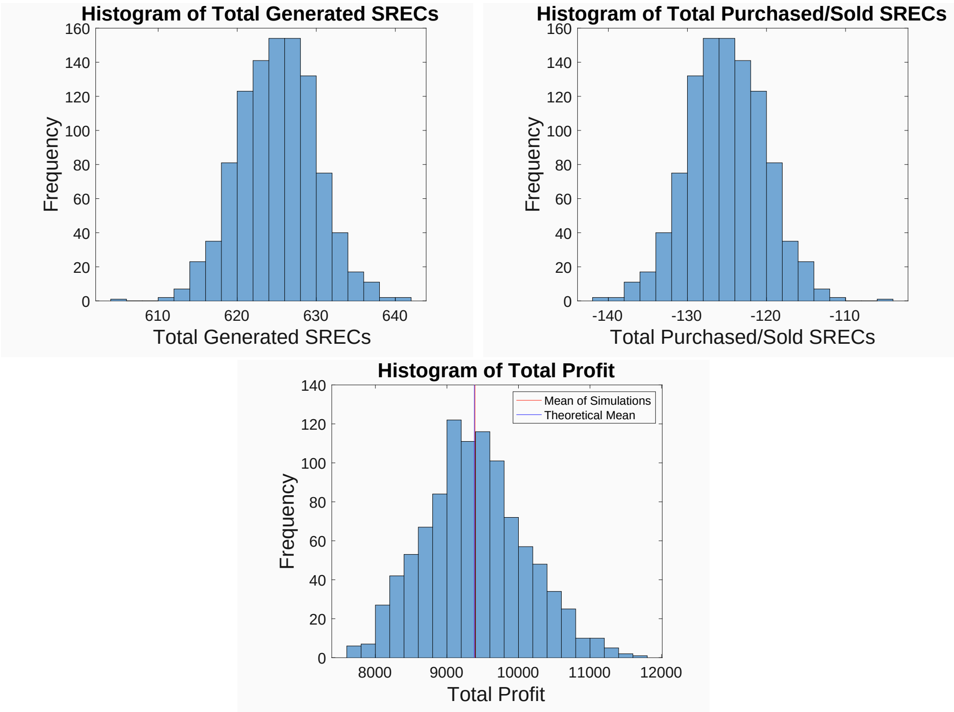

Figure 3. *Histogram of total generation, total trading, and profit over 1000 simulations*

We note that the mean profit of our simulations lies on top of the theoretical expected profit, represented by the value function evaluated at $t = 1, b = b_0, S = S_0$, which suggests our calculations are internally consistent. As the nature of the distribution of b_T would imply, we can also see that the histograms of $\int_0^T g_u du$ and $\int_0^T \Gamma_u du$ (if flipped horizontally) would have the exact same shape. That is, the instances of increased (decreased) total generation correspond exactly to increased (decreased) total SRECs sold, as the firm always ends up with 500 SRECs.

This is more apparent in Figure 4, where we plot $\int_0^T g_u du$ against $\int_0^T \Gamma_u du$ for each path.

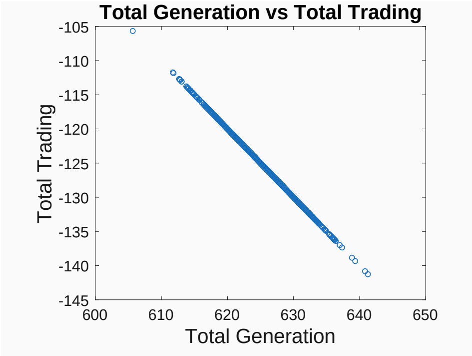

Figure 4. *Scatterplot of total generation vs total trading*

Between their symmetrical histograms, lack of skewness, and lack of excess kurtosis, there is some support for these quantities being normal. Examining their QQ-plots (in Figure 5), we observe that some deviation from normality exists in the tails of these quantities.

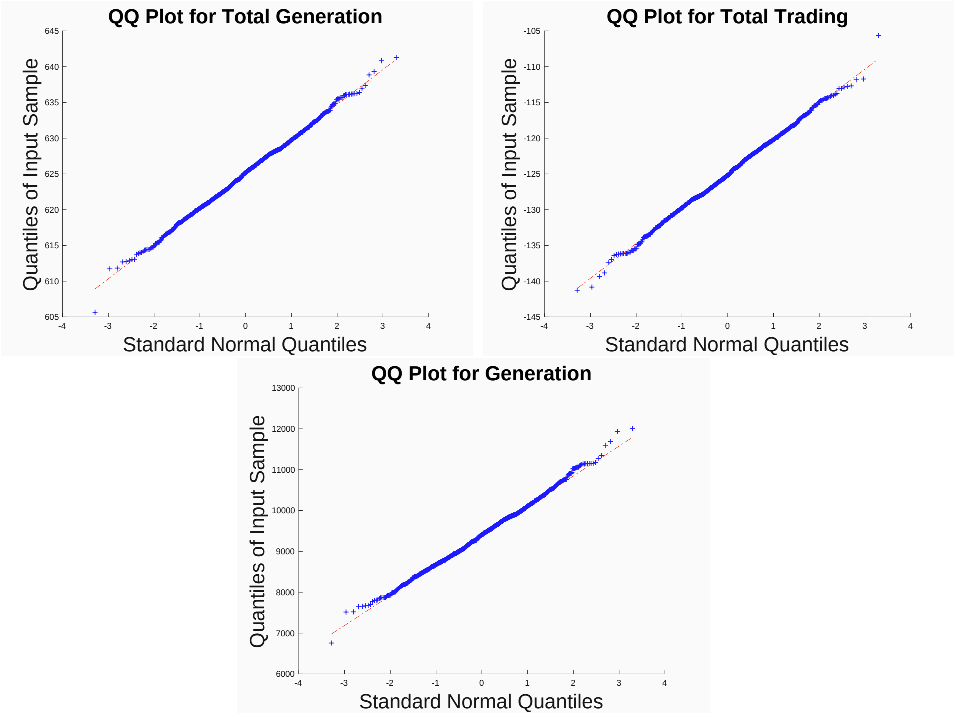

Figure 5. *QQ Plots of total generation, total trading, and profit over 1000 simulations*

5.3. Comparison With Other Strategies. A natural question to arise is how the optimal strategy suggested by the numerical solution to the control problem compares with other intuitive strategies. In particular, we compare the optimal strategy (for the parameter choice in Section 5.1) with the following strategies:

1. A constant generation strategy where $g_t = 500, \Gamma_t = 0$ at all time-steps t (recall $R = 500, h(t) = 500$). Referred to as 'No Trade' strategy.
2. A constant strategy where the firm follows the mean behaviour elucidated in Section 5.2. That is, they produce constantly such that $\int_0^T g_u du = 625$ and $\int_0^T \Gamma_u du = -125$. Referred to as 'Mean Behaviour' strategy.

It can be shown that the Mean Behaviour strategy proposed is the optimal strategy for this parameter set if we restrict the firm to constant behaviours for their controls (i.e. $g_t = g, \Gamma_t = \Gamma$ for every $t \in [0, T]$) and require that they exactly comply with the requirement $(g + \Gamma = R)$[5]. See Appendix A for details.

Over 1000 simulated paths of S with $S_0 = 150$, we calculate the profit from each strategy, as well as other summary statistics, detailed in the table below (we note that these are a separate set of simulations to those carried out in the previous section). The strategy suggested as the output to the dynamic program is referred to as the 'Output' strategy.

Strategy	Mean Profit	Std. Dev. of Profit	Q1 Profit	Q3 Profit
Output	9.38e+03	723.56	8.92e+03	9.85e+03
No Trade	0	0	0	0
Mean Behaviour	9.36e+03	724.05	8.91e+03	9.83e+03

Table 2
Summary of various strategies

The No Trade strategy is trivial. As there is no randomness associated with generation, the profit from the No Trade strategy is wholly deterministic. In particular, it makes sense that the profit is 0 in this case, based on the cost function and the parameters we chose (in particular, that $h(t) = 500, R = 500$). This means the firm can generate SRECs 'cost-free' (in line with their baseline production) and produce enough to comply with the requirement. However, it is clear that this strategy is not optimal for this parameter set. It is worth noting that there is a parameter set for which this strategy is optimal, specifically, if γ was taken towards ∞.

The Output and Mean Behaviour strategies have more competitive results. In order to understand the performance of these strategies further, we explore these two strategies in Figure 6 by looking at the histogram of the differences in profit between them across sample paths.

From Figure 6, we observe that the Output strategy outperformed the Mean Behaviour strategy in every sample path of S. Intuitively, this means that the firm adjusting their behaviour depending on the evolution of S provides value over a static optimal control.

5.4. Parameter Sensitivity. As in any numerical model, parameter choices are of utmost importance. As such, it is critical to understand how modifying the parameters modifies the output of the model.

[5] It is clear that any strategy with $g + \Gamma \neq R$ is not optimal for these parameters. In a one period model with $h(t) = R$, a strategy with $g + \Gamma > R$ implies the firm spends money to generate additional SRECs, some of which expire valueless. If $g + \Gamma < R$, the firm incurs non-compliance penalties that cannot be made up for by sales, as $S < P$

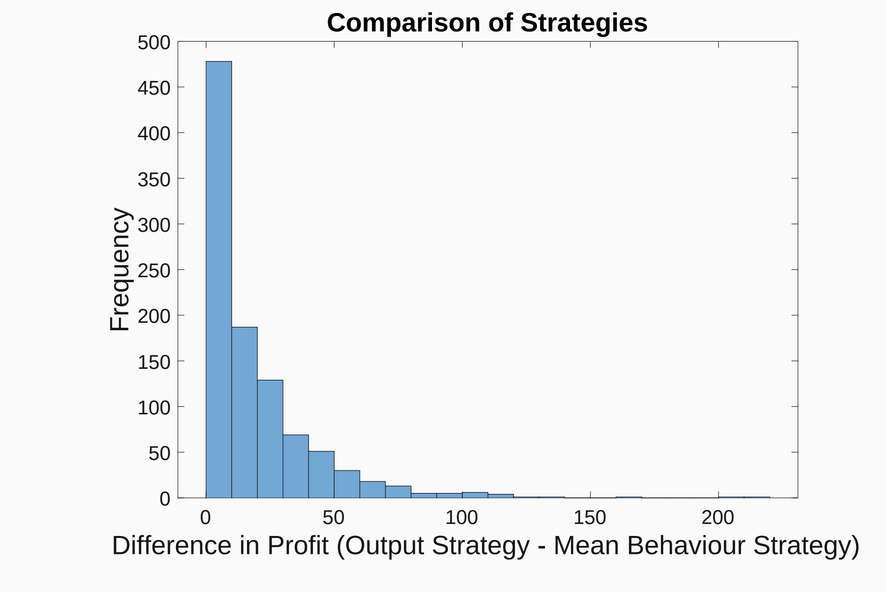

Figure 6. *Comparison between strategies*

5.4.1. Activating Price Impacts. One parameter choice made in previous subsections was that $\eta = \psi = 0$. This was the assumption that the regulated firm whose behaviour we optimize is a small firm in the SREC market and that they have no impact on the overall SREC price (S). We analyze the firm's behaviour when these are non-zero in order to understand how this unique aspect of our model influences their controls. In particular, we choose $\eta = 0.01, \psi = 0.005$ as reasonable parameters that demonstrate the impact of our model choice.

Comparing Figure 7 with Figure 1, we see that the optimal controls have the same basic shape, but with a few new interesting properties. The optimal controls at each of the 'plateaus' now vary slightly with time-step. This was not the case when price impacts of controls were not activated, as in Figure 1. Specifically, at high values of b, firms generate less and purchase more (equivalently, sell less) at earlier time-steps than they do at later time-steps. This is to lower the impact of their behaviour on S, and in particular, to try and keep S as high as possible in order to capitalize on future sales. The inverse behaviour happens for low values of b. Firms generate more and purchase more in order to push the price down and make compliance more attainable. These effects are proportional to the magnitude of η and ψ.

Naturally, comparing the optimal behaviour with price impacts active to the optimal behaviour with price impacts inactive is of importance. To do so, we replicate Figure 2 in Figure 8, for price impacts being both active and inactive (with the same random seed for the path of S, with $S_0 = 150, b_0 = 0$).

Both with and without price impacts, the firm accumulates SRECs at the same constant rate. However, the lower level of generation and selling is consistent across the compliance period. The shape of the optimal controls is broadly similar, reflecting the same features discussed regarding Figure 2. In particular, they accumulate banked SRECs at identical rates. However, we see that the firm under price impacts will generate and sell less in order

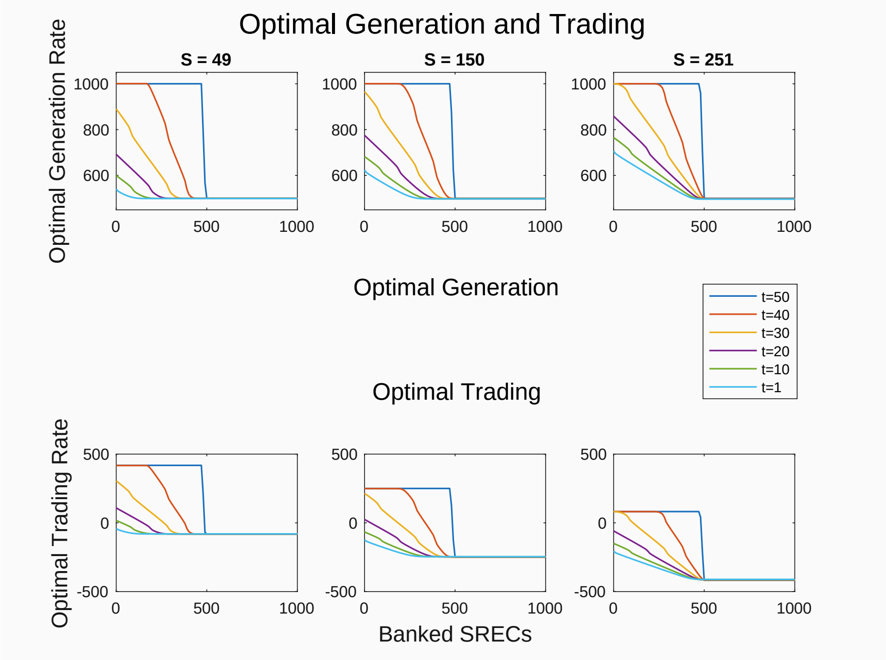

Figure 7. *Optimal firm behaviour as a function of banked SRECs across various time-steps and SREC market prices*

to mitigate their price impact and keep prices high. Nonetheless, the deviation between the paths of S is also clear, with the price impacts resulting in a lower SREC price due to the firm behaviour. Finally, we see that the firm generates less revenue when price impacts are active, as the market will always move against their behaviour.

We also produce the same summary statistics after simulating many paths in the same manner with price impacts activated.

Statistic	Mean	SD	1st Quartile	3rd Quartile	Skewness	Kurtosis
b_T	500.00	9.20e-07	500.00	500.00	3.09	14.52
$\int_0^T g_u du$	622.47	4.73	619.13	625.66	0.05	2.94
$\int_0^T \Gamma_u du$	-122.47	4.73	-125.66	-119.13	-0.05	2.94
Profit	9.13e+03	700.85	8.63e+0.3	9.59e+03	0.18	2.98

Table 3

Summary statistics for 1000 simulations of sample path of S, with price impacts activated

There are some clear similarities to the quantities in Table 1, as all of the main properties discussed previously continue to hold. As in the case without price impacts, banked SRECs are almost universally equal to the requirement R. In comparison to the figures in Table 1, we see that the firm generates and sells less, which is also reflected in Figure 8. Once again, the total generation, trading, and profit exhibit little skewness or excess kurtosis.

As before, we observe that the histograms of $\int_0^T g_u du$ and $\int_0^T \Gamma_u du$ (if flipped horizontally)

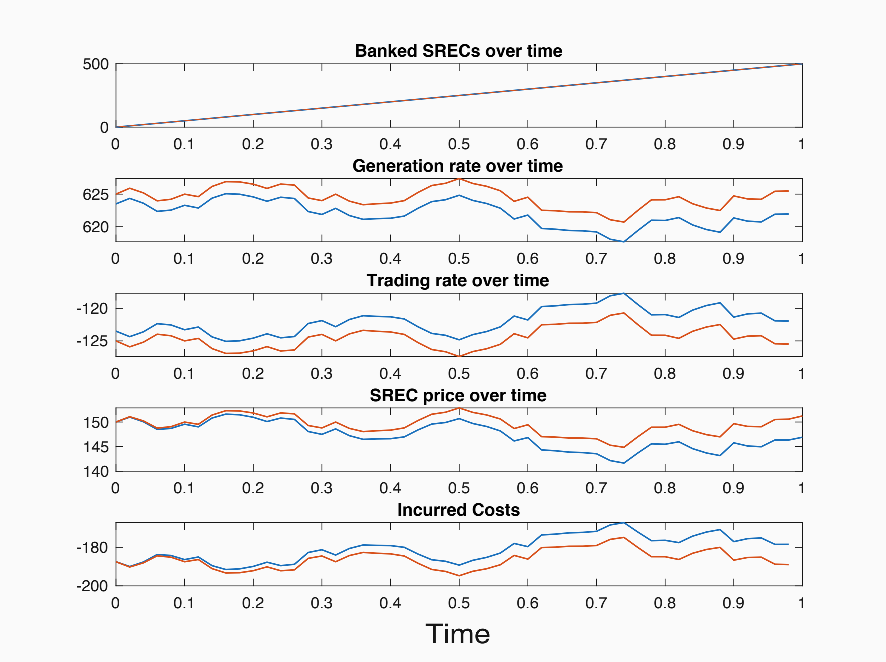

Figure 8. *Optimal firm behaviour with $S_0 = 150, b_0 = 0$ with price impacts active (blue) and inactive (red)*

would have the same shape. This means that total generation and total trading existing on the line $500 - \int_0^T g_u du - \int_0^T \Gamma_u du = 0$ as seen in Figure 11.

We display the QQ plots of each of these distributions, shown in Figure 10. As in the no price impact case, these quantities exhibit some evidence of normality.

5.4.2. Sensitivity to $h(t)$. We have predominantly explored the numerical solution to this optimal control problem from the perspective of a firm with baseline SREC generation of $h(t) = R$. However, in reality, a wide array of LSEs are regulated by SREC systems, with varying levels of investment into solar and thus, capability to generate SRECs. As such, it is important to consider the optimal behaviours of firms with various levels of production capability.

In particular, this has important ramifications from the perspective of a regulator in charge of market design. Ultimately, the goal of SREC systems is to promote investment into solar energy generation. Consequently, the amount of SRECs generated by a regulated firm is a quantity that would be monitored carefully by regulators and market designers. To briefly study the impact of $h(t)$ on total SRECs generated by a firm, we simulate 1000 paths of S starting from S_0, with a regulated firm behaving optimally at each time-step. We examine how the distribution of $\int_0^T g_u du$ changes across varying levels of $h(t)$ for the firm, with all other parameters reprising their value from Section 5.1. Specifically, we look at (constant) values of $h(t)$ of $0.75R$, R, and $1.25R$ (corresponding to values of $375, 500$, and 625, respectively). We also consider various initialization points of the path of S: $S_0 = 49, 150, 251$ (which represent low, medium, and high prices of S respectively).

From Table 4, we confirm the relatively obvious fact that a firm with a higher baseline will

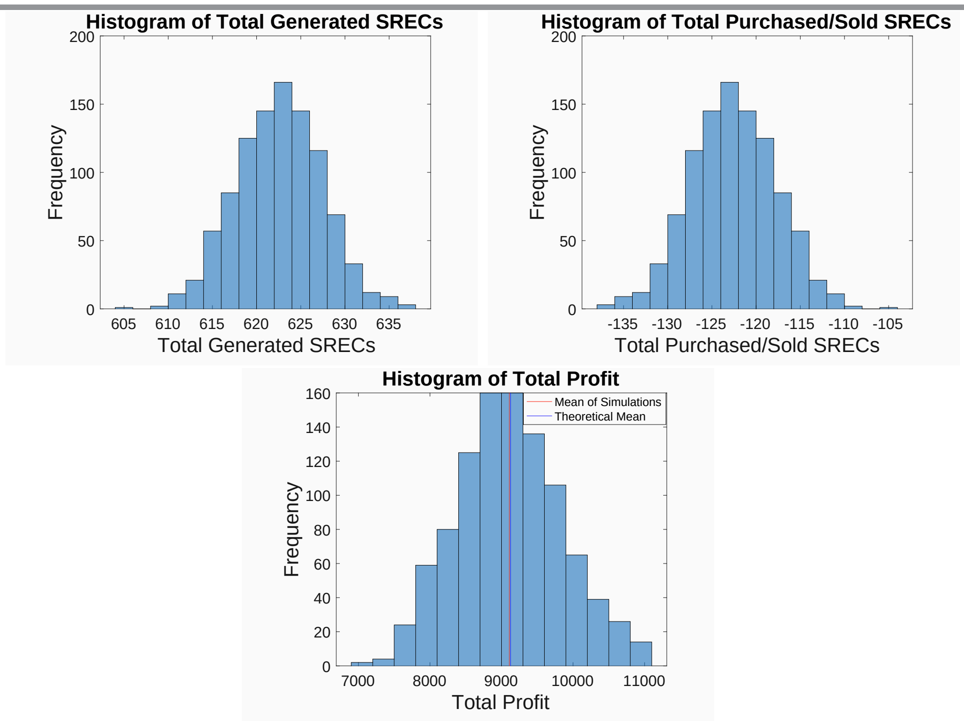

Figure 9. *Histogram of total generation, total trading, and profit over 1000 simulations with price impacts activated*

$h(t)$	$S_0 = 49$		$S_0 = 150$		$S_0 = 251$	
	$\mathbb{E}[\int_0^T g_u du]$	$\mathrm{SD}(\int_0^T g_u du)$	$\mathbb{E}[\int_0^T g_u du]$	$\mathrm{SD}(\int_0^T g_u du)$	$\mathbb{E}[\int_0^T g_u du]$	$\mathrm{SD}(\int_0^T g_u du)$
$0.75R$	478.45	4.78	562.45	4.59	646.48	4.95
R	541.01	4.79	624.95	4.59	708.98	4.95
$1.25R$	625.00	0.00	687.45	4.59	771.48	4.95

Table 4
Total generation across varying levels of $h(t)$ and S_0

produce more SRECs on average given the same level of S_0 than a firm with a lower baseline. It is also apparent that firms produce more as S_0 increases, which is consistent with previous results in this work.

Note also that in every scenario, firms produce above their baseline production. The only scenario in which a firm produces exactly its baseline occurs with a high baseline firm with low SREC prices. In this case, the firm produces its baseline and sells as much as possible given the SREC price and cost parameters, as it is all but guaranteed compliance.

We also note that the low-production firm is incentivized to increase production over the course of the period through the requirement R to a more substiantial degree than the other firms, regardless of S_0. This is evidenced by the fact that their optimal generation is further above their baseline than the mid-production and high-production firms across all values of S_0.

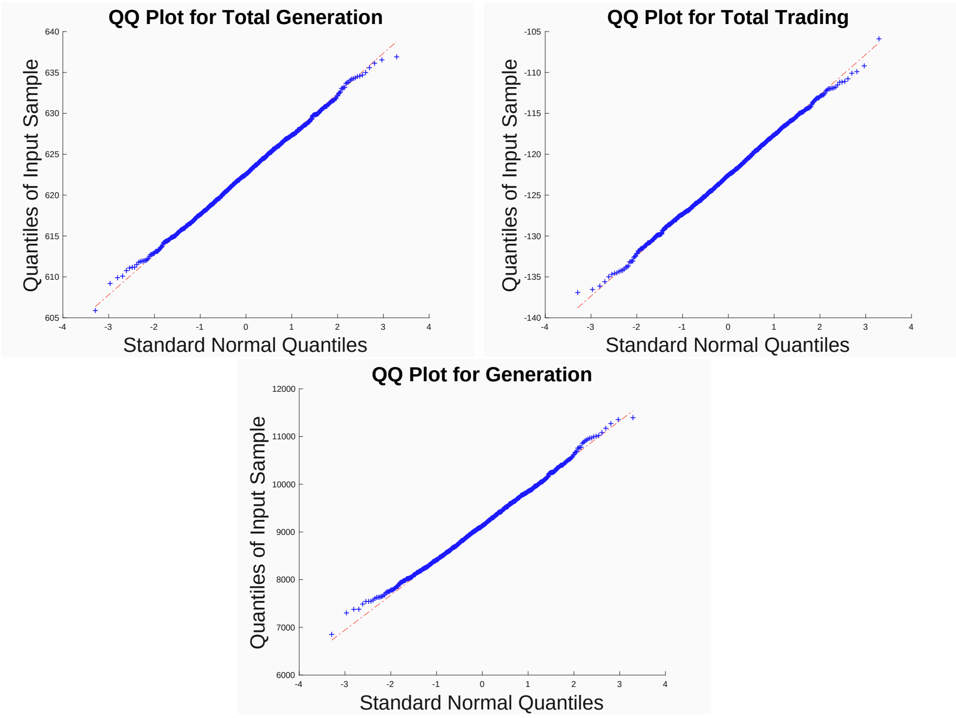

Figure 10. *Histogram of total generation, total trading, and profit over 1000 simulations with price impacts activated*

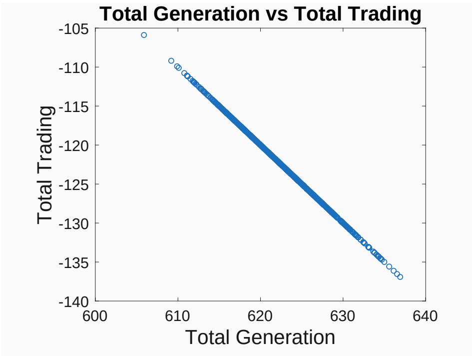

Figure 11. *Scatterplot of total generation vs total trading*

That is, they respond to what is an 'ambitious' requirement from their perspective by investing into generation, which is the ultimate goal of SREC markets. This provides some evidence that setting R to be significantly above economy-wide baseline generation can incite higher degrees of investment into SREC generation. However, this is a very simplistic setting and analysis that does not consider the impact to the firm's profit, political and lobbying pressures against high requirements, and other feasibility conditions. This remains an interesting area for future work which we hope to continue to build towards, with the model presented in this work being a significant first step.

5.4.3. Sensitivity to ζ, γ. Finally, we explore sensitivity to two particularly important parameters: ζ and γ, which represent the generation cost parameter and trading speed penalty parameter respectively. In the figure below, we show how optimal behaviours change for various values of ζ and γ, at a given time-step ($t = 50$) and SREC price level ($S_n = 150$).

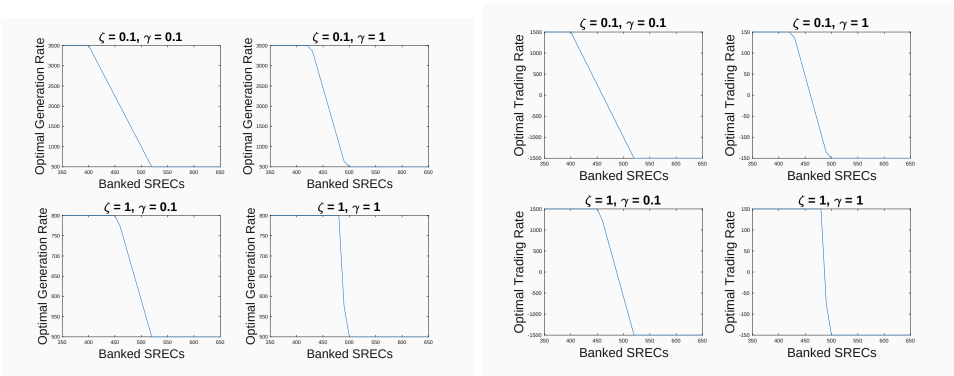

Figure 12. *Optimal generation and trading for differing levels of ζ, γ*

Both optimal generation and trading respond in very similar ways. First, we notice that a change in γ impacts the level of optimal trading significantly, while a change in ζ impacts the level of optimal generation. This is in accordance with the fact that the former represents the trading speed penalty and the latter represents the generation penalty.

Additionally, it is clear that lowering both ζ and γ widens the range over which the firm's optimal behaviour has non-zero derivative in b. Recall from Section 5.1 that the regimes observed in the optimal behaviours correspond to the marginal benefit of the firm holding an additional SREC. In particular, the optimal behaviour having non-zero derivative corresponds to the values of b, S where the marginal benefit of an additional SREC is between 0 and P. As such, the expansion of this range is the result of lowered costs changing the values of b for which this property of marginal benefit occurs.

6. Conclusion.

In this work, We characterize the optimal behaviour of a single regulated LSE in a a single-period SREC market. In particular, we characterize their optimal generation and trading behaviour as the solution to a continuous time stochastic control problem. We propose a model for SREC prices and present optimality conditions and set of coupled FBSDEs whose solution corresponds with the optimal controls. We also numerically solve for the system in a discrete time setting, providing intuition and reasoning for the resulting optimal behaviour, including detailed analysis of various sample paths, strategies, and parameter groups.

Many further extensions are possible. We have only briefly discussed the framework of an SREC system with multiple periods, which would undoubtedly change the firm's optimal behaviour in non-terminal compliance periods. Furthermore, interactions between agents are a critical component of real SREC markets that are largely ignored in this single-firm setup. In particular incorporating partial information of firms would be a very challenging but mathematically interesting problem that would more closely mimic the realities of SREC markets. Improved calibration to real world parameters would also increase the applicability of this work for use by regulated firms and regulators.

However, even our simple model reveals salient facts about the nature of these systems and how firms should behave when regulated by them. Our model reveals that the optimal generation and trading of regulated firms broadly exists in three regimes, depending on the marginal benefit received from holding an additional SREC. We observe that a firm's trading behaviour is more sensitive to changes in S than its generation behaviour, and that higher SREC prices imply greater generation and lower purchasing (more selling). We also show that the optimal behaviour outperforms the optimal constant behaviour strategy for our parameter choice, and study the effects of including price impacts in our model, as well as sensitivity to other parameters.

In providing these results, we have produced a framework and numerical solution that would be of use for both regulated firms and regulatory bodies who both have immense interest in understanding the optimal behaviour of regulated LSEs in these systems.

Appendix A. Optimal Constant Behaviour Strategy.

A sub-problem to the one considered in prior subsections is how a firm should behave if it is restricted to behaving in a constant manner over the course of the compliance period. This provides insight into how their controls will change with respect to parameter changes. As such, consider a regulated firm that is optimizing their behaviour at $t = 0$, with $b_0 = 0$ and $T = 1$. For simplicity, further assume that $h(t), \mu(t), \sigma(t)$ are constants. That is, $h(t) = h, \mu(t) = \mu, \sigma(t) = \sigma$. In particular, we consider the case where we constrain $g + \Gamma = R$

Proposition A.1 (Optimal Constant Behaviours).

Consider a single firm that is regulated in a single-period SREC market, with the following additional assumptions:

- $h(t) = h, \mu(t) = \mu, \sigma(t) = \sigma$
- $b_0 = 0, T = 1$
- *The controls g_t, Γ_t must be constant across the period (i.e. $g_t = g, \Gamma_t = \Gamma$ for all $t \in [0, T]$*
- $g + \Gamma = R$

Therefore, the firm aims to maximize the following:

$$J(g, \Gamma) = \mathbb{E}\left[-\int_0^T \frac{\zeta}{2}(g-h)^2 du - \int_0^T \Gamma S_u^{g,\Gamma} du - \int_0^T \frac{\gamma}{2}\Gamma^2 du\right] \tag{A.1}$$

The optimal control is given by $(g^*, \Gamma^*) = \left(\frac{2S_0+\mu+R(\psi+2\eta+2\gamma)+2\zeta h}{2(\eta+\gamma+\psi+\zeta)}, -\frac{2S_0+\mu-R(\psi+2\zeta)+2\zeta h}{2(\eta+\gamma+\psi+\zeta)}\right)$

Proof. The RHS of (A.1) is identical to the RHS of (2.4) updated for the additional assumptions made (note that we can remove the non-compliance penalty as the constraint $g + \Gamma = R$ ensures compliance). This is a constrained optimization problem, which we solve through the use of Lagrange multipliers.

$$
\begin{aligned}
J(g,\Gamma) &= \mathbb{E}\left[-\int_0^T \frac{\zeta}{2}(g-h)^2 du - \int_0^T \Gamma S_u^{g,\Gamma} du - \int_0^T \frac{\gamma}{2}\Gamma^2 du\right] \\
&= -\frac{\zeta}{2}(g-h)^2 - \Gamma\mathbb{E}\left[\int_0^T S_u^{g,\Gamma} du\right] - \frac{\gamma}{2}\Gamma^2 \\
&= -\frac{\zeta}{2}(g-h)^2 - \Gamma\int_0^T \mathbb{E}[S_u] du - \frac{\gamma}{2}\Gamma^2 && \text{(Fubini's Theorem)} \\
&= -\frac{\zeta}{2}(g-h)^2 - \Gamma\int_0^T (S_0 + (\mu + \eta\Gamma - \psi g)u) du - \frac{\gamma}{2}\Gamma^2 && \text{(From the definition of } S_t \text{ in (2.2))} \\
&= -\frac{\zeta}{2}(g-h)^2 - \Gamma(S_0 + \frac{\mu}{2}) - \frac{\Gamma^2}{2}(\eta+\gamma) + \frac{\psi}{2} g\Gamma
\end{aligned}
\tag{A.2}
$$

We introduce λ as an auxiliary variable, and define:

$$
\mathcal{L}(g,\Gamma,\lambda) := -\frac{\zeta}{2}(g-h)^2 - \Gamma(S_0 + \frac{\mu}{2}) - \frac{\Gamma^2}{2}(\eta+\gamma) + \frac{\psi}{2} g\Gamma - \lambda(g+\Gamma-R) \tag{A.3}
$$

Set $\nabla\mathcal{L} = 0$ to obtain the following system of equations, which is a necessary condition for a candidate optimizer to satisfy:

$$
-\zeta(g-h) + \frac{\psi}{2}\Gamma - \lambda = 0 \tag{A.4}
$$

$$
-S_0 - \frac{\mu}{2} - (\eta+\gamma)\Gamma + \frac{\psi}{2} g - \lambda = 0 \tag{A.5}
$$

$$
-g - \Gamma + R = 0 \tag{A.6}
$$

This is a system of three equations and three unknowns. The solution to this system is given by (with λ omitted):

$$
g = \frac{2S_0 + \mu + R(\psi + 2\eta + 2\gamma) + 2\zeta h}{2(\eta+\gamma+\psi+\zeta)} \qquad \Gamma = -\frac{2S_0 + \mu - R(\psi + 2\zeta) + 2\zeta h}{2(\eta+\gamma+\psi+\zeta)} \tag{A.7}
$$

■

Since J is concave and the equality constraint is linear, a point satisfying the necessary conditions for an optimizer is in fact the global optimizer (see [5]). Therefore, g^*, Γ^* take the forms seen in (A.7)

In Section 5.3, we consider the Mean Behaviour strategy given by $g = 625, \Gamma = -125$. By substituting the applicable parameters from Section 5.1 into (A.7), we see that these values coincide with the optimal behaviours given by A.1. For these parameters, it can also be shown that the Mean Behaviour strategy is optimal among all constant behaviour strategies.

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