HERA : Antenna Design Discussion
Hydrogen Epoch of Reionization Array

1 Cost Vs. Sensitivity

We weigh the cost and sensitivity benefits of the HERA antenna and array. In order to accomplish this we define our figure of merit as the inverse of the Cost (C) times the sensitivity (Δ_N^2):

$$F.O.M = [C \times \Delta_N^2]^{-1}. \tag{1}$$

The goal is to maximize the figure of merit.

Our sensitivity is proportional to k^3, N^{-1}, $u^{\frac{1}{2}}$, and $\Omega^{\frac{5}{2}}$. N is the number of antennas, u is the baseline length, and Ω is the beam size. Therefore, the sensitivity goes as

$$\begin{aligned} &\Delta_N^2 \propto k^3 N^{-1} u^{\frac{1}{2}} \Omega^{\frac{5}{4}} \\ \Rightarrow\ &\Delta_N^2 \propto k^3 N^{-1} D^{\frac{1}{2}} (D^{-2})^{\frac{5}{4}} \\ \Rightarrow\ &\Delta_N^2 \propto k^3 N^{-1} D^{-2} \end{aligned} \tag{2}$$

In general, the minimum k mode we can work at is

$$k_{min} = k_H + \Delta k_{fg}, \tag{3}$$

where k_H is the k-mode given by the geometrical horizon limit of our baseline, and k_{fg} is the width of the foregrounds in k-space. Assuming that our antennas are placed next to each other,

$$k_H = \frac{B}{c}\frac{dk}{d\eta}, \tag{4}$$

where B is the baseline length (in our case $B =$ Diameter,D) and $\frac{dk}{d\eta}$ is some cosmological constant of proportionality that depends on the redshift.

We will now look at a few limiting cases of k and cost, C. In k-space our cases are

1. $k_H \gg k_{fg} \Rightarrow \Delta_N^2 \propto D^1 N^{-1}$
2. $k_{fg} \gg k_H \Rightarrow \Delta_N^2 \propto D^{-2} N^{-1}$
3. $k_H \approx k_{fg} \Rightarrow \Delta_N^2 \propto D^1 N^{-1}$
4. $k > k_{min}$.

The cost models are defined in terms of C. The total cost is proportional to

$$C \propto C_0 + C_1 N + C_2 N^2 \tag{5}$$

where $C_0 \propto D^0$, $C_1 \propto D^2$, and $C_2 \propto D^0$, and N is the number of antennas. The limiting cases for the cost are when

A. Linearly dominated : $C \approx C_1 N \approx D^2 N$

B. Quadratic in N : $C \approx C_2 N^2$

C. Equal contributions : $C_0 \approx C_1 N \approx C_2 N^2$

Table 1: This table shows the proportionality's of the FOM as a function of the diameter (D) and the number of antennas N. Each of the numbers (top row) and letters (left column) signifies the regime we are in, as given in the text.

	1	2	3	4
A	D^{-3}	constant	D^{-3}	constant
B	$D^{-1}N^{-1}$	$N^{-1}D^2$	$D^{-1}N^{-1}$	$N^{-1}D^2$
C	$(DN^{-1}+D^3+DN)^{-1}$	$(N^{-1}D^{-2}+1+N^{-1}D^2)$	$(DN^{-1}+D^3+DN)^{-1}$	$(N^{-1}D^{-2}+1+N^{-1}D^2)$

Table **??** gives a break down of every possible scenario. It provides the FOM (in proportionality's) as a function of the diameter and number of elements.

Now consider the case where we are in the fixed cost regime so that the figure of merit is proportional to the inverse of the sensitivity. PAPER falls in the case A, the linearly dominated regime and our cost goes as $C \propto D^2N$ (This is not quite true and the real cost is somewhere between constant and linearly dominated, most likely $< N^{-.5}$). Since our cost is constant, we can vary our diameter as $N^{-\frac{1}{2}}$, or vary the number of antennas as D^{-2}. If we look at the horizon dominated case, then the figure of merit says that we should increase the number of antennas to get the highest sensitivity for our fixed cost. This is because, $D \propto N^{-\frac{1}{2}} \Rightarrow N^{\frac{3}{2}}$, therefore increasing the number of antennas will maximize the FOM. However, if we are foreground dominated then the FOM is constant and we want to twiddle the knobs so that we have the $D \propto N^{-\frac{1}{2}}$. Further discussion of this scenario is required. Note, that the scenarios repeat with cases 3 and 4.

It is also useful to think of the scenario where we have a fixed number of antennas. Keeping N fixed and noting that in reality the C_1 term does not really depend on the diameter as D^2. It is more or less constant. Hence, the cost is constant. Therefore, the figure of merit is proportional to DN^{-1}, which encourages us to increase the diameter of our dishes to maximize the FOM.

2 The Parabolic Dish

This element concept is novel, but bears similarities to a drift-scan observing mode explored on the GMRT, and to the cylinder dish concept being explored for CHIME.

In order for a parabolic (or cylindrical) dish to work for 21cm intensity mapping, the focal length and dish diameter must be carefully controlled to avoid reflections that enter at a significant delay, and thus modulate foregrounds to corrupt scales at the corresponding $k_{\parallel}$. Based on recent work characterizing foregrounds, it seems reasonable to ask that a dish not create more spectral structure than the shortest ($8\lambda \approx 15$m) baselines that have been shown to be well-behaved. Beyond limiting the dish diameter to 15m, this requirement restricts the focal length, since once of the primary resonances in the dish will be the "narcissistic waves" that arise between the primary and

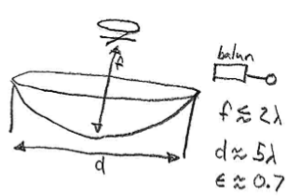

Figure 1

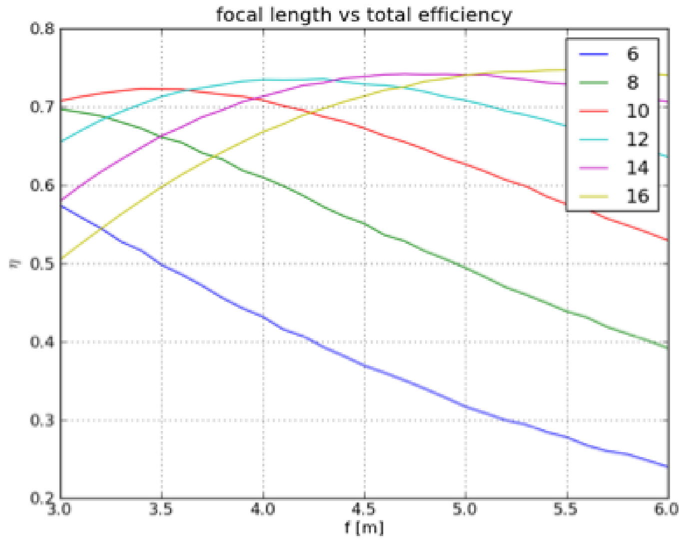

Figure 2: Total efficiency as a function of focal length, for various diameters of parabolic dishes (in meters). Choosing a fiducial focal length of $f = 3.5$m, dish diameters of $10 \leq d \leq 12$m yield efficiencies of $\eta \sim 0.7$.

secondary reflector and/or the primary reflector and the antenna feed. These latter waves are a consequence of the necessarily imperfect match of the feed electronics to the impedance of free space.

To mitigate narcissistic reflections, one typically adds an attenuator or "splash cone" directly below the feed. If we posit that reflections must be suppressed by -60dB at delays corresponding to the time it takes to travel 15m, and if we assume that reflections are attenuated by a factor A for each crossing of the attenuator placed below the feed, then the number of allowed reflections, R, is given by

$$R = \text{floor}\left(\frac{-60\text{dB}}{2A}\right). \tag{6}$$

Thus, for a feed height (or focal length), f, we have

$$2Rf < 15\text{m}. \tag{7}$$

For a moderate attenuation value of $A = -15$dB, we have $f < 4$m.

As shown in Figure **??**, for a chosen focal length of $f = 3.5$m, dish diameters in the range of 10 to 12m yield efficiencies of $\epsilon \approx 0.7$. Hence, the total effective collecting area of a dish that has been tuned for 21cm intensity mapping at EoR frequencies is of order 80m^2. The corresponding effective beam area at 150 MHz (noting the issue described in §**??**) is approximately

$$\Omega' \approx 0.06 \text{ sr}. \tag{8}$$

In addition to the substantial collecting area that this design provides, the relatively narrow field of view associated with a larger dish may go a long way toward mitigating some of the polarization leakage problems that are just on the horizon of being discovered with current designs. However, without concrete measurements, this is largely speculation.

NEED REVISION: A diameter of 10m corresponds to a most efficient focal length of 3.60m instead of 3.50m, 4.5m for 14m diameter.

2.1 The Knee

The eor power spectrum has an upward slope for low k-modes which levels off around $k = 0.15$. It would be nice to work inside this k-mode to say something interesting about eor. This poses a problem due to the fact that without knowing the width of our foregrounds, we can't say for sure which k-modes are corrupted. We also don't want to limit the size of our antenna too much so as to decrease our sensitivity.

The corresponding k-mode for a dish diameter of 10 m is 0.016 and for a dish diameter of 14 m is 0.023 (see Equation **??**). Hence, we have some room for the foreground width's. But, the problem is that we do not know precisely what they are.

In addition to the width of the foreground and the maximum delay of the baseline, narcissistic reflections add into our k budget. For a 10 and 14 meter diameter dish we expect to have a maximum efficiency at a focal length of 3.5 and 4.5 m, respectively. Nominally, we would like these reflections to be 60dB down upon entering the feed. Postulating that each reflection reduces the power by 15 dB, we need four reflections for the right signal to enter our feed. This corresponds to a delay of 45.9 (k=0.023) and 59.0 (k=0.030) ns for focal lengths of 3.5 and 4.5 meters, respectively. Therefore, our budget comes out the $k_{10} = 0.039$ and $k_{14} = 0.053$.

As long as the width of the foregrounds are less than $\Delta k < 0.1$, we are in good shape to working in the knee.

3 Construction

See contruction$_journal.tex$