$$y = f(x) = \sigma(W^L \cdots \sigma(W^2 \sigma(W^1 x + b^1) + b^2) + \cdots + b^L)$$

$$\theta = \{W^1, b^1, W^2, b^2, \cdots, W^L, b^L\}$$

$$W^L = \begin{bmatrix} w_{11}^L & w_{12}^L & \cdots \\ w_{21}^L & w_{22}^L & \cdots \\ \vdots & & \ddots \end{bmatrix} \qquad b^L = \begin{bmatrix} \vdots \\ b^L \\ \vdots \end{bmatrix}$$

$$\nabla C(\theta) = \begin{bmatrix} \vdots \\ \frac{\partial C(\theta)}{\partial w_{ij}} \\ \vdots \\ \frac{\partial C(\theta)}{\partial b_i^L} \end{bmatrix}$$

① [illegible]传递算法

$$z^L = W^L a^{L-1} + b^L$$

$$a^L = \sigma(z^L)$$

$$\Rightarrow a^L = \sigma(W^L a^{L-1} + b^L)$$

② 进行梯度下降

$$w_1^{(t+1)} = w_1^{(t)} - \eta \frac{\partial C(\theta^{(t)})}{\partial w_1}$$

Ps: $$\frac{\partial C(\theta)}{\partial w_1} = 2(\sigma(Wx+b) - \hat{y}) \cdot [1 - \sigma(Wx+b)]\sigma(Wx+b) x_1$$

$$\frac{\partial C(\theta)}{\partial b} = 2(\sigma(Wx+b) - \hat{y}) [1 - \sigma(Wx+b)]\sigma(Wx+b) \cdot 1$$

$$\frac{\partial C(\theta)}{\partial w_{ij}^l} = \frac{\partial C(\theta)}{\partial z_i^l} \cdot \underbrace{\frac{\partial z_i^l}{\partial w_{ij}^l}}$$

拆成两部分

$$\frac{\partial z_i^l}{\partial w_{ij}} = \begin{cases} x_j, & l=1 \\ a_j^{l-1}, & l>1 \end{cases} \Rightarrow \text{前向传播}$$

$$\frac{\partial C(\theta)}{\partial z_i^l} \Rightarrow \text{后向传播}: \delta_i^l:$$

① 反向传播是从~~前向~~后往前推.

② 先计算 δ^L

③ 根据 $\delta^{L+1} \rightarrow \delta^L$

(L为最后一层)

① 计算 δ^L

$$a_i^L = \sigma(z_i^L)$$

激活函数.

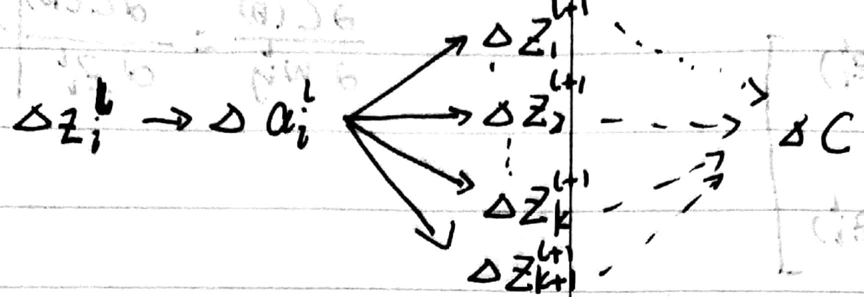
$$\delta^L = \frac{\partial C}{\partial z_i^L} = \frac{\partial C}{\partial y_i} \cdot \frac{\partial y_i}{\partial z_i^L}$$

$$= \frac{\partial C}{\partial y_i} \cdot \sigma'(z_i^L)$$

$$\nabla C(y) = \begin{bmatrix} \frac{\partial C}{\partial y_1} \\ \vdots \\ \frac{\partial C}{\partial y_i} \\ \vdots \end{bmatrix}$$

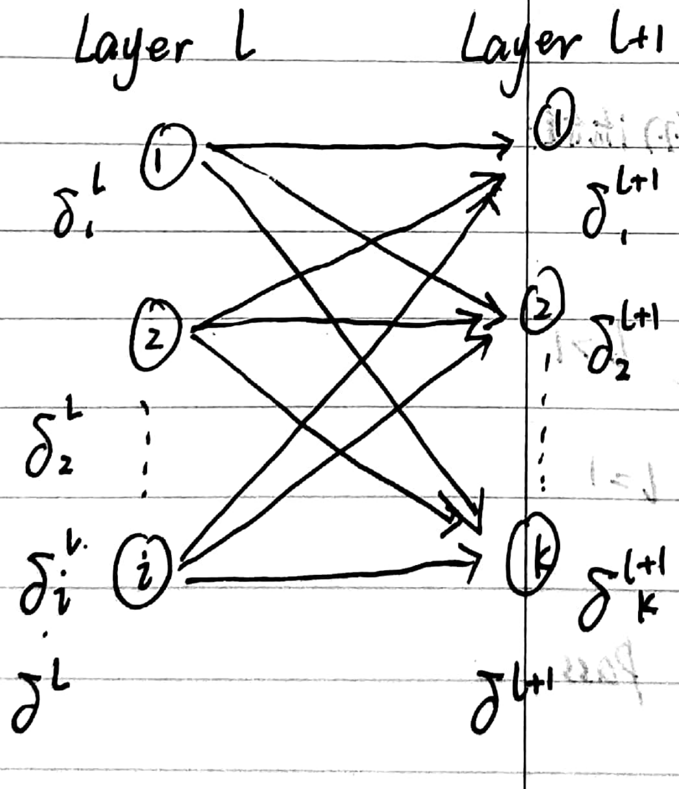
$$\sigma'(z^L) = \begin{bmatrix} \sigma'(z_1^L) \\ \vdots \\ \sigma'(z_i^L) \end{bmatrix}$$

② 计算 δ^l 根据 δ^{l+1}



$$\delta_i^l = \frac{\partial C}{\partial z_i^l} = \sum_k \left(\frac{\partial C}{\partial z_k^{l+1}} \cdot \frac{\partial z_k^{l+1}}{\partial a_i^l} \cdot \frac{\partial a_i^l}{\partial z_i^l} \right)$$

$$= \frac{\partial a_i^l}{\partial z_i^l} \cdot \sum_k \underbrace{\left(\frac{\partial C}{\partial z_k^{l+1}} \cdot \frac{\partial z_k^{l+1}}{\partial a_i^l} \right)}_{\delta_i^{l+1}}$$



$$\delta_i^l = \sigma'(z_i^l) \frac{\partial \sum (w_{ki}^{l+1} a_i^l + b_k^{l+1})}{\partial a_i^l} \cdot \delta_k^{l+1}$$

$$= \sigma'(z_i^l) \cdot \sum_k w_{ki}^{l+1} \delta_k^{l+1}$$

$$\Rightarrow \delta_i^l = \sigma'(z_i^l) \cdot \sum_k w_{ki}^{l+1} \delta_k^{l+1}$$

$$\delta_i^l = \sigma'(z_i) \cdot \sum_k w_{ki}^{l+1} \cdot \delta_k^{l+1}$$

$$\frac{\partial C(\theta)}{\partial w_{ij}^l} = \frac{\partial C(\theta)}{\partial z_i^l} \boxed{\frac{\partial z_i^l}{\partial w_{ij}^l}}$$

$$\sigma'(z^l) = \begin{bmatrix} \sigma'(z_1^l) \\ \vdots \\ \sigma'(z_i^l) \\ \vdots \end{bmatrix}$$

$$\delta^l = \sigma'(z^l) \odot (W^{l+1})^T \cdot \delta^{l+1}$$

综上得：

① 先计算 δ^L： $\delta^L = \sigma'(z^L) \odot \nabla C(y)$

② 根据 $\delta^{l+1} \to \delta$ $\delta^l = \sigma'(z^l) \odot (W^{l+1})^T \delta^{l+1}$

BP的流程：

① 先前向传播：

$$\frac{\partial z_i^l}{\partial w_{ij}^l} = \begin{cases} a_j^{l-1}, & l > 1 \\ x_j, & l = 1 \end{cases}$$

先前向： Forward Pass

$$z^1 = W^1 x + b^1 \qquad a^1 = \sigma(z^1)$$

$$\vdots$$

$$z^l = W^l a^{l-1} + b^l \qquad a^l = \sigma(z^l)$$

$$\vdots$$

$$\frac{\partial C(b)}{\partial z_i^l} = \delta_i^l$$

$$\delta^L = \sigma'(z^L) \odot \nabla C(y)$$

$$\delta^{L-1} = \sigma'(z^{L-1}) \odot (W^L)^T \delta^L$$

$$\vdots$$

$$\delta^l = \sigma'(z^l) \odot (W^{l+1}) \delta^{l+1}$$

$$\vdots$$