The Concrete Floor Problem

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Abstract

The U.S. Department of Housing and Urban Development (HUD) researches ways to reduce heating costs for home owners, including the use of the ambient temperature to maintain a comfortable average temperature of concrete slab foundations. Using the heat diffusion partial differential equation, the average temperature of the concrete slab was modeled over time. Modeling the ambient temperature with a sinusoidal function allows for modeling the variation in temperature through the a single day and through a year. Less temperate climates require the use of external heating sources to maintain a comfortable average temperature; thus, the slab temperature was modeled using two different types of external heat: a fireplace and baseboards.

Contents

	Ambient Temperature	Comfort Zone
High	85	76
Average	72.5	70.5
Low	60	65

Table 1: *The ambient and comfort temperatures for a temparate climate.*

1 Introduction

The U.S. Department of Housing and Urban Development (HUD) is constantly trying to find ways to minimize the use and ultimately the cost of heating and cooling in housing. Through new construction techniques, constrictors have started to depend on ambient temperature variation to maintain comfortable temperature levels in concrete foundations. Many single story houses in temperate climates do not require or rely on external heating and cooling, as the small temperature variation in ambient temperature causes the concrete slab foundation to slightly vary in temperature. The goal of this project is to determine if the average temperature of the concrete slab can be maintained within comfort temperature levels throughout the year, and what shapes and sizes of concrete slabs will permit a constant temperature.

2 Modeling in Temperate Climate

The ambient temperature variation and predetermined comfort temperatures for the concrete slabs are seen in Table 1.

The temperature of the concrete slab was modeled over the course of 60 days or approximately two months using the two dimensional heat diffusion equation,

$$\frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{1}$$

where T is the temperature of the concrete slab and $k = 1.3e-6\ m^2/s$. Using finite differences to approximate the derivatives in Equation 2, the temperature of the concrete slab was modeled for two months, or approximately 60 days, in the future.

The boundary conditions of the ambient temperature were modeled according to the equation suggested by Ali et. al:

$$A_t = \frac{A_{max} + A_{min}}{2} - \left(\frac{A_{max} - A_{min}}{2}\right) sin\left(\frac{2\pi(t+3)}{24}\right) \tag{2}$$

where A_t is the ambient temperature at hour of the day t, $A_max = 85$, and $A_min = 60$. Equation 2 assumes the minimum temperature occurs at 3am every day, and thus the maximum temperature will occur at 3pm each day. Figure 1 shows how the ambient temperature changes through each day, as modeled by Equation 2.

It is important to note for this model that $dx = dy = 0.25m$ and $dt = (60)(60)(3.25) = 11700sec$, as a dt value larger than 3.25 days causes the model to become unstable.

2.1 Results

To determine if the ambient temperature was enough to maintain a constant comfortable temperature in the concrete slab, the model was tested on several slab sizes and different initial temperature conditions. Table 2 shows the average temperature of the concrete slab after 60

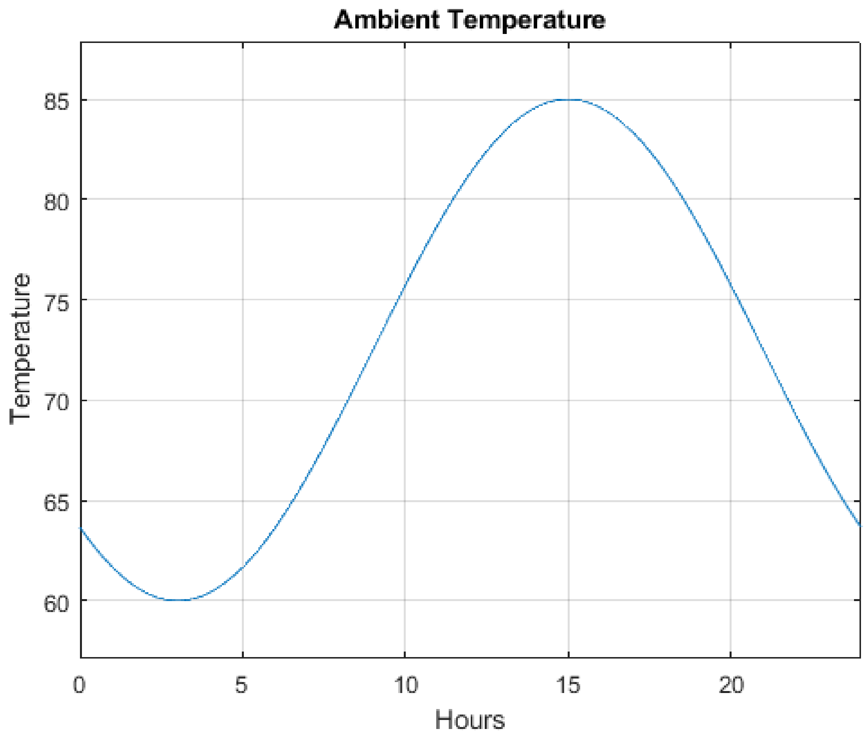

Figure 1: *The ambient temperature throughout one day, as modeled by Equation 2.*

days for different slab sizes and initial conditions. The initial conditions used are the minimum ambient temperature and the average comfort temperature. The Joint Center for Housing Studies approximates the size of a single family house to be 2426 square feet. Assuming the house is two stories, and converting the area into meters, yields a slab with an area of 112.69 square meters. By rounding dimensions to the nearest meter, possible two story home sizes around the median house size would have slabs with dimensions of $11m \times 11m$, $12m \times 10m$, and $8m \times 14m$. If the house is only one level, the dimension of the concrete slab would be about $15m \times 15m$.

Notice how the ambient temperature causes the temperature of the slab to rise, even when below the minimum comfort temperature. Figure 2 shows the temperature of the concrete slab after 60 days and the average temperature of the slab over the course of 60 days. As expected the average temperature of the slab oscillates due to Equation 2.

Figure 2 gives the impression that slab temperature may be approaching a constant tem-

Size of Slab	Starting Temperature	Average Temperature After 60 days
11m x 11m	60	69.95
11m x 11m	70.5	72.36
12m x 10m	60	70.03
12m x 10m	70.5	72.38
8m x 14m	60	70.66
8m x 14m	70.5	72.50
15m x 15m	60	67.93
15m x 15m	70.5	70.50

Table 2: *The average temperature of the concrete slab after 60 days for different slab sizes and initial conditions.*

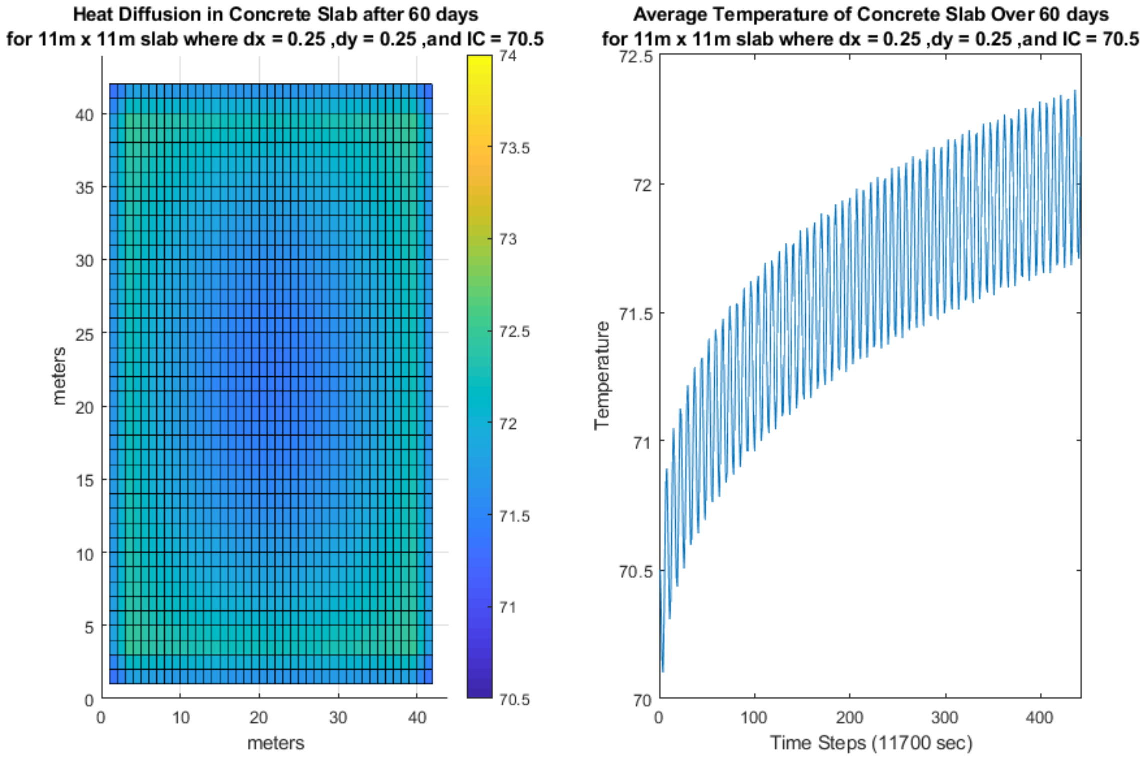

Figure 2: *The temperature of the concrete slab after 60 days and the average temperature of the slab over the course of 60 days.*

perature. Figure 3 shows the concrete slab does approach an asymptote around 72.5 degrees, but oscillates around this temperature.

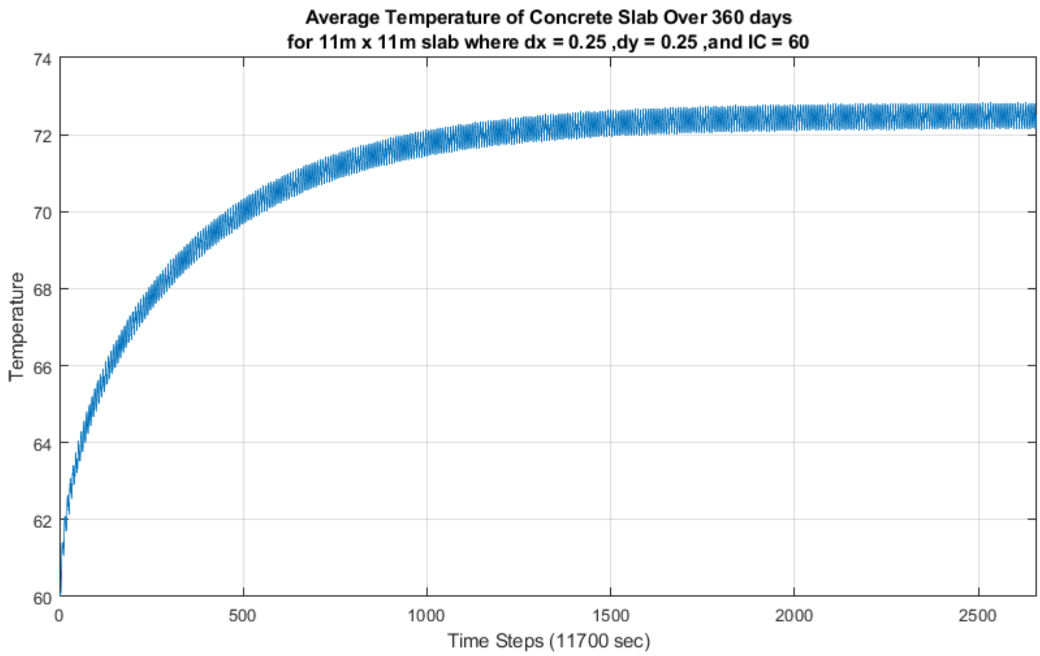

Figure 3: *The average temperature of a 11m x 11m concrete slab over 240 days.*

Overall, the relative size of the slab does not affect the ability for the ambient temperature to maintain a comfortable temperature in a temperate climate.

	Jan	Feb	Mar	Apr	May	Jun
Average High (F)	33	39	48	58	67	76
Average Low (F)	13	17	24	32	41	49
	Jul	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
Average High (F)	86	85	73	59	43	32
Average Low (F)	54	52	43	33	22	12

Table 3: *The average low and high temperatures for Helena, Montana, as reported by U.S. Climate Data.*

3 Modeling in Montana

To adapt the model to less temperate climate, such as Helena, Montana, Equation 2 was adapted to consider the average minimum and maximum temperature of each month in Helena reported by the U.S. Climate Data, as seen in Table 3. Each month, the T_{min} and T_{max} in Equation 2 changes to the average low and high, respectively, for that month. Figure 4 shows the ambient temperature over the course of 1 day in January, February, June, July, August, and December in Helena, Montana.

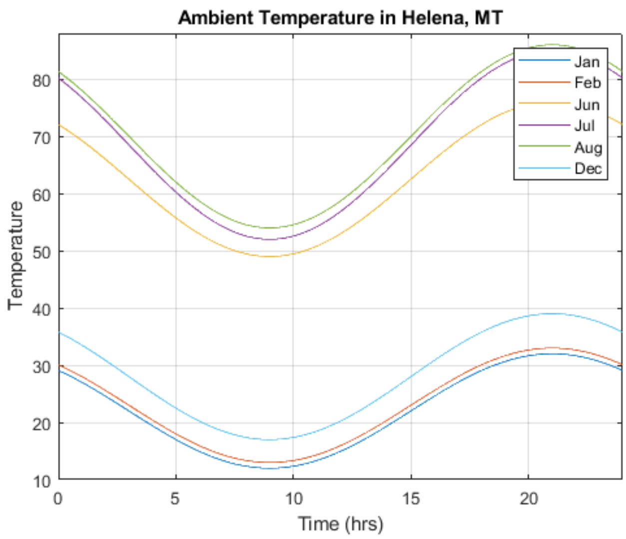

Figure 4: *The ambient temperature over the course of 1 day in January, February, June, July, August, and December in Helena, Montana, as modeled using Equation 2 and the temperatures in Table 3.*

Due to the large variation in the ambient temperature through the year, the concrete slab will not remain at a comfortable level without an external heat source. To attempt to maintain a constant average temperature two additional models were created which simulate the use of a fireplace and baseboard heat. These models use the same parameters and are the same as the prior model in a temperate climate, with the exception of when the average concrete slab temperature falls below 65 degrees.

3.1 Fireplace

The increase in the concrete slab temperature every 3.25 hours (the value of dt) due a fireplace can be modeled by,

$$F(x,y) = \left(\frac{dx}{85}\right)\left(\left(\frac{-(x-2)^2}{2}\right) - \left(y - \frac{{x_{max}}^2}{2}\right) + 85\right) \tag{3}$$

where $F(x,y)$ is the increase in temperature at (x,y), x_{max} is the size of the house in the x direction and the heat coming from the fireplace is assumed to be a constant 85 degrees. Figure 5 shows a contour of Equation 3 for a $11m \times 11m$ concrete slab. Notice by the temperature increase, it is implied the fireplace exists in the bottom left corner of the house. For this example, it is assumed the fireplace is an automatic starting fireplace.

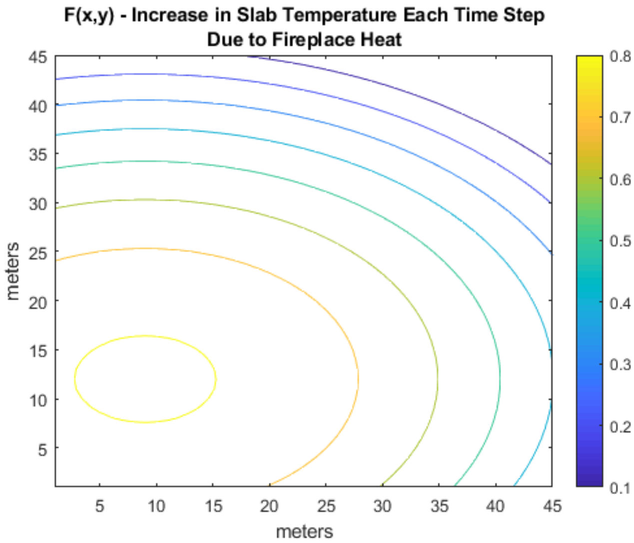

Figure 5: *A contour of Equation 3, which models the increase in the concrete slab temperature each time step due to the heat output of the fireplace.*

If the average temperature of the concrete slab falls below the minimum comfort temperature, then the temperature of the concrete slab as a result of heating can be modeled by

$$T(x,y,n) = T(x,y,n) + F(x,y) \tag{4}$$

where $T(x,y,n)$ is the temperature of the concrete slab at point (x,y) at time step n.

Figure 6 shows a surface plot of the temperature of $11m \times 11m$ concrete slab after 365 days. Notice how the center of the slab is extremely warm and exceeds the maximum comfort temperature. This follows intuition, as houses that use fireplaces as a primary heat source experience different temperatures in each room, as the fireplace heats primarily the room it is in. This large variance in temperature within the house causes the average temperature of the slab to be below the minimum comfort temperature, despite the extremely warm center. It is important to note that due to the variation in temperatures daily, the ambient temperature is not warm long enough to heat the concrete slab due to cold evenings; thus, the fireplace is required run every day through the year. As a result, the inability to keep the whole slab at a comfortable level causes the makes the use of fireplace for primary heat source for a house is poor decision, though a fireplace would work well for heating only one room.

3.2 Baseboard Heating

Since the use of a fireplace was unsuccessful in maintaining a comfortable temperature, this model uses baseboard heat as the primary source of external heat. Like the fireplace model,

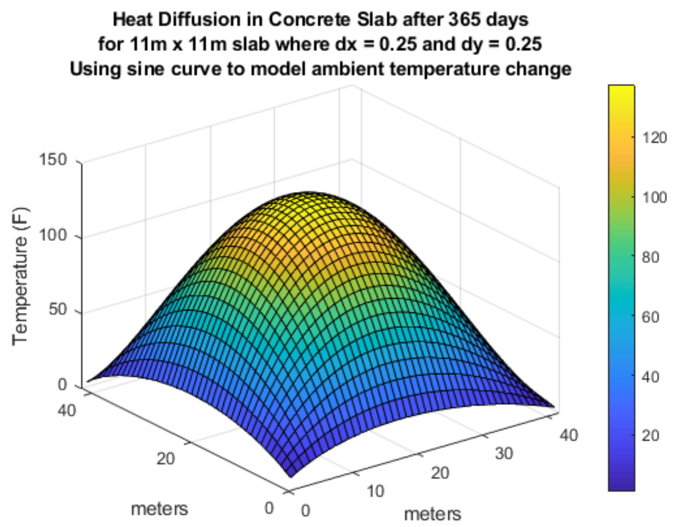

Figure 6: .

this model only differs from the temperate climate model when the average temperature of the slab falls below the minimum comfort level.

The baseboard heaters are modeled by simply changing the boundary conditions to the expected temperature at the baseboards of 85 degrees. Figure 7 shows the Boolean array $B(x, y)$ that model position of the baseboard heaters. The array $B(x, y)$ is filled with zeros, except at the location of the baseboards where the value is one.

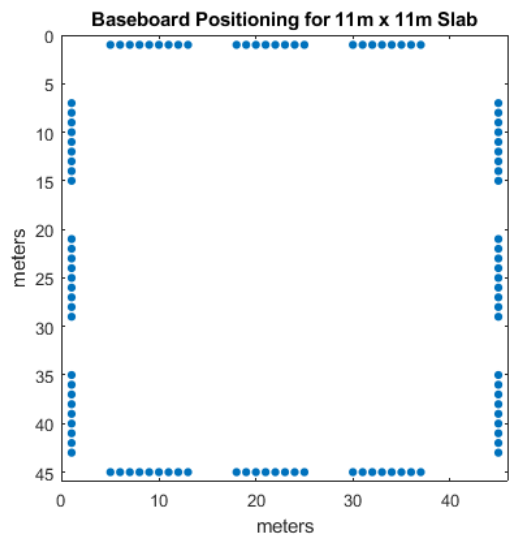

Figure 7: *The Boolean array* $B(x, y)$ *that describes whether or not a baseboard exists at point* (x, y)*.*

If the average slab temperature drops below the minimum comfort temperature, the new boundary conditions for the model for the next time step are

$$T(x, y, n) = T(x, y, n) - T(x, y, n) * B(x, y) + 85B(x, y) \tag{5}$$

where $T(x, y, n)$ is the temperature of the concrete slab at point (x, y) at time step n and $B(x, y)$ is the Boolean variable that describes if a baseboard exists at point (x, y).

Figure 8 shows the ambient temperature over a year. The colored points distinguish whether the baseboard heaters came on to heat a $11m \times 11m$ slab that started with an initial temperature of 60 degrees. This models follows intuition, as the baseboards do not turn on as often or at all during the summer months, but is constantly on during the winter months.

Further, Figure 8 shows the average temperature of a $11m \times 11m$. Similarly, as expected the average temperature of the concrete slab is higher in the summer than the rest of the year when it remains near 65 degrees, the minimum comfort level, due to the dependency on the baseboards for heat. Further, baseboard heat can maintain a comfortable temperature for a $15m \times 15m$, though the baseboards turn on more often than with smaller slabs, as seen in Figure 9.

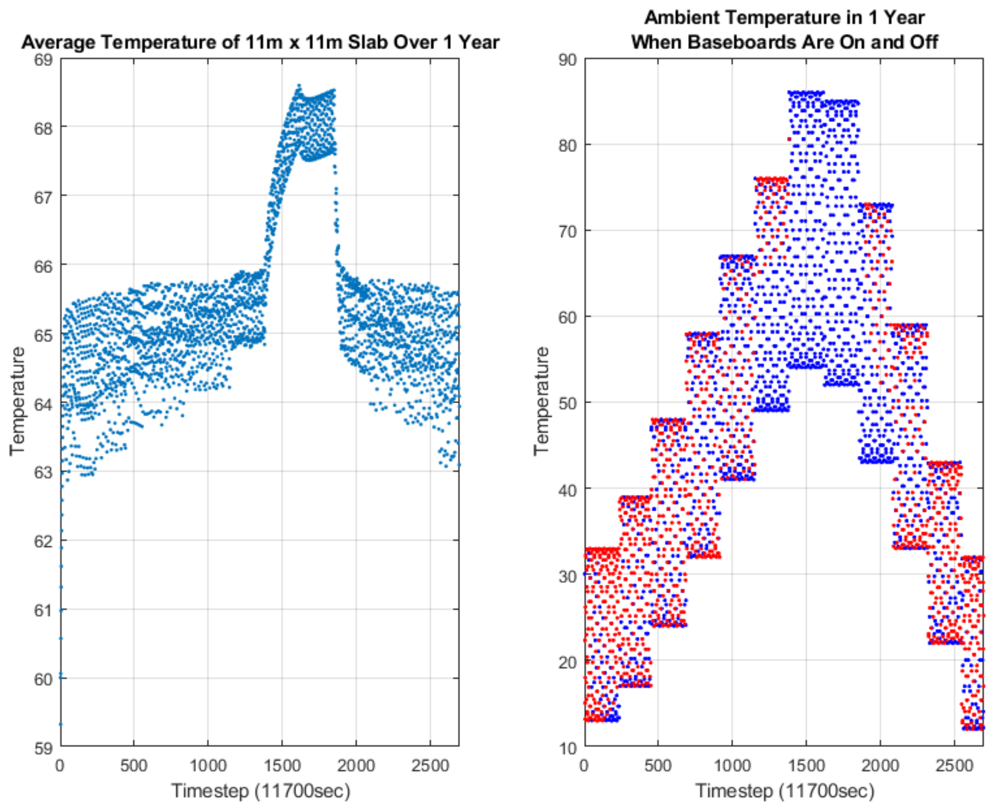

Figure 8: *Whether the baseboards turned on or off during the one year simulation, as well as the average temperature of a 11m x 11m concrete slab during the one year simulation.*

4 Conclusion

In a temperate climate, the heat diffusion resulting from the ambient temperature is enough to maintain a comfortable concrete slab temperature. However, when living in a less temperate climate where the temperature each day and month vary greatly, maintaining a comfortable average temperature of the slab depends on the use of external heat sources. As shown above, the use of a fireplace for an external heat source would work well for a single room, but is very inefficient for a whole house, as the fireplace would need to run most of the year due to the high temperature variation in the slab. However, the use of baseboards is an efficient way to maintain a comfortable temperature, as they can more uniformly heat the area, and thus do not run all year.

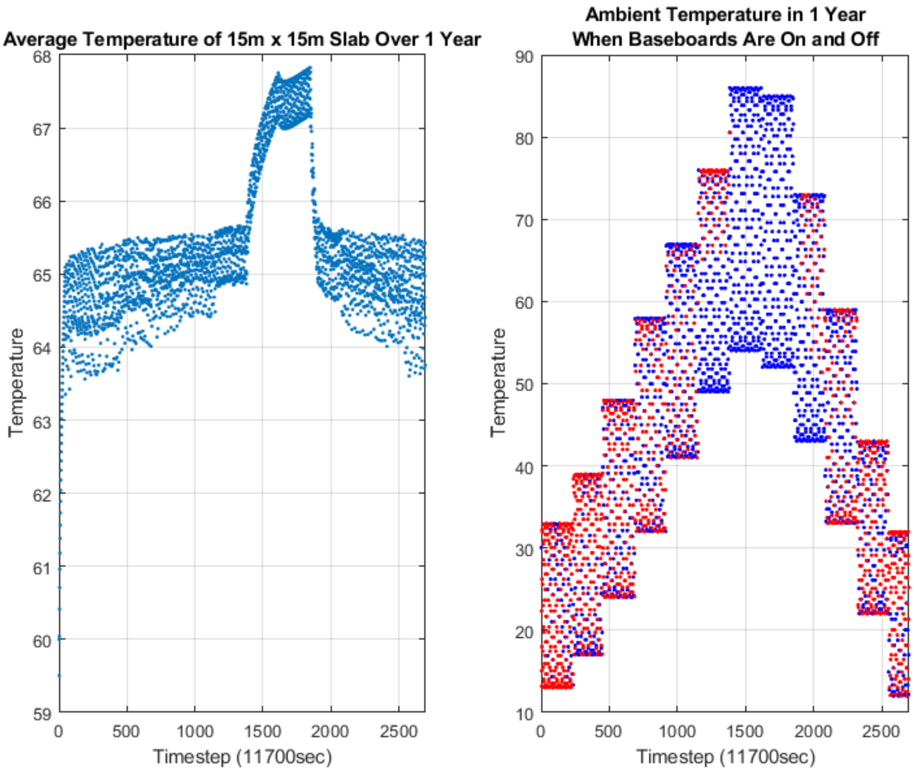

Figure 9: *Whether the baseboards turned on or off during the one year simulation, as well as the average temperature of a 15m x 15m concrete slab during the one year simulation.*

5 Works Cited

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