National Tsing Hua University - Duracell

### data structure

### Sparse Table

```
1 #include <bits/stdc++.h>
using namespace std;
3 int n;
4 int v[1000009];
5 int sparse[22][1000009];
6 // O(nlogn) preprocess O(1)Query
7 // sp[x][y] is the answer from (v[x], v[x+2^y-1])
  inline void init()
       for (int i = 0; i < n; ++i)
           sparse[0][i] = v[i];
       for (int j = 1; (1 << j) <= n; ++j)
12
           for (int i = 0; i + (1 << j) <= n; ++i)
           sparse[j][i] = min(
           sparse[j - 1][i],
16
           sparse[j - 1][i + (1 << (j - 1)])
   inline int query(int 1, int r)
21
^{22}
       int k = __lg(r - l + 1);
23
       return min(sparse[k][l], sparse[k][r - (1 << k) + 1]);
```

### segment Tree

```
1 #define LL long long
2 #define IL(X) ((X << 1) + 1)
3 #define IR(X) ((X << 1) + 2)
4 #define MAXN 500005
5 // add tag
6 // tag += tag
7 // val += tag*size
   struct segID{
       struct Node{
           LL val;
           LL lazy_tag;
13
           int size;
14
15
       LL dataseq[MAXN];
       Node seq[MAXN * 4 + 5];
       void pull(int index){
           seq[index].val = seq[IL(index)].val + seq[IR(index)].
       void push(int index){
^{21}
           seq[IL(index)].lazy_tag += seq[index].lazy_tag;
22
           seq[IL(index)].val += seq[index].lazy tag * seq[IL(
                index)].size;
           seq[IR(index)].lazy_tag += seq[index].lazy_tag;
           seq[IR(index)].val += seq[index].lazy_tag * seq[IR(
                index)].size;
           seq[index].lazy_tag = 0;
```

```
int f[MAX N];
void build(int L, int R, int index){
                                                                   void init(int N){
    if(L == R){
        seq[index].val = dataseq[L];
                                                            10
        seq[index].size = 1;
                                                            11
        seq[index].lazy tag = 0;
                                                            12
        return:
                                                            13
                                                            14
    int M = (L + R) / 2;
                                                            15
                                                                   int find(int v){
    build(L, M, IL(index));
                                                            16
    build(M + 1, R, IR(index));
    seq[index].size = seq[IL(index)].size + seq[IL(index) 18
         l.size;
    pull(index):
                                                            20
                                                                   bool same(int a, int b){
                                                            21
                                                            22
void modify(int 1, int r, int L, int R, int index, long
                                                                   void Union(int a, int b){
                                                            23
    long Add){
                                                            24
    if(1 == L \&\& r == R){
                                                            25
        seq[index].lazy tag += Add;
                                                            26
        seq[index].val += Add * seq[index].size;
                                                            27
        return:
                                                            28
                                                            29
                                                            30
    push(index);
   int M = (L + R) / 2;
                                                            31
                                                            32 };
    if(r \leftarrow M)
        modify(l, r, L, M, IL(index), Add);
   else if(1 > M){
        modify(l, r, M + 1, R, IR(index), Add);
                                                                    geometry
        modify(1, M, L, M, IL(index), Add);
        modify(M + 1, r, M + 1, R, IR(index), Add);
```

### 2.1 closest point

for (int i = 0; i < N; i++){

f[i] = i;

if(f[v] == v)

return v;

if(!same(a,b)){

return f[v] = find(f[v]);

return find(a) == find(b);

// f[find(a)] = find(b);

f[f[b]] = f[a];

rank[a]++;

if(rank[a] < rank[b])</pre>

swap(a, b);

rank[i] = 1;

```
1 template <typename T>
2 T ClosestPairSquareDistance(typename vector<Point<T>>::
       iterator 1.
                               typename vector<Point<T>>::
                                    iterator r)
       auto delta = numeric limits<T>::max();
      if (r - 1 > 1)
           auto m = 1 + (r - 1 >> 1);
           nth_element(l, m, r); // Lexicographical order in
               default
11
           delta = min(ClosestPairSquareDistance<T>(1, m),
                       ClosestPairSquareDistance<T>(m, r));
12
13
           auto square = [&](T y) { return y * y; };
14
           auto sgn = [=](T a, T b) {
15
               return square(a - b) <= delta ? 0 : a < b ? -1 :</pre>
           vector<Point<T>> x near[2];
17
           copy_if(1, m, back_inserter(x_near[0]), [=](Point<T>
               a) {
               return sgn(a.x, x) == 0;
           });
           copy_if(m, r, back_inserter(x_near[1]), [=](Point<T>
               a) {
               return sgn(a.x, x) == 0;
           for (int i = 0, j = 0; i < x near[0].size(); ++i)
```

# 1.3 disjointset

pull(index);

push(index);

 $else if(1 > M){$ 

 $if(r \leftarrow M)$ 

}else{

 $if(1 == L \&\& r == R){$ 

int M = (L + R) / 2;

return seq[index].val;

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72

73

74

75

76

78 };

```
19
1 | #include <algorithm>
                                                                    20
2 using namespace std;
                                                                    21
3 #define MAX N 200005
  struct disjointset
                                                                    22
                                                                    23
      int rank[MAX N];
```

long long Query(int 1, int r, int L, int R, int index){

return Query(1, r, L, M, IL(index));

return Query(1, M, L, M, IL(index)) +

Query(M + 1, r, M + 1, R, IR(index));

return Query(l, r, M + 1, R, IR(index));

```
while (j < x near[1].size() and</pre>
                                                                      40
                       sgn(x_near[1][j].y, x_near[0][i].y) == -1) 41
                    ++j;
                for (int k = j; k < x near[1].size() and
                                 sgn(x_near[1][k].y, x_near[0][i]. 44
                                      v) == 0:
                     ++k)
                                                                      46
                    delta = min(delta, (x_near[0][i] - x_near[1][
                         k]).norm());
                                                                      49
           inplace merge(l, m, r, [](Point<T> a, Point<T> b) {
                return a.y < b.y;</pre>
                                                                      51
           });
38
                                                                      52
39
                                                                      53
40
       return delta;
                                                                      54
                                                                      55
                                                                      57
                                                                      58
```

1 template <typename T, typename Real = double>

```
34
           return 1.ori(st) * 1.ori(ed) < 0 and ori(1.st) * ori(</pre>
                1.ed) < 0;
36
37
38
       bool isPointOnSegProperly(const Point<T> p) const
39
           return ori(p) == 0 and ((st - p) & (ed - p)) < 0;
40
41
42
43
       Point<Real> getIntersection(const Line<Real> 1)
44
           Line<Real> h = *this;
45
46
           return 1((1.st ^ h.vec()) / (h.vec() ^ 1.vec()));
47
48
       Point<Real> projection(const Point<T> p) const
49
50
           return operator()(((p - st) & vec()) / (Real)(vec().
51
                norm()));
52
53
```

#### 2.2 points

1 template <tvpename T>

```
2 struct Point
      T x, y;
       Point(): x(0), y(0) {}
       Point(const T x, const T y) : x(x), y(y) {}
       template <class F>
       explicit operator Point<F>() const
           return Point<F>((F)x, (F)y);
12
13
       Point operator+(const Point b) const
14
           return Point(x + b.x, y + b.y);
15
                                                                   10
       Point operator-(const Point b) const
                                                                   11
                                                                   12
           return Point(x - b.x, y - b.y);
                                                                   13
                                                                   14
       template <class F>
                                                                   15
       Point<F> operator*(const F fac)
22
                                                                   16
23
           return Point<F>(x * fac, y * fac);
24
25
26
       template <class F>
       Point<F> operator/(const F fac)
27
28
           return Point<F>(x / fac, y / fac);
29
30
31
       T operator&(const Point b) const { return x * b.x + y * b
32
       // 內 積 運 算 子
33
       T operator^(const Point b) const \{ return x * b.y - y * b _{28}
34
            .x; }
       // 外 積 運 算 子
                                                                   30
       bool operator == (const Point b) const
                                                                   31
37
                                                                   32
           return x == b.x and y == b.y;
```

#### 2.3 lines

```
struct Line
    Point<T> st, ed;
    Point<T> vec() const { return ed - st; }
    T ori(const Point<T> p) const { return (ed - st) ^ (p -
    Line(const Point\langle T \rangle x, const Point\langle T \rangle y) : st(x), ed(y)
         {}
    template <class F>
    operator Line<F>() const
        return Line<F>((Point<F>)st, (Point<F>)ed);
    // sort by arg, the left is smaller for parallel lines
    bool operator<(Line B) const</pre>
        Point<T> a = vec(), b = B.vec();
        auto sgn = [](const Point<T> t) { return (t.y == 0 ? 17
             t.x : t.y) < 0; };
        if (sgn(a) != sgn(b))
             return sgn(a) < sgn(b);</pre>
        if (abs(a ^ b) == 0)
             return B.ori(st) > 0;
        return (a ^ b) > 0;
    // Regard a line as a function
    template <typename F>
    Point<F> operator()(const F x) const
        return Point<F>(st) + vec() * x;
    bool isSegProperIntersection(const Line 1) const
```

#### 2.4 geometry template

```
1 // import from https://codeforces.com/blog/entry/48122
2 #include <iostream>
  using namespace std;
  template <class F>
  struct Point {
    Point(): x(0), y(0) {}
    Point(const F& x, const F& y) : x(x), y(y) {}
    void swap(Point& other) { using std::swap; swap(x, other.x)
         ; swap(y, other.y); }
    template <class F1> explicit operator Point<F1> () const {
      return Point<F1>(static cast<F1>(x), static cast<F1>(y));
     template <class F1> Point& operator = (const Point<F1>&
      x = other.x; y = other.y; return *this; }
     template <class F1> Point& operator += (const Point<F1>&
      x += other.x; y += other.y; return *this; }
     template <class F1> Point& operator -= (const Point<F1>&
      x -= other.x; y -= other.y; return *this; }
     template <class F1> Point& operator *= (const F1& factor) {
      x *= factor; y *= factor; return *this; }
    template <class F1> Point& operator /= (const F1& factor) {
      x /= factor; y /= factor; return *this; }
22
23 };
24
  template <class F> int read(Point<F>& point) { return read(
       point.x, point.y) / 2; }
  template <class F> int write(const Point<F>& point) { return
       write(point.x, point.y); }
  template <class F> istream& operator >> (istream& is, Point<F
       >& point) {
    return is >> point.x >> point.y; }
```

```
30 template <class F> ostream& operator << (ostream& os, const
       Point<F>& point) {
    return os << point.x << ' ' << point.y; }</pre>
   template <class F> inline Point<F> makePoint(const F& x,
       const F& y) { return Point<F>(x, y); }
  template <class F> void swap(Point<F>& lhs. Point<F>& rhs) {
       lhs.swap(rhs); }
  #define FUNC1(name, arg, expr) \
   template <class F> inline auto name(const arg) -> decltype(
       expr) { return expr: }
  #define FUNC2(name, arg1, arg2, expr) \
39 template <class F1, class F2> \
40 inline auto name(const arg1, const arg2) -> decltype(expr) {
       return expr; }
41 #define FUNC3(name, arg1, arg2, arg3, expr) \
42 template <class F1, class F2, class F3> \
43 inline auto name(const arg1, const arg2, const arg3) ->
       decltype(expr) { return expr; }
  FUNC1(operator -, Point<F>& point, makePoint(-point.x, -point
46 FUNC2(operator +, Point<F1>& lhs, Point<F2>& rhs, makePoint(
       lhs.x + rhs.x, lhs.y + rhs.y)
47 FUNC2(operator -, Point<F1>& lhs, Point<F2>& rhs, makePoint(
       lhs.x - rhs.x, lhs.y - rhs.y))
48 FUNC2(operator *, F1& factor, Point<F2>& rhs, makePoint(
       factor * rhs.x, factor * rhs.y))
49 FUNC2(operator *, Point<F1>& lhs, F2& factor, makePoint(lhs.x
         * factor, lhs.y * factor))
  FUNC2(operator /, Point<F1>& lhs, F2& factor, makePoint(lhs.x
        / factor, lhs.y / factor))
  FUNC2(operator *, Point<F1>& lhs, Point<F2>& rhs, lhs.x * rhs
       .x + lhs.y * rhs.y)
  FUNC2(operator ^, Point<F1>& lhs, Point<F2>& rhs, lhs.x * rhs 11
       .y - lhs.y * rhs.x)
                                                                 12
   // < 0 if rhs <- lhs counter-clockwise, 0 if collinear, > 0
       if clockwise.
56 FUNC2(ccw, Point<F1>& lhs, Point<F2>& rhs, rhs ^ lhs)
  FUNC3(ccw, Point<F1>& lhs, Point<F2>& rhs, Point<F3>& origin, 17
        ccw(lhs - origin, rhs - origin))
  FUNC2(operator ==, Point<F1>& lhs, Point<F2>& rhs, lhs.x ==
       rhs.x && lhs.y == rhs.y)
  FUNC2(operator !=, Point<F1>& lhs, Point<F2>& rhs, !(lhs ==
                                                                 24
62 FUNC2(operator <, Point<F1>& lhs, Point<F2>& rhs,
      lhs.y < rhs.y \mid (lhs.y == rhs.y && lhs.x < rhs.x))
64 FUNC2(operator >, Point<F1>& lhs, Point<F2>& rhs, rhs < lhs)
  FUNC2(operator <=, Point<F1>& lhs, Point<F2>& rhs, !(lhs >
66 FUNC2(operator >=, Point<F1>& lhs, Point<F2>& rhs, !(lhs <
  // Angles and rotations (counter-clockwise).
69 FUNC1(angle, Point<F>& point, atan2(point.y, point.x))
  FUNC2(angle, Point<F1>& lhs, Point<F2>& rhs, atan2(lhs ^ rhs, 35
        1hs * rhs))
71 FUNC3(angle, Point<F1>& lhs, Point<F2>& rhs, Point<F3>&
                                                                 37
         angle(lhs - origin, rhs - origin))
73 FUNC3(rotate, Point<F1>& point, F2& angleSin, F3& angleCos,
```

```
FUNC3(rotate, Point<F1>& point, F2& angle, Point<F3>& origin,
        origin + rotate(point - origin, angle))
  FUNC1(perp, Point<F>& point, makePoint(-point.v, point.x))
  // Distances.
  FUNC1(abs, Point<F>& point, point * point)
  FUNC1(norm, Point<F>& point, sqrt(abs(point)))
  FUNC2(dist, Point<F1>& lhs, Point<F2>& rhs, norm(lhs - rhs))
  FUNC2(dist2, Point<F1>& lhs, Point<F2>& rhs, abs(lhs - rhs))
  FUNC2(bisector, Point<F1>& lhs, Point<F2>& rhs, lhs * norm(
       rhs) + rhs * norm(lhs))
  #undef FUNC1
89 #undef FUNC2
90 #undef FUNC3
```

makePoint(angleCos \* point.x - angleSin \* point.y,

angleSin \* point.x + angleCos \* point.y))

#### 2.5 cp geometry.

angle), cos(angle)))

```
1 #include <bits/stdc++.h>
2 using namespace std:
  const double eps = 1e-6;
  inline int fcmp(const double &a, const double &b){
      if(fabs(a - b) < eps)</pre>
         return 0;
     return ((a - b > 0.0) * 2) - 1;
  template <typename T>
  struct Point
      Point(): x(0), y(0) {}
      Point(const T x, const T y) : x(x), y(y) {}
      template <class F>
      explicit operator Point<F>() const
          return Point<F>((F)x, (F)y);
      Point operator+(const Point b) const
          return Point(x + b.x, y + b.y);
      Point operator-(const Point b) const
         return Point(x - b.x, y - b.y);
      template <class F>
      Point<F> operator*(const F fac)
          return Point<F>(x * fac, y * fac);
      template <class F>
      Point<F> operator/(const F fac)
          return Point<F>(x / fac, y / fac);
```

```
T operator&(const Point b) const { return x * b.x + y * b
                                                                          .v; }
FUNC2(rotate, Point<F1>& point, F2& angle, rotate(point, sin(42))
                                                                     // dot operator
                                                                     T operator^(const Point b) const { return x * b.y - y * b
                                                                     // cross operator
                                                              45
                                                                     bool operator==(const Point b) const
                                                               46
                                                              47
                                                                          return x == b.x and y == b.y;
                                                              48
                                                              49
                                                                     bool operator<(const Point b) const</pre>
                                                              50
                                                                          return x == b.x ? y < b.y : x < b.x;
                                                              51
                                                                     } // 字 典 序
                                                                     Point operator-() const { return Point(-x, -y); }
                                                               55
                                                                     T norm() const { return *this & *this; } // 歐 式 長
                                                                          度平方
                                                                     Point prep() const { return Point(-y, x); } // 左 旋 直
                                                                     template <class F>
                                                                     friend istream &operator>>(istream &is, Point<F> &pt);
                                                                     template<class F>
                                                                     friend ostream &operator<<((ostream &os, const Point<F> &
                                                               61 };
                                                                 template <class F>
                                                                 ostream &operator<<((ostream &os, const Point<F> &pt)
                                                                     return os << "(" << pt.x << " " << pt.y << ")";
                                                               66
                                                               67 template <class F>
                                                               68 istream & operator >> (istream & is, Point < F > & pt)
                                                                     return is >> pt.x >> pt.y;
                                                              71
                                                               72 template <class F>
                                                                 bool collinearity(const Point<F>& p1, const Point<F>& p2,
                                                                      const Point<F>& p3){
                                                                     return (p1 - p3) ^ (p2 - p3) == 0;
                                                                     return fcmp((p1 - p3) ^{\circ} (p2 - p3), 0.0) == 0;
                                                              77 // check co-line first. properly
                                                               78 template<class F>
                                                               79 inline bool btw(const Point<F>& p1, const Point<F>& p2, const
                                                                       Point<F>& p3){
                                                                     return fcmp((p1 - p3) & (p2 - p3), 0.0) <= 0;
                                                               81
                                                               83 // is p3 on (p1, p2)?
                                                               84 template<class F>
                                                               85 inline bool pointOnSegment(const Point<F>& p1, const Point<F</pre>
                                                                      >& p2, const Point<F>& p3){
                                                                     return collinearity(p1, p2, p3) && btw(p1, p2, p3);
                                                               87
                                                                 template <typename T, typename Real = double>
                                                                 struct Line
                                                              91
                                                                     Point<T> st. ed:
                                                                     Point<T> vec() const { return ed - st; }
                                                                     T ori(const Point<T> p) const { return (ed - st) ^ (p -
                                                                     int orint(const Point<T> p) const{
                                                                          T a = this->ori(p);
```

```
return (fcmp(a, 0.0)); // 1 on posi-side // -1 nega- 152
                 side
            // a little bit useless?
                                                                      153
99
                                                                      154
        Line(const Point\langle T \rangle x, const Point\langle T \rangle y) : st(x), ed(y)
100
                                                                      155
101
        template <class F>
                                                                      157
        operator Line<F>() const
102
                                                                      158
103
                                                                      159
            return Line<F>((Point<F>)st, (Point<F>)ed);
                                                                      160
104
105
106
                                                                      161
        // sort by arg, the left is smaller for parallel lines
107
                                                                      162
108
        bool operator<(Line B) const
109
110
            Point<T> a = vec(), b = B.vec();
            auto sgn = [](const Point<T> t) { return (t.y == 0 ? 165
111
                 t.x : t.y) < 0; };
            if (sgn(a) != sgn(b))
112
                return sgn(a) < sgn(b);</pre>
113
                                                                      167
114
            if (abs(a ^ b) == 0)
                                                                      168
                return B.ori(st) > 0;
115
                                                                      169
116
            return (a ^ b) > 0:
117
                                                                      171
118
        // Regard a line as a function
119
        template <typename F>
120
        Point(F) operator()(const F x) const // A + AB * x = the _{174}
121
             point position.
122
                                                                      175 };
123
            return Point<F>(st) + vec() * x;
124
125
                                                                      178
        bool isSegProperIntersection(const Line 1) const
126
127
            return l.ori(st) * l.ori(ed) < 0 and ori(l.st) * ori(180
128
                 1.ed) < 0;
129
130
        bool isSegIntersection(const Line 1)const{
            Line<Real> h = *this;
131
            // hst = 1, hed = 2, lst = 3, led = 4
132
                                                                      185
            double hlst = h.ori(l.st);
133
                                                                      186
            double hled = h.ori(l.ed);
134
                                                                      187
            double lhst = 1.ori(h.st):
135
                                                                      188
            double lhed = 1.ori(h.ed);
136
                                                                      189
            if(fcmp(hlst, 0.0) == 0 \&\& fcmp(hled, 0.0) == 0)
137
                 return h.isPointOnSeg(l.st) || h.isPointOnSeg(l. 191
138
                      ed) || 1.isPointOnSeg(h.st) || 1.
                      isPointOnSeg(h.ed);
139
                                                                      192
            return fcmp(hlst * hled, 0.0) <= 0 && fcmp(lhst *
140
                                                                      193
                 lhed, 0.0) <= 0;
                                                                      194
                                                                      195
142
                                                                      196
143
        bool isPointOnSegProperly(const Point<T> p) const
                                                                      197
144
            return fcmp(ori(p), 0.0) == 0 and fcmp(((st - p) & (_{199}
145
                 ed - p)), 0.0) < 0;
                                                                      200
                                                                      201
147
        bool isPointOnSeg(const Point<T>p) const{
                                                                      202
            return fcmp(ori(p), 0.0) == 0 and fcmp((st - p) & (ed<sub>203</sub>
148
                   - p), 0.0) <= 0;
        Real disP2Line(const Point<T> p) const
151
```

```
return Line<double>(projection(p), Point<double>(p)).
             vec().norm();
    // notice if you should check Segment intersect or not;
    // be careful divided by 0
    Point<Real> getIntersection(Line<Real> 1)
        Line<Real> h = *this;
          return 1(((1.st - h.st)^ h.vec()) / (h.vec() ^ 1.
     vec())); // use operator()
        Real hlst = -h.ori(l.st):
        Real hled = h.ori(l.ed);
        return ((1.st * hled) + (1.ed * hlst)) / (hlst + hled
        // 需要確認+-號的合理性
        // Area of triangle(l.st, h.st, h.ed) divided by Area
             of Quadrilateral(h.st, l.st, h.ed, l.ed)
    Point<Real> projection(const Point<T> p) const
        return operator()(((p - st) & vec()) / (Real)(vec().
             norm()));
    template <class F>
    friend ostream &operator<<((ostream &os, const Line<F> &1)
template <class F>
ostream &operator<<(ostream &os, const Line<F> &1)
   return os << "(" << l.st.x << ", " << l.st.y << ") to ('
        << l.ed.x << ", " << l.ed.y << ")";
template <class F>
using Polygon = vector<Point<F>>>:
template <class F>
Polygon<F> getConvexHull(Polygon <F> points) {
   sort(points.begin(), points.end());
    Polygon<F> CH;
    CH.reserve(points.size() + 1); // for what ??
    for (int round = 0: round < 2: round++){</pre>
        int start = CH.size();
        for (Point<int> &pt: points) {
            while (CH.size() - start >= 2 && Line<F>(CH[CH.
                 size() - 2], CH[CH.size() - 1]).ori(pt) <=
                 0) // ? Line is different than senpai's .
                CH.pop back();
            CH.emplace back(pt);
        CH.pop_back();
        reverse(points.begin(), points.end());
    if (CH.size() == 2 && CH[0] == CH[1])
        CH.pop back();
    return CH;
```

# 3 geometry/Convex\_Hull

#### 3.1 Andrew's Monotone Chain

```
1 // Andrew's Monotone Chiain
2 template <class F>
  using Polygon = vector<Point<F>>;
  template <class F>
  Polygon<F> getConvexHull(Polygon<F> points)
      sort(begin(points), end(points));
      Polygon<F> hull;
      hull.reserve(points.size() + 1);
      for (int phase = 0; phase < 2; ++phase)</pre>
13
           auto start = hull.size();
14
           for (auto &point : points)
15
               while (hull.size() >= start + 2 &&
                      Line<F>(hull.back(), hull[hull.size() -
                           2]).ori(point) <= 0)
                   hull.pop_back();
               // whenever point is at the RIGHT(NEGATIVE) part
                   of the line(hull[size - 1], hull[size-2])
               // pop the last point because it causes concave
                   hull
               hull.push back(point);
           hull.pop back();
           reverse(begin(points), end(points));
      if (hull.size() == 2 and hull[0] == hull[1])
           hull.pop back();
      return hull;
```

### 4 graph

### 4.1 Kosaraju\_for\_SCC

```
1 | #include <vector>
2 #include <stack>
3 using namespace std:
 4 #define MAX N 200005
  class Kosaraju for SCC{
       int NodeNum;
       vector<vector<int>> G;
       vector<vector<int>> GT:
       stack<int> st;
       vector<bool> visited:
       vector<int> scc;
       int sccID;
13
14 public:
       void init(int N){
           NodeNum = N;
```

```
G.clear();
           G.resize(N + 5);
19
           GT.clear();
           GT.resize(N + 5);
20
21
           while(!st.empty())
22
               st.pop();
23
           visited.clear():
           visited.resize(N + 5, false);
24
25
           scc.clear();
26
           scc.resize(N + 5);
27
           sccID = 1;
28
       void addEdge(int w, int v){
29
30
           G[w].emplace back(v):
31
           GT[v].emplace back(w);
32
       void DFS(bool isG, int v, int k = -1){
33
           visited[v] = true;
34
           scc[v] = k;
35
           vector<vector<int>> &dG = (isG ? G : GT);
36
37
           for(int w: dG[v])
38
39
               if(!visited[w]){
40
                   DFS(isG, w, k);
41
42
           if(isG){
43
               st.push(v);
44
45
46
47
       void Kosaraju(int N){
48
           visited.clear();
           visited.resize(N + 5, false);
49
           for (int i = 1; i <= N; i++){
50
               if(!visited[i])
51
                   DFS(true, i);
52
53
54
           visited.clear();
55
           visited.resize(N + 5, false);
           while(!st.empty()){
56
               if(!visited[st.top()])
57
                   DFS(false, st.top(), sccID++);
58
59
               st.pop();
60
61
62
       vector<vector<int>> generateReG(){
63
           vector<vector<int>> reG;
           reG.resize(sccID);
64
           for (int i = 1; i <= NodeNum; i++){</pre>
                for(int w: G[i]){
                  if(scc[i] == scc[w])
                       continue;
                  reG[scc[i]].emplace back(scc[w]);
71
           return reG;
72
73
74 };
```

## 4.2 Tarjan\_for\_BridgeCC

```
_{\rm 1} // BCC for bridge connected component _{\rm 2} // by sylveon a.k.a LFsWang
```

```
3 #include <vector>
4 #include <stack>
  #include <algorithm>
6 using namespace std;
  #define MAX N 200005
  int timestamp = 1;
9 int bccid = 1:
10 int D[MAX N];
11 int L[MAX N];
12 int bcc[MAX N];
13 stack<int> st;
  vector<int> adi[MAX N]:
   bool inSt[MAX N];
15
   void DFS(int v, int fa) { //call DFS(v,v) at first
17
       D[v] = L[v] = timestamp++; //timestamp > 0
18
       st.emplace(v);
19
       for (int w:adj[v]) {
20
           if( w==fa ) continue;
21
           if (!D[w]) \{ // D[w] = 0 \text{ if not visited}
22
23
               DFS(w,v);
24
               L[v] = min(L[v], L[w]);
25
26
           L[v] = min(L[v], D[w]);
27
28
       if (L[v]==D[v]) {
           bccid++;
29
30
           int x;
31
32
               x = st.top(); st.pop();
               bcc[x] = bccid;
33
34
           } while (x!=v);
35
36
       return ;
```

### 4.3 Tarian for AP Bridge

```
1 #include <vector>
2 #include <utility>
3 using namespace std;
  #define MAX_N 200005;
  #define enp pair<int, int> // edge-weight, node-index
  #define con pair<int, int> // connection
  class tarjan{
       vector<vector<int>> G; // adjacency List
       vector<int> D; // visit or visited and D-value
       vector<int> L; // for L-value
11
       vector<con> edgeBridge;
       vector<int> APnode;
       int timestamp;
15
       tarjan(int size = 1){
16
           timestamp = 0;
17
          G.resize(size);
18
          D.resize(size, 0);
19
          L.resize(size, 0);
           edgeBridge.clear();
20
21
           APnode.clear();
22
23
       void init(int size = 1){
24
           tarjan(size);
```

```
void addedge(int u, int v)
      { // undirected graph
27
          G[u].push_back(v);
28
          G[v].push back(u);
29
30
      void DFS(int v, int pa){ // init: call DFS(v,v)
31
          D[v] = L[v] = timestamp++:
32
33
          int Childcount = 0;
          bool isAP = false;
34
35
          for(int w: G[v]){
              if(w == pa)
36
37
                  continue:
38
              if(!D[w]){ // 用 D[w] == 0 if not visited
39
                  DFS(w, v);
                  Childcount++;
40
                  if(D[v] <= L[w])</pre>
41
                      isAP = true; // 結 論 2 對於除了 root 點
42
                           以外的所有點 v·v 點在 G 上為 AP 的
                           充要條件為其在 T 中至少有一個子節點
                          w 滿足 D(v) ≤ L(w)
                  if(D[v] < L[w])
43
                      edgeBridge.emplace_back(v,w);// 結 論 3
44
                           對於包含 r 在內的所有點 v 和 v 在 T
                           中的子節點 w,邊 e(v,w) 在圖 G 中為
                          bridge 的充要條件為 D(v) < L(w)。
                  L[v] = min(L[v], L[w]);
46
              L[v] = min(L[v], D[w]);
47
48
          if(v == pa && Childcount < 2)</pre>
49
              isAP = false;
50
51
          if(isAP)
              APnode.emplace_back(v);
52
53
54 };
```

### 4.4 Tarjan\_for\_SCC

```
1 // by atsushi
2 #include <vector>
3 #include <stack>
  using namespace std;
  class tarjan_for_SCC{
  private:
       vector<vector<int>>> G; // adjacency list
       vector<int> D;
       vector<int> L:
       vector<int> sccID;
       stack<int> st; // for SccID
       vector<bool> inSt;
       vector<vector<int>> reG;
       int timeStamp, sccIDstamp;
   public:
       void init(int size = 1){
17
           G.clear();
           G.resize(size + 3);
20
           D.clear();
21
           D.resize(size + 3, 0);
22
           L.clear();
           L.resize(size + 3, 0);
```

#### sccID.clear(); sccID.resize(size + 3, 0); 25 while(!st.empty()) 26 27 st.pop(); inSt.clear(); 28 29 inSt.resize(size + 3, false); 30 reG.clear(): sccIDstamp = timeStamp = 1; 31 32 33 void addEdge(int from, int to){ 34 G[from].emplace back(to); 35 void DFS(int v, int pa){ //call DFS(v,v) at first 36 37 D[v] = L[v] = timeStamp++; //timestamp > 038 st.push(v); 39 inSt[v] = true; 40 for(int w: G[v]){ // directed graph don't need w == 41 $if(!D[w]){ // D[w] = 0 if not visited}$ 42 43 DFS(w, v);L[v] = min(L[v], L[w]);44 }else if(inSt[w]) { /\* w has been visited. if we don't add this, the L[v] will think 47 that v can back to node whose index less 19 inSt[w] is true that v -> w is a cross edge opposite it's a forward edge L[v] = min(L[v], D[w]); // why D[w] insteadof L[w]?? 52 53 $if(D[v] == L[v]){$ 54 int w; 55 do{ 56 w = st.top();sccID[w] = sccIDstamp; // scc ID for this pooint at which SCC inSt[w] = false; } while (w != v); sccIDstamp++; 62 63 65 void generateReG(int N = 1){ reG.clear(); 66 reG.resize(sccIDstamp); for (int i = 1; i <= N; i++){ for(int w: G[i]){ if(sccID[i] == sccID[w]) 70 72 reG[sccID[i]].emplace\_back(sccID[w]); } 75 bool visited(int v){ 77 return D[v]; 79 };

# 5 graph/Bipartite

#### 5.1 konig algorithm

```
1 #include <vector>
  #include <cstring>
   using namespace std;
   // V times DFS O(EV)
  vector<int> V[205];
  // V[i]記錄了左半邊可以配到右邊的那些點
  int match[205]; // A<=B</pre>
9 | // match[i] 記錄了右半邊配對到左半邊的哪個點
10 bool used[205];
  int n;
12
  bool dfs(int v)
13
       for(int e:V[v])
14
15
           if( used[e] ) continue;
16
           used[e] = true;
17
           if( match[e] == -1 || dfs( match[e] ) )
18
               match[e] = v;
               return true;
       return false;
^{24}
25
   int konig()
27
28
       memset(match,-1,sizeof(match));
29
       int ans=0;
       for(int i=1;i<=n;++i)</pre>
33
           memset(used, 0, sizeof(used));
34
35
           if( dfs(i) )
               ans++;
36
37
38
39
       return ans;
```

### 5.2 Kuhn-Munkres

```
My[y]=x,Mx[x]=y;
15
16 }
   void bfs(int st){
     for(int i=1; i<=n; ++i)</pre>
       Sy[i] = INF, vx[i]=vy[i]=0;
     queue<int> q; q.push(st);
21
     for(;;){
       while(q.size()){
22
23
         int x=q.front(); q.pop();
24
25
         for(int y=1; y<=n; ++y) if(!vy[y]){</pre>
26
           LL t = lx[x]+ly[y]-g[x][y];
27
           if(t==0){
28
              pa[y]=x;
29
              if(!My[y]){augment(y);return;}
              vy[y]=1,q.push(My[y]);
30
           }else if(Sy[y]>t) pa[y]=x,Sy[y]=t;
31
32
33
34
       LL cut = INF;
       for(int y=1; y<=n; ++y)</pre>
35
36
         if(!vy[y]&&cut>Sy[y]) cut=Sy[y];
37
       for(int j=1; j<=n; ++j){
         if(vx[j]) lx[j] -= cut;
38
39
         if(vy[j]) ly[j] += cut;
40
         else Sy[j] -= cut;
41
42
       for(int y=1; y<=n; ++y){</pre>
43
         if(!vy[y]&&Sy[y]==0){
44
           if(!My[y]){augment(y);return;}
45
           vy[y]=1, q.push(My[y]);
46
47
48
49
50
   LL KM(){
     memset(My,0,sizeof(int)*(n+1));
     memset(Mx,0,sizeof(int)*(n+1));
     memset(ly,0,sizeof(LL)*(n+1));
     for(int x=1; x<=n; ++x){</pre>
54
       1x[x] = -INF;
       for(int y=1; y<=n; ++y)</pre>
57
         lx[x] = max(lx[x],g[x][y]);
58
     for(int x=1; x<=n; ++x) bfs(x);</pre>
     for(int y=1; y<=n; ++y) ans+=g[My[y]][y];</pre>
     return ans;
```

### 6 graph/Flow

#### 6.1 Ford Fulkerson

```
#include <vector>
#include <tuple>
#include <cstring>
using namespace std;
// O((V+E)F)
```

```
7 #define maxn 101
                                                                7 | Graph C, F, R; // 容量上限、流量、剩餘容量
   // remember to change used into the maxNode size -- kattis
                                                                  bool visit[MAXN]; // BFS經過的點
       elementary math
                                                                  int path[MAXN]; // BFS tree
9 bool used[MAXN];
                                                                10 | int flow[MAXN]: // 源點到各點的流量瓶頸
10 int End;
vector<int> V[MAXN];
                                                                  int BFS(int s, int t) // 源點與匯點
12 vector<tuple<int, int>> E;
                                                                13
13
                                                                      memset(visit, false, sizeof(visit));
                                                                14
   // x=>y 可以流 C
   // if undirected or 2-direc edge, bakcward Capacity become C; _{16}^{\,\,}
                                                                      queue<int> 0; // BFS queue
  // Graph build by edge array
                                                                      visit[s] = true;
17 // 反向邊的編號只要把自己的編號 xor 1 就能取得
                                                                18
                                                                      path[s] = s;
   void add_edge(int x, int y,int c)
                                                                      flow[s] = 1e9;
                                                                19
19
                                                                      Q.push(s);
      V[x].emplace_back( E.size() );
20
                                                                21
21
      E.emplace_back(y,c);
                                                                22
                                                                      while (!Q.empty())
      V[y].emplace_back( E.size() );
^{22}
                                                                23
      E.emplace back(x,0);
23
                                                                24
                                                                          int i = Q.front(); Q.pop();
^{24}
                                                                25
                                                                           for (int j=0; j<100; ++j)</pre>
  int dfs(int v, int f)
25
                                                                              // 剩餘網路找擴充路徑
26
                                                                27
                                                                              if (!visit[j] && R[i][j] > 0)
      if( v==End ) return f;
27
                                                                28
28
      used[v] = true;
                                                                                  visit[j] = true;
29
      int e,w;
                                                                                  path[j] = i;
      for( int eid : V[v] )
                                                                                  // 一邊找最短路徑,一邊計算流量瓶頸。
                                                                31
31
                                                                32
                                                                                  flow[j] = min(flow[i], R[i][j]);
          tie(e,w) = E[eid];
32
                                                                                  Q.push(j);
                                                                33
          if( used[e] || w==0 ) continue;
33
                                                                34
                                                                35
                                                                                  if (j == t) return flow[t];
          w = dfs(e, min(w,f));
                                                                36
          if( w>0 )
                                                                37
                                                                38
                                                                      return 0; // 找不到擴充路徑了,流量為零。
              // 更新流量
                                                                39
              get<1>(E[eid ]) -= w;
                                                                40
              get<1>(E[eid^1]) += w;
                                                                   int Edmonds Karp(int s, int t)
                                                                41
              return w:
                                                                42
42
                                                                43
                                                                      memset(F, 0, sizeof(F));
43
                                                                44
                                                                      memcpy(R, C, sizeof(C));
      return 0;// Fail!
                                                                45
45
                                                                      int f, df; // 最大流的流量、擴充路徑的流量
   int ffa(int s,int e)
                                                                47
                                                                       for (f=0; df=BFS(s, t); f+=df)
47
                                                                48
                                                                          // 更新擴充路徑上每一條邊的流量
      int ans = 0, f;
                                                                           for (int i=path[t], j=t; i!=j; i=path[j=i])
                                                                49
      End = e;
49
                                                                50
      while(true)
50
                                                                              F[i][j] = F[i][j] + df;
                                                                51
                                                                52
                                                                              F[j][i] = -F[i][j];
52
          memset(used, 0, sizeof(used));
                                                                              R[i][j] = C[i][j] - F[i][j];
                                                                53
          f = dfs(s, INT_MAX);
53
                                                                54
                                                                              R[j][i] = C[j][i] - F[j][i];
          if( f<=0 ) break;</pre>
                                                                55
55
          ans += f;
                                                                56
                                                                      return f;
56
57
      return ans;
```

### 6.2 Edmonds-Karp-adjmax

```
1  // O((V+E)VE) · 簡單寫成 O(VE²)
2  #include <cstring>
3  #include <queue>
4  using namespace std;
5  #define maxn 100
6  typedef int Graph[MAXN][MAXN]; // adjacency matrix
```

### 6.3 Dinic\_algorithm

```
#include <bits/stdc++.h>
using namespace std;
// O(V^2E) O(VE) finding argument path
// if unit capacity network then O(min(V^3/2, E^1/2) E)
// solving bipartite matching O(E V^1/2) better than konig and flow(EV)
#define MAXN 101
```

```
7 #define INT MAX 10000000
8 int End, dist[MAXN];
  vector<tuple<int, int, int>> V[MAXN];
10 // vertex-index, cap, the index of the reverse edge
  void addEdge(int u, int v, int c){
      V[u].emplace back(v, c, V[v].size());
      V[v].emplace_back(u, 0, V[u].size() - 1);
14
15 bool bfs(int s) {
       memset(dist, -1, sizeof(dist));
16
17
       queue<int> qu;
       qu.emplace(s);
18
       dist[s]=0;
19
20
21
       while( !qu.empty() ) {
22
           int S = qu.front(); qu.pop();
           for(auto &p : V[S]) {
23
               int E, C;
24
25
               tie(E, C, ignore) = p;
               if( dist[E]==-1 && C!=0 ) {
26
27
                   dist[E]=dist[S]+1;
                   qu.emplace(E);
28
29
30
31
       return dist[End] != -1;
32
33
  int dfs(int v, int f) {
34
35
       int e,w,rev;
       if( v==End || f==0 ) return f;
37
       for( auto &t : V[v] )
38
39
           tie(e,w,rev) = t;
           if( dist[e]!=dist[v]+1 || w==0 )
40
41
               continue;
42
43
           w = dfs(e, min(w,f));
44
           if( w>0 )
45
46
               get<1>(t) -= w;
47
               get<1>(V[e][rev]) += w;
               return w;
48
49
50
       dist[v] = -1; //優化, 這個點沒用了
       return 0;// Fail!
53
  int dinic(int s,int e)
54
55
       int ans = 0, f;
56
57
       End = e;
58
       while(bfs(s))
59
           while( f = dfs(s, INT MAX) )
60
61
               ans += f;
62
63
       return ans;
```

### 6.4 Edmonds\_Karp\_2

```
#include <bits/stdc++.h>
struct Edge{
```

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static const int MAXN=440;

static const TP INF=999999999;

```
graph/Matching
       int from, to, cap, flow;
                                                                        struct edge{
       Edge(int u, int v, int c, int f):from(u), to(v), cap(c),
                                                                          int v,pre;
            flow(f){}
                                                                          TP r,cost;
                                                                          edge(int v,int pre,TP r,TP cost):v(v),pre(pre),r(r),cost(
5 };
6 const maxn = 200005;
                                                                                                                                         7.1 blossom matching
   struct EdmondsKarp{
       int n. m:
                                                                   11
                                                                        int n,S,T;
       vector<Edge> edges;
                                                                        TP dis[MAXN],PIS,ans;
                                                                   12
                                                                        bool vis[MAXN];
10
       vector<int> G[maxn];
                                                                   13
       int a[maxn];
                                                                        vector<edge> e;
11
                                                                   14
                                                                                                                                       1 // by jinkela
12
       int p[maxn];
                                                                   15
                                                                        int g[MAXN];
                                                                                                                                       2 // 最大圖匹配
       void init(int n){
                                                                        void init(int n){
13
                                                                   16
                                                                                                                                       3 // O(V^2(V+E))
           for (int i = 0; i < n; i++)
                                                                   17
                                                                          memset(g,-1,sizeof(int)*((n=_n)+1));
14
                                                                                                                                       4 #define MAXN 505
15
               G[i].clear():
                                                                   18
                                                                          e.clear():
                                                                                                                                       5 int n; //1-base
16
           edges.clear();
                                                                   19
                                                                                                                                       6 vector<int> g[MAXN];
                                                                   20
                                                                        void add edge(int u,int v,TP r,TP cost,bool directed=false)
17
                                                                                                                                       7 int MH[MAXN]; //output MH
       void AddEdge(int from, int to, int cap){
18
                                                                                                                                       8 int pa[MAXN],st[MAXN],S[MAXN],v[MAXN],t;
           edges.push_back(Edge(from, to, cap, 0));
19
                                                                   21
                                                                          e.push_back(edge(v,g[u],r,cost));
                                                                                                                                       9 int lca(int x,int y){
                                                                          g[u]=e.size()-1;
           edges.push_back(Edge(to, from, 0, 0)) // 反向弧
                                                                   22
20
                                                                                                                                           for(++t;;swap(x,y)){
                                                                   23
                                                                          e.push back(
21
           m = edges.size();
                                                                                                                                             if(!x) continue;
                                                                          edge(u,g[v],directed?0:r,-cost));
                                                                   24
22
           G[from].push back(m - 2);
                                                                                                                                             if(v[x]==t) return x;
                                                                   25
                                                                          g[v]=e.size()-1;
23
           G[to].push back(m - 1);
                                                                                                                                             v[x] = t;
                                                                   26
24
                                                                                                                                             x = st[pa[MH[x]]];
                                                                   27
                                                                        TP augment(int u, TP CF){
       int Maxflow(int s, int t){
25
                                                                                                                                      15
                                                                          if(u==T||!CF)return ans+=PIS*CF,CF;
                                                                   28
           int flow = 0;
26
                                                                                                                                      16
                                                                   29
                                                                          vis[u]=1:
           for (;;){
27
                                                                                                                                         #define qpush(x) q.push(x),S[x]=0
                                                                          TP r=CF,d;
                                                                   30
               memset(a, 0, sizeof(a));
                                                                                                                                         void flower(int x,int y,int l,queue<int>&q){
                                                                   31
                                                                          for(int i=g[u];~i;i=e[i].pre){
29
               queue<int> 0;
                                                                                                                                           while(st[x]!=1){
                                                                            if(e[i].r&&!e[i].cost&&!vis[e[i].v]){
                                                                   32
               Q.push(s);
                                                                                                                                             pa[x]=y;
                                                                   33
                                                                              d=augment(e[i].v,min(r,e[i].r));
31
               a[s] = INF;
                                                                                                                                             if(S[y=MH[x]]==1)qpush(y);
                                                                                                                                      21
                                                                   34
                                                                              e[i].r-=d;
               while(!Q.empty()){
32
                                                                                                                                             st[x]=st[y]=1, x=pa[y];
                                                                   35
                                                                              e[i^1].r+=d;
                   int x = Q.front();
                                                                                                                                      23
                                                                   36
                                                                              if(!(r-=d))break;
                   Q.pop();
                                                                                                                                      ^{24}
                                                                   37
                   for (int i = 0; i < G[x].size(); i++){</pre>
                                                                                                                                         bool bfs(int x){
                       Edge &e = edges[G[x][i]];
                                                                   38
                                                                                                                                           iota(st+1, st+n+1, 1);
                                                                          return CF-r;
                                                                   39
                       if(!a[e.to] && e.cap > e.flow){
                                                                                                                                           memset(S+1,-1,sizeof(int)*n);
                           p[e.to] = G[x][i];
                                                                   40
                                                                                                                                           queue<int>q; qpush(x);
                           a.[e.to] = min(a[x], e.cap - e.flow); 41
                                                                        bool modlabel(){
                                                                                                                                           while(q.size()){
                                                                          for(int u=0;u<=n;++u)dis[u]=INF;</pre>
                           0.push(e.to);
                                                                                                                                             x=q.front(),q.pop();
                                                                          static deque<int>q;
                                                                   43
                                                                                                                                      31
                                                                                                                                             for(int y:g[x]){
                                                                          dis[T]=0,q.push back(T);
                                                                   44
                                                                                                                                               if(S[y]==-1){
                                                                                                                                      32
                                                                          while(q.size()){
                   if(a[t])
                                                                   45
                                                                                                                                                 pa[y]=x,S[y]=1;
                                                                            int u=q.front();q.pop_front();
                                                                   46
                       break:
                                                                                                                                      34
                                                                                                                                                 if(!MH[y]){
                                                                   47
                                                                                                                                      35
                                                                                                                                                   for(int lst;x;y=lst,x=pa[y])
                                                                            for(int i=g[u];~i;i=e[i].pre){
                                                                   48
                                                                                                                                                     lst=MH[x],MH[x]=y,MH[y]=x;
                                                                                                                                      36
           if(!a[t])
                                                                   49
                                                                              if(e[i^1].r&&(dt=dis[u]-e[i].cost)<dis[e[i].v]){</pre>
                                                                                                                                                   return 1;
                                                                                                                                      37
                                                                   50
                                                                                if((dis[e[i].v]=dt)<=dis[q.size()?q.front():S]){</pre>
           for (int u = t; u != s; u = edges[p[u]].from){
                                                                                  q.push_front(e[i].v);
                                                                                                                                                 qpush(MH[y]);
                                                                                                                                      39
                                                                                }else q.push_back(e[i].v);
               edges[p[u]].flow += a[t];
                                                                                                                                      40
                                                                                                                                               }else if(!S[y]&&st[y]!=st[x]){
                                                                   53
               edges[p[u] ^ 1].flow -= a[t];
                                                                                                                                                 int l=lca(y,x);
                                                                                                                                      41
                                                                   54
52
                                                                                                                                      42
                                                                                                                                                  flower(y,x,1,q),flower(x,y,1,q);
                                                                   55
53
           flow += a[t];
                                                                                                                                      43
                                                                   56
                                                                          for(int u=0;u<=n;++u)</pre>
54
                                                                                                                                      44
       return flow;
                                                                   57
                                                                            for(int i=g[u];~i;i=e[i].pre)
55
                                                                                                                                      45
                                                                   58
                                                                              e[i].cost+=dis[e[i].v]-dis[u];
                                                                                                                                           return 0;
                                                                                                                                      46
                                                                   59
                                                                          return PIS+=dis[S], dis[S]<INF;</pre>
                                                                                                                                      47
                                                                   60
                                                                                                                                         int blossom(){
                                                                        TP mincost(int s,int t){
                                                                                                                                           memset(MH+1,0,sizeof(int)*n);
  6.5 MinCostMaxFlow
                                                                          S=s,T=t;
                                                                                                                                           int ans=0;
                                                                          PIS=ans=0:
                                                                                                                                           for(int i=1; i<=n; ++i)</pre>
                                                                          while(modlabel()){
                                                                                                                                             if(!MH[i]&&bfs(i)) ++ans;
                                                                            do memset(vis,0,sizeof(bool)*(n+1));
1 // by jinkela
                                                                   65
                                                                                                                                      53
                                                                                                                                           return ans;
2 template<typename TP>
                                                                            while(augment(S,INF));
                                                                                                                                      54
3 struct MCMF{
                                                                          }return ans;
```

68 69 };

# $8 \quad \operatorname{graph/Minimum\_Spanning\_Tree}_{\scriptscriptstyle{21}}^{\scriptscriptstyle{00}}$

```
1 | #include <vector>
2 #include <queue>
3 #include <utility>
4 using namespace std;
5 #define enp pair<int, int> // pair<edge_val, node>
6 int prim_pq(vector<vector<enp>> E){
       vector<bool> vis;
       vis.resize(E.size(), false);
       vis[0] = true;
       priority_queue<enp> pq;
       for(auto e: E[0]){
12
           pq.emplace(-e.first, e.second);
13
14
       int ans = 0; // min value for MST
       while(pq.size()){
           int w, v; // edge-weight, vertex index
           tie(w, v) = pq.top();
           pq.pop();
19
           if(vis[v])
20
               continue;
           w = -w:
           vis[v] = true;
           ans += w;
           for(auto e: E[v]){
               pg.emplace(-e.first, e.second);
26
       return ans;
```

#### 8.2 Kruskal

prim

8.1

```
1 #include <tuple>
2 #include <vector>
3 #include <algorithm>
4 #include <numeric> // for iota(first, last, val) setting
       iterator value
  using namespace std;
   struct DSU // disjoint set no rank-comp-merge
      vector<int> fa;
      DSU(int n) : fa(n) { iota(fa.begin(), fa.end(), 0); } //
           auto fill fa from 0 to n-1
      int find(int x) { return fa[x] == x ? x : fa[x] = find(fa
       void merge(int x, int y) { fa[find(x)] = find(y); }
14 int kruskal(int V, vector<tuple<int, int, int>> E) // save
       all edges into E, instead of saving graph via adjacency
15
       sort(E.begin(), E.end());
      DSU dsu(V);
       int mcnt = 0;
      int ans = 0;
```

```
for (auto e : E)
{
   int w, u, v; // w for start, u for des, v for val
   tie(w, u, v) = e;
   if (dsu.find(u) == dsu.find(v))
        continue;
   dsu.merge(u, v);
   ans += w;
   if (++mcnt == V - 1)
        break;
}
```

### 9 graph/Shortest\_Path

#### 9.1 dijkstra

return ans:

23

24

26

27

28

29

30

31

```
1 | #include <iostream>
  #include <vector>
  #include <queue>
  #include <utility>
   using namespace std;
   #define con pair<int, int> // first for distance, second for
   vector<vector<con>> Graph; //
   vector<int> dis; // distance;
   int main(){
10
11
       priority queue<con, vector<con>, greater<con>> pq;
12
       dis[0] = 0;
       pq.emplace(con(0, 0));
13
       while(pq.size()){
14
15
           con cur = pq.top();
16
           pq.pop();
17
           if(cur.first != dis[cur.second])
18
19
           for(auto it: Graph[cur.second]){
20
               if(cur.first + it.first < dis[it.second]){</pre>
                    dis[it.second] = cur.first + it.first;
21
                    pq.emplace(dis[it.second], it.second);
22
23
24
25
       return 0;
26
```

### 9.2 dijkstra-alrightchiu-version

```
1 // C++ code
2 #include <iostream>
3 #include <vector>
4 #include <list>
5 #include <utility> // for std::pair<>
6 #include <iomanip> // for std::setw()
7 #include <cmath> // for std::floor
8 // #include "Priority_Queue_BinaryHeap.h"
```

```
10 const int Max Distance = 100;
  class Graph SP{
                               // SP serves as Shortest Path
12 private:
13
       int num vertex;
       std::vector<std::list<std::pair<int,int>>> AdjList;
14
       std::vector<int> predecessor, distance;
16
       std::vector<bool> visited:
   public:
17
18
       Graph SP():num vertex(0){};
19
       Graph SP(int n):num vertex(n){
20
           AdjList.resize(num vertex);
21
22
       void AddEdge(int from, int to, int weight);
23
       void PrintDataArray(std::vector<int> array);
24
       void PrintIntArray(int *array);
25
       void InitializeSingleSource(int Start);
                                                    // 以Start作
       void Relax(int X, int Y, int weight);
                                                    // edge方向:
           from X to Y
28
       void Diikstra(int Start = 0);
                                             // 需要Min-Priority
       friend class BinaryHeap;
                                            // 以Binary Heap實現
30
           Min-Priority Queue
31
  void Graph SP::Dijkstra(int Start){
       InitializeSingleSource(Start);
       BinaryHeap minQueue(num vertex);
                                          // object of min queue
       minOueue.BuildMinHeap(distance):
       visited.resize(num vertex, false); // initializa
           visited[] as {0,0,0,...,0}
       while (!minQueue.IsHeapEmpty()) {
42
           int u = minQueue.ExtractMin();
           for (std::list<std::pair<int, int>>::iterator itr =
43
                AdjList[u].begin();
                itr != AdjList[u].end(); itr++) {
44
45
               Relax(u, (*itr).first, (*itr).second);
47
               minQueue.DecreaseKey((*itr).first, distance[(*itr
                    ).first]); // definition at alrightchiu's
                    priority queue self-version
48
49
       std::cout << "\nprint predecessor:\n";</pre>
       PrintDataArray(predecessor);
       std::cout << "\nprint distance:\n";</pre>
       PrintDataArray(distance);
54
  void Graph SP::InitializeSingleSource(int Start){
       distance.resize(num vertex);
       predecessor.resize(num vertex);
       for (int i = 0; i < num_vertex; i++) {</pre>
           distance[i] = Max_Distance;
61
62
           predecessor[i] = -1;
63
64
       distance[Start] = 0;
65
  void Graph SP::Relax(int from, int to, int weight){
```

```
24
       if (distance[to] > distance[from] + weight) {
           distance[to] = distance[from] + weight;
69
70
           predecessor[to] = from;
71
72
   void Graph SP::AddEdge(int from, int to, int weight){
74
75
       AdjList[from].push back(std::make pair(to,weight));
                                                                      27
76
                                                                      28
   void Graph SP::PrintDataArray(std::vector<int> array){
                                                                      29
       for (int i = 0; i < num vertex; i++)</pre>
78
                                                                      30
           std::cout << std::setw(4) << i;</pre>
79
                                                                      31
80
       std::cout << std::endl:</pre>
       for (int i = 0; i < num vertex; i++)</pre>
82
           std::cout << std::setw(4) << array[i];</pre>
                                                                      33
       std::cout << std::endl:</pre>
83
84
                                                                      34
   int main(){
85
86
                                                                      35
       Graph_SP g9(6);
87
       g9.AddEdge(0, 1, 8);g9.AddEdge(0, 5, 1);
88
89
       g9.AddEdge(1, 0, 3);g9.AddEdge(1, 2, 1);
       g9.AddEdge(2, 0, 5);g9.AddEdge(2, 3, 2);g9.AddEdge(2, 4,
90
       g9.AddEdge(3, 1, 4);g9.AddEdge(3, 2, 6);g9.AddEdge(3, 4,
91
            7);g9.AddEdge(3, 5, 3);
                                                                      41
       g9.AddEdge(5, 3, 2);g9.AddEdge(5, 4, 8);
92
                                                                      42
93
                                                                      43
94
       g9.Dijkstra(0);
95
                                                                      44
96
       return 0;
                                                                      45
                                                                      46
```

#### 9.3 bellman-Ford

```
51
1 // C++ code
                                                                  52
2 #include <iostream>
                                                                  53
3 #include <vector>
                                                                  54
4 #include <list>
                                                                  55
5 #include <utility>
                               // for std::pair<>
                                                                  56
6 #include <iomanip>
                               // for std::setw()
                                                                  57
                                                                  58
  const int Max Distance = 100;
9 class Graph_SP{
                               // SP serves as Shortest Path
10 private:
                                                                  61
       int num vertex:
       std::vector<std::list<std::pair<int,int>>> AdjList;
12
      std::vector<int> predecessor, distance;
13
                                                                  64
14
   public:
                                                                  65
15
      Graph SP():num_vertex(0){};
                                                                  66
      Graph SP(int n):num vertex(n){
16
           AdjList.resize(num_vertex);
17
18
                                                                  69
19
       void AddEdge(int from, int to, int weight);
      void PrintDataArray(std::vector<int> array);
20
      void InitializeSingleSource(int Start);
                                                   // 以Start作
            為起點
       void Relax(int X, int Y, int weight);
                                                    // 對edge(X,Y 74
            ) 進 行 Relax
      bool BellmanFord(int Start = 0);
                                                    // 以Start作
            為起點
```

```
// if there 78 void Graph SP::Relax(int from, int to, int weight){
                                                     is
                                                     negative 80
                                                                     if (distance[to] > distance[from] + weight) {
                                                      cycle,
                                                             81
                                                                          distance[to] = distance[from] + weight;
                                                     return
                                                                          predecessor[to] = from;
                                                               82
                                                     false
                                                               83
};
                                                               84
                                                                 void Graph SP::AddEdge(int from, int to, int weight){
                                                               85
bool Graph SP::BellmanFord(int Start){
                                                                     AdjList[from].push back(std::make pair(to,weight));
                                                               87
    InitializeSingleSource(Start);
                                                                 int main(){
                                                               89
    for (int i = 0; i < num_vertex-1; i++) {</pre>
                                                               90
        // |V-1| 次的iteration
                                                               91
                                                                     Graph SP g7(6);
                                                               92
                                                                     g7.AddEdge(0, 1, 5);
        // for each edge belonging to E(G)
                                                               93
                                                                     g7.AddEdge(1, 4, -4);g7.AddEdge(1, 2, 6);
        for (int j = 0; j < num \ vertex; j++) {
                                                                     g7.AddEdge(2, 4, -3);g7.AddEdge(2, 5, -2);
                                                               94
             // 把AdjList最外層的vector走一遍
                                                                     g7.AddEdge(3, 2, 4);
            for (std::list<std::pair<int,int> >::iterator itr
                                                                     g7.AddEdge(4, 3, 1);g7.AddEdge(4, 5, 6);
                  = AdjList[j].begin();
                                                               97
                                                                     g7.AddEdge(5, 0, 3);g7.AddEdge(5, 1, 7);
                 itr != AdjList[j].end(); itr++) {
                                                               98
                      // 各個vector中,所有edge走一遍
                                                                     if (g7.BellmanFord(0))
                                                               99
                Relax(j, (*itr).first, (*itr).second);
                                                              100
                                                                          std::cout << "There is no negative cycle.\n";</pre>
                                                              101
        }
                                                              102
                                                                          std::cout << "There is negative cycle.\n";</pre>
                                                              103
                                                              104
                                                                     return 0;
    // check if there is negative cycle
                                                              105
    for (int i = 0; i < num vertex; i++) {</pre>
        for (std::list<std::pair<int,int> >::iterator itr =
             AdjList[i].begin();
             itr != AdjList[i].end(); itr++) {
                                                                 9.4 Floyd-Warshall
            if (distance[(*itr).first] > distance[i]+(*itr).
                 second) { // i是from, *itr是to
```

return false;

// print predecessor[] & distance[]

for (int i = 0; i < num\_vertex; i++)</pre>

for (int i = 0; i < num vertex; i++)</pre>

std::cout << std::endl << std::endl;</pre>

void Graph SP::InitializeSingleSource(int Start){

for (int i = 0; i < num vertex; i++) {</pre>

distance[i] = Max\_Distance;

std::cout << std::setw(4) << i;

void Graph SP::PrintDataArray(std::vector<int> array){

std::cout << std::setw(4) << array[i];</pre>

std::cout << "predecessor[]:\n";</pre>

PrintDataArray(predecessor);

std::cout << "distance[]:\n";</pre>

PrintDataArray(distance);

std::cout << std::endl;</pre>

distance.resize(num vertex);

predecessor[i] = -1;

distance[Start] = 0;

predecessor.resize(num vertex);

47

48

49

50

}

return true;

```
1 // C++ code
2 // by alrightchiu
3 // all pairs shortest path
 4 #include <iostream>
5 #include <vector>
6 #include <iomanip>
                           // for setw()
  const int MaxDistance = 1000;
9 class Graph SP AllPairs{
10 private:
11
       int num_vertex;
       std::vector< std::vector<int> > AdjMatrix, Distance,
           Predecessor;
13 public:
       Graph_SP_AllPairs():num_vertex(0){};
       Graph SP AllPairs(int n);
       void AddEdge(int from, int to, int weight);
16
17
       void PrintData(std::vector< std::vector<int> > array);
18
       void InitializeData():
19
       void FloydWarshall();
20
^{21}
  Graph SP AllPairs::Graph SP AllPairs(int n):num vertex(n){
       // Constructor, initialize AdjMatrix with 0 or
            MaxDistance
24
       AdjMatrix.resize(num vertex);
       for (int i = 0; i < num vertex; i++) {</pre>
26
           AdjMatrix[i].resize(num vertex, MaxDistance);
27
           for (int j = 0; j < num vertex; j++) {
28
               if (i == j){
                   AdjMatrix[i][j] = 0;
29
```

```
Graph SP AllPairs g10(4);
                                                                                                                                                   count = num vertex-1,
                                                                                                                                                                                 // count 為
                                                                           g10.AddEdge(0, 1, 2);g10.AddEdge(0, 2, 6);g10.AddEdge(0,
32
                                                                                                                                                        topologicalsort[] 的 index
33
                                                                                                                                                   i = Start:
                                                                                                                                        46
   void Graph SP AllPairs::InitializeData(){
                                                                           g10.AddEdge(1, 2, -2);g10.AddEdge(1, 3, 3);
                                                                                                                                        47
                                                                           g10.AddEdge(2, 0, 4);g10.AddEdge(2, 3, 1);
       Distance.resize(num vertex);
                                                                                                                                        48
                                                                                                                                               for (int j = 0; j < num vertex; j++) {
       Predecessor.resize(num vertex);
                                                                                                                                                   if (color[i] == 0) {
37
                                                                           g10.FlovdWarshall():
                                                                                                                                                       DFSVisit_TS(array, color, discover, finish, i,
38
       for (int i = 0; i < num vertex; i++) {</pre>
                                                                                                                                                            time, count):
           Distance[i].resize(num vertex);
39
                                                                           return 0;
                                                                                                                                        51
           Predecessor[i].resize(num vertex, -1);
                                                                                                                                                   i = j;
                                                                                                                                        52
           for (int j = 0; j < num_vertex; j++) {
   Distance[i][j] = AdjMatrix[i][j];</pre>
                                                                                                                                        53
42
                                                                                                                                               std::cout << "\nprint discover time:\n";</pre>
               if (Distance[i][j] != 0 && Distance[i][j] !=
                                                                                                                                               PrintIntArray(discover);
                                                                      9.5 shortest-path on DAG
                    MaxDistance) {
                                                                                                                                               std::cout << "\nprint finish time:\n";</pre>
                   Predecessor[i][j] = i;
                                                                                                                                               PrintIntArray(finish);
                                                                                                                                        58
46
                                                                                                                                           void Graph SP::DFSVisit TS(int *array, int *color, int *
                                                                    1 // C++ code
47
                                                                     2 // O(V+E)
                                                                                                                                                discover,
                                                                                                                                                                       int *finish, int vertex, int &time
                                                                    3 #include <iostream>
   void Graph SP AllPairs::FloydWarshall(){
                                                                     4 #include <vector>
                                                                                                                                                                            , int &count){
                                                                      #include <list>
                                                                                                                                        61
       InitializeData():
51
                                                                                                                                               color[vertex] = 1; // set gray
                                                                      #include <utility>
                                                                                                    // for std::pair<>
52
                                                                                                                                               discover[vertex] = ++time;
                                                                      #include <iomanip>
                                                                                                    // for std::setw()
53
       std::cout << "initial Distance[]:\n";</pre>
                                                                                                                                               for (std::list<std::pair<int,int>>::iterator itr =
       PrintData(Distance);
54
                                                                                                                                                    AdjList[vertex].begin();
                                                                       const int Max Distance = 100:
       std::cout << "\ninitial Predecessor[]:\n";</pre>
55
                                                                                                                                                    itr != AdjList[vertex].end(); itr++) {
                                                                       class Graph SP{
                                                                                                    // SP serves as Shortest Path
       PrintData(Predecessor);
56
                                                                                                                                                   if (color[(*itr).first] == 0) {
                                                                      private:
57
                                                                    12
                                                                           int num_vertex;
                                                                                                                                        67
                                                                                                                                                       predecessor[(*itr).first] = vertex;
       for (int k = 0; k < num vertex; k++) {
58
                                                                           std::vector<std::list<std::pair<int,int>>> AdjList;
                                                                                                                                                       DFSVisit TS(array, color, discover, finish, (*itr
           std::cout << "\nincluding vertex(" << k << "):\n";</pre>
                                                                                                                                                            ).first, time, count);
                                                                           std::vector<int> predecessor, distance;
60
           for (int i = 0; i < num vertex; i++) {</pre>
                                                                                                                                        69
                                                                       public:
                                                                    15
               for (int j = 0; j < num_vertex; j++) {</pre>
                                                                           Graph SP():num vertex(0){};
                                                                                                                                        70
                   if ((Distance[i][i] > Distance[i][k]+Distance 17
62
                                                                           Graph SP(int n):num vertex(n){
                                                                                                                                               color[vertex] = 2: // set black
                                                                               AdjList.resize(num_vertex);
                                                                                                                                               finish[vertex] = ++time;
                         && (Distance[i][k] != MaxDistance)) {
                                                                                                                                               array[count--] = vertex;
                                                                                                                                                                                    // 產生Topological
                       Distance[i][j] = Distance[i][k]+Distance[ 20
                                                                           void AddEdge(int from, int to, int weight);
                                                                                                                                                   Sort
                                                                           void PrintDataArray(std::vector<int> array);
                        Predecessor[i][j] = Predecessor[k][j];
                                                                           void PrintIntArray(int *array);
                                                                                                                                           void Graph_SP::DAG_SP(int Start){
                   }
                                                                                                                                               InitializeSingleSource(Start);
                                                                                                                                                                                     // distance[].
                                                                           void InitializeSingleSource(int Start);
                                                                                                                         // 以Start作 77
                                                                                為起點
                                                                                                                                                    predecessor[]的initialization
           // print data after including new vertex and updating
                                                                                                                         // 對edge(X,Y <sup>78</sup>
                                                                           void Relax(int X, int Y, int weight);
                 the shortest paths
                                                                                                                                               int topologicalsort[num vertex];
           std::cout << "Distance[]:\n";</pre>
                                                                                ) 進行Relax
                                                                                                                                               GetTopologicalSort(topologicalsort, Start);
                                                                                                                                        80
           PrintData(Distance);
                                                                                                                             // 需要
                                                                           void DAG SP(int Start = 0):
           std::cout << "\nPredecessor[]:\n";</pre>
72
                                                                                                                                               for (int i = 0; i < num vertex; i++) {</pre>
                                                                                DFS,加一個額外的Linked list
73
           PrintData(Predecessor);
                                                                                                                                                   int v = topologicalsort[i];
74
                                                                           void GetTopologicalSort(int *array, int Start);
                                                                                                                                                   for (std::list<std::pair<int, int>>::iterator itr =
                                                                           void DFSVisit_TS(int *array, int *color, int *discover,
                                                                                                                                                        AdjList[v].begin();
   void Graph SP AllPairs::PrintData(std::vector< std::vector<</pre>
                                                                                             int *finish, int vertex, int &time, int
                                                                                                                                                        itr != AdjList[v].end(); itr++) {
       int> > array){
                                                                                                  &count);
                                                                                                                                                       Relax(v, (*itr).first, (*itr).second);
                                                                                                                                        86
                                                                    31
                                                                                                                                        87
       for (int i = 0; i < num_vertex; i++){</pre>
           for (int j = 0; j < num_vertex; j++) {</pre>
                                                                       void Graph SP::GetTopologicalSort(int *array, int Start){
                                                                                                                                               std::cout << "\nprint predecessor:\n";</pre>
               std::cout << std::setw(5) << array[i][j];</pre>
                                                                                                                                               PrintDataArray(predecessor);
                                                                           int color[num_vertex], discover[num_vertex], finish[
                                                                                                                                        91
                                                                                                                                               std::cout << "\nprint distance:\n";</pre>
           std::cout << std::endl;</pre>
                                                                                num vertex];
                                                                                                                                        92
                                                                                                                                               PrintDataArray(distance);
                                                                                                                                        93
                                                                           for (int i = 0; i < num_vertex; i++) {</pre>
                                                                                                                                           void Graph_SP::PrintDataArray(std::vector<int> array){
   void Graph SP AllPairs::AddEdge(int from, int to, int weight) 38
                                                                               color[i] = 0;
                                                                                                                                               for (int i = 0; i < num vertex; i++)</pre>
                                                                               discover[i] = 0;
                                                                                                                                                   std::cout << std::setw(4) << i;
                                                                                                                                        96
       AdjMatrix[from][to] = weight;
                                                                               finish[i] = 0;
                                                                                                                                               std::cout << std::endl;</pre>
                                                                    41
                                                                               predecessor[i] = -1;
                                                                                                                                               for (int i = 0; i < num vertex; i++)</pre>
                                                                    42
                                                                                                                                                   std::cout << std::setw(4) << array[i];</pre>
   int main(){
                                                                    43
                                                                                                                                               std::cout << std::endl;</pre>
                                                                           int time = 0,
```

```
void Graph SP::PrintIntArray(int *array){
        for (int i = 0; i < num vertex; i++)</pre>
103
104
            std::cout << std::setw(4) << i;</pre>
        std::cout << std::endl;</pre>
105
106
        for (int i = 0; i < num vertex; i++)</pre>
107
            std::cout << std::setw(4) << arrav[i]:</pre>
        std::cout << std::endl;</pre>
108
109
    void Graph SP::InitializeSingleSource(int Start){
110
111
        distance.resize(num vertex):
112
        predecessor.resize(num_vertex);
113
114
115
        for (int i = 0; i < num \ vertex; i++) {
            distance[i] = Max Distance;
116
            predecessor[i] = -1;
117
118
        distance[Start] = 0;
119
120
    void Graph SP::Relax(int from, int to, int weight){
121
122
123
        if (distance[to] > distance[from] + weight) {
            distance[to] = distance[from] + weight;
124
125
            predecessor[to] = from;
126
127
    void Graph SP::AddEdge(int from, int to, int weight){
128
        AdjList[from].push_back(std::make_pair(to,weight));
129
130
131
    int main(){
132
133
        Graph SP g8(7);
134
        g8.AddEdge(0, 1, 3);g8.AddEdge(0, 2, -2);
135
        g8.AddEdge(1, 3, -4);g8.AddEdge(1, 4, 4);
136
137
        g8.AddEdge(2, 4, 5);g8.AddEdge(2, 5, 6);
138
        g8.AddEdge(3, 5, 8);g8.AddEdge(3, 6, 2);
        g8.AddEdge(4, 3, -3);g8.AddEdge(4, 6, -2);
139
140
        g8.AddEdge(5, 6, 2);
141
        g8.DAG_SP(0);
                              // 以vertex(0)作為起點
142
143
        return 0;
144
145 }
```

# graph/Tree

#### 10.1 Lowest Common Ancestor

```
1 #define MAXN 200005
2 #define MAXLOG 200
3 int D[MAXN];
4 int P[MAXLOG][MAXLOG];
5 #include <cmath>
6 #include <algorithm>
vsing namespace std;
8 #define MAXN 200005
9 #define MAXLOG 200
10 int N = MAXN;
int lgN = log(N) / log(2);
```

```
12 int D[MAXN];
                                                                     10 pair<int, int> Tree Centroid(int v, int pa)
   int P[MAXLOG][MAXLOG];
                                                                     11 | {
   int LCA(int u, int v)
14
15
                                                                     13
16
       if (D[u] > D[v])
17
           swap(u, v);
18
       int s = D[v] - D[u]; // adjust D until D[v] = D[u]
19
       for (int i = 0; i <= lgN; ++i) // 調整他們到二進位數一樣
           if (s & (1 << i))</pre>
                                                                     19
               v = P[v][i];
                                                                     20
       if (u == v)
                                                                     21
           return v;
                                                                     22
       // because they are at same depth
                                                                     23
       // jump up if they are different
                                                                     24
       // think about that if P[u][i] == P[v][i]
                                                                     25
       // then that point must be the ancestor of LCA or LCA
       // by this, we will stop at LCA's child
       for (int i = lgN; i >= 0; --i)
                                                                     28
           if (P[u][i] != P[v][i])
                                                                     29
               u = P[u][i];
                                                                     31
                                                                        // Tree_Centroid2
35
               v = P[v][i];
                                                                       vector<int> V[10005];
36
                                                                       int N;
37
       return P[u][0];
                                                                        int center, csize;
38
                                                                        int dfs(int v, int fa)
39
                                                                     36
40
   void ComputeP()
                                                                     37
41
42
       for (int i = 0; i < lgN; ++i) // to lgN enough
43
44
                                                                     41
45
           for (int x = 0: x < n: ++x)
                                                                     42
46
                                                                     43
47
                if (P[x][i] == -1)
                                                                     44
                   P[x][i + 1] = -1;
48
                                                                     45
49
                    P[x][i + 1] = P[P[x][i]][i]; // equal to move <sub>47</sub>
50
                          on the parent direction
                    // And P[x][i] move 2 ^ n steps to a parent
51
                                                                    49
                         we call it y
                                                                     50
52
                    // P[y][i] means continue move 2 ^ n step
                                                                     51
                         from y to a parent we call z
                                                                     52
                    // so the total equal to move 2 ^ n * 2 ^ n
                                                                     53
                         steps from x to z
                                                                     54
                    // which is move 2 ^{\circ} (n + 1) steps to z
54
55
56
```

#### 10.2 Tree Centroid

```
1 #include <utility>
2 #include <vector>
3 #include <algorithm>
  using namespace std;
  int subTsize[200005];
8 vector<int> adi[200005];
9 int n; // n for node num ??
```

### 11.1 hashingVec

```
1 #include < bits / stdc++.h>
2 struct VectorHash {
      size t operator()(const std::vector<int>& v) const {
          std::hash<int> hasher;
          size t seed = 0;
          for (const int& i : v) {
              seed ^{=} hasher(i) + 0x9e3779b9 + (seed<<6) + (
```

// return (最 大 子 樹 節 點 數 , 節 點ID)

for (size\_t i = 0; i < adj[v].size(); ++i)</pre>

res = min(res, Tree Centroid(x, v));

max\_subT = max(max\_subT, subTsize[x]);

// min because all res will be greater than n/2;

int max\_subT = 0; // 最大子樹節點數

subTsize[v] += subTsize[x];

// the min one is the tree centroid

int x = adj[v][i];

continue:

if(x == pa)

the biggest

return res;

int sz = 1;

int maxsub = 0;

for(int u:V[v])

sz += sub;

if (maxsub<csize)</pre>

center = v;

csize = maxsub:

if (u==fa)continue;

maxsub = max(maxsub, N-sz);

int sub = dfs(u, v);

maxsub = max(maxsub, sub);

pair<int, int> res(INT MAX, -1); // ans: tree cnetroid

res = min(res, make\_pair(max(max\_subT, n - subTsize[v]),

v)); // (n - subTsize[v]) for maybe parent tree is

subTsize[v] = 1;

# 11 hashing

return sz;

```
return seed;
11 };
12 std::unordered set<std::vector<int>, VectorHash> H;
```

### 12 number theory

#### 12.1 Fib

```
1 #include <bits/stdc++.h>
using namespace std;
3 // Cassini's identity : F_{n-1}F_{n+1} - F_{n^2} = (-1)^n
4 // The "addition" rule : F {n+k} = F kF {n+1}+F {k-1}F n
5 / / k = n, F_{2n} = F_n*(F_{n+1} + F_{n-1})
6 // F \{2k\} = F k*(2F \{K+1\}-F k)
7 // F_{2k+1} = F_{K+1}^2 + F_k^2
  pair<int, int> fib (int n) {
       if (n == 0)
           return {0, 1};
11
       auto p = fib(n >> 1);
       int c = p.first * (2 * p.second - p.first);
12
13
       int d = p.first * p.first + p.second * p.second;
14
       if (n & 1)
15
           return {d, c + d};
16
       else
17
           return {c, d};
```

### 12.2 BigInterger

```
1 #include <vector>
2 #include <string>
3 #include <iostream>
4 #include <cmath>
5 #include <algorithm>
6 #include <cstdio>
7 #include <cstring>
  using namespace std;
9 struct BigInteger{
       static const int BASE = 100000000;
       static const int WIDTH = 8;
       vector<int> s;
13
       BigInteger(long long num = 0) { *this = num; }
       BigInteger operator = (long long num) {
           s.clear();
           do{
               s.push back(num % BASE);
               num /= BASE:
20
           } while (num > 0);
           return *this:
22
23
       BigInteger operator = (const string& str){
25
           s.clear();
           int x, len = (str.length() - 1) / WIDTH + 1;
26
           for (int i = 0; i < len;i++){</pre>
```

```
int start = max(0, end - WIDTH);
        sscanf(str.substr(start, end - start).c str(), "% 91
        s.push_back(x);
    return *this:
                                                             93
BigInteger operator+ (const BigInteger b) const{
    BigInteger c;
                                                             95
    c.s.clear():
    for(int i=0,g=0;;i++){
                                                             97
        if(g== 0 && i >=s.size() && i >=b.s.size())
                                                             98
                                                             99
        int x = g;
                                                             100
        if(i<s.size()) x+=s[i];</pre>
                                                            101
        if(i<b.s.size()) x+=b.s[i];</pre>
                                                            102
        c.s.push_back(x % BASE);
                                                            103
        g = x/BASE;
                                                            104
                                                            105
    return c;
                                                            106
                                                            107
BigInteger operator+=(const BigInteger& b){
                                                            108
    *this = *this + b;
                                                             109
    return *this;
                                                             110
BigInteger operator* (const BigInteger b)const{
    BigInteger c;
    c.s.clear();
    long long mul;
    for (int i = 0;i < s.size(); i++)</pre>
        long long carry = 0;
        for (int g = 0; g < b.s.size();g++){</pre>
            mul = (long long)(s[i]) * (long long)(b.s[g]) 4 int gcd(int a, int b, int& x, int& y) {
            mul += carry;
            if(i + g < c.s.size()){
                 c.s[i+g] += mul % BASE;
            }else{
                 c.s.push back(mul % BASE);
                                                             11
            carry = mul / BASE;
                                                             12
                                                             13
    for (int i = 0; i < c.s.size(); i++){
                                                             15
        if(c.s[i] >= BASE){
            if(i + 1 < c.s.size()){
                 c.s.push back(c.s[i] / BASE);
            }else{
                 c.s[i + 1] += c.s[i] / BASE;
                                                             19
                                                             20
            c.s[i] %= BASE;
                                                             21
                                                             22
   }
                                                             23
    return c;
bool operator< (const BigInteger& b) const{</pre>
    if(s.size() != b.s.size()) return s.size() < b.s.size 27</pre>
         ();
                                                             28
    for(int i=s.size() -1; i>=0;i--)
        if(s[i] != b.s[i]) return s[i] < b.s[i];</pre>
    return false; // Equal
```

int end = str.length() - i \* WIDTH;

29

30

31 32

33

34

35

36

37

38

39

40

41

42

43

44

45

46

47

48

49

50

51

52

53

54

55

56

57

58

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61

62

63

64

65

66

67

68

69

70

71

72

73

75

80

81

82

83

85

87

```
bool operator> (const BigInteger& b) const{return b < *</pre>
         this;}
    bool operator<= (const BigInteger& b) const {return !(b<*
    bool operator>=(const BigInteger& b) const {return !(*
         this < b);}
    bool operator!=(const BigInteger& b) const {return b< *</pre>
         this || *this < b;}
    bool operator == (const BigInteger& b)const {return !(b<*
         this) && !(*this<b);}
ostream& operator<< (ostream &out, const BigInteger& x){
    out << x.s.back();</pre>
    for (int i = x.s.size() - 2; i >= 0; i--){}
        char buf[20];
        sprintf(buf,"%08d",x.s[i]);
        for(int j = 0;j<strlen(buf);j++) out << buf[j];</pre>
    return out;
istream& operator>> (istream &in, BigInteger & x){
    string s;
    if(!(in >> s)) return in;
    x = s;
    return in;
```

#### 12.3 gcds

```
1 #include <bits/stdc++.h>
using namespace std;
3 // O(log(min(a, b)))
      if (b == 0) {
          x = 1;
          y = 0;
          return a;
      int x1, y1;
      int d = gcd(b, a % b, x1, y1);
      x = y1;
      y = x1 - y1 * (a / b);
      return d;
  bool find_any_solution(int a, int b, int c, int &x0, int &y0,
        int &g) {
      g = gcd(abs(a), abs(b), x0, y0);
      if (c % g) {
          return false;
      x0 *= c / g;
      v0 *= c / g;
      if (a < 0) x0 = -x0;
      if (b < 0) y0 = -y0;
      return true:
30 // finding all solution
31 void shift_solution(int & x, int & y, int a, int b, int cnt)
      x += cnt * b;
```

```
y -= cnt * a;
34
35
   int find all solutions(int a, int b, int c, int minx, int
       maxx, int miny, int maxy) {
37
       int x, y, g;
38
       if (!find_any_solution(a, b, c, x, y, g))
           return 0;
39
40
       b /= g;
41
42
       int sign a = a > 0 ? +1 : -1;
43
       int sign_b = b > 0 ? +1 : -1;
44
45
46
       shift_solution(x, y, a, b, (minx - x) / b);
47
       if (x < minx)</pre>
           shift_solution(x, y, a, b, sign_b);
48
       if (x > maxx)
49
50
           return 0;
       int 1x1 = x;
51
52
       shift_solution(x, y, a, b, (maxx - x) / b);
53
54
       if (x > maxx)
55
           shift_solution(x, y, a, b, -sign_b);
56
       int rx1 = x;
57
       shift_solution(x, y, a, b, -(miny - y) / a);
58
59
       if (y < miny)</pre>
60
           shift_solution(x, y, a, b, -sign_a);
61
       if (y > maxy)
62
           return 0;
63
       int 1x2 = x;
64
65
       shift_solution(x, y, a, b, -(maxy - y) / a);
       if (y > maxy)
66
           shift_solution(x, y, a, b, sign_a);
67
       int rx2 = x;
68
69
70
       if (1x2 > rx2)
           swap(1x2, rx2);
72
       int 1x = max(1x1, 1x2);
       int rx = min(rx1, rx2);
74
75
       if (1x > rx)
76
           return 0;
       return (rx - lx) / abs(b) + 1;
78
79 // smallest possible val
80 // x' + y' = x + y + k(b-a)g, minimize b-a
```

#### 12.4 nCr

```
using i64 = long long;
#define maxn 300005
i64 fact[MAXN], tcaf[MAXN];

#define P 998244353
#define REP1(i, n) for (int i = 1; i <= (int)(n); ++i)
#define REP(i, n) for (int i = (int)(n) - 1; i >= 0; --i)
void init(int n){
fact[0] = 1;
for (int i = 1; i <= n; i++)
fact[i] = i * fact[i - 1] % P;</pre>
```

```
for (int i = n; i >= 0; --i)
13
           tcaf[i] = deg(fact[i], -1);
14
15
16
   i64 deg(i64 x, i64 d) {
17
18
       if (d < 0) d += P - 1:
       i64 y = 1;
19
20
       while (d) {
21
           if (d & 1) (y *= x) %= P;
22
           d /= 2;
23
           (x *= x) %= P:
24
25
       return y;
26
28
   i64 cnk(int n, int k) {
       if (k < 0 \mid | k > n) return 0;
29
       return fact[n] * tcaf[k] % P * tcaf[n - k] % P;
30
```

### 13 string

#### 13.1 KMP

```
1 #include <iostream>
  #include <string>
3 #include <regex>
4 #include <vector>
  using namespace std;
  // T for Text, P for Patterm
  vector <int> build_kmp(const string &P) {
       vector <int> f(P.size());
       int fp = f[0] = -1;
       for (int i = 1; i < P.size(); ++i) {</pre>
11
           while (~fp && P[fp + 1] != P[i])
12
               fp = f[fp];
           if (P[fp + 1] == P[i])
13
14
               ++fp;
15
           f[i] = fp;
16
17
18
       return f;
19
  vector <int> kmp match(vector <int> fail, const string &P,
        const string &T)
21
22
       vector <int> res; // start from these points
       const int n = P.size();
       for (int j = 0, i = -1; j < T.size(); ++j) {
           while (~i && T[j] != P[i + 1])
25
26
               i = fail[i];
           if (P[i + 1] == T[j])
               ++i;
           if (i == n - 1)
30
               res.push back(j - n + 1), i = fail[i];
31
       return res;
33
  int main(){
       char control;
```

```
string test patterm = "a";
       string test text = "abcdabcdabceabcd";
37
       // for testing
38
39
       cout << "Do you want to Enter by ys?(y/n)";
       cin >> control;
40
       if(control == 'y' || control == 'Y'){
41
42
           cout << "Enter text:";</pre>
           cin >> test text;
43
44
           cout << "Enter patterm:";</pre>
45
           cin >> test_patterm;
46
47
       vector<int> V = build_kmp(test_patterm);
48
49
       vector<int> Ans = kmp match(V, test patterm, test text);
50
       cout << '\n';</pre>
51
       for(auto it = V.begin(); it != V.end(); ++it)
           cout << *it << ' ';
52
       cout << '\n';</pre>
53
54
       for(auto it = Ans.begin(); it != Ans.end(); ++it)
           cout << *it <<' ';
55
56
       return 0;
57
```

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