Population range estimation with animal tracking data

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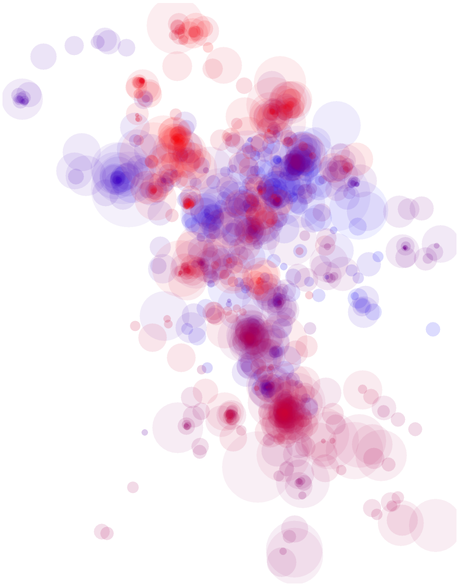
Overview

- Introduction
- Kernel density estimation
- Validation
- Discussion



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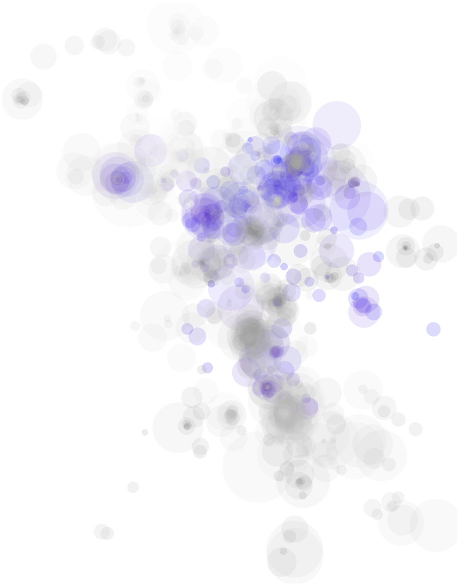

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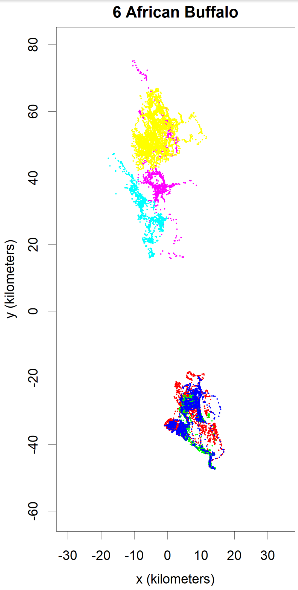

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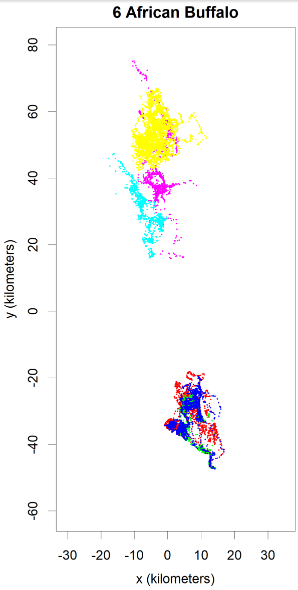

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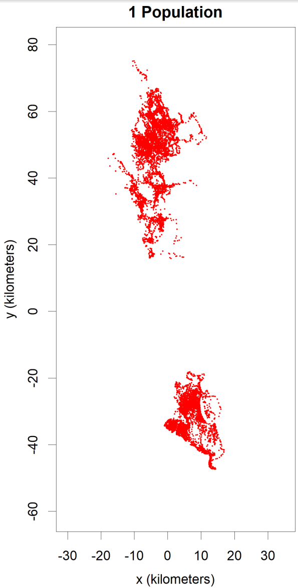

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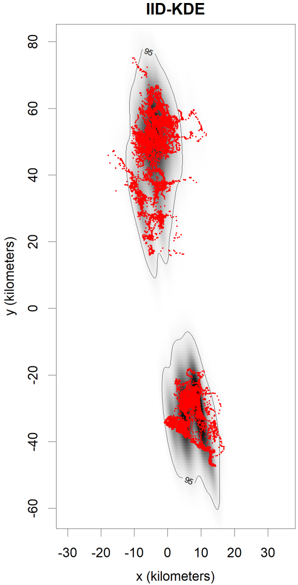

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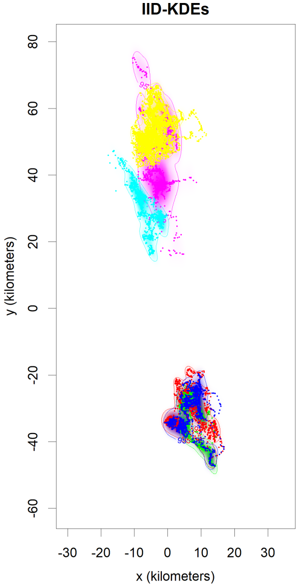

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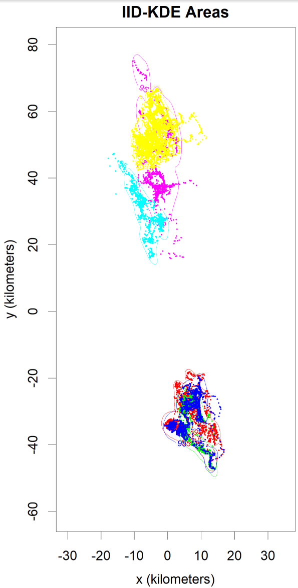

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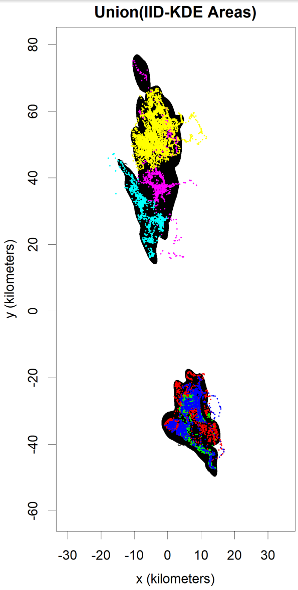

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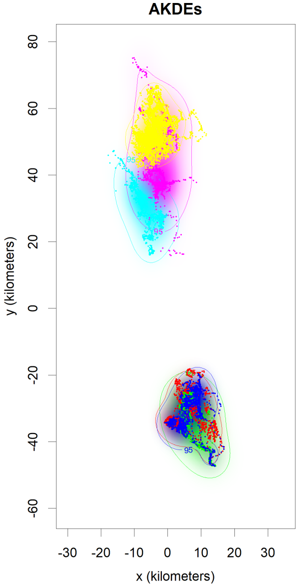

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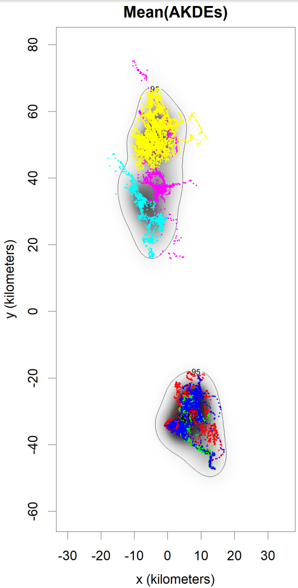

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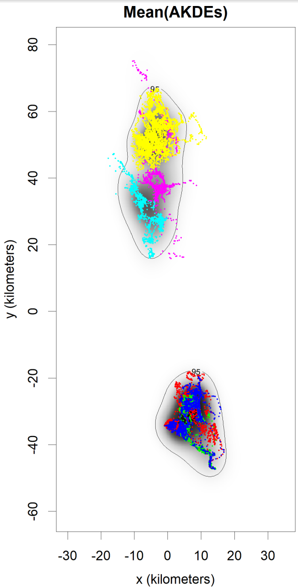

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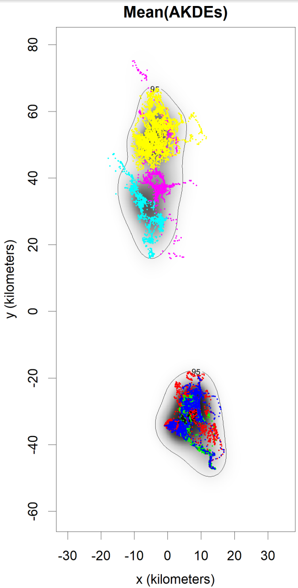

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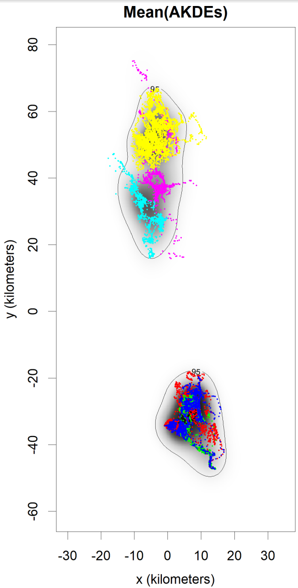

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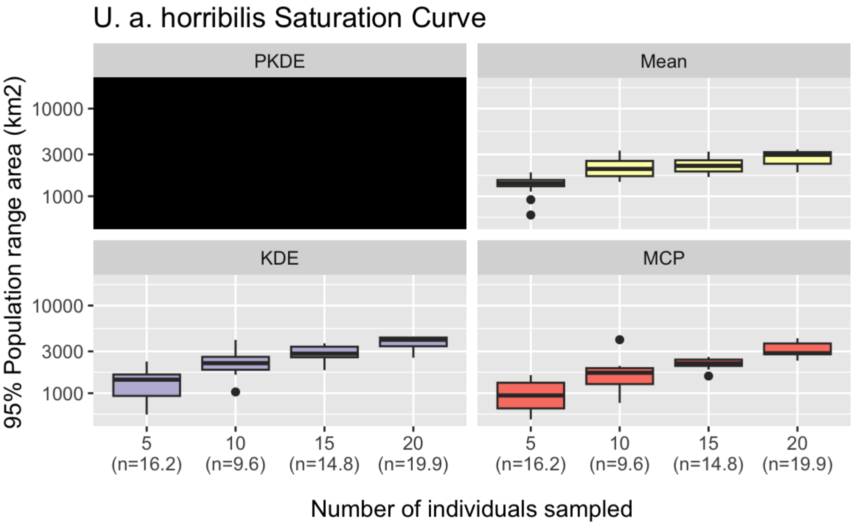

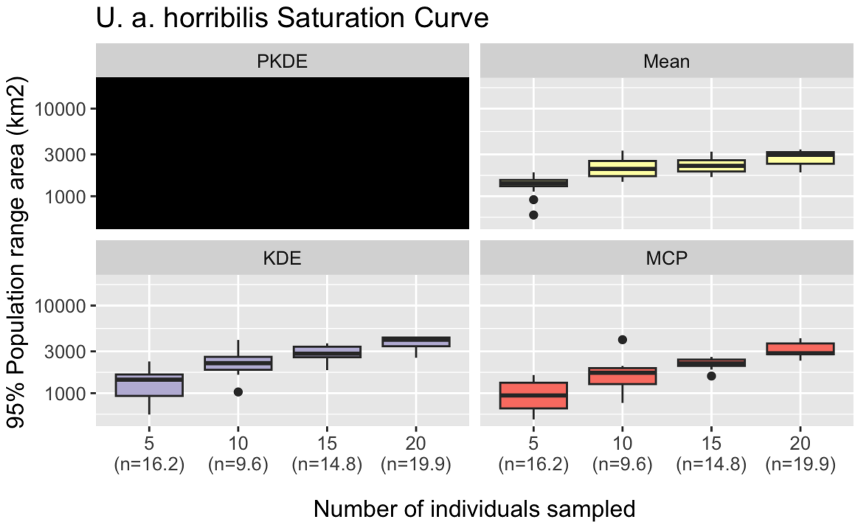

This is bias from not modeling population variance

Transition

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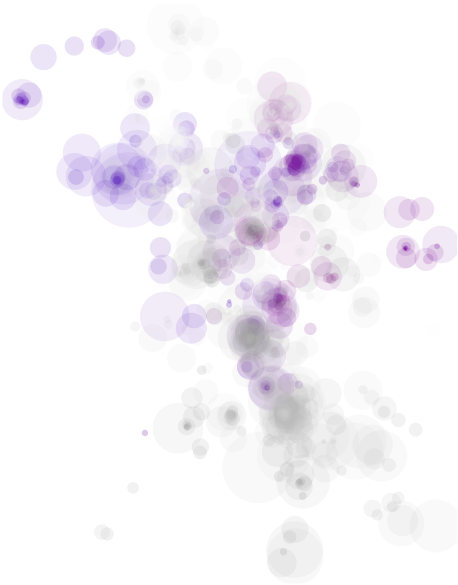

Kernel Kernel Density Estimation

A KDE is a :

$\hat{p}(\mathbf{x}|\mathbf{H}) =$

$\hat{p}(\cdot)$: probability density estimate

$\mathbf{x}$: location of interest

Kernel Kernel Density Estimation

A KDE is a weighted average of :

$$\hat{p}(\mathbf{x}|\mathbf{H}) = \sum_{t}^{n} w(t)$$

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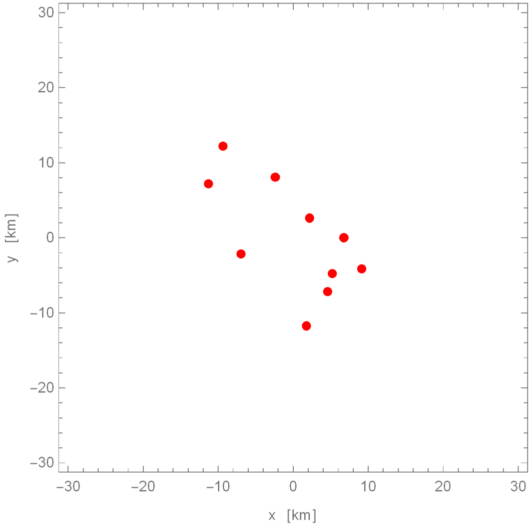

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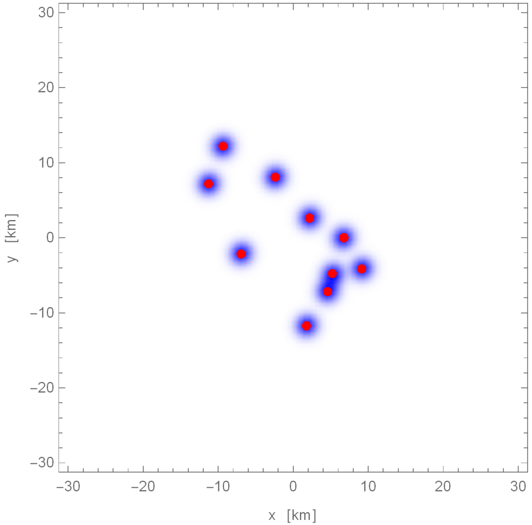

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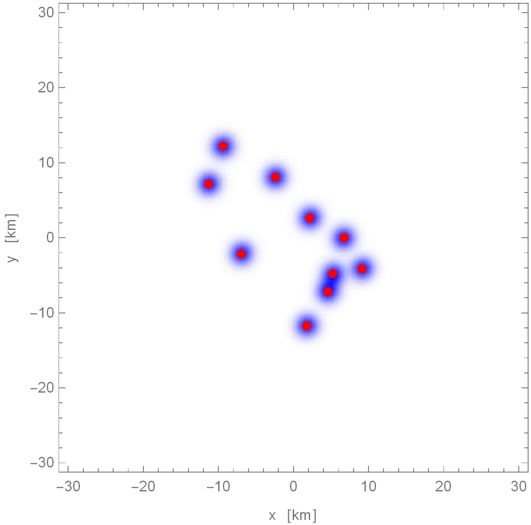

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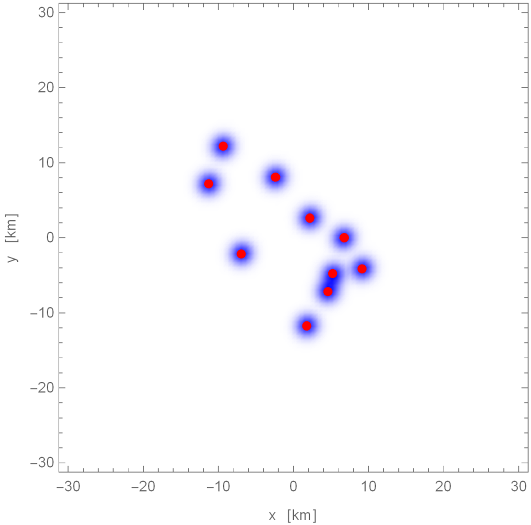

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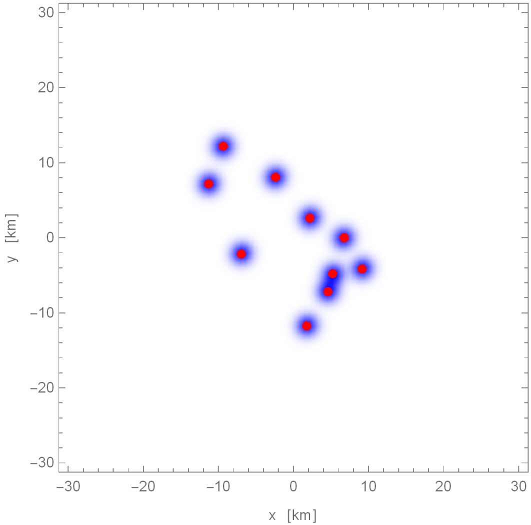

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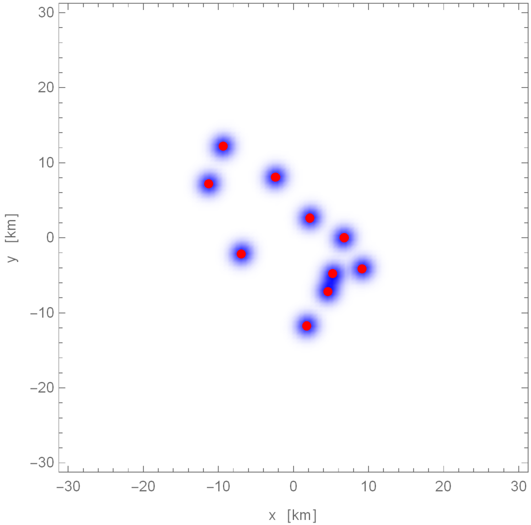

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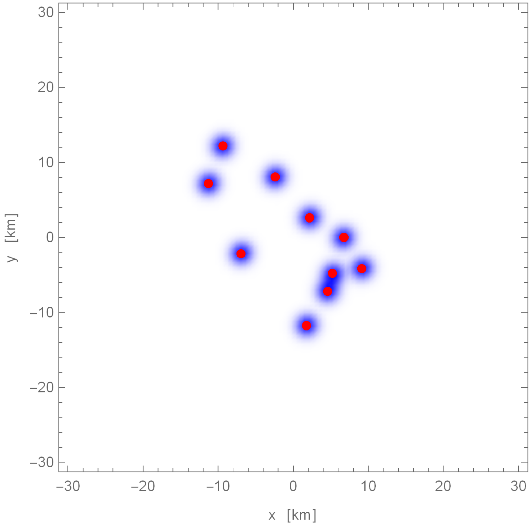

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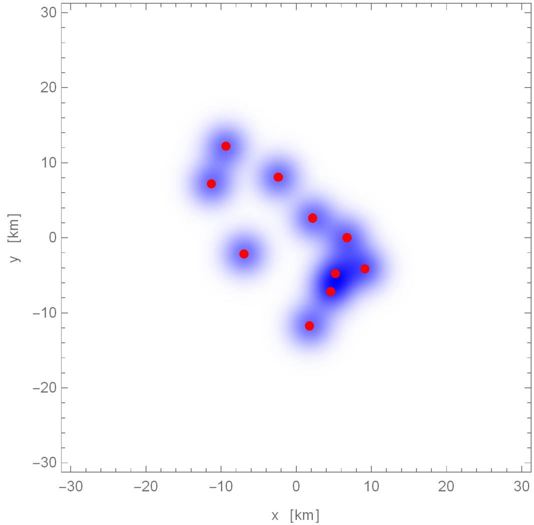

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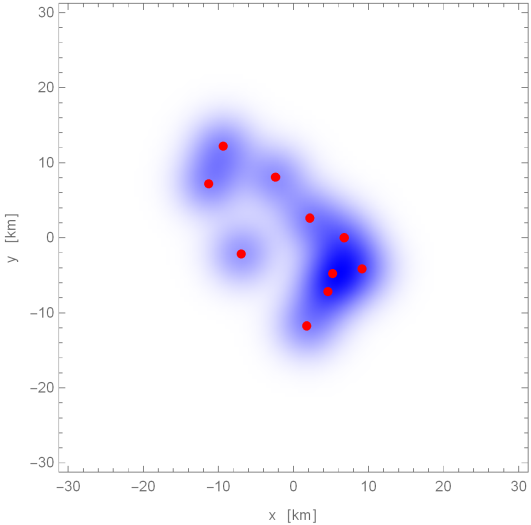

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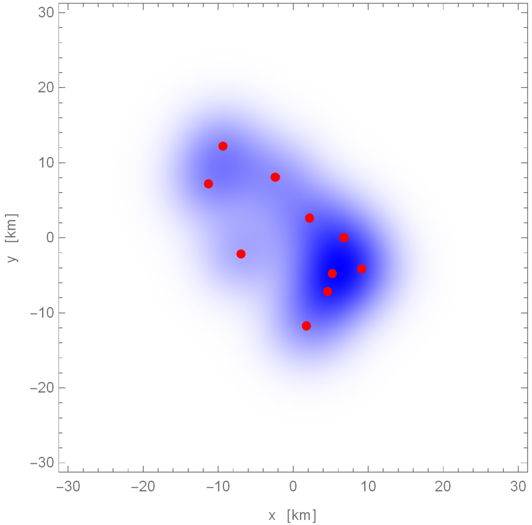

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- bandwidth optimization (Turlach 1993)

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Autocorrelated Kernel Density Estimation

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- bandwidth optimization (Turlach 1993)
- AKDE (Fleming, Fagan, et al. 2015)
- bias-corrected AKDEc (Fleming and Calabrese 2017)
- (MISE) optimally weighted wAKDE (Fleming, Sheldon, et al. 2018)

Population kernel density estimation

$$\mathrm{MISE}[\mathbf{H}] = \mathrm{E}\left[\iint \left(\hat{p}_{\text{pop.}}(\mathbf{x}|\mathbf{H}) - p_{\text{pop.}}(\mathbf{x}) \right)^2 d\mathbf{x} \right]$$

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- Bandwidth assumptions:

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$$\hat{p}_{\text{pop.}}(\mathbf{x}|\mathbf{H}) = \sum_{\text{ind.}} \sum_{t} w_{\text{ind.}}(t)\, \kappa(\mathbf{x} - \mathbf{x}_{\text{ind.}}(t)|\mathbf{H}_{\text{ind.}})$$

- $p_{\mathrm{pop.}}(\mathbf{x})$ requires a *hierarchical* approximation
- Individual weight assumptions:
  - Uniform: $\sum_t w_{\text{ind.}}(t) = \frac{1}{\sum_{\mathrm{ind.}}}$
  - MISE-optimized $\sum_t w_{\text{ind.}}(t)$ (tends to zero-weight overlapped individuals)
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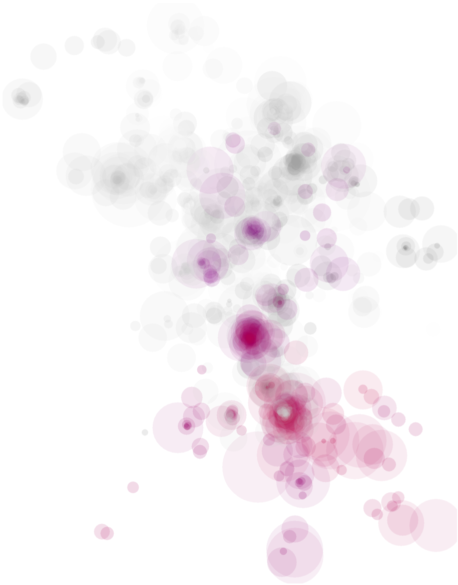
Transition

- Introduction
- Kernel density estimation
- Validation
- Discussion



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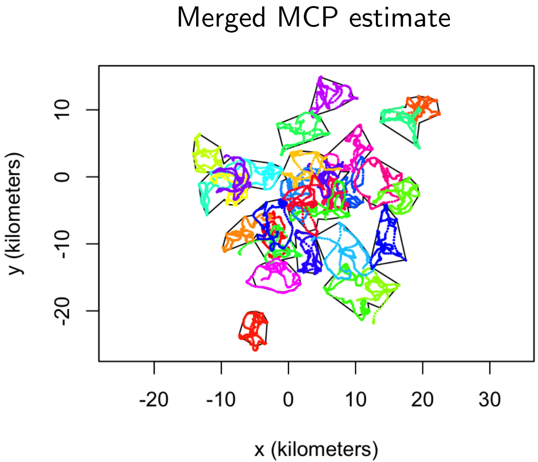
Gayatri Anand (UMD Ph.D.)

Validation: Simulations



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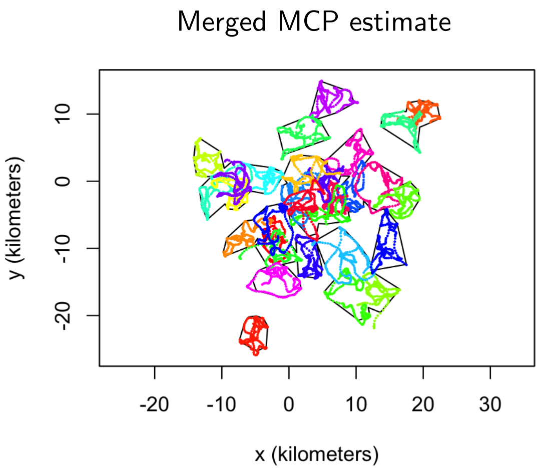

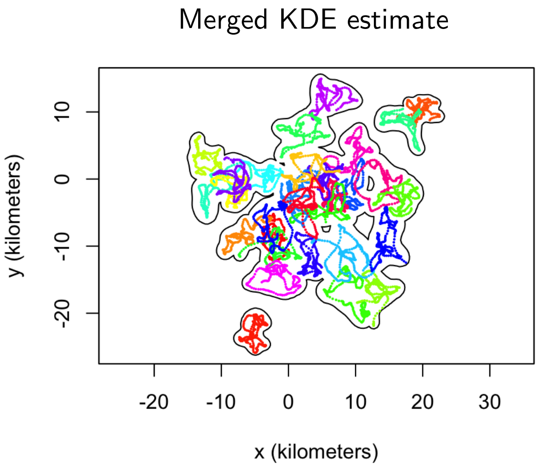

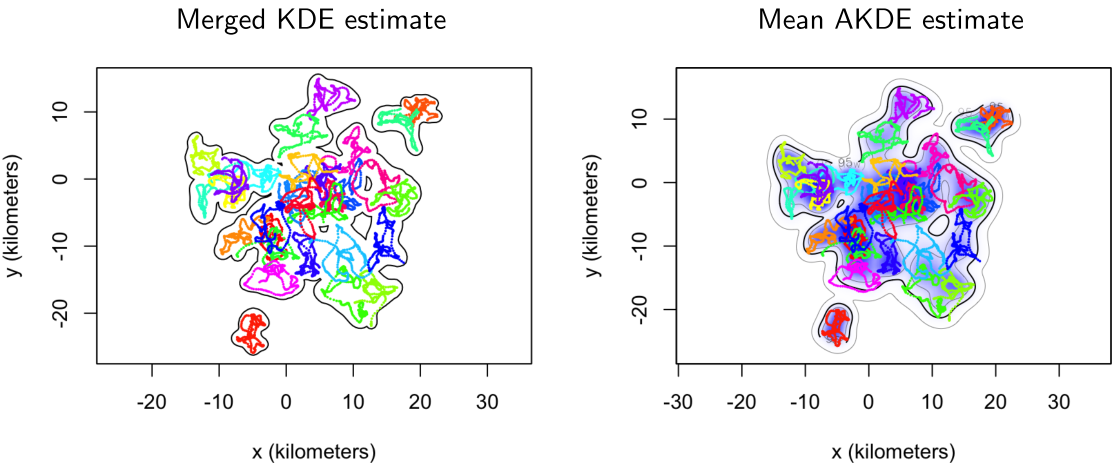
Merged KDE estimate
y (kilometers)
10
0
-10
-20
-20
-10
0
10
20
30
x (kilometers)
Mean AKDE estimate
y (kilometers)
10
0
-10
-20
-30
-20
-10
0
10
20
30
x (kilometers)

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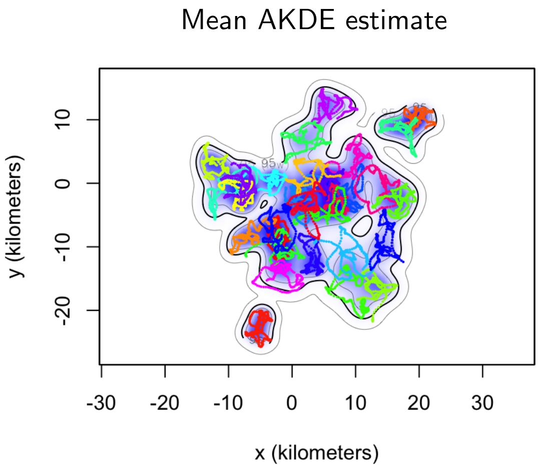

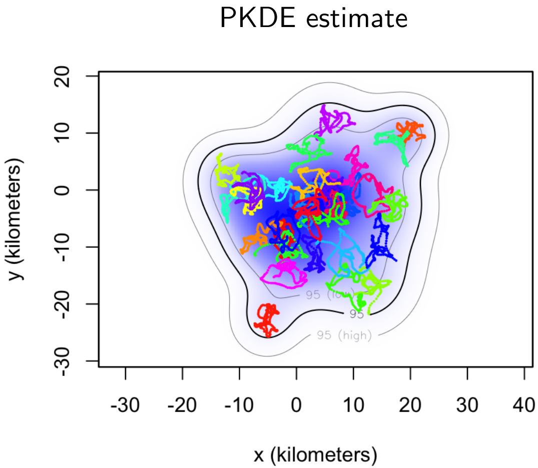

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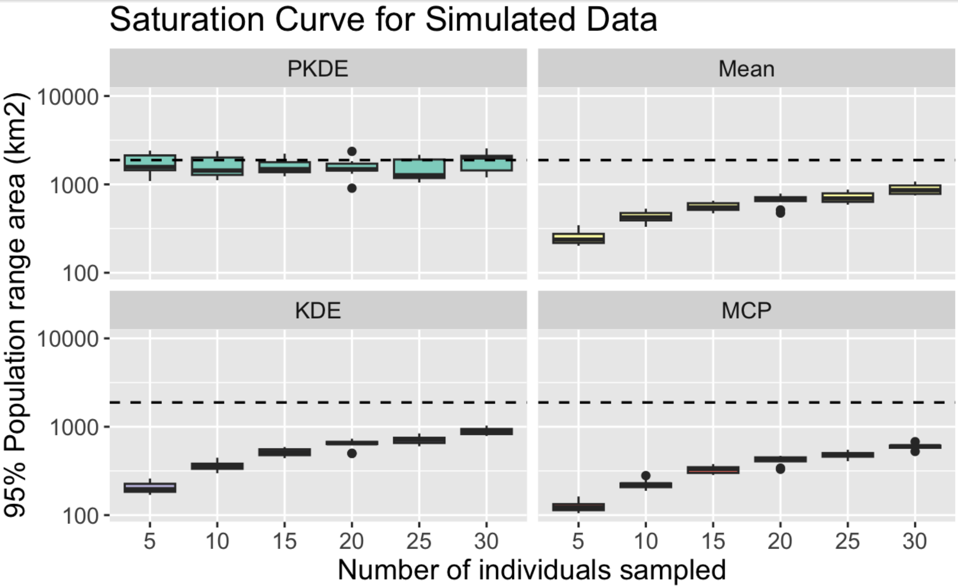
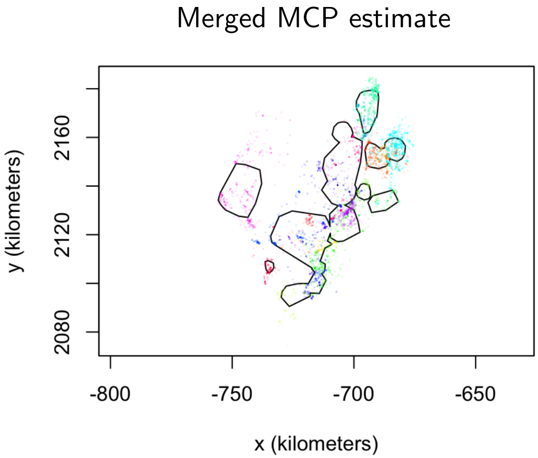


Validation: Empirical (*Ursus arctos horribilis*)



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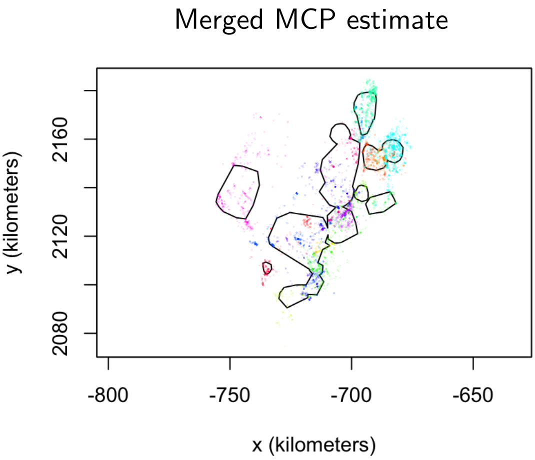

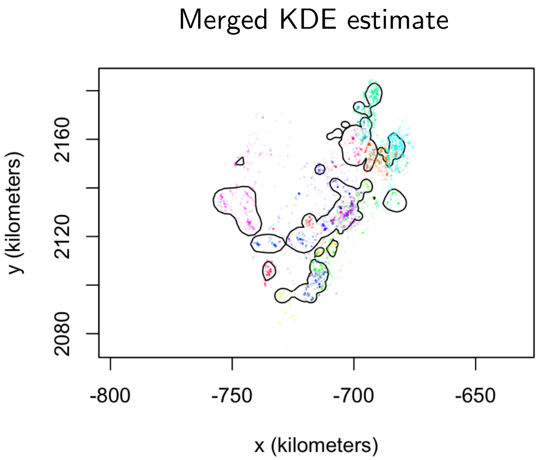

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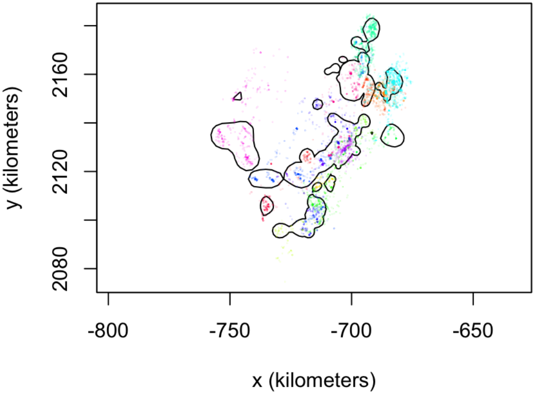
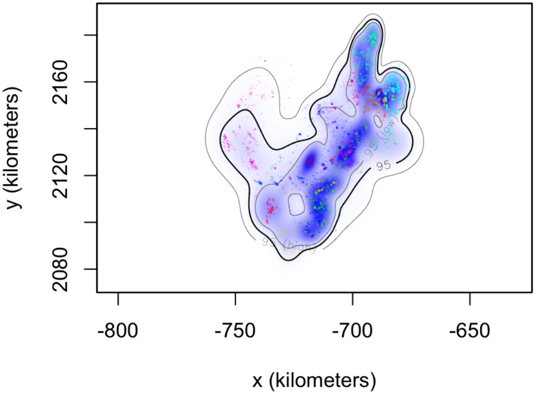
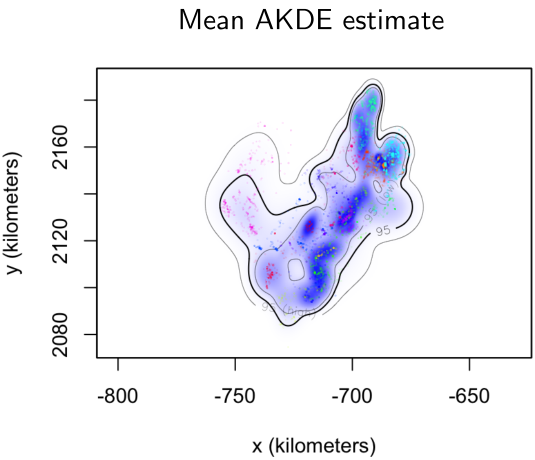
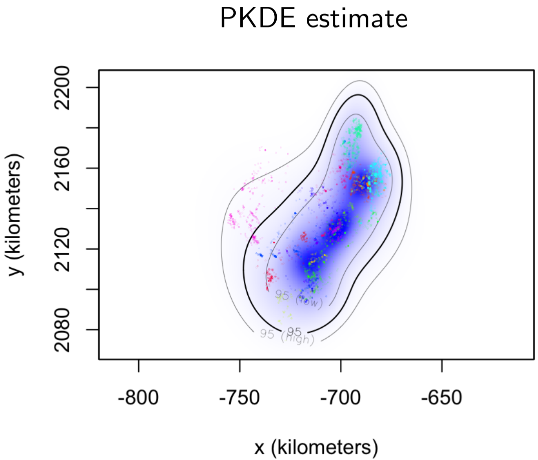
Merged KDE estimate



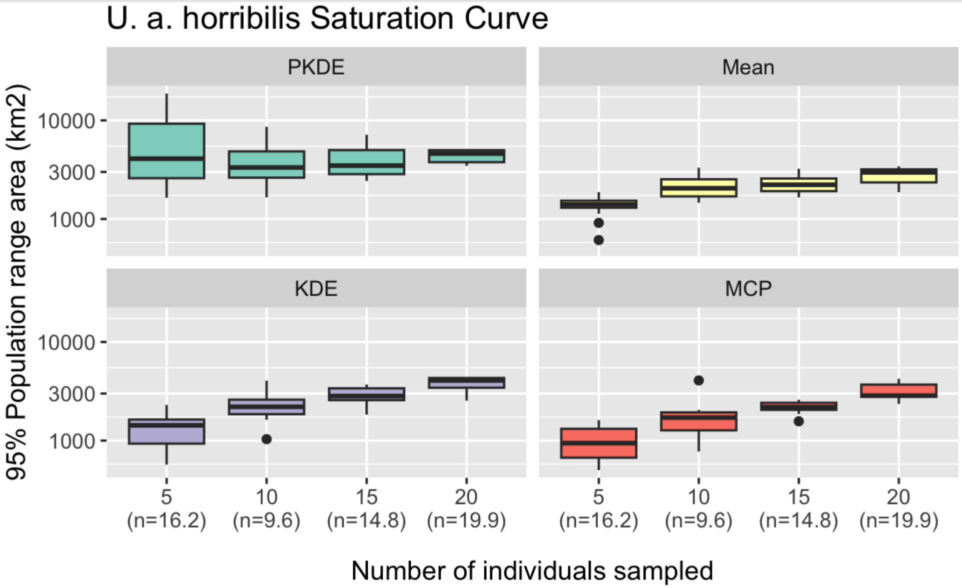
Mean AKDE estimate




Mean AKDE estimate
y (kilometers)
2080
2120
2160
-800
-750
-700
-650
x (kilometers)
95
95 (low)
95 (high)


PKDE estimate
y (kilometers)
2080
2120
2160
2200
-800
-750
-700
-650
x (kilometers)
95
95 (low)
95 (high)

Validation: Empirical (*Ursus arctos horribilis*)



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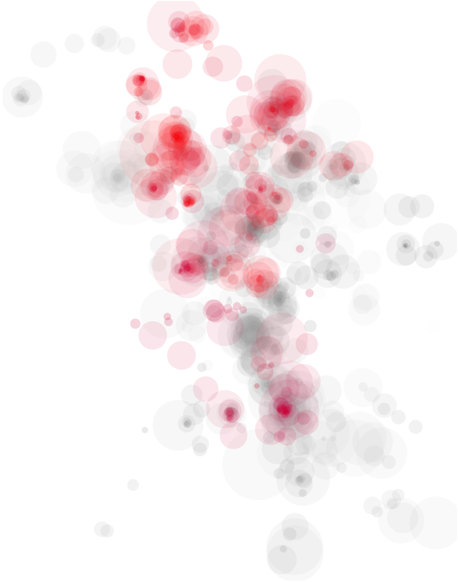

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  $\sum_t w_{\text{ind.}}(t) \sim 1 + \frac{1}{\text{DOF}[\text{Area}_{\text{ind.}}]}$
  - Parallels the problem of down-weighting low-quality (large HDOP) locations in home-range estimation

Thank you

- In collaboration with: Gayatri Anand, William F. Fagan, Justin M. Calabrese, etc.
- Funded by and supported by:

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