

Introduction to continuous-time movement modeling

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University of Central Florida

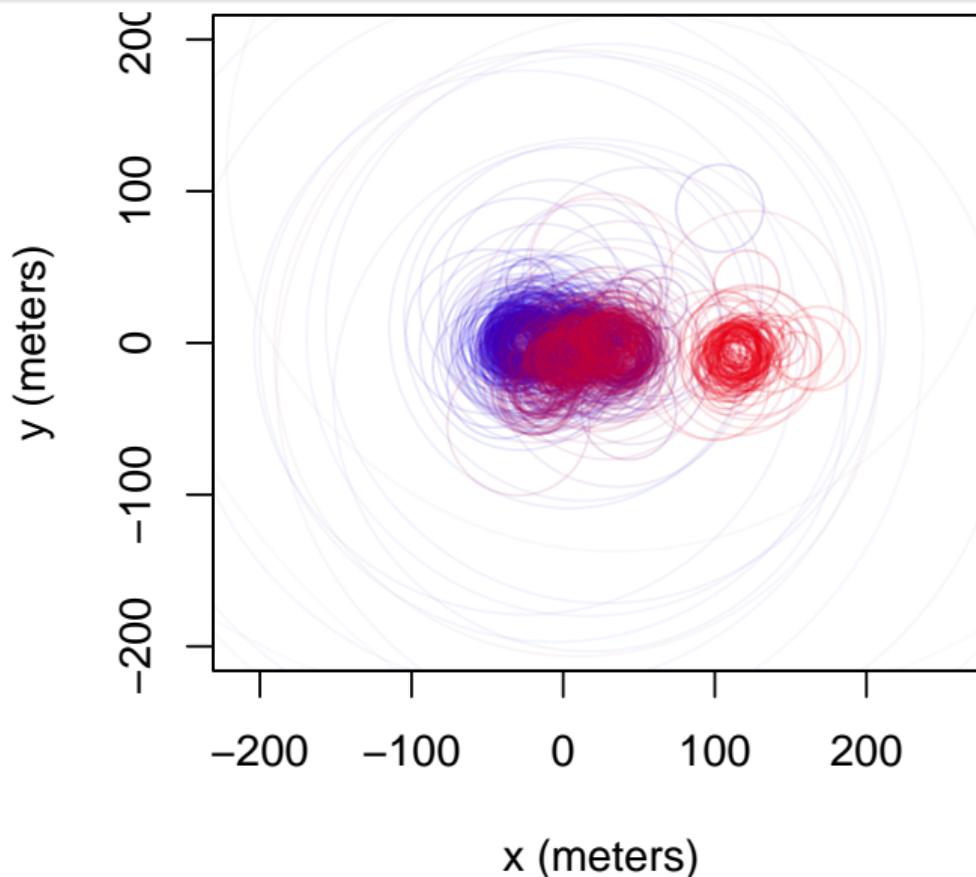


CASUS

2024-06-19

Animal tracking data are complex

(*Glyptemys insculpta*)



Animal tracking data analysis goals in this workshop

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- Account for autocorrelation

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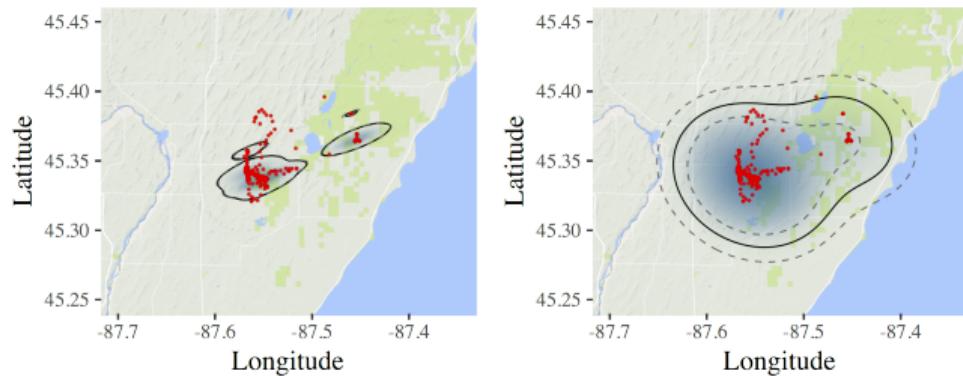
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- Reducing bias and error as much as possible (MVU, BLUE, debiased estimators)

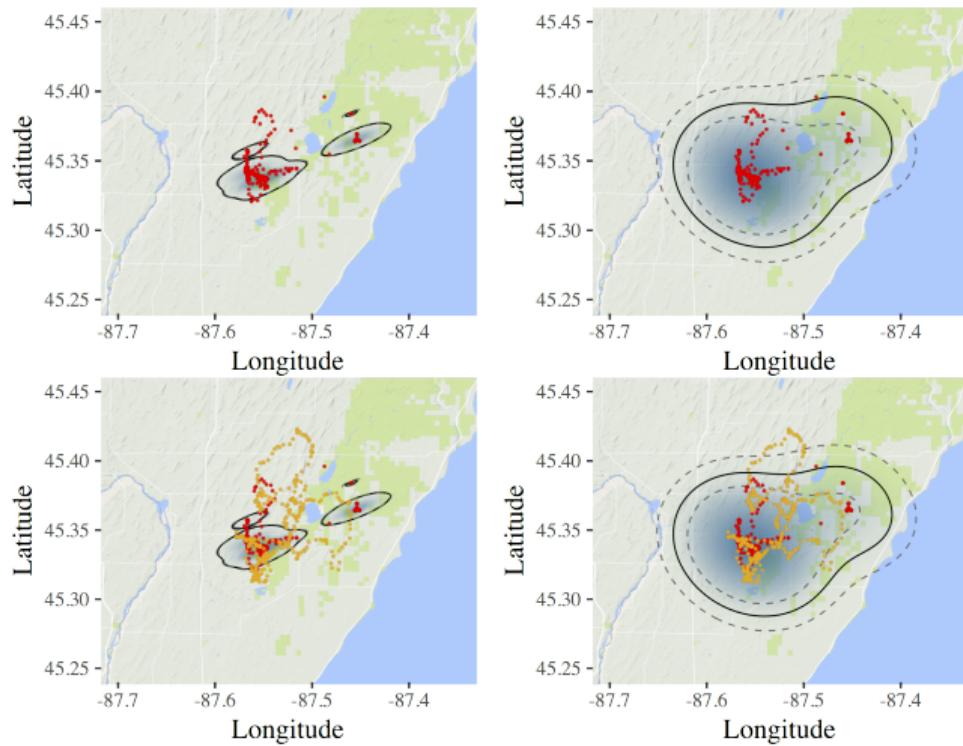
Motivating example: Neglecting autocorrelation in home-range estimation



● GPS-tracked black bear

(Noonan, Tucker, Fleming, et al., Ecological Monographs. 2019)

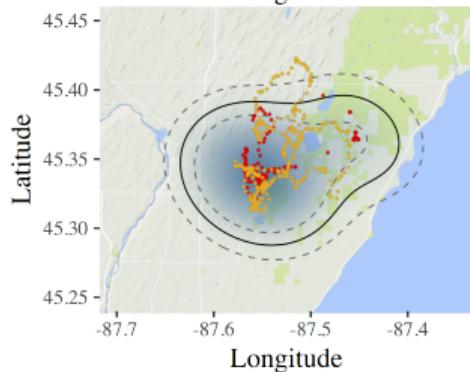
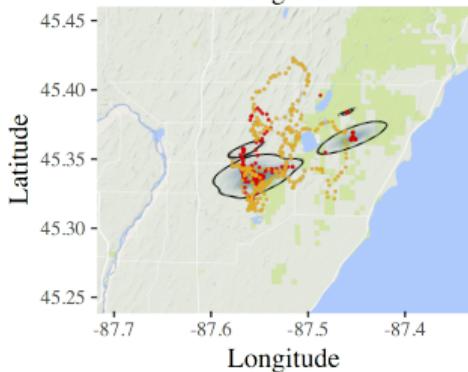
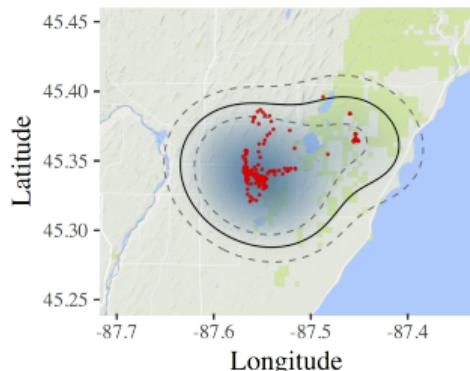
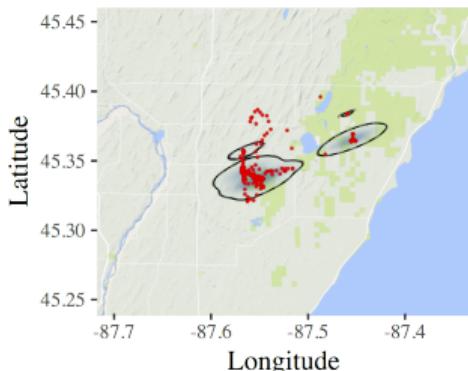
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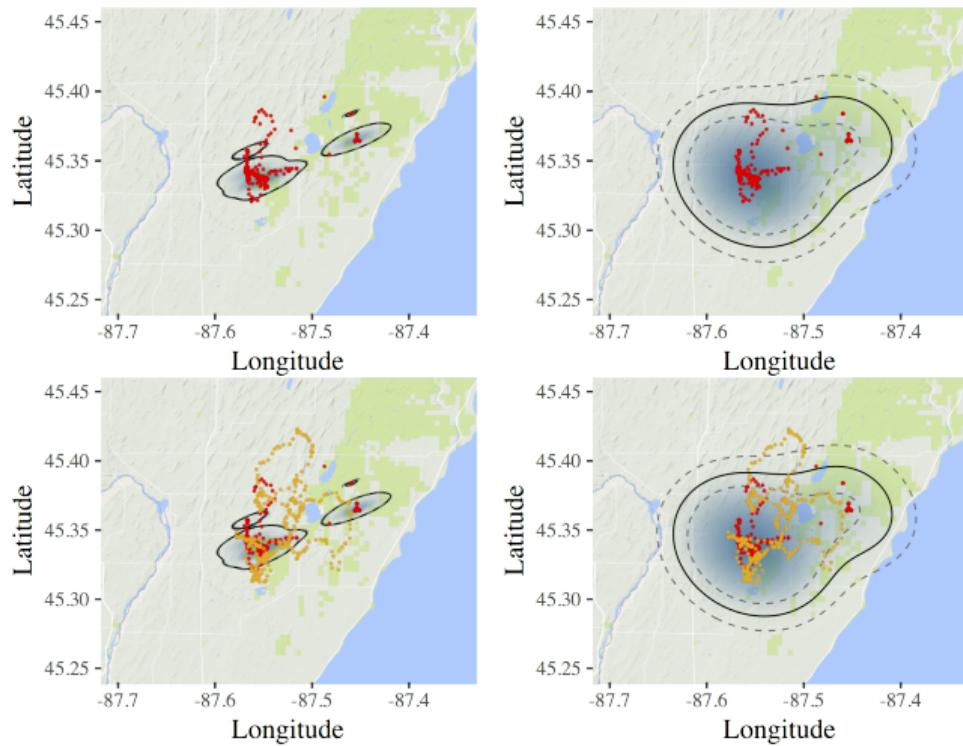
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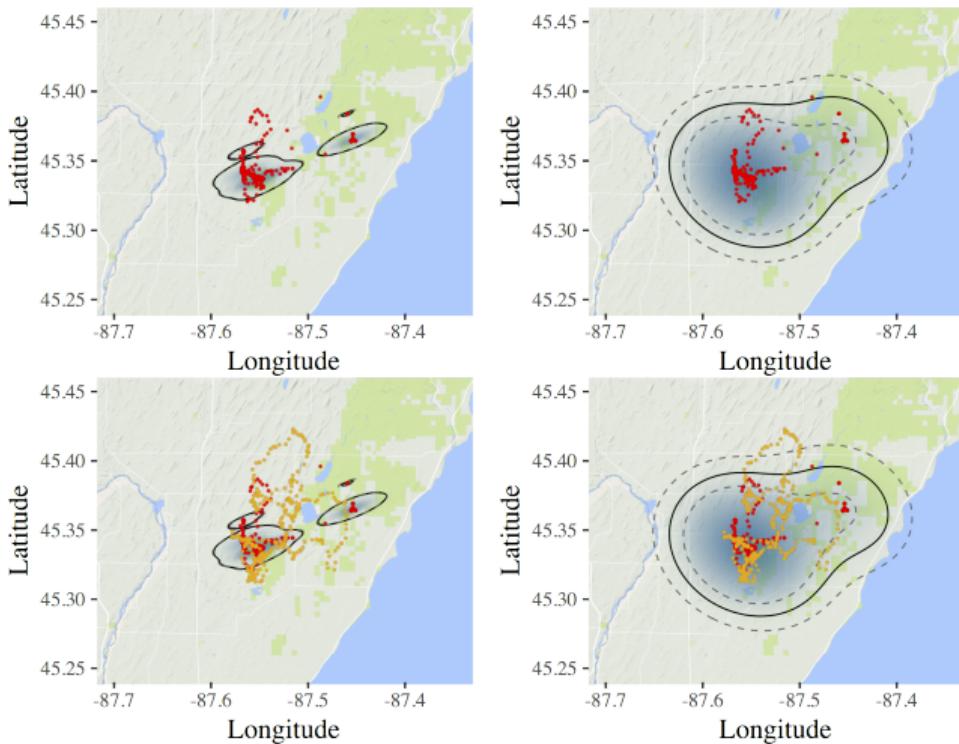
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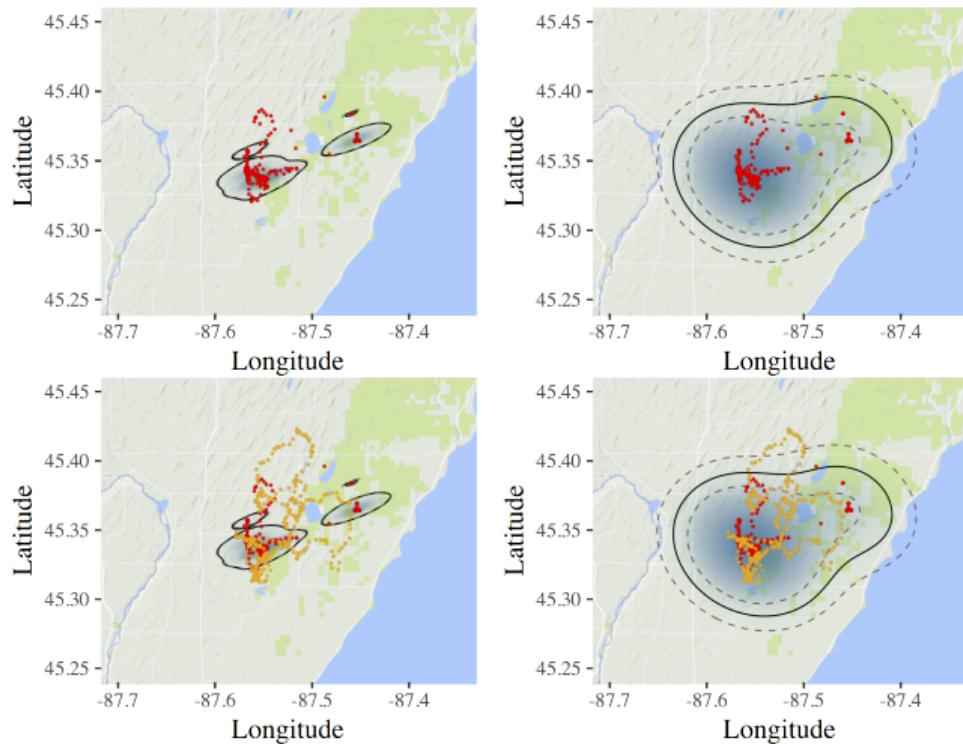
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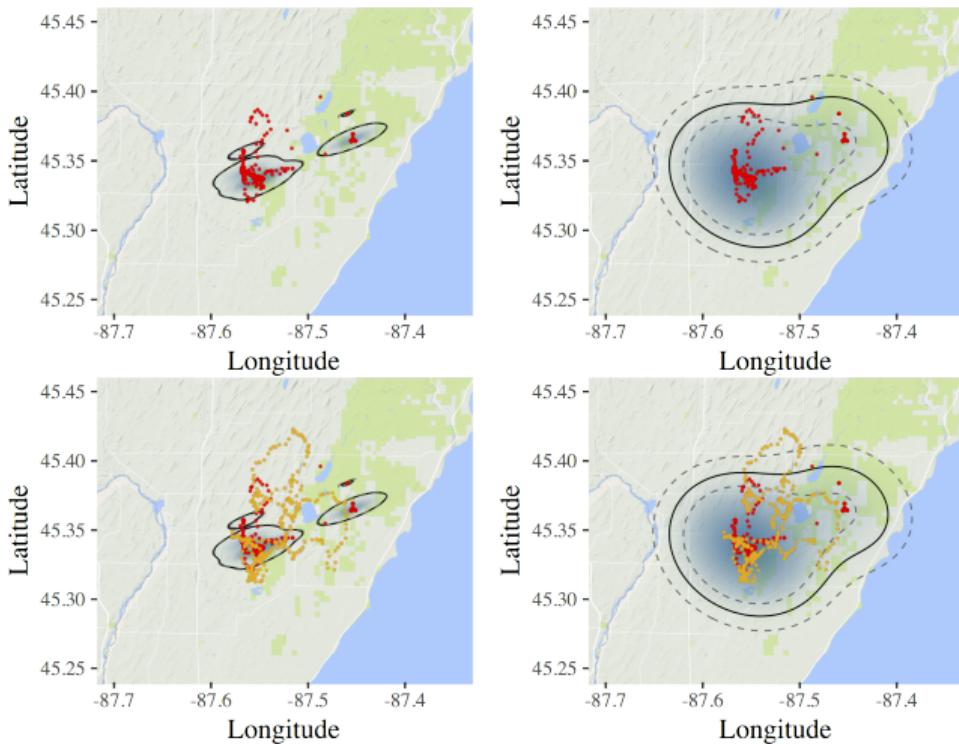
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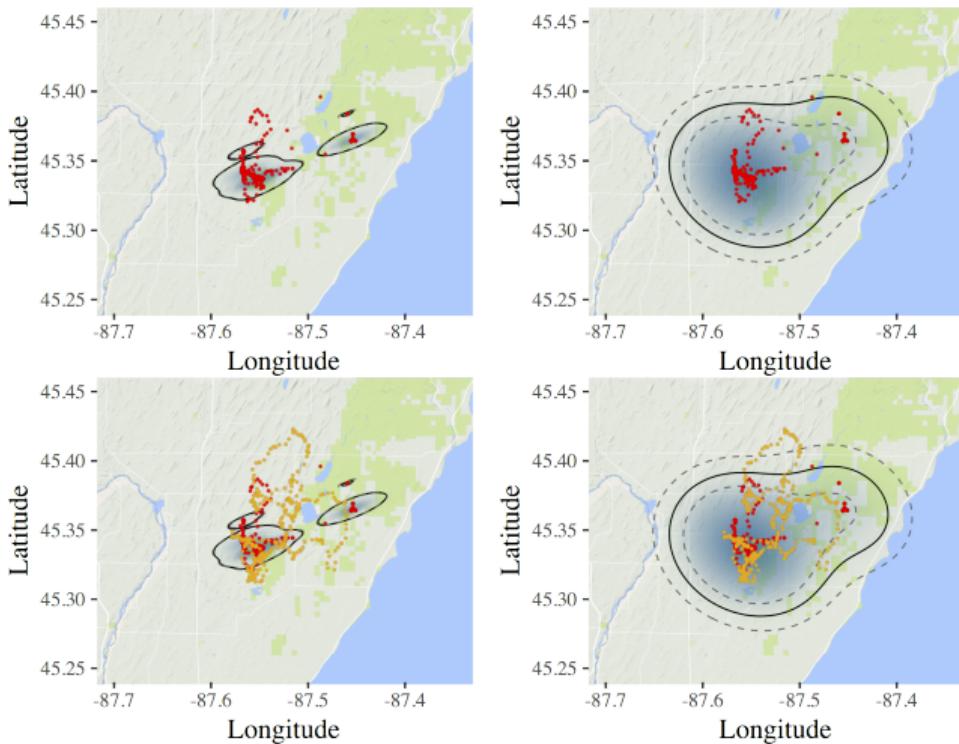
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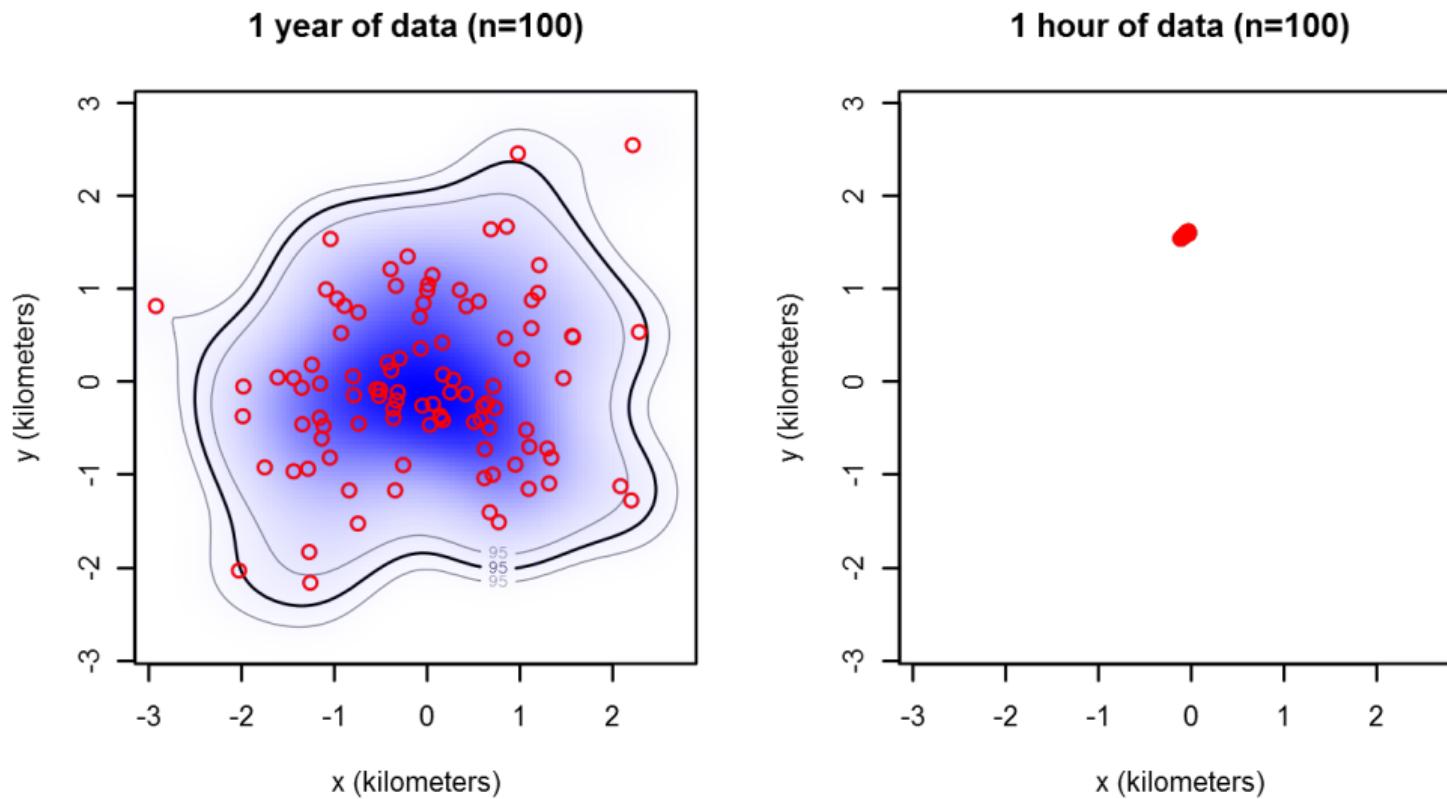
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- Q2: Does this happen in practice?
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(Noonan, Tucker, Fleming, et al., Ecological Monographs. 2019)

Q1: Why does this happen?



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- Generalized to autocorrelated data (Fleming et al., Ecology 2015; Fleming and Calabrese, Methods in Ecology and Evolution 2017; Fleming et al., Ecological Applications 2018)

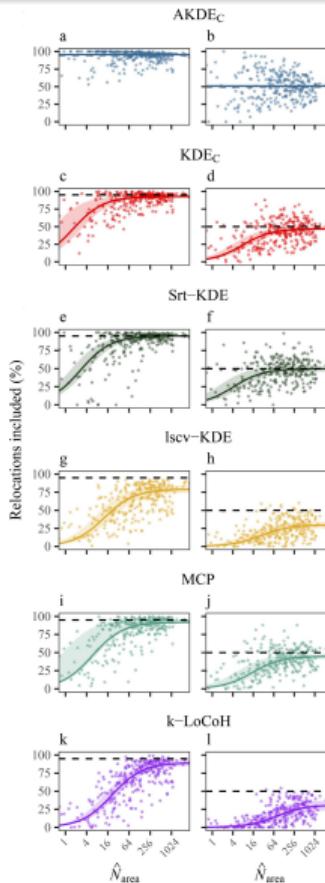
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- Largest comparative analysis to date: 369 individuals, 27 species
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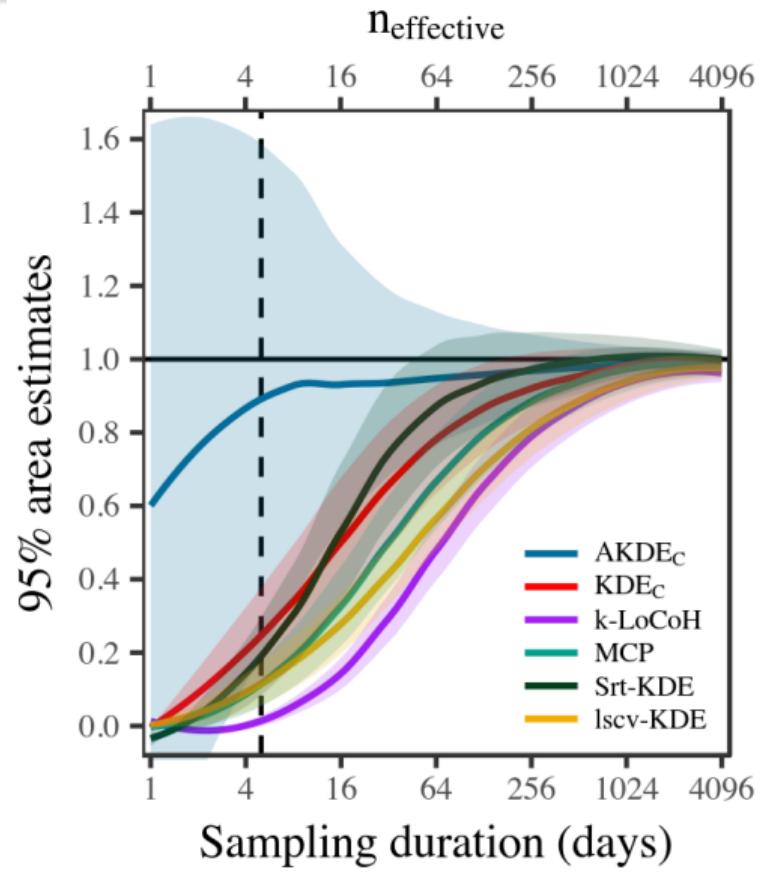
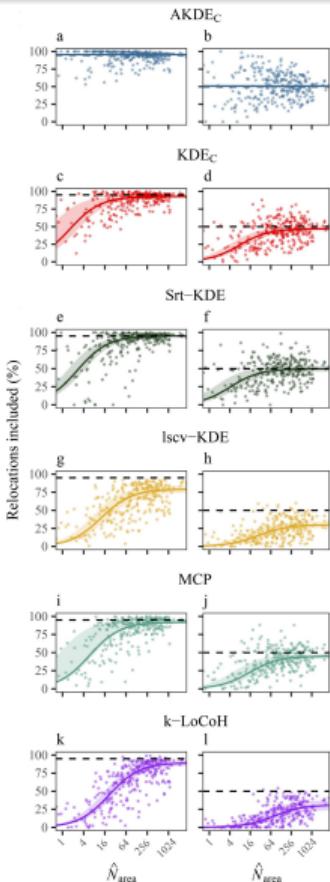
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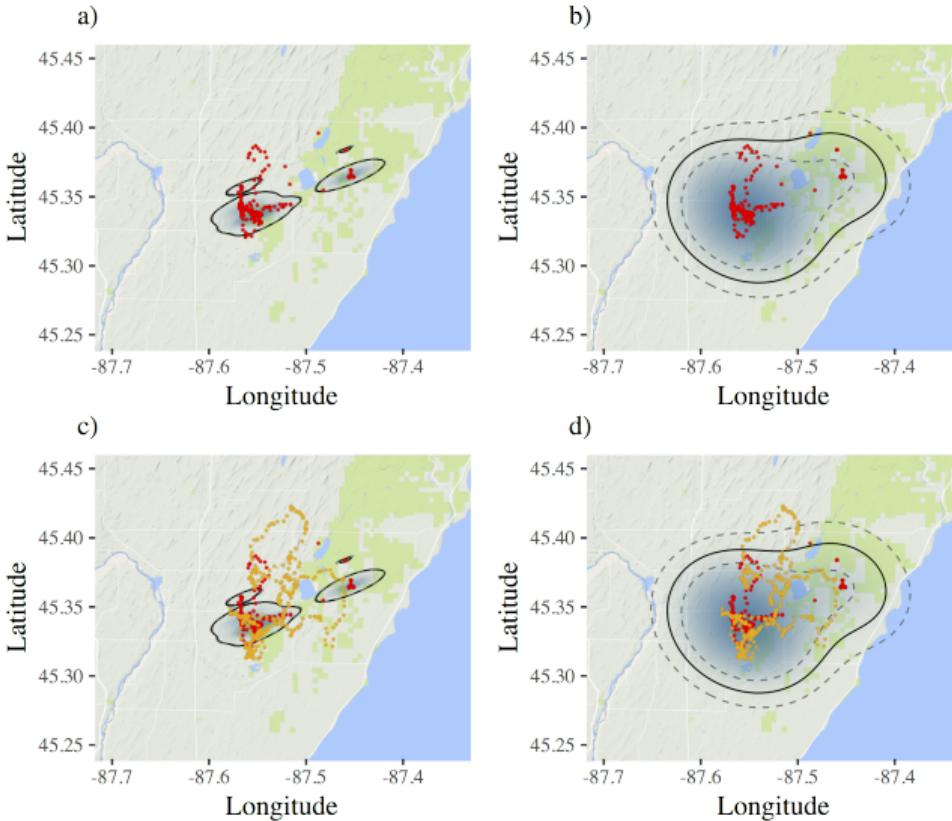
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- Bias is worse for larger species (Noonan, Fleming, et al., Conservation Biology 2020)

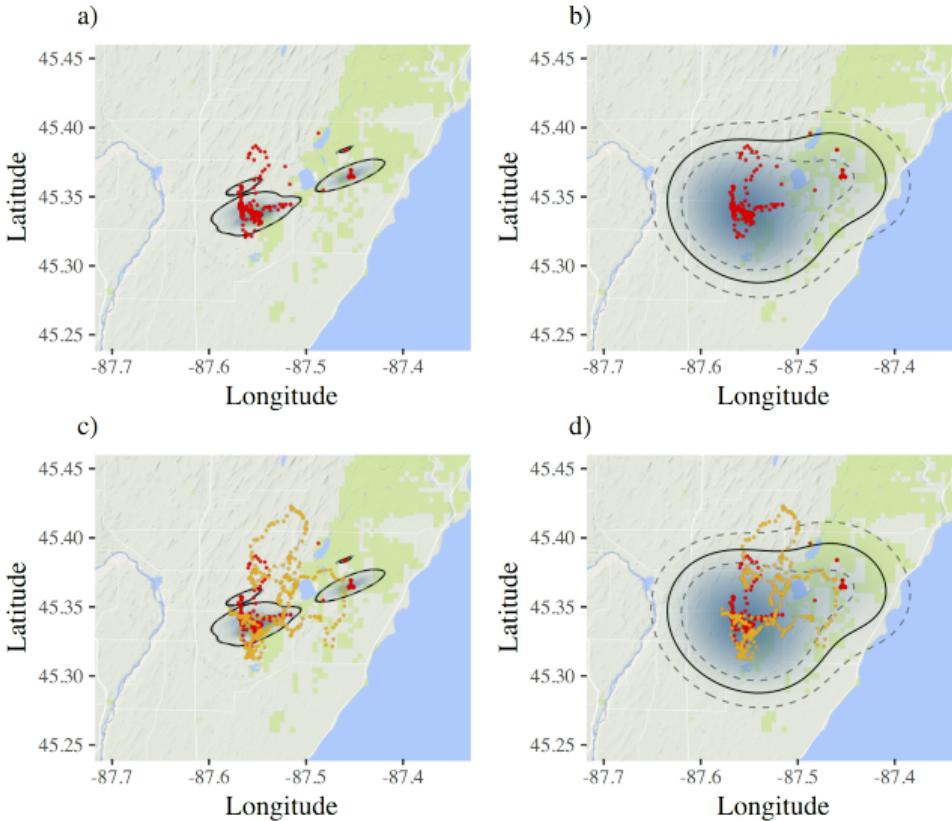
The underestimation of animal space use, the solution



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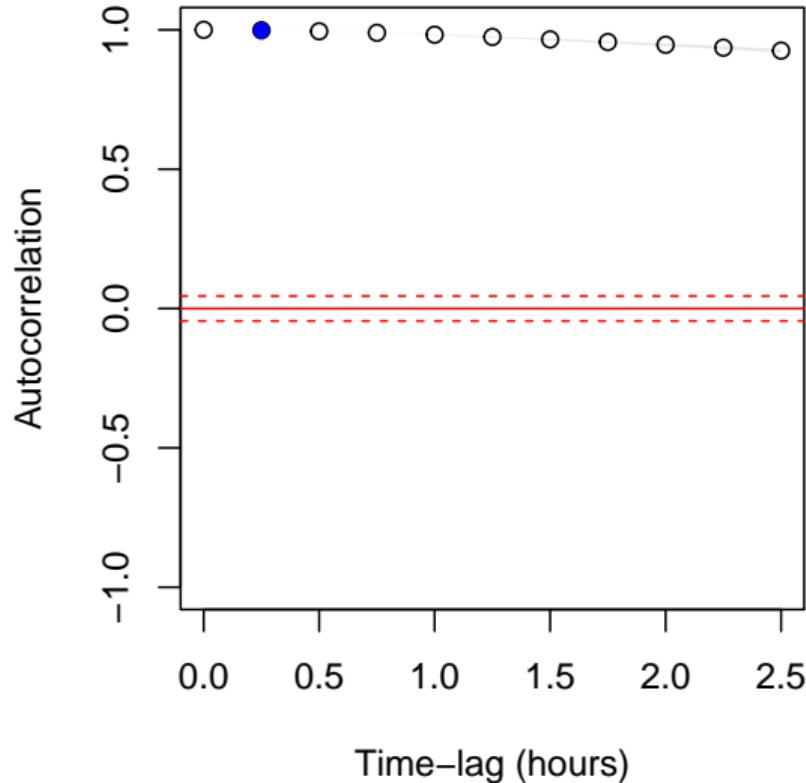
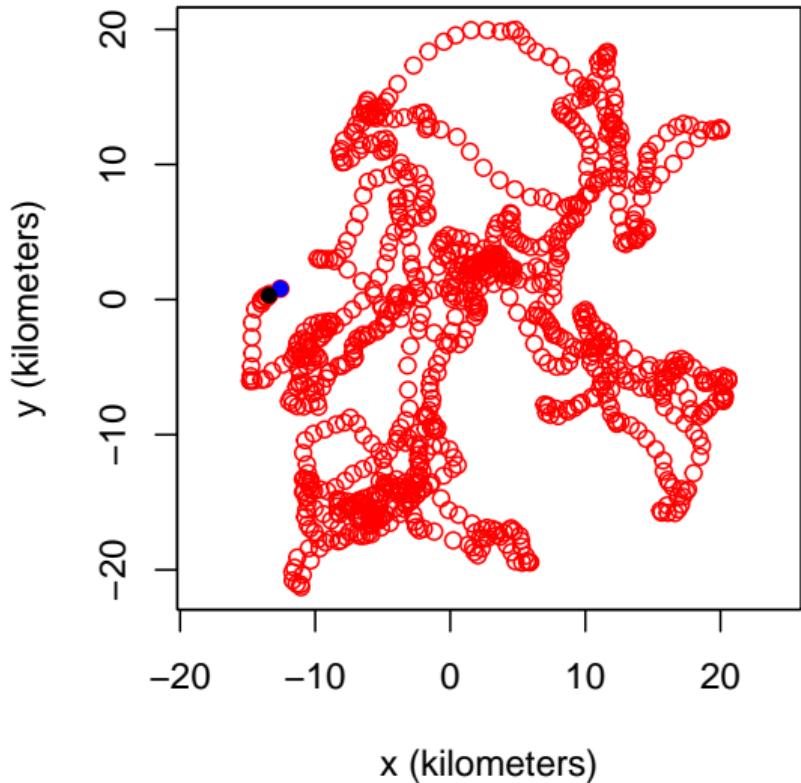
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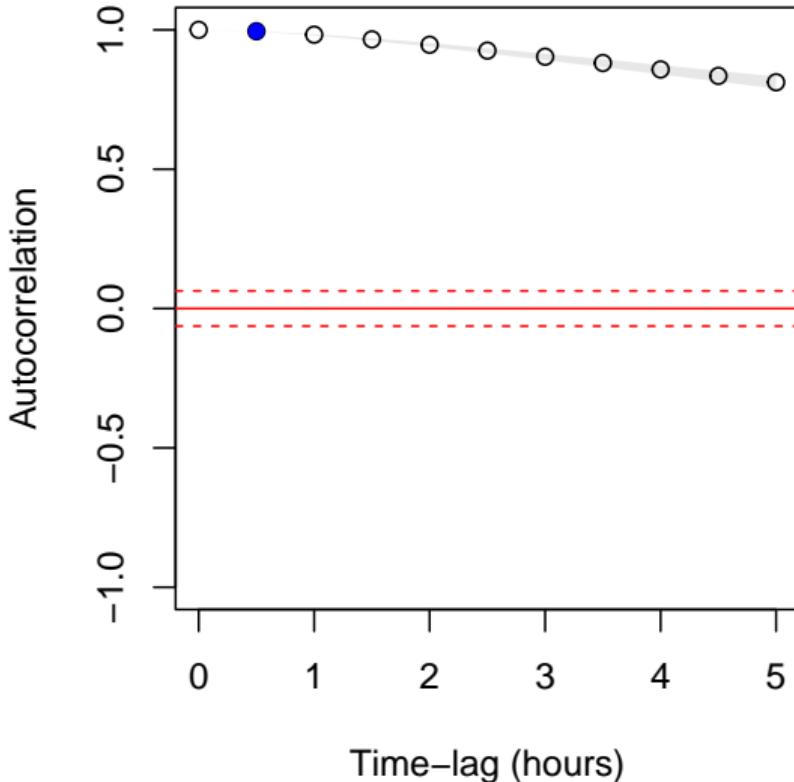
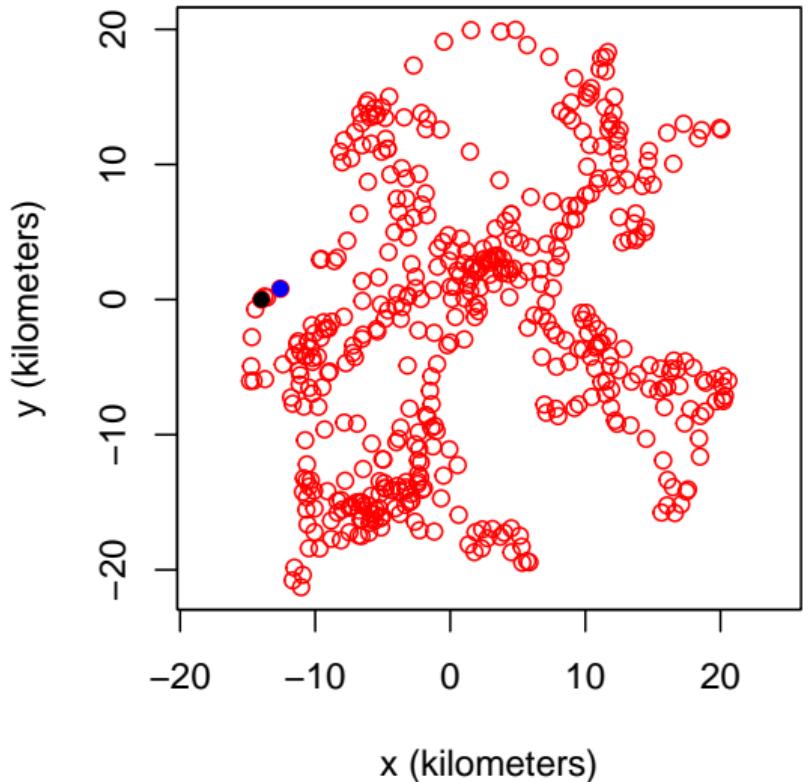
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- A3: Continuous-time stochastic process models of the *autocorrelation*

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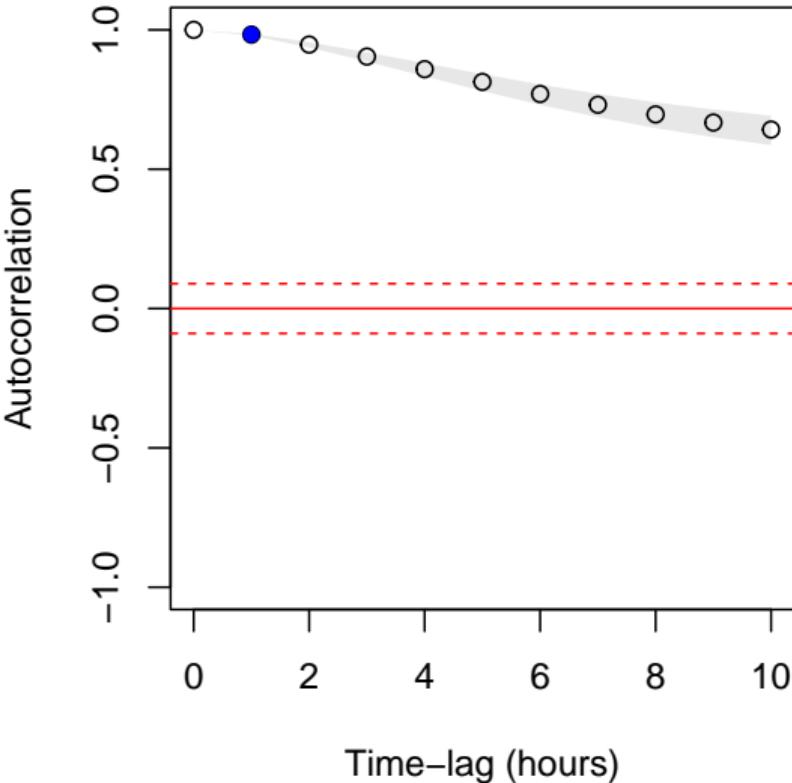
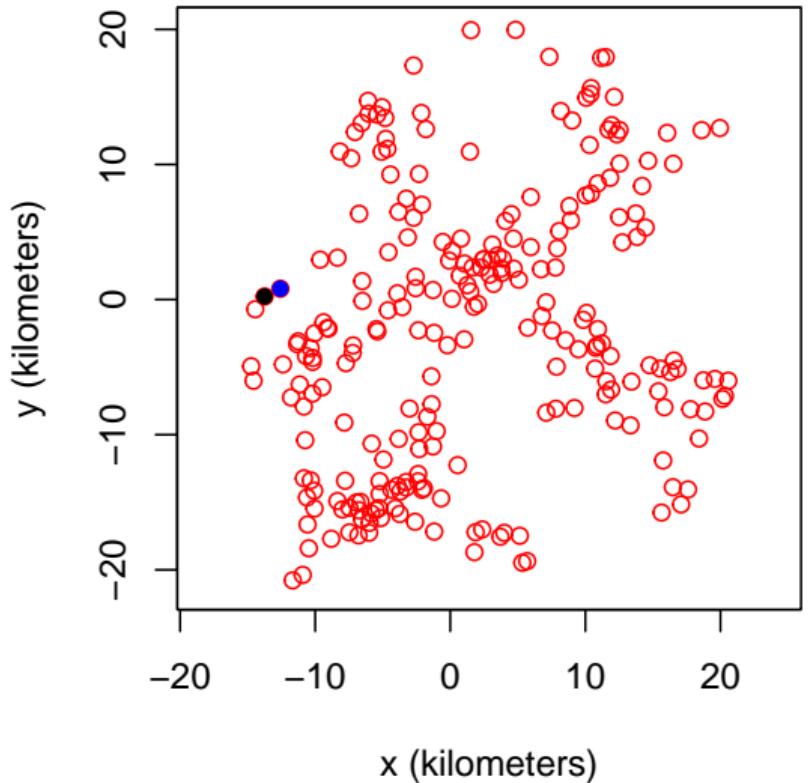
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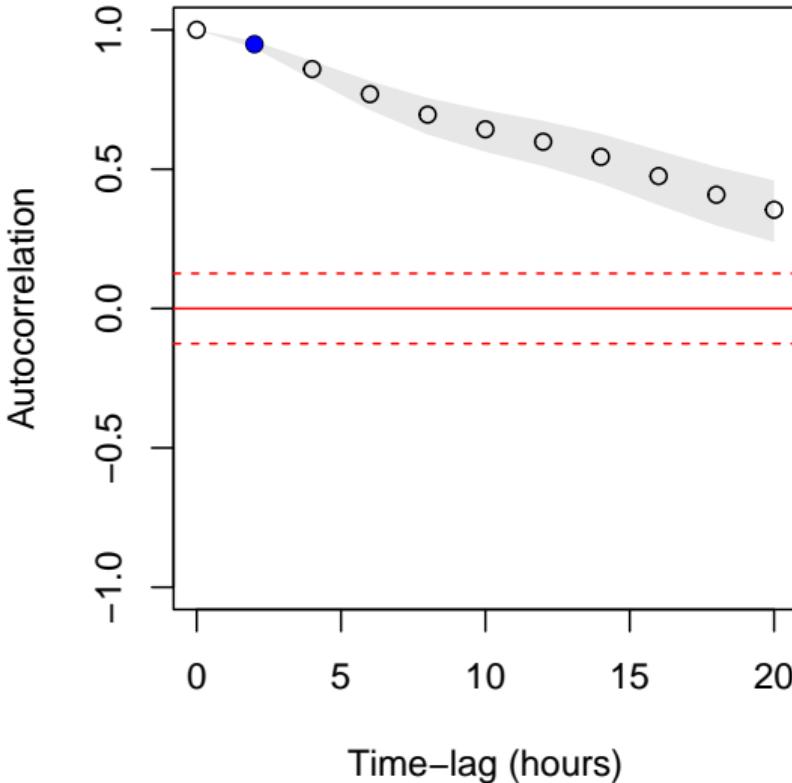
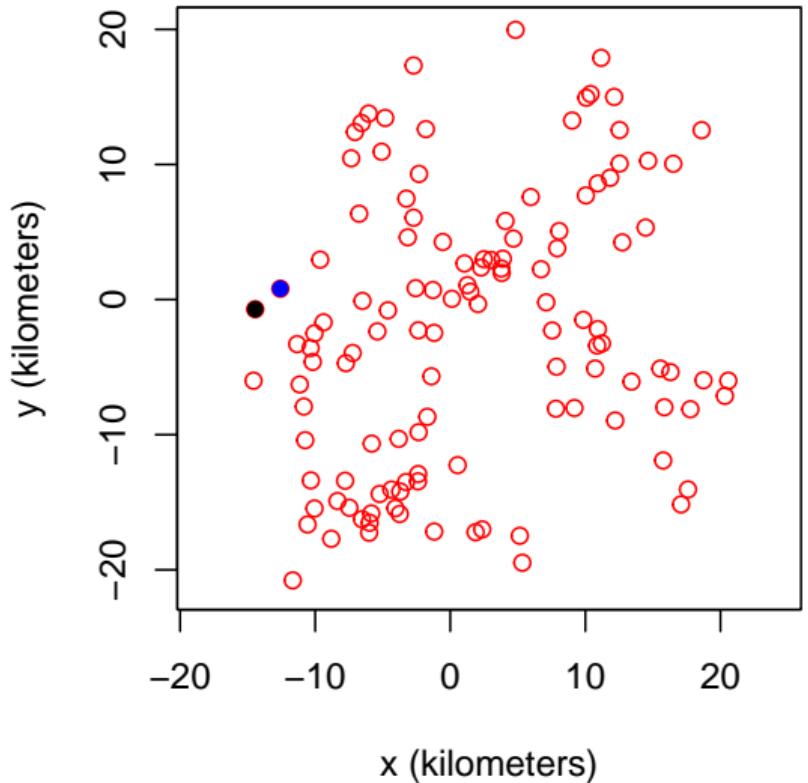
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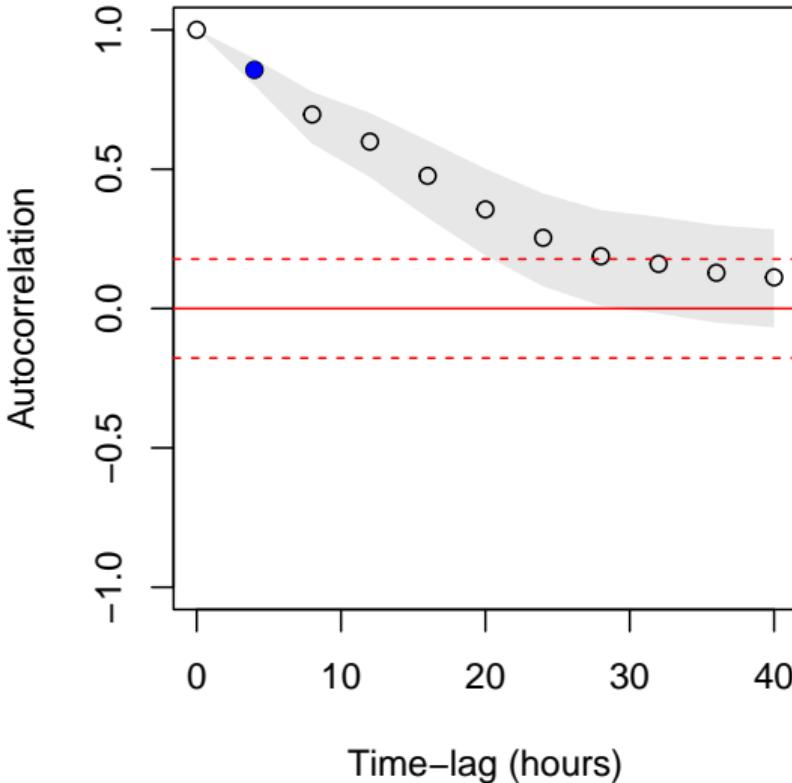
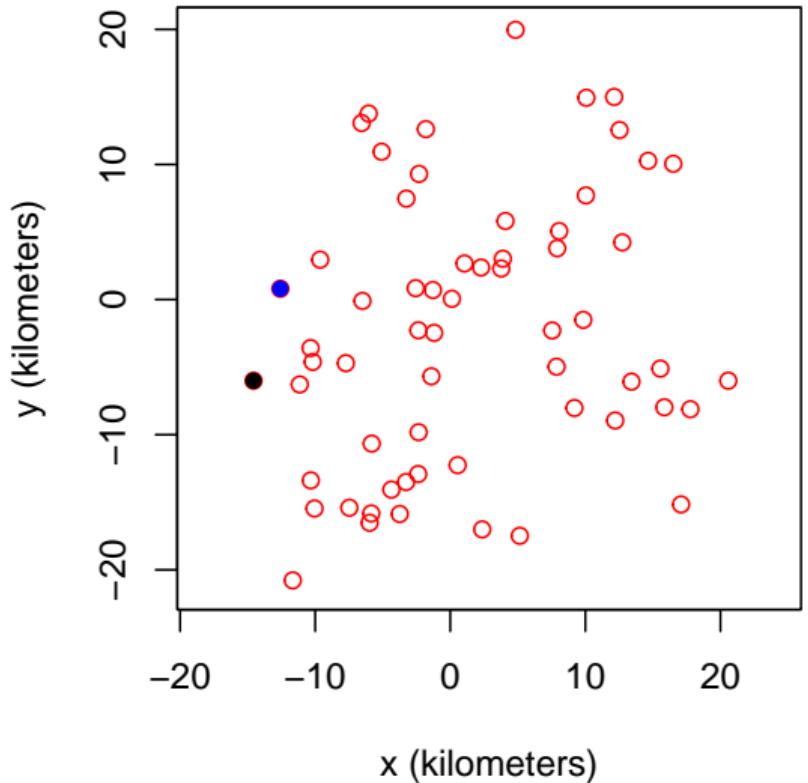
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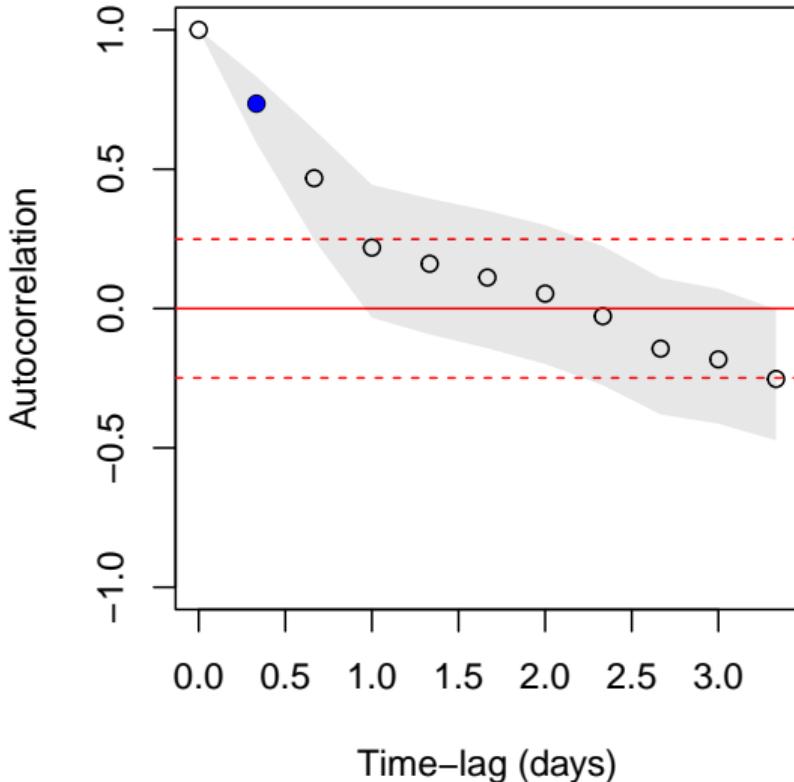
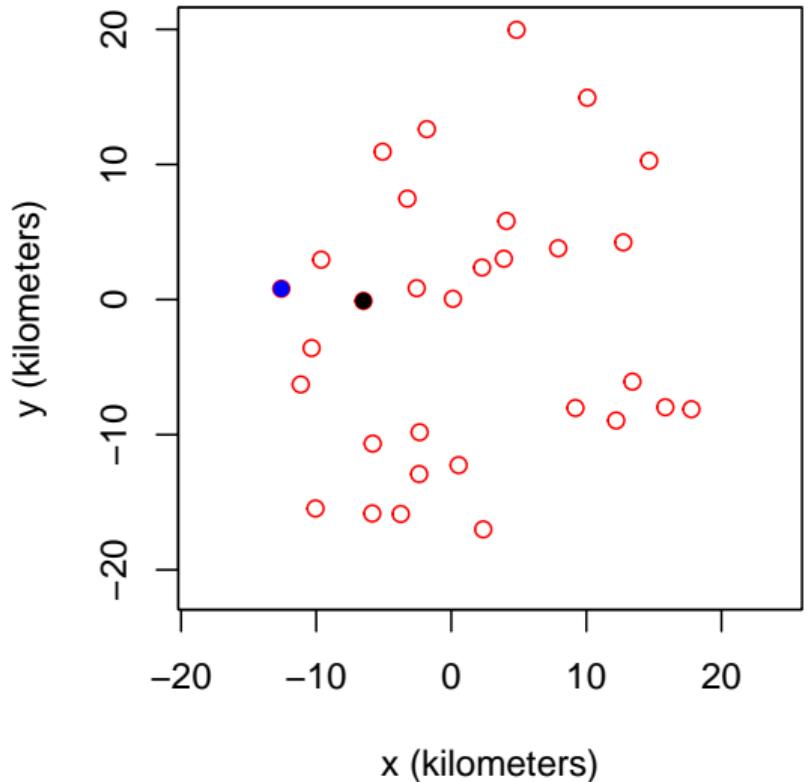
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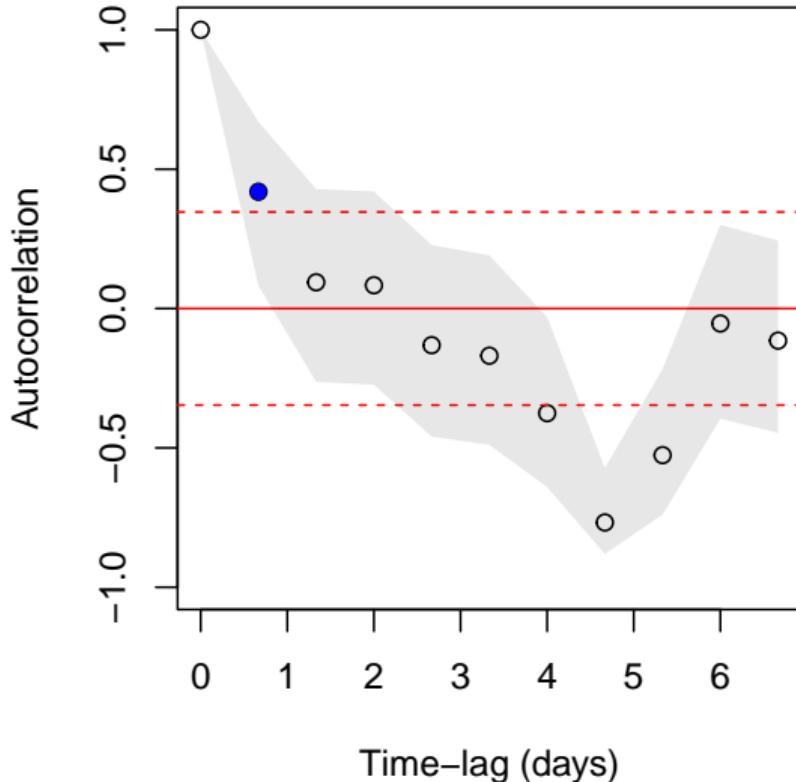
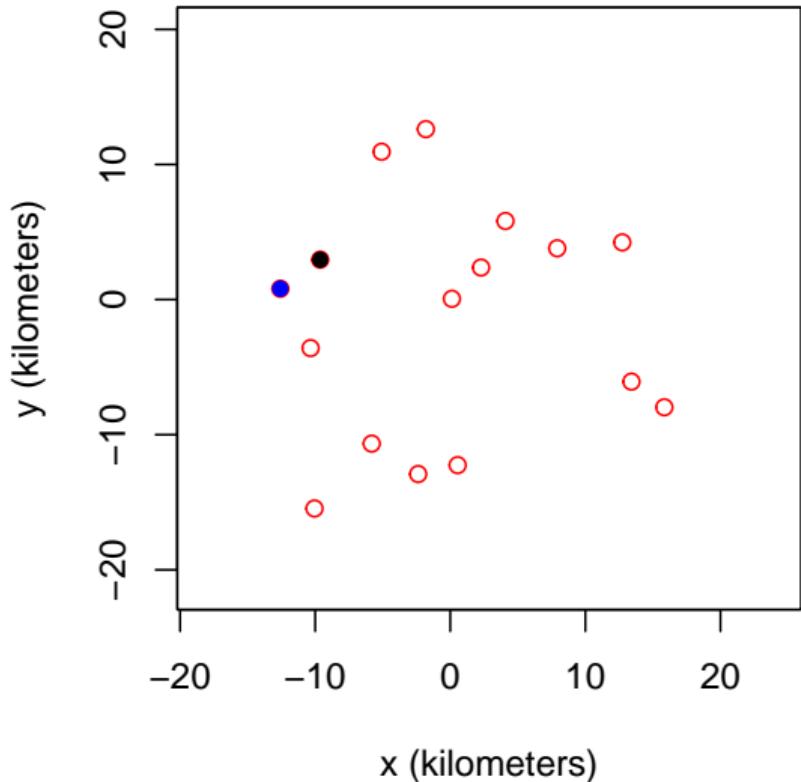
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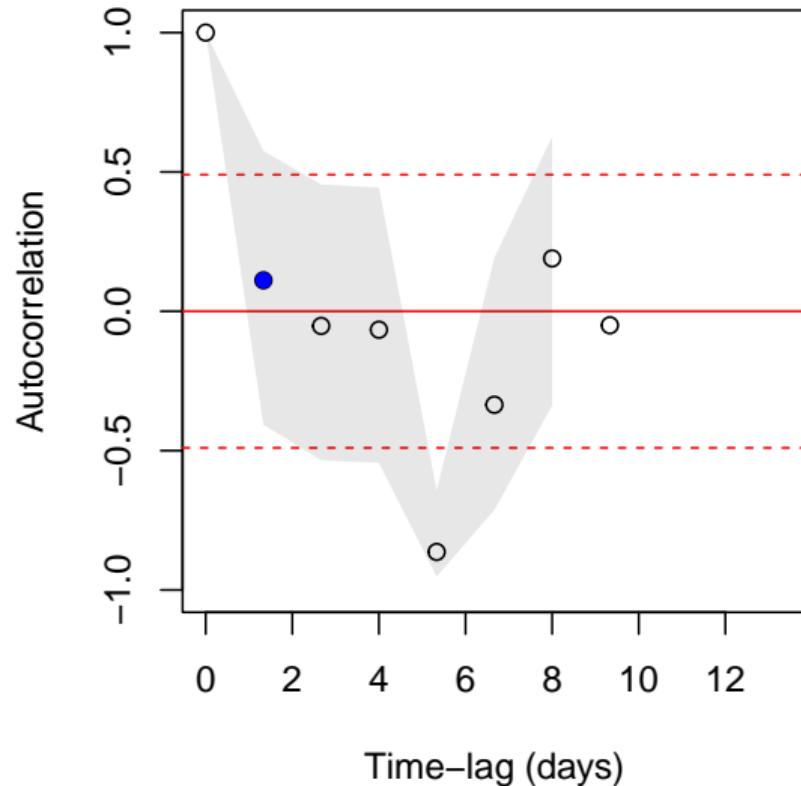
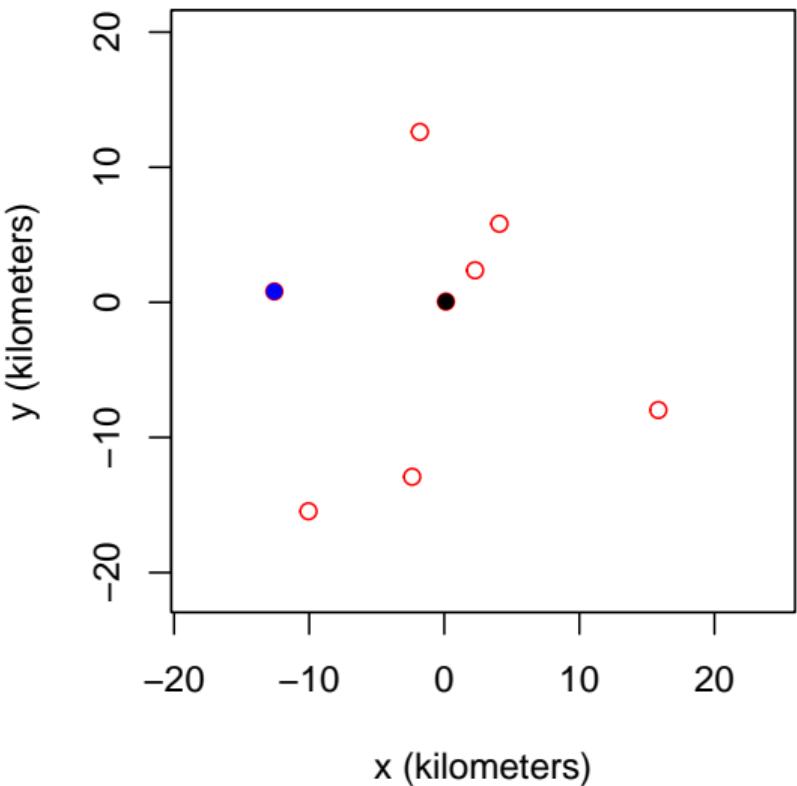
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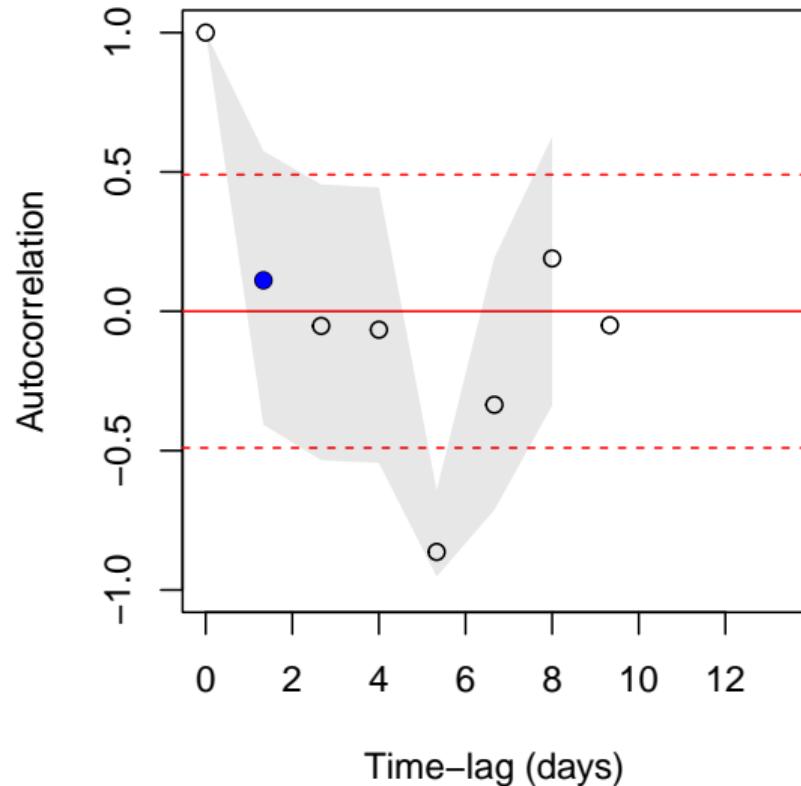
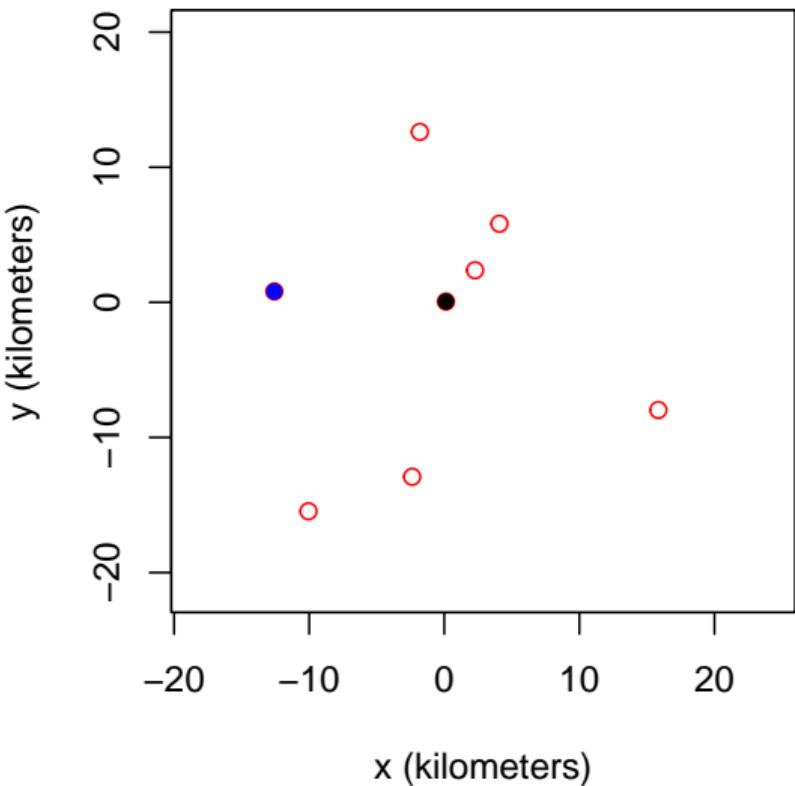
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...you might have to thin a lot to reach IID

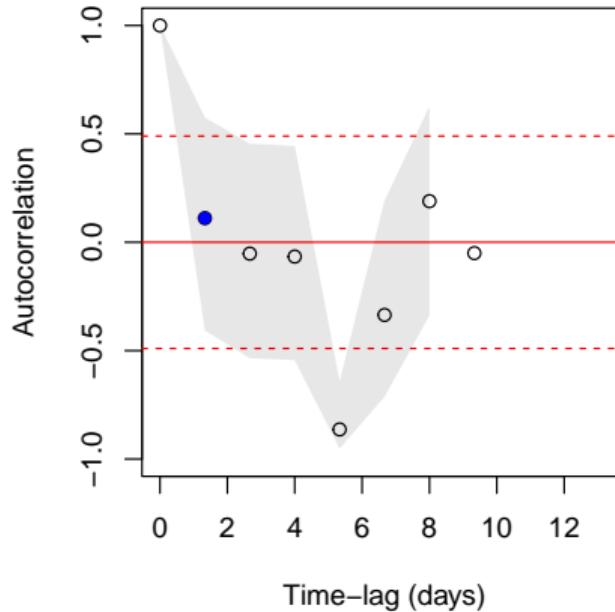
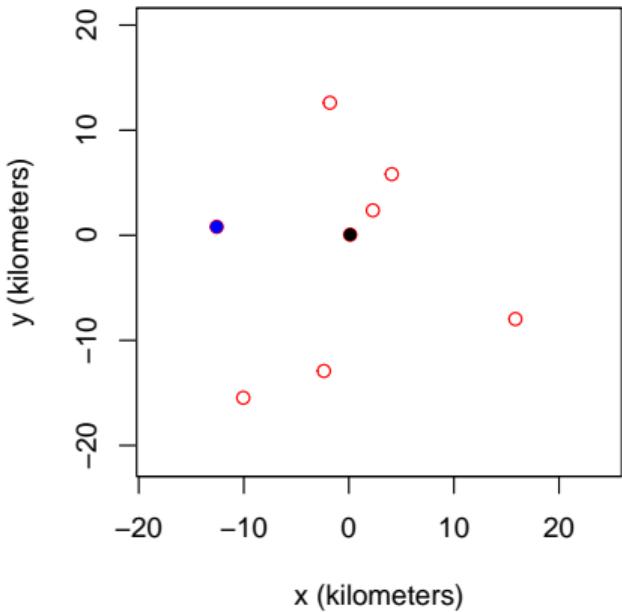


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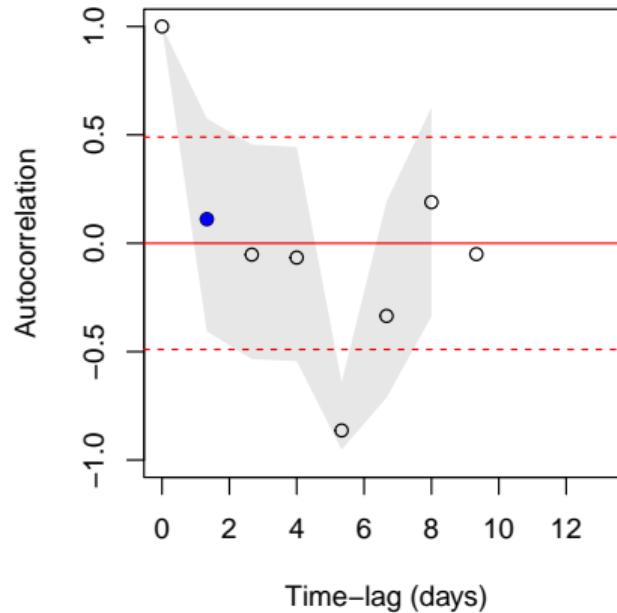
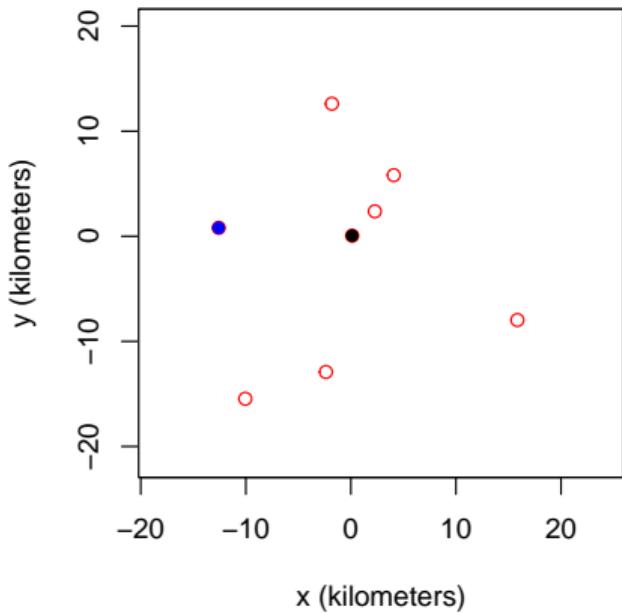
What about speed estimation with these data?

Autocorrelation is information



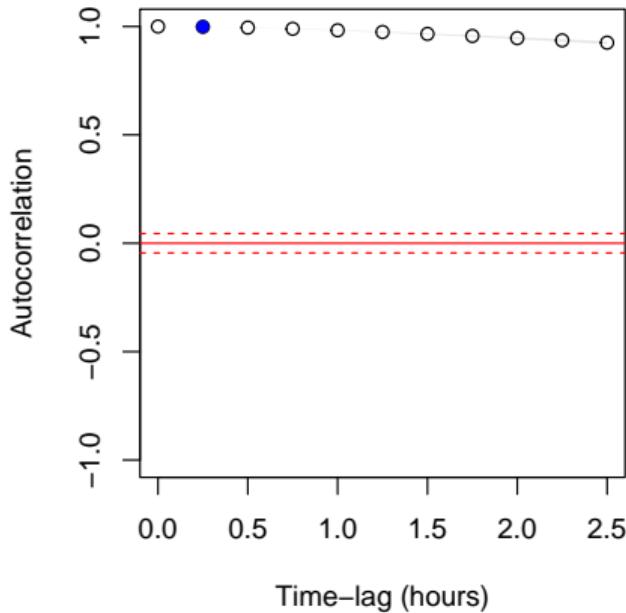
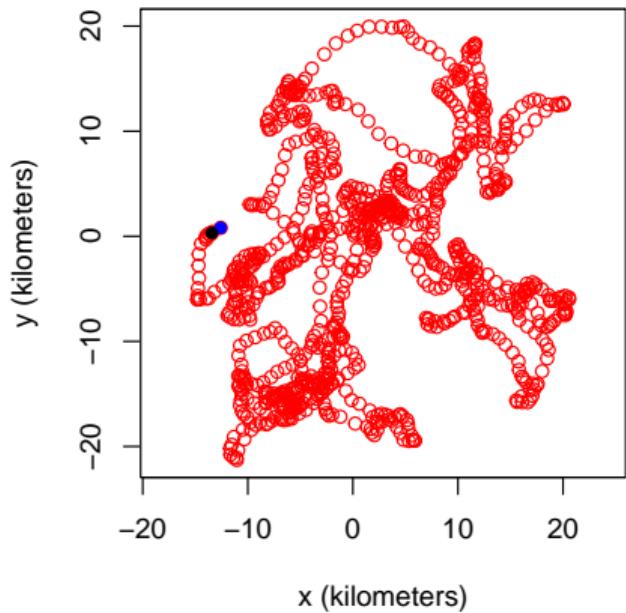
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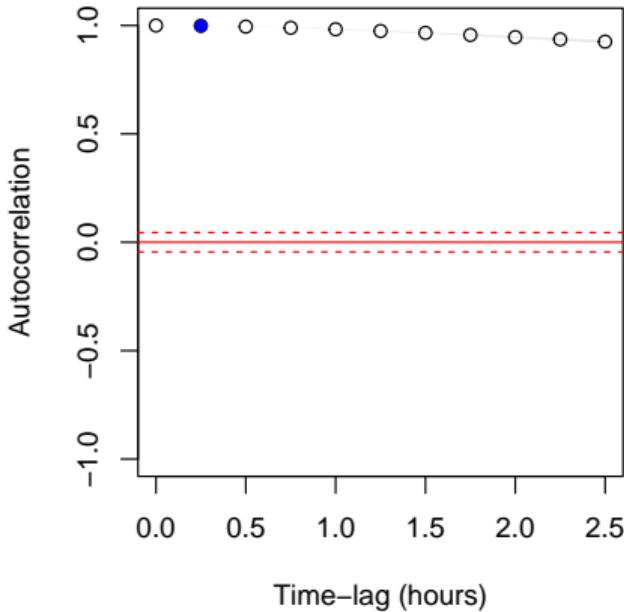
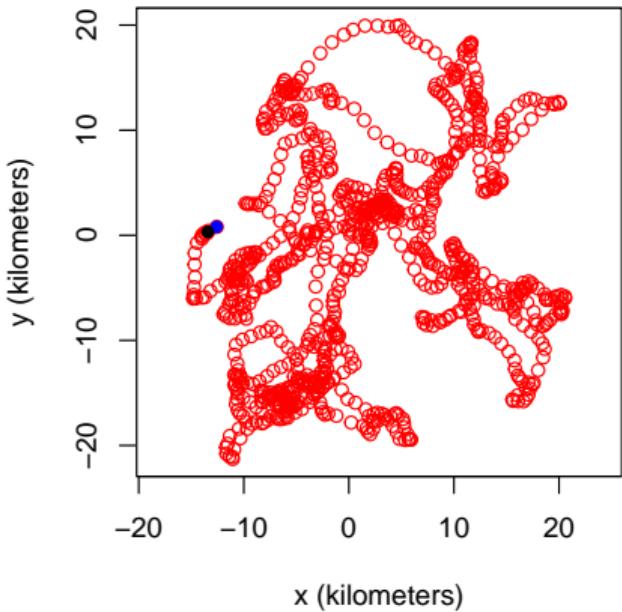
- Convenient for home-range analysis
- Worthless for speed/distance estimation

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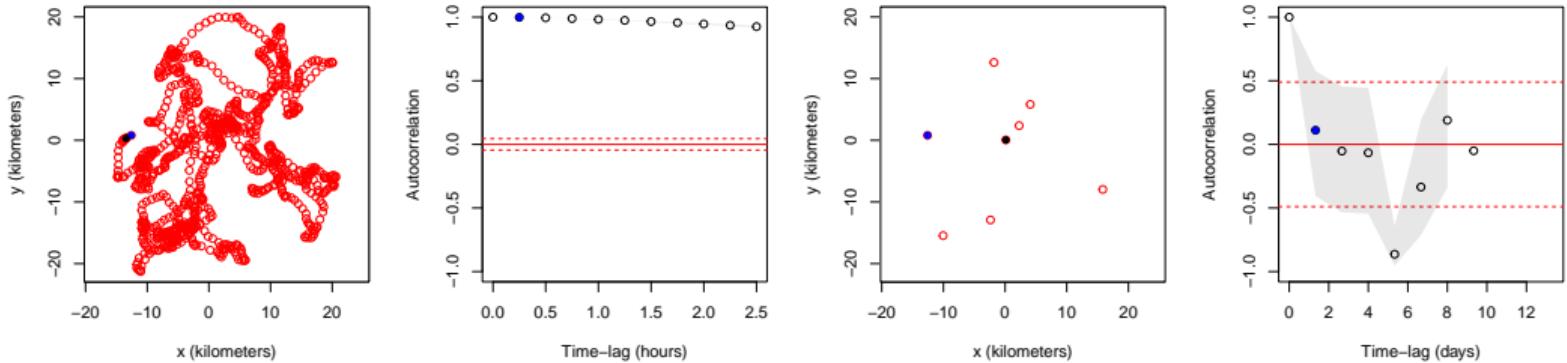
- Inconvenient for (conventional) home-range analysis

Autocorrelation is information



- Inconvenient for (conventional) home-range analysis
- Great for speed/distance estimation

Objective



We want methods that can handle whatever autocorrelation is present in the data

Why continuous time?

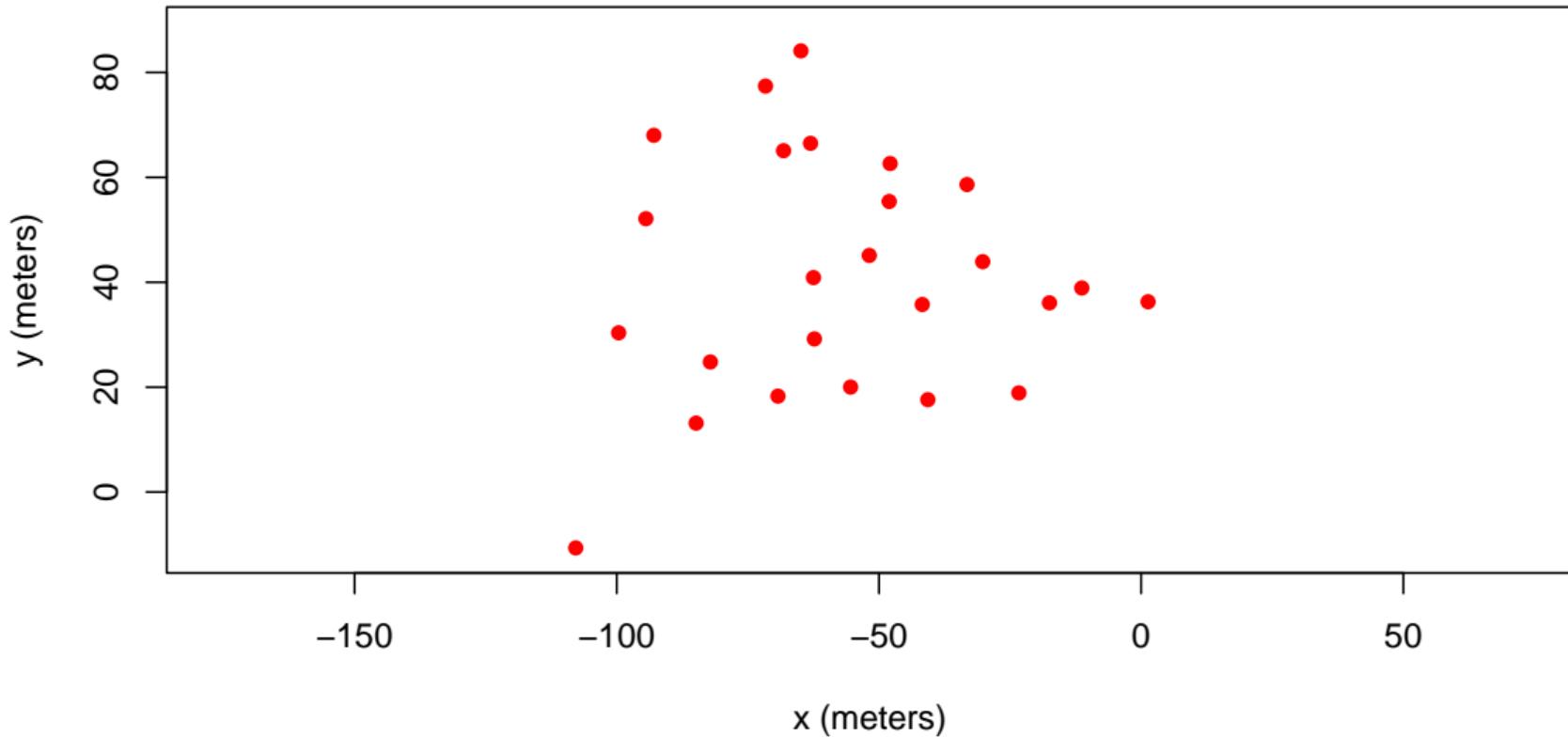
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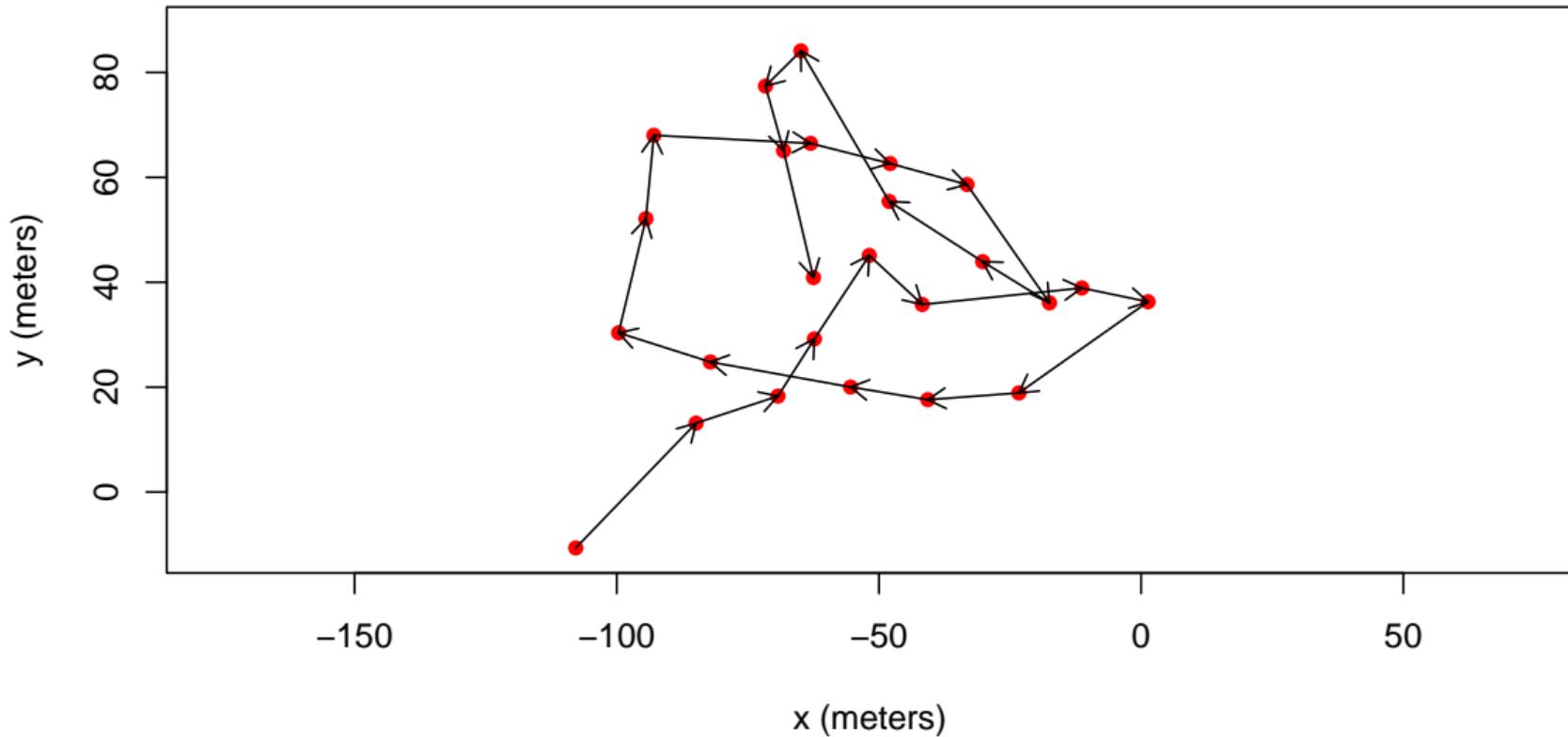
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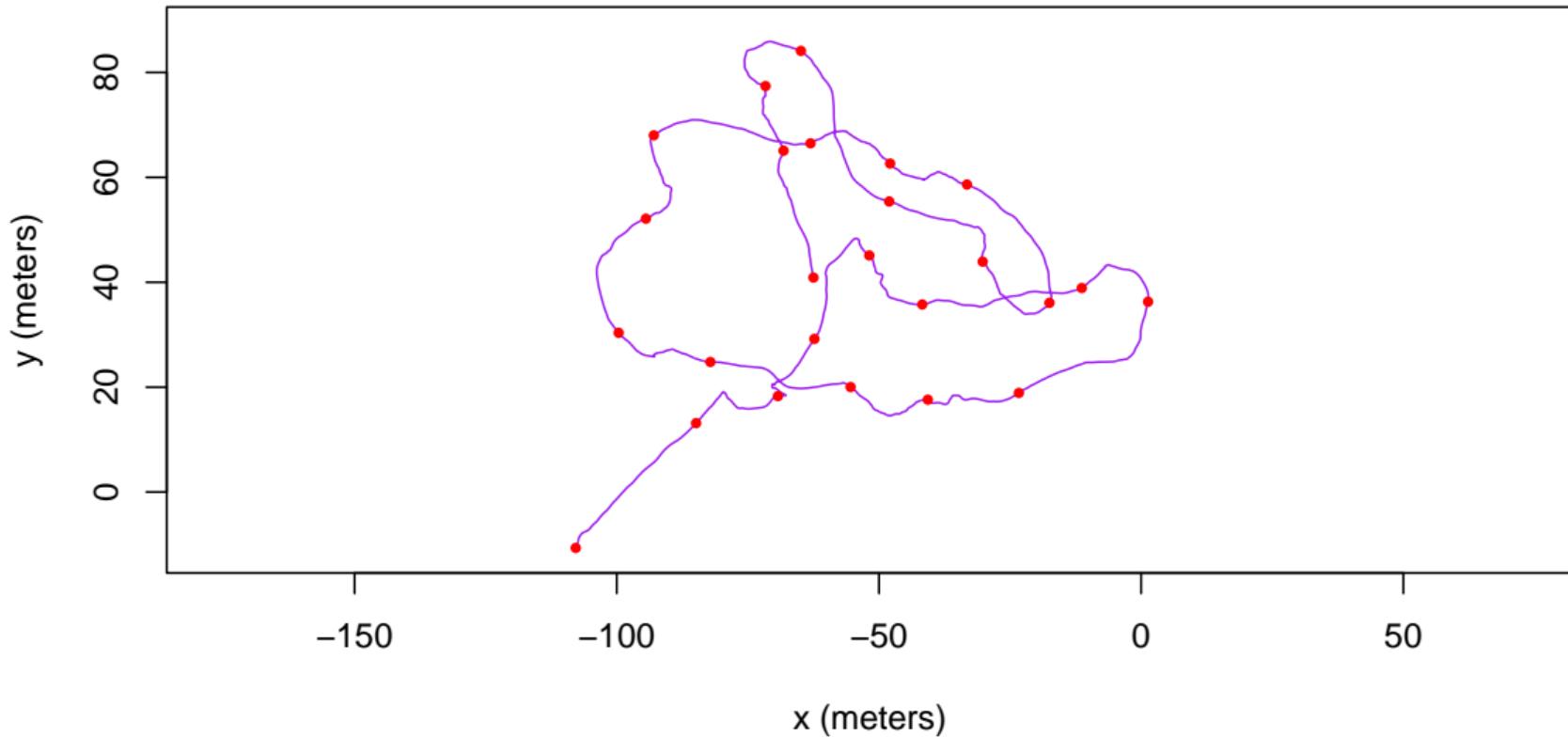
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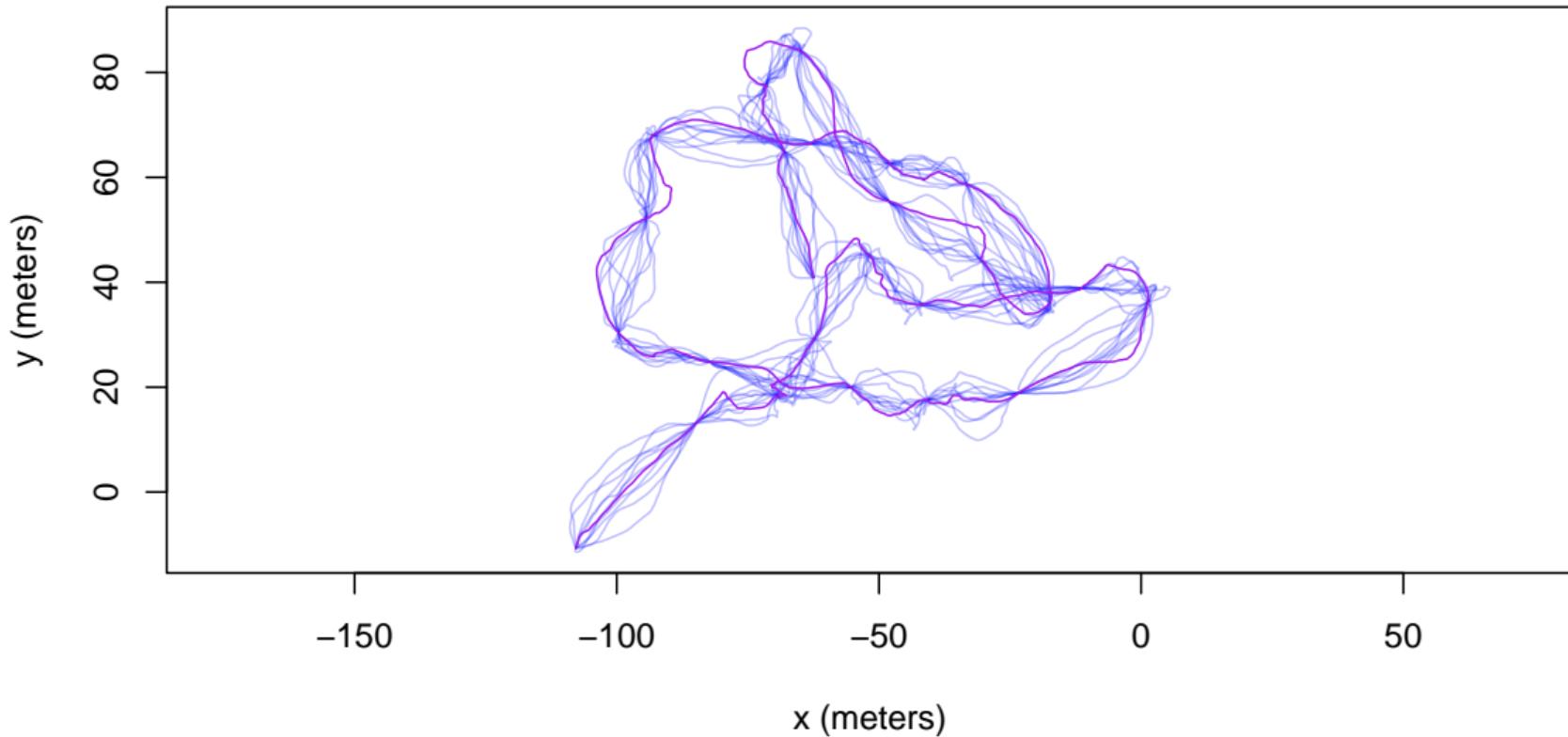
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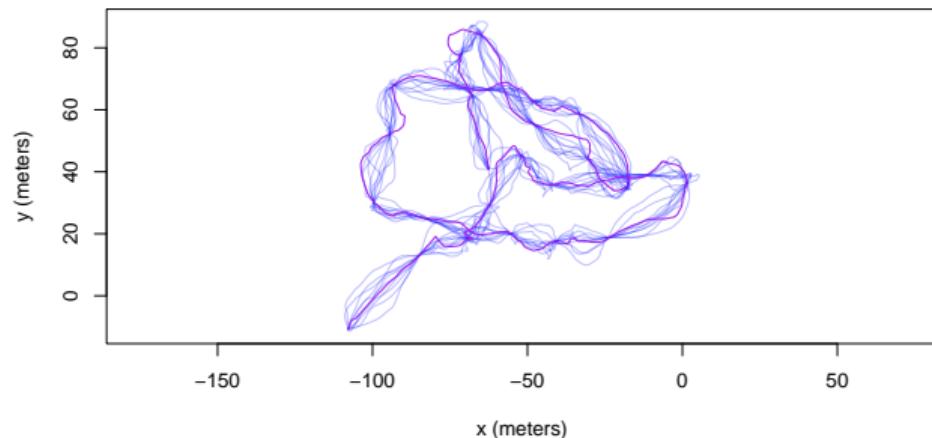
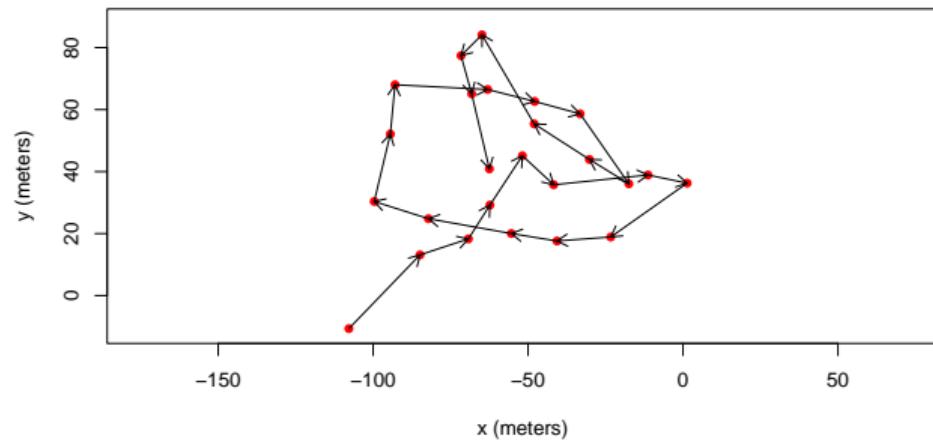
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 - Location error is easy to model (versus CRWs)

Motivating example: Neglecting autocorrelation in speed estimation



Building-block continuous-time stochastic process models

Building-block continuous-time stochastic process models

- Independent locations

Building-block continuous-time stochastic process models

- Independent locations (KDE,MCP,RSF,...)

Building-block continuous-time stochastic process models

- Independent locations (KDE,MCP,RSF,...)
- Brownian motion

Building-block continuous-time stochastic process models

- Independent locations (KDE,MCP,RSF,...)
- Brownian motion (Brownian bridge)

Building-block continuous-time stochastic process models

- Independent locations (KDE,MCP,RSF,...)
- Brownian motion (Brownian bridge)
- Ornstein-Uhlenbeck motion

Building-block continuous-time stochastic process models

- Independent locations (KDE,MCP,RSF,...)
- Brownian motion (Brownian bridge)
- Ornstein-Uhlenbeck motion
- Integrated Ornstein-Uhlenbeck motion

Building-block continuous-time stochastic process models

- Independent locations (KDE,MCP,RSF,...)
- Brownian motion (Brownian bridge)
- Ornstein-Uhlenbeck motion
- Integrated Ornstein-Uhlenbeck motion (crawl)

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- OUF motion

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Building-block continuous-time stochastic process models

- Independent locations ($<1\%$)
- Brownian motion
- Ornstein-Uhlenbeck motion
- Integrated Ornstein-Uhlenbeck motion
- OUF motion

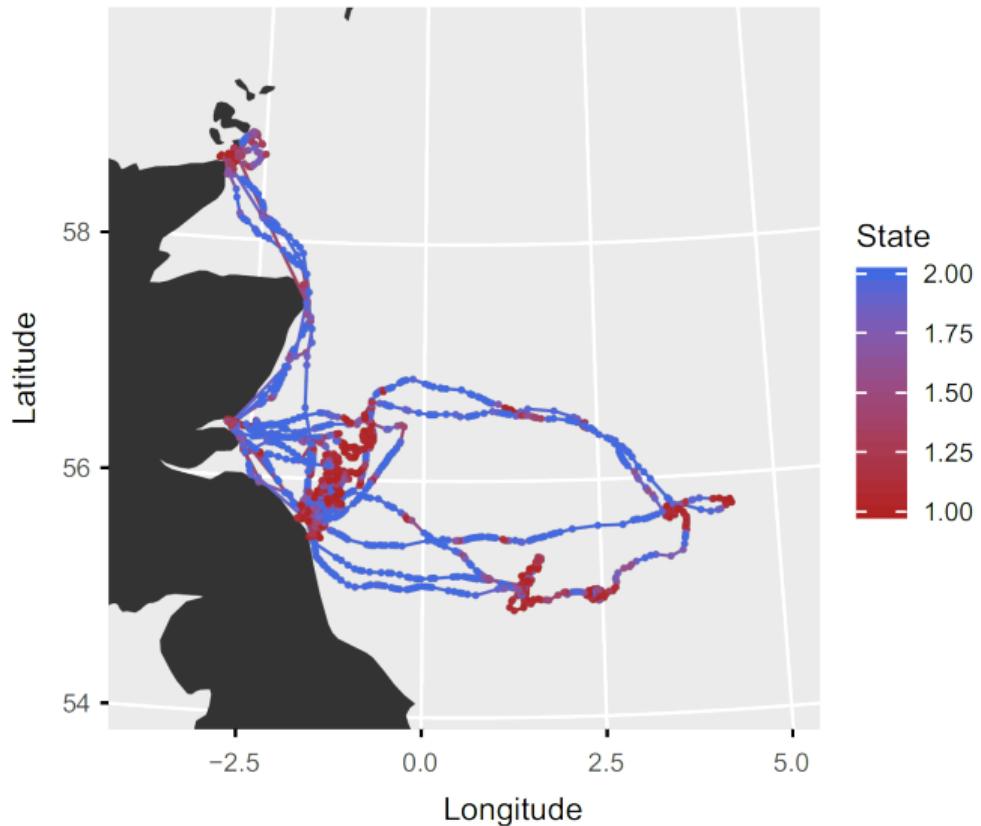
Building-block continuous-time stochastic process models

- Independent locations ($<1\%$)
- Brownian motion
- Ornstein-Uhlenbeck motion ($<30\%$)
- Integrated Ornstein-Uhlenbeck motion
- OUF motion

Building-block continuous-time stochastic process models

- Independent locations ($<1\%$)
- Brownian motion
- Ornstein-Uhlenbeck motion ($<30\%$)
- Integrated Ornstein-Uhlenbeck motion
- OUF motion ($\sim 70\%$)

Coming-soon: Continuous-time behavioral switching models



Michelot & Blackwell (2019)

Building-block continuous-time stochastic process models

- Independent locations ($<1\%$)
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Don't assume a model, select a model

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Don't assume a model, select a model

Let's dive into R `ctmmlearn "ctmm_intro.R"`