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1.1 default code

```
#pragma GCC optimize("O3,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
#include <bits/stdc++.h>
using namespace std;
ios_base::sync_with_stdio(0); cin.tie(0); cout.tie(0);
```

```
set nu rnu ts=4 sw=4 bs=2 ai hls cin mouse=a
color default
sy on
inoremap {<CR> {<CR>}<C-o>0
inoremap jk <Esc>
nnoremap J 5j
nnoremap K 5k
nnoremap run :w<bar>!g++ -std=c++14 -DLOCAL -Wfatal-
errors -o test "%" && echo "done." && time ./test<
CR>
```

1.3 Increase Stack Size (linux)

```
#include <sys/resource.h>
void increase_stack_size() {
    const rlim_t ks = 64*1024*1024;
    struct rlimit rl;
    int res=getrlimit(RLIMIT_STACK, &rl);
    if(res==0){
        if(rl.rlim_cur<ks){
            rl.rlim_cur=ks;
            res=setrlimit(RLIMIT_STACK, &rl);
        }
    }
}
```

1.4 Misc

```

編譯參數: -std=c++11 -Wall -Wshadow (-fsanitize=
undefined)

mt19937 gen(chrono::steady_clock::now().
    time_since_epoch().count());
int randint(int lb, int ub)
{ return uniform_int_distribution<int>(lb, ub)(gen); }

#define SECs ((double)clock() / CLOCKS_PER_SEC)
double startTime;
bool TIME() { // 比最大可執行時間小一點
    return SECs - startTime > 0.8;
}

int main() {
    startTime = SECs;
}

struct KeyHasher {
    size_t operator()(const Key& k) const {
        return k.first + k.second * 100000;
    } };
typedef unordered_map<Key, int, KeyHasher> map_t;

__builtin_popcountll // 二進位有幾個1
__builtin_clzll // 左起第一個1之前的個數
__builtin_parityll // 1的個數的奇偶性
__builtin_mul_overflow(a,b,&h) // a*b是否溢位

```

1.5 check

```
for ((i=0;;i++))
do
    echo "$i"
    python3 gen.py > input
    ./ac < input > ac.out
    ./wa < input > wa.out
    diff ac.out wa.out || break
done
```

2 flow

2.1 ISAP $O(V^3)$

```
struct Maxflow {
    static const int MAXV = 20010;
    static const int INF = 1000000;
    struct Edge {
        int v, c, r;
        Edge(int _v, int _c, int _r):
            v(_v), c(_c), r(_r) {}
    };
    int s, t;
    vector<Edge> G[MAXV*2];
    int iter[MAXV*2], d[MAXV*2], gap[MAXV*2], tot;
    void init(int x) {
        tot = x+2;
        s = x+1, t = x+2;
        for(int i = 0; i <= tot; i++) {
            G[i].clear();
            iter[i] = d[i] = gap[i] = 0;
        }
    }
    void addEdge(int u, int v, int c) {
        G[u].push_back(Edge(v, c, SZ(G[v])));
        G[v].push_back(Edge(u, 0, SZ(G[u]) - 1));
    }
    int dfs(int p, int flow) {
        if(p == t) return flow;
        for(int &i = iter[p]; i < SZ(G[p]); i++) {
            Edge &e = G[p][i];
            if(e.c > 0 && d[p] == d[e.v]+1) {
                int f = dfs(e.v, min(flow, e.c));
                if(f) {
                    e.c -= f;
                    G[e.v][e.r].c += f;
                    return f;
                }
            }
        }
        if(--gap[d[p]] == 0) d[s] = tot;
        else {
            d[p]++;
            iter[p] = 0;
            ++gap[d[p]];
        }
        return 0;
    }
    int solve() {
        int res = 0;
        gap[0] = tot;
        for(res = 0; d[s] < tot; res += dfs(s, INF));
        return res;
    }
    void reset() {
        for(int i=0;i<=tot;i++) {
            iter[i]=d[i]=gap[i]=0;
        }
    }
} fflow;
```

2.2 MinCostFlow

```
struct zkwflow{
    static const int maxN=10000;
    struct Edge{ int v,f,re; ll w;};
    int n,s,t,ptr[maxN]; bool vis[maxN]; ll dis[maxN];
    vector<Edge> E[maxN];
    void init(int _n,int _s,int _t){
        n=_n,s=_s,t=_t;
```

```
for(int i=0;i<n;i++) E[i].clear();
}
void addEdge(int u,int v,int f,ll w){
    E[u].push_back({v,f,(int)E[v].size(),w});
    E[v].push_back({u,0,(int)E[u].size()-1,-w});
}
bool SPFA(){
    fill_n(dis,n,LLONG_MAX); fill_n(vis,n,false);
    queue<int> q; q.push(s); dis[s]=0;
    while (!q.empty()){
        int u=q.front(); q.pop(); vis[u]=false;
        for(auto &it:E[u]){
            if(it.f>0&&dis[it.v]>dis[u]+it.w){
                dis[it.v]=dis[u]+it.w;
                if(!vis[it.v]){
                    vis[it.v]=true; q.push(it.v);
                }
            }
        }
    }
    return dis[t]!=LLONG_MAX;
}
int DFS(int u,int nf){
    if(u==t) return nf;
    int res=0; vis[u]=true;
    for(int &i=ptr[u];i<(int)E[u].size();i++){
        auto &it=E[u][i];
        if(it.f>0&&dis[it.v]==dis[u]+it.w&&!vis[it.v]){
            int tf=DFS(it.v,min(nf,it.f));
            res+=tf,nf-=tf,it.f-=tf;
            E[it.v][it.re].f+=tf;
            if(nf==0){ vis[u]=false; break; }
        }
    }
    return res;
}
pair<int,ll> flow(){
    int flow=0; ll cost=0;
    while (SPFA()){
        fill_n(ptr,n,0);
        int f=DFS(s,INT_MAX);
        flow+=f; cost+=dis[t]*f;
    }
    return{ flow,cost };
} // reset: do nothing
} flow;
```

2.3 Dinic $O(V^2E)$

```
#define SZ(x) (int)x.size()
#define PB push_back
struct Dinic{
    struct Edge{ int v,f,re; };
    int n,s,t,level[MXN];
    vector<Edge> E[MXN];
    void init(int _n, int _s, int _t){
        n = _n; s = _s; t = _t;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void add_edge(int u, int v, int f){
        E[u].PB({v,f,SZ(E[v])});
        E[v].PB({u,0,SZ(E[u])-1});
    }
    bool BFS(){
        for (int i=0; i<n; i++) level[i] = -1;
        queue<int> que;
        que.push(s);
        level[s] = 0;
        while (!que.empty()){
            int u = que.front(); que.pop();
            for (auto it : E[u]){
                if (it.f > 0 && level[it.v] == -1){
                    level[it.v] = level[u]+1;
                    que.push(it.v);
                }
            }
        }
        return level[t] != -1;
    }
    int DFS(int u, int nf){
        if (u == t) return nf;
        int res = 0;
        for (auto &it : E[u]){
            if (it.f > 0 && level[it.v] == level[u]+1){
                int tf = DFS(it.v, min(nf,it.f));
```

```

    res += tf; nf -= tf; it.f -= tf;
    E[it.v][it.re].f += tf;
    if (nf == 0) return res;
  } }
  if (!res) level[u] = -1;
  return res;
}
int flow(int res=0){
  while ( BFS() )
    res += DFS(s,2147483647);
  return res;
} }flow;

```

2.4 Kuhn Munkres 最大完美二分匹配 $O(n^3)$

```

struct KM{ // max weight, for min negate the weights
  int n, mx[MXN], my[MXN], pa[MXN];
  ll g[MXN][MXN], lx[MXN], ly[MXN], sy[MXN];
  bool vx[MXN], vy[MXN];
  void init(int _n) { // 1-based
    n = _n;
    for(int i=1; i<=n; i++) fill(g[i], g[i]+n+1, 0);
  }
  void addEdge(int x, int y, ll w) {g[x][y] = w;}
  void augment(int y) {
    for(int x, z; y; y = z)
      x=pa[y], z=mx[x], my[y]=x, mx[x]=y;
  }
  void bfs(int st) {
    for(int i=1; i<=n; ++i) sy[i]=INF, vx[i]=vy[i]=0;
    queue<int> q; q.push(st);
    for(;;) {
      while(q.size()) {
        int x=q.front(); q.pop(); vx[x]=1;
        for(int y=1; y<=n; ++y) if(!vy[y]){
          ll t = lx[x]+ly[y]-g[x][y];
          if(t==0){
            pa[y]=x;
            if(!my[y]){augment(y);return;}
            vy[y]=1, q.push(my[y]);
          }else if(sy[y]>t) pa[y]=x, sy[y]=t;
        }
      }
      ll cut = INF;
      for(int y=1; y<=n; ++y)
        if(!vy[y]&&cut>sy[y]) cut=sy[y];
      for(int j=1; j<=n; ++j){
        if(vx[j]) lx[j] -= cut;
        if(vy[j]) ly[j] += cut;
        else sy[j] -= cut;
      }
      for(int y=1; y<=n; ++y) if(!vy[y]&&sy[y]==0){
        if(!my[y]){augment(y);return;}
        vy[y]=1, q.push(my[y]);
      }
    }
  }
  ll solve(){
    fill(mx, mx+n+1, 0); fill(my, my+n+1, 0);
    fill(ly, ly+n+1, 0); fill(lx, lx+n+1, -INF);
    for(int x=1; x<=n; ++x) for(int y=1; y<=n; ++y)
      lx[x] = max(lx[x], g[x][y]);
    for(int x=1; x<=n; ++x) bfs(x);
    ll ans = 0;
    for(int y=1; y<=n; ++y) ans += g[my[y]][y];
    return ans;
  } }graph;

```

2.5 SW min-cut (不限 S-T 的 min-cut) $O(V^3)$

```

// global min cut
struct SW{ //  $O(V^3)$ 
  int n, vst[MXN], del[MXN];
  int edge[MXN][MXN], wei[MXN];
  void init(int _n){
    n = _n; FZ(edge); FZ(del);
  }
  void addEdge(int u, int v, int w){
    edge[u][v] += w; edge[v][u] += w;
  }
  void search(int &s, int &t){

```

```

    FZ(vst); FZ(wei);
    s = t = -1;
    while (true){
      int mx=-1, cur=0;
      for (int i=0; i<n; i++)
        if (!del[i] && !vst[i] && mx<wei[i])
          cur = i, mx = wei[i];
      if (mx == -1) break;
      vst[cur] = 1;
      s = t; t = cur;
      for (int i=0; i<n; i++)
        if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
    }
  }
  int solve(){
    int res = 2147483647;
    for (int i=0,x,y; i<n-1; i++){
      search(x,y);
      res = min(res,wei[y]);
      del[y] = 1;
      for (int j=0; j<n; j++)
        edge[x][j] = (edge[j][x] += edge[y][j]);
    }
    return res;
  } }graph;

```

2.6 Max flow with lower/upper bound

```

// flow use ISAP
// Max flow with lower/upper bound on edges
// source = 1, sink = n
int in[ N ], out[ N ];
int l[ M ], r[ M ], a[ M ], b[ M ]; // 0-base, a下界, b上界
int solve(){
  flow.init( n ); // n為點的數量, m為邊的數量, 點是1-base
  for( int i = 0 ; i < m ; i ++ ){
    in[ r[ i ] ] += a[ i ];
    out[ l[ i ] ] += a[ i ];
    flow.addEdge( l[ i ], r[ i ], b[ i ] - a[ i ] );
    // flow from l[i] to r[i] must in [a[i], b[i]]
  }
  int nd = 0;
  for( int i = 1 ; i <= n ; i ++ ){
    if( in[ i ] < out[ i ] ){
      flow.addEdge( i, flow.t, out[ i ] - in[ i ] );
      nd += out[ i ] - in[ i ];
    }
    if( out[ i ] < in[ i ] )
      flow.addEdge( flow.s, i, in[ i ] - out[ i ] );
  }
  // original sink to source
  flow.addEdge( n, 1, INF );
  if( flow.maxflow() != nd )
    return -1; // no solution
  int ans = flow.G[ 1 ].back().c; // source to sink
  flow.G[ 1 ].back().c = flow.G[ n ].back().c = 0;
  // take out super source and super sink
  for( size_t i = 0 ; i < flow.G[ flow.s ].size() ; i ++ ){
    flow.G[ flow.s ][ i ].c = 0;
    Edge &e = flow.G[ flow.s ][ i ];
    flow.G[ e.v ][ e.r ].c = 0;
  }
  for( size_t i = 0 ; i < flow.G[ flow.t ].size() ; i ++ ){
    flow.G[ flow.t ][ i ].c = 0;
    Edge &e = flow.G[ flow.t ][ i ];
    flow.G[ e.v ][ e.r ].c = 0;
  }
  flow.addEdge( flow.s, 1, INF );
  flow.addEdge( n, flow.t, INF );
  flow.reset();
  return ans + flow.maxflow();
}

```

2.7 Flow Method

Maximize $c^T x$ subject to $Ax \leq b$, $x \geq 0$;
with the corresponding symmetric dual problem,
Minimize $b^T y$ subject to $A^T y \geq c$, $y \geq 0$.

Maximize $c^T x$ subject to $Ax \leq b$;
with the corresponding asymmetric dual problem,
Minimize $b^T y$ subject to $A^T y = c$, $y \geq 0$.

Minimum vertex cover on bipartite graph =
Maximum matching on bipartite graph

Minimum edge cover on bipartite graph =
vertex number - Minimum vertex cover(Maximum matching)

Independent set on bipartite graph =
vertex number - Minimum vertex cover(Maximum matching)

找出最小點覆蓋。做完dinic之後，從源點dfs只走還有流量的邊。左邊沒被走到的點跟右邊被走到的點就是答案。其他點為最大獨立集

Maximum density subgraph ($\sum W_e + \sum W_v$) / $|V|$

Binary search on answer:

For a fixed D, construct a Max flow model as follow:

Let S be Sum of all weight(or inf)

1. from source to each node with cap = S
2. For each (u,v,w) in E, $(u \rightarrow v, \text{cap}=w)$, $(v \rightarrow u, \text{cap}=w)$
3. For each node v, from v to sink with cap = $S + 2 * D - \text{deg}[v] - 2 * (W \text{ of } v)$

where $\text{deg}[v] = \sum \text{weight of edge associated with } v$
If $\text{maxflow} < S * |V|$, D is an answer.

Requiring subgraph: all vertex can be reached from source with edge whose cap > 0.

3 Math

3.1 FFT

```
// const int MAXN = 262144;
// (must be 2^k)
// before any usage, run pre_fft() first
typedef long double ld;
typedef complex<ld> cplx; //real(), imag()
const ld PI = acos(-1);
const cplx I(0, 1);
cplx omega[MAXN+1];
void pre_fft(){
    for(int i=0; i<=MAXN; i++){
        omega[i] = exp(i * 2 * PI / MAXN * I);
    }
}
// n must be 2^k
void fft(int n, cplx a[], bool inv=false){
    int basic = MAXN / n;
    int theta = basic;
    for (int m = n; m >= 2; m >= 1) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            cplx w = omega[inv ? MAXN - (i*theta%MAXN) : i*theta%MAXN];
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                cplx x = a[j] - a[k];
                a[j] += a[k];
                a[k] = w * x;
            }
        }
        theta = (theta * 2) % MAXN;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^ k); k >= 1);
        if (j < i) swap(a[i], a[j]);
    }
    if(inv) for (i = 0; i < n; i++) a[i] /= n;
```

```
}
cplx arr[MAXN+1];
inline void mul(int _n, ll a[], int _m, ll b[], ll ans[]){
    int n=1, sum=_n+_m-1;
    while(n<sum)
        n<=1;
    for(int i=0; i<n; i++) {
        double x=(i<_n?a[i]:0), y=(i<_m?b[i]:0);
        arr[i]=complex<double>(x+y, x-y);
    }
    fft(n, arr);
    for(int i=0; i<n; i++)
        arr[i]=arr[i]*arr[i];
    fft(n, arr, true);
    for(int i=0; i<sum; i++)
        ans[i]=(long long int)(arr[i].real()/4+0.5);
}
```

3.2 $O(1)$ mul

```
LL mul(LL x, LL y, LL mod){
    LL ret=x*y-(LL)((long double)x/mod*y)*mod;
    // LL ret=x*y-(LL)((long double)x*y/mod+0.5)*mod;
    return ret<0?ret+mod:ret;
}
```

3.3 Faulhaber $(\sum_{i=1}^n i^p)$

```
/* faulhaber' s formula -
 * cal power sum formula of all p=1~k in  $O(k^2)$  */
#define MAXK 2500
const int mod = 1000000007;
int b[MAXK]; // bernoulli number
int inv[MAXK+1]; // inverse
int cm[MAXK+1][MAXK+1]; // combinactories
int co[MAXK][MAXK+2]; // coefficient of  $x^j$  when  $p=i$ 
inline int getinv(int x) {
    int a=x, b=mod, a0=1, a1=0, b0=0, b1=1;
    while(b) {
        int q, t;
        q=a/b; t=b; b=a-b*q; a=t;
        t=b0; b0=a0-b0*q; a0=t;
        t=b1; b1=a1-b1*q; a1=t;
    }
    return a0<0?a0+mod:a0;
}
inline void pre() {
    /* combinational */
    for(int i=0; i<=MAXK; i++) {
        cm[i][0]=cm[i][i]=1;
        for(int j=1; j<i; j++)
            cm[i][j]=add(cm[i-1][j-1], cm[i-1][j]);
    }
    /* inverse */
    for(int i=1; i<=MAXK; i++) inv[i]=getinv(i);
    /* bernoulli */
    b[0]=1; b[1]=getinv(2); // with b[1] = 1/2
    for(int i=2; i<MAXK; i++) {
        if(i&1) { b[i]=0; continue; }
        b[i]=1;
        for(int j=0; j<i; j++)
            b[i]=sub(b[i], mul(cm[i][j], mul(b[j], inv[i-j+1])));
    }
    /* faulhaber */
    // sigma_x=1~n {x^p} =
    // 1/(p+1) * sigma_j=0~p {C(p+1, j)*B_j*n^(p-j+1)}
    for(int i=1; i<MAXK; i++) {
        co[i][0]=0;
        for(int j=0; j<i; j++)
            co[i][i-j+1]=mul(inv[i+1], mul(cm[i+1][j], b[j]));
    }
}
/* sample usage: return f(n,p) = sigma_x=1~n (x^p) */
inline int solve(int n, int p) {
```

```

int sol=0,m=n;
for(int i=1;i<=p+1;i++) {
    sol=add(sol,mul(co[p][i],m));
    m = mul(m, n);
}
return sol;
}

```

3.4 Chinese Remainder

```

LL x[N],m[N];
LL CRT(LL x1, LL m1, LL x2, LL m2) {
    LL g = __gcd(m1, m2);
    if((x2 - x1) % g) return -1; // no sol
    m1 /= g; m2 /= g;
    pair<LL,LL> p = gcd(m1, m2);
    LL lcm = m1 * m2 * g;
    LL res = p.first * (x2 - x1) * m1 + x1;
    return (res % lcm + lcm) % lcm;
}
LL solve(int n){ // n>=2, be careful with no solution
    LL res=CRT(x[0],m[0],x[1],m[1]),p=m[0]/__gcd(m[0],m[1])*m[1];
    for(int i=2;i<n;i++){
        res=CRT(res,p,x[i],m[i]);
        p=p/__gcd(p,m[i])*m[i];
    }
    return res;
}

```

3.5 Miller Rabin

```

// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pirmses <= 13
// n < 2^64               7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
LL magic[] = {}
bool witness(LL a, LL n, LL u, int t){
    if(!a) return 0;
    LL x=mypow(a,u,n);
    for(int i=0;i<t;i++){
        LL nx=mul(x,x,n);
        if(nx==1&&x!=1&&x!=n-1) return 1;
        x=nx;
    }
    return x!=1;
}
bool miller_rabin(LL n) {
    int s=(magic number size)
    // iterate s times of witness on n
    if(n<2) return 0;
    if(!(n&1)) return n == 2;
    ll u=n-1; int t=0;
    // n-1 = u*2^t
    while(!(u&1)) u>>=1, t++;
    while(s--){
        LL a=magic[s]%n;
        if(witness(a,n,u,t)) return 0;
    }
    return 1;
}

```

3.6 Pollard Rho

```

// does not work when n is prime 0(n^(1/4))
LL f(LL x, LL mod){ return add(mul(x,x,mod),1,mod); }
LL pollard_rho(LL n) {
    if(!(n&1)) return 2;
    while(true){
        LL y=2, x=rand()%(n-1)+1, res=1;
        for(int sz=2; res==1; sz*=2) {
            for(int i=0; i<sz && res<=1; i++) {
                x = f(x, n);

```

```

        res = __gcd(abs(x-y), n);
    }
    y = x;
    if (res!=0 && res!=n) return res;
} }

```

3.7 Josephus Problem

```

int josephus(int n, int m){ //n人 每m次
    int ans = 0;
    for (int i=1; i<=n; ++i)
        ans = (ans + m) % i;
    return ans;
}

```

3.8 Matrix

```

//矩陣乘法
for(int i = 0; i < n; i++){
    for(int j = 0; j < n; j++){
        for(int k = 0; k < n; k++){
            ret[i][j] += a[i][k] * b[k][j];
        }
    }
}
//矩陣快速幂
int base[2][2] = { {1, 1}, {1, 0} }, ans[2][2] = { {1, 0}, {0, 1} };
};

int mypow(int y){
    while(y){
        if( y&1 ) { ans = mul(ans, base); } //實作矩陣乘法
        base = mul(base, base); //實作矩陣乘法
        y >>= 1;
    }
    return ans[0][0];
}

```

3.9 Gaussian Elimination

```

const int GAUSS_MOD = 100000007LL;
struct GAUSS{
    int n;
    vector<vector<int>> v;
    int ppow(int a, int k){
        if(k == 0) return 1;
        if(k % 2 == 0) return ppow(a * a % GAUSS_MOD, k >> 1);
        if(k % 2 == 1) return ppow(a * a % GAUSS_MOD, k >> 1) * a % GAUSS_MOD;
    }
    vector<int> solve(){
        vector<int> ans(n);
        REP(now, 0, n){
            REP(i, now, n) if(v[now][now] == 0 && v[i][now] != 0)
                swap(v[i], v[now]); // det = -det;
            if(v[now][now] == 0) return ans;
            int inv = ppow(v[now][now], GAUSS_MOD - 2);
            REP(i, 0, n) if(i != now){
                int tmp = v[i][now] * inv % GAUSS_MOD;
                REP(j, now, n + 1) (v[i][j] += GAUSS_MOD - tmp * v[now][j] % GAUSS_MOD) %= GAUSS_MOD;
            }
        }
        REP(i, 0, n) ans[i] = v[i][n + 1] * ppow(v[i][i], GAUSS_MOD - 2) % GAUSS_MOD;
        return ans;
    }
    // gs.v.clear(), gs.v.resize(n, vector<int>(n + 1, 0));
} gs;

```

3.10 Inverse Matrix

```
int GAUSS_MOD;
struct GAUSS{
    int n;
    vector<vector<int>> > v;
    vector<vector<int>> > rev;
    int mul(int x,int y,int mod){
        int ret=x*y-(int)((long double)x/mod*y)*mod;
        return ret<0?ret+mod:ret;
    }
    int ppow(int a, int b){//res=(a^b)%m
        int res=1, k=a;
        while(b){
            if((b&1)) res=mul(res,k,GAUSS_MOD)%GAUSS_MOD;
            k=mul(k,k,GAUSS_MOD)%GAUSS_MOD;
            b>>=1;
        }
        return res%GAUSS_MOD;
    }
    bool solve(){
        for(int now = 0; now < n; now++){
            int ch;
            for(ch = now; ch < n && !v[ch][now]; ch++);
            if(ch >= n) return 0;
            for(int i = now; i < n; i++) if(v[now][now] == 0
                && v[i][now] != 0){
                swap(v[i], v[now]); // det = -det;
                swap(rev[i], rev[now]);
            }
            if(v[now][now] == 0) return 0;
            int inv = ppow(v[now][now], GAUSS_MOD - 2);
            for(int i = 0; i < n; i++) if(i != now){
                int tmp = v[i][now] * inv % GAUSS_MOD;
                for(int j = 0; j < n; j++) {
                    (v[i][j] += GAUSS_MOD - tmp * v[now][j] %
                        GAUSS_MOD) %= GAUSS_MOD;
                    (rev[i][j] += GAUSS_MOD - tmp * rev[now][j] %
                        GAUSS_MOD) %= GAUSS_MOD;
                }
            }
        }
        return 1;
    }
} gs;

signed main(){
    int n, p; //n*n matrix, MOD=p
    cin>>n>>p; //if(!n && !p) return 0;
    GAUSS_MOD = p; gs.n = n;
    gs.v.clear(), gs.v.resize(n + 1, vector<int>(n + 2, 0));
    gs.rev.clear(), gs.rev.resize(n + 1, vector<int>(n + 2, 0));
    for(int i = 0; i < n; i++){
        for(int j = 0; j < n; j++){
            cin>>gs.v[i][j];
            if(i == j) gs.rev[i][j] = 1;
        }
    }
    if(!gs.solve()) cout << "singular\n";
    else{
        for(int i = 0; i < n; i++){
            int inv = gs.ppow(gs.v[i][i], p - 2);
            for(int j = 0; j < n; j++){
                cout << (gs.rev[i][j] * inv % p) << " ";
                cout<<"\n";
            }
        }
        cout << "\n";
    }
}
```

3.11 模反元素

```
long long inv(long long a,long long m){
    long long x,y;
    long long d=exgcd(a,m,x,y);
    if(d==1) return (x+m)%m;
    else return -1; //-1為無解
}
```

3.12 ax+by=gcd

```
PII gcd(int a, int b){
    if(b == 0) return {1, 0};
    PII q = gcd(b, a % b);
    return {q.second, q.first - q.second * (a / b)};
}

int exgcd(int a,int b,long long &x,long long &y) {
    if(b == 0){x=1,y=0;return a;}
    int now=exgcd(b,a%b,y,x);
    y-=a/b*x;
    return now;
}
```

3.13 Discrete sqrt

```
void calch(LL &t, LL &h, const LL p) {
    LL tmp=p-1; for(t=0;(tmp&1)==0;tmp/=2) t++; h=tmp;
}
// solve equation x^2 mod p = a
bool solve(LL a, LL p, LL &x, LL &y) {
    if(p == 2) { x = y = 1; return true; }
    int p2 = p / 2, tmp = mypow(a, p2, p);
    if (tmp == p - 1) return false;
    if ((p + 1) % 4 == 0) {
        x=mypow(a,(p+1)/4,p); y=p-x; return true;
    } else {
        LL t, h, b, pb; calch(t, h, p);
        if (t >= 2) {
            do {b = rand() % (p - 2) + 2;
                } while (mypow(b, p / 2, p) != p - 1);
            pb = mypow(b, h, p);
            int s = mypow(a, h / 2, p);
            for (int step = 2; step <= t; step++) {
                int ss = (((LL)(s * s) % p) * a) % p;
                for(int i=0;i<t-step;i++) ss=mul(ss,ss,p);
                if (ss + 1 == p) s = (s * pb) % p;
                pb = ((LL)pb * pb) % p;
            } x = ((LL)s * a) % p; y = p - x;
        } return true;
    }
}
```

3.14 Prefix Inverse

```
void solve( int m ){
    inv[ 1 ] = 1;
    for( int i = 2 ; i < m ; i ++ )
        inv[ i ] = ((LL)(m - m / i) * inv[m % i]) % m;
}
```

3.15 Roots of Polynomial 找多項式的根

```
const double eps = 1e-12;
const double inf = 1e+12;
double a[ 10 ], x[ 10 ]; // a[0..n](coef) must be filled
// degree of polynomial must be filled
int sign( double x ){return (x < -eps)?(-1):(x>eps);}
double f(double a[], int n, double x){
    double tmp=1,sum=0;
    for(int i=0;i<n;i++){
        sum=sum+a[i]*tmp; tmp=tmp*x;
    }
    return sum;
}
double binary(double l,double r,double a[],int n){
    int sl=sign(f(a,n,l)),sr=sign(f(a,n,r));
    if(sl==0) return l; if(sr==0) return r;
    if(sl*sr>0) return inf;
    while(r-l>eps){
        double mid=(l+r)/2;
        int ss=sign(f(a,n,mid));
        if(ss==0) return mid;
        if(ss*sl>0) l=mid; else r=mid;
    }
}
```



```

    return l;
}
void solve(int n, double a[], double x[], int &nx){
    if(n==1){ x[1]=-a[0]/a[1]; nx=1; return; }
    double da[10], dx[10]; int ndx;
    for(int i=n; i>=1; i--){ da[i-1]=a[i]*i;
    solve(n-1, da, dx, ndx);
    nx=0;
    if(ndx==0){
        double tmp=binary(-inf, inf, a, n);
        if (tmp<inf) x[++nx]=tmp;
        return;
    }
    double tmp;
    tmp=binary(-inf, dx[1], a, n);
    if(tmp<inf) x[++nx]=tmp;
    for(int i=1; i<=ndx-1; i++){
        tmp=binary(dx[i], dx[i+1], a, n);
        if(tmp<inf) x[++nx]=tmp;
    }
    tmp=binary(dx[ndx], inf, a, n);
    if(tmp<inf) x[++nx]=tmp;
} // roots are stored in x[1..nx]

```

3.16 Combination theorem

```

const ll mod = 1e9 + 7;
ll fac[(int)2e6 + 1], inv[(int)2e6 + 1];
ll getinv(ll a){ return qpow(a, mod-2); }
void init(int n){
    fac[0] = 1;
    for(int i = 1; i <= n; i++){
        fac[i] = fac[i-1] * i % mod;
    }
    inv[n] = getinv(fac[n]);
    for(int i = n - 1; i >= 0; i--){
        inv[i] = inv[i + 1] * (i + 1) % mod;
    }
}
ll C(int n, int m){
    if(m > n) return 0;
    return fac[n] * inv[m] % mod * inv[n-m] % mod;
}

```

3.17 Primes

```

/* 12721, 13331, 14341, 75577, 123457, 222557, 556679
* 999983, 1097774749, 1076767633, 100102021, 999997771
* 1001010013, 1000512343, 987654361, 999991231
* 999888733, 98789101, 987777733, 999991921, 1010101333
* 1010102101, 1000000000039, 100000000000037
* 2305843009213693951, 4611686018427387847
* 9223372036854775783, 18446744073709551557 */
int mu[ N ], p_tbl[ N ];
vector<int> primes;
void sieve() {
    mu[ 1 ] = p_tbl[ 1 ] = 1;
    for( int i = 2 ; i < N ; i ++ ){
        if( !p_tbl[ i ] ){
            p_tbl[ i ] = i;
            primes.push_back( i );
            mu[ i ] = -1;
        }
        for( int p : primes ){
            int x = i * p;
            if( x >= M ) break;
            p_tbl[ x ] = p;
            mu[ x ] = -mu[ i ];
            if( i % p == 0 ){
                mu[ x ] = 0;
                break;
            }
        }
    }
}
vector<int> factor( int x ){
    vector<int> fac{ 1 };
    while( x > 1 ){
        int fn = SZ(fac), p = p_tbl[ x ], pos = 0;
        while( x % p == 0 ){
            x /= p;

```

```

        for( int i = 0 ; i < fn ; i ++ )
            fac.PB( fac[ pos ++ ] * p );
    } }
    return fac;
}

```

3.18 Phi

```

ll phi(ll n){ // 計算小於n的數中與n互質的有幾個
    ll res = n, a=n;
    for(ll i=2; i*i<=a; i++){ // O(sqrt(N))
        if(a%i==0){
            res = res/i*(i-1);
            while(a%i==0) a/=i;
        }
    }
    if(a>1) res=res/a*(a-1);
    return res;
}

```

3.19 Result

- Lucas' Theorem :
For $n, m \in \mathbb{Z}^*$ and prime P , $C(m, n) \bmod P = \prod C(m_i, n_i)$ where m_i is the i -th digit of m in base P .
- Stirling approximation :
$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n e^{\frac{1}{12n}}$$
- Stirling Numbers(permutation $|P| = n$ with k cycles):
 $S(n, k) = \text{coefficient of } x^k \text{ in } \Pi_{i=0}^{n-1} (x+i)$
- Stirling Numbers(Partition n elements into k non-empty set):
$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$
- Pick' s Theorem : $A = i + b/2 - 1$
在二維座標平面中畫上網格，對於任何簡單多邊形
 A : 面積、 i : 內部的格點數、 b : 邊上的格點數
- Catalan number : $C_n = \binom{2n}{n} / (n+1)$
$$C_n^{n+m} - C_{n+1}^{n+m} = (m+n)! \frac{n-m+1}{n+1} \quad \text{for } n \geq m$$

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1 \quad \text{and} \quad C_{n+1} = 2 \binom{2n+1}{n+2} C_n$$

$$C_0 = 1 \quad \text{and} \quad C_{n+1} = \sum_{i=0}^n C_i C_{n-i} \quad \text{for } n \geq 0$$
- Euler Characteristic:
planar graph: $V - E + F - C = 1$
convex polyhedron: $V - E + F = 2$
 V, E, F, C : number of vertices, edges, faces(regions), and components
- Kirchhoff's theorem :
 $A_{ii} = \deg(i), A_{ij} = (i, j) \in E ? -1 : 0$, Deleting any one row, one column, and cal the $\det(A)$
- Polya' theorem (c is number of color, m is the number of cycle size):
$$\left(\sum_{i=1}^m c^{gcd(i, m)} \right) / m$$
- Burnside lemma:
$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$
- 錯排公式: (n 個人中，每個人皆不再原來位置的組合數):
$$dp[0] = 1; dp[1] = 0;$$

$$dp[i] = (i-1) * (dp[i-1] + dp[i-2]);$$
- Bell 數 (有 n 個人，把他們拆組的方法總數) :
$$B_0 = 1$$

$$B_n = \sum_{k=0}^n s(n, k) \quad (\text{second - stirling})$$

$$B_{n+1} = \sum_{k=0}^n \binom{n}{k} B_k$$
- Wilson's theorem :
$$(p-1)! \equiv -1 \pmod{p}$$
- Fermat's little theorem :
$$a^p \equiv a \pmod{p}$$
- Euler's totient function:
$$A^{B^C} \bmod p = \text{pow}(A, \text{pow}(B, C, p-1)) \bmod p$$
- 歐拉函數降冪公式:
$$A^B \bmod C = A^{B \bmod \phi(C) + \phi(C)} \bmod C$$
- 用歐拉函數求模反元素:
如果 a 和 n 互質，則 a 對 n 的模反元素
$$a^{-1} \equiv a^{\phi(n)-1} \pmod{n}$$
- 6 的倍數:
$$(a-1)^3 + (a+1)^3 + (-a)^3 + (-a)^3 = 6a$$

4 Geometry

4.1 definition

```
typedef long double ld;
const ld eps = 1e-8;
int dcmp(ld x) {
    if(abs(x) < eps) return 0;
    else return x < 0 ? -1 : 1;
}
struct Pt {
    ld x, y;
    Pt(ld _x=0, ld _y=0):x(_x), y(_y) {}
    Pt operator+(const Pt &a) const {
        return Pt(x+a.x, y+a.y); }
    Pt operator-(const Pt &a) const {
        return Pt(x-a.x, y-a.y); }
    Pt operator*(const ld &a) const {
        return Pt(x*a, y*a); }
    Pt operator/(const ld &a) const {
        return Pt(x/a, y/a); }
    ld operator*(const Pt &a) const {
        return x*a.x + y*a.y; }
    ld operator^(const Pt &a) const {
        return x*a.y - y*a.x; }
    bool operator<(const Pt &a) const {
        return x < a.x || (x == a.x && y < a.y); }
    //return dcmp(x-a.x) < 0 || (dcmp(x-a.x) == 0 &&
        dcmp(y-a.y) < 0); }
    bool operator==(const Pt &a) const {
        return dcmp(x-a.x) == 0 && dcmp(y-a.y) == 0; }
};
ld norm2(const Pt &a) {
    return a*a; }
ld norm(const Pt &a) {
    return sqrt(norm2(a)); }
Pt perp(const Pt &a) {
    return Pt(-a.y, a.x); }
Pt rotate(const Pt &a, ld ang) {
    return Pt(a.x*cos(ang)-a.y*sin(ang), a.x*sin(ang)+a.y*cos(ang)); }
struct Line {
    Pt s, e, v; // start, end, end-start
    ld ang;
    Line(Pt _s=Pt(0, 0), Pt _e=Pt(0, 0)):s(_s), e(_e) { v
        = e-s; ang = atan2(v.y, v.x); }
    bool operator<(const Line &l) const {
        return ang < l.ang; }
};
struct Circle {
    Pt o; ld r;
    Circle(Pt _o=Pt(0, 0), ld _r=0):o(_o), r(_r) {}
};
```

4.2 Intersection of 2 lines

```
Pt LLIntersect(Line a, Line b) {
    Pt p1 = a.s, p2 = a.e, q1 = b.s, q2 = b.e;
    ld f1 = (p2-p1)^(q1-p1), f2 = (p2-p1)^(p1-q2), f;
    if(dcmp(f=f1+f2) == 0)
        return dcmp(f1)?Pt(NAN,NAN):Pt(INFINITY,INFINITY);
    return q1*(f2/f) + q2*(f1/f);
}
```

4.3 halfPlaneIntersection

```
// 0(nlogn)
// 傳入 vector<Line>
// (半平面為點 st 往 ed 的逆時針方向)
// 回傳值為形成的凸多邊形的頂點 vector

// 對於點或線的解，將 '>' 改為 '>='
bool onleft(Line L, Pt p) { return dcmp(L.v ^ (p - L.s))
    > 0; }

// 假設線段是有交點的
```

```
vector<Pt> HPI(vector<Line> &L) {
    sort(L.begin(), L.end()); // 按角度排序
    int n = L.size(), fir, las;
    Pt *p = new Pt[n];
    Line *q = new Line[n];
    q[fir = las = 0] = L[0];
    for (int i = 1; i < n; i++) {
        while (fir < las && !onleft(L[i], p[las - 1])) las--;
        while (fir < las && !onleft(L[i], p[fir])) fir++;
        q[++las] = L[i];
        if (dcmp(q[las].v ^ q[las - 1].v) == 0) {
            las--;
            if (onleft(q[las], L[i].s)) q[las] = L[i];
        }
        if (fir < las)
            p[las - 1] = LLIntersect(q[las - 1], q[las]);
    }
    while (fir < las && !onleft(q[fir], p[las - 1])) las--;
    if (las - fir <= 1) return {};
    p[las] = LLIntersect(q[las], q[fir]);
    int m = 0;
    vector<Pt> ans(las - fir + 1);
    for (int i = fir; i <= las; i++) ans[m++] = p[i];
    return ans;
}
```

4.4 Convex Hull

```
double cross(Pt o, Pt a, Pt b){
    return (a-o) ^ (b-o);
}
vector<Pt> convex_hull(vector<Pt> pt){
    sort(pt.begin(), pt.end());
    int top=0;
    vector<Pt> stk(2*pt.size());
    for (int i=0; i<(int)pt.size(); i++){
        while (top >= 2 && cross(stk[top-2], stk[top-1], pt[i])
            <= 0)
            top--;
        stk[top++] = pt[i];
    }
    for (int i=pt.size()-2, t=top+1; i>=0; i--){
        while (top >= t && cross(stk[top-2], stk[top-1], pt[i])
            <= 0)
            top--;
        stk[top++] = pt[i];
    }
    stk.resize(top-1);
    return stk;
}
```

4.5 Convex Hull trick

```
/* Given a convexhull, answer queries in O(lgN)
CH should not contain identical points, the area should
be > 0, min pair(x, y) should be listed first */
double det( const Pt& p1 , const Pt& p2 )
{ return p1.X * p2.Y - p1.Y * p2.X; }
struct Conv{
    int n;
    vector<Pt> a;
    vector<Pt> upper, lower;
    Conv(vector<Pt> _a) : a(_a){
        n = a.size();
        int ptr = 0;
        for(int i=1; i<n; ++i) if (a[ptr] < a[i]) ptr = i;
        for(int i=0; i<=ptr; ++i) lower.push_back(a[i]);
        for(int i=ptr; i<n; ++i) upper.push_back(a[i]);
        upper.push_back(a[0]);
    }
    int sign( LL x ){ // fixed when changed to double
        return x < 0 ? -1 : x > 0; }
    pair<LL,int> get_tang(vector<Pt> &conv, Pt vec){
        int l = 0, r = (int)conv.size() - 2;
        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;
```



```

    if(sign(det(conv[mid+1]-conv[mid],vec))>0)r=mid;
    else l = mid;
}
return max(make_pair(det(vec, conv[r]), r),
           make_pair(det(vec, conv[0]), 0));
}
void upd_tang(const Pt &p, int id, int &i0, int &i1){
    if(det(a[i0] - p, a[id] - p) > 0) i0 = id;
    if(det(a[i1] - p, a[id] - p) < 0) i1 = id;
}
void bi_search(int l, int r, Pt p, int &i0, int &i1){
    if(l == r) return;
    upd_tang(p, l % n, i0, i1);
    int sl=sign(det(a[l % n] - p, a[(l + 1) % n] - p));
    for( ; l + 1 < r; ) {
        int mid = (l + r) / 2;
        int smid=sign(det(a[mid%n]-p, a[(mid+1)%n]-p));
        if (smid == sl) l = mid;
        else r = mid;
    }
    upd_tang(p, r % n, i0, i1);
}
int bi_search(Pt u, Pt v, int l, int r) {
    int sl = sign(det(v - u, a[l % n] - u));
    for( ; l + 1 < r; ) {
        int mid = (l + r) / 2;
        int smid = sign(det(v - u, a[mid % n] - u));
        if (smid == sl) l = mid;
        else r = mid;
    }
    return l % n;
}
// 1. whether a given point is inside the CH
bool contain(Pt p) {
    if (p.X < lower[0].X || p.X > lower.back().X)
        return 0;
    int id = lower_bound(lower.begin(), lower.end(), Pt
        (p.X, -INF)) - lower.begin();
    if (lower[id].X == p.X) {
        if (lower[id].Y > p.Y) return 0;
    }else if(det(lower[id-1]-p,lower[id]-p)<0)return 0;
    id = lower_bound(upper.begin(), upper.end(), Pt(p.X
        , INF), greater<Pt>()) - upper.begin();
    if (upper[id].X == p.X) {
        if (upper[id].Y < p.Y) return 0;
    }else if(det(upper[id-1]-p,upper[id]-p)<0)return 0;
    return 1;
}
// 2. Find 2 tang pts on CH of a given outside point
// return true with i0, i1 as index of tangent points
// return false if inside CH
bool get_tang(Pt p, int &i0, int &i1) {
    if (contain(p)) return false;
    i0 = i1 = 0;
    int id = lower_bound(lower.begin(), lower.end(), p)
        - lower.begin();
    bi_search(0, id, p, i0, i1);
    bi_search(id, (int)lower.size(), p, i0, i1);
    id = lower_bound(upper.begin(), upper.end(), p,
        greater<Pt>()) - upper.begin();
    bi_search((int)lower.size() - 1, (int)lower.size()
        - 1 + id, p, i0, i1);
    bi_search((int)lower.size() - 1 + id, (int)lower.
        size() - 1 + (int)upper.size(), p, i0, i1);
    return true;
}
// 3. Find tangent points of a given vector
// ret the idx of vertex has max cross value with vec
int get_tang(Pt vec){
    pair<LL, int> ret = get_tang(upper, vec);
    ret.second = (ret.second+(int)lower.size()-1)%n;
    ret = max(ret, get_tang(lower, vec));
    return ret.second;
}
// 4. Find intersection point of a given line
// return 1 and intersection is on edge (i, next(i))
// return 0 if no strictly intersection
bool get_intersection(Pt u, Pt v, int &i0, int &i1){
    int p0 = get_tang(u - v), p1 = get_tang(v - u);
    if(sign(det(v-u,a[p0]-u))*sign(det(v-u,a[p1]-u))<0){
        if (p0 > p1) swap(p0, p1);
        i0 = bi_search(u, v, p0, p1);

```

```

        i1 = bi_search(u, v, p1, p0 + n);
        return 1;
    }
    return 0;
}
};

```

4.6 Intersection of 2 segments

```

int ori( const Pt& o , const Pt& a , const Pt& b ){
    LL ret = ( a - o ) ^ ( b - o );
    return (ret > 0) - (ret < 0);
}
// p1 == p2 || q1 == q2 need to be handled
bool banana( const Pt& p1 , const Pt& p2 ,
              const Pt& q1 , const Pt& q2 ){
    if( ( ( p2 - p1 ) ^ ( q2 - q1 ) ) == 0 ){ // parallel
        if( ori( p1 , p2 , q1 ) ) return false;
        return ( ( p1 - q1 ) * ( p2 - q1 ) ) <= 0 ||
               ( ( p1 - q2 ) * ( p2 - q2 ) ) <= 0 ||
               ( ( q1 - p1 ) * ( q2 - p1 ) ) <= 0 ||
               ( ( q1 - p2 ) * ( q2 - p2 ) ) <= 0;
    }
    return (ori( p1, p2, q1 ) * ori( p1, p2, q2 )<=0) &&
           (ori( q1, q2, p1 ) * ori( q1, q2, p2 )<=0);
}

```

4.7 Point In Polygon

```

int ptInPoly(vector<Pt> ps,Pt p){
    int c=0;
    for(int i=0;i<ps.size();++i){
        int a=i,b=(i+1)%ps.size(); Line l(ps[a],ps[b]);
        Pt q=l.s+l.v*((l.v*(p-l.s))/norm2(l.v)); // project
        if(norm(p-q)<eps&&onseg(q,l)) return 1; // boundary
        if(dcmp(ps[a].y-ps[b].y)==0&&dcmp(ps[a].y-p.y)==0)
            continue;
        if(ps[a].y>ps[b].y) swap(a,b);
        if(ps[a].y<=p.y&&p.y<ps[b].y&&p.x<=ps[a].x+(ps[b].x
            -ps[a].x)/(ps[b].y-ps[a].y)*(p.y-ps[a].y)) ++c;
    }
    return (c&1)*2; // 0: outside, 1: boundary, 2: inside
} // check whether a point is in a polygon

```

4.8 Tangent line of two circles

```

vector<Line> go( const Cir& c1 , const Cir& c2 , int
    sign1 ){
    // sign1 = 1 for outer tang, -1 for inner tang
    vector<Line> ret;
    double d_sq = norm2( c1.0 - c2.0 );
    if( d_sq < eps ) return ret;
    double d = sqrt( d_sq );
    Pt v = ( c2.0 - c1.0 ) / d;
    double c = ( c1.R - sign1 * c2.R ) / d;
    if( c * c > 1 ) return ret;
    double h = sqrt( max( 0.0 , 1.0 - c * c ) );
    for( int sign2 = 1 ; sign2 >= -1 ; sign2 -= 2 ){
        Pt n = { v.X * c - sign2 * h * v.Y ,
                v.Y * c + sign2 * h * v.X };
        Pt p1 = c1.0 + n * c1.R;
        Pt p2 = c2.0 + n * ( c2.R * sign1 );
        if( fabs( p1.X - p2.X ) < eps and
            fabs( p1.Y - p2.Y ) < eps )
            p2 = p1 + perp( c2.0 - c1.0 );
        ret.push_back( { p1 , p2 } );
    }
    return ret;
}

```

4.9 Minimum distance of two convex

```
double TwoConvexHullMinDis(Pt P[], Pt Q[], int n, int m){
    int mn=0, mx=0; double tmp, ans=1e9;
    for(int i=0; i<n; ++i) if(P[i].y<P[mn].y) mn=i;
    for(int i=0; i<m; ++i) if(Q[i].y>Q[mx].y) mx=i;
    P[n]=P[0]; Q[m]=Q[0];
    for (int i=0; i<n; ++i) {
        while(tmp=((Q[mx+1]-P[mn+1])^(P[mn]-P[mn+1]))>((Q[mx]-P[mn+1])^(P[mn]-P[mn+1]))) mx=(mx+1)%m;
        if(tmp<0) // pt to segment distance
            ans=min(ans, dis(Line(P[mn], P[mn+1]), Q[mx]));
        else // segment to segment distance
            ans=min(ans, dis(Line(P[mn], P[mn+1]), Line(Q[mx], Q[mx+1])));
        mn=(mn+1)%n;
    }
    return ans;
}
```

4.10 Area of Rectangles

```
struct AreaofRectangles{
#define cl(x) (x<<1)
#define cr(x) (x<<1|1)
    ll n, id, sid;
    pair<ll, ll> tree[MXN<<3]; // count, area
    vector<ll> ind;
    tuple<ll, ll, ll> scan[MXN<<1];
    void pull(int i, int l, int r){
        if(tree[i].first) tree[i].second = ind[r+1] - ind[l];
        else if(l != r){
            int mid = (l+r)>>1;
            tree[i].second = tree[cl(i)].second + tree[cr(i)].second;
        }
        else tree[i].second = 0;
    }
    void upd(int i, int l, int r, int ql, int qr, int v){
        if(ql <= l && r <= qr){
            tree[i].first += v;
            pull(i, l, r); return;
        }
        int mid = (l+r)>>1;
        if(ql <= mid) upd(cl(i), l, mid, ql, qr, v);
        if(qr > mid) upd(cr(i), mid+1, r, ql, qr, v);
        pull(i, l, r);
    }
    void init(int _n){
        n = _n; id = sid = 0;
        ind.clear(); ind.resize(n<<1);
        fill(tree, tree+(n<<2), make_pair(0, 0));
    }
    void addRectangle(int lx, int ly, int rx, int ry){
        ind[id++] = lx; ind[id++] = rx;
        scan[sid++] = make_tuple(ly, 1, lx, rx);
        scan[sid++] = make_tuple(ry, -1, lx, rx);
    }
    ll solve(){
        sort(ind.begin(), ind.end());
        ind.resize(unique(ind.begin(), ind.end()) - ind.begin());
        sort(scan, scan + sid);
        ll area = 0, pre = get<0>(scan[0]);
        for(int i = 0; i < sid; i++){
            auto [x, v, l, r] = scan[i];
            area += tree[1].second * (x-pre);
            upd(1, 0, ind.size()-1, lower_bound(ind.begin(), ind.end(), l)-ind.begin(), lower_bound(ind.begin(), ind.end(), r)-ind.begin()-1, v);
            pre = x;
        }
        return area;
    }
} rect;
```

4.11 Min dist on Cuboid

```
typedef LL T;
T r;
void turn(T i, T j, T x, T y, T z,
          T x0, T y0, T L, T W, T H) {
    if (z==0) { T R = x*x+y*y; if (R<r) r=R; return; }
    if(i>=0 && i< 2) turn(i+1, j, x0+L+z, y, x0+L-x,
                          x0+L, y0, H, W, L);
    if(j>=0 && j< 2) turn(i, j+1, x, y0+W+z, y0+W-y,
                          x0, y0+W, L, H, W);
    if(i<=0 && i>-2) turn(i-1, j, x0-z, y, x-x0,
                          x0-H, y0, H, W, L);
    if(j<=0 && j>-2) turn(i, j-1, x, y0-z, y-y0,
                          x0, y0-H, L, H, W);
}
T solve(T L, T W, T H,
        T x1, T y1, T z1, T x2, T y2, T z2){
    if( z1!=0 && z1!=H ){
        if( y1==0 || y1==W )
            swap(y1,z1), swap(y2,z2), swap(W,H);
        else swap(x1,z1), swap(x2,z2), swap(L,H);
    }
    if (z1==H) z1=0, z2=H-z2;
    r=INF; turn(0,0,x2-x1,y2-y1,z2,-x1,-y1,L,W,H);
    return r;
}
```

4.12 Heart of Triangle

```
Pt inCenter( Pt &a, Pt &b, Pt &c) { // 內心
    double a = norm(B-C), b = norm(C-A), c = norm(A-B);
    return (A * a + B * b + C * c) / (a + b + c);
}
Pt circumCenter( Pt &a, Pt &b, Pt &c) { // 外心
    Pt bb = b - a, cc = c - a;
    double db=norm2(bb), dc=norm2(cc), d=2*(bb ^ cc);
    return a-Pt(bb.Y*dc-cc.Y*db, cc.X*db-bb.X*dc) / d;
}
Pt othroCenter( Pt &a, Pt &b, Pt &c) { // 垂心
    Pt ba = b - a, ca = c - a, bc = b - c;
    double Y = ba.Y * ca.Y * bc.Y,
           A = ca.X * ba.Y - ba.X * ca.Y,
           x0 = (Y+ca.X*ba.Y*b.X-ba.X*ca.Y*c.X) / A,
           y0 = -ba.X * (x0 - c.X) / ba.Y + ca.Y;
    return Pt(x0, y0);
}
```

5 Graph

5.1 DSU 並查集 & MST

```
struct DSU { // 並查集
    vector<int> fa, sz;
    DSU(int n = 0) : fa(n), sz(n, 1) {
        iota(fa.begin(), fa.end(), 0);
    }
    int Find(int x) { // 路徑壓縮
        while (x != fa[x])
            x = fa[x] = fa[fa[x]];
        return x;
    }
    bool Merge(int x, int y) { // 合併
        x = Find(x), y = Find(y);
        if (x == y) return false; // 是否為連通
        if (sz[x] > sz[y]) swap(x, y);
        fa[x] = y;
        sz[y] += sz[x];
        return true;
    }
};
int MST(int n, int m, vector<tuple<int, int, int>> &edge) {
    // 0 base
    sort(edge.begin(), edge.end());
    DSU dsu(n+1); // 初始化並查集
```

```

int res = 0, flag=1; // 最小生成樹邊權和
for (auto &[w, u, v] : edge)
    if(dsu.Merge(u, v)) {
        res += w; //合併並統計答案
        //graph[u].push_back({v,w});
        //graph[v].push_back({u,w});
    }
    //else edges.push_back({w,u,v});
return res;
}
int main(){
    int n, m; //點數, 邊數
    cin >> n >> m;
    vector<tuple<int, int, int>> edge(m);
    for (auto &[w, u, v] : edge) cin >> u >> v >> w;
    cout << MST(n, m, edge);
}

```

5.2 Lowest Common Ancestor $O(\lg n)$

```

struct LCA {
    int n, ti, lgN;
    int anc[MXN + 5][__lg(MXN) + 1] = {0};
    int MaxLength[MXN][__lg(MXN) + 1] = {0};
    int time_in[MXN] = {0};
    int time_out[MXN] = {0};
    LCA(int _n, int f):n(_n), ti(0), lgN(__lg(n)) {
        dfs(f, f, 0);
        build();
    }
    void dfs(int now, int f, int len_to_father) { // dfs
        for (auto i : graph[now]) {
            if (i.first == f) continue;
            dfs(i.first, now, i.second);
        }
        time_out[now] = ti;
    }
    void build() { // build anc[], MaxLength[]
        for (int i = 1; i <= lgN; ++i) {
            for (int u = 1; u <= n; ++u) {
                anc[u][i] = anc[anc[u][i - 1]][i - 1];
                MaxLength[u][i] = max(MaxLength[u][i - 1],
                                     MaxLength[anc[u][i - 1]][i - 1]);
            }
        }
    }
    bool isAncestor(int x, int y) {
        if (time_in[x] <= time_in[y] && time_out[x] >=
            time_out[y]) return true;
        return false;
    }
    int getLCA(int u, int v) {
        if (isAncestor(u, v)) return u;
        if (isAncestor(v, u)) return v;
        for (int i = lgN; i >= 0; --i) {
            if (!isAncestor(anc[u][i], v)) {
                u = anc[u][i];
            }
        }
        return anc[u][0];
    }
    int getMAX(int u, int v) { //獲得路徑上最大邊權
        int lca = getLCA(u, v);
        int maxx = -1;
        for (int i = lgN; i >= 0; --i) {
            // u to lca
            if (!isAncestor(anc[u][i], lca)) {
                maxx = max(maxx, MaxLength[u][i]);
                u = anc[u][i];
            }
            // v to lca
            if (!isAncestor(anc[v][i], lca)) {
                maxx = max(maxx, MaxLength[v][i]);
                v = anc[v][i];
            }
        }
    }
}

```

```

}
}
if (u != lca) maxx = max(maxx, MaxLength[u][0]);
if (v != lca) maxx = max(maxx, MaxLength[v][0]);
return maxx;
}
};

```

5.3 Hamiltonian path $O(n^2 2^n)$

```

//dp[i][j] = 目前在j節點走過{i}節點的最短路徑
for(int i=1; i < (1 << n); i++) {
    for(int j = 1; j < n; j++) {
        if(!((1 << j) & i) && (i & 1)) {
            for(int k = 0; k < n; k++) {
                if(j == k) continue;
                if((1 << k) & i) dp[j][i|(1 << j)] =
                    min(dp[j][i|(1 << j)], dp[k][i] + dis[k][j]);
            }
        }
    }
}
}

```

5.4 MaximumClique 最大團

```

#define N 111
struct MaxClique { // 0-base
    typedef bitset<N> Int;
    Int linkto[N], v[N];
    int n;
    void init(int _n) {
        n = _n;
        for(int i = 0; i < n; i++) {
            linkto[i].reset(); v[i].reset();
        }
    }
    void addEdge(int a, int b) {
        v[a][b] = v[b][a] = 1;
    }
    int popcount(const Int& val) {
        return val.count();
    }
    int lowbit(const Int& val) {
        return val._Find_first();
    }
    int ans, stk[N];
    int id[N], di[N], deg[N];
    Int cans;
    void maxclique(int elem_num, Int candi) {
        if(elem_num > ans) {
            ans = elem_num; cans.reset();
            for(int i = 0; i < elem_num; i++)
                cans[id[stk[i]]] = 1;
        }
        int potential = elem_num + popcount(candi);
        if(potential <= ans) return;
        int pivot = lowbit(candi);
        Int smaller_candi = candi & (~linkto[pivot]);
        while(smaller_candi.count() && potential > ans) {
            int next = lowbit(smaller_candi);
            candi[next] = !candi[next];
            smaller_candi[next] = !smaller_candi[next];
            potential--;
            if(next == pivot || (smaller_candi & linkto[next]).count()) {
                stk[elem_num] = next;
                maxclique(elem_num + 1, candi & linkto[next]);
            }
        }
    }
    void solve() {
        for(int i = 0; i < n; i++) {
            id[i] = i; deg[i] = v[i].count();
        }
        sort(id, id + n, [&](int id1, int id2) {
            return deg[id1] > deg[id2];
        });
        for(int i = 0; i < n; i++) di[id[i]] = i;
        for(int i = 0; i < n; i++)
            for(int j = 0; j < n; j++)
                if(v[i][j]) linkto[di[i]][di[j]] = 1;
        Int cand; cand.reset();
        for(int i = 0; i < n; i++) cand[i] = 1;
        ans = 1;
        cans.reset(); cans[0] = 1;
    }
}

```

```

    maxclique(0, cand);
    return ans;
} }solver;

```

5.5 MaximalClique 極大團

```

#define N 80
struct MaxClique{ // 0-base
    typedef bitset<N> Int;
    Int lnk[N] , v[N];
    int n;
    void init(int _n){
        n = _n;
        for(int i = 0 ; i < n ; i++){
            lnk[i].reset(); v[i].reset();
        }
    }
    void addEdge(int a , int b)
    { v[a][b] = v[b][a] = 1; }
    int ans , stk[N], id[N] , di[N] , deg[N];
    Int cans;
    void dfs(int elem_num, Int candi, Int ex){
        if(candi.none() && ex.none()){
            cans.reset();
            for(int i = 0 ; i < elem_num ; i++){
                cans[id[stk[i]]] = 1;
            }
            ans = elem_num; // cans is a maximal clique
            return;
        }
        int pivot = (candi ^ ex).Find_first();
        Int smaller_candi = candi & (~lnk[pivot]);
        while(smaller_candi.count()){
            int nxt = smaller_candi.Find_first();
            candi[nxt] = smaller_candi[nxt] = 0;
            ex[nxt] = 1;
            stk[elem_num] = nxt;
            dfs(elem_num+1, candi & lnk[nxt], ex & lnk[nxt]);
        }
    }
    int solve(){
        for(int i = 0 ; i < n ; i++){
            id[i] = i; deg[i] = v[i].count();
        }
        sort(id , id + n , [&](int id1, int id2){
            return deg[id1] > deg[id2]; });
        for(int i = 0 ; i < n ; i++) di[id[i]] = i;
        for(int i = 0 ; i < n ; i++){
            for(int j = 0 ; j < n ; j++){
                if(v[i][j]) lnk[di[i]][di[j]] = 1;
            }
        }
        ans = 1; cans.reset(); cans[0] = 1;
        dfs(0, Int(string(n, '1')), 0);
        return ans;
    }
} }solver;

```

5.6 BCC based on vertex 點雙聯通分量

```

#define PB push_back
#define REP(i, n) for(int i = 0; i < n; i++)
struct BccVertex {
    int n, nScc, step, dfn[MXN], low[MXN];
    vector<int> E[MXN], sccv[MXN];
    int top, stk[MXN];
    void init(int _n) { // 初始化n點
        n = _n; nScc = step = 0;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void addEdge(int u, int v) // 無向邊
    { E[u].PB(v); E[v].PB(u); }
    void DFS(int u, int f) {
        dfn[u] = low[u] = step++;
        stk[top++] = u;
        for (auto v:E[u]) {
            if (v == f) continue;
            if (dfn[v] == -1) {
                DFS(v, u);
                low[u] = min(low[u], low[v]);
            }
            if (low[v] >= dfn[u]) {
                int z;
                sccv[nScc].clear();
                do {

```

```

                    z = stk[--top];
                    sccv[nScc].PB(z);
                } while (z != v);
                sccv[nScc++].PB(u);
            }
        } else
            low[u] = min(low[u], dfn[v]);
    }
}
vector<vector<int>> solve() { // 回傳(size=2 橋, size
    >2 點雙聯通分量)
    vector<vector<int>> res;
    for (int i=0; i<n; i++)
        dfn[i] = low[i] = -1;
    for (int i=0; i<n; i++)
        if (dfn[i] == -1) {
            top = 0;
            DFS(i, i);
        }
    REP(i, nScc) res.PB(sccv[i]);
    return res;
}
}graph;

```

5.7 Strongly Connected Component 強連通分量

```

#define PB push_back
#define FZ(x) memset(x, 0, sizeof(x)) //fill zero
struct Scc{
    int n, nScc, vst[MXN], bln[MXN];
    vector<int> E[MXN], rE[MXN], vec;
    void init(int _n){
        n = _n;
        for (int i=0; i<MXN; i++)
            E[i].clear(), rE[i].clear();
    }
    void addEdge(int u, int v){
        E[u].PB(v); rE[v].PB(u);
    }
    void DFS(int u){
        vst[u]=1;
        for (auto v : E[u]) if (!vst[v]) DFS(v);
        vec.PB(u);
    }
    void rDFS(int u){
        vst[u] = 1; bln[u] = nScc;
        for (auto v : rE[u]) if (!vst[v]) rDFS(v);
    }
    void solve(){
        nScc = 0;
        vec.clear();
        FZ(vst);
        for (int i=0; i<n; i++)
            if (!vst[i]) DFS(i);
        reverse(vec.begin(), vec.end());
        FZ(vst);
        for (auto v : vec)
            if (!vst[v]){
                rDFS(v); nScc++;
            }
    }
};

```

5.8 Maximum General graph Matching

```

// should shuffle vertices and edges
const int N=100005, E=(2e5)*2+40;
struct Graph{ // 1-based; match: i <-> lnk[i]
    int to[E], bro[E], head[N], e, lnk[N], vis[N], stp, n;
    void init(int _n){
        stp=0; e=1; n=_n;
        for(int i=1; i<=n; i++) head[i]=lnk[i]=vis[i]=0;
    }
    void add_edge(int u, int v){
        to[e]=v, bro[e]=head[u], head[u]=e++;
        to[e]=u, bro[e]=head[v], head[v]=e++;
    }
};

```

```

bool dfs(int x){
    vis[x]=stp;
    for(int i=head[x];i;i=bro[i]){
        int v=to[i];
        if(!lnk[v]){ lnk[x]=v,lnk[v]=x; return true; }
    }
    for(int i=head[x];i;i=bro[i]){
        int v=to[i];
        if(vis[lnk[v]]<stp){
            int w=lnk[v]; lnk[x]=v,lnk[v]=x,lnk[w]=0;
            if(dfs(w)) return true;
            lnk[w]=v,lnk[v]=w,lnk[x]=0;
        }
    }
    return false;
}
int solve(){
    int ans=0;
    for(int i=1;i<=n;i++) if(!lnk[i]) stp++,ans+=dfs(i);
    return ans;
}
}graph;

```

5.9 Min Mean Cycle

```

/* minimum mean cycle O(VE) */
struct MMC{
#define E 101010
#define V 1021
#define inf 1e9
#define eps 1e-6
    struct Edge { int v,u; double c; };
    int n, m, prv[V][V], prve[V][V], vst[V];
    Edge e[E];
    vector<int> edgeID, cycle, rho;
    double d[V][V];
    void init( int _n )
    { n = _n; m = 0; }
    // WARNING: TYPE matters
    void addEdge( int vi , int ui , double ci )
    { e[ m ++ ] = { vi , ui , ci }; }
    void bellman_ford() {
        for(int i=0; i<n; i++) d[0][i]=0;
        for(int i=0; i<n; i++) {
            fill(d[i+1], d[i+1]+n, inf);
            for(int j=0; j<m; j++) {
                int v = e[j].v, u = e[j].u;
                if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
                    d[i+1][u] = d[i][v]+e[j].c;
                    prv[i+1][u] = v;
                    prve[i+1][u] = j;
                }
            }
        }
    }
    double solve(){
        // returns inf if no cycle, mmc otherwise
        double mmc=inf;
        int st = -1;
        bellman_ford();
        for(int i=0; i<n; i++) {
            double avg=-inf;
            for(int k=0; k<n; k++) {
                if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])/(n-k));
                else avg=max(avg,inf);
            }
            if (avg < mmc) tie(mmc, st) = tie(avg, i);
        }
        fill(vst,0); edgeID.clear(); cycle.clear(); rho.clear();
        for (int i=n; !vst[st]; st=prv[i--][st]) {
            vst[st]++;
            edgeID.PB(prve[i][st]);
            rho.PB(st);
        }
        while (vst[st] != 2) {
            if(rho.empty()) return inf;
            int v = rho.back(); rho.pop_back();
            cycle.PB(v);
            vst[v]++;
        }
    }
}

```

```

reverse(ALL(edgeID));
edgeID.resize(SZ(cycle));
return mmc;
} }mmc;

```

5.10 Directed Graph Min Cost Cycle

```

// works in O(N M)
#define INF 100000000000000LL
#define N 5010
#define M 200010
struct edge{
    int to; LL w;
    edge(int a=0, LL b=0): to(a), w(b){}
};
struct node{
    LL d; int u, next;
    node(LL a=0, int b=0, int c=0): d(a), u(b), next(c){}
}b[M];
struct DirectedGraphMinCycle{
    vector<edge> g[N], grev[N];
    LL dp[N][N], p[N], d[N], mu;
    bool inq[N];
    int n, bn, bsz, hd[N];
    void b_insert(LL d, int u){
        int i = d/mu;
        if(i >= bn) return;
        b[++bsz] = node(d, u, hd[i]);
        hd[i] = bsz;
    }
    void init( int _n ){
        n = _n;
        for( int i = 1 ; i <= n ; i ++ )
            g[ i ].clear();
    }
    void addEdge( int ai , int bi , LL ci )
    { g[ai].push_back(edge(bi,ci)); }
    LL solve(){
        fill(dp[0], dp[0]+n+1, 0);
        for(int i=1; i<=n; i++){
            fill(dp[i+1], dp[i+1]+n+1, INF);
            for(int j=1; j<=n; j++) if(dp[i-1][j] < INF){
                for(int k=0; k<(int)g[j].size(); k++){
                    dp[i][g[j][k].to] =min(dp[i][g[j][k].to],
                                            dp[i-1][j]+g[j][k].w);
                }
            }
            mu=INF; LL bunbo=1;
            for(int i=1; i<=n; i++) if(dp[n][i] < INF){
                LL a=-INF, b=1;
                for(int j=0; j<=n-1; j++) if(dp[j][i] < INF){
                    if(a*(n-j) < b*(dp[n][i]-dp[j][i])){
                        a = dp[n][i]-dp[j][i];
                        b = n-j;
                    }
                }
                if(mu*b > bunbo*a)
                    mu = a, bunbo = b;
            }
            if(mu < 0) return -1; // negative cycle
            if(mu == INF) return INF; // no cycle
            if(mu == 0) return 0;
            for(int i=1; i<=n; i++){
                for(int j=0; j<(int)g[i].size(); j++){
                    g[i][j].w *= bunbo;
                    memset(p, 0, sizeof(p));
                    queue<int> q;
                    for(int i=1; i<=n; i++){
                        q.push(i);
                        inq[i] = true;
                    }
                    while(!q.empty()){
                        int i=q.front(); q.pop(); inq[i]=false;
                        for(int j=0; j<(int)g[i].size(); j++){
                            if(p[g[i][j].to] > p[i]+g[i][j].w-mu){
                                p[g[i][j].to] = p[i]+g[i][j].w-mu;
                                if(!inq[g[i][j].to]){
                                    q.push(g[i][j].to);
                                    inq[g[i][j].to] = true;
                                }
                            }
                        }
                    }
                    for(int i=1; i<=n; i++) grev[i].clear();
                    for(int i=1; i<=n; i++)

```



```

    for(int j=0; j<(int)g[i].size(); j++){
        g[i][j].w += p[i]-p[g[i][j].to];
        grev[g[i][j].to].push_back(edge(i, g[i][j].w));
    }
    LL mlldc = n*mu;
    for(int i=1; i<=n; i++){
        bn=mlldc/mu, bsz=0;
        memset(hd, 0, sizeof(hd));
        fill(d+i+1, d+n+1, INF);
        b_insert(d[i]=0, i);
        for(int j=0; j<=bn-1; j++) for(int k=hd[j]; k; k=
            b[k].next){
            int u = b[k].u;
            LL du = b[k].d;
            if(du > d[u]) continue;
            for(int l=0; l<(int)g[u].size(); l++) if(g[u][l]
                .to > i){
                if(d[g[u][l].to] > du + g[u][l].w){
                    d[g[u][l].to] = du + g[u][l].w;
                    b_insert(d[g[u][l].to], g[u][l].to);
                } }
            for(int j=0; j<(int)grev[i].size(); j++) if(grev[i]
                [j].to > i)
                mlldc=min(mlldc, d[grev[i][j].to] + grev[i][j].w);
        }
        return mlldc / bunbo;
    } }graph;

```

5.11 K-th Shortest Path

```

// time: O(|E| \lg |E| + |V| \lg |V| + K)
// memory: O(|E| \lg |E| + |V|)
struct KSP{ // 1-base
    struct nd{
        int u, v; ll d;
        nd(int ui = 0, int vi = 0, ll di = INF)
        { u = ui; v = vi; d = di; }
    };
    struct heap{
        nd* edge; int dep; heap* chd[4];
    };
    static int cmp(heap* a, heap* b)
    { return a->edge->d > b->edge->d; }
    struct node{
        int v; ll d; heap* H; nd* E;
        node(){}
        node(ll _d, int _v, nd* _E)
        { d = _d; v = _v; E = _E; }
        node(heap* _H, ll _d)
        { H = _H; d = _d; }
        friend bool operator<(node a, node b)
        { return a.d > b.d; }
    };
    int n, k, s, t;
    ll dst[ N ];
    nd *nxt[ N ];
    vector<nd*> g[ N ], rg[ N ];
    heap *nullNd, *head[ N ];
    void init( int _n, int _k, int _s, int _t ){
        n = _n; k = _k; s = _s; t = _t;
        for( int i = 1 ; i <= n ; i ++ ){
            g[ i ].clear(); rg[ i ].clear();
            nxt[ i ] = NULL; head[ i ] = NULL;
            dst[ i ] = -1;
        }
    }
    void addEdge( int ui, int vi, ll di ){
        nd* e = new nd(ui, vi, di);
        g[ ui ].push_back( e );
        rg[ vi ].push_back( e );
    }
    queue<int> dfsQ;
    void dijkstra(){
        while(dfsQ.size()) dfsQ.pop();
        priority_queue<node> Q;
        Q.push(node(0, t, NULL));
        while (!Q.empty()){
            node p = Q.top(); Q.pop();
            if(dst[p.v] != -1) continue;
            dst[ p.v ] = p.d;
            nxt[ p.v ] = p.E;

```

```

            dfsQ.push( p.v );
            for(auto e: rg[ p.v ])
                Q.push(node(p.d + e->d, e->u, e));
        } }
        heap* merge(heap* curNd, heap* newNd){
            if(curNd == nullNd) return newNd;
            heap* root = new heap;
            memcpy(root, curNd, sizeof(heap));
            if(newNd->edge->d < curNd->edge->d){
                root->edge = newNd->edge;
                root->chd[2] = newNd->chd[2];
                root->chd[3] = newNd->chd[3];
                newNd->edge = curNd->edge;
                newNd->chd[2] = curNd->chd[2];
                newNd->chd[3] = curNd->chd[3];
            }
            if(root->chd[0]->dep < root->chd[1]->dep)
                root->chd[0] = merge(root->chd[0], newNd);
            else
                root->chd[1] = merge(root->chd[1], newNd);
            root->dep = max(root->chd[0]->dep, root->chd[1]->
                dep) + 1;
            return root;
        }
        vector<heap*> V;
        void build(){
            nullNd = new heap;
            nullNd->dep = 0;
            nullNd->edge = new nd;
            fill(nullNd->chd, nullNd->chd+4, nullNd);
            while(not dfsQ.empty()){
                int u = dfsQ.front(); dfsQ.pop();
                if(!nxt[ u ]) head[ u ] = nullNd;
                else head[ u ] = head[nxt[ u ]->v];
                V.clear();
                for( auto&& e : g[ u ] ){
                    int v = e->v;
                    if( dst[ v ] == -1 ) continue;
                    e->d += dst[ v ] - dst[ u ];
                    if( nxt[ u ] != e ){
                        heap* p = new heap;
                        fill(p->chd, p->chd+4, nullNd);
                        p->dep = 1;
                        p->edge = e;
                        V.push_back(p);
                    }
                }
                if(V.empty()) continue;
                make_heap(V.begin(), V.end(), cmp);
            }
            #define L(X) ((X<<1)+1)
            #define R(X) ((X<<1)+2)
            for( size_t i = 0 ; i < V.size() ; i ++ ){
                if(L(i) < V.size()) V[i]->chd[2] = V[L(i)];
                else V[i]->chd[2]=nullNd;
                if(R(i) < V.size()) V[i]->chd[3] = V[R(i)];
                else V[i]->chd[3]=nullNd;
            }
            head[u] = merge(head[u], V.front());
        }
        vector<ll> ans;
        void first_K(){
            ans.clear();
            priority_queue<node> Q;
            if( dst[ s ] == -1 ) return;
            ans.push_back( dst[ s ] );
            if( head[s] != nullNd )
                Q.push(node(head[s], dst[s]+head[s]->edge->d));
            for( int _ = 1 ; _ < k and not Q.empty() ; _ ++ ){
                node p = Q.top(); Q.pop();
                ans.push_back( p.d );
                if(head[ p.H->edge->v ] != nullNd){
                    q.H = head[ p.H->edge->v ];
                    q.d = p.d + q.H->edge->d;
                    Q.push(q);
                }
            }
            for( int i = 0 ; i < 4 ; i ++ )
                if( p.H->chd[ i ] != nullNd ){
                    q.H = p.H->chd[ i ];
                    q.d = p.d - p.H->edge->d + p.H->chd[ i ]->
                        edge->d;
                    Q.push( q );
                }
        }
        void solve(){ // ans[i] stores the i-th shortest path

```

```

    dijkstra();
    build();
    first_K(); // ans.size() might less than k
} }solver;

```

5.12 Floryd Warshall

```

for( int k=0 ; k < n ; k++ )
    for( int i=0 ; i < n ; i++ )
        for( int j=0 ; j < n ; j++ )
            if( dis[i][j] > dis[i][k]+dis[k][j] && dis[i][k]
                < INF && dis[k][j] < INF )
                dis[i][j]=dis[i][k]+dis[k][j];
for( int i=0 ; i < n ; i++ )
    for( int j=0 ; j < n ; j++ )
        for( int k=0 ; k < n && dis[i][j] != negINF ; k++ )
            if( dis[k][k] < 0 && dis[i][k] != INF && dis[k]
                [j] != INF )
                dis[i][j]=negINF;

```

5.13 SPFA

```

#define MXN 200005
struct SPFA{
    int n;
    LL inq[MXN], len[MXN];
    vector<LL> dis;
    vector<pair<int, LL>> edge[MXN];
    void init(int _n){
        n = _n;
        dis.clear(); dis.resize(n, 1e18);
        for(int i = 0; i < n; i++){
            edge[i].clear();
            inq[i] = len[i] = 0;
        }
        void addEdge(int u, int v, LL w){
            edge[u].push_back({v, w});
        }
        vector<LL> solve(int st = 0){
            deque<int> dq; //return {-1} if has negative cycle
            dq.push_back(st); //otherwise return dis from st
            inq[st] = 1; dis[st] = 0;
            while(!dq.empty()){
                int u = dq.front(); dq.pop_front();
                inq[u] = 0;
                for(auto [to, d] : edge[u]){
                    if(dis[to] > d+dis[u]){
                        dis[to] = d+dis[u];
                        len[to] = len[u]+1;
                        if(len[to] > n) return {-1};
                        if(inq[to]) continue;
                        if(!dq.empty() && dis[dq.front()] > dis[to]?
                            dq.push_front(to) : dq.push_back(to));
                        inq[to] = 1;
                    }
                }
                return dis;
            }
        }
    } }spfa;

```

5.14 Tree Hash

```

//限定root = 1
//從 dfs(1,1) 開始
int subtree_sz[MXN];
vector<int> edge[MXN];

int dfs(int u, int f) {
    vector<pair<int, int>> h;
    subtree_sz[u] = 1;
    for (int child : edge[u]) {
        if (child == f) continue;
        int tmp = dfs(child, u);
        h.push_back(make_pair(tmp, subtree_sz[child]));
        subtree_sz[u] += subtree_sz[child];
    }
    sort(h.begin(), h.end());

```

```

    int ret = subtree_sz[u];
    for (auto v : h) {
        ret = ((ret * p) % MOD + v.first) % MOD;
        ret = ret * v.second % MOD;
    }
    return ret;
}

```

5.15 HeavyLightDecomposition

```

// 詢問,修改複雜度  $O(\log^2 n)$ 
// 1-base

int sz[MXN], dep[MXN], son[MXN], fa[MXN];

// 第一次 dfs
// 找重兒子 需要紀錄當前節點的子樹大小(sz)、深度(dep)、
// 重兒子(son)、父節點(fa)
// 沒有子節點 son[x] = 0
void dfs_sz(int x, int f, int d) { //當前節點 x · 父節
    點 f · 深度 d
    sz[x] = 1; dep[x] = d; fa[x] = f;
    for(int i : edge[x]) {
        if(i == f) continue;
        dfs_sz(i, x, d+1);
        sz[x] += sz[i];
        if(sz[son[x]] < sz[i]) son[x] = i;
    }
}

// 第二次 dfs
int top[MXN]; // 每個節點所在的鏈的頂端節點
int dfn[MXN]; // 節點編號,編號為在線段樹上的位置
int rnk[MXN]; // 編號為哪個節點
int bottom[MXN]; // 維護每個節點的子樹中最大 dfn 編號
int cnt = 0;
int dfs_hld(int x, int f){
    top[x] = (son[fa[x]] == x ? top[fa[x]] : x);
    rnk[cnt] = x;
    bottom[x] = dfn[x] = cnt++;
    if(son[x]) bottom[x] = max(bottom[x], dfs_hld(
        son[x], x)); // 更新子樹最大編號
    for(int i : edge[x]){
        if(i == f || i == son[x]) continue;
        bottom[x] = max(bottom[x], dfs_hld(i, x)); //
            更新子樹最大編號
    }
    return bottom[x];
}

// 求出 lca
// 不斷跳鏈 · 直到 u,v 跳到同一條鏈上為止
// 每次跳鏈選所在的鏈頂端深度較深的一端往上跳
int getLca(int u, int v) {
    while(top[u] != top[v]){
        if(dep[top[u]] > dep[top[v]])
            u = fa[top[u]];
        else
            v = fa[top[v]];
    }
    return dep[u] > dep[v] ? v : u;
}

```

```

// 路徑權重總和
int query(int u, int v) {
    int ret = 0;
    while(top[u] != top[v]){
        if (dep[top[u]] > dep[top[v]]){
            ret += segtree.query(dfn[top[u]], dfn[u]);
            u = fa[top[u]];
        }
        else{
            ret += segtree.query(dfn[top[v]], dfn[v]);
            v = fa[top[v]];
        }
    }
    // 最後到同一條鏈上
    ret += segtree.query(min(dfn[u], dfn[v]), max(dfn[u],
        dfn[v]));
}

```

```

    return ret;
}

```

5.16 差分約束

約束條件 $V_j - V_i \leq W$ addEdge(V_i, V_j, W) and run bellman-ford or spfa

6 String

6.1 PalTree $O(n)$

```

// state[i]代表第i個字元為結尾的最長回文編號
// len[s]是對應的回文長度
// num[s]是有幾個回文後綴
// cnt[s]是這個回文子字串在整個字串中的出現次數
// fail[s]是他長度次長的回文後綴 · aba的fail是a
const int MXN = 1000010;
struct PalT{
    int nxt[MXN][26], fail[MXN], len[MXN];
    int tot, lst, n, state[MXN], cnt[MXN], num[MXN];
    int diff[MXN], sfail[MXN], fac[MXN], dp[MXN];
    char s[MXN] = {-1};
    int newNode(int l, int f){
        len[tot] = l, fail[tot] = f, cnt[tot] = num[tot] = 0;
        memset(nxt[tot], 0, sizeof(nxt[tot]));
        diff[tot] = (l > 0 ? l - len[f] : 0);
        sfail[tot] = (l > 0 && diff[tot] == diff[f] ? sfail[f] : f);
        return tot++;
    }
    int getfail(int x){
        while(s[n - len[x] - 1] != s[n]) x = fail[x];
        return x;
    }
    int getmin(int v){
        dp[v] = fac[n - len[sa[v]] - diff[v]];
        if(diff[v] == diff[fail[v]])
            dp[v] = min(dp[v], dp[fail[v]]);
        return dp[v] + 1;
    }
    int push(){
        int c = s[n] - 'a', np = getfail(lst);
        if(!lst || !nxt[np][c]){
            lst = newNode(len[np] + 2, getfail(fail[np])[c]);
            nxt[np][c] = lst; num[lst] = num[fail[lst]] + 1;
        }
        fac[n] = n;
        for(int v = lst; len[v] > 0; v = sfail[v])
            fac[n] = min(fac[n], getmin(v));
        return ++cnt[lst], lst;
    }
    void init(const char *_s){
        tot = lst = n = 0;
        newNode(0, 1), newNode(-1, 1);
        for(; *_s; ++_s) s[n + 1] = *_s, ++n, state[n - 1] = push();
        for(int i = tot - 1; i > 1; i--) cnt[fail[i]] += cnt[i];
    }
}palt;

```

6.2 Longest Increasing Subsequence

```

vector<int> getLIS(vector<int> a){
    vector<int> lis;
    for(int i : a){
        if(lis.empty() || lis.back() < i)    lis.push_back(i);
        else    *lower_bound(lis.begin(), lis.end(), i) = i;
    }
    return lis;
}

```

6.3 Longest Common Subsequence $O(nlgn)$

```

int LCS(string& s1, string& s2) {
    vector<int> p[128]; // 假設字元範圍為 0 ~ 127
    for (int i = 0; i < s2.size(); ++i) p[s2[i]].push_back(i);
    vector<int> v;
    v.push_back(-1);

    for (int i = 0; i < s1.size(); ++i)
        for (int j = p[s1[i]].size() - 1; j >= 0; --j) {
            int n = p[s1[i]][j];

            if (n > v.back())
                v.push_back(n);
            else
                *lower_bound(v.begin(), v.end(), n) = n;
        }
    return v.size() - 1;
}

```

6.4 KMP

```

/* len-failure[k]:
在k結尾的情況下，這個子字串可以由開頭
長度為(len-failure[k])的部分重複出現來表達

failure[k]為次長相同前綴後綴
如果我們不只想求最多，而且以0-base做為考量
，那可能的長度由大到小會是
failuer[k]、failure[failuer[k]-1]
、failure[failure[failuer[k]-1]-1]..
直到有值為0為止 */
int failure[MXN];
vector<int> KMP(string& t, string& p){
    vector<int> ret;
    if (p.size() > t.size()) return;
    for (int i = 1, j = failure[0] = -1; i < p.size(); ++i){
        while (j >= 0 && p[j + 1] != p[i])
            j = failure[j];
        if (p[j + 1] == p[i]) j++;
        failure[i] = j;
    }
    for (int i = 0, j = -1; i < t.size(); ++i){
        while (j >= 0 && p[j + 1] != t[i])
            j = failure[j];
        if (p[j + 1] == t[i]) j++;
        if (j == p.size() - 1){
            ret.push_back(i - p.size() + 1);
            j = failure[j];
        }
    }
}

```

6.5 SAIS $O(n)$

```

/** SA · 將字串的所有後綴排序後的數組 */
/* SA[i]儲存排序後第i小的後綴從哪裡開始 */
/** H[i]為第i小的字串跟第i-1小的LCP */
/** 註：LCP(Longest Common Prefix) */
/** ex: S = "babd", SA[0] = 1("abd") */
/** SA[1] = 0("babd"), SA[2] = 2("bd") */
/** H[0] = 0, H[1] = 0, H[2] = 1("b") */
/* 傳入參數: ip 陣列放字串 · len為字串長度 */
/* 需保證ip[len]為0，且字串裡的元素不為0 */
const int N = 300010;
struct SA{
#define REP(i,n) for (int i=0; i<int(n); i++)
#define REP1(i,a,b) for (int i=a; i<=int(b); i++)
    bool _t[N*2];
    int _s[N*2], _sa[N*2], _c[N*2], x[N], _p[N], _q[N*2],
        hei[N], r[N];
    int operator [] (int i){ return _sa[i]; }
    void build(int *_s, int n, int m){
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    }
}

```

```

void mkhei(int n){
    REP(i,n) r[_sa[i]] = i;
    hei[0] = 0;
    REP(i,n) if(r[i]) {
        int ans = i>0 ? max(hei[r[i-1]] - 1, 0) : 0;
        while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans++;
        hei[r[i]] = ans;
    }
}
void sais(int *s, int *sa, int *p, int *q, bool *t,
    int *c, int n, int z){
    bool uniq = t[n-1] = true, neq;
    int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
        lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
    memcpy(x, c, sizeof(int) * z); \
    XD; \
    memcpy(x + 1, c, sizeof(int) * (z - 1)); \
    REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[sa[i]-1]]++ = sa[i]-1; \
    memcpy(x, c, sizeof(int) * z); \
    for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i]-1]) sa[--x[sa[i]-1]] = sa[i]-1;
    MS0(c, z);
    REP(i,n) uniq &= ++c[s[i]] < 2;
    REP(i,z-1) c[i+1] += c[i];
    if(uniq) { REP(i,n) sa[--c[s[i]]] = i; return; }
    for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s[i+1] ? t[i+1] : s[i]<s[i+1]);
    MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[--x[s[i]]]=p[q[i]=nn++]=i);
    REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1]) {
        neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa[i])*sizeof(int));
        ns[q[lst=sa[i]]]=nmzx+=neq;
    }
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx + 1);
    MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[s[p[nsa[i]]]]] = p[nsa[i]]);
}
}
int H[ N ], SA[ N ];
void suffix_array(int* ip, int len) {
    // should padding a zero in the back
    // ip is int array, len is array length
    // ip[0..n-1] != 0, and ip[len] = 0
    ip[len++] = 0;
    sa.build(ip, len, 128);
    for (int i=0; i<len; i++) {
        H[i] = sa.hei[i + 1];
        SA[i] = sa._sa[i + 1];
    }
    // resulting height, sa array \in [0,len)
}

```

6.6 Z Value $O(n)$

```

//z[i] = lcp(s[1...n-1],s[i...n-1])
int z[ MAXN ];
void Z_value(const string& s) {
    int i, j, left, right, len = s.size();
    left=right=0; z[0]=len;
    for(i=1;i<len;i++) {
        j=max(min(z[i-left],right-i),0);
        for(;i+j<len&&s[i+j]==s[j];j++);
        z[i]=j;
        if(i+z[i]>right) {
            right=i+z[i];
            left=i;
        }
    }
}

```

6.7 Manacher Algorithm $O(n)$

```

// 求以每個字元為中心的最長回文半徑
// 頭尾以及每個字元間都加入一個
// 沒出現過的字元 · 這邊以' @ '為例

```

```

// s為傳入的字串 · len為字串長度
// z為儲存答案的陣列 (有包含'@'要小心)
// ex: s = "abaac" -> "@a@b@a@a@c@"
// z = [12141232121]
void z_value_pal(char *s,int len,int *z){
    len=(len<1)+1;
    for(int i=len-1;i>=0;i--){
        s[i]=i&1?s[i>>1]:'@';
        z[0]=1;
        for(int i=1,l=0,r=0;i<len;i++){
            z[i]=i<r?min(z[l+l-i],r-i):1;
            while(i-z[i]>=0&&i+z[i]<len&&s[i-z[i]]==s[i+z[i]])
                ++z[i];
            if(i+z[i]>r) l=i,r=i+z[i];
        }
    }
}

```

6.8 Smallest Rotation

```

//rotate(begin(s),begin(s)+minRotation(s),end(s))
int minRotation(string s) {
    int a = 0, N = s.size(); s += s;
    rep(b,0,N) rep(k,0,N) {
        if(a+k == b || s[a+k] < s[b+k])
            {b += max(0, k-1); break;}
        if(s[a+k] > s[b+k]) {a = b; break;}
    } return a;
}

```

6.9 Cyclic LCS

```

#define L 0
#define LU 1
#define U 2
const int mov[3][2]={0,-1, -1,-1, -1,0};
int al,bl;
char a[ MAXL*2 ],b[ MAXL*2 ]; // 0-indexed
int dp[ MAXL*2 ][ MAXL ];
char pred[ MAXL*2 ][ MAXL ];
inline int lcs_length(int r) {
    int i=r+al,j=bl,l=0;
    while(i>r) {
        char dir=pred[i][j];
        if(dir==LU) l++;
        i+=mov[dir][0];
        j+=mov[dir][1];
    }
    return l;
}
inline void reroot(int r) { // r = new base row
    int i=r,j=1;
    while(j<=bl&&pred[i][j]!=LU) j++;
    if(j>bl) return;
    pred[i][j]=L;
    while(i<2*al&&j<=bl) {
        if(pred[i+1][j]==U) {
            i++;
            pred[i][j]=L;
        } else if(j<bl&&pred[i+1][j+1]==LU) {
            i++;
            j++;
            pred[i][j]=L;
        } else {
            j++;
        }
    }
}
int cyclic_lcs() {
    // a, b, al, bl should be properly filled
    // note: a WILL be altered in process
    // -- concatenated after itself
    char tmp[ MAXL ];
    if(al>bl) {
        swap(al,bl);
        strcpy(tmp,a);
        strcpy(a,b);
        strcpy(b,tmp);
    }
    strcpy(tmp,a);
    strcat(a,tmp);
    // basic lcs
}

```

```

for(int i=0;i<=2*al;i++) {
    dp[i][0]=0;
    pred[i][0]=U;
}
for(int j=0;j<=bl;j++) {
    dp[0][j]=0;
    pred[0][j]=L;
}
for(int i=1;i<=2*al;i++) {
    for(int j=1;j<=bl;j++) {
        if(a[i-1]==b[j-1]) dp[i][j]=dp[i-1][j-1]+1;
        else dp[i][j]=max(dp[i-1][j],dp[i][j-1]);
        if(dp[i][j-1]==dp[i][j]) pred[i][j]=L;
        else if(a[i-1]==b[j-1]) pred[i][j]=LU;
        else pred[i][j]=U;
    }
}
// do cyclic lcs
int clcs=0;
for(int i=0;i<al;i++) {
    clcs=max(clcs,lcs_length(i));
    reroot(i+1);
}
// recover a
a[al]='\0';
return clcs;
}

```

6.10 Hash

```

//字串雜湊前的idx是0-base · 雜湊後為1-base
//即區間為 [0,n-1] -> [1,n]
//若要取得區間[L,R]的值得
//H[R] - H[L-1] * p^(R-L+1)
//cmp為比較從i開始長度為len的字串和
//從j開始長度為len的字串是否相同
#define x first
#define y second
pair<int,int> Hash[MXN];
void build(const string& s){
    pair<int,int> val = make_pair(0,0);
    Hash[0]=val;
    for(int i=1; i<=s.size(); i++){
        val.x = (val.x * P1 + s[i-1]) % MOD;
        val.y = (val.y * P2 + s[i-1]) % MOD;
        Hash[i] = val;
    }
}
bool cmp( int i, int j, int len ) {
    return ((Hash[i+len-1].x-Hash[i-1].x*qpow(P1,len)%
        MOD+MOD)%MOD == (Hash[j+len-1].x-Hash[j-1].x*
        qpow(P1,len)%MOD+MOD)%MOD)
        && ((Hash[i+len-1].y-Hash[i-1].y*qpow(P2,len)%MOD+
        MOD)%MOD == (Hash[j+len-1].y-Hash[j-1].y*qpow(
        P2,len)%MOD+MOD)%MOD);
}

```

7 Data Structure

7.1 Segment tree

```

// !!!注意build()時初始化用的陣列也是1-base
#define cl(x) (x*2)
#define cr(x) (x*2+1)

struct segmentTree {
    int n;
    vector<int> seg, tag, cov;
    segmentTree( int _n ): n(_n) {
        seg=tag=cov=vector<int>(n*4,0);
    }
    void push( int i, int L, int R ) {
        if( cov[i] ) {
            seg[i]=cov[i]*(R-L+1);
            if( L < R ) {
                cov[cl(i)]=cov[cr(i)]=cov[i];

```

```

                tag[cl(i)]=tag[cr(i)]=0;
            }
            cov[i]=0;
        }
        if( tag[i] ) {
            seg[i]+=tag[i]*(R-L+1);
            if( L < R ) {
                tag[cl(i)]=tag[i];
                tag[cr(i)]=tag[i];
            }
            tag[i]=0;
        }
    }
    void pull( int i, int L, int R ) {
        if( L >= R ) return;
        int mid=(L+R)>>1;
        push(cl(i),L,mid);
        push(cr(i),mid+1,R);
        seg[i]=seg[cl(i)]+seg[cr(i)];
    }
    void build( vector<int>& arr, int i=1, int L=1, int
        R=-1 ) {
        if( R == -1 ) R=n;
        if( L == R ) {
            seg[i]=arr[L];
            return;
        }
        int mid=(L+R)>>1;
        build(arr,cl(i),L,mid);
        build(arr,cr(i),mid+1,R);
        pull(i,L,R);
    }
    int query( int rL, int rR, int i=1, int L=1, int R
        =-1 ) {
        if( R == -1 ) R=n;
        push(i,L,R);
        if( rL <= L && R <= rR ) return seg[i];
        int mid=(L+R)>>1, ret=0;
        if( rL <= mid ) ret+=query(rL,rR,cl(i),L,mid);
        if( mid < rR ) ret+=query(rL,rR,cr(i),mid+1,R);
        return ret;
    }
    void update( int rL, int rR, int val, int i=1, int
        L=1, int R=-1 ) {
        if( R == -1 ) R=n;
        push(i,L,R);
        if( rL <= L && R <= rR ) {
            tag[i]=val;
            return;
        }
        int mid=(L+R)>>1;
        if( rL <= mid ) update(rL,rR,val,cl(i),L,mid);
        if( mid < rR ) update(rL,rR,val,cr(i),mid+1,R);
        pull(i,L,R);
    }
    void cover( int rL, int rR, int val, int i=1, int L
        =1, int R=-1 ) {
        if( R == -1 ) R=n;
        push(i,L,R);
        if( rL <= L && R <= rR ) {
            cov[i]=val;
            return;
        }
        int mid=(L+R)>>1;
        if( rL <= mid ) cover(rL,rR,val,cl(i),L,mid);
        if( mid < rR ) cover(rL,rR,val,cr(i),mid+1,R);
        pull(i,L,R);
    }
};

```

7.2 持久化 SMT

```

struct node{
    node *l, *r;
    int val;
};

vector<node *> ver;
int arr[MXN] = {0};

```



```
//0-base
struct SegmentTree{
    int n;
    node *root;
    void build(int _n){
        n = _n;
        root = build(0, n-1);
    }
    node* build(int L, int R){
        node *x = new node();
        if(L == R){ x->val = arr[L]; return x;}
        int mid = (L+R)/2;
        x->l = build(L, mid);
        x->r = build(mid+1, R);
        x->val = x->l->val + x->r->val;
        return x;
    }
    int query(node *ro, int L, int R){return query(ro, 0, n-1, L, R);}
    int query(int L, int R){return query(root, 0, n-1, L, R);}
    int query(node *x, int L, int R, int recL, int recR){
        if(recL <= L && R <= recR) return x->val;
        int mid = (L+R)/2, res = 0;
        if(recL <= mid) res += query(x->l, L, mid, recL, recR);
        if(mid < recR) res += query(x->r, mid+1, R, recL, recR);
        return res;
    }
    void update(int pos, int v){update(root, 0, n-1, pos, v);}
    void update(node *x, int L, int R, int pos, int v){
        if(L == R){ x->val = v; arr[L] = v; return;}
        int mid = (L+R)/2;
        if(pos <= mid) update(x->l, L, mid, pos, v);
        else update(x->r, mid+1, R, pos, v);
        x->val = x->l->val + x->r->val;
    }
    node *update_ver(node *pre, int l, int r, int pos, int v){
        int v){
            node *x = new node(); //當前位置建立新節點
            if(l == r){
                x->val = v;
                return x;
            }
            int mid = (l+r)>>1;
            if(pos <= mid){ //更新左邊
                x->l = update_ver(pre->l, l, mid, pos, v); //左邊節點連向新節點
                x->r = pre->r; //右邊連到原本的右邊
            }
            else{ //更新右邊
                x->l = pre->l; //左邊連到原本的左邊
                x->r = update_ver(pre->r, mid+1, r, pos, v); //右邊節點連向新節點
            }
            x->val = x->l->val + x->r->val;
            return x;
        }
    } seg;

    void add_ver(int x,int v){ //修改位置 x 的值为 v
        ver.push_back(seg.update_ver(ver.back(), 0, seg.n-1, x, v));
    }
}
```

7.3 Trie

```
struct trie{
    trie *nxt[26];
    int cnt; //紀錄有多少個字串以此節點結尾
    int sz; //有多少字串的前綴包括此節點
    trie():cnt(0),sz(0){
        memset(nxt,0,sizeof(nxt));
    }
};

trie *root = new trie(); //創建新的字典樹

void insert(string& s){
```

```
trie *now = root; // 每次從根結點出發
for(auto i:s){
    now->sz++;
    if(now->nxt[i-'a'] == NULL){
        now->nxt[i-'a'] = new trie();
    }
    now = now->nxt[i-'a']; //走到下一個字母
}
now->cnt++; now->sz++;
}

int query_prefix(string& s){ //查詢有多少前綴為 s
    trie *now = root; // 每次從根結點出發
    for(auto i:s){
        if(now->nxt[i-'a'] == NULL){
            return 0;
        }
        now = now->nxt[i-'a'];
    }
    return now->sz;
}

int query_count(string& s){ //查詢字串 s 出現次數
    trie *now = root; // 每次從根結點出發
    for(auto i:s){
        if(now->nxt[i-'a'] == NULL){
            return 0;
        }
        now = now->nxt[i-'a'];
    }
    return now->cnt;
}
```

7.4 Treap (interval reverse)

//拆出[a,b]區間就如同下面所展示先使用splitByTh()拆出左右,再把左區間拆成l, m最後merge()回去
//反轉區間時又記得使用^=可以直接反轉01

//treap拆區間時從後面拆是因為這樣[a,b]的關係
//不用重新考慮·要是先拆前面b的位置會變成b-a+1
//splitByTh(root,a-1,l,m);
//splitByTh(m,b-a+1,m,r);

```
mt19937 gen(chrono::steady_clock::now().
    time_since_epoch().count());
struct Treap {
    int key, pri, sz, tag, sum;
    Treap *L, *R;
    Treap( int val ) {
        sum=key=val, pri=gen(), sz=1, tag=0;
        L=R=NULL;
    };
};
int Size( Treap *a ) { return !a?0:a->sz; }
void pull( Treap *a ) {
    a->sz=Size(a->L)+Size(a->R)+1;
    a->sum=a->key;
    if( a->L ) a->sum+=a->L->sum;
    if( a->R ) a->sum+=a->R->sum;
}
void push( Treap *a ) {
    if( a && a->tag ) {
        swap(a->L,a->R);
        if( a->L ) a->L->tag^=1;
        if( a->R ) a->R->tag^=1;
        a->tag=0;
    }
}
Treap *merge(Treap *a, Treap *b) {
    if( !a || !b ) return a?a:b;
    push(a), push(b);
    if( a->pri > b->pri ) {
        a->R=merge(a->R,b);
        pull(a); return a;
    }
    b->L=merge(a,b->L);
    pull(b); return b;
}
void print(Treap *a) {
    if( !a ) return;
```

```

push(a);
print(a->L);
cout.put(a->key);
print(a->R);
}
Treap *buildTreap( int n, string& str ) {
    Treap *root=NULL;
    for( int i=0 ; i < n ; i++ )
        root=merge(root,new Treap(str[i]));
    return root;
}
void splitbyk( Treap *x, int k, Treap *&a, Treap *&b )
{
    if(!x) a=b=NULL;
    else if( x->key <= k ) {
        a=x;
        splitbyk(x->R,k,a->R,b);
        pull(a);
    }
    else {
        b=x;
        splitbyk(x->L,k,a,b->L);
        pull(b);
    }
}
void splitByTh( Treap *x, int k, Treap *&a, Treap *&b )
{
    if( !x ) { a=b=NULL; return; }
    push(x);
    if( Size(x->L)+1 <= k ) {
        a=x;
        splitByTh(x->R,k-Size(x->L)-1,a->R,b);
        pull(a);
    }
    else {
        b=x;
        splitByTh(x->L,k,a,b->L);
        pull(b);
    }
}
signed main() {
    string str;
    int n, m;
    cin>>n>>m>>str;
    Treap *root;
    root=buildTreap(n,str);
    for( int i=0 ; i < m ; i++ ) {
        int a, b;
        cin>>a>>b;
        Treap *l, *m, *r;
        splitByTh(root,b,l,r);
        splitByTh(l,a-1,l,m);
        m->tag^=1;
        root=merge(l,merge(m,r));
    }
    print(root);
}

```

7.5 Treap (interval erase)

```

//區間移除使用bitset維護區間值
mt19937 gen(chrono::steady_clock::now().
    time_since_epoch().count());
struct Treap {
    char key;
    int pri, sz;
    bitset<128> tag;
    Treap *L, *R;
    Treap( char val ) {
        key=val, pri=gen(), sz=1;
        L=R=NULL;
        tag.set(key);
    };
};
int Size( Treap *a ) { return !a?a->sz;}
void pull( Treap *a ) {
    if( !a ) return;
    a->sz=Size(a->L)+Size(a->R)+1;
    a->tag=a->tag.reset();
    a->tag=a->tag.set(a->key);
    if( a->L ) a->tag|=a->L->tag;

```

```

    if( a->R ) a->tag|=a->R->tag;
}
Treap *merge( Treap *a, Treap *b ) {
    if( !a || !b ) return a?a:b;
    if( a->pri > b->pri ) {
        a->R=merge(a->R,b);
        pull(a);
        return a;
    }
    b->L=merge(a,b->L);
    pull(b);
    return b;
}
Treap *buildTreap( int n, string& str ) {
    Treap *root=NULL;
    for( int i=0 ; i < n ; i++ )
        root=merge(root,new Treap(str[i]));
    return root;
}
void print( Treap *a ) {
    if( !a ) return;
    print(a->L);
    cout.put(a->key);
    print(a->R);
}
void splitByTh( Treap *x, int k, Treap *&a, Treap *&b )
{
    if( !x ) { a=b=NULL; return; }
    if( Size(x->L)+1 <= k ) {
        a=x;
        splitByTh(x->R,k-Size(x->L)-1,a->R,b);
        pull(a);
    }
    else {
        b=x;
        splitByTh(x->L,k,a,b->L);
        pull(b);
    }
}
void erase( Treap *&x, char ch ) {
    if( !x || !x->tag.test(ch) ) return;
    erase(x->L,ch);
    erase(x->R,ch);
    if( x->key == ch ) {
        Treap *l=x->L, *r=x->R;
        x=NULL;
        x=merge(l,r);
    }
    pull(x);
}
signed main() {
    string str;
    int n, m;
    cin>>n>>m>>str;
    Treap *root;
    root=buildTreap(n,str);
    for( int i=0 ; i < m ; i++ ) {
        char c;
        int a, b;
        cin>>a>>b>>c;
        Treap *l, *m, *r;
        if( !root || !root->tag.test(c) ) continue;
        splitByTh(root,b,l,r);
        splitByTh(l,a-1,l,m);
        if( m || !m->tag.test(c) ) erase(m,c);
        root=merge(l,merge(m,r));
    }
    print(root);
}

```

7.6 BIT

```
#define lowbit(x) (x&-x)
struct BIT {
    int n;
    vector<int> bit;
    BIT( int t ) {
        n=t;
        bit=vector<int>(n+1,0);
    }
    ~BIT() {bit.clear();}
    void update( int pos, int val ) {
        while ( pos <= n ) {
            bit[pos]+=val;
            pos+=lowbit(pos);
        }
    }
    void range_update( int L, int R, int val ) {
        update(L,val);
        update(R+1,-val);
    }
    int query( int pos ) {
        int res=0;
        while( pos ) {
            res+=bit[pos];
            pos-=lowbit(pos);
        }
        return res;
    }
    int range_query( int L, int R ) {
        return query(R)-query(L-1);
    }
};
```

7.7 Black Magic

```
#include <bits/extc++.h>
using namespace __gnu_pbds;
typedef tree<int,null_type,less<int>,rb_tree_tag,
    tree_order_statistics_node_update> set_t;
// tree<int,null_type,less_equal<int>,rb_tree_tag,
//    tree_order_statistics_node_update> s;
#include <ext/pb_ds/assoc_container.hpp>
typedef cc_hash_table<int,int> umap_t;
// gp_hash_table<int, int>
typedef priority_queue<int> heap;
#include<ext/rope>
using namespace __gnu_cxx;
int main(){
    // Insert some entries into s.
    set_t s; s.insert(12); s.insert(505);
    // The order of the keys should be: 12, 505.
    assert(*s.find_by_order(0) == 12);
    assert(*s.find_by_order(3) == 505);
    // The order of the keys should be: 12, 505.
    assert(s.order_of_key(12) == 0);
    assert(s.order_of_key(505) == 1);
    // Erase an entry.
    s.erase(12);
    // The order of the keys should be: 505.
    assert(*s.find_by_order(0) == 505);
    // The order of the keys should be: 505.
    assert(s.order_of_key(505) == 0);

    heap h1 , h2; h1.join( h2 );

    rope<char> r[ 2 ];
    r[ 1 ] = r[ 0 ]; // persistenet
    string t = "abc";
    r[ 1 ].insert( 0 , t.c_str() );
    r[ 1 ].erase( 1 , 1 );
    cout << r[ 1 ].substr( 0 , 2 );
}
```

8 Others

8.1 SOS dp

```
for(int i = 0; i<(1<<N); ++i)
    F[i] = A[i];
for(int i = 0; i < N; ++i) for(int mask = 0; mask < (1<<N); ++mask){
    if(mask & (1<<i))
        F[mask] += F[mask^(1<<i)];
}
```

8.2 Max subrectangle

```
const int N = 1e5+5;
int n, a[N], l[N], r[N];
long long ans;
int main() {
    while (cin>>n) {
        ans = 0;
        for (int i = 1; i <= n; i++) cin>>a[i], l[i] = r[i] = i;
        for (int i = 1; i <= n; i++)
            while (l[i] > 1 && a[i] <= a[l[i] - 1]) l[i] = l[l[i] - 1];
        for (int i = n; i >= 1; i--)
            while (r[i] < n && a[i] <= a[r[i] + 1]) r[i] = r[r[i] + 1];
        for (int i = 1; i <= n; i++)
            ans = max(ans, (long long)(r[i] - l[i] + 1) * a[i]);
        cout<<ans<<"\n";
    }
}
```

8.3 De Bruijn sequence

```
// return cyclic array of length k^n such that every
// array of length n using 0~k-1 appears as a subarray.
vector<int> DeBruijn(int k,int n){
    if(k==1) return {0};
    vector<int> aux(k*n),res;
    function<void(int,int)> f=[&](int t,int p)->void{
        if(t>n){ if(n%p==0)
            for(int i=1;i<=p;++i) res.push_back(aux[i]);
        }
        aux[t]=aux[t-p]; f(t+1,p);
        for(aux[t]=aux[t-p]+1;aux[t]<k;++aux[t]) f(t+1,t);
    };
    f(1,1); return res;
}
```

8.4 CDQ 分治

```
//cdq分治使用的結構u, v, w為排序物的三個維度
//ans記錄了有幾項三維都小於等於自己
//cnt記錄了相同物有幾個，在使用cdq之前必先去重，
//並且將相同元素紀錄至cnt中，可使用map來做到這步
//cdq使用的BIT就是普通求和的BIT，大小就開維度的
//值域範圍，若值域大於2e6則要先進行離散化
struct triple {int u, v, w, ans, cnt;};
int n, k;
BIT *bt;
void cdq( int L, int R, vector<triple>& arr ) {
    if( R-L <= 1 ) return;
    int mid=L+R>>1;
    vector<triple> temp;
    cdq(L,mid,arr), cdq(mid,R,arr);
    for( int i=L, j=mid; i < mid || j < R; ){
```

```

    for( i < mid && ( j >= R || arr[i].v <= arr[j].v )
        ; i++ ) {
        bt->update(arr[i].w,arr[i].cnt);
        temp.push_back(arr[i]);
    }
    if( j < R ) {
        arr[j].ans+=bt->query(arr[j].w);
        temp.push_back(arr[j]);
        j++;
    }
}
for( int i=L ; i < mid ; i++ )
    bt->update(arr[i].w,-arr[i].cnt);
copy(temp.begin(),temp.end(),arr.begin()+L);
}
signed main()
{
    cin>>n>>k;
    map<tuple<int,int,int>,int> mp;
    vector<int> res(n,0);
    vector<triple> arr;
    bt=new BIT(k+1);
    for( int i=0 ; i < n ; i++ ) {
        int x, y, z;
        cin>>x>>y>>z;
        mp[{x,y,z}]++;
    }
    for( auto t : mp )
        arr.push_back({get<0>(t.first),get<1>(t.first),get
            <2>(t.first),0,t.second});
    cdq(0,arr.size(),arr);
    for( auto &[x,y,z,a,b] : arr ) res[a+b-1]+=b;
    for( int i : res ) cout<<i<<"\n";
}

```

8.5 3D LIS

```

#define lowbit(x) (x&-x)
const int MAXN=1e5+5;
struct BIT {
    int n;
    vector<int> bit;
    BIT( int _n ):n(_n), bit(_n+1,0) {}
    int query( int x ) {
        int res=0;
        for( ; x > 0 ; x-=lowbit(x) ) res=max(res,bit[x]);
        return res;
    }
    void update( int x, int val ) {
        for( ; x <= n ; x+=lowbit(x) ) {
            if( val < 0 ) bit[x]=0;
            else bit[x]=max(bit[x],val);
        }
    }
}bt(MAXN);
struct triple {
    int u, v, w, ans, cnt;
    bool operator<( triple b ) { return u<b.u; }
};
bool cmp( triple a, triple b ) {return a.v<b.v;}
void cdq( int L, int R, vector<triple>& arr ) {
    if( R-L <= 1 ) return;
    int mid=L+R>>1;
    cdq(L,mid,arr);
    sort(arr.begin()+L,arr.begin()+mid,cmp);
    sort(arr.begin()+mid,arr.begin()+R,cmp);
    for( int i=L, j=mid ; i < mid || j < R ; ) {
        for( ; i < mid && ( j >= R || arr[i].v < arr[j].v )
            ; i++ ) bt.update(arr[i].w,arr[i].ans);
        if( j < R ) {
            arr[j].ans=max(bt.query(arr[j].w-1)+1,arr[j].ans);
            j++;
        }
    }
    for( int i=L ; i < mid ; i++ ) bt.update(arr[i].w,-1);
    sort(arr.begin()+L,arr.begin()+R);
    cdq(mid,R,arr);
}

```

```

signed main()
{
    ios_base::sync_with_stdio(0); cin.tie(0); cout.tie(0);
    ;
    int n, res=0;
    cin>>n;
    vector<int> ls;
    vector<triple> arr;
    for( int i=0 ; i < n ; i++ ) {
        int a, b;
        cin>>a>>b;
        arr.push_back({i,a,b,1,1}); //{第一維,第二維,第三維,
            答案,數量}
        ls.push_back(b);
    }
    sort(ls.begin(),ls.end());
    ls.resize(unique(ls.begin(),ls.end())-ls.begin());
    for( auto &t : arr ) t.w=lower_bound(ls.begin(),ls.
        end(),t.w)-ls.begin()+1;
    n=arr.size();
    cdq(0,n,arr);
    for( int i=0 ; i < n ; i++ ) res=max(res,arr[i].ans);
    cout<<res<<"\n";
}

```

8.6 Aho-Corasick

```

struct AAutomata{
    struct Node{
        int cnt,i;
        Node *go[26], *fail, *dic;
        Node (){
            cnt = 0; fail = 0; dic = 0; i = 0;
            memset(go,0,sizeof(go));
        }
    }pool[1048576],*root;
    int nMem,n_pattern;
    Node* new_Node(){
        pool[nMem] = Node();
        return &pool[nMem++];
    }
    void init() {
        nMem=0;root=new_Node();n_pattern=0;
        add("");
    }
    void add(const string &str) { insert(root,str,0); }
    void insert(Node *cur, const string &str, int pos){
        for(int i=pos;i<str.size();i++){
            if(!cur->go[str[i]-'a'])
                cur->go[str[i]-'a'] = new_Node();
            cur=cur->go[str[i]-'a'];
        }
        cur->cnt++; cur->i=n_pattern++;
    }
    void make_fail(){
        queue<Node*> que;
        que.push(root);
        while (!que.empty()){
            Node* fr=que.front(); que.pop();
            for (int i=0; i<26; i++){
                if (fr->go[i]){
                    Node *ptr = fr->fail;
                    while (ptr && !ptr->go[i]) ptr = ptr->fail;
                    fr->go[i]->fail=ptr=(ptr?ptr->go[i]:root);
                    fr->go[i]->dic=(ptr->cnt?ptr:ptr->dic);
                    que.push(fr->go[i]);
                }
            }
        }
    }
    void query(string s){
        Node *cur=root;
        for(int i=0;i<(int)s.size();i++){
            while(cur&&!cur->go[s[i]-'a']) cur=cur->fail;
            cur=(cur?cur->go[s[i]-'a']:root);
            if(cur->i>=0) ans[cur->i]++;
            for(Node *tmp=cur->dic;tmp;tmp=tmp->dic)
                ans[tmp->i]++;
        } // ans[i] : number of occurrence of pattern i
    }
}AC;

```





