

Contents

1 Basic	1	7 Data Structure	19
1.1 default code	1	7.1 Segment tree	19
1.2 .vimrc	1	7.2 持久化 SMT	20
1.3 Increase Stack Size (linux)	1	7.3 持久化並查集	20
1.4 Misc	1	7.4 Trie	21
1.5 check	2	7.5 Treap (interval reverse)	21
1.6 python-related	2	7.6 BIT	22
2 flow	2	7.7 Black Magic	22
2.1 ISAP $O(V^3)$	2	8 Others	22
2.2 MinCostFlow	2	8.1 SOS dp	22
2.3 Dinic $O(V^2E)$	3	8.2 De Bruijn sequence	22
2.4 Kuhn Munkres 最大完美二分匹配 $O(n^3)$	3	8.3 CDQ 分治	22
2.5 Flow Method	3	8.4 3D LIS	23
3 Math	4	8.5 Ternary Search	23
3.1 FFT	4	8.6 Max Subrectangle	23
3.2 Faulhaber ($\sum_{i=1}^n i^p$)	4	8.7 Maximal Rectangle	23
3.3 Chinese Remainder	4	8.8 p-Median	23
3.4 Miller Rabin	5	8.9 Tree Knapsack	24
3.5 Pollard Rho	5	8.10 質數個數	24
3.6 Josephus Problem	5	8.11 AC-Automaton	24
3.7 Gaussian Elimination	5		
3.8 Inverse Matrix	5		
3.9 模反元素	6		
3.10 ax+by=gcd	6		
3.11 Discrete sqrt	6		
3.12 Prefix Inverse	6		
3.13 Roots of Polynomial 找多項式的根	6		
3.14 Combination theorem	6		
3.15 Primes	7		
3.16 Phi	7		
3.17 Int Sqrt	7		
3.18 Result	7		
4 Geometry	7		
4.1 definition	7		
4.2 Intersection of 2 lines	8		
4.3 halfPlaneIntersection	8		
4.4 Convex Hull	8		
4.5 Convex Hull trick	8		
4.6 Intersection of 2 segments	9		
4.7 Intersection of Polygon and Circle	9		
4.8 Circle cover	9		
4.9 Tangent line of two circles	10		
4.10 Minimum distance of two convex	10		
4.11 Poly Union	10		
4.12 Minkowski sum	10		
4.13 Area of Rectangles	10		
4.14 Min dist on Cuboid	11		
4.15 Distance of Line and Point	11		
4.16 Angle of two vector	11		
4.17 極角排序	11		
4.18 Heart of Triangle	11		
5 Graph	11		
5.1 Lowest Common Ancestor $O(\lg n)$	11		
5.2 Hamiltonian path $O(n^2 2^n)$	12		
5.3 Maximum Clique 最大團	12		
5.4 Maximal Clique 極大團	12		
5.5 BCC based on vertex 點雙聯通分量	13		
5.6 Strongly Connected Component 強連通分量	13		
5.7 Manhattan MST	13		
5.8 Min Mean Cycle	13		
5.9 Directed Graph Min Cost Cycle	14		
5.10 Dominator Tree	15		
5.11 K-th Shortest Path	15		
5.12 Floyd Warshall	16		
5.13 Minimum Steiner Tree	16		
5.14 虛樹	16		
5.15 Tree Hash	16		
5.16 HeavyLight Decomposition	17		
5.17 Graph Theorem	17		
6 String	17		
6.1 PalTree $O(n)$	17		
6.2 Longest Increasing Subsequence	17		
6.3 Longest Common Subsequence $O(n \lg n)$	17		
6.4 KMP	18		
6.5 SAIS $O(n)$	18		
6.6 Z Value $O(n)$	18		
6.7 Manacher Algorithm $O(n)$	18		
6.8 Smallest Rotation	19		
6.9 Cyclic LCS	19		
6.10 Hash	19		

1 Basic

1.1 default code

```
#pragma GCC optimize("O3,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
#include <bits/stdc++.h>
using namespace std;
ios_base::sync_with_stdio(0); cin.tie(0); cout.tie(0);
```

1.2 .vimrc

```
set nu rnu ts=4 sw=4 bs=2 ai hls cin mouse=a
color default
sy on
inoremap {<CR> {<CR>}<C-o>0
inoremap jk <Esc>
nnoremap J 5j
nnoremap K 5k
nnoremap run :w<bar>!g++ -std=c++14 -DLOCAL -Wfatal-
errors -o test "%" && echo "done." && time ./test<
CR>
```

1.3 Increase Stack Size (linux)

```
#include <sys/resource.h>
void increase_stack_size() {
    const rlim_t ks = 64*1024*1024;
    struct rlimit rl;
    int res=getrlimit(RLIMIT_STACK, &rl);
    if(res==0){
        if(rl.rlim_cur<ks){
            rl.rlim_cur=ks;
            res=setrlimit(RLIMIT_STACK, &rl);
        } } }
```

1.4 Misc

```
編譯參數: -std=c++14 -Wall -Wshadow (-fsanitize=
undefined)

mt19937 gen(chrono::steady_clock::now().
    time_since_epoch().count());
int randint(int lb, int ub)
{ return uniform_int_distribution<int>(lb, ub)(gen); }

#define SECs ((double)clock() / CLOCKS_PER_SEC)
double startTime;
bool TIME() { // 比最大可執行時間小一點
    return SECs - startTime > 0.8;
}
int main() {
    startTime = SECs;
}
```

```

struct KeyHasher {
    size_t operator()(const Key& k) const {
        return k.first + k.second * 100000;
    } };
typedef unordered_map<Key,int,KeyHasher> map_t;
// builtin function 可以代的值為int32
__builtin_popcountll    // 二進位有幾個1
__builtin_clzll         // 左起第一個1之前0的個數
__builtin_parityll      // 1的個數的奇偶性
__builtin_mul_overflow(a,b,&h) // a*b是否溢位

```

1.5 check

```

for ((i=0;;i++))
do
    echo "$i"
    python3 gen.py > input
    ./ac < input > ac.out
    ./wa < input > wa.out
    diff ac.out wa.out || break
done

```

1.6 python-related

```

parser:
int(eval(num.replace("/", "")))

from fractions import Fraction
from decimal import Decimal, getcontext, ROUND_HALF_UP,
    ROUND_CEILING, ROUND_FLOOR
getcontext().prec = 250 # set precision
getcontext().rounding = ROUND_HALF_UP

itwo = Decimal(0.5)
two = Decimal(2)

format(x, '0.10f') # set precision

N = 200
def angle(cosT):
    """given cos(theta) in decimal return theta"""
    for i in range(N):
        cosT = ((cosT + 1) / two) ** itwo
        sinT = (1 - cosT * cosT) ** itwo
        return sinT * (2 ** N)
pi = angle(Decimal(-1))

"""round to 2 decimal places"""
sum = Decimal(input())
sum.quantize(Decimal('.00'), ROUND_HALF_UP)

"""Fraction"""
x = Fraction(1, 3) # 1/3
x.as_integer_ratio() # (1, 3)

"""input list of integers"""
arr = list(map(int, input().split()))

```

2 flow

2.1 ISAP $O(V^3)$

```

struct Maxflow {
    static const int MAXV = 20010;
    static const int INF = 1000000;
    struct Edge {
        int v, c, r;
        Edge(int _v, int _c, int _r):
            v(_v), c(_c), r(_r) {}
    };
    int s, t;
    vector<Edge> G[MAXV*2];
    int iter[MAXV*2], d[MAXV*2], gap[MAXV*2], tot;

```

```

void init(int x) {
    tot = x+2;
    s = x+1, t = x+2;
    for(int i = 0; i <= tot; i++) {
        G[i].clear();
        iter[i] = d[i] = gap[i] = 0;
    }
}
void addEdge(int u, int v, int c) {
    G[u].push_back(Edge(v, c, SZ(G[v])));
    G[v].push_back(Edge(u, 0, SZ(G[u]) - 1));
}
int dfs(int p, int flow) {
    if(p == t) return flow;
    for(int &i = iter[p]; i < SZ(G[p]); i++) {
        Edge &e = G[p][i];
        if(e.c > 0 && d[p] == d[e.v]+1) {
            int f = dfs(e.v, min(flow, e.c));
            if(f) {
                e.c -= f;
                G[e.v][e.r].c += f;
                return f;
            }
        }
    }
    if( (--gap[d[p]]) == 0) d[s] = tot;
    else {
        d[p]++;
        iter[p] = 0;
        ++gap[d[p]];
    }
    return 0;
}
int solve() {
    int res = 0;
    gap[0] = tot;
    for(res = 0; d[s] < tot; res += dfs(s, INF));
    return res;
}
void reset() {
    for(int i=0;i<=tot;i++) {
        iter[i]=d[i]=gap[i]=0;
    }
}
} } }flow;

```

2.2 MinCostFlow

```

struct zkwflow{
    static const int maxN=10000;
    struct Edge{ int v,f,re; ll w;};
    int n,s,t,ptr[maxN]; bool vis[maxN]; ll dis[maxN];
    vector<Edge> E[maxN];
    void init(int _n,int _s,int _t){
        n=_n,s=_s,t=_t;
        for(int i=0;i<n;i++) E[i].clear();
    }
    void addEdge(int u,int v,int f,ll w){
        E[u].push_back({v,f,(int)E[v].size(),w});
        E[v].push_back({u,0,(int)E[u].size()-1,-w});
    }
    bool SPFA(){
        fill_n(dis,n,LLONG_MAX); fill_n(vis,n,false);
        queue<int> q; q.push(s); dis[s]=0;
        while (!q.empty()){
            int u=q.front(); q.pop(); vis[u]=false;
            for(auto &it:E[u]){
                if(it.f>0&&dis[it.v]>dis[u]+it.w){
                    dis[it.v]=dis[u]+it.w;
                    if(!vis[it.v]){
                        vis[it.v]=true; q.push(it.v);
                    }
                }
            }
        }
        return dis[t]!=LLONG_MAX;
    }
    int DFS(int u,int nf){
        if(u==t) return nf;
        int res=0; vis[u]=true;
        for(int &i=ptr[u];i<(int)E[u].size();i++){
            auto &it=E[u][i];
            if(it.f>0&&dis[it.v]==dis[u]+it.w&&!vis[it.v]){
                int tf=DFS(it.v,min(nf,it.f));
                res+=tf,nf-=tf,it.f-=tf;
                E[it.v][it.re].f+=tf;
                if(nf==0){ vis[u]=false; break; }
            }
        }
    }
}

```

```

    }
    return res;
}
pair<int,ll> flow(){
    int flow=0; ll cost=0;
    while (SPFA()){
        fill_n(ptr,n,0);
        int f=DFS(s,INT_MAX);
        flow+=f; cost+=dis[t]*f;
    }
    return{ flow,cost };
} // reset: do nothing
} flow;

```

2.3 Dinic $O(V^2E)$

```

#define SZ(x) (int)x.size()
#define PB push_back
struct Dinic{
    struct Edge{ int v,f,re; };
    int n,s,t,level[MXN];
    vector<Edge> E[MXN];
    void init(int _n, int _s, int _t){
        n = _n; s = _s; t = _t;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void add_edge(int u, int v, int f){
        E[u].PB({v,f,SZ(E[v])});
        E[v].PB({u,0,SZ(E[u])-1});
    }
    bool BFS(){
        for (int i=0; i<n; i++) level[i] = -1;
        queue<int> que;
        que.push(s);
        level[s] = 0;
        while (!que.empty()){
            int u = que.front(); que.pop();
            for (auto it : E[u]){
                if (it.f > 0 && level[it.v] == -1){
                    level[it.v] = level[u]+1;
                    que.push(it.v);
                }
            }
        }
        return level[t] != -1;
    }
    int DFS(int u, int nf){
        if (u == t) return nf;
        int res = 0;
        for (auto &it : E[u]){
            if (it.f > 0 && level[it.v] == level[u]+1){
                int tf = DFS(it.v, min(nf,it.f));
                res += tf; nf -= tf; it.f -= tf;
                E[it.v][it.re].f += tf;
                if (nf == 0) return res;
            }
        }
        if (!res) level[u] = -1;
        return res;
    }
    int flow(int res=0){
        while (BFS())
            res += DFS(s,2147483647);
        return res;
    }
} flow;

```

2.4 Kuhn Munkres 最大完美二分匹配 $O(n^3)$

```

struct KM{ // max weight, for min negate the weights
    int n, mx[MXN], my[MXN], pa[MXN];
    ll g[MXN][MXN], lx[MXN], ly[MXN], sy[MXN];
    bool vx[MXN], vy[MXN];
    void init(int _n) { // 1-based
        n = _n;
        for(int i=1; i<=n; i++) fill(g[i], g[i]+n+1, 0);
    }
    void addEdge(int x, int y, ll w) {g[x][y] = w;}
    void augment(int y) {
        for(int x, z; y = z)
            x=pa[y], z=mx[x], my[y]=x, mx[x]=y;
    }
}

```

```

void bfs(int st) {
    for(int i=1; i<=n; ++i) sy[i]=INF, vx[i]=vy[i]=0;
    queue<int> q; q.push(st);
    for(;;) {
        while(q.size()) {
            int x=q.front(); q.pop(); vx[x]=1;
            for(int y=1; y<=n; ++y) if(!vy[y]){
                ll t = lx[x]+ly[y]-g[x][y];
                if(t==0){
                    pa[y]=x;
                    if(!my[y]){augment(y);return;}
                    vy[y]=1, q.push(my[y]);
                }else if(sy[y]>t) pa[y]=x,sy[y]=t;
            }
        }
        ll cut = INF;
        for(int y=1; y<=n; ++y)
            if(!vy[y]&&cut>sy[y]) cut=sy[y];
        for(int j=1; j<=n; ++j){
            if(vx[j]) lx[j] -= cut;
            if(vy[j]) ly[j] += cut;
            else sy[j] -= cut;
        }
        for(int y=1; y<=n; ++y) if(!vy[y]&&sy[y]==0){
            if(!my[y]){augment(y);return;}
            vy[y]=1, q.push(my[y]);
        }
    }
} ll solve(){
    fill(mx, mx+n+1, 0); fill(my, my+n+1, 0);
    fill(ly, ly+n+1, 0); fill(lx, lx+n+1, -INF);
    for(int x=1; x<=n; ++x) for(int y=1; y<=n; ++y)
        lx[x] = max(lx[x], g[x][y]);
    for(int x=1; x<=n; ++x) bfs(x);
    ll ans = 0;
    for(int y=1; y<=n; ++y) ans += g[my[y]][y];
    return ans;
} }graph;

```

2.5 Flow Method

Maximize $c^T x$ subject to $Ax \leq b, x \geq 0$;
with the corresponding symmetric dual problem,
Minimize $b^T y$ subject to $A^T y \geq c, y \geq 0$.

Maximize $c^T x$ subject to $Ax \leq b$;
with the corresponding asymmetric dual problem,
Minimize $b^T y$ subject to $A^T y = c, y \geq 0$.

Minimum vertex cover on bipartite graph =
Maximum matching on bipartite graph

Minimum edge cover on bipartite graph =
vertex number - Minimum vertex cover(Maximum matching)

König Theorem

最小點覆蓋：選出最少的點，滿足每條邊至少有一個端點被選
二分圖中，最小點覆蓋 = 最大匹配

Independent set on bipartite graph =
vertex number - Minimum vertex cover(Maximum matching)

二分圖中，最大獨立集 = n - 最小點覆蓋

找出最小點覆蓋，做完dinic之後，

從源點dfs只走還有流量的邊，

左邊沒被走到的點跟右邊被走到的點就是答案，

其他點為最大獨立集

最大閉包(最大權閉合子圖)

源點連到所有正權點流量為點權

所有負權點連到匯點流量為點權(絕對值)

所有圖上的邊權重為 INF

路徑覆蓋數量

把每個點拆成 入點 和 出點，轉化為二分圖

原圖頂點數 - 二分圖最大匹配數

Maximum density subgraph $(\sum W_e + \sum W_v) / |V|$

Binary search on answer:

For a fixed D, construct a Max flow model as follow:

Let S be Sum of all weight(or inf)
 1. from source to each node with $\text{cap} = S$
 2. For each (u,v,w) in E , $(u \rightarrow v, \text{cap}=w)$, $(v \rightarrow u, \text{cap}=w)$
 3. For each node v , from v to sink with $\text{cap} = S + 2 * D - \text{deg}[v]$
 where $\text{deg}[v] = \sum \text{weight of edge associated with } v$
 If $\text{maxflow} < S * |V|$, D is an answer.

Requiring subgraph: all vertex can be reached from source with edge whose $\text{cap} > 0$.

3 Math

3.1 FFT

```
// const int MXN = 262144 (MXN must be 2^k)
// before any usage, run pre_fft() first
typedef long double ld;
typedef complex<ld> cplx; //real() ,imag()
const ld PI = acos(-1);
const cplx I(0, 1);

struct FFT{
    cplx omega[MXN+1];
    FFT(){ //pre_fft
        for(int i=0; i<=MXN; i++)
            omega[i] = exp(i * 2 * PI / MXN * I);
    }
    // n must be 2^k
    void fft(int n, cplx a[], bool inv=false){
        int basic = MXN / n;
        int theta = basic;
        for (int m = n; m >= 2; m >>= 1) {
            int mh = m >> 1;
            for (int i = 0; i < mh; i++) {
                cplx w = omega[inv ? MXN - (i * theta % MXN) : i * theta % MXN];
                for (int j = i; j < n; j += m) {
                    int k = j + mh;
                    cplx x = a[j] - a[k];
                    a[j] += a[k];
                    a[k] = w * x;
                }
            }
            theta = (theta * 2) % MXN;
        }
        int i = 0;
        for (int j = 1; j < n - 1; j++) {
            for (int k = n >> 1; k > (i ^ k); k >>= 1);
            if (j < i) swap(a[i], a[j]);
        }
        if(inv) for (i = 0; i < n; i++) a[i] /= n;
    }
    cplx arr[MXN+1];
    inline void mul(int _n, ll a[], int _m, ll b[], ll ans[])
    {
        int n=1, sum=_n+_m-1;
        while(n<sum)
            n<<=1;
        for(int i=0; i<n; i++) {
            double x=(i<_n?a[i]:0), y=(i<_m?b[i]:0);
            arr[i]=complex<double>(x+y, x-y);
        }
        fft(n, arr);
        for(int i=0; i<n; i++)
            arr[i]=arr[i]*arr[i];
        fft(n, arr, true);
        for(int i=0; i<sum; i++)
            ans[i]=(long long)(arr[i].real()/4+0.5);
    }
}fft;
```

3.2 Faulhaber ($\sum_{i=1}^n i^p$)

```
/* faulhaber' s formula -
 * cal power sum formula of all p=1~k in O(k^2) */
#define MAXK 2500
const int mod = 1000000007;
int b[MAXK]; // bernoulli number
int inv[MAXK+1]; // inverse
int cm[MAXK+1][MAXK+1]; // combinactories
int co[MAXK][MAXK+2]; // coeeficient of x^j when p=i
inline int getinv(int x) {
    int a=x, b=mod, a0=1, a1=0, b0=0, b1=1;
    while(b) {
        int q, t;
        q=a/b; t=b; b=a-b*q; a=t;
        t=b0; b0=a0-b0*q; a0=t;
        t=b1; b1=a1-b1*q; a1=t;
    }
    return a0<0?a0+mod:a0;
}
inline void pre() {
    /* combinational */
    for(int i=0; i<=MAXK; i++) {
        cm[i][0]=cm[i][i]=1;
        for(int j=1; j<i; j++)
            cm[i][j]=add(cm[i-1][j-1], cm[i-1][j]);
    }
    /* inverse */
    for(int i=1; i<=MAXK; i++) inv[i]=getinv(i);
    /* bernoulli */
    b[0]=1; b[1]=getinv(2); // with b[1] = 1/2
    for(int i=2; i<=MAXK; i++) {
        if(i&1) { b[i]=0; continue; }
        b[i]=1;
        for(int j=0; j<i; j++)
            b[i]=sub(b[i], mul(cm[i-1][j], mul(b[j], inv[i-j+1])));
    }
    /* faulhaber */
    // sigma_x=1~n {x^p} =
    // 1/(p+1) * sigma_j=0~p {C(p+1, j)*Bj*n^(p-j+1)}
    for(int i=1; i<=MAXK; i++) {
        co[i][0]=0;
        for(int j=0; j<=i; j++)
            co[i][i-j+1]=mul(inv[i+1], mul(cm[i+1][j], b[j]));
    }
}
/* sample usage: return f(n,p) = sigma_x=1~n (x^p) */
inline int solve(int n, int p) {
    int sol=0, m=n;
    for(int i=1; i<=p+1; i++) {
        sol=add(sol, mul(co[p][i], m));
        m = mul(m, n);
    }
    return sol;
}
```

3.3 Chinese Remainder

```
LL x[N], m[N];
LL CRT(LL x1, LL m1, LL x2, LL m2) {
    LL g = __gcd(m1, m2);
    if((x2 - x1) % g) return -1; // no sol
    m1 /= g; m2 /= g;
    pair<LL, LL> p = gcd(m1, m2);
    LL lcm = m1 * m2 * g;
    LL res = p.first * (x2 - x1) * m1 + x1;
    return (res % lcm + lcm) % lcm;
}
LL solve(int n) { // n>=2, be careful with no solution
    LL res=CRT(x[0], m[0], x[1], m[1]), p=m[0]/__gcd(m[0], m[1])*m[1];
    for(int i=2; i<n; i++){
        res=CRT(res, p, x[i], m[i]);
        p=p/__gcd(p, m[i])*m[i];
    }
    return res;
}
```

3.4 Miller Rabin

```
// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pirmses <= 13
// n < 2^64              7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
LL mul(LL x, LL y, LL mod){
    LL ret=x*y-(LL)((long double)x/mod*y)*mod;
    // LL ret=x*y-(LL)((long double)x*y/mod+0.5)*mod;
    return ret<0?ret+mod:ret;
}

LL magic[]={}
bool witness(LL a, LL n, LL u, int t){
    if(!a) return 0;
    LL x=myspow(a,u,n);
    for(int i=0; i<t; i++){
        LL nx=mul(x,x,n);
        if(nx==1&&x!=1&&x!=n-1) return 1;
        x=nx;
    }
    return x!=1;
}

bool miller_rabin(LL n) {
    int s=(magic number size)
    // iterate s times of witness on n
    if(n<2) return 0;
    if(!(n&1)) return n == 2;
    ll u=n-1; int t=0;
    // n-1 = u*2^t
    while(!(u&1)) u>>=1, t++;
    while(s--){
        LL a=magic[s]%n;
        if(witness(a,n,u,t)) return 0;
    }
    return 1;
}
```

3.5 Pollard Rho

```
// does not work when n is prime 0(n^(1/4))
LL f(LL x, LL mod){ return add(mul(x,x,mod),1,mod); }
LL pollard_rho(LL n) {
    if(!(n&1)) return 2;
    while(true){
        LL y=2, x=rand()%(n-1)+1, res=1;
        for(int sz=2; res==1; sz*=2) {
            for(int i=0; i<sz && res<=1; i++) {
                x = f(x, n);
                res = __gcd(abs(x-y), n);
            }
            y = x;
        }
        if (res!=0 && res!=n) return res;
    }
}
```

3.6 Josephus Problem

```
int josephus(int n, int m){ //n人每m次
    int ans = 0;
    for (int i=1; i<=n; ++i)
        ans = (ans + m) % i;
    return ans;
}
```

3.7 Gaussian Elimination

```
const int GAUSS_MOD = 100000007LL;
struct GAUSS{
    int n;
    vector<vector<int>>> v;
    int ppow(int a, int k){
```

```
    if(k == 0) return 1;
    if(k % 2 == 0) return ppow(a * a % GAUSS_MOD, k >> 1);
    if(k % 2 == 1) return ppow(a * a % GAUSS_MOD, k >> 1) * a % GAUSS_MOD;
}

vector<int> solve(){
    vector<int> ans(n);
    REP(now, 0, n){
        REP(i, now, n) if(v[now][now] == 0 && v[i][now] != 0)
            swap(v[i], v[now]); // det = -det;
        if(v[now][now] == 0) return ans;
        int inv = ppow(v[now][now], GAUSS_MOD - 2);
        REP(i, 0, n) if(i != now){
            int tmp = v[i][now] * inv % GAUSS_MOD;
            REP(j, now, n + 1) (v[i][j] += GAUSS_MOD - tmp * v[now][j] % GAUSS_MOD) %= GAUSS_MOD;
        }
        REP(i, 0, n) ans[i] = v[i][n + 1] * ppow(v[i][i], GAUSS_MOD - 2) % GAUSS_MOD;
        return ans;
    }
    // gs.v.clear(), gs.v.resize(n, vector<int>(n + 1, 0));
} gs;
```

3.8 Inverse Matrix

```
int GAUSS_MOD;
struct GAUSS{
    int n;
    vector<vector<int>> > v;
    vector<vector<int>> > rev;
    int mul(int x, int y, int mod){
        int ret=x*y-(int)((long double)x/mod*y)*mod;
        return ret<0?ret+mod:ret;
    }
    int ppow(int a, int b){//res=(a^b)%m
        int res=1, k=a;
        while(b){
            if(b&1) res=mul(res,k,GAUSS_MOD)%GAUSS_MOD;
            k=mul(k,k,GAUSS_MOD)%GAUSS_MOD;
            b>>=1;
        }
        return res%GAUSS_MOD;
    }
    bool solve(){
        for(int now = 0; now < n; now++){
            int ch;
            for(ch = now; ch < n && !v[ch][now]; ch++);
            if(ch >= n) return 0;
            for(int i = now; i < n; i++) if(v[now][now] == 0 && v[i][now] != 0){
                swap(v[i], v[now]); // det = -det;
                swap(rev[i], rev[now]);
            }
            if(v[now][now] == 0) return 0;
            int inv = ppow(v[now][now], GAUSS_MOD - 2);
            for(int i = 0; i < n; i++) if(i != now){
                int tmp = v[i][now] * inv % GAUSS_MOD;
                for(int j = 0; j < n; j++) {
                    (v[i][j] += GAUSS_MOD - tmp * v[now][j] % GAUSS_MOD) %= GAUSS_MOD;
                    (rev[i][j] += GAUSS_MOD - tmp * rev[now][j] % GAUSS_MOD) %= GAUSS_MOD;
                }
            }
        }
        return 1;
    }
} gs;

signed main(){
    int n, p; //n*n matrix, MOD=p
    cin>>n>>p; //if(!n && !p) return 0;
    GAUSS_MOD = p; gs.n = n;
    gs.v.clear(), gs.v.resize(n + 1, vector<int>(n + 2, 0));
```

```

gs.rev.clear() , gs.rev.resize(n + 1, vector<int>(n +
    2 , 0));
for(int i = 0; i < n; i++){
    for(int j = 0; j < n; j++){
        cin>>gs.v[i][j];
        if(i == j) gs.rev[i][j] = 1;
    }
}
if(!gs.solve()) cout << "singular\n";
else{
    for(int i = 0; i < n; i++){
        int inv = gs.ppow(gs.v[i][i] , p - 2);
        for(int j = 0; j < n; j++){
            cout << (gs.rev[i][j] * inv % p) << " ";
            cout<<"\n";
        }
    }
    cout << "\n";
}

```

3.9 模反元素

```

long long inv(long long a,long long m){
    long long x,y;
    long long d=exgcd(a,m,x,y);
    if(d==1) return (x+m)%m;
    else return -1; //-1為無解
}

```

3.10 $ax+by=gcd$

```

PII gcd(int a, int b){
    if(b == 0) return {1, 0};
    PII q = gcd(b, a % b);
    return {q.second, q.first - q.second * (a / b)};
}

int exgcd(int a,int b,long long &x,long long &y) {
    if(b == 0){x=1,y=0;return a;}
    int now=exgcd(b,a%b,y,x);
    y-=a/b*x;
    return now;
}

```

3.11 Discrete sqrt

```

void calcH(LL &t, LL &h, const LL p) {
    LL tmp=p-1; for(t=0;(tmp&1)==0;tmp/=2) t++; h=tmp;
}
// solve equation  $x^2 \bmod p = a$ 
bool solve(LL a, LL p, LL &x, LL &y) {
    if(p == 2) { x = y = 1; return true; }
    int p2 = p / 2, tmp = mypow(a, p2, p);
    if (tmp == p - 1) return false;
    if ((p + 1) % 4 == 0) {
        x=mypow(a,(p+1)/4,p); y=p-x; return true;
    } else {
        LL t, h, b, pb; calcH(t, h, p);
        if (t >= 2) {
            do {b = rand() % (p - 2) + 2;
                while (mypow(b, p / 2, p) != p - 1);
                pb = mypow(b, h, p);
            } int s = mypow(a, h / 2, p);
            for (int step = 2; step <= t; step++) {
                int ss = (((LL)(s * s) % p) * a) % p;
                for(int i=0;i<t-step;i++) ss=mul(ss,ss,p);
                if (ss + 1 == p) s = (s * pb) % p;
                pb = ((LL)pb * pb) % p;
            } x = ((LL)s * a) % p; y = p - x;
        } return true;
    }
}

```

3.12 Prefix Inverse

```

void solve( int m ){
    inv[ 1 ] = 1;
    for( int i = 2 ; i < m ; i ++ )
        inv[ i ] = ((LL)(m - m / i) * inv[m % i]) % m;
}

```

3.13 Roots of Polynomial 找多項式的根

```

const double eps = 1e-12;
const double inf = 1e+12;
double a[ 10 ], x[ 10 ]; // a[0..n](coef) must be
    filled
int n; // degree of polynomial must be filled
int sign( double x ){return (x < -eps)?(-1):(x>eps);}
double f(double a[], int n, double x){
    double tmp=1,sum=0;
    for(int i=0;i<=n;i++)
        { sum=sum+a[i]*tmp; tmp=tmp*x; }
    return sum;
}

double binary(double l,double r,double a[],int n){
    int sl=sign(f(a,n,l)),sr=sign(f(a,n,r));
    if(sl==0) return l; if(sr==0) return r;
    if(sl*sr>0) return inf;
    while(r-l>eps){
        double mid=(l+r)/2;
        int ss=sign(f(a,n,mid));
        if(ss==0) return mid;
        if(ss*sl>0) l=mid; else r=mid;
    }
    return l;
}

void solve(int n,double a[],double x[],int &nx){
    if(n==1){ x[1]=-a[0]/a[1]; nx=1; return; }
    double da[10], dx[10]; int ndx;
    for(int i=n;i>=1;i--) da[i-1]=a[i]*i;
    solve(n-1,da,dx,ndx);
    nx=0;
    if(ndx==0){
        double tmp=binary(-inf,inf,a,n);
        if (tmp<inf) x[++nx]=tmp;
        return;
    }
    double tmp;
    tmp=binary(-inf,dx[1],a,n);
    if(tmp<inf) x[++nx]=tmp;
    for(int i=1;i<=ndx-1;i++){
        tmp=binary(dx[i],dx[i+1],a,n);
        if(tmp<inf) x[++nx]=tmp;
    }
    tmp=binary(dx[ndx],inf,a,n);
    if(tmp<inf) x[++nx]=tmp;
} // roots are stored in x[1..nx]

```

3.14 Combination thearom

```

const ll mod = 1e9 + 7;
ll fac[(int)2e6 + 1], inv[(int)2e6 + 1];
ll getinv(ll a){ return qpow(a, mod-2); }
void init(int n){
    fac[0] = 1;
    for(int i = 1; i <= n; i++){
        fac[i] = fac[i-1] * i % mod;
    }
    inv[n] = getinv(fac[n]);
    for(int i = n - 1; i >= 0; i--){
        inv[i] = inv[i + 1] * (i + 1) % mod;
    }
}

ll C(int n, int m){
    if(m > n) return 0;
    return fac[n] * inv[m] % mod * inv[n-m] % mod;
}

```


3.15 Primes

```

/* 12721, 13331, 14341, 75577, 123457, 222557, 556679
 * 999983, 1097774749, 1076767633, 100102021, 999997771
 * 1001010013, 1000512343, 987654361, 999991231
 * 999888733, 98789101, 987777733, 999991921, 1010101333
 * 1010102101, 10000000000039, 1000000000000037
 * 2305843009213693951, 4611686018427387847
 * 9223372036854775783, 18446744073709551557 */
int mu[ N ], p_tbl[ N ];
vector<int> primes;
void sieve() {
    mu[ 1 ] = p_tbl[ 1 ] = 1;
    for( int i = 2 ; i < N ; i ++ ){
        if( !p_tbl[ i ] ){
            p_tbl[ i ] = i;
            primes.push_back( i );
            mu[ i ] = -1;
        }
        for( int p : primes ){
            int x = i * p;
            if( x >= M ) break;
            p_tbl[ x ] = p;
            mu[ x ] = -mu[ i ];
            if( i % p == 0 ){
                mu[ x ] = 0;
                break;
            }
        }
    }
}
vector<int> factor( int x ){
    vector<int> fac{ 1 };
    while( x > 1 ){
        int fn = SZ(fac), p = p_tbl[ x ], pos = 0;
        while( x % p == 0 ){
            x /= p;
            for( int i = 0 ; i < fn ; i ++ )
                fac.PB( fac[ pos ++ ] * p );
        }
    }
    return fac;
}

```

3.16 Phi

```

ll phi(ll n){ // 計算小於n的數中與n互質的有幾個
    ll res = n, a=n;
    for(ll i=2;i*i<=a;i++){ // O(sqrtN)
        if(a%i==0){
            res = res/i*(i-1);
            while(a%i==0) a/=i;
        }
    }
    if(a>1) res=res/a*(a-1);
    return res;
}

```

3.17 Int Sqrt

```

LL intSqrt(LL S) { //return origin val when S <= 0
    if (S <= 0) return S;
    LL x = S;
    for (LL nx;;x = nx){
        nx = (x+S/x)>>1LL;
        if(nx >= x) break;
    }
    return x;
}

```

3.18 Result

- Lucas' Theorem :
For $n, m \in \mathbb{Z}^*$ and prime P , $C(m, n) \bmod P = \prod (C(m_i, n_i))$ where m_i is the i -th digit of m in base P .
- Stirling approximation :
$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n e^{\frac{1}{12n}}$$
- Stirling Numbers(permutation $|P| = n$ with k cycles):
$$S(n, k) = \text{coefficient of } x^k \text{ in } \prod_{i=0}^{n-1} (x+i)$$

- Stirling Numbers(Partition n elements into k non-empty set):

$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

- Pick' s Theorem : $A = i + b/2 - 1$
在二維座標平面中畫上網格，對於任何簡單多邊形
 A : 面積、 i : 內部的格點數、 b : 邊上的格點數
- Catalan number : $C_n = \binom{2n}{n} / (n+1)$
$$C_n^{n+m} - C_{n+1}^{n+m} = (m+n)! \frac{n-m+1}{n+1} \quad \text{for } n \geq m$$

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)n!}$$

$$C_0 = 1 \quad \text{and} \quad C_{n+1} = 2 \left(\frac{2n+1}{n+2} \right) C_n$$

$$C_0 = 1 \quad \text{and} \quad C_{n+1} = \sum_{i=0}^n C_i C_{n-i} \quad \text{for } n \geq 0$$

- Euler Characteristic:
planar graph: $V - E + F - C = 1$
convex polyhedron: $V - E + F = 2$
 V, E, F, C : number of vertices, edges, faces(regions), and components

- Kirchhoff's theorem :
 $A_{ii} = \deg(i), A_{ij} = (i, j) \in E ? -1 : 0$, Deleting any one row, one column, and cal the $\det(A)$

- Polya' theorem (c is number of color, m is the number of cycle size):
$$\left(\sum_{i=1}^m c^{gcd(i, m)} \right) / m$$

- Burnside lemma:
$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

- 錯排公式: (n 個人中，每個人皆不再原來位置的組合數):
$$dp[0] = 1; dp[1] = 0;$$

$$dp[i] = (i-1) * (dp[i-1] + dp[i-2]);$$

- Bell 數 (有 n 個人，把他們拆組的方法總數) :
$$B_0 = 1$$

$$B_n = \sum_{k=0}^n s(n, k) \quad (\text{second - stirling})$$

$$B_{n+1} = \sum_{k=0}^n \binom{n}{k} B_k$$

- Wilson's theorem :
$$(p-1)! \equiv -1 \pmod{p}$$

- Fermat's little theorem :
$$a^p \equiv a \pmod{p}$$

- Euler's totient function:
$$A^{B^C} \bmod p = \text{pow}(A, \text{pow}(B, C, p-1)) \bmod p$$

- 歐拉函數降冪公式:
$$A^B \bmod C = A^{B \bmod \phi(C) + \phi(C)} \bmod C$$

- 用歐拉函數求模反元素:
如果 a 和 n 互質，則 a 對 n 的模反元素
$$a^{-1} \equiv a^{\phi(n)-1} \pmod{n}$$

- 6 的倍數:
$$(a-1)^3 + (a+1)^3 + (-a)^3 + (-a)^3 = 6a$$

- 上高斯 (向上取整):
$$\lceil \frac{a}{b} \rceil = \frac{a+b-1}{b}$$

- 點到直線距離公式:
$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

4 Geometry

4.1 definition

```

#define all(a) a.begin(),a.end()
ostream& operator<<(ostream& os, const Pt& pt) {
    return os << "(" << pt.x << ", " << pt.y << ")";
}
typedef long double ld;
const ld eps = 1e-8;
const ld pi = acos(-1);
int dcmp(ld x) {
    if(abs(x) < eps) return 0;
    else return x < 0 ? -1 : 1;
}
struct Pt {
    ld x, y;
    Pt(ld _x=0, ld _y=0):x(_x), y(_y) {}
    Pt operator+(const Pt &a) const {
        return Pt(x+a.x, y+a.y);
    }
    Pt operator-(const Pt &a) const {

```

```

    return Pt(x-a.x, y-a.y); }
Pt operator*(const ld &a) const {
    return Pt(x*a, y*a); }
Pt operator/(const ld &a) const {
    return Pt(x/a, y/a); }
ld operator*(const Pt &a) const {
    return x*a.x + y*a.y; }
ld operator^(const Pt &a) const {
    return x*a.y - y*a.x; }
bool operator<(const Pt &a) const {
    return x < a.x || (x == a.x && y < a.y); }
//return dcmp(x-a.x) < 0 || (dcmp(x-a.x) == 0 &&
    dcmp(y-a.y) < 0); }
bool operator==(const Pt &a) const {
    return dcmp(x-a.x) == 0 && dcmp(y-a.y) == 0; }
};
ld norm2(const Pt &a) {
    return a*a; }
ld norm(const Pt &a) {
    return sqrt(norm2(a)); }
Pt perp(const Pt &a) {
    return Pt(-a.y, a.x); }
Pt rotate(const Pt &a, ld ang) {
    return Pt(a.x*cos(ang)-a.y*sin(ang), a.x*sin(ang)+a.y
        *cos(ang)); }
bool collinear(Pt a, Pt b, Pt c) { return ((b - a) ^ (c
    - a)) == 0; }
struct Circle {
    Pt o; ld r;
    Circle(Pt _o=Pt(0, 0), ld _r=0):o(_o), r(_r) {}
};

```

4.2 Intersection of 2 lines

```

// NAN(parallel), INF(overlapping)
Pt LLIntersect(Line a, Line b) {
    Pt p1 = a.s, p2 = a.e, q1 = b.s, q2 = b.e;
    ld f1 = (p2-p1)^(q1-p1), f2 = (p2-p1)^(p1-q2), f;
    if(dcmp(f=f1+f2) == 0)
        return dcmp(f1)?Pt(NAN,NAN):Pt(INFINITY,INFINITY);
    return q1*(f2/f) + q2*(f1/f);
}

```

4.3 halfPlaneIntersection

```

// O(nlogn)
// 傳入 vector<Line>
// (半平面為點 st 往 ed 的逆時針方向)
// 回傳值為形成的凸多邊形的頂點 vector
// assume that Lines intersect
vector<Pt> HPI(vector<Line> P) {
    sort(P.begin(), P.end(), [&](Line l, Line m) {
        if (argcmp(l.v, m.v)) return true;
        if (argcmp(m.v, l.v)) return false;
        return PtSide(l.s, m) > 0;
    });
    int n = P.size(), l = 0, r = -1;
    for (int i = 0; i < n; i++) {
        if (i and !argcmp(P[i-1].v, P[i].v)) continue;
        while (l < r and PtSide(LLIntersect(P[r-1], P[r]),
            P[i]) <= 0) r--;
        while (l < r and PtSide(LLIntersect(P[l], P[l+1]),
            P[i]) <= 0) l++;
        P[++r] = P[i];
    }
    while (l < r and PtSide(LLIntersect(P[r-1], P[r]),
        P[l]) <= 0) r--;
    while (l < r and PtSide(LLIntersect(P[l], P[l+1]),
        P[r]) <= 0) l++;
    if (r - l <= 1 or !argcmp(P[l].v, P[r].v))
        return {}; // empty
    if (PtSide(LLIntersect(P[l], P[r]), P[l+1]) <= 0) {
        assert(0);
        return {}; // infinity
    }
    vector<Line> lns = vector(P.begin() + l, P.begin()
        + r + 1);
}

```

```

lns.push_back(lns[0]);
vector<Pt> hpi;
for(int i = 1; i < lns.size(); i++) hpi.push_back(
    LLIntersect(lns[i-1], lns[i]));
return hpi;
}

```

4.4 Convex Hull

```

double cross(Pt o, Pt a, Pt b){
    return (a-o) ^ (b-o);
}
vector<Pt> convex_hull(vector<Pt> pt){
    sort(pt.begin(),pt.end());
    int top=0;
    vector<Pt> stk(2*pt.size());
    for (int i=0; i<(int)pt.size(); i++){
        while (top >= 2 && cross(stk[top-2],stk[top-1],pt[i]
            ]) <= 0) // 如果想要有點共線的點 · 把 <= 改成 <
            top--;
        stk[top++] = pt[i];
    }
    for (int i=pt.size()-2, t=top+1; i>=0; i--){
        while (top >= t && cross(stk[top-2],stk[top-1],pt[i]
            ]) <= 0)
            top--;
        stk[top++] = pt[i];
    }
    stk.resize(top-1);
    return stk;
}

```

4.5 Convex Hull trick

```

struct Convex {
    int n;
    vector<Pt> A, V, L, U;
    Convex(const vector<Pt> &A) : A(_A), n(_A.size()) {
        // n >= 3
        auto it = max_element(all(A));
        L.assign(A.begin(), it + 1);
        U.assign(it, A.end()), U.push_back(A[0]);
        for (int i = 0; i < n; i++) {
            V.push_back(A[(i + 1) % n] - A[i]);
        }
    }
    int PtSide(Pt p, Line L) {
        return dcmp((L.b - L.a)^(p - L.a));
    }
    int inside(Pt p, const vector<Pt> &h, auto f) {
        auto it = lower_bound(all(h), p, f);
        if (it == h.end()) return 0;
        if (it == h.begin()) return p == *it;
        return 1 - dcmp((p - *prev(it))^( *it - *prev(it)))
            ;
    }
    // 1. whether a given point is inside the CH
    // ret 0: out, 1: on, 2: in
    int inside(Pt p) {
        return min(inside(p, L, less{}), inside(p, U,
            greater{}));
    }
    static bool cmp(Pt a, Pt b) { return dcmp(a ^ b) > 0; }
}
// 2. Find tangent points of a given vector
// ret the idx of far/closer tangent point
int tangent(Pt v, bool close = true) {
    assert(v != Pt{});
    auto l = V.begin(), r = V.begin() + L.size() - 1;
    if (v < Pt{}) l = r, r = V.end();
    if (close) return (lower_bound(l, r, v, cmp) - V.
        begin()) % n;
    return (upper_bound(l, r, v, cmp) - V.begin()) % n;
}
// 3. Find 2 tang pts on CH of a given outside point
// return index of tangent points
// return {-1, -1} if inside CH
array<int, 2> tangent2(Pt p) {
}

```



```

array<int, 2> t{-1, -1};
if (inside(p) == 2) return t;
if (auto it = lower_bound(all(L), p); it != L.end()
    and p == *it) {
    int s = it - L.begin();
    return {(s + 1) % n, (s - 1 + n) % n};
}
if (auto it = lower_bound(all(U), p, greater{}); it
    != U.end() and p == *it) {
    int s = it - U.begin() + L.size() - 1;
    return {(s + 1) % n, (s - 1 + n) % n};
}
for (int i = 0; i != t[0]; i = tangent((A[t[0] = i]
    - p), 0));
for (int i = 0; i != t[1]; i = tangent((p - A[t[1]
    = i]), 1));
return t;
}
int find(int l, int r, Line L) {
    if (r < l) r += n;
    int s = PtSide(A[l % n], L);
    return *ranges::partition_point(views::iota(l, r),
        [&](int m) {
            return PtSide(A[m % n], L) == s;
        }) - 1;
};
// 4. Find intersection point of a given line
// intersection is on edge (i, next(i))
vector<int> intersect(Line L) {
    int l = tangent(L.a - L.b), r = tangent(L.b - L.a);
    if(PtSide(A[l], L) == 0) return {l};
    if(PtSide(A[r], L) == 0) return {r};
    if (PtSide(A[l], L) * PtSide(A[r], L) > 0) return
        {};
    return {find(l, r, L) % n, find(r, l, L) % n};
}
};

```

4.6 Intersection of 2 segments

```

int ori( const Pt& o , const Pt& a , const Pt& b ){
    LL ret = ( a - o ) ^ ( b - o );
    return (ret > 0) - (ret < 0);
}
// p1 == p2 || q1 == q2 need to be handled
bool banana( const Pt& p1 , const Pt& p2 ,
    const Pt& q1 , const Pt& q2 ){
    if( ( ( p2 - p1 ) ^ ( q2 - q1 ) ) == 0 ){ // parallel
        if( ori( p1 , p2 , q1 ) ) return false;
        return ( ( p1 - q1 ) * ( p2 - q1 ) ) <= 0 ||
            ( ( p1 - q2 ) * ( p2 - q2 ) ) <= 0 ||
            ( ( q1 - p1 ) * ( q2 - p1 ) ) <= 0 ||
            ( ( q1 - p2 ) * ( q2 - p2 ) ) <= 0;
    }
    return (ori( p1, p2, q1 ) * ori( p1, p2, q2 ) <= 0) &&
        (ori( q1, q2, p1 ) * ori( q1, q2, p2 ) <= 0);
}

```

4.7 Intersection of Polygon and Circle

```

ld PCIntersect(vector<Pt> v, Circle cir) {
    for(int i = 0 ; i < (int)v.size() ; ++i) v[i] = v[i]
        - cir.o;
    ld ans = 0, r = cir.r;
    int n = v.size();
    for(int i = 0 ; i < n ; ++i) {
        Pt pa = v[i], pb = v[(i+1)%n];
        if(norm(pa) < norm(pb)) swap(pa, pb);
        if(dcmp(norm(pb)) == 0) continue;
        ld s, h, theta;
        ld a = norm(pb), b = norm(pa), c = norm(pb-pa);
        ld cosB = (pb*(pb-pa))/a/c, B = acos(cosB);
        if(cosB > 1) B = 0;
        else if(cosB < -1) B = PI;
        ld cosC = (pa*pb)/a/b, C = acos(cosC);
        if(cosC > 1) C = 0;
        else if(cosC < -1) C = PI;
        if(a > r) {

```

```

            s = (C/2)*r*r;
            h = a*b*sin(C)/c;
            if(h < r && B < PI/2) s -= (acos(h/r)*r*r - h*
                sqrt(r*r-h*h));
        }
        else if(b > r) {
            theta = PI - B - asin(sin(B)/r*a);
            s = 0.5*a*r*sin(theta) + (C-theta)/2*r*r;
        }
        else s = 0.5*sin(C)*a*b;
        ans += abs(s)*dcmp(v[i]^v[(i+1)%n]);
    }
    return abs(ans);
}

```

4.8 Circle cover

```

#define N 1021
#define D long double
struct CircleCover{
    int C; Circle c[ N ]; //填入C(圖數量),c(圖陣列)
    bool g[ N ][ N ], overlap[ N ][ N ];
    // Area[i] : area covered by at least i circles
    D Area[ N ];
    void init( int _C ){ C = _C; }
    bool Cinter( Circle& a , Circle& b , Pt& p1 , Pt& p2
        ){
        Pt o1 = a.o , o2 = b.o;
        D r1 = a.r , r2 = b.r;
        if( norm( o1 - o2 ) > r1 + r2 ) return {};
        if( norm( o1 - o2 ) < max(r1, r2) - min(r1, r2) )
            return {};
        D d2 = ( o1 - o2 ) * ( o1 - o2 );
        D d = sqrt(d2);
        if( d > r1 + r2 ) return false;
        Pt u=(o1+o2)*0.5 + (o1-o2)*((r2*r2-r1*r1)/(2*d2));
        D A=sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2+d));
        Pt v=Pt( o1.y-o2.y , -o1.x + o2.x ) * A / (2*d2);
        p1 = u + v; p2 = u - v;
        return true;
    }
}
struct Teve {
    Pt p; D ang; int add;
    Teve() {}
    Teve(Pt _a, D _b, int _c):p(_a), ang(_b), add(_c){}
    bool operator<(const Teve &a)const {
        return ang < a.ang;
    }
}eve[ N * 2 ];
// strict: x = 0, otherwise x = -1
bool disjunct( Circle& a, Circle& b, int x )
{return dcmp( norm( a.o - b.o ) - a.r - b.r ) > x;}
bool contain( Circle& a, Circle& b, int x )
{return dcmp( a.r - b.r - norm( a.o - b.o ) ) > x;}
bool contain(int i, int j){
    /* c[j] is non-strictly in c[i]. */
    return (dcmp(c[i].r - c[j].r) > 0 ||
        (dcmp(c[i].r - c[j].r) == 0 && i < j) ) &&
        contain(c[i], c[j], -1);
}
void solve(){
    for( int i = 0 ; i <= C + 1 ; i ++ )
        Area[ i ] = 0;
    for( int i = 0 ; i < C ; i ++ )
        for( int j = 0 ; j < C ; j ++ )
            overlap[i][j] = contain(i, j);
    for( int i = 0 ; i < C ; i ++ )
        for( int j = 0 ; j < C ; j ++ )
            g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                disjunct(c[i], c[j], -1));
    for( int i = 0 ; i < C ; i ++ ){
        int E = 0, cnt = 1;
        for( int j = 0 ; j < C ; j ++ )
            if( j != i && overlap[j][i] )
                cnt ++;
        for( int j = 0 ; j < C ; j ++ )
            if( i != j && g[i][j] ){
                Pt aa, bb;
                Cinter(c[i], c[j], aa, bb);
                D A=atan2(aa.y - c[i].o.y, aa.x - c[i].o.x);
                D B=atan2(bb.y - c[i].o.y, bb.x - c[i].o.x);

```

4.9 Tangent line of two circles

4.10 Minimum distance of two convex

4.11 Poly Union

```

return (p.x-p1.x)/(p2.x-p1.x);
}
double polyUnion(int n){ //py[0~n-1] must be filled
    int i,j,ii,jj,ta,tb,r,d; double z,w,s,sum=0,tc,td;
    for(i=0;i<n;i++) py[i][py[i].n]=py[i][0];
    for(i=0;i<n;i++){
        for(ii=0;ii<py[i].n;ii++){
            r=0;
            c[r++]=make_pair(0.0,0); c[r++]=make_pair(1.0,0);
            for(j=0;j<n;j++){
                if(i==j) continue;
                for(jj=0;jj<py[j].n;jj++){
                    ta=dcmp(tri(py[i][ii],py[i][ii+1],py[j][jj]))
                    ;
                    tb=dcmp(tri(py[i][ii],py[i][ii+1],py[j][jj
                    +1]));
                    if(ta==0 && tb==0){
                        if((py[j][jj+1]-py[j][jj])*(py[i][ii+1]-py[i
                        ][ii])>0&&j<i){
                            c[r++]=make_pair(segP(py[j][jj],py[i][ii
                            ],py[i][ii+1]),1);
                            c[r++]=make_pair(segP(py[j][jj+1],py[i][
                            ii],py[i][ii+1]),-1);
                        }
                    }else if(ta>=0 && tb<0){
                        tc=tri(py[j][jj],py[j][jj+1],py[i][ii]);
                        td=tri(py[j][jj],py[j][jj+1],py[i][ii+1]);
                        c[r++]=make_pair(tc/(tc-td),1);
                    }else if(ta<0 && tb>=0){
                        tc=tri(py[j][jj],py[j][jj+1],py[i][ii]);
                        td=tri(py[j][jj],py[j][jj+1],py[i][ii+1]);
                        c[r++]=make_pair(tc/(tc-td),-1);
                    } } }
            sort(c,c+r);
            z=min(max(c[0].first,0.0),1.0); d=c[0].second; s
            =0;
            for(j=1;j<r;j++){
                w=min(max(c[j].first,0.0),1.0);
                if(!d) s+=w-z;
                d+=c[j].second; z=w;
            }
            sum+=(py[i][ii]^py[i][ii+1])*s;
        } }
    return sum/2;
}

```

4.12 Minkowski sum

```
// P, Q, R(return) are counterclockwise order convex
    polygon
#define all(a) a.begin(),a.end()
vector<Pt> Minkowski(vector<Pt> P, vector<Pt> Q) {
    auto cmp = [&](Pt a, Pt b) {
        return Pt{a.y, a.x} < Pt{b.y, b.x};
    };
    auto reorder = [&](auto &R) {
        rotate(R.begin(), min_element(all(R), cmp), R.
            end());
        R.push_back(R[0]), R.push_back(R[1]);
    };
    const int n = P.size(), m = Q.size();
    reorder(P), reorder(Q);
    vector<Pt> R;
    for (int i = 0, j = 0, s; i < n or j < m; ) {
        R.push_back(P[i] + Q[j]);
        s = dcmp((P[i + 1] - P[i]) ^ (Q[j + 1] - Q[j]));
        if (s >= 0) i++;
        if (s <= 0) j++;
    }
    rotate(R.begin(), min_element(all(R)), R.end());
    return R;
}
```

4.13 Area of Rectangles

```
struct AreaofRectangles{
#define cl(x) (x<<1)
```

```

#define cr(x) (x<<11)
ll n, id, sid;
pair<ll,ll> tree[MXN<<3]; // count, area
vector<ll> ind;
tuple<ll,ll,ll> scan[MXN<<1];
void pull(int i, int l, int r){
    if(tree[i].first) tree[i].second = ind[r+1] - ind[l];
    else if(l != r){
        int mid = (l+r)>>1;
        tree[i].second = tree[cl(i)].second + tree[cr(i)].second;
    }
    else tree[i].second = 0;
}
void upd(int i, int l, int r, int ql, int qr, int v){
    if(ql <= l && r <= qr){
        tree[i].first += v;
        pull(i, l, r); return;
    }
    int mid = (l+r) >> 1;
    if(ql <= mid) upd(cl(i), l, mid, ql, qr, v);
    if(qr > mid) upd(cr(i), mid+1, r, ql, qr, v);
    pull(i, l, r);
}
void init(int _n){
    n = _n; id = sid = 0;
    ind.clear(); ind.resize(n<<1);
    fill(tree, tree+(n<<2), make_pair(0, 0));
}
void addRectangle(int lx, int ly, int rx, int ry){
    ind[id++] = lx; ind[id++] = rx;
    scan[sid++] = make_tuple(ly, 1, lx, rx);
    scan[sid++] = make_tuple(ry, -1, lx, rx);
}
ll solve(){
    sort(ind.begin(), ind.end());
    ind.resize(unique(ind.begin(), ind.end()) - ind.begin());
    sort(scan, scan + sid);
    ll area = 0, pre = get<0>(scan[0]);
    for(int i = 0; i < sid; i++){
        auto [x, v, l, r] = scan[i];
        area += tree[1].second * (x-pre);
        upd(1, 0, ind.size()-1, lower_bound(ind.begin(), ind.end(), l)-ind.begin(), lower_bound(ind.begin(), ind.end(), r)-ind.begin()-1, v);
        pre = x;
    }
    return area;
}
} rect;

```

4.14 Min dist on Cuboid

```

typedef LL T;
T r;
void turn(T i, T j, T x, T y, T z, T x0, T y0, T L, T W, T H) {
    if (z==0) { T R = x*x+y*y; if (R<r) r=R; return; }
    if(i>=0 && i<2) turn(i+1, j, x0+L+z, y, x0+L-x, x0+L, y0, H, W, L);
    if(j>=0 && j<2) turn(i, j+1, x, y0+W+z, y0+W-y, x0, y0+W, L, H, W);
    if(i<=0 && i>-2) turn(i-1, j, x0-z, y, x-x0, x0-H, y0, H, W, L);
    if(j<=0 && j>-2) turn(i, j-1, x, y0-z, y-y0, x0, y0-H, L, H, W);
}
T solve(T L, T W, T H, T x1, T y1, T z1, T x2, T y2, T z2){
    if( z1!=0 && z1!=H ){
        if( y1==0 || y1==W )
            swap(y1,z1), swap(y2,z2), swap(W,H);
        else swap(x1,z1), swap(x2,z2), swap(L,H);
    }
    if (z1==H) z1=0, z2=H-z2;
    r=INF; turn(0,0,x2-x1,y2-y1,z2,-x1,-y1,L,W,H);
    return r;
}

```

```

}

```

4.15 Distance of Line and Point

```

ld Distance_of_Line_and_Point(Line l, Pt p) {
    ld cross_product = abs((p - l.s) ^ l.v);
    ld line_length = sqrtl(l.v * l.v);
    return cross_product / line_length;
}

```

4.16 Angle of two vector

```

// radian of OA and OB (directed angle)
ld Angle_of_two_vector(Pt A, Pt B, Pt O) {
    ld a = (A - O) * (B - O);
    ld b = (A - O) ^ (B - O);
    ld theta = atan2(b, a);
    return theta;
}

```

4.17 極角排序

```

//極角排序
//atan2(y, x) version
// p is reference point
// 180 度開始，逆時針排序，剛好在 180 度會排最後
bool cmp(Pt &lhs, Pt rhs) {
    return atan2((lhs - p).y, (lhs - p).x) < atan2((rhs - p).y, (rhs - p).x);
}

//cross product version
// p is reference point
// 270 度開始，逆時針排序，剛好在 270 度會排最後
bool cmp(const Pt& lhs, const Pt& rhs) {
    if ((lhs < p) ^ (rhs < p)) return (lhs < p) < (rhs < p);
    return ((lhs - p) ^ (rhs - p)) > 0;
}

```

4.18 Heart of Triangle

```

Pt inCenter( Pt &A, Pt &B, Pt &C) { // 內心
    double a = norm(B-C), b = norm(C-A), c = norm(A-B);
    return (A * a + B * b + C * c) / (a + b + c);
}
Pt circumCenter( Pt &a, Pt &b, Pt &c) { // 外心
    Pt bb = b - a, cc = c - a;
    double db=norm2(bb), dc=norm2(cc), d=2*(bb ^ cc);
    return a-Pt(bb.Y*dc-cc.Y*db, cc.X*db-bb.X*dc) / d;
}
Pt othroCenter( Pt &a, Pt &b, Pt &c) { // 垂心
    Pt ba = b - a, ca = c - a, bc = b - c;
    double Y = ba.Y * ca.Y * bc.Y,
        A = ca.X * ba.Y - ba.X * ca.Y,
        x0= (Y+ca.X*ba.Y*b.X-ba.X*ca.Y*c.X) / A,
        y0= -ba.X * (x0 - c.X) / ba.Y + ca.Y;
    return Pt(x0, y0);
}

```

5 Graph

5.1 Lowest Common Ancestor $O(\lg n)$

```

struct LCA {
    int n, ti, lgN;
    int anc[MXN + 5][__lg(MXN) + 1] = {0};
    int MaxLength[MXN][__lg(MXN) + 1] = {0};
    int time_in[MXN] = {0};
    int time_out[MXN] = {0};
}

```

```

LCA(int _n, int f):n(_n), ti(0), lgN(__lg(n)) {
    dfs(f, f, 0);
    build();
}
void dfs(int now, int f, int len_to_father) { // dfs
    for anc, time, Lenth
    ti++;
    anc[now][0] = f;
    time_in[now] = ti;
    MaxLength[now][0] = len_to_father;
    for (auto i : graph[now]) {
        if (i.first == f) continue;
        dfs(i.first, now, i.second);
    }
    time_out[now] = ti;
}
void build() { // build anc[], MaxLength[]
    for (int i = 1; i <= lgN; ++i) {
        for (int u = 1; u <= n; ++u) {
            anc[u][i] = anc[anc[u][i - 1]][i - 1];
            MaxLength[u][i] = max(MaxLength[u][i - 1],
                                MaxLength[anc[u][i - 1]][i - 1]);
            // dis[u][i] += dis[anc[u][i - 1]][i - 1]
            // + dis[u][i - 1];
        }
    }
}
bool isAncestor(int x, int y) {
    return time_in[x] <= time_in[y] && time_out[x] >=
        time_out[y];
}
int getLCA(int u, int v) {
    if (isAncestor(u, v)) return u;
    if (isAncestor(v, u)) return v;
    for (int i = lgN; i >= 0; --i) {
        if (!isAncestor(anc[u][i], v)) {
            u = anc[u][i];
        }
    }
    return anc[u][0];
}
int getMax(int u, int v) { //獲得路徑上最大邊權
    int lca = getLCA(u, v);
    int maxx = -1;
    for (int i = lgN; i >= 0; --i) {
        // u to lca
        if (!isAncestor(anc[u][i], lca)) {
            maxx = max(maxx, MaxLength[u][i]);
            u = anc[u][i];
        }

        // v to lca
        if (!isAncestor(anc[v][i], lca)) {
            maxx = max(maxx, MaxLength[v][i]);
            v = anc[v][i];
        }
    }
    if (u != lca) maxx = max(maxx, MaxLength[u][0]);
    if (v != lca) maxx = max(maxx, MaxLength[v][0]);
    return maxx;
}
};

```

5.2 Hamiltonian path $O(n^2 2^n)$

```

//dp[i][j] = 目前在j節點走過{i}節點的最短路徑
for(int i=1 ; i < (1 << n) ; i++) {
    for(int j = 1 ; j < n ; j++) {
        if(!((1 << j) & i)&&(i&1)) {
            for( int k = 0 ; k < n ; k++) {
                if(j == k) continue;
                if( (1<<k)&i ) dp[j][i|(1<<j)] =
                    min(dp[j][i|(1<<j)], dp[k][i]+dis[k][j]);
            }
        }
    }
}
}

```

5.3 MaximumClique 最大團

```

#define N 111
struct MaxClique{ // 0-base
    typedef bitset<N> Int;
    Int linkto[N] , v[N];
    int n;
    void init(int _n){
        n = _n;
        for(int i = 0 ; i < n ; i++){
            linkto[i].reset(); v[i].reset();
        }
    }
    void addEdge(int a , int b)
    { v[a][b] = v[b][a] = 1; }
    int popcount(const Int& val)
    { return val.count(); }
    int lowbit(const Int& val)
    { return val._Find_first(); }
    int ans , stk[N];
    int id[N] , di[N] , deg[N];
    Int cans;
    void maxclique(int elem_num, Int candi){
        if(elem_num > ans){
            ans = elem_num; cans.reset();
            for(int i = 0 ; i < elem_num ; i++){
                cans[id[stk[i]]] = 1;
            }
        }
        int potential = elem_num + popcount(candi);
        if(potential <= ans) return;
        int pivot = lowbit(candi);
        Int smaller_candi = candi & (~linkto[pivot]);
        while(smaller_candi.count() && potential > ans){
            int next = lowbit(smaller_candi);
            candi[next] = !candi[next];
            smaller_candi[next] = !smaller_candi[next];
            potential--;
            if(next == pivot || (smaller_candi & linkto[next]
                                ).count()){
                stk[elem_num] = next;
                maxclique(elem_num + 1, candi & linkto[next]);
            }
        }
    }
    int solve(){
        for(int i = 0 ; i < n ; i++){
            id[i] = i; deg[i] = v[i].count();
        }
        sort(id , id + n , [&](int id1, int id2){
            return deg[id1] > deg[id2]; });
        for(int i = 0 ; i < n ; i++) di[id[i]] = i;
        for(int i = 0 ; i < n ; i++)
            for(int j = 0 ; j < n ; j++)
                if(v[i][j]) linkto[di[i]][di[j]] = 1;
        Int cand; cand.reset();
        for(int i = 0 ; i < n ; i++) cand[i] = 1;
        ans = 1;
        cans.reset(); cans[0] = 1;
        maxclique(0, cand);
        return ans;
    }
} solver;

```

5.4 MaximalClique 極大團

```

#define N 80
struct MaxClique{ // 0-base
    typedef bitset<N> Int;
    Int lnk[N] , v[N];
    int n;
    void init(int _n){
        n = _n;
        for(int i = 0 ; i < n ; i++){
            lnk[i].reset(); v[i].reset();
        }
    }
    void addEdge(int a , int b)
    { v[a][b] = v[b][a] = 1; }
    int ans , stk[N], id[N] , di[N] , deg[N];
    Int cans;
    void dfs(int elem_num, Int candi, Int ex){
        if(candi.none()&&ex.none()){
            cans.reset();
            for(int i = 0 ; i < elem_num ; i++)

```

```

    cans[id[stk[i]]] = 1;
    ans = elem_num; // cans is a maximal clique
    return;
}
int pivot = (candilex)._Find_first();
Int smaller_candi = candi & (~lnk[pivot]);
while(smaller_candi.count()){
    int nxt = smaller_candi._Find_first();
    candi[nxt] = smaller_candi[nxt] = 0;
    ex[nxt] = 1;
    stk[elem_num] = nxt;
    dfs(elem_num+1, candi&lnk[nxt], ex&lnk[nxt]);
}
}
int solve(){
    for(int i = 0; i < n; i++){
        id[i] = i; deg[i] = v[i].count();
    }
    sort(id, id + n, [&](int id1, int id2){
        return deg[id1] > deg[id2]; });
    for(int i = 0; i < n; i++) di[id[i]] = i;
    for(int i = 0; i < n; i++)
        for(int j = 0; j < n; j++)
            if(v[i][j]) lnk[di[i]][di[j]] = 1;
    ans = 1; cans.reset(); cans[0] = 1;
    dfs(0, Int(string(n, '1')), 0);
    return ans;
}
} solver;

```

5.5 BCC based on vertex 點雙聯通分量

```

#define PB push_back
#define REP(i, n) for(int i = 0; i < n; i++)
struct BccVertex {
    int n, nScc, step, dfn[MXN], low[MXN];
    vector<int> E[MXN], sccv[MXN];
    int top, stk[MXN];
    void init(int _n) { // 初始化n點
        n = _n; nScc = step = 0;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void addEdge(int u, int v) // 無向邊
    { E[u].PB(v); E[v].PB(u); }
    void DFS(int u, int f) {
        dfn[u] = low[u] = step++;
        stk[top++] = u;
        for (auto v:E[u]) {
            if (v == f) continue;
            if (dfn[v] == -1) {
                DFS(v, u);
                low[u] = min(low[u], low[v]);
                if (low[v] >= dfn[u]) {
                    int z;
                    sccv[nScc].clear();
                    do {
                        z = stk[--top];
                        sccv[nScc].PB(z);
                    } while (z != v);
                    sccv[nScc++].PB(u);
                }
            } else
                low[u] = min(low[u], dfn[v]);
        }
    }
    vector<vector<int>> solve() { // 回傳(size=2 橋, size
        >2 點雙聯通分量)
        vector<vector<int>> res;
        for (int i=0; i<n; i++)
            dfn[i] = low[i] = -1;
        for (int i=0; i<n; i++)
            if (dfn[i] == -1) {
                top = 0;
                DFS(i, i);
            }
        REP(i, nScc) res.PB(sccv[i]);
        return res;
    }
} graph;

```

5.6 Strongly Connected Component 強連通分量

```

#define PB push_back
#define FZ(x) memset(x, 0, sizeof(x)) //fill zero
struct Scc{
    int n, nScc, vst[MXN], bln[MXN];
    vector<int> E[MXN], rE[MXN], vec;
    void init(int _n){
        n = _n;
        for (int i=0; i<MXN; i++)
            E[i].clear(), rE[i].clear();
    }
    void addEdge(int u, int v){
        E[u].PB(v); rE[v].PB(u);
    }
    void DFS(int u){
        vst[u]=1;
        for (auto v : E[u]) if (!vst[v]) DFS(v);
        vec.PB(u);
    }
    void rDFS(int u){
        vst[u] = 1; bln[u] = nScc;
        for (auto v : rE[u]) if (!vst[v]) rDFS(v);
    }
    void solve(){
        nScc = 0;
        vec.clear();
        FZ(vst);
        for (int i=0; i<n; i++)
            if (!vst[i]) DFS(i);
        reverse(vec.begin(), vec.end());
        FZ(vst);
        for (auto v : vec)
            if (!vst[v]){
                rDFS(v); nScc++;
            }
    }
};

```

5.7 ManhattanMST

```

//return {{u,v},w}: u <-> v (w), 需要再手動去重
//need Point definition
vector<pair<pair<int,int>, int>> ManhattanMST(vector<Pt
> P) {
    vector<int> id(P.size());
    iota(id.begin(), id.end(), 0);
    vector<pair<pair<int,int>, int>> edg;
    for (int k = 0; k < 4; k++) {
        sort(id.begin(), id.end(), [&](int i, int j) {
            return (P[i] - P[j]).x < (P[j] - P[i]).y;
        });
        map<int, int> sweep;
        for (int i : id) {
            auto it = sweep.lower_bound(-P[i].y);
            while (it != sweep.end()) {
                int j = it->second;
                Pt d = P[i] - P[j];
                if (d.y > d.x) break;
                edg.push_back({{i, j}, d.x + d.y});
                it = sweep.erase(it);
            }
            sweep[-P[i].y] = i;
        }
        for (Pt &p : P) {
            if (k % 2) p.x = -p.x;
            else swap(p.x, p.y);
        }
    }
    return edg;
}

```

5.8 Min Mean Cycle

```

/* minimum mean cycle O(VE) */
struct MMC{

```



```

#define E 101010
#define V 1021
#define inf 1e9
#define eps 1e-6
struct Edge { int v,u; double c; };
int n, m, prv[V][V], prve[V][V], vst[V];
Edge e[E];
vector<int> edgeID, cycle, rho;
double d[V][V];
void init( int _n )
{ n = _n; m = 0; }
// WARNING: TYPE matters
void addEdge( int vi , int ui , double ci )
{ e[ m ++ ] = { vi , ui , ci }; }
void bellman_ford() {
    for(int i=0; i<n; i++) d[0][i]=0;
    for(int i=0; i<n; i++) {
        fill(d[i+1], d[i+1]+n, inf);
        for(int j=0; j<m; j++) {
            int v = e[j].v, u = e[j].u;
            if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
                d[i+1][u] = d[i][v]+e[j].c;
                prv[i+1][u] = v;
                prve[i+1][u] = j;
            }
        }
    }
}
double solve(){
    // returns inf if no cycle, mmc otherwise
    double mmc=inf;
    int st = -1;
    bellman_ford();
    for(int i=0; i<n; i++) {
        double avg=-inf;
        for(int k=0; k<n; k++) {
            if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])/(n-k));
            else avg=max(avg,inf);
        }
        if (avg < mmc) tie(mmc, st) = tie(avg, i);
    }
    fill(vst,0); edgeID.clear(); cycle.clear(); rho.clear();
    for (int i=n; !vst[st]; st=prv[i--][st]) {
        vst[st]++;
        edgeID.PB(prve[i][st]);
        rho.PB(st);
    }
    while (vst[st] != 2) {
        if(rho.empty()) return inf;
        int v = rho.back(); rho.pop_back();
        cycle.PB(v);
        vst[v]++;
    }
    reverse(ALL(edgeID));
    edgeID.resize(SZ(cycle));
    return mmc;
} }mmc;

```

5.9 Directed Graph Min Cost Cycle

```

// works in O(N M)
#define INF 100000000000000LL
#define N 5010
#define M 200010
struct edge{
    int to; LL w;
    edge(int a=0, LL b=0): to(a), w(b){}
};
struct node{
    LL d; int u, next;
    node(LL a=0, int b=0, int c=0): d(a), u(b), next(c){}
}b[M];
struct DirectedGraphMinCycle{
    vector<edge> g[N], grev[N];
    LL dp[N][N], p[N], d[N], mu;
    bool inq[N];
    int n, bn, bsz, hd[N];
    void b_insert(LL d, int u){
        int i = d/mu;
        if(i >= bn)return;
        b[++bsz] = node(d, u, hd[i]);
    }

```

```

        hd[i] = bsz;
    }
    void init( int _n ){
        n = _n;
        for( int i = 1 ; i <= n ; i ++ )
            g[ i ].clear();
    }
    void addEdge( int ai , int bi , LL ci )
    { g[ai].push_back(edge(bi,ci)); }
    LL solve(){
        fill(dp[0], dp[0]+n+1, 0);
        for(int i=1; i<=n; i++){
            fill(dp[i+1], dp[i+1]+n+1, INF);
            for(int j=1; j<=n; j++) if(dp[i-1][j] < INF){
                for(int k=0; k<(int)g[j].size(); k++){
                    dp[i][g[j][k].to] = min(dp[i][g[j][k].to],
                        dp[i-1][j]+g[j][k].w);
                }
            }
            mu=INF; LL bunbo=1;
            for(int i=1; i<=n; i++) if(dp[n][i] < INF){
                LL a=-INF, b=1;
                for(int j=0; j<=n-1; j++) if(dp[j][i] < INF){
                    if(a*(n-j) < b*(dp[n][i]-dp[j][i])){
                        a = dp[n][i]-dp[j][i];
                        b = n-j;
                    }
                }
                if(mu*b > bunbo*a)
                    mu = a, bunbo = b;
            }
            if(mu < 0) return -1; // negative cycle
            if(mu == INF) return INF; // no cycle
            if(mu == 0) return 0;
            for(int i=1; i<=n; i++){
                for(int j=0; j<(int)g[i].size(); j++){
                    g[i][j].w *= bunbo;
                }
            }
            memset(p, 0, sizeof(p));
            queue<int> q;
            for(int i=1; i<=n; i++){
                q.push(i);
                inq[i] = true;
            }
            while(!q.empty()){
                int i=q.front(); q.pop(); inq[i]=false;
                for(int j=0; j<(int)g[i].size(); j++){
                    if(p[g[i][j].to] > p[i]+g[i][j].w-mu){
                        p[g[i][j].to] = p[i]+g[i][j].w-mu;
                        if(!inq[g[i][j].to]){
                            q.push(g[i][j].to);
                            inq[g[i][j].to] = true;
                        }
                    }
                }
            }
            for(int i=1; i<=n; i++) grev[i].clear();
            for(int i=1; i<=n; i++){
                for(int j=0; j<(int)g[i].size(); j++){
                    g[i][j].w += p[i]-p[g[i][j].to];
                    grev[g[i][j].to].push_back(edge(i, g[i][j].w));
                }
            }
            LL mldc = n*mu;
            for(int i=1; i<=n; i++){
                bn=mldc/mu, bsz=0;
                memset(hd, 0, sizeof(hd));
                fill(d+i+1, d+i+1+n+1, INF);
                b_insert(d[i]=0, i);
                for(int j=0; j<=bn-1; j++) for(int k=hd[j]; k; k=
                    b[k].next){
                    int u = b[k].u;
                    LL du = b[k].d;
                    if(du > d[u]) continue;
                    for(int l=0; l<(int)g[u].size(); l++) if(g[u][l].to > i){
                        if(d[g[u][l].to] > du + g[u][l].w){
                            d[g[u][l].to] = du + g[u][l].w;
                            b_insert(d[g[u][l].to], g[u][l].to);
                        }
                    }
                }
                for(int j=0; j<(int)grev[i].size(); j++) if(grev[i][j].to > i)
                    mldc=min(mldc,d[grev[i][j].to] + grev[i][j].w);
            }
            return mldc / bunbo;
        }
    } }graph;

```

5.10 DominatorTree

```

struct DominatorTree{ // O(N)
#define REP(i,s,e) for(int i=(s);i<=(e);i++)
#define REPD(i,s,e) for(int i=(s);i>=(e);i--)
    int n , m , s;
    vector< int > g[ MAXN ] , pred[ MAXN ];
    vector< int > cov[ MAXN ];
    int dfn[ MAXN ] , nfd[ MAXN ] , ts;
    int par[ MAXN ]; //idom[u] s到u的最後一個必經點
    int sdom[ MAXN ] , idom[ MAXN ];
    int mom[ MAXN ] , mn[ MAXN ];
    inline bool cmp( int u , int v )
    { return dfn[ u ] < dfn[ v ] ; }
    int eval( int u ){
        if( mom[ u ] == u ) return u;
        int res = eval( mom[ u ] );
        if(cmp( sdom[ mn[ mom[ u ] ] ] , sdom[ mn[ u ] ] ))
            mn[ u ] = mn[ mom[ u ] ];
        return mom[ u ] = res;
    }
    void init( int _n , int _m , int _s ){
        ts = 0; n = _n; m = _m; s = _s;
        REP( i , 1 , n ) g[ i ].clear(), pred[ i ].clear();
    }
    void addEdge( int u , int v ){
        g[ u ].push_back( v );
        pred[ v ].push_back( u );
    }
    void dfs( int u ){
        ts++;
        dfn[ u ] = ts;
        nfd[ ts ] = u;
        for( int v : g[ u ] ) if( dfn[ v ] == 0 ){
            par[ v ] = u;
            dfs( v );
        }
    }
    void build(){
        REP( i , 1 , n ){
            dfn[ i ] = nfd[ i ] = 0;
            cov[ i ].clear();
            mom[ i ] = mn[ i ] = sdom[ i ] = i;
        }
        dfs( s );
        REPD( i , n , 2 ){
            int u = nfd[ i ];
            if( u == 0 ) continue;
            for( int v : pred[ u ] ) if( dfn[ v ] ){
                eval( v );
                if( cmp( sdom[ mn[ v ] ] , sdom[ u ] ) )
                    sdom[ u ] = sdom[ mn[ v ] ];
            }
            cov[ sdom[ u ] ].push_back( u );
            mom[ u ] = par[ u ];
            for( int w : cov[ par[ u ] ] ){
                eval( w );
                if( cmp( sdom[ mn[ w ] ] , par[ u ] ) )
                    idom[ w ] = mn[ w ];
                else idom[ w ] = par[ u ];
            }
            cov[ par[ u ] ].clear();
        }
        REP( i , 2 , n ){
            int u = nfd[ i ];
            if( u == 0 ) continue;
            if( idom[ u ] != sdom[ u ] )
                idom[ u ] = idom[ idom[ u ] ];
        }
    } } domT;

```

5.11 K-th Shortest Path

```

// time: O(|E| \lg |E| + |V| \lg |V| + K)
// memory: O(|E| \lg |E| + |V|)
struct KSP{ // 1-base
    struct nd{
        int u, v; ll d;
        nd(int ui = 0, int vi = 0, ll di = INF)
        { u = ui; v = vi; d = di; }
    };
};

```

```

struct heap{
    nd* edge; int dep; heap* chd[4];
};
static int cmp(heap* a, heap* b)
{ return a->edge->d > b->edge->d; }
struct node{
    int v; ll d; heap* H; nd* E;
    node(){
        node(ll _d, int _v, nd* _E)
        { d = _d; v = _v; E = _E; }
        node(heap* _H, ll _d)
        { H = _H; d = _d; }
        friend bool operator<(node a, node b)
        { return a.d > b.d; }
    };
    int n, k, s, t;
    ll dst[ N ];
    nd *nxt[ N ];
    vector<nd*> g[ N ], rg[ N ];
    heap *nullNd, *head[ N ];
    void init( int _n , int _k , int _s , int _t ){
        n = _n; k = _k; s = _s; t = _t;
        for( int i = 1 ; i <= n ; i ++ ){
            g[ i ].clear(); rg[ i ].clear();
            nxt[ i ] = NULL; head[ i ] = NULL;
            dst[ i ] = -1;
        }
    }
    void addEdge( int ui , int vi , ll di ){
        nd* e = new nd(ui, vi, di);
        g[ ui ].push_back( e );
        rg[ vi ].push_back( e );
    }
    queue<int> dfsQ;
    void dijkstra(){
        while(dfsQ.size()) dfsQ.pop();
        priority_queue<node> Q;
        Q.push(node(0, t, NULL));
        while (!Q.empty()){
            node p = Q.top(); Q.pop();
            if(dst[p.v] != -1) continue;
            dst[ p.v ] = p.d;
            nxt[ p.v ] = p.E;
            dfsQ.push( p.v );
            for(auto e: rg[ p.v ] )
                Q.push(node(p.d + e->d, e->u, e));
        }
    }
    heap* merge(heap* curNd, heap* newNd){
        if(curNd == nullNd) return newNd;
        heap* root = new heap;
        memcpy(root, curNd, sizeof(heap));
        if(newNd->edge->d < curNd->edge->d){
            root->edge = newNd->edge;
            root->chd[2] = newNd->chd[2];
            root->chd[3] = newNd->chd[3];
            newNd->edge = curNd->edge;
            newNd->chd[2] = curNd->chd[2];
            newNd->chd[3] = curNd->chd[3];
        }
        if(root->chd[0]->dep < root->chd[1]->dep)
            root->chd[0] = merge(root->chd[0], newNd);
        else
            root->chd[1] = merge(root->chd[1], newNd);
        root->dep = max(root->chd[0]->dep, root->chd[1]->
            dep) + 1;
        return root;
    }
    vector<heap*> V;
    void build(){
        nullNd = new heap;
        nullNd->dep = 0;
        nullNd->edge = new nd;
        fill(nullNd->chd, nullNd->chd+4, nullNd);
        while(not dfsQ.empty()){
            int u = dfsQ.front(); dfsQ.pop();
            if(!nxt[ u ]) head[ u ] = nullNd;
            else head[ u ] = head[nxt[ u ]->v];
            V.clear();
            for( auto&& e : g[ u ] ){
                int v = e->v;
                if( dst[ v ] == -1 ) continue;
                e->d += dst[ v ] - dst[ u ];
                if( nxt[ u ] != e ){

```

```

    heap* p = new heap;
    fill(p->chd, p->chd+4, nullNd);
    p->dep = 1;
    p->edge = e;
    V.push_back(p);
} }
if(V.empty()) continue;
make_heap(V.begin(), V.end(), cmp);
#define L(X) ((X<<1)+1)
#define R(X) ((X<<1)+2)
for( size_t i = 0 ; i < V.size() ; i ++ ){
    if(L(i) < V.size()) V[i]->chd[2] = V[L(i)];
    else V[i]->chd[2]=nullNd;
    if(R(i) < V.size()) V[i]->chd[3] = V[R(i)];
    else V[i]->chd[3]=nullNd;
}
head[u] = merge(head[u], V.front());
} }
vector<ll> ans;
void first_K(){
    ans.clear();
    priority_queue<node> Q;
    if( dst[ s ] == -1 ) return;
    ans.push_back( dst[ s ] );
    if( head[s] != nullNd )
        Q.push(node(head[s], dst[s]+head[s]->edge->d));
    for( int _ = 1 ; _ < k and not Q.empty() ; _ ++ ){
        node p = Q.top(), q; Q.pop();
        ans.push_back( p.d );
        if(head[ p.H->edge->v ] != nullNd){
            q.H = head[ p.H->edge->v ];
            q.d = p.d + q.H->edge->d;
            Q.push(q);
        }
        for( int i = 0 ; i < 4 ; i ++ )
            if( p.H->chd[ i ] != nullNd ){
                q.H = p.H->chd[ i ];
                q.d = p.d - p.H->edge->d + p.H->chd[ i ]->
                    edge->d;
                Q.push( q );
            }
    }
}
void solve(){ // ans[i] stores the i-th shortest path
    dijkstra();
    build();
    first_K(); // ans.size() might less than k
} } solver;

```

5.12 Floryd Warshall

```

for( int k=0 ; k < n ; k++ )
    for( int i=0 ; i < n ; i++ )
        for( int j=0 ; j < n ; j++ )
            if( dis[i][j] > dis[i][k]+dis[k][j] && dis[i][k]
                < INF && dis[k][j] < INF )
                dis[i][j]=dis[i][k]+dis[k][j];
for( int i=0 ; i < n ; i++ )
    for( int j=0 ; j < n ; j++ )
        for( int k=0 ; k < n && dis[i][j] != negINF ; k++ )
            if( dis[k][k] < 0 && dis[i][k] != INF && dis[k]
                ][j] != INF )
                dis[i][j]=negINF;

```

5.13 Minimum Steiner Tree

```

// Minimum Steiner Tree 重要點的mst
//  $O(V^3 \log V + V^2 \log V)$ 
struct SteinerTree{
#define V 33
#define T 8
#define INF 1023456789
    int n , dst[V][V] , dp[1 << T][V] , tdst[V];
    void init( int _n ){
        n = _n;
        for( int i = 0 ; i < n ; i ++ ){
            for( int j = 0 ; j < n ; j ++ )
                dst[ i ][ j ] = INF;
            dst[ i ][ i ] = 0;

```

```

} }
void add_edge( int ui , int vi , int wi ){
    dst[ ui ][ vi ] = min( dst[ ui ][ vi ] , wi );
    dst[ vi ][ ui ] = min( dst[ vi ][ ui ] , wi );
}
void shortest_path(){ // using spfa may faster
    for( int k = 0 ; k < n ; k ++ )
        for( int i = 0 ; i < n ; i ++ )
            for( int j = 0 ; j < n ; j ++ )
                dst[ i ][ j ] = min( dst[ i ][ j ] ,
                    dst[ i ][ k ] + dst[ k ][ j ] );
} // call shorest_path before solve
int solve( const vector<int>& ter ){
    int t = (int)ter.size();
    for( int i = 0 ; i < ( 1 << t ) ; i ++ )
        for( int j = 0 ; j < n ; j ++ )
            dp[ i ][ j ] = INF;
    for( int i = 0 ; i < n ; i ++ )
        dp[ 0 ][ i ] = 0;
    for( int msk = 1 ; msk < ( 1 << t ) ; msk ++ ){
        if( msk == ( msk & (-msk) ) ){
            int who = __lg( msk );
            for( int i = 0 ; i < n ; i ++ )
                dp[ msk ][ i ] = dst[ ter[ who ] ][ i ];
            continue;
        }
        for( int i = 0 ; i < n ; i ++ )
            for( int submsk = ( msk - 1 ) & msk ; submsk ;
                submsk = ( submsk - 1 ) & msk )
                dp[ msk ][ i ] = min( dp[ msk ][ i ] ,
                    dp[ submsk ][ i ] +
                    dp[ msk ^ submsk ][ i ] );
        for( int i = 0 ; i < n ; i ++ ){
            tdst[ i ] = INF;
            for( int j = 0 ; j < n ; j ++ )
                tdst[ i ] = min( tdst[ i ] ,
                    dp[ msk ][ j ] + dst[ j ][ i ] );
        }
        for( int i = 0 ; i < n ; i ++ )
            dp[ msk ][ i ] = tdst[ i ];
    }
    int ans = INF;
    for( int i = 0 ; i < n ; i ++ )
        ans = min( ans , dp[ ( 1 << t ) - 1 ][ i ] );
    return ans;
} } solver;

```

5.14 虛樹

```

vector<int> virTree(vector<int> ver, LCA &lca) {
    auto cmp = [&](int u, int v){return time_in[u] <
        time_in[v];};
    sort(ver.begin(), ver.end(), cmp); //用dfn排序
    vector<int> res(ver.begin(), ver.end());
    for(int i = 1; i < ver.size(); i++){
        res.push_back(lca.getLCA(ver[i-1], ver[i])); //把
        LCA丟進虛樹內
    }
    sort(res.begin(), res.end(), cmp); //再用dfn排序
    res.erase(unique(res.begin(), res.end()), res.end());
    //丟掉重複的點
    return res;
}

```

5.15 Tree Hash

```

map<vector<int>, int> id;
int dfs(int x, int f){
    vector<int> sub;
    for( int v : edge[x] ){
        if (v != f)
            sub.push_back(dfs(v, x));
    }
    sort(sub.begin(), sub.end());
    if (!id.count(sub))
        id[sub] = id.size();
    return id[sub];
}

```

5.16 HeavyLightDecomposition

```
// 詢問,修改複雜度  $O(\log^2 n)$ 
// 1-base

int sz[MXN], dep[MXN], son[MXN], fa[MXN];

// 第一次 dfs
// 找重兒子 需要紀錄當前節點的子樹大小(sz)、深度(dep)、
// 重兒子(son)、父節點(fa)
// 沒有子節點 son[x] = 0
void dfs_sz(int x, int f, int d) { //當前節點 x · 父節點 f · 深度 d
    sz[x] = 1; dep[x] = d; fa[x] = f;
    for(int i : edge[x]) {
        if(i == f) continue;
        dfs_sz(i, x, d+1);
        sz[x] += sz[i];
        if(sz[son[x]] < sz[i]) son[x] = i;
    }
}

// 第二次 dfs
int top[MXN]; // 每個節點所在的鏈的頂端節點
int dfn[MXN]; // 節點編號,編號為在線段樹上的位置
int rnk[MXN]; // 編號為哪個節點
int bottom[MXN]; // 維護每個節點的子樹中最大 dfn 編號
int cnt = 0;
int dfs_hld(int x, int f){
    top[x] = (son[fa[x]] == x ? top[fa[x]] : x);
    rnk[cnt] = x;
    bottom[x] = dfn[x] = cnt++;
    if(son[x]) bottom[x] = max(bottom[x], dfs_hld(son[x], x)); // 更新子樹最大編號
    for(int i : edge[x]){
        if(i == f || i == son[x]) continue;
        bottom[x] = max(bottom[x], dfs_hld(i, x)); // 更新子樹最大編號
    }
    return bottom[x];
}

// 求出 lca
// 不斷跳鏈 · 直到 u,v 跳到同一條鏈上為止
// 每次跳鏈選所在的鏈頂端深度較深的一端往上跳
int getLca(int u, int v) {
    while(top[u] != top[v]){
        if(dep[top[u]] > dep[top[v]])
            u = fa[top[u]];
        else
            v = fa[top[v]];
    }
    return dep[u] > dep[v] ? v : u;
}

// 路徑權重總和
int query(int u, int v) {
    int ret = 0;
    while(top[u] != top[v]){
        if(dep[top[u]] > dep[top[v]]){
            ret += segtree.query(dfn[top[u]], dfn[u]);
            u = fa[top[u]];
        }
        else{
            ret += segtree.query(dfn[top[v]], dfn[v]);
            v = fa[top[v]];
        }
    }
    // 最後到同一條鏈上
    ret += segtree.query(min(dfn[u], dfn[v]), max(dfn[u], dfn[v]));
    return ret;
}
```

5.17 Graph Thearom

- 差分約束條件:
約束條件 $V_j - V_i \leq W$ addEdge(V_i, V_j, W) and run bellman-ford or spfa

- 龜兔賽跑演算法:
開始賽跑, 兔子一次走兩格、烏龜一次走一格直到他們相遇停止
此時讓兔子返回起始點, 兩者以相同走一格的速度繼續前進, 他們就會在環入口會合
- 2-SAT 條件:
滿足 $(x_1 \text{ or } y_1) \text{ and } (x_2 \text{ or } y_2) \text{ and } \dots$ 對於一個限制 $(x \text{ or } y)$ · 則加兩條邊 $x \rightarrow \neg y, y \rightarrow \neg x$

6 String

6.1 PalTree $O(n)$

```
// state[i]代表第i個字元為結尾的最長回文編號
// len[s]是對應的回文長度
// num[s]是有幾個回文後綴
// cnt[s]是這個回文子字串在整個字串中的出現次數
// fail[s]是他長度次長的回文後綴 · aba的fail是a
const int MXN = 1000010;
struct PalT{
    int nxt[MXN][26], fail[MXN], len[MXN];
    int tot, lst, n, state[MXN], cnt[MXN], num[MXN];
    int diff[MXN], sfail[MXN], fac[MXN], dp[MXN];
    char s[MXN] = {"-1"};
    int newNode(int l, int f){
        len[tot] = l, fail[tot] = f, cnt[tot] = num[tot] = 0;
        memset(nxt[tot], 0, sizeof(nxt[tot]));
        diff[tot] = (l > 0 ? l - len[f] : 0);
        sfail[tot] = (l > 0 && diff[tot] == diff[f] ? sfail[f] : f);
        return tot++;
    }
    int getfail(int x){
        while(s[n-len[x]-1] != s[n]) x = fail[x];
        return x;
    }
    int getmin(int v){
        dp[v] = fac[n-len[sfail[v]]-diff[v]];
        if(diff[v] == diff[fail[v]])
            dp[v] = min(dp[v], dp[fail[v]]);
        return dp[v]+1;
    }
    int push(){
        int c = s[n] - 'a', np = getfail(lst);
        if(!(lst = nxt[np][c])){
            lst = newNode(len[np]+2, getfail(fail[np]));
            nxt[np][c] = lst; num[lst] = num[fail[lst]]+1;
        }
        fac[n] = n;
        for(int v = lst; len[v] > 0; v = sfail[v])
            fac[n] = min(fac[n], getmin(v));
        return ++cnt[lst], lst;
    }
    void init(const char *_s){
        tot = lst = n = 0;
        newNode(0, 1), newNode(-1, 1);
        for(; _s[n];) s[n+1] = _s[n], ++n, state[n-1] = push();
        for(int i = tot-1; i > 1; i--) cnt[fail[i]] += cnt[i];
    }
}palt;
```

6.2 Longest Increasing Subsequence

```
vector<int> getLIS(vector<int> a){
    vector<int> lis;
    for(int i : a){
        if(lis.empty() || lis.back() < i) lis.push_Back(i);
        else *lower_bound(lis.begin(), lis.end(), i) = i;
    }
    return lis;
}
```

6.3 Longest Common Subsequence $O(n \lg n)$

```
int LCS(string& s1, string& s2) {
    vector<int> p[128]; // 假設字元範圍為 0 ~ 127
    for (int i = 0; i < s2.size(); ++i) p[s2[i]].
        push_back(i);
    vector<int> v;
    v.push_back(-1);

    for (int i = 0; i < s1.size(); ++i)
        for (int j = p[s1[i]].size() - 1; j >= 0; --j) {
            int n = p[s1[i]][j];

            if (n > v.back())
                v.push_back(n);
            else
                *lower_bound(v.begin(), v.end(), n) = n;
        }
    return v.size() - 1;
};
```

6.4 KMP

/* len-failure[k]:
在k結尾的情況下，這個子字串可以由開頭
長度為(len-failure[k])的部分重複出現來表達

failure[k]為次長相同前綴後綴
如果我們不只想求最多，而且以0-base做為考量
，那可能的長度由大到小會是
failure[k]、failure[failuer[k]-1]
、failure[failure[failuer[k]-1]-1]..
直到有值為0為止 */

```
int failure[MXN];
vector<int> KMP(string& t, string& p) {
    vector<int> ret;
    if(p.size() > t.size()) return ret;
    for(int i = 1, j = failure[0] = -1; i < p.size(); i++) {
        while(j >= 0 && p[j + 1] != p[i]) j = failure[j];
        if(p[j + 1] == p[i]) j++;
        failure[i] = j;
    }
    for(int i = 0, j = -1; i < t.size(); i++) {
        while(j >= 0 && p[j + 1] != t[i]) j = failure[j];
        if(p[j + 1] == t[i]) j++;
        if(j == p.size() - 1) {
            ret.push_back(i - p.size() + 1);
            j = failure[j];
        }
    }
    return ret;
}
```

6.5 SAIS $O(n)$

```
/* SA：將字串的所有後綴排序後的數組 */
/* SA[i]儲存排序後第i小的後綴從哪裡開始 */
/* H[i]為第i小的字串跟第i-1小的LCP */
/* 註：LCP(Longest Common Prefix) */
/* ex:S = "babd", SA[0] = 1("abd") */
/* SA[1] = 0("babd"), SA[2] = 2("bd") */
/* H[0] = 0, H[1] = 0, H[2] = 1("b") */
/* 傳入參數:ip 陣列放字串，len為字串長度 */
/* 需保證ip[len]為0，且字串裡的元素不為0 */
const int N = 300010;
struct SA{
#define REP(i,n) for (int i=0; i<int(n); i++)
#define REP1(i,a,b) for (int i=(a); i<=int(b); i++)
    bool _t[N*2];
    int _s[N*2], _sa[N*2], _c[N*2], x[N], _p[N], _q[N*2],
        hei[N], r[N];
    int operator [] (int i){ return _sa[i]; }
    void build(int *s, int n, int m){
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    }
};
```

```
void mkhei(int n){
    REP(i,n) r[_sa[i]] = i;
    hei[0] = 0;
    REP(i,n) if(r[i]) {
        int ans = i>0 ? max(hei[r[i-1]] - 1, 0) : 0;
        while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans++;
        hei[r[i]] = ans;
    }
}

void sais(int *s, int *sa, int *p, int *q, bool *t,
    int *c, int n, int z){
    bool uniq = t[n-1] = true, neq;
    int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
        lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
    memcpy(x, c, sizeof(int) * z); \
    XD; \
    memcpy(x + 1, c, sizeof(int) * (z - 1)); \
    REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[s[sa[i]
        ]-1]]++ = sa[i]-1; \
    memcpy(x, c, sizeof(int) * z); \
    for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i]
        ]-1]) sa[--x[s[sa[i]-1]]] = sa[i]-1;
    MS0(c, z);
    REP(i,n) uniq &= ++c[s[i]] < 2;
    REP(i,z-1) c[i+1] += c[i];
    if (uniq) { REP(i,n) sa[--c[s[i]]] = i; return; }
    for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s[i
        +1] ? t[i+1] : s[i]<s[i+1]);
    MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[--x[s[i
        ]]] = p[q[i]=nn++] = i);
    REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1]) {
        neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa
            [i])*sizeof(int));
        ns[q[lst=sa[i]]]=nmzx+=neq;
    }
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx
        + 1);
    MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[s[p[
        nsa[i]]]]] = p[nsa[i]]);
}

}sa;
int H[ N ], SA[ N ];
void suffix_array(int* ip, int len) {
    // should padding a zero in the back
    // ip is int array, len is array length
    // ip[0..n-1] != 0, and ip[len] = 0
    ip[len++] = 0;
    sa.build(ip, len, 128);
    for (int i=0; i<len; i++) {
        H[i] = sa.hei[i + 1];
        SA[i] = sa._sa[i + 1];
    }
    // resulting height, sa array \in [0,len)
}
```

6.6 Z Value $O(n)$

```
//z[i] = lcp(s[1...n-1],s[i...n-1])
int z[MXN];
void Z_value(const string& s) {
    int i, j, left, right, len = s.size();
    left=right=0; z[0]=len;
    for(i=1;i<len;i++) {
        j=max(min(z[i-left],right-i),0);
        for(;i+j<len&&s[i+j]==s[j];j++);
        z[i]=j;
        if(i+z[i]>right) {
            right=i+z[i];
            left=i;
        }
    }
}
```

6.7 Manacher Algorithm $O(n)$

```
// 求以每個字元為中心的最長回文半徑
// 頭尾以及每個字元間都加入一個
```



```
// 沒出現過的字元 · 這邊以' @'為例
// s為傳入的字串 · len為字串長度
// z為儲存答案的陣列 (有包含'@'要小心)
// ex: s = "abaac" -> "@a@b@a@a@c@"
// z = [12141232121]
void z_value_pal(char *s,int len,int *z){
    len=(len<<1)+1;
    for(int i=len-1;i>=0;i--){
        s[i]=i&1?s[i>>1]:'@';
        z[0]=1;
        for(int i=1,l=0,r=0;i<len;i++){
            z[i]=i<r?min(z[l+l-i],r-i):1;
            while(i-z[i]>=0&&i+z[i]<len&&s[i-z[i]]==s[i+z[i]])
                ++z[i];
            if(i+z[i]>r) l=i,r=i+z[i];
        }
    }
}
```

6.8 Smallest Rotation

```
//rotate(begin(s),begin(s)+minRotation(s),end(s))
int minRotation(string s) {
    int a = 0, N = s.size(); s += s;
    rep(b,0,N) rep(k,0,N) {
        if(a+k == b || s[a+k] < s[b+k])
            {b += max(0, k-1); break;}
        if(s[a+k] > s[b+k]) {a = b; break;}
    } return a;
}
```

6.9 Cyclic LCS

```
#define L 0
#define LU 1
#define U 2
const int mov[3][2]={0,-1, -1,-1, -1,0};
int al,bl;
char a[MAXL*2],b[MAXL*2]; // 0-indexed
int dp[MAXL*2][MAXL];
char pred[MAXL*2][MAXL];
inline int lcs_length(int r) {
    int i=r+al,j=bl,l=0;
    while(i>r) {
        char dir=pred[i][j];
        if(dir==LU) l++;
        i+=mov[dir][0];
        j+=mov[dir][1];
    }
    return l;
}
inline void reroot(int r) { // r = new base row
    int i=r,j=1;
    while(j<=bl&&pred[i][j]!=LU) j++;
    if(j>bl) return;
    pred[i][j]=L;
    while(i<2*al&&j<=bl) {
        if(pred[i+1][j]==U) {
            i++;
            pred[i][j]=L;
        } else if(j<bl&&pred[i+1][j+1]==LU) {
            i++;
            j++;
            pred[i][j]=L;
        } else {
            j++;
        }
    }
}
int cyclic_lcs() {
    // a, b, al, bl should be properly filled
    // note: a WILL be altered in process
    // -- concatenated after itself
    char tmp[MAXL];
    if(al>bl) {
        swap(al,bl);
        strcpy(tmp,a);
        strcpy(a,b);
        strcpy(b,tmp);
    }
    strcpy(tmp,a);
    strcat(a,tmp);
}
```

```
// basic lcs
for(int i=0;i<=2*al;i++) {
    dp[i][0]=0;
    pred[i][0]=U;
}
for(int j=0;j<=bl;j++) {
    dp[0][j]=0;
    pred[0][j]=L;
}
for(int i=1;i<=2*al;i++) {
    for(int j=1;j<=bl;j++) {
        if(a[i-1]==b[j-1]) dp[i][j]=dp[i-1][j-1]+1;
        else dp[i][j]=max(dp[i-1][j],dp[i][j-1]);
        if(dp[i][j-1]==dp[i-1][j]) pred[i][j]=L;
        else if(a[i-1]==b[j-1]) pred[i][j]=LU;
        else pred[i][j]=U;
    }
}
// do cyclic lcs
int clcs=0;
for(int i=0;i<al;i++) {
    clcs=max(clcs,lcs_length(i));
    reroot(i+1);
}
// recover a
a[al]='\0';
return clcs;
}
```

6.10 Hash

```
//字串雜湊前的idx是0-base · 雜湊後為1-base
//即區間為 [0,n-1] -> [1,n]
//若要取得區間[L,R]的值得則
//H[R] - H[L-1] * p^(R-L+1)
//cmp為比較從i開始長度為len的字串和
//(h[i+len-1] - h[i-1] * qpow(p, len) % mod1 + mod1)
//從j開始長度為len的字串是否相同
#define x first
#define y second
pair<int,int> Hash[MxN];
void build(const string& s){
    pair<int,int> val = make_pair(0,0);
    Hash[0]=val;
    for(int i=1; i<=s.size(); i++){
        val.x = (val.x * P1 + s[i-1]) % MOD;
        val.y = (val.y * P2 + s[i-1]) % MOD;
        Hash[i] = val;
    }
}
bool cmp( int i, int j, int len ) {
    return ((Hash[i+len-1].x-Hash[i-1].x*qpow(P1,len)%MOD+MOD)%MOD == (Hash[j+len-1].x-Hash[j-1].x*qpow(P1,len)%MOD+MOD)%MOD)
    && ((Hash[i+len-1].y-Hash[i-1].y*qpow(P2,len)%MOD+MOD)%MOD == (Hash[j+len-1].y-Hash[j-1].y*qpow(P2,len)%MOD+MOD)%MOD);
}
```

7 Data Structure

7.1 Segment tree

```
// !!!注意build()時初始化用的陣列也是1-base
// !!!query(0, 0) 會報錯
#define cl(x) (x*2)
#define cr(x) (x*2+1)
struct segmentTree {
    int n;
    vector<int> seg, tag, cov;
    segmentTree(int _n): n(_n) {
        seg = tag = cov = vector<int>(n * 4, 0);
    }
    void push(int i, int L, int R) {
        if(cov[i]) {

```

```

        seg[i] = cov[i] * (R - L + 1);
        if(L < R) {
            cov[cl(i)] = cov[cr(i)] = cov[i];
            tag[cl(i)] = tag[cr(i)] = 0;
        }
        cov[i] = 0;
    }
    if(tag[i]) {
        seg[i] += tag[i] * (R - L + 1);
        if(L < R) {
            tag[cl(i)] += tag[i];
            tag[cr(i)] += tag[i];
        }
        tag[i] = 0;
    }
}

void pull(int i, int L, int R) {
    if(L >= R) return;
    int mid = L + R >> 1;
    push(cl(i), L, mid);
    push(cr(i), mid + 1, R);
    seg[i] = seg[cl(i)] + seg[cr(i)];
}

void build(vector<int>& arr, int i = 1, int L = 1,
int R = -1) {
    if(R == -1) R = n;
    if(L == R) return void(seg[i] = arr[L]);
    int mid = L + R >> 1;
    build(arr, cl(i), L, mid);
    build(arr, cr(i), mid + 1, R);
    pull(i, L, R);
}

int query(int rL, int rR, int i = 1, int L = 1, int
R = -1) {
    if(R == -1) R = n;
    push(i, L, R);
    if(rL <= L && R <= rR) return seg[i];
    int mid = L + R >> 1, ret = 0;
    if(rL <= mid) ret += query(rL, rR, cl(i), L,
mid);
    if(mid < rR) ret += query(rL, rR, cr(i), mid +
1, R);
    return ret;
}

void update(int rL, int rR, int val, int i = 1, int
L = 1, int R = -1) {
    if(R == -1) R = n;
    push(i, L, R);
    if(rL <= L && R <= rR) return void(tag[i] = val
);
    int mid = L + R >> 1;
    if(rL <= mid) update(rL, rR, val, cl(i), L, mid
);
    if(mid < rR) update(rL, rR, val, cr(i), mid +
1, R);
    pull(i, L, R);
}

void cover(int rL, int rR, int val, int i = 1, int
L = 1, int R = -1) {
    if(R == -1) R = n;
    push(i, L, R);
    if(rL <= L && R <= rR) return void(cov[i] = val
);
    int mid = L + R >> 1;
    if(rL <= mid) cover(rL, rR, val, cl(i), L, mid
);
    if(mid < rR) cover(rL, rR, val, cr(i), mid +
1, R);
    pull(i, L, R);
}
};

/* Test Case:
4
1 2 3 4
5
2 1 3
1 1 3 1
2 1 3
1 1 4 1
2 1 4
*/

```

7.2 持久化 SMT

```

struct node{
    node *l, *r;
    int val;
};

vector<node*> ver;
int arr[MXN] = {0};

//0-base
struct SegmentTree{
    int n;
    node *root;
    void build(int _n){
        n = _n;
        root = build(0, n-1);
    }
    node* build(int L, int R){
        node *x = new node();
        if(L == R){ x->val = arr[L]; return x;}
        int mid = (L+R)/2;
        x->l = build(L, mid);
        x->r = build(mid + 1, R);
        x->val = x->l->val + x->r->val;
        return x;
    }
    int query(node *ro, int L, int R){return query(ro, 0,
n-1, L, R);}
    int query(int L, int R){return query(root, 0, n-1, L,
R);}
    int query(node *x, int L, int R, int recL, int recR){
        if(recL <= L && R <= recR) return x->val;
        int mid = (L+R)/2, res = 0;
        if(recL <= mid) res += query(x->l, L, mid, recL,
recR);
        if(mid < recR) res += query(x->r, mid+1, R, recL,
recR);
        return res;
    }
    void update(int pos, int v){update(root, 0, n-1, pos,
v);}
    void update(node *x, int L, int R, int pos, int v){
        if(L == R){ x->val = v; arr[L] = v; return;}
        int mid = (L+R)/2;
        if(pos <= mid) update(x->l, L, mid, pos, v);
        else update(x->r, mid+1, R, pos, v);
        x->val = x->l->val + x->r->val;
    }
    node *update_ver(node *pre, int l, int r, int pos,
int v){
        node *x = new node(); //當前位置建立新節點
        if(l == r){
            x->val = v;
            return x;
        }
        int mid = (l+r)>>1;
        if(pos <= mid){ //更新左邊
            x->l = update_ver(pre->l, l, mid, pos, v); //左邊
            節點連向新節點
            x->r = pre->r; //右邊連到原本的右邊
        }
        else{ //更新右邊
            x->l = pre->l; //左邊連到原本的左邊
            x->r = update_ver(pre->r, mid+1, r, pos, v); //
            右邊節點連向新節點
        }
        x->val = x->l->val + x->r->val;
        return x;
    }
} seg;

void add_ver(int x, int v){ //修改位置 x 的值為 v
    ver.push_back(seg.update_ver(ver.back(), 0, seg.n
-1, x, v));
}

```

7.3 持久化並查集

```

struct DSU {

```

```

int n;
vector<int> fa, sz;
vector<tuple<int, int, int, int>> ver;
DSU(int _n): n(_n), fa(n), sz(n, 1) {
    iota(fa.begin(), fa.end(), 0);
}
int find(int x) {
    return fa[x] == x ? x : find(fa[x]);
}
void merge(int x, int y) {
    x = find(x), y = find(y);
    if(sz[x] < sz[y]) swap(x, y);
    ver.push_back({x, sz[x], y, fa[y]});
    if(x == y) return;
    sz[x] += sz[y];
    fa[y] = x;
}
void undo() {
    if(ver.empty()) return;
    auto [x, szx, y, fy] = ver.back();
    ver.pop_back();
    sz[x] = szx;
    fa[y] = fy;
}
};

```

7.4 Trie

```

struct trie{
    trie *nxt[26];
    int cnt; //紀錄有多少個字串以此節點結尾
    int sz; //有多少字串的前綴包括此節點
    trie():cnt(0),sz(0){
        memset(nxt,0,sizeof(nxt));
    }
};

trie *root = new trie(); //創建新的字典樹

void insert(string& s){
    trie *now = root; //每次從根結點出發
    for(auto i:s){
        now->sz++;
        if(now->nxt[i-'a'] == NULL){
            now->nxt[i-'a'] = new trie();
        }
        now = now->nxt[i-'a']; //走到下一個字母
    }
    now->cnt++; now->sz++;
}

int query_prefix(string& s){ //查詢有多少前綴為 s
    trie *now = root; //每次從根結點出發
    for(auto i:s){
        if(now->nxt[i-'a'] == NULL){
            return 0;
        }
        now = now->nxt[i-'a'];
    }
    return now->sz;
}

int query_count(string& s){ //查詢字串 s 出現次數
    trie *now = root; //每次從根結點出發
    for(auto i:s){
        if(now->nxt[i-'a'] == NULL){
            return 0;
        }
        now = now->nxt[i-'a'];
    }
    return now->cnt;
}

```

7.5 Treap (interval reverse)

//拆出[a,b]區間就如同下面所展示先使用splitByTh()拆出
//左右,再把左區間拆成l, m最後merge()回去

```

//反轉區間時又記得使用^=可以直接反轉01

//treap拆區間時從後面拆是因為這樣[a,b]的關係
//不用重新考慮·要是先拆前面b的位置會變成b-a+1
//0-base
//splitByTh(root,a-1,l,m);
//splitByTh(m,b-a+1,m,r);

mt19937 gen(chrono::steady_clock::now().
    time_since_epoch().count());
struct Treap {
    int key, pri, sz, tag, sum;
    Treap *L, *R;
    Treap( int val ) {
        sum=key=val, pri=gen(), sz=1, tag=0;
        L=R=NULL;
    };
};
int Size( Treap *a ) { return !a?0:a->sz; }
void pull( Treap *a ) {
    a->sz=Size(a->L)+Size(a->R)+1;
    a->sum=a->key;
    if( a->L ) a->sum+=a->L->sum;
    if( a->R ) a->sum+=a->R->sum;
}
void push( Treap *a ) {
    if( a && a->tag ) {
        swap(a->L,a->R);
        if( a->L ) a->L->tag^=1;
        if( a->R ) a->R->tag^=1;
        a->tag=0;
    }
}
Treap *merge(Treap *a, Treap *b) {
    if( !a || !b ) return a?b;
    push(a), push(b);
    if( a->pri > b->pri ) {
        a->R=merge(a->R,b);
        pull(a); return a;
    }
    b->L=merge(a,b->L);
    pull(b); return b;
}
void print(Treap *a) {
    if( !a ) return;
    push(a);
    print(a->L);
    cout.put(a->key);
    print(a->R);
}
Treap *buildTreap( int n, string& str ) {
    Treap *root=NULL;
    for( int i=0; i < n; i++ )
        root=merge(root,new Treap(str[i]));
    return root;
}
void splitbyk( Treap *x, int k, Treap *&a, Treap *&b )
{
    if(!x) a=b=NULL;
    else if( x->key <= k ) {
        a=x;
        splitbyk(x->R,k,a->R,b);
        pull(a);
    }
    else {
        b=x;
        splitbyk(x->L,k,a,b->L);
        pull(b);
    }
}
void splitByTh( Treap *x, int k, Treap *&a, Treap *&b )
{
    if( !x ) { a=b=NULL; return; }
    push(x);
    if( Size(x->L)+1 <= k ) {
        a=x;
        splitByTh(x->R,k-Size(x->L)-1,a->R,b);
        pull(a);
    }
    else {
        b=x;
        splitByTh(x->L,k,a,b->L);
        pull(b);
    }
}

```

```

}
}
signed main() {
    string str;
    int n, m;
    cin >> n >> m >> str;
    Treap *root;
    root = buildTreap(n, str);
    for (int i = 0; i < m; i++) {
        int a, b;
        cin >> a >> b;
        Treap *l, *m, *r;
        splitByTh(root, b, l, r);
        splitByTh(l, a - 1, l, m);
        m->tag ^= 1;
        root = merge(l, merge(m, r));
    }
    print(root);
}

```

7.6 BIT

```

#define lowbit(x) (x&-x)
struct BIT {
    int n;
    vector<int> bit;
    BIT(int _n):n(_n), bit(n + 1) {}
    void update(int x, int val) {
        for (; x <= n; x += lowbit(x)) bit[x] += val;
    }
    void update(int L, int R, int val) {
        update(L, val), update(R + 1, -val);
    }
    int query(int x) {
        int res = 0;
        for (; x; x -= lowbit(x)) res += bit[x];
        return res;
    }
    int query(int L, int R) {
        return query(R) - query(L - 1);
    }
};

```

7.7 Black Magic

```

#include <bits/extc++.h>
using namespace __gnu_pbds;
typedef tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> set_t;
tree<int, null_type, less_equal<int>, rb_tree_tag,
    tree_order_statistics_node_update> mt_t;
#include <ext/pb_ds/assoc_container.hpp>
typedef cc_hash_table<int, int> umap_t;
// gp_hash_table<int, int>
typedef priority_queue<int> heap;
#include <ext/rope>
using namespace __gnu_cxx;
int main() {
    // Insert some entries into s.
    set_t s; s.insert(12); s.insert(505);
    // The order of the keys should be: 12, 505.
    assert(*s.find_by_order(0) == 12);
    assert(*s.find_by_order(3) == 505);
    // The order of the keys should be: 12, 505.
    assert(s.order_of_key(12) == 0);
    assert(s.order_of_key(505) == 1);
    // Erase an entry.
    s.erase(12);
    // The order of the keys should be: 505.
    assert(*s.find_by_order(0) == 505);
    // The order of the keys should be: 505.
    assert(s.order_of_key(505) == 0);
    // if we want to delete less_equal tag tree
    mt_t.erase(mt_t.find_by_order(mt_t.order_of_key(val))
    );
    heap h1, h2; h1.join(h2);
}

```

```

rope<char> r[ 2 ];
r[ 1 ] = r[ 0 ]; // persistenet
string t = "abc";
r[ 1 ].insert( 0 , t.c_str() );
r[ 1 ].erase( 1 , 1 );
cout << r[ 1 ].substr( 0 , 2 );
}

```

8 Others

8.1 SOS dp

```

for (int i = 0; i < (1 << N); ++i)
    F[i] = A[i];
for (int i = 0; i < N; ++i) for (int mask = 0; mask < (1 <<
    N); ++mask) {
    if (mask & (1 << i))
        F[mask] += F[mask ^ (1 << i)];
}

```

8.2 De Bruijn sequence

```

// return cyclic array of length k^n such that every
// array of length n using 0~k-1 appears as a subarray.
vector<int> DeBruijn(int k, int n) {
    if (k == 1) return {0};
    vector<int> aux(k * n), res;
    function<void(int, int)> f = [&](int t, int p) -> void {
        if (t > n) { if (n % p == 0)
            for (int i = 1; i <= p; ++i) res.push_back(aux[i]);
        } else {
            aux[t] = aux[t - p]; f(t + 1, p);
            for (aux[t] = aux[t - p] + 1; aux[t] < k; ++aux[t]) f(t + 1, t);
        }
    };
    f(1, 1); return res;
}

```

8.3 CDQ 分治

```

//cdq分治使用的結構u, v, w為排序物的三個維度
//ans記錄了有幾項三維都小於等於自己
//cnt記錄了相同物有幾個，在使用cdq之前必先去重，
//並且將相同元素紀錄至cnt中，可使用map來做到這步
//cdq使用的BIT就是普通求和的BIT，大小就開維度的
//值域範圍，若值域大於2e6則要先進行離散化
struct triple {int u, v, w, ans, cnt;};
BIT *bt;
void cdq(int L, int R, vector<triple>& arr) {
    if (R - L <= 1) return;
    int mid = L + R >> 1;
    vector<triple> temp;
    cdq(L, mid, arr), cdq(mid, R, arr);
    for (int i = L, j = mid; i < mid || j < R; ) {
        for (; i < mid && (j >= R || arr[i].v <= arr[j].v);
            i++) {
            bt->update(arr[i].w, arr[i].cnt);
            temp.push_back(arr[i]);
        }
        if (j < R) {
            arr[j].ans += bt->query(arr[j].w);
            temp.push_back(arr[j]);
            j++;
        }
    }
    for (int i = L; i < mid; i++)
        bt->update(arr[i].w, -arr[i].cnt);
    copy(temp.begin(), temp.end(), arr.begin() + L);
}
signed main()
{
    // n 個數 k 值域範圍
    int n, k;
}

```

```

cin >> n >> k;
map<tuple<int, int, int>, int> mp;
vector<int> res(n, 0);
vector<triple> arr;
bt = new BIT(k + 1);
for(int i = 0; i < n; i++) {
    int x, y, z;
    cin >> x >> y >> z;
    mp[{x, y, z}]++;
}
for(auto t : mp)
    arr.push_back({get<0>(t.first), get<1>(t.first),
        get<2>(t.first), 0, t.second});
cdq(0, arr.size(), arr);
for(auto &[x,y,z,a,b] : arr) res[a + b - 1] += b;
for(int i : res) cout << i << '\n';
}

```

8.4 3D LIS

```

#define lowbit(x) (x&-x)
const int MAXN=1e5+5;
struct BIT {
    int n;
    vector<int> bit;
    BIT( int _n ):n(_n), bit(_n+1,0) {}
    int query( int x ) {
        int res=0;
        for(; x > 0 ; x-=lowbit(x) ) res=max(res,bit[x]);
        return res;
    }
    void update( int x, int val ) {
        for(; x <= n ; x+=lowbit(x) ) {
            if( val < 0 ) bit[x]=0;
            else bit[x]=max(bit[x],val);
        }
    }
}bt(MAXN);
struct triple {
    int u, v, w, ans, cnt;
    bool operator<( triple b ) { return u<b.u; }
};
bool cmp( triple a, triple b ) {return a.v<b.v;}
void cdq( int L, int R, vector<triple>& arr ) {
    if( R-L <= 1 ) return;
    int mid=L+R>>1;
    cdq(L,mid,arr);
    sort(arr.begin()+L,arr.begin()+mid,cmp);
    sort(arr.begin()+mid,arr.begin()+R,cmp);
    for( int i=L, j=mid ; i < mid || j < R ; ) {
        for(; i < mid && ( j >= R || arr[i].v < arr[j].v )
            ; i++ ) bt.update(arr[i].w,arr[i].ans);
        if( j < R ) {
            arr[j].ans=max(bt.query(arr[j].w-1)+1,arr[j].ans)
            ;
            j++;
        }
    }
    for( int i=L ; i < mid ; i++ ) bt.update(arr[i].w,-1)
    ;
    sort(arr.begin()+L,arr.begin()+R);
    cdq(mid,R,arr);
}
signed main()
{
    ios_base::sync_with_stdio(0); cin.tie(0); cout.tie(0)
    ;
    int n, res=0;
    cin>>n;
    vector<int> ls;
    vector<triple> arr;
    for( int i=0 ; i < n ; i++ ) {
        int a, b;
        cin>>a>>b;
        arr.push_back({i,a,b,1,1}); //{第一維,第二維,第三維,
            答案,數量}
        ls.push_back(b);
    }
    sort(ls.begin(),ls.end());
    ls.resize(unique(ls.begin(),ls.end())-ls.begin());
}

```

```

for( auto &t : arr ) t.w=lower_bound(ls.begin(),ls.
    end(),t.w)-ls.begin()+1;
n=arr.size();
cdq(0,n,arr);
for( int i=0 ; i < n ; i++ ) res=max(res,arr[i].ans);
cout<<res<<'\n';
}

```

8.5 Ternary Search

```

while(L <= R) {
    int ml = L + (R - L) / 3, mr = R - (R - L) / 3;
    if(L == R) return L;
    else if( checker(ml) < checker(mr) ) L = ml + 1;
    else R = mr - 1;
}

```

8.6 Max Subrectangle

```

const int N = 1e5+5;
int n, a[N],l[N], r[N];
long long ans;
int main() {
    while( cin>>n ) {
        ans = 0;
        for( int i = 1; i <= n; i++ ) cin>>a[i], l[i] = r[i]
            = i;
        for( int i = 1; i <= n; i++ )
            while( l[i] > 1 && a[i] <= a[l[i] - 1] ) l[i] = l[
                l[i] - 1];
        for( int i = n; i >= 1; i-- )
            while( r[i] < n && a[i] <= a[r[i] + 1] ) r[i] = r[
                r[i] + 1];
        for( int i = 1; i <= n; i++ )
            ans = max(ans, (long long)(r[i] - l[i] + 1) * a[i]
                );
        cout<<ans<<'\n';
    }
}

```

8.7 Maximal Rectangle

```

const int MXN = 300;
int maximalRectangle(vector<vector<char>>& matrix) {
    int a[MXN][], l[MXN][], r[MXN][];
    int n = matrix.size(), m = matrix[0].size(), ans =
        0;
    for(int i = 1; i <= n; i++) {
        for(int j = 1; j <= m; j++) l[j] = r[j] = j;
        char c;
        for(int j = 1; j <= m; j++) { //對每一個直行做
            統計·若是上一個a[j]也是1則會變成2
            c = matrix[i - 1][j - 1];
            if( c == '1' ) a[j]++;
            else if( c == '0' ) a[j] = 0;
        }
        for(int j = 1; j <= m; j++) while(l[j] != 1 &&
            a[l[j] - 1] >= a[j]) l[j] = l[l[j] - 1];
        for(int j = m; j >= 1; j--) while(r[j] != m &&
            a[r[j] + 1] >= a[j]) r[j] = r[r[j] + 1];
        for(int j = 1; j <= m; j++) ans = max(ans, (r[j]
            - l[j] + 1) * a[j]);
    }
    return ans;
}

```

8.8 p-Median

```

for(int i = 1; i <= n; i++) {
    for(int j = i; j <= n; j++) {
        dis[i][j] = 0;
        for(int k = i; k <= j; k++) dis[i][j] += abs(arr[k]
            - arr[i + j >> 1]);
    }
}

```



```

    if(i == 1) dp[i][j] = dis[i][j];
}
}
for(int i = 2; i <= p; i++) {
    for(int j = i; j <= n; j++) {
        dp[i][j] = INF;
        for(int k = i; k <= j; k++) {
            if(dp[i][j] > dp[i - 1][k - 1] + dis[k][j]) {
                dp[i][j] = dp[i - 1][k - 1] + dis[k][j];
                fa[i][j] = k - 1;
            }
        }
    }
}
}

```

8.9 Tree Knapsack

```

int dfs(int u) {
    int p = 1;
    dp[u][1] = s[u];
    for (int v : edge[u]) {
        int siz = dfs(v);
        for (int i = min(p, m + 1); i; i--)
            for (int j = 1; j <= siz && i + j <= m + 1; j++)
                dp[u][i + j] = max(dp[u][i + j], dp[u][i] + dp[v][j]);
        p += siz;
    }
    return p;
}

```

8.10 質數個數

- 10^2 內有 25 個質數
- 10^3 內有 168 個質數
- 10^4 內有 1229 個質數
- 10^5 內有 9592 個質數
- 10^6 內有 78498 個質數
- 10^7 內有 664579 個質數
- 10^8 內有 5761455 個質數
- 10^9 內有 50847534 個質數
- 10^{12} 內有 37607912018 個質數
- 10^{18} 內有 24739954287740860 個質數

8.11 AC-Automaton

```

// 1-based
// n is the number of patterns
struct Automaton {
    static const int MXN = 1e6;
    int n, cnt, vis[MXN], rev[MXN], indeg[MXN], ans[MXN];
    queue<int> q;
    struct trie_node {
        vector<int> son;
        int fail, flag, ans;
        trie_node(): son(27), fail(0), flag(0) {}
    } trie[MXN];
    void init(int _n) {
        n = _n, cnt = 1;
        for (int i = 1; i <= n; i++) vis[i] = 0;
    }
    // insert a string s with number num
    // num is the index of the pattern
    void insert(string s, int num) {
        int u = 1, len = s.size();
        for (int i = 0; i < len; i++) {
            int v = s[i] - 'a';
            if (!trie[u].son[v]) trie[u].son[v] = ++cnt;
            u = trie[u].son[v];
        }
        trie[u].flag = num;
    }
}

```

```

}
if (!trie[u].flag) trie[u].flag = num;
rev[num] = trie[u].flag;
}
void getfail() {
    for (int i = 0; i < 26; i++) trie[0].son[i] = 1;
    q.push(1);
    trie[1].fail = 0;
    while (q.size()) {
        int u = q.front(); q.pop();
        int Fail = trie[u].fail;
        for (int i = 0; i < 26; i++) {
            int v = trie[u].son[i];
            if (!v) {
                trie[u].son[i] = trie[Fail].son[i];
                continue;
            }
            trie[v].fail = trie[Fail].son[i];
            indeg[trie[Fail].son[i]]++;
            q.push(v);
        }
    }
}
void topu() {
    for (int i = 1; i <= cnt; i++)
        if (!indeg[i]) q.push(i);
    while (q.size()) {
        int fr = q.front(); q.pop();
        vis[trie[fr].flag] = trie[fr].ans;
        int u = trie[fr].fail;
        trie[u].ans += trie[fr].ans;
        if (!--indeg[u]) q.push(u);
    }
}
void query(string &s) {
    int u = 1, len = s.size();
    for (int i = 0; i < len; i++) u = trie[u].son[s[i] - 'a'], trie[u].ans++;
}
void solve(string &s) {
    getfail();
    query(s);
    topu();
    for (int i = 1; i <= n; i++) ans[i] = vis[rev[i]];
}
} AC;

```

