Supplemental Note to Section 7.1

1 When X_i are **normal** and σ^2 is **known**: (CI)

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1) \text{ exactly}$$

- See Example 7.1-1 and Example 7.1-2.
- Note: exactly normal.
  Thus, n can be small or large.
- Theory: exact normal distribution.

2 When X_i are **normal** and σ^2 is **unknown**: (CI)

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t(\text{df} = n-1) \text{ exactly}$$

- See Example 7.1-5.
- Note: exact t-distribution.
  Thus, n can be small or large.
- Theory: exact t-distribution.

3 When X_i are **not necessarily normal** ($n \ge 30$) and σ^2 is **known**: (approximate CI)

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \overset{\bullet}{\sim} N(0,1)$$

- See Example 7.1-3.
- Theory: CLT.

4 When X_i are **not necessarily normal** ($n \ge 30$) and σ^2 is **unknown**: (approximate CI)

$$Z = \frac{\bar{X} - \mu}{S/\sqrt{n}} \overset{\bullet\bullet}{\sim} N(0,1)$$

- See Example 7.1-4.
- Theory: CLT + Slutzky.

■ When X_i are **not necessarily normal** ($n < 30$) and σ^2 is **unknown**: (approximate CI)

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \overset{\bullet\bullet\bullet}{\sim} t(\text{df} = n-1)$$

- Note: if $n \ge 30$, then use 4.
- Theory: CLT + Slutzky + Rule of thumb.

Supplemental Note to Section 7.2

1 When $X_i \sim N(\mu_x, \sigma_x^2)$ for $i = 1, 2, \ldots, n_x$ and $Y_j \sim N(\mu_y, \sigma_y^2)$ for $j = 1, 2, \ldots, n_y$ (σ_x^2 and σ_y^2 are **known**):

$$Z = \frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\sigma_x^2/n_x + \sigma_y^2/n_y}} \sim N(0,1) \text{ exactly}$$

- See Example 7.2-1.
- Note: exactly normal.
  Thus, n_x and n_y can be small or large.
- Theory: exact normal distribution.

2 When $X_i \sim N(\mu_x, \sigma_x^2)$ for $i = 1, 2, \ldots, n_x$ and $Y_j \sim N(\mu_y, \sigma_y^2)$ for $j = 1, 2, \ldots, n_y$ ($\sigma_x^2 = \sigma_y^2 = \sigma^2$ but σ^2 is **unknown**) :

$$T = \frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{S_p^2\,(1/n_x + 1/n_y)}} \sim t(\text{df}) \text{ exactly}$$

where df $= (n_x - 1) + (n_y - 1) = n_x + n_y - 2$ and

$$S_p^2 = \frac{(n_x - 1)S_x^2 + (n_y - 1)S_y^2}{(n_x - 1) + (n_y - 1)}.$$

- See Example 7.2-2.
- Note: exact t-distribution.
  Thus, n_x and n_y can be small or large.
- Theory: exact t-distribution.

3 When X_i are **not necessarily normal** for $i = 1, 2, \ldots, n_x$ ($n_x \ge 30$) and Y_j are **not necessarily normal** for $j = 1, 2, \ldots, n_y$ ($n_y \ge 30$) (σ_x^2 and σ_y^2 are **unknown**):

$$Z = \frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{S_x^2/n_x + S_y^2/n_y}} \overset{\bullet\bullet}{\sim} N(0,1)$$

- See *Exercise* 7.2-5
- Theory: CLT + Slutzky.

4 When $X_i \sim N(\mu_x, \sigma_x^2)$ for $i = 1, 2, \ldots, n_x$ ($n_x < 30$) and $Y_j \sim N(\mu_y, \sigma_y^2)$ for $j = 1, 2, \ldots, n_y$ ($n_y < 30$) (σ_x^2 and σ_y^2 are **unknown**):

$$T = \frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{S_x^2/n_x + S_y^2/n_y}} \overset{\bullet}{\sim} t(\text{df} = r),$$

where

$$r = \left\lfloor \frac{\left(\frac{S_x^2}{n_x} + \frac{S_y^2}{n_y}\right)^2}{\frac{1}{n_x - 1}\left(\frac{S_x^2}{n_x}\right)^2 + \frac{1}{n_y - 1}\left(\frac{S_y^2}{n_y}\right)^2} \right\rfloor$$

- See Example 7.2-3.
- Theory: Welch approximation.