

# Basics on Statistical Simulation

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## 1 Definition of Statistical Simulation

## 2 Problems can be solved by statistical simulations

- Estimation of the probability when tossing a die or dice
- Ratio of the circumference of a circle

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# Definition of Statistical Simulation

## Definition (wikipedia)

Monte Carlo methods (or Monte Carlo experiments) are a broad class of computational algorithms that rely on **repeated random sampling** to obtain numerical results. Their essential idea is using randomness to solve problems that might be **deterministic in principle**. They are often used in physical and mathematical problems and are most useful when it is difficult or impossible to use other approaches. *Monte Carlo methods are mainly used in three(actually much more?) problem classes*: optimization, numerical integration, and generating draws from a probability distribution.

- Monte Carlo methods  $\implies$  statistical or stochastic simulation.
- It can solve **stochastic** and **deterministic** problems.
- More applications in statistics: power, distribution, efficiency, bias-correction, etc.

# Tips for Simulation

From my teaching experience, many students are confused with:

- sample size  $n$  and
- the iteration (replication) number  $I$ .

- Sample size  $n$  is in general the number of random variable(s).
- The iteration number  $I$  is the number of repeated experiments.

# Problems can be solved by statistical simulations

In this class, we will solve probability and mathematics problems by statistical simulations.

- ① Probability: estimation of the probability when tossing a die or dice.
- ② Mathematics: the ratio of the circumference of a circle,  $\pi = 3.14\dots$

# (1) Estimation of the probability

**We are interested in the probability when tossing a single die.**

## R Code for tossing a fair die

```
set.seed(1)
ITER = 10000
x = sample(1:6, size=ITER, replace=TRUE)
table(x) / ITER
table( factor(x, levels=1:6)) / ITER # cosmetic
```

**What is the prob. of sum of two outcomes when tossing two dice.**

## R Code for tossing two dice (sum of two)

```
set.seed(1)
ITER = 10000
x = sample(1:6, size=ITER, replace=TRUE)
y = sample(1:6, size=ITER, replace=TRUE)
table(x+y) / ITER
table( factor(x+y, levels=1:12)) / ITER # cosmetic
```

# (1) Estimation of the probability

Homework: 1. Find the probability of the difference of two dice.  
2. What if a die is not fair?



## (2) Ratio of the circumference of a circle, $\pi = 3.14 \dots$

The idea is very simple — consider a dart game.

The probability that a pin drops in a circle when a pin was thrown at random is given by:

$$\frac{\pi}{4} = \frac{\text{Area of unit circle}}{\text{Area of square}} \approx \frac{\# \text{ of pins in a circle}}{\# \text{ of all the pins}} = \hat{p}.$$

Thus, we have  $\pi \approx 4 \times \hat{p}$ .

◇ NB: Avoid using **for()** loop in R.

### R Code for finding $\pi = 3.14 \dots$

```
set.seed(1)
ITER = 100000
x = runif(ITER, min=-1, max=1)
y = runif(ITER, min=-1, max=1)
4 * sum (x^2 + y^2 < 1) / ITER
```

(2) Ratio of the circumference of a circle,  $\pi = 3.14 \dots$