Basics on Statistical Simulation

Chanseok Park (박찬석)

Applied Statistics Laboratory
Department of Industrial Engineering
Pusan National University

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Overview

1. Definition of Statistical Simulation

2. Problems can be solved by statistical simulations
    - Estimation of the probability when tossing a die or dice
    - Ratio of the circumference of a circle

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Definition of Statistical Simulation

Definition (wikipedia)

Monte Carlo methods (or Monte Carlo experiments) are a broad class of computational algorithms that rely on **repeated random sampling** to obtain numerical results. Their essential idea is using randomness to solve problems that might be **deterministic in principle**. They are often used in physical and mathematical problems and are most useful when it is difficult or impossible to use other approaches. *Monte Carlo methods are mainly used in three(~~actually much more~~?) problem classes*: optimization, numerical integration, and generating draws from a probability distribution.

- Monte Carlo methods $\Longrightarrow$ statistical or stochastic simulation.
- It can solve **stochastic** and **deterministic** problems.
- More applications in statistics: power, distribution, efficiency, bias-correction, etc.

Tips for Simulation

From my teaching experience, many students are confused with:

- sample size n and
- the iteration (replication) number I.

- Sample size n is in general the number of random variable(s).
- The iteration number I is the number of repeated experiments.

Problems can be solved by statistical simulations

In this class, we will solve probability and mathematics problems by statistical simulations.

1. Probability: estimation of the probability when tossing a die or dice.
2. Mathematics: the ratio of the circumference of a circle, $\pi = 3.14\ldots$

(1) Estimation of the probability

We are interested in the probability when tossing a single die.

R Code for tossing a fair die

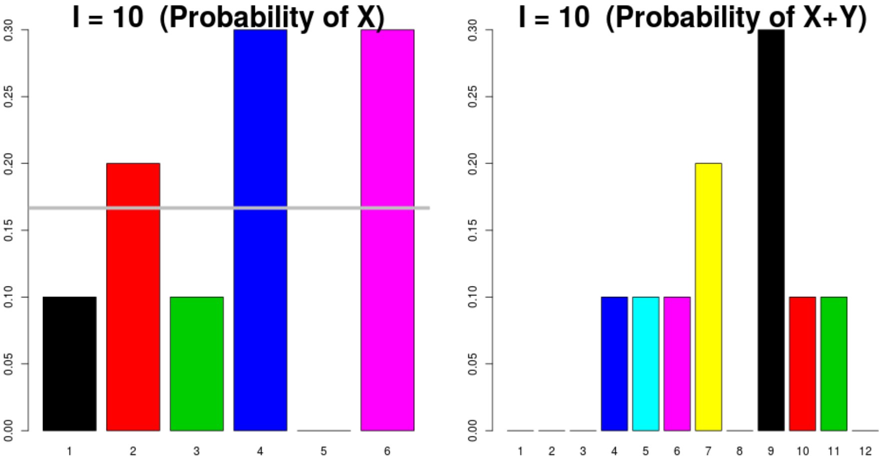
```
set.seed(1)
ITER = 10000
x = sample(1:6, size=ITER, replace=TRUE)
table(x) / ITER
table( factor(x, levels=1:6)) / ITER  # cosmetic
```

What is the prob. of sum of two outcomes when tossing two dice.

R Code for tossing two dice (sum of two)

```
set.seed(1)
ITER = 10000
x = sample(1:6, size=ITER, replace=TRUE)
y = sample(1:6, size=ITER, replace=TRUE)
table(x+y) / ITER
table( factor(x+y, levels=1:12)) / ITER  # cosmetic
```

(1) Estimation of the probability



~~Homework~~: 1. Find the probability of the difference of two dice.

2. What if a die is not fair?

(2) Ratio of the circumference of a circle, $\pi = 3.14\ldots$

The idea is very simple — consider a dart game.
The probability that a pin drops in a circle when a pin was thrown at random is given by:

$$\frac{\pi}{4} = \frac{\text{Area of unit circle}}{\text{Area of square}} \approx \frac{\#\text{ of pins in a circle}}{\#\text{ of all the pins}} = \hat{p}.$$

Thus, we have $\pi \approx 4 \times \hat{p}$.
$\Diamond$ NB: Avoid using for() loop in R.

R Code for finding $\pi = 3.14\ldots$

```
set.seed(1)
ITER = 100000
x = runif(ITER, min=-1, max=1)
y = runif(ITER, min=-1, max=1)
4 * sum (x^2 + y^2 < 1) / ITER
```

(2) Ratio of the circumference of a circle, $\pi = 3.14\ldots$