

## Supplemental Note to Section 7.1

- 1 When  $X_i$  are **normal** and  $\sigma^2$  is **known**: (CI)

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1) \text{ exactly}$$

- See Example 7.1-1 and Example 7.1-2.
- Note: exactly normal.  
Thus,  $n$  can be small or large.
- Theory: exact normal distribution.

- 2 When  $X_i$  are **normal** and  $\sigma^2$  is **unknown**: (CI)

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t(\text{df} = n - 1) \text{ exactly}$$

- See Example 7.1-5.
- Note: exact  $t$ -distribution.  
Thus,  $n$  can be small or large.
- Theory: exact  $t$ -distribution.

- 3 When  $X_i$  are **not necessarily normal** ( $n \geq 30$ ) and  $\sigma^2$  is **known**: (approximate CI)

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \overset{\bullet}{\sim} N(0, 1)$$

- See Example 7.1-3.
- Theory: CLT.

- 4 When  $X_i$  are **not necessarily normal** ( $n \geq 30$ ) and  $\sigma^2$  is **unknown**: (approximate CI)

$$Z = \frac{\bar{X} - \mu}{S/\sqrt{n}} \overset{\bullet}{\sim} N(0, 1)$$

- See Example 7.1-4.
- Theory: CLT + **Slutzky**.

- When  $X_i$  are **not necessarily normal** ( $n < 30$ ) and  $\sigma^2$  is **unknown**: (approximate CI)

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \overset{\bullet}{\sim} t(\text{df} = n - 1)$$

- Note: if  $n \geq 30$ , then use 4.
- Theory: CLT + **Slutzky** + **Rule of thumb**.

## Supplemental Note to Section 7.2

- 1 When  $X_i \sim N(\mu_x, \sigma_x^2)$  for  $i = 1, 2, \dots, n$  and  $Y_j \sim N(\mu_y, \sigma_y^2)$  for  $j = 1, 2, \dots, m$  ( $\sigma_x^2$  and  $\sigma_y^2$  are **known**):

$$Z = \frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\sigma_x^2/n + \sigma_y^2/m}} \sim N(0, 1) \text{ exactly}$$

- See Example 7.2-1.
- Note: exactly normal.  
Thus,  $n$  and  $m$  can be small or large.
- Theory: exact normal distribution.

- 2 When  $X_i \sim N(\mu_x, \sigma_x^2)$  for  $i = 1, 2, \dots, n$  and  $Y_j \sim N(\mu_y, \sigma_y^2)$  for  $j = 1, 2, \dots, m$  ( $\sigma_x^2 = \sigma_y^2 = \sigma^2$  but  $\sigma^2$  is **unknown**):

$$T = \frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{S_p^2(1/n + 1/m)}} \sim t(\text{df}) \text{ exactly}$$

where  $\text{df} = (n - 1) + (m - 1)$  and

$$S_p^2 = \frac{(n - 1)S_x^2 + (m - 1)S_y^2}{(n - 1) + (m - 1)}.$$

- See Example 7.2-2.
- Note: exact  $t$ -distribution.  
Thus,  $n$  and  $m$  can be small or large.
- Theory: exact  $t$ -distribution.

- 3 When  $X_i$  are **not necessarily normal** for  $i = 1, 2, \dots, n$  ( $n \geq 30$ ) and  $Y_j$  are **not necessarily normal** for  $j = 1, 2, \dots, m$  ( $m \geq 30$ ) ( $\sigma_x^2$  and  $\sigma_y^2$  are **unknown**):

$$Z = \frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{S_x^2/n + S_y^2/m}} \overset{\bullet}{\sim} N(0, 1)$$

- Theory: CLT + **Slutzky**.

- 4 When  $X_i \sim N(\mu_x, \sigma_x^2)$  for  $i = 1, 2, \dots, n$  ( $n < 30$ ) and  $Y_j \sim N(\mu_y, \sigma_y^2)$  for  $j = 1, 2, \dots, m$  ( $m < 30$ ) ( $\sigma_x^2$  and  $\sigma_y^2$  are **unknown**):

$$T = \frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{S_x^2/n + S_y^2/m}} \overset{\bullet}{\sim} t(\text{df} = r),$$

where

$$r = \left\lfloor \frac{\left(\frac{S_x^2}{n} + \frac{S_y^2}{m}\right)^2}{\frac{1}{n-1}\left(\frac{S_x^2}{n}\right)^2 + \frac{1}{m-1}\left(\frac{S_y^2}{m}\right)^2} \right\rfloor$$

- See Example 7.2-3.
- Theory: **Welch** approximation.