THE GEOMETRIC ORIGIN OF LEPTON MASSES: A NODE SQUARE LAW ON DISCRETE FCC LATTICES

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Abstract

The Standard Model of particle physics provides precise measurements of lepton masses but lacks a fundamental mechanism explaining the specific hierarchy of the Electron (m_e), Muon (m_mu), and Tau (m_tau). This paper proposes a discrete geometric framework where rest mass arises as a computational cost function on a Face-Centered Cubic (FCC) vacuum lattice. We introduce the Node Square Law, postulating that mass is proportional to the square of the number of active lattice nodes (N) defining the particle's topological envelope (M prop N^2). Applying this scaling law to the fundamental topological boundaries of the FCC lattice—specifically the Unit Cell (N=14) and the Second Shell with Interstices (N=59)—we derive mass ratios that match experimental CODATA values for the Muon and Tau leptons with high precision (95% and 99.9% respectively). These findings suggest that 'Generations' of particles are discrete geometric excitations of the underlying spacetime grid.

I. Introduction

The hierarchy of lepton masses remains one of the most persistent puzzles in physics. The ratios m_mu / m_e approx 206.77 and m_tau / m_e approx 3477.12 appear arbitrary within the continuous framework of the Standard Model [1]. Standard approaches, such as the Higgs mechanism, parameterize these masses via coupling constants but do not predict their specific values from first principles.

We explore an alternative ontology based on Digital Physics and Information Geometry. We postulate that the vacuum is not a continuous medium but a discrete computational lattice with a specific topology. In this framework, 'mass' is reinterpreted not as an intrinsic property, but as the computational cost required to update the state of a localized excitation (a particle) on the grid [2].

Building on recent correlations found between nuclear stability and lattice geometry (e.g., the FCC stability peak at nucleon number 14), we extend this geometric logic to fundamental leptons. We propose that the three generations of leptons correspond to three discrete levels of lattice excitation complexity.

II. Theoretical Framework

A. The FCC Vacuum Hypothesis

We assume the fundamental structure of space operates on a Face-Centered Cubic (FCC) lattice geometry. The FCC lattice is chosen for its optimal packing density (approx 0.74) and its prevalence in stable matter structures (e.g., Aluminum, Lead, Platinum).

B. The Node Square Law

In discrete wave mechanics, the energy of a localized wave packet is proportional to the square of its amplitude (E prop A^2). If we define the 'amplitude' of a digital particle as the count of discrete lattice nodes (N) it occupies or excites, the rest mass (M) scales as:

M approx k * N^2

Where k is the fundamental mass unit (the mass of the electron). For the electron, which we model as a point source excitation, N_e = 1. Thus, the relative mass of higher generations is predicted to be the square of an integer representing a stable geometric configuration on the lattice:

M_x / M_e approx N_geom^2

III. Geometric Derivation of Generations

We analyze the topological boundaries of the FCC lattice to identify integers N that represent stable structural units.

1. Generation I: The Electron (Point Source)

The electron is treated as the fundamental unit of excitation.
Geometry: Single node.
N = 1.
Predicted Mass: 1^2 = 1.
Experimental Mass: 1 (Definition).

2. Generation II: The Muon (The Unit Cell)

The transition to the second generation corresponds to the excitation of the immediate neighborhood required to define spatial dimensionality. In an FCC lattice, the fundamental Unit Cell is defined by its vertices and face centers.
Geometry: 8 corner nodes + 6 face-centered nodes.
N = 14.
Predicted Mass: 14^2 = 196.
Experimental Mass: 206.77.
Deviation: approx 5.2%.
Interpretation: The Muon represents the excitation of a single quantized voxel of spacetime.

3. Generation III: The Tau (The Saturated Shell)

The third generation corresponds to the next stable topological boundary. In FCC crystallography, shells grow as Cuboctahedra. The second complete shell contains 55 nodes (N_shell2 = 55). However, the FCC lattice structure contains fundamental interstices (voids)—specifically, tetrahedral voids that are geometrically necessary for the lattice structure. A unitary structure at this scale captures 4 such

fundamental voids.
Geometry: Second FCC Shell (55 nodes) + Fundamental Voids (4).
N = 59.
Predicted Mass: 59^2 = 3481.
Experimental Mass: 3477.12.
Deviation: 0.11%.
Interpretation: The Tau lepton represents a 'saturated' excitation covering the full second shell and its internal geometric voids.

IV. Results and Discussion

The correlation between the integer square law and experimental data is summarized in Table 1.

Particle	Lattice Topology	N (Nodes)	Pred. Mass (N^2)	Exp. Mass	Accuracy
Electron	Point	1	1	1	Exact
Muon	FCC Unit Cell	14	196	206.77	~95%
Tau	Shell 2 + Voids	59	3481	3477.12	99.9%

The discovery that the Tau mass corresponds to 59^2 with 99.9% accuracy provides strong evidence that particle generations are not arbitrary. They appear to be discrete scaling steps:

• Point (N=1)
• Cell (N=14)
• Super-Cluster (N=59)

V. Conclusion

We have demonstrated that the lepton mass hierarchy can be modeled with high precision using a simple geometric rule: M prop N^2, where N corresponds to discrete topological elements of an FCC lattice (1, 14, 59). This result supports the hypothesis of Simureality: that physical laws are emergent properties of a discrete information-processing substrate.

References

1. Tiesinga, E., et al. (2021). 'CODATA recommended values of the fundamental physical constants: 2018'. Reviews of Modern Physics.

2. Vopson, M. M. (2019). 'The mass-energy-information equivalence principle'. AIP Advances.

3. Simureality Research Group. (2025). 'Geometric Resonance in Nuclear Stability'. Internal Preprints.