COMP3766 Assignment 3
Forward Kinematics
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1. **(10 marks)** The RRRP SCARA robot of Figure 1 is shown in its zero position.

   (a) Determine the end-effector zero position configuration M

   (b) Construct configuration tables for the screw axes, listing $\mathcal{S}_i = (\omega_i, v_i)$ and $\mathcal{B}_i = (\omega_i, v_i)$, for the screw axes $\mathcal{S}_i$ in $\{0\}$, and the screw axes $\hat{\mathcal{B}}_i$ in $\{b\}$.

   (c) For $\ell_0 = \ell_1 = \ell_2 = 1$ and the joint variable values $\theta = (0, \pi/2, -\pi/2, 1)$, calculate by hand the end-effector configurations $T \in SE(3)$ in both $\{0\}$ and $\{b\}$. Show your workings.

   (d) Confirm that they agree with each other.

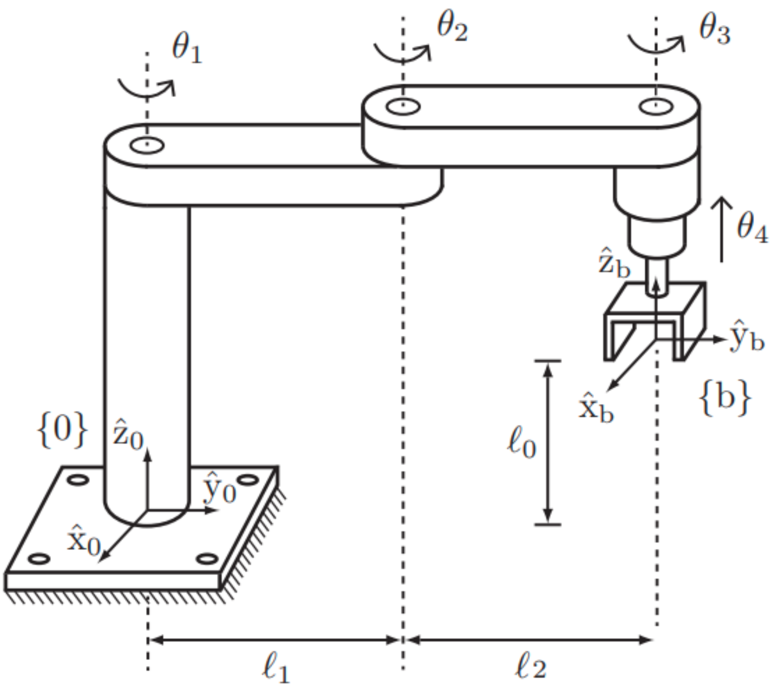

Figure 1: RRRP SCARA robot in its zero position.

2. **(10 marks)** The PUMA robot of Figure 2 is shown in its zero position.

$$T_{b1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_{b2} = \begin{bmatrix} 0 & 0 & -1 & -0.150 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_{b3} = \begin{bmatrix} 0 & 0 & -1 & -0.150 \\ 0 & 1 & 0 & 0.430 \\ 1 & 0 & 0 & 0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{b4} = \begin{bmatrix} 1 & 0 & 0 & -0.150 \\ 0 & 1 & 0 & 0.86 \\ 0 & 0 & 1 & 0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_{b5} = \begin{bmatrix} 1 & 0 & 0 & -0.150 \\ 0 & 0 & 1 & 0.86 \\ 0 & -1 & 0 & 0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_{b6} = \begin{bmatrix} 0 & 0 & 1 & -0.150 \\ -1 & 0 & 0 & 0.86 \\ 0 & -1 & 0 & 0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The given matrices are the transformation of each link in zero position configuration.

(a) Find the transformation matrix of each link with respect to their parent-link; in other words, find $T_{12}, T_{23}, T_{34}, T_{45}, T_{56}$. (Hint: This will be a lot of calculations if you really want to calculate, but some of these matrices can be intuitively determined if you understand transformation matrix)

(b) Extract the rotation matrices from the transformation matrices you found in (a) and convert them to roll-pitch-yaw

(c) Determine the end-effector zero position configuration M

(d) v_i is calculated using the formula $v_i = -\omega \times q$, meaning we have to do cross-product, which you can do by hand or computer. Which numpy function can be used to calculate cross-product?

(e) Construct configuration tables for the screw axes, listing $\mathcal{B}_i = (\omega_i, v_i)$, for the screw axes $\hat{\mathcal{B}}_i$ in the end-effector (link-6) frame.

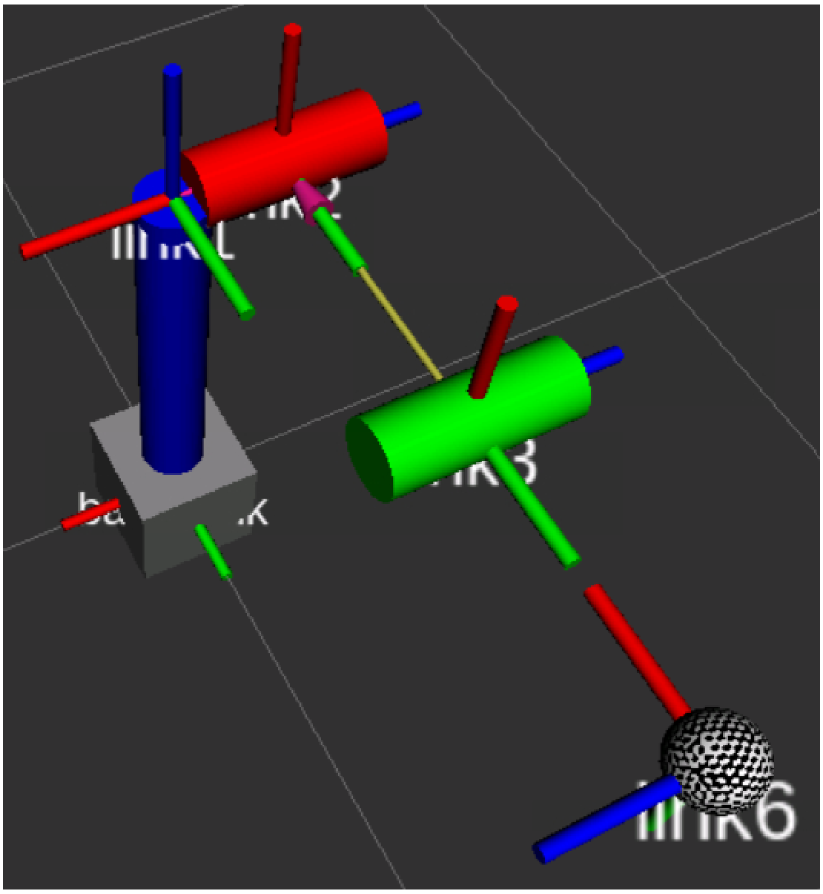

Figure 2: PUMA robot in its zero position.

3. **(10 marks)** The spherical wrist in Figure 3 robot has the joint-3 with rpy="0 -1.57 0".

(a) Convert the rpy="0 -1.57 0" to rotation matrix.

(b) Is the rotation matrix same as the rotation matrix of $\{5\}$ in reference to $\{4\}$?

(c) We want the rotation matrix of joint-3 to be:

$$\begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Convert this rotation matrix to roll-pitch-yaw.

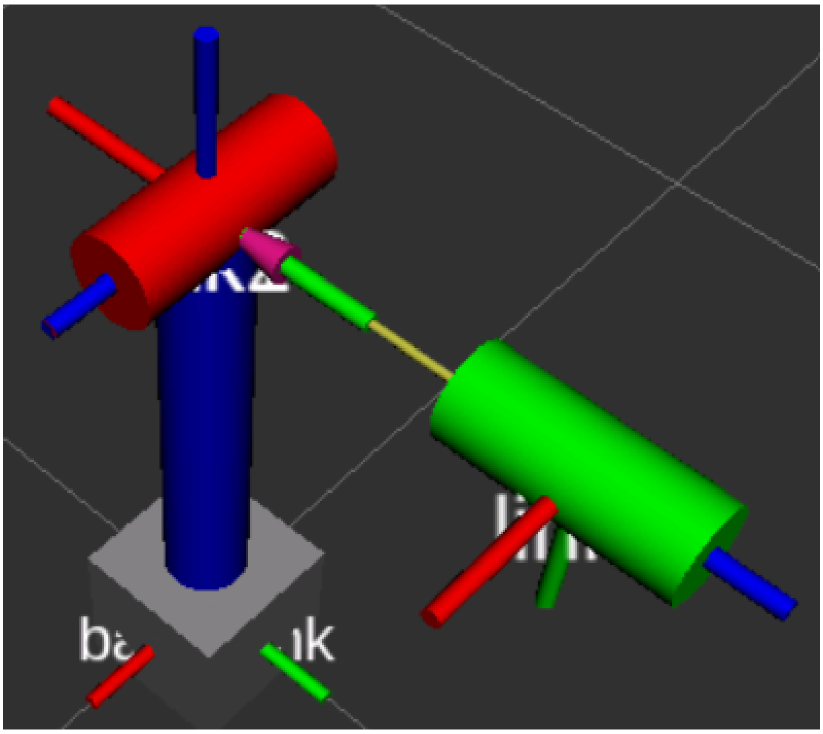

Figure 3: A spherical wrist robot in its zero position.