Q2a) We are given transformation from base to link in 0 config. We want to find the relative transformation b/w the consecutive links - T_{12}, T_{23}, T_{34}, T_{45}, T_{56}

$$T_{12} = T_{b1}^{-1} \cdot T_{b2}$$

Given $T = \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix}$, $T^{-1} = \begin{bmatrix} R^T & -R^T P \\ 0 & 1 \end{bmatrix}$

$$T_{b1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_{b2} = \begin{bmatrix} 0 & 0 & -1 & -0.15 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{b1}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{12} = T_{b1}^{-1} \cdot T_{b2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & -1 & -0.15 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{12} = \begin{bmatrix} 0 & 0 & -1 & -0.15 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We repeat these for the rest

$$T_{23} = T_{b2}^{-1} \cdot T_{b3}$$

$$T_{b2} = \begin{bmatrix} 0 & 0 & -1 & -0.15 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{b3} = \begin{bmatrix} 0 & 0 & -1 & -0.15 \\ 0 & 1 & 0 & 0.43 \\ 1 & 0 & 0 & 0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{b2}^{-1} = \begin{bmatrix} 0 & 0 & 1 & -0.67 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -0.15 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{b2}^{-1} \cdot T_{b3} = \begin{bmatrix} 0 & 0 & 1 & -0.67 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -0.15 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & -0.15 \\ 0 & 1 & 0 & 0.43 \\ 1 & 0 & 0 & 0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= T_{23} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.43 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{34} = T_{b3}^{-1} \cdot T_{b4}$$

$$T_{b3} = \begin{bmatrix} 0 & 0 & -1 & -0.15 \\ 0 & 1 & 0 & 0.43 \\ 1 & 0 & 0 & 0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_{b4} = \begin{bmatrix} 1 & 0 & 0 & -0.15 \\ 0 & 1 & 0 & 0.86 \\ 0 & 0 & 1 & 0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{b3}^{-1} = \begin{bmatrix} 0 & 0 & 1 & -0.67 \\ 0 & 1 & 0 & -0.43 \\ -1 & 0 & 0 & -0.15 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{34} = T_{b3}^{-1} \cdot T_{b4} = \begin{bmatrix} 0 & 0 & 1 & -0.67 \\ 0 & 1 & 0 & -0.43 \\ -1 & 0 & 0 & -0.15 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & -0.15 \\ 0 & 1 & 0 & 0.86 \\ 0 & 0 & 1 & 0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0.43 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{45} = T_{b4}^{-1} \cdot T_{b5}$$

$$T_{b4} = \begin{bmatrix} 1 & 0 & 0 & -0.15 \\ 0 & 1 & 0 & 0.86 \\ 0 & 0 & 1 & 0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_{b5} = \begin{bmatrix} 1 & 0 & 0 & -0.15 \\ 0 & 0 & 1 & 0.86 \\ 0 & -1 & 0 & 0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{b4}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0.15 \\ 0 & 1 & 0 & -0.86 \\ 0 & 0 & 1 & -0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{45} = \begin{bmatrix} 1 & 0 & 0 & 0.15 \\ 0 & 1 & 0 & -0.86 \\ 0 & 0 & 1 & -0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & -0.15 \\ 0 & 0 & 1 & 0.86 \\ 0 & -1 & 0 & 0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{56} = T_{b5}^{-1} \cdot T_{b6}$$

$$T_{b5} = \begin{bmatrix} 1 & 0 & 0 & -0.15 \\ 0 & 0 & 1 & 0.86 \\ 0 & -1 & 0 & 0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_{b6} = \begin{bmatrix} 0 & 0 & 1 & -0.15 \\ -1 & 0 & 0 & 0.86 \\ 0 & -1 & 0 & 0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{b5}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0.15 \\ 0 & 0 & -1 & 0.67 \\ 0 & 1 & 0 & -0.86 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 & -0.15 \\ -1 & 0 & 0 & 0.86 \\ 0 & -1 & 0 & 0.67 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) $R = R_z(yaw) \cdot R_y(pitch) \cdot R_x(roll)$

Given $R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$ we Know that

$\rightarrow$ pitch $= \theta = \arcsin(-r_{31})$
$\rightarrow$ roll $= \phi = \arctan2(r_{32}, r_{33})$
$\rightarrow$ yaw $= \varphi = \arctan2(r_{21}, r_{11})$

Doing so, we get the following values :

Joint 1 rpy : 0, 0, 0

Joint 2 rpy : 0, -1.57, 0

Joint 3 rpy : 0, 0, 0

Joint 4 rpy : 0, 1.57, 0

Joint 5 rpy : -1.57, 0, 0

Joint 6 rpy : 0, 1.57, 0

c) The full transformation from base to end effector is

$$T_{1\to6} = T_{12} \cdot T_{23} \cdot T_{34} \cdot T_{45} \cdot T_{56}$$

Multiplying the matrices from a) together, we get

$$\begin{bmatrix} 0 & 0 & 1 & -0.15 \\ -1 & 0 & 0 & 0.86 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The end-effector is $\begin{bmatrix} -0.15 \\ 0.86 \\ 0 \end{bmatrix}$

d) To find the cross product in NumPy, we can use

```
np.cross(a, b)
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e) To construct the configuration tables for the screw axes $B_i = (\omega_i, v_i)$, we need

- ↳ the screw axes $S_i = (\omega_i, v_i)$ in base frame
- ↳ the transformation matrix $T = T_{0 \to 6}$ from the base frame
- ↳ the adjoint representation of the inverse of the transformation matrix

$$\hat{B}_i = Ad_{T^{-1}} \cdot \hat{S}_i$$

Following all the steps, we get the following:

ω	r
(0, -1, 0)	(0.15, 0, -0.864)
(0, 0, -1)	(0, 0.864, 0)
(0, 0, -1)	(0, 0.432, 0)
(0, -1, 0)	(0, 0, 0)
(-1, 0, 0)	(0, 0, 0)
(0, 0, 1)	(0, 0, 0)