Assignment 3 Solution Scheme

April 11, 2025

Question 1

This questions is unchanged from the existing Assignment-3; refer to the instructor's solution

Question 2

Part (a)

We are given transformations from base to link in the 0 configuration. We want to find the relative transformation between the consecutive links - T_{12}, T_{23}, T_{34}, T_{45}, T_{56}

We know the following:

$$T = \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix}, \quad T^{-1} = \begin{bmatrix} R^T & -R^T p \\ 0 & 1 \end{bmatrix}$$

and

$$T_{xy} = T_{bx}^{-1} \cdot T_{by}$$

Going through with the calculations, we get the following answers:

$$T_{12} = \begin{bmatrix} 0 & 0 & -1 & -0.15 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_{23} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.43 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0.43 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T_{45} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_{56} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Part (b)

To find the RPY values, we know:

$$R = R_z(yaw) \cdot R_y(pitch) \cdot R_x(roll)$$

Given

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

we know that

$$\text{pitch} = \theta = \arcsin(-r_{31})$$
$$\text{roll} = \phi = \arctan 2(r_{32}, r_{33})$$
$$\text{yaw} = \psi = \arctan 2(r_{21}, r_{11})$$

Going through with the calculations, we get the following values:

$$\text{Joint 1 RPY: } 0, 0, 0$$
$$\text{Joint 2 RPY: } 0, -1.57, 0$$
$$\text{Joint 3 RPY: } 0, 0, 0$$
$$\text{Joint 4 RPY: } 0, 1.57, 0$$
$$\text{Joint 5 RPY: } -1.57, 0, 0$$
$$\text{Joint 6 RPY: } 0, 1.57, 0$$

Part (c)

We know that the full transformation from the base to the end effector is:

$$T_{1->6} = T_{12} \cdot T_{23} \cdot T_{34} \cdot T_{45} \cdot T_{56}$$

Performing the calculation using the matrices derived in a), we get:

$$\begin{bmatrix} 0 & 0 & 1 & -0.15 \\ -1 & 0 & 0 & 0.86 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore, we know the end effector orientation is:

$$\begin{bmatrix} -0.15 \\ 0.86 \\ 0 \end{bmatrix}$$

Part (d)

To find the cross product in NumPy, we can use:

np.cross(a,b)

Part (e)

To construct the configuration tables for the screw axes $B_i = (w_i, v_i)$, we need:

- The screw axes $S_i = (w_i, v_i)$, in the base frame:
- The transformation matrix $T = T_{0->6}$ from the base frame
- The adjoint representation of the inverse of the transformation matrix

$$\hat{B}_i = \mathrm{Ad}_{T^{-1}} \cdot \hat{S}_i$$

Following all the steps, we get the following:

ω	v
$(0, -1, 0)$	$(0.15, 0, -0.864)$
$(0, 0, -1)$	$(0, 0.864, 0)$
$(0, 0, -1)$	$(0, 0.432, 0)$
$(0, -1, 0)$	$(0, 0, 0)$
$(-1, 0, 0)$	$(0, 0, 0)$
$(0, 0, 1)$	$(0, 0, 0)$

Question 3

Part (a)

$$R = \begin{bmatrix} 6.1232e-17 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 6.1232e-17 \end{bmatrix}$$

Part (b)

No.

Part (c)

Following the same steps from Q2b, we get:

$$0, -1.57, 1.57$$