CG1 公式汇总

1. 基础公式

1.1. 光追基本 1

$$\vec{r}(t) = o + t\vec{d}$$

1.2. 基本 Ray Tracing

(1) Radiant energy Q

$$E_P = h \cdot f = h \cdot \frac{c}{\lambda}, \quad Q = \sum E_P$$

Unit: R: [J] or [Ws]

(2) Radiant flux Φ

$$\Phi = \frac{dQ}{dt}$$

Unit: R: [W], P: [lm]

(3) Intensity I

$$I = \frac{d\Phi}{d\omega}$$

Unit: R: [W/sr], P: [lm/sr]

(4) Radiosity B

$$B = \frac{d\Phi_{\text{out}}}{dA}$$

Unit: R: [W/m^2], P: [lm/m^2]

(5) Irradiance E

$$E = \frac{d\Phi_{\text{in}}}{dA}$$

Unit: R: [W/m^2], P: [lm/m^2]

Isotropic Light Source

$$E(x) = \frac{\Phi_g \cos(\theta)}{4\pi r^2}$$

Radiance L

$$L = \frac{\Phi}{d\Omega' \, dA}$$

Unit: R: [W/(sr·m^2)], P: [lm/(sr·m^2)]

2. 其它公式

2.1. 关于 Shading

$$\omega, \Omega = \int_{\Phi_0}^{\Phi_1} d\Phi \int_{\theta_0(\Phi)}^{\theta_1(\Phi)} \sin(\theta)\, d\theta$$

$$\Delta\omega, \Delta\Omega \approx \frac{\Delta A \cos(\theta)}{r^2}, \quad d^2\Phi = L(x, \theta, \Phi)\, dA \cos\theta\, \omega$$

θ 是光线与法线夹角

Irradiance

$$E = \frac{d\Phi}{dA}, \quad d\Phi = \left[\int_\Omega L_i(x, \theta, \omega) \cos\theta \, d\omega\right] dA$$

$$E = \int_\Omega L_i(x, \theta, \Phi) \cos\theta \, d\omega = \int_0^{\pi/2} \int_0^{2\pi} L_i(x, \theta, \Phi) \cos\theta \sin\theta \, d\theta \, d\Phi$$

Radiosity

$$B = \frac{d\Phi}{dA}, \quad d\Phi = \left[\int_\Omega L_o(x, \theta, \omega) \cos\theta \, d\omega\right] dA$$

$$E = \int_\Omega L_o(x, \theta, \Phi) \cos\theta \, d\omega = \int_0^{\pi/2} \int_0^{2\pi} L_o(x, \theta, \Phi) \cos\theta \sin\theta \, d\theta \, d\Phi$$

Albedo ρ

$$\rho = \frac{\Phi_{\text{out}}}{\Phi_{\text{in}}}$$

统一 Irradiance 公式:

$$L_o(\omega_o) = L_e(\omega_o) + \int_\Omega f_r(\omega_i \to \omega_o)\, L(\omega_i) \cos(\theta_i)\, d\omega_i$$

1) 面光（平行光源）:

$$dE(\theta_i, \Phi_i) = L_i(\theta_i, \Phi_i) \cos(\Phi_i)\, d\omega_i$$

2) 点光源，随距离衰减:

$$dE(\omega_i) = L_{ps}\, \delta(\omega_{ps} - \omega_i) \frac{1}{r^2} \cos(\Phi_i)\, d\omega_i$$

2.2. BRDF Measurement

$$f_r(\theta_o, \Phi_o, \theta_{in}, \Phi_{in}) = \frac{L_o(\theta_o, \Phi_o)}{dE_i(\theta_i, \Phi_i)}, \quad \rho_{dh}(\theta_i, \Phi_i) = \int_\Omega f_r \cos\theta_o \, d\omega_o$$

2.3. Fresnel Equations

$$R_\perp = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}, \quad R_\parallel = \frac{n_1 \cos\theta_2 - n_2 \cos\theta_1}{n_1 \cos\theta_2 + n_2 \cos\theta_1}.$$

$$T_\perp = \frac{2\, n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2}, \quad T_\parallel = \frac{2\, n_1 \cos\theta_1}{n_1 \cos\theta_2 + n_2 \cos\theta_1}.$$

金属:

$$R(\theta_i, \lambda) = \frac{1}{2} \frac{\sin^2(\theta_i - \theta_t)}{\sin^2(\theta_i + \theta_t)} \left(1 + \frac{\cos^2(\theta_i + \theta_t)}{\cos^2(\theta_i - \theta_t)}\right), \quad \sin\theta_i = \frac{\sin\theta_i}{n_\lambda}$$

2.4. Lambert's Law

$$E = \frac{\Phi \cos\theta}{A}$$

Lambert Surface (Isotropic)

$$\Phi = L\, r^2 \pi \, d\Omega, \quad \Phi = L \cos(\Phi)\, r^2 \pi \, d\Omega$$