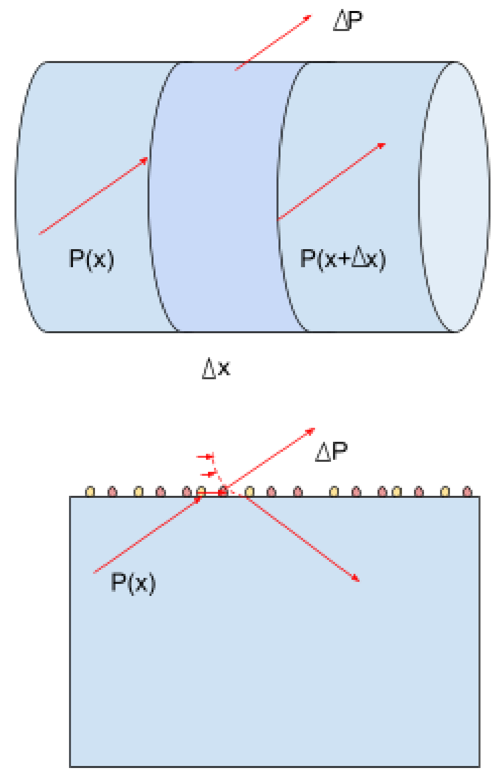
Inserting loss by nanoparticles in MMI calculations

Concept and model

Figure 1:

An infinitesimal segment, of width Δx, of the fiber showing the relation between the power of a mode entering and exiting the segment. The presence of nanoparticles on top of the fiber surface causes a loss, ΔP, from the power leaving the segment.

This loss is due to interaction of the evanescent beam in the cladding region with the monolayer of the nanoparticles. Hence, the amount of power lost by the NPs on the surface should be correlated to the portion of the modal power in the cladding region.



Mathematically, we can write the power loss in the segment in terms of the input power, $P(x)$, and the NPs layer attenuation coefficient, αthat is calculated using Mei scattering.

$$\Delta P(x) =- \eta\alpha\Delta x \cdot P(x) \qquad (1)$$

$$\frac{\Delta P(x)}{P(x)} =- \eta\alpha\Delta x \qquad (2)$$

When working at the limit $\Delta x \rightarrow 0$, the difference is replaced by differentiation

$$\frac{dP(x)}{P(x)} =- \eta\alpha dx \rightarrow ln(P) - ln(P_o) =- \eta\alpha x \qquad (3)$$

$$P(x) = P_o \cdot exp(- \eta\alpha x) \qquad (4)$$

Solving equation 4, gives the well known Beer’s law.

$$\frac{P(x)}{P_o} = exp(-\ \eta\alpha x) \tag{5}$$

Here η is the portion of mode power in the cladding region.

$$\eta = \frac{P_{cladd}}{P_{total}} \tag{6}$$

For linearly polarized radial modes, the field inside the fiber is written as

$$E(r) = J_0(\frac{ur}{a})/J_0(u) \qquad r \le a \tag{7a}$$

$$E(r) = K_0(\frac{wr}{a})/K_0(w) \qquad r > a \tag{7b}$$

Where

$$u^2 = k_o^2 a^2 n_{core}^2 - \beta^2 \tag{8a}$$

$$w^2 = \beta^2 - k_o^2 a^2 n_{clad}^2 \tag{8b}$$

$$v^2 = u^2 + w^2 \tag{8c}$$

And a is the radius of the core. The fraction of the power in the cladding is then

$$\eta = \frac{\int_{r=a}^{\infty} J_0^2(u)\cdot K_0^2(wr/a)dr}{\int_{r=0}^{a} K_0^2(w)\cdot J_0^2(ur/a)dr + \int_{r=a}^{\infty} J_0^2(u)\cdot K_0^2(wr/a)dr} \tag{9}$$

For the upper limit in equation 9, instead of infinity, it might be more accurate to set the limit at the thickness of the nanoparticles layer. If a monolayer of particles are considered, then the thickness equals the diameter of the particle. For simplicity, one can define a thickness d_{NP} as the thickness of the film. Hence, equation 9 can be set as

$$\eta = \frac{\int_{r=a}^{d_{NP}} J_0^2(u)\cdot K_0^2(wr/a)dr}{\int_{r=0}^{a} K_0^2(w)\cdot J_0^2(ur/a)dr + \int_{r=a}^{\infty} J_0^2(u)\cdot K_0^2(wr/a)dr} \tag{10}$$

An approximate form of the power fraction is defined in Snyder and Love (Optical Waveguide Theory.)

$$\eta = 1 - \frac{u^2}{v^2}\left\{\frac{w^2}{u^2} + \frac{K_0^2(w)}{K_1^2(w)}\right\} \tag{11}$$

Using either equations 10 or 11, for each mode there is a different power fraction in the cladding. This portion of the power will contribute to the loss due to Mei scattering. Hence, in the MMI equation, we can add the following term in the equation

$$E_{MMI}(r,z) = \sum_{j=1}^{N_P} a_j \psi_j(r) exp(i\beta_j z) exp(-\ \eta_j \alpha z/2) \tag{12}$$

In equation 12, N_P is the total number of guided modes when neglecting the presence of the nanoparticles film. The constant a_j is the amplitude coefficient for each mode and $\psi_j(r)$ is the mode radial profile. Finally, β_j is the longitudinal propagation constant for the jth mode.

Evaluation of the power ratio

Numerical evaluation of equation 10, requires solving it in discrete form. First point to consider here is that the infinity upper limit in the denominator can be replaced by a finite value over which the extension of the evanescent wave vanishes. This can be a range of a few microns for weakly guided fiber. So, let's set this limit as $a_{max} = M \cdot \Delta r$ where *M* is the number of samples and Δr is the step size in the radial direction. Equation 10 can then be written as

$$\eta_j = \frac{\sum_{m=0}^{M_{NP}} K_0^2(w_j \cdot (a + m \cdot \Delta r)/a)}{\left(K_0(w_j)/J_0(u_j)\right)^2 \sum_{n=0}^{N} J_0^2(u_j \cdot n \cdot \Delta r/a) + \sum_{m=0}^{M} K_0^2(w_j \cdot (a + m \cdot \Delta r)/a)} \tag{13}$$

In the numerator, the upper limit of the summation is $M_{NP} = d_{NP}/\Delta r$.