Calculating the Guided Modes in Optical Fibers and Waveguides

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***Abstract*—A new method is presented for calculating the fundamental mode and all the higher order guided modes in optical fibers and waveguides. This method, which is referred to as "slowly decaying imaginary distance beam propagation method," has been demonstrated to be very accurate and efficient by applying it to analyze well-known guided wave devices.**

***Index Terms*—Beam propagation method (BPM), eigenvalues, finite-difference method, imaginary distance BPM, mode solving techniques, optical fibers, optical waveguides.**

I. Introduction

BEAM propagation method (BPM)-based mode solving methods are important among the different types of mode solving techniques to calculate the modes in optical fibers and waveguides. In one method, BPM is combined with Fourier transform along the propagation direction to calculate the mode properties of optical fibers [1], [2]. In [3], it is demonstrated that in the imaginary distance propagation procedure the fundamental mode will dominate after a sufficiently long propagation distance. In [4], a generalization of the imaginary distance propagation method to vector mode calculation is described. However, the imaginary distance propagation method introduced in [4] can only be used to calculate the fundamental mode. Several methods have been reported to extend the imaginary distance propagation method to calculate higher order modes [5]–[7]. In the method described in [5], the higher order modes are obtained by removing contributions from all the lower order modes. This means that to compute a higher order mode, the complete set of all the lower order modes have to be known. Therefore, the accuracy of the solution depends on the accuracy of all the previously calculated lower order modes. Furthermore, the subtraction may be done imperfectly, leaving residual traces of the lower order modes. The method in [6] is for calculating the higher order modes directly. However, it is based on properly choosing parameters so that one individual mode is amplified faster than other modes, and it requires the solution of a nearly singular problem. In the method described in [7], the governing equation is modified for the imaginary distance BPM. However, when numerically solving the governing equation, there is a restriction on the largest allowed propagation step size to guarantee stability, and a calculation using coarser discrete space coordinates is conducted first, and the output of this calculation is used as the input for a second calculation using finer discrete space coordinates. Furthermore, the correlation function and least squares Prony method have to be used to obtain the propagation constants after conducting the standard BPM calculation on the real axis. Therefore, this method is complicated to implement. A method based on the combination of BPM with matrix pencil method [8] can also be used to calculate the higher order modes. This method requires the application of BPM and matrix pencil method, and then, it is still relatively complicated to implement.

In this paper, a new method that is referred to as "slowly decaying imaginary distance BPM (SD-ID-BPM)" is presented, for calculating the fundamental mode and all the higher order guided modes directly and very efficiently. A new governing equation is proposed. Both the propagation constant and the field amplitude pattern of the targeted mode are directly and simultaneously obtained, by simply solving the governing equation using the standard finite-difference method assuming that the z-coordinate is imaginary. To obtain a high-order mode, no knowledge of the lower order modes is required. During the calculation, all the modes decay in different rates and the targeted mode decays much slower than all the other modes. The SD-ID-BPM has the advantages of being easy and straightforward to implement and being efficient in computation. This method will also be shown to be very stable numerically and to be very accurate. The finite-difference method using the Crank–Nicholson scheme [9]–[11] is used to solve the governing equation in this paper, and it is widely used in standard BPM calculation. On the other hand, the governing equation may also be solved by other numerical methods instead of finite-difference method, such as the finite-element method. The method is presented using the general vector formulation, which can be easily changed to semivector or scalar formulations when appropriate. In Section II, the theory of this method is described. In Section III, the application of this method to a typical step index fiber is presented and the calculated results are compared to the analytic solution for validation. The assessment of the convergence speed of the calculation is also presented in Section III. Section IV is the conclusion.

Manuscript received January 25, 2007; revised April 30, 2007. This work was supported by the DoD JTO MRI under Contract W911NF-05-1-0517.

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Digital Object Identifier 10.1109/JLT.2007.902102

II. THEORY

The vector formulation of the paraxial wave equation can be written as [9]

$$\frac{\partial \Psi}{\partial z} = -j \cdot H \cdot \Psi \tag{1}$$

where

$$H = \begin{pmatrix} P_{xx} & P_{xy} \\ P_{yx} & P_{yy} \end{pmatrix} \quad \Psi = \begin{pmatrix} A_x \\ A_y \end{pmatrix} \tag{2}$$

and A_x and A_y are the x and y components of the slow amplitude of the electric field, respectively. P_{xx}, P_{xy}, P_{yx}, and P_{yy} are defined as [9]

$$\begin{aligned} P_{xx}A_x &= \frac{1}{2k_0 n_0}\left\{\frac{\partial^2 A_x}{\partial y^2} + \frac{\partial}{\partial x}\left[\frac{1}{\varepsilon_{zz}}\frac{\partial(\varepsilon_{xx}A_x)}{\partial x}\right]\right. \\ &\qquad\left. + k_0^2 \cdot \left[\varepsilon_{xx} - n_0^2\right] \cdot A_x\right\} \\ P_{xy}A_y &= \frac{1}{2k_0 n_0}\left\{k_0^2 \cdot \varepsilon_{xy} \cdot A_y + \frac{\partial}{\partial x}\left[\frac{1}{\varepsilon_{zz}}\frac{\partial(\varepsilon_{yy}A_y)}{\partial y}\right]\right. \\ &\qquad\left. - \frac{\partial^2 A_y}{\partial x \partial y}\right\} \\ P_{yx}A_x &= \frac{1}{2k_0 n_0}\left\{k_0^2 \cdot \varepsilon_{yx} \cdot A_x + \frac{\partial}{\partial y}\left[\frac{1}{\varepsilon_{zz}}\frac{\partial(\varepsilon_{xx}A_x)}{\partial x}\right]\right. \\ &\qquad\left. - \frac{\partial^2 A_x}{\partial x \partial y}\right\} \\ P_{yy}A_y &= \frac{1}{2k_0 n_0}\left\{\frac{\partial^2 A_y}{\partial x^2} + \frac{\partial}{\partial y}\left[\frac{1}{\varepsilon_{zz}}\frac{\partial(\varepsilon_{yy}A_y)}{\partial y}\right]\right. \\ &\qquad\left. + k_0^2 \cdot \left[\varepsilon_{yy} - n_0^2\right] \cdot A_y\right\}. \end{aligned} \tag{3}$$

Here, k_0 is the wavenumber in vacuum, and n_0 is the reference refractive index, which can be chosen to be the index in either the cladding or the core for weakly guiding fibers or waveguides.

In the common imaginary distance propagation method, z is substituted by $j \cdot z'$ in (1).

In the SD-ID-BPM, the proposed governing equation is

$$\frac{\partial \Psi}{\partial z} = -j \cdot \frac{(H - \alpha \cdot I) \cdot (\alpha \cdot I - H)}{c} \cdot \Psi \tag{4}$$

where I is a 2×2 identity matrix, and α and c are scalar constants with the dimension of units per meter. The operator H in (4) is identical to that in (1). Ψ in (4) is still denoted as in (2), but it does not physically correspond to the slow amplitude of the electric field (field amplitude) any more. However, Ψ in (4) will be shown to accurately represent the pattern of the field amplitude of the targeted mode after the solution converges. Although (4) does not correspond to any physical system, it will be shown to be of great help in calculating the modes in optical guided wave devices. The constant c is introduced in (4) to adjust the order of magnitude of the product of the operators.

For z invariant structures, H is independent on z-coordinate, and then, the solution of (4) can be written as

$$\Psi(x, y, z) = e^{-j \cdot \frac{(H-\alpha \cdot I)\cdot(\alpha \cdot I - H)}{c} \cdot z} \cdot \Psi(x, y, z = 0). \tag{5}$$

Any arbitrary input can be expanded in the eigenfunctions of H as

$$\Psi(x, y, 0) = \sum_m a_m \phi_m(x, y) \tag{6}$$

and

$$H \cdot \phi_m(x, y) = \lambda_m \phi_m(x, y) \tag{7}$$

where $\phi_m(x, y) = \begin{pmatrix} \phi_m^x(x, y) \\ \phi_m^y(x, y) \end{pmatrix}$ is the eigenfunction corresponding to the mth mode, and its corresponding eigenvalue is λ_m

$$\lambda_m = \beta_m - k_0 n_0 \tag{8}$$

for all the guided modes in weakly guiding structures if the reference index n_0 is chosen to be in between the maximum and minimum indexes in the considered guiding structure. Here, β_m is the propagation constant of any guided mode. The eigenvalues of the unconfined modes are smaller than those of the guided modes. In this paper, only the guided modes are considered. Both the eigenfunctions and propagation constants are unknown and need to be determined for the targeted guided modes. Applying (6) and (7) in (5) results in

$$\Psi(x, y, z) = \sum_m a_m e^{j \cdot \frac{(\lambda_m - \alpha)^2}{c} \cdot z} \phi_m(x, y). \tag{9}$$

It is assumed that z is imaginary in (4), and then, it is written as $z = j \cdot z'$, where z' is real. Equation (9) can then be written as

$$\Psi(x, y, z') = \sum_m a_m e^{-\frac{(\lambda_m - \alpha)^2}{c} \cdot z'} \phi_m(x, y). \tag{10}$$

If (4) is numerically solved step by step along the z-axis assuming $z = j \cdot z'$, and using the same input as in (6), the solution of Ψ is equal to that in (10). This means that all the modes exponentially decay when the step-by-step numerical calculation or propagation is performed along z'. Different modes decay in different rates, and the mode with the eigenvalue that is closest to the numerical value of α decays much slower than the other modes and will dominate when z' is large enough, if this mode is present in the initial input. For this reason, this method is referred to as "SD-ID-BPM." If several modes are degenerate and have the same propagation constant, the eventual converged solution is a superposition of these degenerate modes if all these degenerate modes are present in the initial input.

When z' is large enough, (10) can be accurately approximated as

$$\Psi(x,y,z') = a_i e^{-\frac{(\lambda_i-\alpha)^2}{c}\cdot z'}\phi_i(x,y) \tag{11}$$

where $\phi_i(x,y)$ is the eigenfunction of the dominant mode, and λ_i is the corresponding eigenvalue. Then, the propagation constant of the dominant mode is calculated according to

$$\beta_\pm = k_0 n_0 + \alpha \pm \sqrt{-\frac{c}{\Delta z'}\cdot \ln\left[\frac{A_x(x_0,y_0,z'+\Delta z')}{A_x(x_0,y_0,z')}\right]} \tag{12}$$

if A_x is the major component of Ψ, which is the solution of (4) using the step-by-step numerical calculation. If A_y is the major component in the solution, A_x shall be replaced by A_y in (12). If the total powers in the A_x and A_y components are not very different, either A_x or A_y can be used in (12). x_0 or y_0 are the coordinates for some specific point in the transverse plane. As a result, when the propagation along z' is performed for sufficiently long distance, the field amplitude pattern and propagation constant of the dominant mode are simultaneously obtained. It has to be noticed that there are two possible solutions in (12), and only one of them is the right solution. To find the right solution, α has to be set to $\beta_+ - k_0 n_0$ and $\beta_- - k_0 n_0$, respectively, and the numerical solution of (4) has to be repeated for the two values of α, respectively. It is best to use the output of the previous calculation as the input for the new calculation since the solution $\Psi(x,y,z')$ contains only the dominant mode essentially for sufficiently large z'. For α set to the right solution, the new version of $\beta_+ - \beta_-$ is expected to approach zero, which is generally not exactly zero in actual calculation but is much smaller than the previous version of $\beta_+ - \beta_-$. In this way, the right solution can be found in generally two repetitions of the calculation.

The propagation distance needed for convergence is related to the numerical value of the constant c chosen in the calculation, since it determines the decaying rates of all the modes in the propagation as can be seen from (10). The targeted mode decays much slower than all the other modes. If c is not in (4), the typical step size in z' has to be extremely small due to the introduction of a product of the differential operators in (4).

The finite-difference method using the Crank–Nicholson scheme [9]–[11] is used to solve (4) step by step along the imaginary z-axis. The continuous space coordinates are replaced by their discrete versions. The multiplication of the operators $(H - \alpha \cdot I)\cdot(\alpha \cdot I - H)$ is carried out after H is replaced by its finite-difference approximation, through simple matrix multiplication. The solution of the finite-difference representation of (4) results in the solution of a matrix equation, in which the matrix is very large and sparse. This matrix equation is solved by the MATLAB[1] function "bicgstab" using the biconjugate gradients stabilized method. The numerical values of

$$\Psi = \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

at the boundaries of the transverse computation window are set to zero. Therefore, the window size has to be large enough to ensure that A_x and A_y for the targeted guided mode are completely negligible at the boundaries. In the practical implementation of the method proposed in this paper, the numerical value of the constant c has to be properly chosen for the fast convergence to the dominant mode.

The in-depth analysis of the stability of the SD-ID-BPM has not been attempted yet. However, the representative sample calculations demonstrated that it is very accurate, efficient, and numerically stable for calculating all the guided modes in optical fibers and waveguides.

Considering a step index fiber as a typical example, the strategy to choose the numerical value of α to extract different mode is given as follows: 1) First, set $\alpha = k_0 n_{\mathrm{core}} - k_0 n_0$, and the fundamental mode will be obtained using the method introduced in this paper. This is because the eigenvalue of the fundamental mode is the closest to the chosen value of α. 2) Then, decrease the numerical value of α slowly, and do

TABLE I
EFFECTIVE INDEXES FOR ALL THE GUIDED MODES

LP modes	Vector modes	Effective index calculated using SD-ID-BPM	Analytic solution of the effective index using the dispersion equations of LP modes[12, 13]
LP_{01}	HE_{11}	1.467540	1.467573
LP_{11}	TE_{01}, TM_{01}, HE_{21}	1.465350	1.465430
LP_{21}	EH_{11}, HE_{31}	1.462577	1.462722
LP_{02}	HE_{12}	1.461825	1.461947
LP_{02} (using finer discrete x and y coordinates)	HE_{12}	1.461927	1.461947

[1]MATLAB is a numerical computing environment and programming language created by The MathWorks.

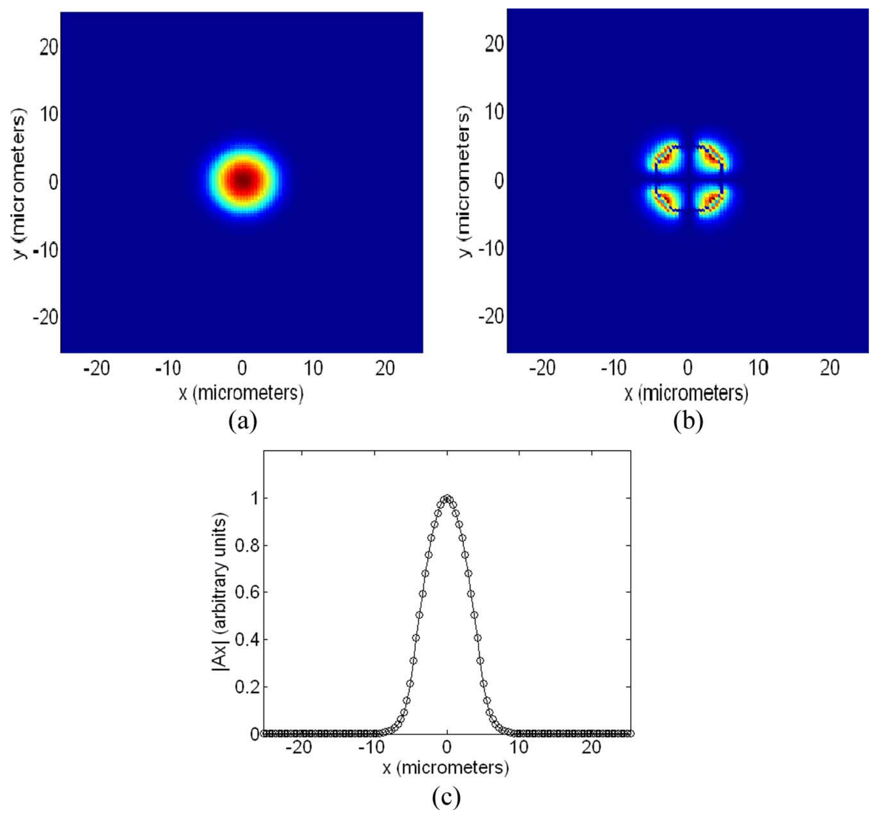

Fig. 1. Field amplitude pattern for the fundamental mode. (a) Numerically calculated $|A_x|$ on the x–y plane. (b) Numerically calculated $|A_y|$ on the x–y plane. (c) (Solid line) Numerically calculated $|A_x|$ together with the (open circles) analytical solution on x-axis for $y = 0$, both are scaled for comparison.

the same imaginary distance propagation calculation until the solution gives a mode different from the fundamental mode. This is the second lowest order mode. 3) Then, decrease the numerical value of α further, and do the calculation until a mode different from the previous mode is obtained. In this way, all the guided modes can be obtained with very good accuracy. To avoid missing any mode, when a mode different from the previous mode is obtained, α can be set to be in the middle of the eigenvalues of the present mode and the previous mode, and the calculation is performed again to make sure that the solution does not give a mode other than the present one or the mode obtained in the previous step. This is particularly important when the mode spacing is very small.

In this paper, only the guided modes in guiding structures with real index are considered. The leaky modes have not been analyzed in details yet.

III. Validation and Assessment

To validate this method, we applied it to calculate the modes in a standard step index fiber. The core index $n_{\rm core} = 1.469$, the index in the cladding $n_{\rm clad} = 1.46$, the wavelength is 1.03 μm, and the radius of the fiber core $a = 5.05$ μm. For this fiber, the weakly guiding approximation is valid. The guided modes supported by this step index fiber include four linearly polarized (LP) modes, which are LP_{01}, LP_{11}, LP_{21}, and LP_{02} modes. The LP_{11} mode corresponds to three nearly degenerate vector modes, which are TE_{01}, TM_{01}, and HE_{21} modes; the LP_{21} mode corresponds to two nearly degenerate vector modes, which are EH_{11} and HE_{31} modes. The analytic solutions for the LP modes are approximate solutions, but they are accurate enough for the considered weakly guiding step index fiber and were used for the validation of the SD-ID-BPM. In our calculation, we did not attempt to resolve each of the several nearly degenerate vector modes corresponding to the LP_{11} and LP_{21} modes, respectively, for the considered fiber, which may require much smaller step sizes in the x and y axes compared with those used in the calculation in this paper. The full-vector formulation was used in the method presented in this paper to numerically calculate the modes in the considered fiber. A_x component of the input was set to be the major component, and A_y component of the input was set to zero.

In the calculation, the reference index was fixed to $n_0 = (n_{\rm core} + n_{\rm clad})/2$, and the constant c was chosen to be $c = 10^6\ m^{-1}$. The number of data points on both the x- and y-axes is 121 for a square computation window with $w_x = w_y = 50.5$ μm. The step size in the z'-axis is $\Delta z' = 10$ μm.

First, α was set to be $k_0 n_{\rm core} - k_0 n_0$. After sufficiently long propagation distance, the solution converged, and $\beta_+/k_0 = 1.4704608$ and $\beta_-/k_0 = 1.4675392$. One of these two numbers is the right solution of the effective index of the fundamental mode. Since the effective index of any guided mode has to satisfy $n_{\rm clad} < n_{\rm eff} < n_{\rm core}$, β_+/k_0 is excluded from the physical solution and β_-/k_0 must be the right solution. To be sure, α was reset to $\alpha = k_0 \cdot 1.4675392 - k_0 n_0$, and the calculation

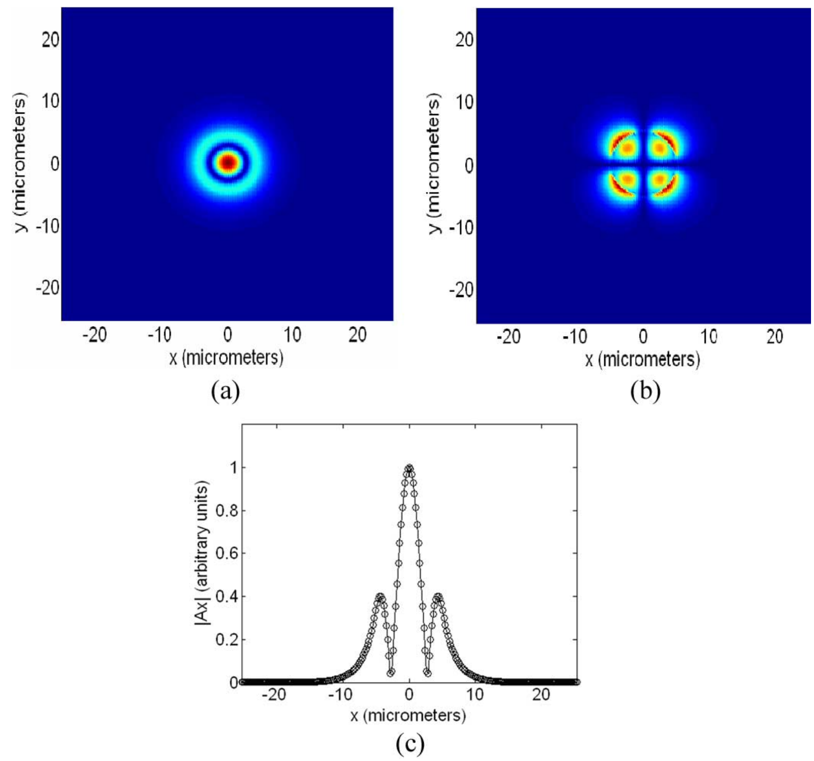

Fig. 2. Field amplitude pattern for the LP_{02} mode. (a) Numerically calculated $|A_x|$ on the x–y plane. (b) Numerically calculated $|A_y|$ on the x–y plane. (c) (Solid line) Numerically calculated $|A_x|$ together with the (open circles) analytical solution on x-axis for $y = 0$, both are scaled for comparison.

was repeated, using the output of the previous calculation as the input. The new version of the calculated $\beta_+/k_0 = 1.4675399$ and $\beta_-/k_0 = 1.4675384$. It can be seen that now $\beta_+ - \beta_-$ is very small compared to the previously calculated version. Therefore, the effective index can be taken as 1.467540 with very good accuracy, and the output gives the field amplitude pattern. This is the HE_{11} (LP_{01}) mode.

Then, α was set to be a smaller value $k_0 \cdot 1.4650 - k_0 n_0$. After the convergence of the calculation, $\beta_+/k_0 = 1.4653477$ and $\beta_-/k_0 = 1.4646523$. Either of these two values could be the right solution. First, α was reset to $\alpha = k_0 \cdot 1.4653477 - k_0 n_0$, and the calculation was repeated, using the previous output as the input. The new version of the calculated $\beta_+/k_0 = 1.4653514$ and $\beta_-/k_0 = 1.4653440$. It can be seen that now $\beta_+ - \beta_-$ is very small compared to the previous version. Therefore, the effective index of this mode can be taken as 1.465350 with very good accuracy. This is the LP_{11} mode. The choice of the input at the initial starting point of the calculation determines which of the three corresponding nearly degenerate vector modes are present in the output. To be sure, α was reset to $\alpha = k_0 \cdot 1.4646523 - k_0 n_0$, and the calculation was repeated again, using the output of the previous calculation as the input. The new version of the calculated $\beta_+/k_0 = 1.4653477$ and $\beta_-/k_0 = 1.4639569$. It can be seen that now $\beta_+ - \beta_-$ is larger than the previously calculated version. Therefore, this is not the right solution.

Next, the numerical value of α was further decreased to be $k_0 \cdot 1.4630 - k_0 n_0$. The calculated effective index is 1.462577. This is the LP_{21} mode.

Then, α was further decreased to be $k_0 \cdot 1.4615 - k_0 n_0$. The calculated effective index is 1.461825. This is the LP_{02} mode. It needs to be noticed that α shall not be set to be too close to $k_0 \cdot n_{\mathrm{clad}} - k_0 n_0$. When α was set to be very close to $k_0 \cdot n_{\mathrm{clad}} - k_0 n_0$, we obtained a mode very similar to a leaky mode. However, the leaky modes need to be analyzed in more details to guarantee good accuracy, including using different boundary conditions.

Table I shows the effective indexes for all the guided modes in the considered fiber, which were calculated using both the SD-ID-BPM and the analytic formula [12], [13]. It can be seen that the numerically calculated results agree with the analytic solution very well. Generally, a more accurate solution can be obtained using smaller step sizes in the x and y axes. To demonstrate this, the number of data points on both the x- and y-axes was changed to be 251 with all other parameters except α unchanged, and the calculation for the LP_{02} mode was conducted again. The calculated effective index using finer discrete x- and y-coordinates is closer to that calculated analytically, as can be seen in Table I. The numerically calculated field amplitude pattern for the fundamental mode (HE_{11}) is shown in Fig. 1. In the converged solution of the fundamental mode, the peak value of $|A_x|$ is more than 100 times of the peak value of $|A_y|$. From Fig. 1(c), it can be seen that the numerically calculated field amplitude pattern for the fundamental mode agrees with the analytic solution [12], [13] very well. The numerically calculated field amplitude pattern for the LP_{02} mode (HE_{12}) is shown in Fig. 2. In the converged solution of the LP_{02} mode, the peak value of $|A_x|$ is also

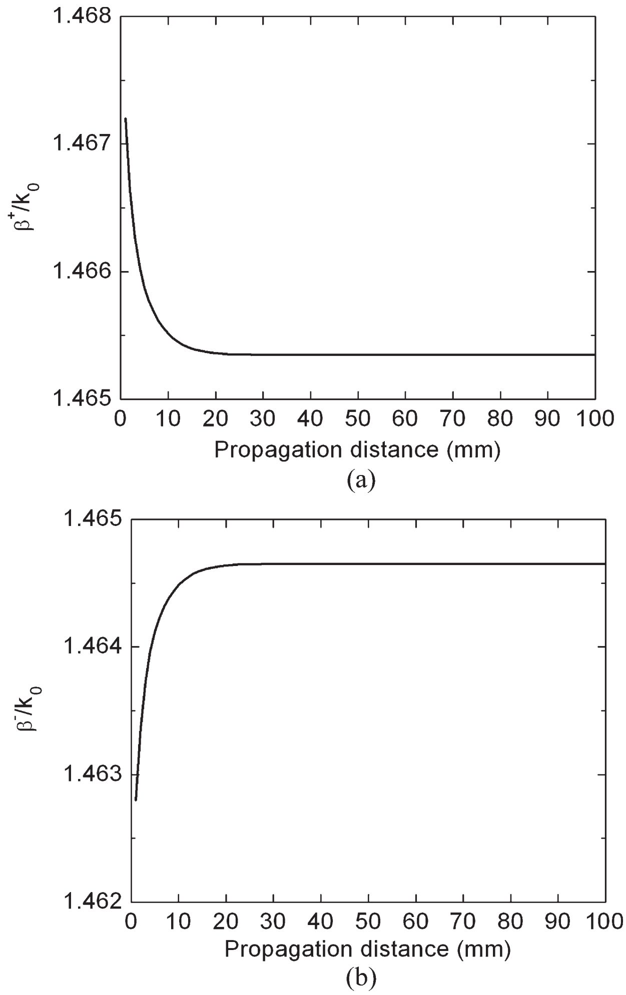

Fig. 3. Convergence of β_+/k_0 and β_-/k_0 for the LP_{11} mode as a function of the propagation distance on the imaginary axis, with α set to be $k_0 \cdot 1.4650 - k_0 n_0$. (a) β_+/k_0. (b) β_-/k_0.

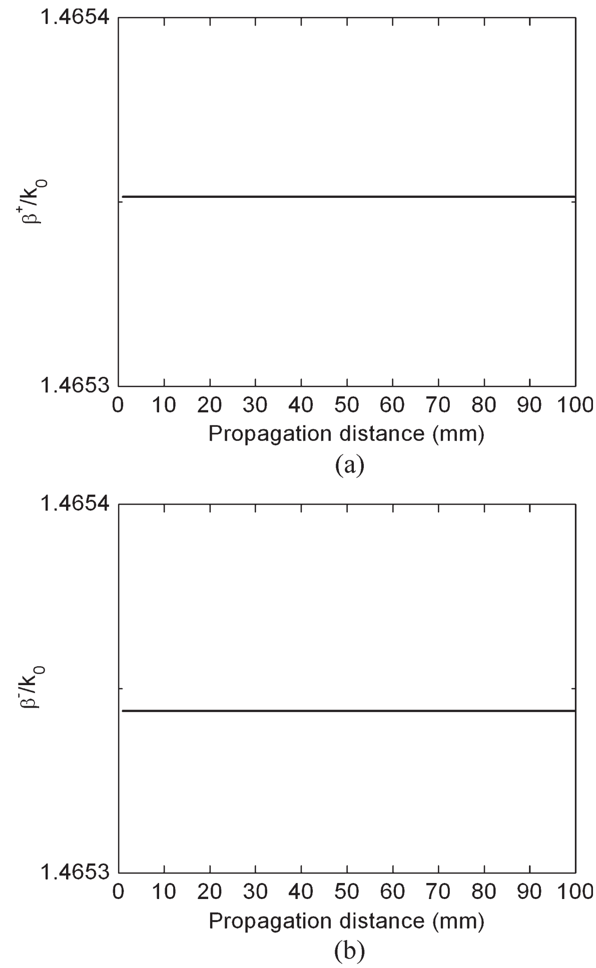

Fig. 4. Convergence of β_+/k_0 and β_-/k_0 for the LP_{11} mode as a function of the propagation distance on the imaginary axis, with α set to be $k_0 \cdot 1.4653477 - k_0 n_0$. (a) β_+/k_0. (b) β_-/k_0.

more than 100 times of the peak value of $|A_y|$. The numerically calculated field amplitude pattern for the LP_{02} mode agrees very well with the analytic solution too, as can be seen from Fig. 2(c).

As a typical example, the convergence of the calculated effective index of the LP_{11} mode as a function of the propagation distance on the imaginary axis was investigated. For this calculation, the number of data points on both the x- and y-axes was changed back to be 121, with all other parameters except α unchanged. First, α was set to be $k_0 \cdot 1.4650 - k_0 n_0$, and the calculation was performed. The convergence of the two possible solutions of the effective index β_+/k_0 and β_-/k_0 is shown in Fig. 3. At the propagation distance of 100 mm, $\beta_+/k_0 = 1.4653477$ and $\beta_-/k_0 = 1.4646523$. It can be seen that both β_+ and β_- converge very well after 100 mm of propagation. Now, it needs to be determined which one of them is the right solution of the propagation constant. α was first reset to $k_0 \cdot 1.4653477 - k_0 n_0$, and the calculation was repeated, using the output of the previous calculation as the input. The convergence of the new version of β_+/k_0 and β_-/k_0 is shown in Fig. 4. At the propagation distance of 100 mm, $\beta_+/k_0 = 1.4653514$ and $\beta_-/k_0 = 1.4653440$. Now, $\beta_+ - \beta_-$ is very small compared to the previous version, and then, the effective index can be taken as 1.465350 with very good accuracy. Then, α was reset to $k_0 \cdot 1.4646523 - k_0 n_0$, and the calculation was repeated using the previous output as the input. The convergence of this version of β_+/k_0 and β_-/k_0 is shown in Fig. 5. At the propagation distance of 100 mm, $\beta_+/k_0 = 1.4653477$ and $\beta_-/k_0 = 1.4639569$. This is not the right solution because now $\beta_+ - \beta_-$ is larger than the previously calculated version.

IV. Conclusion

A new method has been presented for calculating all the guided modes in optical fibers and waveguides. The propagation constant and field amplitude pattern of any targeted guided mode are directly and simultaneously obtained through the calculation. This method has been validated and found to be very accurate. It is also very efficient in computation and straightforward to implement. It shall be very useful when the analytic solutions are not known for fibers or waveguides with complicated index distribution.

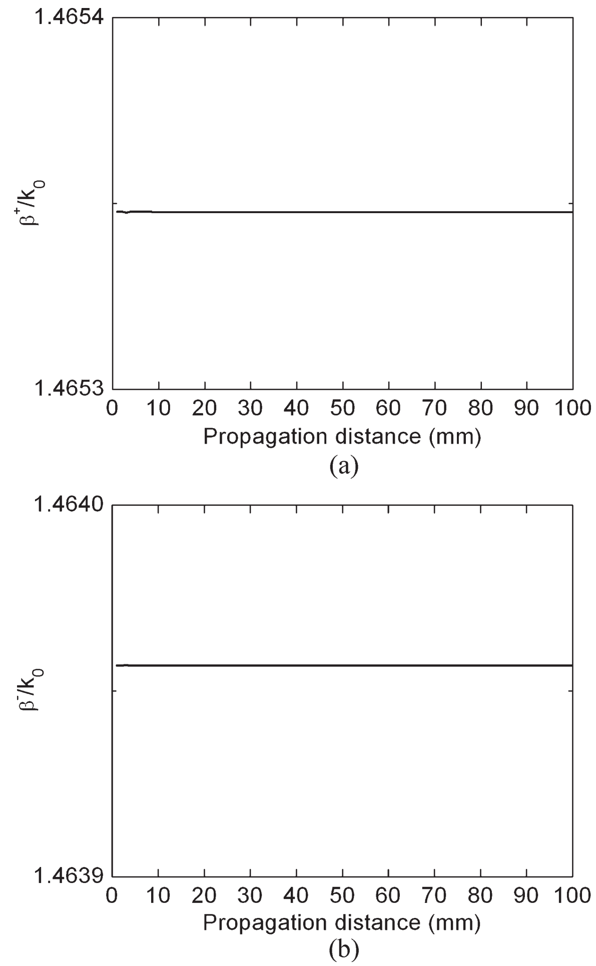

Fig. 5. Convergence of β_+/k_0 and β_-/k_0 for the LP$_{11}$ mode as a function of the propagation distance on the imaginary axis, with α set to be $k_0 \cdot 1.4646523 - k_0 n_0$. (a) β_+/k_0. (b) β_-/k_0.

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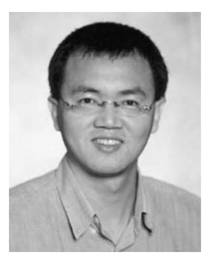

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Michael Bass, photograph and biography not available at the time of publication.