Indian Institute of Technology, Ropar
Department of Mathematics
MA202: Probability and Statistics
2nd Semester of Academic Year 2024-25
Tutorial Sheet 13: t, F-distributions, confidence interval

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1. Consider a random sample $X_1, X_2, \ldots, X_{16}$ taken from a normal distribution with unknown mean μ and unknown variance σ^2. For the observed sample, the sample mean is $\bar{X} = 16.7$, and the sample variance is $S^2 = 7.5$.

   (a) Find a 95% confidence interval for the population mean μ.

   (b) Find a 90% confidence interval for the population variance σ^2.

2. The diameter of steel ball bearings produced by a company is known to be normally distributed. To assess the variation in the diameter, the product manager takes a random sample of 10 ball bearings from a lot having an average diameter of 5.0 cm. The diameters (in cm) of the selected ball bearings are given below:

S. No.	Diameter (cm)
1	5.0
2	5.1
3	5.0
4	5.2
5	4.9
6	5.0
7	5.0
8	5.1
9	5.1
10	5.2

   Find the 95% confidence interval for the variance in the diameter of the steel ball bearings from the lot from which the sample is drawn.

3. A human wishes to estimate the mean of a population using a sample large enough that the probability will be 0.95 that the sample mean will not differ from the population mean by more than 25% of the standard deviation. How large a sample should he take? Given that for $Z \sim N(0, 1)$, $P(|Z| < 1.96) = 0.95$

4. Let X_1, X_2 be i.i.d random variables with common probability density function

$$f(x) = \begin{cases} e^{-(x-\theta)}, & if\, x > 0, \\ 0, & otherwise. \end{cases}$$

   Let $Y = min\{X_1, X_2\}$. Find a confidence interval for θ, of the type

$$[Y - b, Y], \quad 0 \leq b < \infty,$$

   having confidence coefficient 0.95.

5. A random sample X of size one is taken from a distribution with the probability density function

$$f(x;\theta) = \begin{cases} \frac{2x}{\theta^2}, & 0 < x < \theta, \\ 0, & \text{elsewhere.} \end{cases}$$

If $\frac{X}{\theta}$ is used as a pivot for obtaining the confidence interval for θ, then find the 80% confidence interval (confidence limits rounded off to three decimal places) for θ based on the observed sample value $x = 10$.

6. Let $X_1, X_2 \overset{\text{iid}}{\sim} \mathcal{N}(0, 1)$ and $Y_1, Y_2 \overset{\text{iid}}{\sim} \mathcal{N}(1, 1)$. Also X_i and Y_i are independent. Find the distribution of

$$\frac{(Y_1 + Y_2 - 2)^2}{(X_2 - X_1)^2} \text{ and } \frac{X_1 + X_2}{\sqrt{\frac{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}{2}}}.$$

7. It is known that the heights of cadets at a particular training centre follow a normal distribution. A random sample of 6 cadets was selected, and their heights (in inches) were recorded as follows:

$$70, 72, 80, 82, 78, 80$$

Using this sample data, construct a 95% confidence interval for the true mean height of cadets at the training centre.

8. A certain brand of refined oil is packed in tins labeled as 15 kg each. The filling machine is calibrated to maintain this average, with a known standard deviation of $\sigma = 0.30$ kg. A random sample of 200 tins is selected from the production line, and the sample mean weight is found to be 15.25 kg. Construct a 95% confidence interval for the true mean weight of the oil tins.

9. Let X_1, X_2, X_3, X_4 be independent observations from the standard normal distribution $N(0, 1)$. State, giving reasons, the sampling distributions of:

   (a) $U = \dfrac{\sqrt{2}X_3}{\sqrt{X_1^2 + X_2^2}}$

   (b) $V = \dfrac{3X_4^2}{X_1^2 + X_2^2 + X_3^2}$

10. Consider a random sample $X_1, X_2, \ldots, X_n$ of size n from $N(\mu, 16)$ population. If a 95 % confidence interval for μ is $[\bar{X} - 0.98, \bar{X} - 0.98]$. Then find the value of n?

11. Let $X_1, X_2, \ldots, X_n$ be a random sample from Cauchy distribution,

$$f(x) = \frac{1}{\pi(1 + (x - \theta)^2)}, x \in \mathbb{R}$$

where $\theta \in \mathbb{R}$ is an unknown parameter. Prove or disprove that the confidence coefficient for the following confidence interval

$$\left[\frac{X_1 + X_2 + X_3}{3} - \tan\left(\frac{5\pi(1 - \alpha)}{7}\right), \frac{X_1 + X_2 + X_3}{3} + \tan\left(\frac{2\pi(1 - \alpha)}{7}\right)\right]$$

is $1 - \alpha$.

12. Let $X_1, X_2, \ldots, X_n$ be a random sample from

$$f(x, y) = \begin{cases} e^{\theta - x}, & x > \theta \\ 0, & otherwise \end{cases}$$

Find the confidence coefficient for the following confidence interval

$$\left[X_{(1)} - \frac{\ln(4)}{n}, X_{(1)} + \frac{\ln(2)}{n}\right]$$

where $X_{(1)} = \min\{X_1, X_2, \ldots, X_n\}$.

Best Wishes