Indian Institute of Technology, Ropar
Department of Mathematics
MA202: Probability and Statistics
2nd Semester of Academic Year 2024-25
Tutorial Sheet 11: Sampling Distributions

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1. Let $X_1, X_2, \ldots, X_8$ be iid $N(0, \sigma^2)$. Find correlation coefficient between $X_1 + X_2$ and $\sum_{i=1}^{8} X_i$.

2. Let $X_1, X_2, \ldots, X_{100}$ be iid $N(0, 1)$. Find correlation coefficient between $\sum_{i=1}^{98} X_i$ and $\sum_{i=3}^{100} X_i$.

3. Let (X_1, X_2) be a bivariate normal random variable such that

$$E(X_1) = E(X_2) = 0, E(X_1^2) = E(X_2^2) = 1, E(X_1 X_2) = 0.5$$

Find $P(X_1 + 2X_2 > \sqrt{7})$.

4. Let $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ be a random sample from bivariate normal random variable such that

$$E(X_1) = 75, E(Y_1) = 25, Var(X_1) = 36, Var(Y_1) = 16, \rho(X_1, Y_1) = 0.25$$

Find the value of n such that $P(\overline{U} \leq 104) \geq 0.99$ where

$$\overline{U} = \frac{1}{n} \sum_{i=1}^{n} (X_i + Y_i)$$

5. Let $(X_1, Y_1), (X_2, Y_2), \ldots, (X_{20}, Y_{20})$ be a random sample from the $N_2\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \frac{3}{4} \\ \frac{3}{4} & 1 \end{pmatrix}\right)$ distribution. Define $\bar{X} = \frac{1}{20} \sum_{i=1}^{20} X_i$ and $\bar{Y} = \frac{1}{20} \sum_{i=1}^{20} Y_i$. Then $\text{Var}(\bar{X} - \bar{Y})$ is equal to

6. Let $X_1, X_2, \ldots, X_{10}$ be a random sample from an $N(0, \sigma^2)$ distribution, where $\sigma > 0$ is an unknown parameter. For some real constant c, let

$$Y = \frac{c}{10} \sum_{i=1}^{10} |X_i|$$

be an unbiased estimator of σ. Find the value of c.

7. Let $X_1, X_2, \ldots, X_n$ be a random sample from a

$$U\left(\theta + \frac{\sigma}{\sqrt{3}}, \theta + \sqrt{3}\sigma\right)$$

distribution, where $\theta \in \mathbb{R}$ and $\sigma > 0$ are unknown parameters. Let

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i \quad \text{and} \quad S = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2}.$$

Let $\hat{\theta}$ and $\hat{\sigma}$ be the method of moment estimators of θ and σ, respectively. Prove or disprove:

$$2\sqrt{3}\hat{\sigma} + \hat{\theta} = \overline{X} - 4\sqrt{3}S.$$

8. Let $X_1, X_2, \dots, X_{25}$ be a random sample from a $N(5.2, 1)$ distribution. If

$$P\left(\frac{1}{25}\sum_{i=1}^{25} X_i > k\right) = 0.05.$$

Find k?

9. Let $X_1, X_2, \dots, X_n$ be a random sample from a $N(0, 1)$ distribution. Find minimum value of n such that

$$P\left(\frac{1}{n}\sum_{i=1}^{n} X_i > \frac{3}{4}\right) \le 0.05.$$

10. Let $X_1, X_2, \dots, X_n$ be a random sample from $P(\lambda)$. Find the sampling distribution of $\bar{X}$, the sample mean.

11. A random sample of 5 is taken from a normal population with mean 2.5 and variance $\sigma^2 = 36$.

    (a) Find the probability that the sample variance lies between 30 and 44.

    (b) Find the probability that the sample mean lies between 1.3 and 3.5, while the sample variance lies between 30 and 44.

12. A random sample of size n is obtained from a uniform distribution on the interval (0, 1). Show that $\frac{X_{(1)}}{X_{(n)}}$ and $X_{(n)}$ are independent random variables.

13. Let $X_1, \dots, X_n$ be a sample from $N(0, 4)$. Find $P\left(\sum_{i=1}^{5} X_i^2 > 5.75\right)$.

Best Wishes