$$L = \sum_{i=1}^{n} \frac{V_i}{X_i} + \sum_{i=1}^{n} \mu(B - X_i W_i) + \sum_{i=1}^{n} \lambda_i(0 + X_i)$$

$$X_i \geq 0$$
$$-X_i \leq 0$$

$$\frac{\partial L}{\partial X_i} = -\frac{V_i}{X_i^2} + \mu W_i + \lambda_i = 0 \quad \text{①}$$

$$\sum_{i=1}^{n} X_i W_i \leq B \quad \text{②}$$

$$-X_i \leq 0 \quad \text{③}$$

$$\mu(B - X_i W_i) = 0 \quad \text{④}$$

$$\lambda_i X_i = 0 \quad \text{⑤}$$

$$\mu \geq 0 \quad \text{⑥}$$

$$\lambda_i \geq 0 \quad \text{⑦}$$

from ① $\Rightarrow X_i \neq 0$. and ⑦ $\lambda_i \geq 0$, ii from ⑤ $\Rightarrow \lambda_i = 0$.
(division by zero)

i. $\lambda_i = 0$

$$\begin{cases} -\dfrac{V_i}{x_i^2} + \mu W_i = 0 & (1) \\[4mm] \displaystyle\sum_{i=1}^{n} x_i W_i \leqslant B & (2) \\[4mm] x_i \geqslant 0 & (3) \\[4mm] \mu(B - x_i W_i) = 0 & (4) \\[4mm] \mu \geqslant 0 & (5) \end{cases}$$

Case 1. $\mu = 0$.

$$\frac{V_i}{x_i^2} = 0. \qquad \text{Not Possible}.$$

Case 2. $\mu > 0$. $\qquad \dfrac{V_i}{x_i^2} = \mu W_i. \quad \Rightarrow x_i^2 = \dfrac{V_i}{\mu W_i} \Rightarrow x_i = \dfrac{1}{\sqrt{\mu}} \cdot \sqrt{\dfrac{V_i}{W_i}}.$

let x_k a possible point $\Rightarrow x_{\tilde{i}} = 0$ for $\begin{cases} i=1,\cdots n \\ i \neq k. \end{cases}$

ii $\begin{cases} x_k = \dfrac{1}{\sqrt{\mu}} \cdot \sqrt{\dfrac{v_k}{w_k}} \\ \\ x_k = \dfrac{B}{w_k}. \end{cases}$

$$\frac{1}{\mu} \cdot \frac{v_k}{w_k} = \frac{B}{w_k}$$

$$\frac{1}{\mu} = \frac{B}{w_k} \cdot \frac{w_k}{v_k} = \frac{B}{v_k}.$$

$$\Rightarrow \mu = \frac{v_k}{B}.$$

ii $x_k = \dfrac{B}{w_k} = \dfrac{v_k}{B} \cdot N \sqrt{\dfrac{v_k}{w_k}}.$

obj.

there is only one point x_k, minimizes original function.