

Problem Chosen

B

**2024
MCM/ICM
Summary Sheet**

Team Control Number

2421255

my title

Summary

my abstract

Keywords: keyword1; keyword2

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1 Introduction

1.1 Background

In the 21st century, there has been a growing interest in the exploration of the ocean in various countries and regions. Maritime Cruises Mini-Submarines (MCMS), a Greek company, specializes in manufacturing submersibles to take people to the deepest parts of the ocean. They have set their sights on leading tourists on exciting adventures to explore sunken shipwrecks in the depths of the Ionian Sea.

To make their submersible operations a reality, we help MCMS to obtain regulatory approval and establish safety protocols to address potential communication loss and mechanical issues, such as propulsion failure. And we develop a predictive model that can track the submersible's location over time. This model considers factors such as sea floor positioning, buoyancy, currents, sea density, and geography.

1.2 Restatement of the problem

Considering the background information and restricted conditions identified in the problem statement, we need to solve the following problems:

- Create a model to predict location of the submersible overtime, figure out the uncertainties and find what information can the submersible send back with specific equipment to reduce that.
- Determine additional equipment for searching to carry on the host ship considering the possible costs, and recommend devices for rescuing. on both the host ship and rescue ship if necessary.
- Develop a model using location data to recommend deployment points and search patterns to minimize the time to locate a lost submersible.
- Establish a function that relates the probability of finding the submersible to both time and the accumulated search results.
- Extend the model to cover other tourist destinations and adapt it for multiple submersibles moving in the same general vicinity.

1.3 Our work

to my eq (??)

- the angular velocity of the bat,
- the velocity of the ball, and
- the position of impact along the bat.

center of percussion [Brody 1986]

Theorem 1.1. *TEX*

Lemma 1.2. *TEX*.

Proof. The proof of theorem. □

2 Assumptions and Justifications

Considering that practical problems always contain many complex factors, first of all, we need to make reasonable assumptions to simplify the model, and each hypothesis is closely followed by its corresponding explanation.

- **Assumptions 1: Rationalize the assumptions for the submersible by considering it as a point mass.**

When moving in the deep sea, the size and shape of the submersible are negligible compared to the problem we are studying, and the movement of each point on the submersible can be considered to be the same. Therefore, the submersible can be represented by a mass point, following the principle of the center of **mass theorem**.

- **Assumptions 2: Assume that the submarine will no longer provide propulsion.**

Unless under extreme special circumstances, the submersible pilot will cease all operations and await rescue after experiencing distress. Therefore, it can be inferred that the submersible is powerless on its own once in distress.

- **Assumptions 3: The decision factors only include ocean currents, density, and topography.**

Based on the principles of fluid mechanics, the motion of the submersible is primarily influenced by water flow and its own dynamics. Ocean currents play a significant role in governing the movement of water bodies in the ocean, and density is associated with the flow of water bodies. so these factors are emphasized and other factors are ignored.

- **Assumptions 4: For the simplicity of the model, decision factors do not interfere with each other.**

3 Notations

The key mathematical notations used in this paper are listed in Table 1

Symbol	Definitions	Units
g	Gravitational acceleration	m/s^2
ρ	Density	kg/m^3

Table 1: Notations used in this paper

4 Model 1 : Locate the submersible over time

4.1 The uncertainties associated with the predictions

There are many factors that will affect the results of model predictions, mainly the following:

- **Ocean current interference**,The influence of ocean currents on the movement of the submersible, which in turn affects the trajectory.
- **Changes in seawater density**,When the temperature and salinity correlation change, it will affect the density of seawater, thereby affecting the buoyancy and resistance. The buoyancy will change the vertical acceleration, and the resistance will change the velocity in the direction of motion.
- **Measure noise and communication delays**, Due to measure noise and communication delays, navigation accuracy is affected,communication efficiency is reduced ,so the results are interfered.
- **Change in the probability of finding the submersible caused by long search and rescue time**, it will lead to difficulty in accumulating data, decreased search efficiency, uncertainty in mission objectives, changes in communication requirements, and impact on location prediction.
- **Neutral buoyancy condition**,Neutral buoyancy conditions cause drift risk, risk of getting wet, depth changes, environmental variations, and changes in forecast results.

4.2 Density varying with depth

We developed a model to predict how a submersible's position will change over time

In the model, the density of seawater is a crucial factor determining the water resistance and buoyancy of the submarine, which is essential for predicting the trajectory after a submarine accident.The Ionian Sea is small and closed, so horizontal density changes can be ignored. Therefore, the relationship between density and submarine depth is first solved.

The relationship between pressure and density, gravitational acceleration and depth is related by this formula.

$$P = \rho gh \quad (1)$$

On the left side of the equation is the pressure at a certain underwater location, while the right side represents the product of seawater density (ρ), local gravity acceleration (g), and diving depth (h). Since the change in depth is significant during the descent, after consulting the data and calculating, It is known that the product of seawater density (ρ) and local gravity acceleration (g) is approximately 0.10045. Therefore, in the formula, it is reasonable to consider $P = 0.10045h$, therefore we can calculate a certain Pressure corresponding to depth.

By reviewing literature^[2] and analyzing the information, we obtained scatter diagrams illustrating the relationship between seawater salinity and pressure, as well as seawater temperature and pressure. This enables us to determine the corresponding temperature and salinity values for a specific depth.

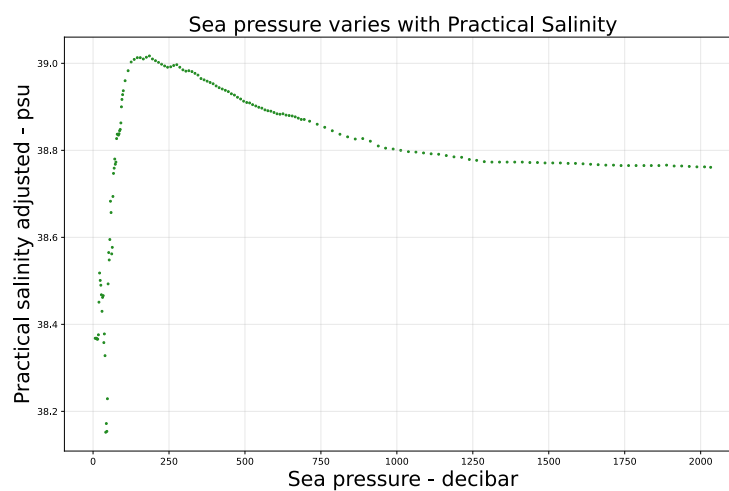


Figure 1: Seawater salinity-pressure scatter point

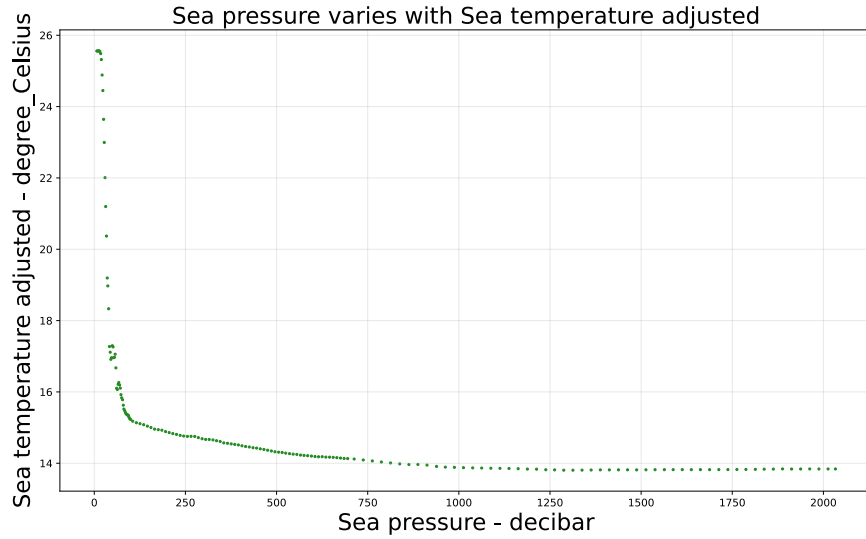


Figure 2: Seawater temperature-pressure scatter plot

After calculating the seawater temperature and salinity, and then considering the relationship between seawater salinity, temperature and seawater density, we can use the Thermodynamic Equation of Seawater to calculate the density of seawater.

$$\rho = \rho_0 + \alpha(T - T_0) + \beta(S - S_0) \quad (2)$$

On the left side of the equation, ρ represents the density at a certain underwater location, ρ_0 is the reference density that is taken as the density of pure water, and $(T - T_0)$ and $(S - S_0)$ denote the temperature and salinity differences from their respective reference values. Moreover, α and β are their respective correlation coefficients, where α is the thermal expansion coefficient and β is the salinity expansion coefficient.

From the formula, we can infer that ρ is determined by the three-dimensional linear relationship between T and S . However, because factors such as sea currents and temperature will stabilize after reaching a certain depth, the change in seawater density no longer shows an obvious linear relationship, so we need to add a correction formula to calculate more accurate value of seawater density.

$$\rho = \begin{cases} \rho_0 + \alpha(T - T_0) + \beta(S - S_0) & \text{if } h < 2,000 \text{ m} \\ C_1 & \text{if } h \geq 2,000 \text{ m} \end{cases} \quad (3)$$

By consulting information and data, we calculated that the density of seawater is 1070 kg/m^3 when the depth is more than 2000 meters. So the density of seawater can be given at a specific depth. Therefore we plotted the density of seawater as a function of depth.

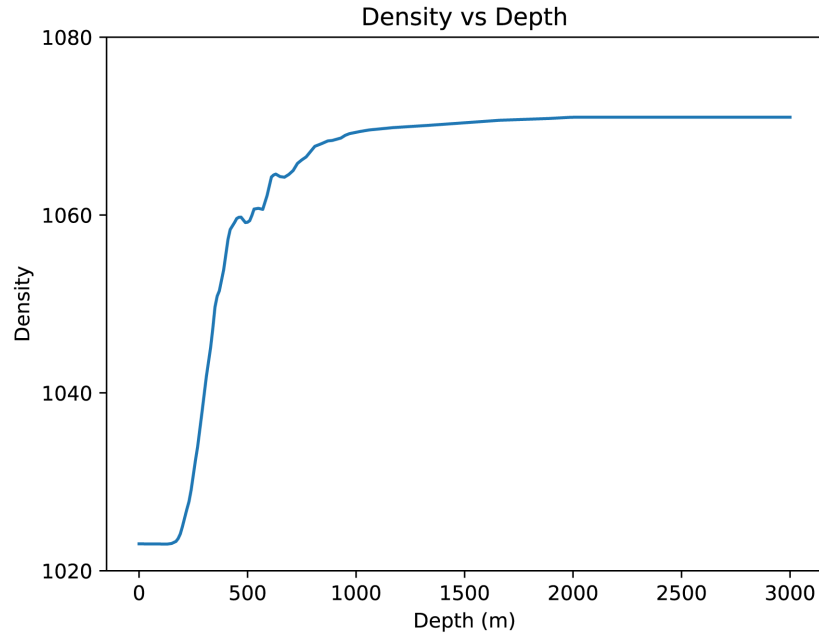


Figure 3: Seawater Density changes with depth

By using linear interpolation at each point on this scatter plot, we can obtain arbitrary continuous density values.

4.3 Model of submersible position

4.3.1 Depth at specific coordinate

Depth computation is a critical part of our model. By referring to geographic data files and data we are able to obtain elevation data on the earth's surface, and also obtain the latitude and longitude of various points. The curvature of the earth's surface can be calculated by simplifying latitude and longitude, and because there are still many factors in the conversion of kilometers and latitude and longitude in actual situations, we give an estimation formula

$$La = La_0 + \frac{Oa}{111.32} \quad (4)$$

$$Lo = Lo_0 + \frac{Og}{111.32 \cos(\text{radians}(La))} \quad (5)$$

where la represents the new latitude, la_0 represents the latitude of the origin position, and Oa represents the offset of the latitude. Lo represents the new longitude, Lo_0 represents the longitude of the origin position, and Og represents the offset of the longitude.

And 1 degree of latitude is approximately 111.32 kilometers. After obtaining the new geographical coordinates, we can calculate the new depth by latitude and longitude. We can obtain the eleva-

tion information of the target location.

4.3.2 Gravity, Buoyancy and Seawater Resistance

Water resistance is a key factor in the movement of submarines in the water, which involves factors such as drag coefficient, reference area and speed.

$$P = \frac{1}{2} C_d \rho A v^2 \quad (6)$$

The formula represents an expression for water resistance. In the formula, water resistance corresponds to P , density to ρ , cross-sectional area to A , and velocity to v .

Gravity and buoyancy are two opposing forces experienced by a submarine moving in the water. It is also a key factor in the movement of submarines in the water. Their relationship are given by the formulae below.

$$G = m g \quad (7)$$

$$F = \rho g V \quad (8)$$

Where gravity corresponds to G , mass corresponds to m , buoyancy corresponds to F , density corresponds to ρ , and volume corresponds to V .

4.3.3 Differential equations for simulating submarine motion

We describe the motion process of the submarine in the water through the calculation of differential equations. We take into account several physical factors, including water resistance, buoyancy and gravity. By solving a system of differential equations, the motion trajectories of submarines at different depths can be simulated, taking into account the influence of terrain elevation on submarine motion and the characteristics of density changes with depth.

For the submarine position x, y, z and its velocity v_x, v_y, v_z . **When the submarine's depth is below or close to the seabed (to reduce errors), the speed and acceleration in all directions are set to zero to simulate the situation when the submarine stops moving. The seabed depth can be known from the data in 4.3.1. When the depth of the submarine is higher than zero, the position and speed are recalculated to simulate the situation when the submarine floats to the surface.** We calculated the displacement velocity in the x and y directions, taking into account the effect of Gaussian noise. We then calculate the velocity and acceleration in the z direction, taking into account the effects of water drag, buoyancy and gravity. In this way, the result of the differential equation is solved, that is, the change of the submarine's position and velocity in time. The formula is as follows

$$\begin{cases} \frac{dv_x}{dt} = -\frac{R \operatorname{sgn}(v_x)}{m} + N_x \\ \frac{dv_y}{dt} = -\frac{R \operatorname{sgn}(v_y)}{m} + N_y \\ \frac{dv_z}{dt} = \frac{g - R \operatorname{sgn}(v_z)}{m} + N_z \end{cases} \quad (9)$$

where v_x , v_y , and v_z represent the speed in the horizontal and vertical directions respectively. dv_x/dt , dv_y/dt , dv_z/dt is the rate of change of the corresponding speed. R is the drag coefficient. m is mass. g is the acceleration due to gravity. $\operatorname{sgn}(a)$ is the symbolic function. N_x , N_y and N_z represent the influence of Gaussian noise. These noises can affect the dynamic behavior of the system.

4.3.4 Estimating a submarine's position in the ocean

To infer the likely location of the submarine in three-dimensional space, we used three-dimensional Bayesian inference. Through the observation data, the corresponding posterior probability distribution is obtained. In the absence of observational information, we make an initial estimate of the submarine's position. We have this formula to calculate the prior probability,

$$P(x, y, z) = \frac{1}{(2\pi)^{3/2} \sqrt{\det(\Sigma_{\text{prior}})}} \exp \left(-\frac{1}{2} (\mathbf{x} - \boldsymbol{\mu}_{\text{prior}})^T \Sigma_{\text{prior}}^{-1} (\mathbf{x} - \boldsymbol{\mu}_{\text{prior}}) \right) \quad (10)$$

where $P(X)$ represents the prior probability of the submarine position. $\det(\text{prior})$ is the determinant of the covariance matrix Σ_{prior} , and x is a column vector containing the variables x , y , and z . $\boldsymbol{\mu}_{\text{prior}}$ is a column vector containing variables x , y , z , representing the mean vector of the distribution. Σ_{prior} is a covariance matrix that represents the covariance relationship between variables x , y , and z . $\Sigma_{\text{prior}}^{-1}$ is the inverse matrix of the covariance matrix, used for matrix inversion operations.

Based on observational data, we estimate the likelihood of a submarine being at a given location. Observation errors are taken into account using a multivariate normal distribution, where the mean is the observation location and the covariance matrix is calculated with the given standard deviation.

$$P(x|X) = \frac{1}{(2\pi\sigma^2)^{3/2}} e^{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T \Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu})} \quad (11)$$

Where X represents a given position, x represents the observed position coordinates, $\boldsymbol{\mu}$ represents the mean of the observed position, Σ represents the covariance matrix, and σ represents the standard deviation of the observation error.

Combining the prior probability and likelihood function, we can calculate the posterior probability. given by the formula. When the observation data, observation error and position range are given, we simulate Bayesian inference to obtain the posterior probability distribution of the submarine's position.

$$P(X|O) = \frac{P(X) \times P(O|X)}{\sum P(X) \times P(O|X)} \quad (12)$$

Where $P(X)$ represents the prior probability. $P(O|X)$ represents the likelihood function.

Based on the position model we established and these formulas, we can infer the possible position of the submarine as it changes over time in three-dimensional space.

4.4 Ways to Reduce Uncertainty

4.4.1 Informations need to be transmitted

Based on the information and literature we reviewed, we believe the message we want the submersible to send is as follows:

- **Sonar Localization Position:** Transmitting sonar localization position allows the main ship to accurately know the submersible's underwater location. This is crucial during accidents for emergency rescue and locating the submersible.
- **Speed:** Conveying information about the submersible's speed enables the main ship to monitor its real-time movement. This is vital for quickly assessing situations during accidents and formulating effective response strategies.
- **Coordinate Information:** Providing coordinate information enables the main ship to accurately determine the submersible's position underwater. This is crucial for navigation, rescue operations, and real-time positioning during accidents.
- **Depth:** Transmitting depth information helps ensure that the submersible operates within a safe depth range. This aids in preventing the submersible from exceeding its design limits, reducing the risk of potential underwater accidents.
- **Density:** Density information provides key data about the surrounding water, essential for assessing underwater environmental conditions, planning tasks, and enhancing submersible safety.
- **Flow Rate:** Understanding the flow rate of the surrounding water assists both the main ship and submersible in better planning actions, especially in complex water flow environments. This helps mitigate risks for the submersible and ensures its safe operation.
- **Environmental Data:** Monitoring seawater salinity and density offers critical insights into water properties. This not only helps avoid unfavorable environmental conditions but also improves the main ship's understanding of the underwater environment, ensuring the submersible's safe operation.
- **System Health Status:** Regularly reporting the submersible's system health status, including sensors, batteries, and other essential components, helps the main ship detect potential issues promptly and prevent system failures.
- **Communication Status:** Regularly sending communication status information ensures effective communication between the submersible and the main ship. Maintaining stable communication is crucial during accidents for rescue and coordination actions.

4.4.2 Additional required equipment

After considering various factors of information transmission, we reviewed the information and literature and concluded that the following equipment is needed:

- **Accelerometer:**The accelerometer is utilized to measure the acceleration of an object. Acceleration is the rate of change of velocity over time, and by integrating acceleration data, velocity can be obtained. Subsequent integration of velocity data provides displacement or the position of the object. This sequential integration accurately reconstructs the object's motion. Moreover, it can be employed for monitoring engine faults and structural fatigue damage.
- **Gyroscope:**The gyroscope, through integrating the measured angular velocity, yields changes in the submarine's orientation relative to the initial position. This information is crucial for navigation and positioning, especially during extended underwater voyages. The angular data provided by the gyroscope enables calculations of the submarine's direction and location, ensuring adherence to the predefined course.
- **Magnetometer:**The magnetometer offers directional information relative to the Earth's magnetic field. This is pivotal for submarine navigation and determining the vessel's orientation, particularly in the absence of other navigation signals. The magnetometer can also assist in underwater communication systems. By introducing subtle variations in the magnetic field, submarines can utilize the magnetometer to detect these changes, enabling low-frequency magnetic induction communication.
- **VLF Bandwidth:**VLF signals exhibit strong propagation capabilities in water, penetrating through the medium and reaching relatively long distances. This allows submarines to employ the VLF band for long-distance underwater communication, as well as for underwater navigation and positioning systems. Leveraging the propagation characteristics of VLF signals underwater, submarines can utilize positioning systems to acquire information about their own location.

5 Search and rescue equipment

5.1 Search equipment

We are committed to ensuring the safety of tourists during underwater tours. In order to carry out rescue quickly and effectively in possible emergencies, when considering the supplies required for search and rescue by the submersible company, we focus on many aspects, including purchase cost, Maintenance cost, usage cost, reliability, accuracy, search and rescue breadth, etc. After reviewing information and collecting prices, we have listed the following search and rescue equipment to choose from. Some price sources^[1].

5.1.1 Above water rescue

- **Search and rescue helicopter:** In emergency rescue situations, especially when reaching the sea or inaccessible areas, helicopters can quickly arrive at the scene and perform search

and rescue missions. Considering that when an accident occurs, it may be very difficult to predict the location of people in distress, especially at sea. The helicopter is equipped with infrared imaging equipment, speakers, search lights and other equipment, which can efficiently search for targets at sea and detect targets under different sea conditions. For example the search radar of the Russian Ka-27 search and rescue helicopter. Although the purchase cost of the helicopter is relatively high (approximately 4,171,560 USD) and there are scheduled and unscheduled maintenance costs, insurance, and flight usage costs (approximately 69,530 USD), making it very expensive, but its reliability and breadth in emergency rescue make it a reasonable choice.

- **GPS Sensor:** Dramatically improves managers' ability to efficiently manage their emergency response teams. It can effectively discover and confirm the location of rescue teams and ships, and confirm the relationship between their locations and the transportation system network in a sea area, providing a new way of working for the entire rescue operation. Automated GPS location information reduces delays in dispatching emergency services. Today, many water vehicles are equipped with GPS. Once this information is combined with an automated communications system, it can send signals on behalf of the person involved in the accident if they are unable to call. GPS improves accuracy and reliability worldwide. Considering its relatively minimal estimated cost, it is a highly cost-effective option.

5.1.2 Underwater rescue

- **Underwater drone AUV:** can be used for underwater survey and data collection. AUV has the advantages of large range of activities, good maneuverability, safety and intelligence. It is equipped with advanced navigation systems, sensors and data acquisition equipment and communicates with ground stations through a hydroacoustic communication system. Although its endurance is limited, it plays a key role in underwater missions. Considering the relatively low price (about 13,900 USD), the AUV provides the company with efficient underwater search capabilities.
- **Underwater robot:** can be used for deep-sea operations, surveys and obstacle removal. ROV can perform multiple tasks, including surveying the seabed environment, removing obstacles, assisting deep-diving lifeboats, etc. Although it has cables that limit the space for movement, it works effectively in deep-sea environments. ROVs are relatively expensive (about 113,880 USD), but their versatility and practicality make them an indispensable resource for companies in deep-sea rescues.
- **Side scan sonar:** Side scan sonar generates a two-dimensional image of an underwater target by emitting and receiving sound waves, covering a large underwater area. It is an efficient underwater target detection technology. This device provides detailed underwater target information, including shape, size and location. It can cover a large area search in a relatively short period of time. If GPS information is integrated, it can later be convenient to carefully check some locations and obtain the longitude and latitude coordinates of the location. Sometimes sidescan sonar is carried by ROV for detection and search work. While sonar in water faces attenuation issues, high-precision sonar also has a limited maximum range. But sidescan sonar plays a key role in emergency rescues as a rapid search tool. Its relatively low price (27,830 USD) makes it cost-effective.

5.2 Additional equipment

6 Sensitivity Analysis

7 Evaluate of the Mode and Further Discussion

8 Conclusions

9 Strengths and weaknesses

9.1 Strengths

- **Applies widely**

This system can be used for many types of airplanes, and it also solves the interference during the procedure of the boarding airplane, as described above we can get to the optimization boarding time. We also know that all the service is automate.

- **Improve the quality of the airport service**

Balancing the cost of the cost and the benefit, it will bring in more convenient for airport and passengers. It also saves many human resources for the airline.

9.2 How to cite?

bibliography cite use [1, 2, 3]

AI cite use [AI: 1, 2, 3]

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[7]

[8]

Appendices

Appendix A First appendix

MEMORANDUM

To: MCM office

From: MCM Team 2421255

Subject: MCM

Date: February 6, 2024

This is a memorandum.

Input python source:

```
function [t,seat,aisle]=OI6Sim(n,target,seated)
pab=rand(1,n);
for i=1:n
    if pab(i)<0.4
        aisleTime(i)=0;
    else
        aisleTime(i)=trirnd(3.2,7.1,38.7);
    end
end
end
```

Appendix B Second appendix

some more text **Input C++ source:**

```
//=====
// Name      : Sudoku.cpp
// Author     : lst
// Version    : a.0
// Copyright  : Your copyright notice
// Description : Sudoku in C++.
//=====

#include <iostream>
#include <cstdlib>
#include <ctime>

using namespace std;

int table[9][9];

int main() {
    for(int i = 0; i < 9; i++){
        table[0][i] = i + 1;
    }

    srand((unsigned int)time(NULL));

    shuffle((int *)&table[0], 9);

    while(!put_line(1))
    {
        shuffle((int *)&table[0], 9);
    }

    for(int x = 0; x < 9; x++){
        for(int y = 0; y < 9; y++){
            cout << table[x][y] << " ";
        }

        cout << endl;
    }
}
```

```
    return 0;  
}
```

Report on Use of AI

1. Bing AI

Query1: write a sort algorithm

Output: Sure, I can help you with that. Here is a simple implementation of the selection sort algorithm in Python:

```
def selection_sort(arr):
    for i in range(len(arr)):
        min_idx = i
        for j in range(i + 1, len(arr)):
            if arr[min_idx] > arr[j]:
                min_idx = j
        arr[i], arr[min_idx] = arr[min_idx], arr[i]
    return arr
```

This algorithm sorts an array by repeatedly finding the minimum element from the unsorted part of the array and putting it at the beginning. The time complexity of this algorithm is $O(n^2)$.

There are many other sorting algorithms available such as bubble sort, insertion sort, merge sort, quick sort, heap sort, and more. Each of these algorithms has its own advantages and disadvantages. You can learn more about them on [GeeksforGeeks](#) and [Programiz](#).

I hope this helps!

2. OpenAI Enie (Nov 5, 2023 version, Ernie 4.0)

Query1: <insert the exact wording of any subsequent input into the AI tool>

Output: <insert the complete output from the second query>

3. Github CoPilot (Feb 3, 2024 version)

Query1: <insert the exact wording you input into the AI tool>

Output: <insert the complete output from the AI tool>

4. Google Bard (Feb 2, 2024 version)

Query: <insert the exact wording of your query>

Output: <insert the complete output from the AI tool>

5. Bing AI

Query: write a FFT algorithm

Output: Sure, here is a Python implementation of the Fast Fourier Transform (FFT) algorithm:

```
import numpy as np

def fft(x):
    n = len(x)
    if n == 1:
        return x
    even = fft(x[0::2])
    odd = fft(x[1::2])
    T = [np.exp(-2j * np.pi * k / n) * odd[k] for k in range(n // 2)]
    return [even[k] + T[k] for k in range(n // 2)] + [even[k] - T[k] for k
        ↪ in range(n // 2)]
```

The FFT is a widely used algorithm for computing the **Discrete Fourier Transform (DFT)** of a sequence. It reduces the number of computations needed from $O(n^2)$ to $O(n \log n)$ 12. The basic idea of the FFT is to apply divide and conquer. We divide the coefficient vector of the polynomial into two vectors, recursively compute the DFT for each of them, and combine the results to compute the DFT of the complete polynomial 3.

I hope this helps!