Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
B		**2421255**

Path planning for submarine search and rescue

Summary

my abstract

Keywords: Randomisierter Algorithm ; Bayesian inference ; The Multivariate Gaussian Distribution ; The analytic hierarchy process

Contents

1 Introduction

1.1 Background

In the 21st century, there has been a growing interest in the exploration of the ocean in various countries and regions. Maritime Cruises Mini-Submarines (MCMS), a Greek company, specializes in manufacturing submersibles to take people to the deepest parts of the ocean. They have set their sights on leading tourists on exciting adventures to explore sunken shipwrecks in the depths of the Ionian Sea.

To make their submersible operations a reality, we help MCMS to obtain regulatory approval and establish safety protocols to address potential communication loss and mechanical issues, such as propulsion failure. And we develop a predictive model that can track the submersible's location over time. This model considers factors such as sea floor positioning, buoyancy, currents, sea density , and geography.

1.2 Restatement of the problem

Considering the background information and restricted conditions identified in the problem statement,we need to solve the following problems:

- Create a model to predict location of the submersible overtime, figure out the uncertain-ties and find what information can the submersible send back with specific equipment to reduce that.
- Determine additional equipment for searching to carry on the host ship considering the possible costs, and recommend devices for rescuing. on both the host ship and rescue ship if necessary.
- Develop a model using location data to recommend deployment points and search patterns to minimize the time to locate a lost submersible.
- Establish a function that relates the probability of finding the submersible to both time and the accumulated search results.
- Extend the model to cover other tourist destinations and adapt it for multiple submersibles moving in the same general vicinity.

1.3 Our work

to my eq (**??**)

2 Assumptions and Justifications

Considering that practical problems always contain many complex factors, first of all, we need to make reasonable assumptions to simplify the model, and each hypothesis is closely followed by its corresponding explanation.

- **Assumptions 1: Rationalize the assumptions for the submersible by considering it as a point mass.**

  When moving in the deep sea, the size and shape of the submersible are negligible compared to the problem we are studying, and the movement of each point on the submersible can be considered to be the same. Therefore, the submersible can be represented by a mass point, following the principle of the center of **mass theorem**.

- **Assumptions 2: Assume that the submarine will no longer provide propulsion.**

  Unless under extreme special circumstances, the submersible pilot will cease all operations and await rescue after experiencing distress. Therefore, it can be inferred that the submersible is powerless on its own once in distress.

- **Assumptions 3: The decision factors only include ocean currents, density, and topography.**

  Based on the principles of fluid mechanics, the motion of the submersible is primarily influenced by water flow and its own dynamics. Ocean currents play a significant role in governing the movement of water bodies in the ocean, and density is associated with the flow of water bodies.so these factors are emphasized and other factors are ignored.

- **Assumptions 4: For the simplicity of the model, decision factors do not interfere with each other.**

3 Notations

Symbol	Definitions	Units
g	Gravitational acceleration	m/s^2
ρ	Density	kg/m^3
L_a	the new latitude	°Decimal Degrees
L_o	the new longitude	°Decimal Degrees
O_a	the offset of the latitude	°Decimal Degrees
O_g	the offset of the longitude	°Decimal Degrees
C_d	fluid resistance coefficient	

Table 1: Notations used in this paper

The key mathematical notations used in this paper are listed in Table 1

4 Model 1 : Locate the submersible over time

4.1 The uncertainties associated with the predictions

There are many factors that will affect the results of model predictions, mainly the following:

- **Ocean current interference**,The influence of ocean currents on the movement of the submersible, which in turn affects the trajectory.
- **Changes in seawater density**,When the temperature and salinity correlation change, it will affect the density of seawater, thereby affecting the buoyancy and resistance. The buoyancy will change the vertical acceleration, and the resistance will change the velocity in the direction of motion.
- **Measure noise and communication delays**, Due to measure noise and communication delays, navigation accuracy is affected,communication efficiency is reduced ,so the results are interfered.
- **Change in the probability of finding the submersible caused by long search and rescue time**, it will lead to difficulty in accumulating data, decreased search efficiency, uncertainty in mission objectives, changes in communication requirements, and impact on location prediction.
- **Neutral buoyancy condition**,Neutral buoyancy conditions cause drift risk, risk of getting wet, depth changes, environmental variations, and changes in forecast results.

4.2 Density varying with depth

We developed a model to predict how a submersible's position will change over time

In the model, the density of seawater is a crucial factor determining the water resistance and buoyancy of the submarine, which is essential for predicting the trajectory after a submarine accident.The Ionian Sea is small and closed, so horizontal density changes can be ignored. Therefore, the relationship between density and submarine depth is first solved.

The relationship between pressure and density, gravitational acceleration and depth is related by this formula.

$$P = \rho g h \tag{1}$$

On the left side of the equation is the pressure at a certain underwater location, while the right side represents the product of seawater density (ρ), local gravity acceleration (g), and diving depth (h). Since the change in depth is significant during the descent, after consulting the data and calculating, It is known that the product of seawater density (ρ) and local gravity acceleration (g) is approximately 0.10045. Therefore, in the formula, it is reasonable to consider P = 0.10045h, therefore we can calculate a certain Pressure corresponding to depth. By reviewing literature[2] and analyzing the information, we obtained scatter diagrams illustrating the relationship between seawater salinity and pressure, as well as seawater temperature and pressure. This enables us to determine the corresponding temperature and salinity values for a specific depth.

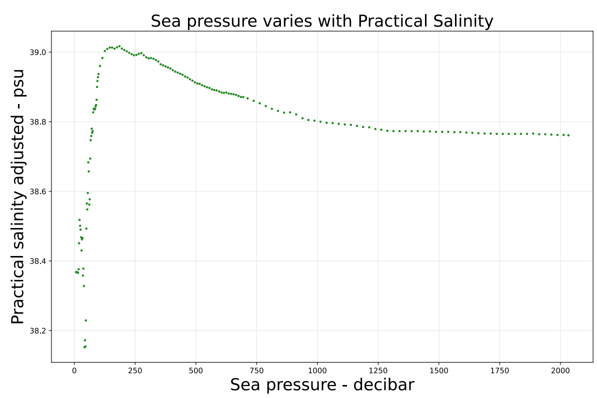

Figure 1: Seawater salinity-pressure scatter point

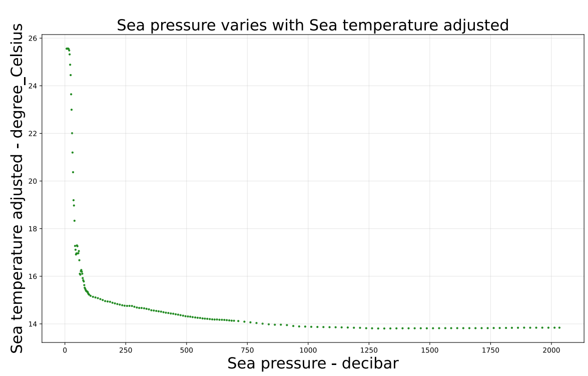

Figure 2: Seawater temperature-pressure scatter plot

After calculating the seawater temperature and salinity, and then considering the relationship between seawater salinity, temperature and seawater density, we can use the Thermodynamic Equation of Seawater to calculate the density of seawater.

$$\rho = \rho_0 + \alpha(T - T_0) + \beta(S - S_0) \tag{2}$$

On the left side of the equation, ρ represents the density at a certain underwater location, ρ_0 is the reference density that is taken as the density of pure water, and $(T - T_0)$ and $(S - S_0)$ denote the temperature and salinity differences from their respective reference values. Moreover, α and β are their respective correlation coefficients, where α is the thermal expansion coefficient and β is the salinity expansion coefficient.

From the formula, we can infer that ρ is determined by the three-dimensional linear relationship between T and S. However, because factors such as sea currents and temperature will stabilize after reaching a certain depth, the change in seawater density no longer shows an obvious linear relationship, so we need to add a correction formula to calculate more accuratele value of seawater density.

$$\rho = \begin{cases} \rho_0 + \alpha(T - T_0) + \beta(S - S_0) & \text{if } h < 2,000\ m \\ C_1 & \text{if } h \geqslant 2,000\ m \end{cases} \tag{3}$$

By consulting information and data, we calculated that the density of seawater is $1070 kg/m^3$ when the depth is more than 2000 meters. So the the density of seawater can be given at a specific depth. Therefore we plotted the density of seawater as a function of depth.

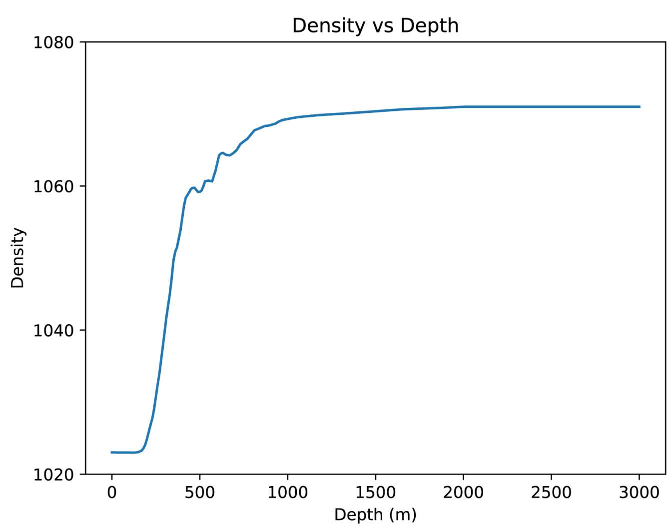

Figure 3: Seawater Density changes with depth

By using linear interpolation at each point on this scatter plot, we can obtain arbitrary continuous density values.

4.3 Model of submersible position

4.3.1 Depth at specific coordinate

Depth computation is a critical part of our model. By referring to geographic data files and data we are able to obtain elevation data on the earth's surface,and also obtain the latitude and longitude of various points. By consulting the elevation data[3], we obtained a depth map of the Ionian Sea.

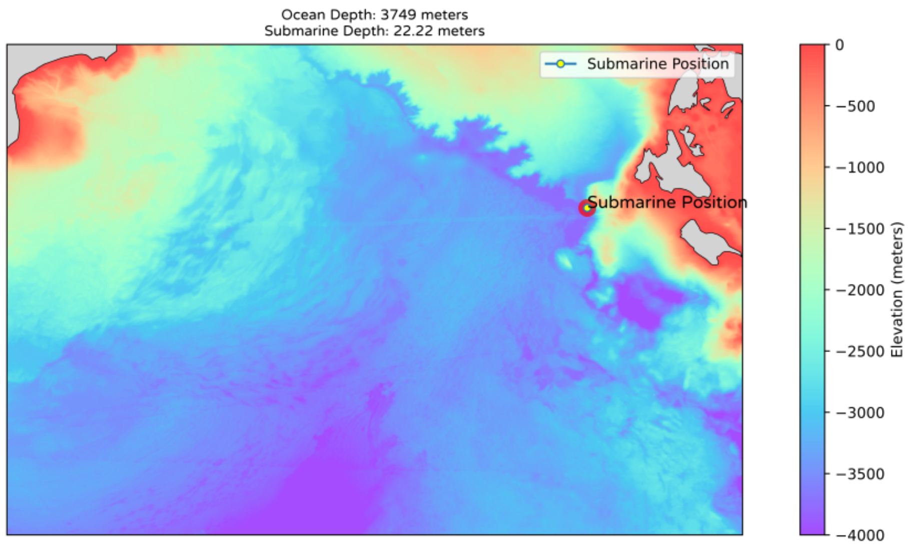

Figure 4: The Depth Map Of The Ionian Sea

The curvature of the earth's surface can be calculated by simplifying latitude and longitude, and because there are still many factors in the conversion of kilometers and latitude and longitude in actual situations, we give an estimation formula

$$La = La_0 + \frac{Oa}{111.32} \tag{4}$$

$$Lo = Lo_0 + \frac{Og}{111.32\ \cos(\text{radians}(La))} \tag{5}$$

where La represents the new latitude, La_0 represents the latitude of the origin position, and Oa represents the offset of the latitude. Lo represents the new longitude, Lo_0 represents the longitude of the origin position, and Og represents the offset of the longitude.

And 1 degree of latitude is approximately 111.32 kilometers. After obtaining the new geographical coordinates, we can calculate the new depth by latitude and longitude.We can obtain the elevation information of the target location.

4.3.2 Gravity, Buoyancy and Seawater Resistance

Water resistance is a key factor in the movement of submarines in the water, which involves factors such as drag coefficient, reference area and speed.

$$P = \frac{1}{2} C_d \, \rho A \, v^2 \tag{6}$$

The formula represents an expression for water resistance. In the formula, water resistance corresponds to P,cross-sectional area to A,and velocity to v.

Gravity and buoyancy are two opposing forces experienced by a submarine moving in the water. It is also a key factor in the movement of submarines in the water. Their relationship are given by the formulae below.

$$G = m \, g \tag{7}$$

$$F = \rho \, g \, V \tag{8}$$

Where gravity corresponds to G, mass corresponds to m, buoyancy corresponds to F, and volume corresponds to V.

4.3.3 Differential equations for simulating submarine motion

We describe the motion process of the submarine in the water through the calculation of differential equations. We take into account several physical factors, including water resistance, buoyancy and gravity. By solving a system of differential equations, the motion trajectories of submarines at different depths can be simulated, taking into account the influence of terrain elevation on submarine motion and the characteristics of density changes with depth.

For the submarine position x, y, z and its velocity v_x, v_y, v_z. **When the submarine's depth is below or close to the seabed (to reduce errors), the speed and acceleration in all directions are set to zero to simulate the situation when the submarine stops moving.The seabed depth can be known from the data in 4.3.1. When the depth of the submarine is higher than zero, the position and speed are recalculated to simulate the situation when the submarine floats to the surface.** We calculated the displacement velocity in the x and y directions, taking into account the effect of Gaussian noise. We then calculate the velocity and acceleration in the z direction, taking into account the effects of water drag, buoyancy and gravity. In this way, the result of the differential equation is solved, that is, the change of the submarine's position and velocity in time. The formula is as follows

$$\begin{cases} \dfrac{dv_x}{dt} = -\dfrac{R \, \mathrm{sgn}(v_x)}{m} + N_x \\ \dfrac{dv_y}{dt} = -\dfrac{R \, \mathrm{sgn}(v_y)}{m} + N_y \\ \dfrac{dv_z}{dt} = \dfrac{g - R \, \mathrm{sgn}(v_z)}{m} + N_z \end{cases} \tag{9}$$

where v_x, v_y, and v_z represent the speed in the horizontal and vertical directions respectively. dv_x/dt, dv_y/dt, dv_z/dt is the rate of change of the corresponding speed. R is the drag coefficient. m is mass. g is the acceleration due to gravity. $sgn(a)$ is the symbolic function.Nx, Ny and Nz represent the influence of Gaussian noise. These noises can affect the dynamic behavior of the system.

4.3.4 Estimating a submarine's position in the ocean

To infer the likely location of the submarine in three-dimensional space, we used three-dimensional Bayesian inference. Through the observation data, the corresponding posterior probability distribution is obtained. In the absence of observational information, we make an initial estimate of the submarine's position. We have this formula to calculate the prior probability,

$$P(x, y, z) = \frac{1}{(2\pi)^{3/2}\sqrt{\det(\Sigma_{\text{prior}})}} e^{-\frac{1}{2}d^2(x,\mu)} \tag{10}$$

where $P(X)$ represents the prior probability of the submarine position.$det(prior)$ is the determinant of the covariance matrix Σprior, and x is a column vector containing the variables x, y, and z. μ_{prior} is a column vector containing variables x, y, z, representing the mean vector of the distribution. The explanation for $e^{-\frac{1}{2}d^2(x,\mu)}$ comes from the literature[7][8].

Based on observational data, we estimate the likelihood of a submarine being at a given location. Observation errors are taken into account using a multivariate normal distribution, where the mean is the observation location and the covariance matrix is calculated with the given standard deviation.

$$P(x|X) = \frac{1}{(2\pi\sigma^2)^{3/2}} e^{-\frac{1}{2}(\mathbf{x}-\mu)^T\Sigma^{-1}(\mathbf{x}-\mu)} \tag{11}$$

Where X represents a given position, x represents the observed position coordinates, μ represents the mean of the observed position, represents the covariance matrix, and σ represents the standard deviation of the observation error.

Combining the prior probability and likelihood function, we can calculate the posterior probability. given by the formula.When the observation data, observation error and position range are given, we simulate Bayesian inference to obtain the posterior probability distribution of the submarine's position.

$$P(X|O) = \frac{P(X) \times P(O|X)}{\sum P(X) \times P(O|X)} \tag{12}$$

Where$P(X)$ represents the prior probability. $P(O|X)$ represents the likelihood function.

Based on the position model we established and these formulas, we can infer the possible position of the submarine as it changes over time in three-dimensional space.

4.4 Ways to Reduce Uncertainty

4.4.1 Informations need to be transmitted

Based on the information and literature we reviewed, we believe the message we want the submersible to send is as follows:

- Sonar Localization Position: Transmitting sonar localization position allows the main ship to accurately know the submersible's underwater location. This is crucial during accidents for emergency rescue and locating the submersible.
- Speed: Conveying information about the submersible's speed enables the main ship to monitor its real-time movement. This is vital for quickly assessing situations during accidents and formulating effective response strategies.
- Coordinate Information: Providing coordinate information enables the main ship to accurately determine the submersible's position underwater. This is crucial for navigation, rescue operations, and real-time positioning during accidents.
- Depth: Transmitting depth information helps ensure that the submersible operates within a safe depth range. This aids in preventing the submersible from exceeding its design limits, reducing the risk of potential underwater accidents.
- Density: Density information provides key data about the surrounding water, essential for assessing underwater environmental conditions, planning tasks, and enhancing submersible safety.
- Flow Rate: Understanding the flow rate of the surrounding water assists both the main ship and submersible in better planning actions, especially in complex water flow environments. This helps mitigate risks for the submersible and ensures its safe operation.
- Environmental Data: Monitoring seawater salinity and density offers critical insights into water properties. This not only helps avoid unfavorable environmental conditions but also improves the main ship's understanding of the underwater environment, ensuring the submersible's safe operation.
- System Health Status: Regularly reporting the submersible's system health status, including sensors, batteries, and other essential components, helps the main ship detect potential issues promptly and prevent system failures.
- Communication Status: Regularly sending communication status information ensures effective communication between the submersible and the main ship. Maintaining stable communication is crucial during accidents for rescue and coordination actions.

4.4.2 Additional required equipment

After considering various factors of information transmission, we reviewed the information and literature and concluded that the following equipment is needed:

- **Inertial Measurement Unit (IMU) :** It offers several advantages for determining the position of underwater vehicles,IMU operates independently, unaffected by external signals. In underwater environments, especially where GPS signals are unavailable, IMU provides an independent means of determining the vehicle's position.IMU continuously and in real-time measures acceleration and angular velocity. Integrating this data allows the underwater vehicle to monitor its velocity and displacement throughout the entire diving process.With gyroscopes and accelerometers, IMU delivers high-precision attitude information, crucial for ensuring the underwater vehicle adheres to the planned trajectory and avoids instability in attitude.

- **Acoustic Positioning Systems (LBL or SBL) :** LBL and SBL acoustic systems offer highly accurate positioning information, allowing you to precisely track the submersible's location. Provides real-time location data, enabling prompt responses to the submersible's current status for ensuring safety.

- **Depth Sensor :** Measures the submersible's depth, providing vertical position information for ensuring operations within a safe depth range. Depth sensors are typically simple and reliable tools, offering accurate depth measurements.

- **Density Sensor :** Density sensors can monitor water density, providing crucial information about the underwater environment and helping anticipate potential changes in water flow and deep-sea conditions. Changes in density can indicate seafloor topography or other variations, assisting in avoiding potential hazards.

- **Flow Meter :** Measures water flow velocity, offering insights into the underwater flow conditions, aiding in planning submersible routes and predicting potential risks. In areas with strong currents, flow meter data can optimize submersible navigation, ensuring that the submersible progresses as planned.

- **VLF Bandwidth :** VLF signals exhibit strong propagation capabilities in water, penetrating through the medium and reaching relatively long distances. This allows submarines to employ the VLF band for long-distance underwater communication, as well as for underwater navigation and positioning systems. Leveraging the propagation characteristics of VLF signals underwater, submarines can utilize positioning systems to acquire information about their own location.

5 Search and rescue equipment

5.1 Search equipment

We are committed to ensuring the safety of tourists during underwater tours. In order to carry out rescue quickly and effectively in possible emergencies, when considering the supplies required for search and rescue by the submersible company, we focus on many aspects, including purchase cost, Maintenance cost, usage cost, reliability, accuracy, search and rescue breadth, etc. After reviewing information and collecting prices, we have listed the following search and rescue equipment to choose from. Some price sources[1].

5.1.1 Above water rescue

- **Search and rescue helicopter:** In emergency rescue situations, especially when reaching the sea or inaccessible areas, helicopters can quickly arrive at the scene and perform search and rescue missions. Considering that when an accident occurs, it may be very difficult to predict the location of people in distress, especially at sea. The helicopter is equipped with infrared imaging equipment, speakers, search lights and other equipment, which can efficiently search for targets at sea and detect targets under different sea conditions. For example the search radar of the Russian Ka-27 search and rescue helicopter[5]. Although the purchase cost of the helicopter is relatively high (approximately 4,171,560 USD) and there are scheduled and unscheduled maintenance costs, insurance, and flight usage costs (approximately 69,530 USD), making it very expensive , but its reliability and breadth in emergency rescue make it a reasonable choice.

- **GPS Sensor:** Dramatically improves managers' ability to efficiently manage their emergency response teams. It can effectively discover and confirm the location of rescue teams and ships, and confirm the relationship between their locations and the transportation system network in a sea area, providing a new way of working for the entire rescue operation. Automated GPS location information[6] reduces delays in dispatching emergency services.Today, many water vehicles are equipped with GPS. Once this information is combined with an automated communications system, it can send signals on behalf of the person involved in the accident if they are unable to call. GPS improves accuracy and reliability worldwide. Considering its relatively minimal estimated cost, it is a highly cost-effective option.

5.1.2 Underwater rescue

- **Underwater drone AUV :** can be used for underwater survey and data collection. AUV has the advantages of large range of activities, good maneuverability, safety and intelligence. It is equipped with advanced navigation systems, sensors and data acquisition equipment and communicates with ground stations through a hydroacoustic communication system. Although its endurance is limited, it plays a key role in underwater missions. Considering the relatively low price (about 13,900 USD), the AUV provides the company with efficient underwater search capabilities.

- **Underwater robot :** can be used for deep-sea operations, surveys and obstacle removal. ROV can perform multiple tasks[4], including surveying the seabed environment, removing obstacles, assisting deep-diving lifeboats, etc. Although it has cables that limit the space for movement, it works effectively in deep-sea environments. ROVs are relatively expensive (about 113,880 USD), but their versatility and practicality make them an indispensable resource for companies in deep-sea rescues.

- **Side scan sonar :** Side scan sonar generates a two-dimensional image of an underwater target by emitting and receiving sound waves, covering a large underwater area. It is an efficient underwater target detection technology. This device provides detailed underwater target information, including shape, size and location. It can cover a large area search in a relatively short period of time. If GPS information is integrated, it can later be convenient to carefully check some locations and obtain the longitude and latitude coordinates of the location. Sometimes sidescan sonar is carried by ROV for detection and search work.While sonar in water faces attenuation issues, high-precision sonar also has a limited maximum range.But sidescan sonar plays a key role in emergency rescues as a rapid search tool. Its relatively low price (27,830 USD) makes it cost-effective.

Taking into account the purchase cost, maintenance cost, use cost, reliability, accuracy and search and rescue breadth of these equipment, these are reasonable and cost-effective choices. We meet multiple needs while maintaining overall cost-effectiveness. The combined use of these devices will provide comprehensive and efficient support for the company's undersea search and rescue missions.

5.2 Additional equipment

In order for the rescue ship to provide more comprehensive and flexible search and rescue services, covering the needs of different emergency situations. After consulting the information and analyzing the information, we concluded that the following additional equipment can be carried:

- **Life raft :** A life raft is an important piece of equipment that provides safe shelter in an emergency. In the event of an emergency, such as a ship in distress, a life raft provides a relatively safe environment for passengers and crew to await rescue. Life rafts are usually designed for rapid deployment and can be quickly launched into the water in an emergency to provide a rapid means of evacuation for rescued persons.

- **Marine search and rescue dogs :** Marine search and rescue dogs have excellent olfactory sensitivity and are able to detect odors in the water, including human scent. This gives them a unique advantage in water search and rescue missions. Marine search and rescue dogs can cover large areas of water and quickly locate trapped people, helping to improve search and rescue efficiency.

  Adaptation to the water environment: Marine search and rescue dogs are trained to adapt to the water environment, including performing tasks in rough seas.

- **Deep diving lifeboat :** Deep submersible lifeboats have the ability to perform rescue missions in deep sea environments. It may involve underwater rescue, and the deep-diving lifeboat can carry divers deep into the water to perform tasks.

  Deep-diving lifeboats can perform a variety of tasks, including investigating the environment around a wrecked submarine, removing obstacles, and assisting divers and equipment. This increases the rescue vessel's adaptability in complex underwater environments.

- **Emergency medical equipment and first aid kit :** Emergency medical equipment includes basic first aid tools, first aid kits, and automated external defibrillators (AEDs), which can provide rapid and effective medical support in emergencies. First aid medicine kits are equipped with a variety of medicines to respond to different types of emergencies. conditions, including pain relief, allergic reactions, heart attacks, and more. A first aid kit containing tourniquets, bandages, disinfectants, and more can be used to treat accidental injuries, helping to relieve the victim's pain and prevent the injury from getting worse. Emergency medical equipment may include simple life support equipment such as breathing bags and endotracheal tubes to provide basic life support until rescue personnel arrive.

By carrying this additional equipment, rescue ships are able to provide more comprehensive and flexible search and rescue services, covering the needs of different emergency situations. The comprehensive use of these devices helps improve search and rescue efficiency, reduce search and rescue time, and protect the safety of the rescued persons to the greatest extent.

6 Wonderful search position model

6.1 Various data of the submersible

To determine the optimal search position and minimize the time required for positioning the lost submersible, we developed a model for optimal search positions. Initially, we need to collect various data related to the submersible and the sea water conditions. Formula (3) enables us to calculate the seawater density . From the seawater salinity-pressure scatter plot and formula (1), we can calculate the seawater salinity.By utilizing formulas (3) (4) (5), we can obtain motion data and position data of the submersible.

6.2 Calculation of Prior Probability and Posterior Probability

Thus, we acquire the initial data for the submarine. in order to infer the optimal search position of the submarine in three-dimensional space, without observation information, we use multivariate normal distribution to calculate the prior probability of the submarine's position. This prior probability is derived from the known best trajectory and we can calculate it using Equation (10) and the picture.

In this way, the model pays more attention to past movement trends and takes into account position uncertainty, making the change of prior probability on the submarine's movement path smoother.

Based on observational data, we estimate the probability of a submarine being at a specific location. Taking into account the observation error, we used a multivariate normal distribution to obtain the position data, using formula (11), where the mean is the observed position and the covariance matrix is calculated with the given standard deviation. In this way, the model takes into account observation errors and can better adapt to actual observation data.

Combining the prior probability and likelihood function, we can calculate the posterior probability, which is given by the following formula.

$$P(X \mid O) = \frac{P(X) \times P(O \mid X)}{\sum P(X) \times P(O \mid X)} \tag{13}$$

where $P(X)$ represents the prior probability, and $P(X \mid O)$ represents the likelihood function.

When provided with observation data, observation errors, and a specified location range, we perform Bayesian inference simulations to derive the posterior probability distribution

of the submarine's location. This crucial step empowers the model to assign weights to the likelihood of various locations based on actual observation data. Consequently, the model becomes more adept at accurately reflecting real-world scenarios, representing the essence of Bayesian inference — the continual update of the model to a more precise state through the integration of observed data. In this manner, we can precisely predict the submarine's movement trajectory and minimize the time required for locating the lost submersible. Subsequently, we determine the interpretation of the probability of locating the submarine as a function of time and cumulative search results.

6.3 Derivation of the Probability Function to be Obtained

Based on time and cumulative search results, we can use Bayesian inference to estimate the probability. We assume that $P(t)$ represents the probability of finding the submersible at time t, $S(t)$ represents the accumulated search results at time t, the predicted position of the submersible is (x_p, z_p), and the initial deployment point of the search equipment is (x_s, z_s), then the probability $P(t)$ of finding the submersible is given by

$$P(t) = \int_f (x, z) \cdot S(t) \cdot dxdz \tag{14}$$

In reality, during the search process, movements occur at discrete intervals, leading to a trajectory that consists of a collection of discrete points, as depicted in the figure below.

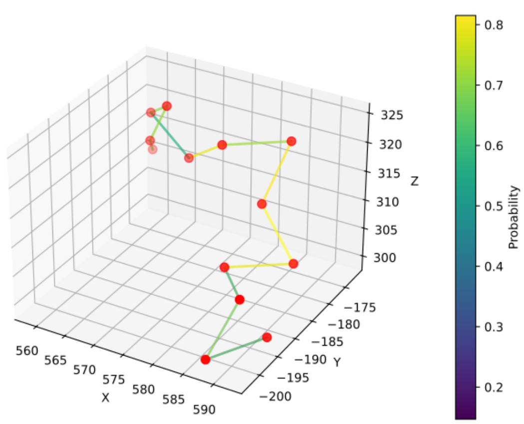

Figure 5: Probability Modelify

Consequently, we express the equation in a summation form to account for this discretization.

$$P(n) = \sum_{i}^{n} f(x, y, z)S(i) \tag{15}$$

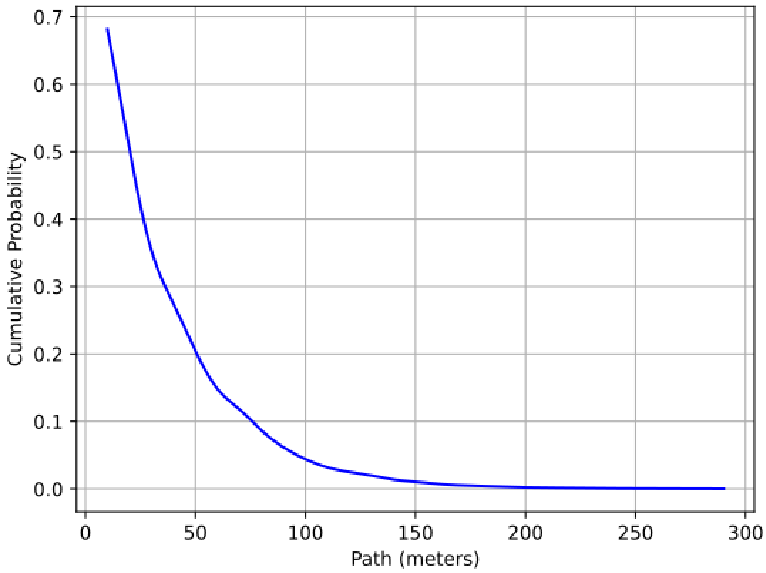

Figure 6: Probability Modelify

7 Sensitivity Analysis

8 Evaluate of the Model and Further Discussion

8.1 Strengths

8.2 Weaknesses

8.3 Further Discussion

8.3.1 Model Imporvements

8.3.2 Model Extensions

9 Conclusions

References

[1] VirtualEXPO GROUP. “NauticEXPO,”n.d .https://www.nauticexpo.com/

[2] Argo (2024). Argo float data and metadata from Global Data Assembly Centre (Argo GDAC) - Snapshot of Argo GDAC of Jannuary 09st 2024. SEANOE. https://doi.org/10.17882/42182#107470 Addison-Wesley Publishing Company, 1986.

[3] Gebco.net. “GEBCO,” n.d. https://www.gebco.net/data_and_products/gridded_bathymetry_data/. gebco.net. https://www.gebco.net/data_and_products/gridded_bathymetry_data/GE

[4] CAO Hong-tao, ZHANG Qi-feng, TANG Shi. Current status and key technologies of Remotely Operated Vehicles in international submarine rescue and deep-sea salvage. Ship Science and Technology, 2021, 43(12): 16-20

[5] Zhihui,YAN Kexue,HUANG Rong.Search and rescue helicopter capability requirement analysis[J].Aeronautical Science & Technology,2021,32(01):60-64

[6] gps.gov. “GPS: The Global Positioning System,” n.d. https://www.gps.gov/applications/safety/.

[7] Do, C. B. (2008). The Multivariate Gaussian Distribution.

[8] Ribeiro, M. I. (2004). Gaussian probability density functions: Properties and error characterization. Institute for Systems and Robotics, Lisboa, Portugal.

[9] sea-eye. “THE SEA-EYE 4,” n.d. https://sea-eye.org/en/sea-eye-4/?new-rescue-ship/spende.

[10] sea-watch.org. “No Rescue without You,” n.d. https://sea-watch.org/en/sea-watch-5/.

Appendices

Appendix A First appendix

MEMORANDUM

To: MCM office

From: MCM Team 2421255

Subject: MCM

Date: February 6, 2024

This is a memorandum.

Appendix B Second appendix

Input python source:

```
function [t,seat,aisle]=OI6Sim(n,target,seated)
pab=rand(1,n);
for i=1:n
    if pab(i)<0.4
        aisleTime(i)=0;
    else
        aisleTime(i)=trirnd(3.2,7.1,38.7);
    end
end
```

Report on Use of AI

1. OpenAl Enie (Nov 5, 2023 version, Ernie 4.0)
   Query1: <insert the exact wording of any subsequent input into the Al tool>
   Output: <insert the complete output from the second query>

2. Bing AI
   Query: write a FFT algorithm
   Output: Sure, here is a Python implementation of the Fast Fourier Transform (FFT) algorithm:

```
import numpy as np

def fft(x):
    n = len(x)
    if n == 1:
        return x
    even = fft(x[0::2])
    odd = fft(x[1::2])
    T = [np.exp(-2j * np.pi * k / n) * odd[k] for k in range(n // 2)]
    return [even[k] + T[k] for k in range(n // 2)] + [even[k] - T[k] for k in range(n // 2)]
```