NUMERICAL METHODS AND ITS APPLICATIONS

What is Numerical method?

- Numerical method is an approach for solving complex mathematical problems using simple arithmetic operations.
- It involves the formulation of model of physical situations that can be solved by arithmetic operations

Need of numerical method

- Mathematical models are a central piece of science and engineering.
- Some models have closed-form solutions, therefore they can be solved analytically. Many models can not be solved analytically or the analytic solution is too costly to be practical.
- All models can be solved computationally and the result may not be the exact answer but it can be useful.

Steps involved to solve

- FORMULATION OF MATHEMATICAL MODEL.
- CONSTRUCTION OF AN APPROPRIATE NUMERICAL METHOD.
- IMPLEMENTATION OF THE NUMERICAL METHOD.
- OBTAIN THE SOLUTION BY SOLVING.
- VALIDATION OF THE SOLUTION.

Why are numerical methods used in Engineering?

- Engineers use mathematical modelling which includes various equations and data to describe and predict the behaviour of systems.
- Computers are widely used which give accurate results and are cheap and affordable to all.
- Many software packages are available that can be used to solve the problems.

APPLICATIONS OF NUMERICAL METHODS

MODELLING USING NUMERICAL METHOD

- Numerical methods are used in modelling to reduce the number of prototypes to be constructed.
- It is also used to get the exact or similar design which is needed in the best way.

Steps to solve:

1. Write your numerical modelling code.
2. Find a simple analytical model and divide into segments.
3. The dimensions are to be specified.

SCIENTIFIC COMPUTING USING NUMERICAL METHODS

- Design and analysis for numerically solving mathematical problems in science and engineering.
- Effect of approximations and performs error analysis.
- Modern simulations of engineering applications.

How it is applied

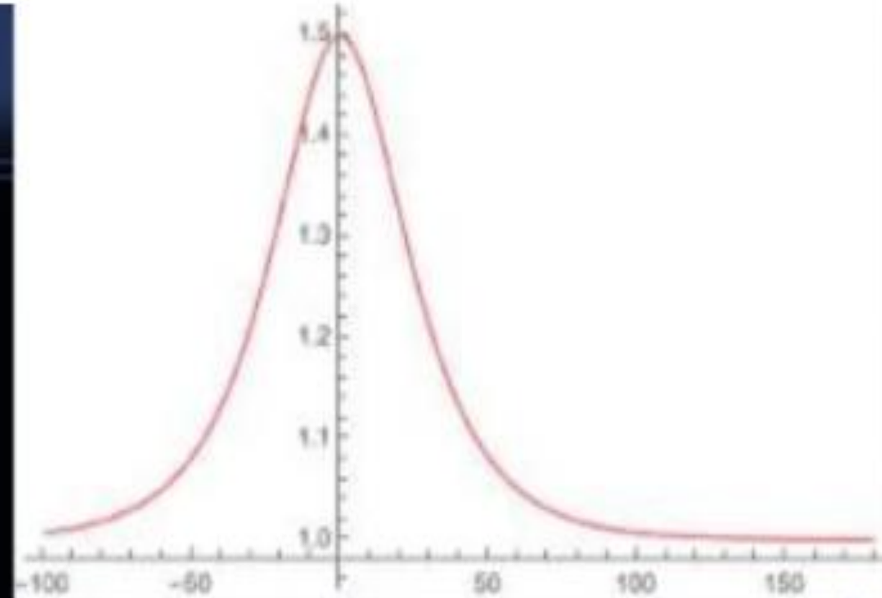
- Develop a mathematical problem with your skills and the requirement.
- Come up with a numerical algorithm.
- Implement the algorithm.
- Run, debug, test the code.
- Visualize and interpret the result.
- Validate the result.

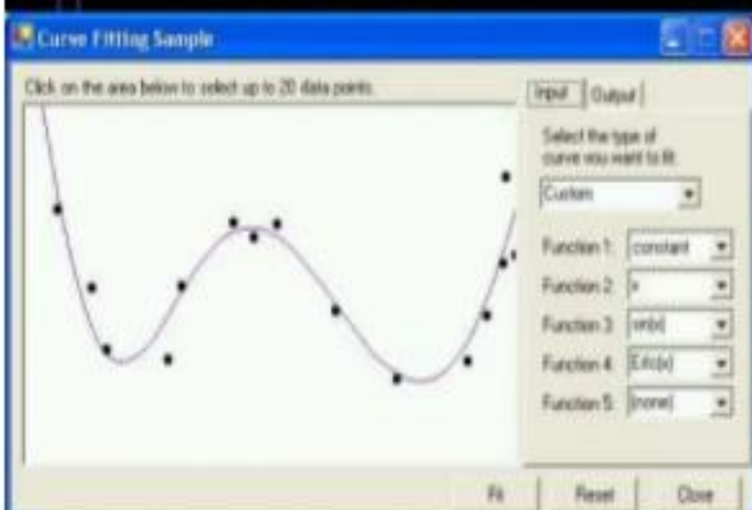
APPLICATION OF NUMERICAL METHODS IN DAY-TO-DAY LIFE

ESTIMATION OF OCEAN CURRENTS

MODELLING OF AIRFLOW OVER AIRPLANES

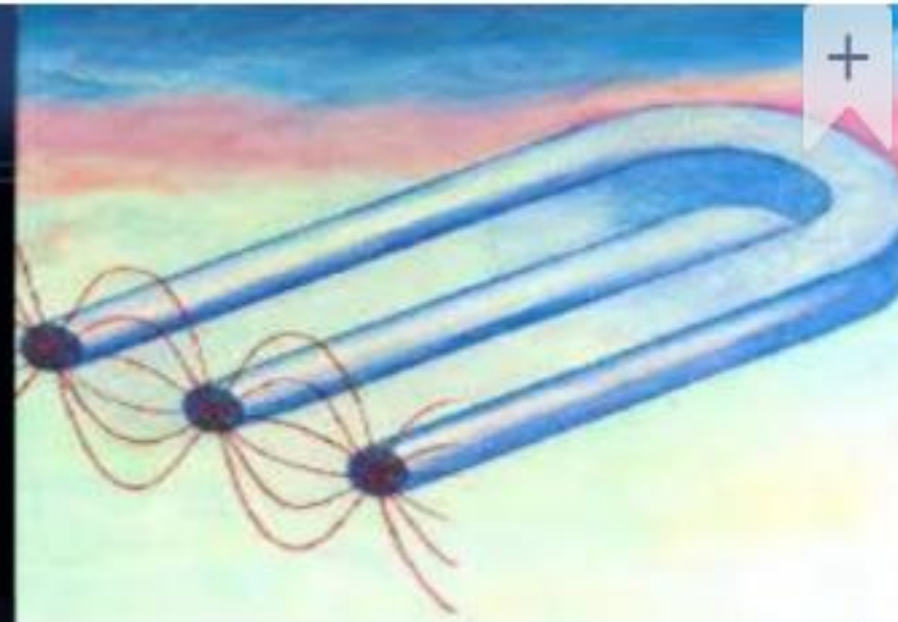
Shock wave





Curve fitting of tabular data

Electromagnetics



Shuttle tank separation

- General solution exists for equations such as

  $$ax^2 + bx + c = 0$$

  The quadratic formula provides a quick answer to *all* quadratic equations.

  However, *no* exact *general solution (formula)* exists for equations with exponents greater than 4.

- Even if “exact” procedures existed, we are stuck with the problem that a computer can only represent a finite number of values… thus, we cannot “validate” our answer because it will not come out exactly
- However we can say how accurate our solution is as compared to the “exact” solution

- *Transcendental equations*: involving geometric functions (*sin*, *cos*), *log*, *exp*. These equations cannot be reduced to solution of a polynomial.
- *Convergence*: we might imagine a "reasonable" procedure for finding solutions, but can we guarantee it terminates?

- **Approximation**: since we cannot have exactness, we specify our *tolerance* to error
- **Convergence**: we also specify how long we are willing to *wait* for a solution
- **Method**: we choose a method easy to implement and yet powerful enough and general
- **Put a human in the loop**: since no general procedure can find roots of complex equations, we let a human specify a *neighbourhood* of a solution

Practical approach - *hand calculations*

- Choose method and initial guess wisely
- Good idea to start with a crude graph. If you are looking for a single root you only need one positive and one negative value
- If even a crude graph is difficult, generate a table of values and plot graph from that.

Practical approach - example

- Example

$$e^{-x} = sin(\pi x/2)$$

- Solve for *x*.
- Graph functions - the crossover points are solutions
- This is equivalent to finding the roots of the difference function:

$$f(x)= e^{-x} - sin(\pi x/2)$$

Module 1:- Numerical Solution of Algebraic and transcendental Equations:

* Roots of the equation:- The function $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$, where n is a +ve integer and $a_n \neq 0$, $a_{n-1}, a_{n-2}, \dots a_0$ are constants is known as a polynomial of degree 'n'. The values of x making $f(x)$ zero are known as zero's or roots of the polynomial $f(x)$.

<u>Note:</u> Every polynomial of degree 'n' has exactly 'n' roots.

<u>Algebraic equation:-</u> The equation of the form $f(x)=0$ is called an algebraic if $f(x)$ is purely a polynomial in x.

Ex: $x+5=0$ $\longrightarrow$ linear equation

$x^2-5x+6=0$ $\longrightarrow$ quadratic equation, ...

<u>Transcendental equation:</u> The equation of the form $f(x)=0$ is called as transcendental if $f(x)$ contains functions such as logarithmic, trigonometric, exponential and so on..

Ex: $\cos x - xe^x = 0$

$\log_{10} x - 1.2 = 0.$

<u>Note:</u> A transcendental equation may have a finite or infinite <u>no</u> real roots and many have no real roots at all.

Solutions of Linear system of equations:

Direct Methods

1. Gauss Elimination method
2. Gauss Jordan method

By applying row operations we can find the Solutions.

3. LU decomposition Method. → Numerical method.

Iterative methods

1. Jacobi's iterative method
2. Gauss-Seidal iterative method.

Importance of LU decomposition method

In many engineering applications, when you solve $Ax=b$ the matrix $A \in R^{N \times N}$ remains unchanged, while the right hand side vector b keeps changing.

The key idea behind solving using the LU factorization is to decouple the factorization phase from the actual solving phase. The factorization phase only needs the matrix A, while the actual solving phase makes use of the factored form of A and the right hand side to solve the linear system. Hence once we have the factorization, we can make use of the factored form of A, to solve for different right hand sides at a relatively moderate computational cost.

If we do Gauss elimination separately for each right hand side vector b_j, then the total cost increases.

Direct method of solution

1.1.3 : LU Decomposition method

LU Decomposition method is based on the fact that " every square matrix A can be expressed as the product of a lower triangular matrix and an upper triangular matrix, provided all the principal minors of A are non-singular".

i.e if $A = [a_{ij}]$ then $a_{11} \neq 0$, $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0, \ldots\ldots$$

such a factorization if exists, it is unique.

Consider the system of equations

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

In matrix form we have $AX = B$ where

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Now let $A = LU$ where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \quad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

Consider $LU = A$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

By matrix multiplication

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + u_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Now compute the elements of L & U in the following order.

(i) From first row ofor U

$$u_{11} = a_{11}, \quad u_{12} = a_{12}, \quad u_{13} = a_{13}$$

(ii) From first column for L

$$l_{21}u_{11} = a_{21} \Rightarrow l_{21} = \frac{a_{21}}{u_{11}}, \quad l_{31}u_{11} = a_{31} \Rightarrow l_{31} = \frac{a_{31}}{u_{11}}$$

(iii) From second row for U

$$l_{21}u_{12} + u_{22} = a_{22} \Rightarrow u_{22} = a_{22} - l_{21}u_{12}$$

$$l_{21}u_{13} + u_{23} = a_{23} \Rightarrow u_{23} = a_{23} - l_{21}u_{13}$$

(iv) From second column for L

$$l_{31}u_{12} + l_{32}u_{22} = a_{32} \Rightarrow l_{32} = \frac{1}{u_{22}}\left[a_{32} - l_{31}u_{12}\right]$$

(v) From third row for U

$$l_{31}u_{13} + l_{32}u_{23} + u_{33} = a_{33} \Rightarrow u_{33} = a_{33} - l_{13}u_{13} - l_{32}u_{23}$$

Now substitute $A = LU$ in equation ①

$\therefore$ eqn ① becomes

$$LUX = B \quad \text{— ②}$$

Let $UX = V$ — ③

$\therefore$ eqn ② becomes

$$LV = B$$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ l_{21}v_1 + v_2 \\ l_{31}v_1 + l_{32}v_2 + v_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$\therefore$ $v_1 = b_1$, $l_{21}v_1 + v_2 = b_2$, $l_{31}v_1 + l_{32}v_2 + v_3 = b_3$

solving these equations we get v_1, v_2, v_3 and substituting in ③ we get.

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$\begin{bmatrix} u_{11}x_1 + u_{12}x_2 + u_{13}x_3 \\ u_{22}x_2 + u_{23}x_3 \\ u_{33}x_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$\Rightarrow u_{11}x_1 + u_{12}x_2 + u_{13}x_3 = v_1$, $u_{22}x_2 + u_{23}x_3 = v_2$, $u_{33}x_3 = v_3$

solving these equations we get x_1, x_2, x_3 which is the solution of given system of equations.

Procedure in brief

Write the given system of equations in the matrix form $AX = B$ — ①

Let $A = LU$ — ②

Find the elements of L & U

using eqn ② in eqn ① we get

$$LUX = B \quad \text{— ③}$$

Now let $UX = V$

$\therefore$ eqn ③ becomes

$$LV = B$$

Find the elements of V and then

Consider $UX = V$ and find elements of X,

$\therefore X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is the solution of given system of equations.

Solved Problems

1) Apply LU decomposition method to solve the equations
$3x+2y+7z=4$, $2x+3y+z=5$, $3x+4y+z=7$

Sol^n: Given
$$3x+2y+7z=4$$
$$2x+3y+z=5$$
$$3x+4y+z=7$$

In matrix form we have $AX=B$ where

$$A=\begin{bmatrix}3&2&7\\2&3&1\\3&4&1\end{bmatrix} \quad X=\begin{bmatrix}x\\y\\z\end{bmatrix} \quad B=\begin{bmatrix}4\\5\\7\end{bmatrix}$$

Let $LU=A$

$$\begin{bmatrix}1&0&0\\l_{21}&1&0\\l_{31}&l_{32}&1\end{bmatrix}\begin{bmatrix}u_{11}&u_{12}&u_{13}\\0&u_{22}&u_{23}\\0&0&u_{33}\end{bmatrix}=\begin{bmatrix}3&2&7\\2&3&1\\3&4&1\end{bmatrix}$$

$$\begin{bmatrix}u_{11}&u_{12}&u_{13}\\l_{21}u_{11}&l_{21}u_{12}+u_{22}&l_{21}u_{13}+u_{23}\\l_{31}u_{11}&l_{31}u_{12}+l_{32}u_{22}&l_{31}u_{13}+l_{32}u_{23}+u_{33}\end{bmatrix}=\begin{bmatrix}3&2&7\\2&3&1\\3&4&1\end{bmatrix}$$

Now from elements of

(i) first row for U : $\boxed{u_{11}=3}$, $\boxed{u_{12}=2}$, $\boxed{u_{13}=7}$

(ii) first column for L : $l_{21}u_{11}=2 \Rightarrow l_{21}=\frac{2}{u_{11}}=\frac{2}{3}$

$l_{31}u_{11}=3 \Rightarrow l_{31}=\frac{3}{u_{11}}=\frac{3}{3}=1$

$\therefore \boxed{l_{21}=\frac{2}{3}}$, $\boxed{l_{31}=1}$

(iii) second row of U : $l_{21}u_{12}+u_{22}=3$

$\frac{2}{3}\cdot 2+u_{22}=3 \Rightarrow \boxed{u_{22}=\frac{5}{3}}$

& $l_{21}u_{13}+u_{23}=1 \Rightarrow \frac{2}{3}\cdot 7+u_{23}=1 \Rightarrow \boxed{u_{23}=-\frac{11}{3}}$

(iv) second column for L

$l_{31}u_{12} + l_{32}u_{22} = 4$

$1(2) + l_{32} \cdot \frac{5}{3} = 4 \Rightarrow$

$\frac{5}{3} l_{32} = 4 - 2 = 2 \Rightarrow \boxed{l_{32} = \frac{6}{5}}$

(v) third row for U

$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 1$

$1(7) + \frac{6}{5}\left(-\frac{11}{3}\right) + u_{33} = 1 \Rightarrow \boxed{u_{33} = -\frac{8}{5}}$

Now substituting $A = LU$ in $AX = B$ we get

$LUX = B.$

and substituting $UX = V$ we get.

$LV = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1 & 6/5 & 1 \end{bmatrix} * \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$$

$\therefore$ we get $\begin{bmatrix} V_1 \\ \frac{2}{3}V_1 + V_2 \\ V_1 + \frac{6}{5}V_2 + V_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$

$\therefore \boxed{V_1 = 4}$, $\frac{2}{3}V_1 + V_2 = 5 \Rightarrow \boxed{V_2 = \frac{7}{3}}$, $V_1 + \frac{6}{5}V_2 + V_3 = 7$

$\Rightarrow \boxed{V_3 = \frac{1}{5}}$

$\therefore UX = V$ becomes

$$\begin{bmatrix} 3 & 2 & 7 \\ 0 & 5/3 & -11/3 \\ 0 & 0 & -8/5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 7/3 \\ 1/5 \end{bmatrix}$$

$\Rightarrow 3x + 2y + 7z = 4$

$5/3\, y - \frac{11}{3} z = 7/3$

$-\frac{8}{5} z = \frac{1}{5} \Rightarrow \boxed{z = -\frac{1}{8}}$

By back substitution we get

$\boxed{y = \frac{9}{8}}$ & $\boxed{x = \frac{7}{8}}$

2) Solve $2x+3y+z=9$, $x+2y+3z=6$, $3x+y+2z=8$ using (22)
LU decomposition method

Soln: Given $2x+3y+z=9$
$x+2y+3z=6$
$3x+y+2z=8$

In matrix form we have $AX=B$ where

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$

Let $LU = A$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

Now from

(i) first row for U : $\boxed{u_{11}=2}$ $\boxed{u_{12}=3}$ $\boxed{u_{13}=1}$

(ii) first column for L : $l_{21}u_{11} = 1 \Rightarrow l_{21} = \frac{1}{u_{11}} = \frac{1}{2}$

& $l_{31}u_{11} = 3 \Rightarrow l_{31} = \frac{3}{u_{11}} = \frac{3}{2}$

$\therefore \boxed{l_{21} = \frac{1}{2}}, \boxed{l_{31} = \frac{3}{2}}$

(iii) second row for U : $l_{21}u_{12} + u_{22} = 2$

$\frac{1}{2}(3) + u_{22} = 2 \Rightarrow \boxed{u_{22} = \frac{1}{2}}$

& $l_{21}u_{13} + u_{23} = 3$

$\frac{1}{2}(1) + u_{23} = 3 \Rightarrow \boxed{u_{23} = \frac{5}{2}}$

(iv) second column for L : $l_{31}u_{12} + l_{32}u_{22} = 1$

$\frac{3}{2}(3) + l_{32}\left(\frac{1}{2}\right) = 1 \Rightarrow \boxed{l_{32} = -7}$

(v) third row for U:

$$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 2$$
$$\frac{3}{2}(1) + (-7)\left(\frac{5}{2}\right) + u_{33} = 2 \Rightarrow \boxed{u_{33} = 18}$$

$$\therefore L = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -7 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{bmatrix}$$

Now substituting $A = LU$ in $AX = B$ we get

$$LUX = B$$

and substituting $UX = V$ we get

$$LV = B$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -7 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$

$\Rightarrow \boxed{V_1 = 9}$ $\quad \frac{1}{2}V_1 + V_2 = 6 \Rightarrow \boxed{V_2 = \frac{3}{2}}$, $\quad \frac{3}{2}V_1 - 7V_2 + V_3 = 8$

$$\Rightarrow \boxed{V_3 = 5}$$

$\therefore$ $UX = V$ becomes

$$\begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 3/2 \\ 5 \end{bmatrix}$$

$$\Rightarrow \quad 2x + 3y + z = 9$$
$$\frac{1}{2}y + \frac{5}{2}z = \frac{3}{2}$$
$$18z = 5$$
$$\Rightarrow z = \frac{5}{18}$$

By back substitution we get $y = \frac{29}{18}$, $x = \frac{35}{18}$

$\therefore$ $x = \frac{35}{18}$, $y = \frac{29}{18}$, $z = \frac{5}{18}$ is the solution of given system of equations

3) solve $10x+y+z=12$, $2x+10y+z=13$, $2x+2y+10z=14$ (24) using LU decomposition method

Soln: Given
$$10x+y+z=12$$
$$2x+10y+z=13$$
$$2x+2y+10z=14$$

In matrix form we have $AX=B$ where

$$A=\begin{bmatrix}10&1&1\\2&10&1\\2&2&10\end{bmatrix}\quad X=\begin{bmatrix}x\\y\\z\end{bmatrix}\quad B=\begin{bmatrix}12\\13\\14\end{bmatrix}$$

Let $LU=A$

$$\begin{bmatrix}1&0&0\\l_{21}&1&0\\l_{31}&l_{32}&1\end{bmatrix}\begin{bmatrix}u_{11}&u_{12}&u_{13}\\0&u_{22}&u_{23}\\0&0&u_{33}\end{bmatrix}=\begin{bmatrix}10&1&1\\2&10&1\\2&2&10\end{bmatrix}$$

$$\begin{bmatrix}u_{11}&u_{12}&u_{13}\\l_{21}u_{11}&l_{21}u_{12}+u_{22}&l_{21}u_{13}+u_{23}\\l_{31}u_{11}&l_{31}u_{12}+l_{32}u_{22}&l_{31}u_{13}+l_{32}u_{23}+u_{33}\end{bmatrix}=\begin{bmatrix}10&1&1\\2&10&1\\2&2&10\end{bmatrix}$$

Now from

(i) first row for U : $\boxed{u_{11}=10}$, $\boxed{u_{12}=1}$, $\boxed{u_{13}=1}$

(ii) first column for L : $l_{21}u_{11}=2 \Rightarrow l_{21}(10)=2 \Rightarrow \boxed{l_{21}=\frac{1}{5}}$

$l_{31}u_{11}=2 \Rightarrow l_{31}(10)=2 \Rightarrow \boxed{l_{31}=\frac{1}{5}}$

(iii) second row for U : $l_{21}u_{12}+u_{22}=10$

$\frac{1}{5}(1)+u_{22}=10 \Rightarrow \boxed{u_{22}=\frac{49}{5}}$

& $l_{21}u_{13}+u_{23}=1$

$\frac{1}{5}(1)+u_{23}=1 \Rightarrow \boxed{u_{23}=\frac{4}{5}}$

(iv) second column for L : $l_{31}u_{12}+l_{32}u_{22}=2$

$\frac{1}{5}(1)+l_{32}\cdot\left(\frac{49}{5}\right)=2 \Rightarrow \boxed{l_{32}=\frac{9}{49}}$

(v) third row for U : $l_{31}u_{13}+l_{32}u_{23}+u_{33}=10$

$\frac{1}{5}(1)+\frac{9}{49}\cdot\frac{4}{5}+u_{33}=10 \Rightarrow \boxed{u_{33}=\frac{473}{49}}$

$\therefore$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{5} & 1 & 0 \\ \frac{1}{5} & \frac{9}{49} & 1 \end{bmatrix} \quad \& \quad U = \begin{bmatrix} 10 & 1 & 1 \\ 0 & \frac{49}{5} & \frac{4}{5} \\ 0 & 0 & \frac{473}{49} \end{bmatrix}$$

Now substituting $A = LU$ in $AX = B$ we get

$$LUX = B$$

and substituting $UX = V$ we get

$$LV = B$$

$$\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{5} & 1 & 0 \\ \frac{1}{5} & \frac{9}{49} & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 13 \\ 14 \end{bmatrix}$$

$\Rightarrow \boxed{V_1 = 12}$, $\frac{1}{5}V_1 + V_2 = 13 \Rightarrow \boxed{V_2 = \frac{53}{5}}$, $\frac{1}{5}V_1 + \frac{9}{49}V_2 + V_3 = 14$

$\Rightarrow \boxed{V_3 = \frac{473}{49}}$

$\therefore$ $UX = V$ becomes

$$\begin{bmatrix} 10 & 1 & 1 \\ 0 & \frac{49}{5} & \frac{4}{5} \\ 0 & 0 & \frac{473}{49} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 53/5 \\ \frac{473}{49} \end{bmatrix}$$

$$\Rightarrow 10x + y + z = 12$$

$$\frac{49}{5}y + \frac{4}{5}z = \frac{53}{5}$$

$$\frac{473}{49}z = \frac{473}{49} \Rightarrow \boxed{z = 1}$$

By back substitution we get $\boxed{y = 1}$, $\boxed{x = 1}$

$\therefore$ $x = 1$, $y = 1$, $z = 1$ is the solution of given system of equations.

Tutorial Problems

Solve the following system of equations by LU decomposition method.

(1) $x + 2y + 3z = 14$,
$2x + 3y + 4z = 20$ Ans: $x=1$, $y=2$, $z=3$
$3x + 4y + z = 14$.

(2) $3x + 2y - 3z = 6$
$2x + 2y + 5z = -3$ Ans: $x=1$, $y=0$, $z=-1$
$x + y - z = 2$.

Iterative method :

The repeated execution of the same process, where at each step the result of the preceding step is used. This is known as iterative process and this process is repeated till the result is obtained to a desired degree of accuracy.

We have the two iterative methods

1. Jacobi's iterative method
2. Gauss- Seidal iterative method.

These two methods are applicable to system of equations in which the numerically large coefficients are along the principal diagonal of the coefficient matrix, associated with the system of equations.

Gauss seidal method is an improved form of Jacobi method, also known as the successive displacement method.

The difference between the Gauss- Seidal and Jacobi methods is that the Jacobi method uses the values obtained from the previous step while the Gauss- Seidal method always applies the latest updated values during the iterative procedures.

ie., Second unknown is determined from the first unknown in the current iteration, the third unknown is determined from the first and second unknowns, etc.,.

1.1.2: Gauss-seidel Iterative Method

The modification of Jacobi's method is the Gauss-seidel iterative method.
In this method the convergence is twice as fast as in Jacobi's method.

Consider the system of equations

$$a_1 x + b_1 y + c_1 z = d_1$$

$$a_2 x + b_2 y + c_2 z = d_2$$

$$a_3 x + b_3 y + c_3 z = d_3$$

The system of equations can be written as

$$x = \frac{1}{a_1}[d_1 - b_1 y - c_1 z]$$

$$y = \frac{1}{b_2}[d_2 - a_2 x - c_2 z]$$

$$z = \frac{1}{c_3}[d_3 - a_3 x - b_3 y]$$

Start with the initial approximation x_0, y_0, z_0

1st iteration : $x_1 = \frac{1}{a_1}[d_1 - b_1 y_0 - c_1 z_0]$

$$y_1 = \frac{1}{b_2}[d_2 - a_2 x_1 - c_2 z_0]$$

$$z_1 = \frac{1}{c_3}[d_3 - a_3 x_1 - b_3 y_1]$$

i.e as soon as a new approximation for an unknown is found, it is immediately used in next step

2nd iteration : $x_2 = \frac{1}{a_1}[d_1 - b_1 y_1 - c_1 z_1]$

$$y_2 = \frac{1}{b_2}[d_2 - a_2 x_2 - c_2 z_1]$$

$$z_2 = \frac{1}{c_3}[d_3 - a_3 x_2 - b_3 y_2]$$

Repeat the process till two consecutive values of x, y, z are same.

Solved Problems

1) Apply Gauss - Seidel iteration method to solve the equations $20x+y-2z=17$, $3x+20y-z=-18$, $2x-3y+20z=25$

Sol^n: Given equations are diagonally dominant

They can be written in the form

$$x = \frac{1}{20}[17 - y + 2z]$$

$$y = \frac{1}{20}[-18 - 3x + z]$$

$$z = \frac{1}{20}[25 - 2x + 3y]$$

Start with the initial approximation $x_0 = y_0 = z_0 = 0$

1st iteration:

$$x_1 = \frac{1}{20}[17 - y_0 + 2z_0] = \frac{1}{20}[17 - 0 + 2(0)] = \frac{17}{20} = 0.8500$$

$$y_1 = \frac{1}{20}[-18 - 3x_1 + z_0] = \frac{1}{20}[-18 - 3(0.8500) + 0] = -1.0275$$

$$z_1 = \frac{1}{20}[25 - 2x_1 + 3y_1] = \frac{1}{20}[25 - 2(0.8500) + 3(-1.0275)]$$
$$= 1.0109$$

2nd iteration:

$$x_2 = \frac{1}{20}[17 - y_1 + 2z_1] = \frac{1}{20}[17 - (-1.0275) + 2(1.0109)] = 1.0025$$

$$y_2 = \frac{1}{20}[-18 - 3x_2 + z_1] = \frac{1}{20}[-18 - 3(1.0025) + 1.0109] = -0.9998$$

$$z_2 = \frac{1}{20}[25 - 2x_2 + 3y_2] = \frac{1}{20}[25 - 2(1.0025) + 3(-0.9998)] = 0.9998$$

3rd iteration:

$$x_3 = \frac{1}{20}[17 - y_2 + 2z_2] = \frac{1}{20}[17 - (-0.9998) + 2(0.9998)] = 1.0000$$

$$y_3 = \frac{1}{20}[-18 - 3x_3 + z_2] = \frac{1}{20}[-18 - 3(1.0000) + 0.9998] = -1.0000$$

$$z_3 = \frac{1}{20}[25 - 2x_3 + 3y_3] = \frac{1}{20}[25 - 2(1.0000) + 3(-1.0000)] = 1.0000$$

4th iteration :

$x_4 = \frac{1}{20}[17 - y_3 + 2z_3] = \frac{1}{20}[17-(-1.0000)+2(1.0000)] = 1.0000$

$y_4 = \frac{1}{20}[-18-3x_4+z_3] = \frac{1}{20}[-18-3(1.0000)+1.0000] = -1.0000$

$z_4 = \frac{1}{20}[25-2x_4+3y_4] = \frac{1}{20}[25-2(1.0000)+3(-1.0000)] = 1.0000$

3rd and 4th iteration values are same

$\therefore x=1, y=-1, z=1$ is the required solution

2) solve $27x+6y-z=85$, $x+y+54z=110$, $6x+15y+2z=72$ using Gauss seidel iteration method correct to three decimal places

Soln: Given equations are not diagonally dominant

$\therefore$ rearranging we get

$27x+6y-z=85$

$6x+15y+2z=72$

$x+y+54z=110$

Now writing these equations in the form

$x = \frac{1}{27}[85-6y+z]$

$y = \frac{1}{15}[72-6x-2z]$

$z = \frac{1}{54}[110-x-y]$

start with initial approximation $x_0=y_0=z_0=0$

1st iteration:

$x_1 = \frac{1}{27}[85-6y_0+z_0] = \frac{1}{27}[85-6(0)+0] = 3.148$

$y_1 = \frac{1}{15}[72-6x_1-2z_0] = \frac{1}{15}[72-6(3.148)-2(0)]$

$= 3.541$

$$z_1 = \frac{1}{54}[110 - x_1 - y_1] = \frac{1}{54}[110 - 3.148 - 3.541] = 1.913$$

2nd iteration:

$$x_2 = \frac{1}{27}[85 - 6y_1 + z_1] = \frac{1}{27}[85 - 6(3.541) + 1.913] = 2.432$$

$$y_2 = \frac{1}{15}[72 - 6x_2 - 2z_1] = \frac{1}{15}[72 - 6(2.432) - 2(1.913)] = 3.572$$

$$z_2 = \frac{1}{54}[110 - x_2 - y_2] = \frac{1}{54}[110 - 2.432 - 3.572] = 1.926$$

3rd iteration:

$$x_3 = \frac{1}{27}[85 - 6y_2 + z_2] = \frac{1}{27}[85 - 6(3.572) + 1.926] = 2.426$$

$$y_3 = \frac{1}{15}[72 - 6x_3 - 2z_2] = \frac{1}{15}[72 - 6(2.426) - 2(1.926)] = 3.573$$

$$z_3 = \frac{1}{54}[110 - x_3 - y_3] = \frac{1}{54}[110 - 2.426 - 3.573] = 1.926$$

4th iteration:

$$x_4 = \frac{1}{27}[85 - 6y_3 + z_3] = \frac{1}{27}[85 - 6(3.573) + 1.926] = 2.425$$

$$y_4 = \frac{1}{15}[72 - 6x_4 - 2z_3] = \frac{1}{15}[72 - 6(2.425) - 2(1.926)] = 3.573$$

$$z_4 = \frac{1}{54}[110 - x_4 - y_4] = \frac{1}{54}[110 - 2.425 - 3.573] = 1.926$$

3rd and 4th iteration values are same

$\therefore$ $x = 2.425$, $y = 3.573$ & $z = 1.926$ is the solution

3) Solve the following equations using Gauss-Seidel (14) iteration method correct to three decimal places. Carry out 5 iterations,

Sum: $2x+y+6z=9$, $8x+3y+2z=13$, $x+5y+z=7$

Soln: Given equations are not diagonally dominant

$\therefore$ rearranging the equations

$$8x+3y+2z=13$$
$$x+5y+z=7$$
$$2x+y+6z=9$$

Now writing the equations in the form

$$x=\frac{1}{8}[13-3y-2z]$$
$$y=\frac{1}{5}[7-x-z]$$
$$z=\frac{1}{6}[9-2x-y]$$

Start with initial approximation $x_0=y_0=z_0=0$

1st iteration:

$$x_1=\frac{1}{8}[13-3y_0-2z_0]=\frac{1}{8}[13-3(0)-2(0)]=\frac{13}{8}=1.625$$
$$y_1=\frac{1}{5}[7-x_1-z_0]=\frac{1}{5}[7-1.625-0]=1.075$$
$$z_1=\frac{1}{6}[9-2x_1-y_1]=\frac{1}{6}[9-2(1.625)-1.075]=0.779$$

2nd iteration

$$x_2=\frac{1}{8}[13-3y_1-2z_1]=\frac{1}{8}[13-3(1.075)-2(0.779)]=1.027$$
$$y_2=\frac{1}{5}[7-x_2-z_1]=\frac{1}{5}[7-1.027-0.779]=1.039$$
$$z_2=\frac{1}{6}[9-2x_2-y_2]=\frac{1}{6}[9-2(1.027)-1.039]=0.985$$

3^{rd} iteration

$$x_3 = \frac{1}{8}\left[13 - 3y_2 - 2z_2\right] = \frac{1}{8}\left[13 - 3(1.039) - 2(0.985)\right] = 0.989$$

$$y_3 = \frac{1}{5}\left[7 - x_3 - z_2\right] = \frac{1}{5}\left[7 - 0.989 - 0.985\right] = 1.005$$

$$z_3 = \frac{1}{6}\left[9 - 2x_3 - y_3\right] = \frac{1}{6}\left[9 - 2(0.989) - 1.005\right] = 1.003$$

4^{th} iteration

$$x_4 = \frac{1}{8}\left[13 - 3y_3 - 2z_3\right] = \frac{1}{8}\left[13 - 3(1.005) - 2(1.003)\right] = 0.997$$

$$y_4 = \frac{1}{5}\left[7 - x_4 - z_3\right] = \frac{1}{5}\left[7 - 1.003 - 1.003\right] = 1.000$$

$$z_4 = \frac{1}{6}\left[9 - 2x_4 - y_4\right] = \frac{1}{6}\left[9 - 2(1.003) - 0.999\right] = \cancel{0.999}\ 1.001$$

5^{th} iteration

$$x_5 = \frac{1}{8}\left[13 - 3y_4 - 2z_4\right] = 1.000$$

$$y_5 = \frac{1}{5}\left[7 - x_5 - z_4\right] = 1.000$$

$$z_5 = \frac{1}{6}\left[9 - 2x_5 - y_5\right] = 1.000$$

6^{th} iteration

$x_6 = 1.000$

$y_6 = 1.000$

$z_6 = 1.000$

From the 6^{th} iteration, the solution of given equations is $x = 1,\ y = 1,\ z = 1$

Tutorial Problems

Solve the following system of equations using Gauss Seidel iterative method to three decimal places.

1) $10x + y + z = 12,\ 2x + 10y + z = 13,\ 2x + 2y + 10z = 14.$

Ans: $x = 1,\ y = 1,\ z = 1$

2) $5x + 2y + z = 12,\ x + 4y + 2z = 15,\ x + 2y + 5z = 20,$ with initial approximation $(1, 0, 3)$.

Ans: $x = 1,\ y = 2,\ z = 3$

Solution of Algebraic and Transcendental Equations:-

For any non-linear single variable equation, we can find atleast one real root.

- Root finding
  - open methods
    - Iterative method
    - Newton-Raphson method
    - Secant method
  - Bracketing methods
    - Bisection method
    - Regula falsi method.

Rate of convergence is a measure of how fast the difference between the solution point and its estimates goes to zero.

① Rate of convergence of Bisection method is 0.5

② Rate of convergence of Regula falsi method is 1.618

③ Rate of convergence of Newton-Raphson method is 2.

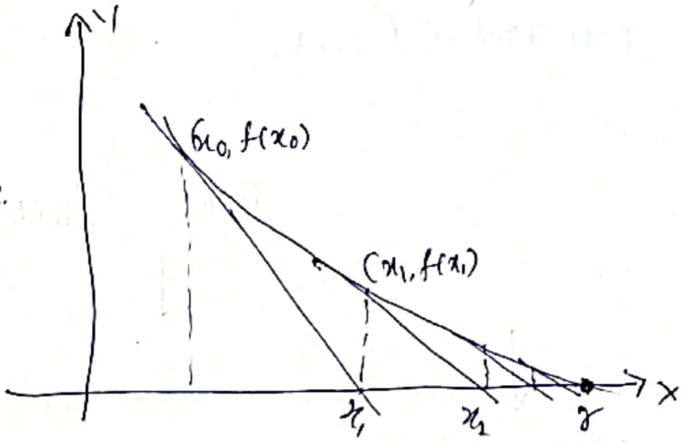
It has quadratic rate of convergence.

<u>Note</u>:- A better and close approximation of the root can be found by using Newton-Raphson method compared to Bisection & Regula falsi method.

Hence the errors in the values of the roots of a fun^n are greatly reduced by this method compared to other methods of finding the roots.

Newton Raphson Method

We assume that f is a differentiable function in some interval $[a,b]$ containing the root.



Let x_0 be an approximation of γ

Then $(x_0, f(x_0))$ is a point on the curve.

Draw the tangent line to the curve at the point $(x_0, f(x_0))$. This line intersects the x-axis at a new point, say x_1

Now x_1 is a better approximation to γ than x_0. We now repeat this process, yielding new pts $x_2, x_3 \dots$ until we are close to γ!

Newton - Raphson iterative formula is given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad \text{where } n = 0, 1, 2, \dots$$

procedure to obtain formula

The equation of the tangent at $(x_0, f(x_0))$ is given by

$$y - f(x_0) = f'(x_0)(x - x_0)$$

where $f'(x_0)$ is the slope of the curve at $(x_0, f(x_0))$

Set $y = 0$, we get the point of intersection of the tangent with x-axis as

$$-\frac{f(x_0)}{f'(x_0)} = x - x_0$$

or $$x = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Iterating this process, we get $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$.

Problems:

1. Use Newton-Raphson method to find a real root of the equation $x^3 - 2x - 5 = 0$ correct to 3 decimal places.

Solution: Let $f(x) = x^3 - 2x - 5$

$f(1) = -6 \quad f(2) = -1 \,(-ve) \quad f(3) = 16 \,(+ve)$.

Here $f(2)$ is negative and $f(3)$ is positive. Hence the root lies between 2 and 3. The root will be in the neighbourhood of 2 and let the approximate root $x_0 = 2$. [Since -1 is close to zero compared to 16]

The first approximation is given by $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

$$\text{i.e.,} \quad x_1 = 2 - \frac{f(2)}{f'(2)}$$

Here $f(x) = x^3 - 2x - 5 \Rightarrow f(2) = -1$

$f'(x) = 3x^2 - 2 \Rightarrow f'(2) = 10$

$$\therefore x_1 = 2 - \frac{(-1)}{10} = 2.1$$

The second approximation is given by $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$$\text{i.e.,} \quad x_2 = 2.1 - \frac{f(2.1)}{f'(2.1)}$$

Here $f(2.1) = (2.1)^3 - 2(2.1) - 5 = 0.061$

$f'(2.1) = 3(2.1)^2 - 2 = 11.23$

$$\therefore x_2 = 2.1 - \frac{(0.061)}{11.23} = 2.0946$$

The third approximation is given by $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

i.e., $x_3 = 2.0946 - \frac{f(2.0946)}{f'(2.0946)}$

Here $f(2.0946) = (2.0946)^3 - 2(2.0946) - 5 = 0.00054155$

$f'(2.0946) = 3(2.0946)^2 - 2 = 11.1620$.

$\therefore x_3 = 2.0946 - \frac{0.00054155}{11.1620} = 2.0946$.

Since x_2 and x_3 are same, stop the process.
Hence the required approximate root correct to 3 decimal places is $\underline{2.095}$

2. Find the positive root of $x^4 - x = 10$ correct to three decimal places, using Newton-Raphson method.

$\underline{\text{Sol}^n}$ Let $f(x) = x^4 - x - 10$.

$f(0) = -10$, $f(1) = -10$ (-ve) $f(2) = 4$ (+ve).

Here $f(1)$ is negative and $f(2)$ is positive. Hence the root lies between 1 and 2. The root will be in the neighbourhood of 2 and let the approximate root $x_0 = 2$. [Since 4 is close to zero compared to -10].

The first approximate is given by $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

i.e., $x_1 = 2 - \frac{f(2)}{f'(2)}$

Here $f(x) = x^4 - x - 10 \Rightarrow f(2) = 4$

$f'(x) = 4x^3 - 1 \Rightarrow f'(2) = 31$

$\therefore\ x_1 = 2 - \frac{4}{31} = 1.871$

The second approximation is given by $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

i.e., $x_2 = 1.871 - \frac{f(1.871)}{f'(1.871)}$

$f(1.871) = (1.871)^4 - (1.871) - 10 = 0.3835$

$f'(1.871) = 4(1.871)^3 - 1 = 25.199$

$\therefore\ x_2 = 1.871 - \frac{0.3835}{25.199} = 1.856.$

The third approximation is given by $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

i.e. $x_3 = 1.856 - \frac{f(1.856)}{f'(1.856)}$

$f(1.856) = (1.856)^4 - (1.856) - 10 = 0.010$

$f'(1.856) = 4(1.856)^3 - 1 = 24.574.$

$\therefore\ x_3 = 1.856 - \frac{0.010}{24.574} = 1.856.$

Since x_2 and x_3 are same, stop the process.

Hence the required root correct to 3 decimal places is

$= 1.856.$

3. Find the real root of the Equation, $3x = \cos x + 1$ using Newton-Raphson method correct to 4 decimal places.

Solution: Let $f(x) = 3x - \cos x - 1$ [Take x in radians]

$f(0) = -2$ (-ve) $f(1) = 1.46$ (+ve)

Here $f(0)$ is negative and $f(1)$ is positive. Hence the root lies between 0 and 1.

Let us take the first approximation x_0 of the root the average of 0 and 1, namely 0.5 i.e, $x_0 = 0.5$

The first approximation is given by $x_1 = x_0 - \frac{f(x_0)}{f'(x_1)}$

i.e., $x_1 = 0.5 - \frac{f(0.5)}{f'(0.5)}$

$f(x) = 3x - \cos x - 1 \Rightarrow f(0.5) = 3(0.5) - \cos(0.5) - 1$

$= -0.3775$

$f'(x) = 3 + \sin x \Rightarrow f'(0.5) = 3 + \sin(0.5) = 3.4794$

$\therefore x_1 = 0.5 - \frac{(-0.3775)}{3.4794} = 0.6085$

The second approximation is given by $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

i.e $x_2 = 0.6085 - \frac{f(0.6085)}{f'(0.6085)}$

$f(0.6085) = 3(0.6085) - \cos(0.6085) - 1 = 0.00493$

$f'(0.6085) = 3 + \sin(0.6085) = 3.5716$

$\therefore x_2 = 0.6085 - \frac{(0.00493)}{3.5716} = 0.6071$

The third approximation is given by $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

i.e., $x_3 = 0.6071 - \frac{f(0.6071)}{f'(0.6071)}$

$f(0.6071) = 3(0.6071) - \cos(0.6071) - 1 = -0.000005884$

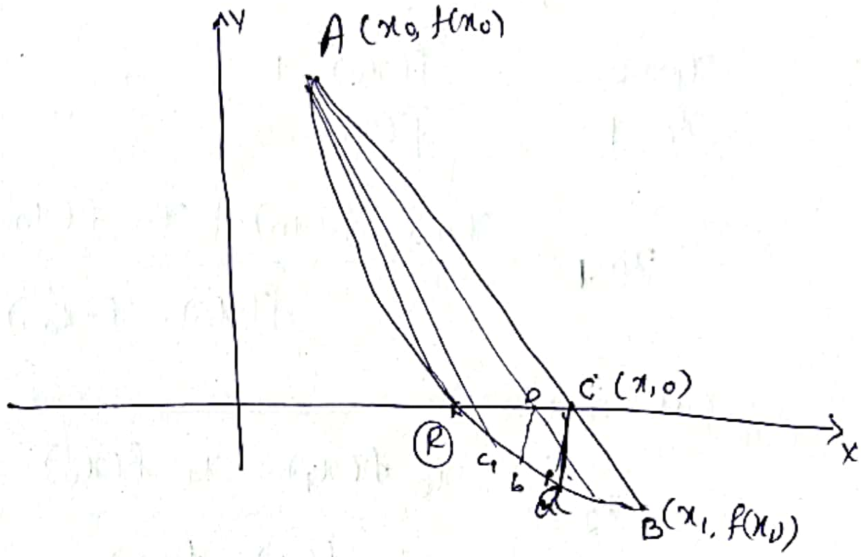
$f'(0.6071) = 3 + \sin(0.6071) = 3.5704$

$\therefore x_3 = 0.6071 - \frac{(-0.000005884)}{3.5704} = 0.6071$

Secant Method: (Chord method)

This method is quite similar to Regula-falsi method except for the condition $f(x_1) f(x_2) < 0$.

Note: Students can refere NPTEL links given in CDCH to learn Regula-falsi method.



Slope of AB = Slope of AC

$$\frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{0 - f(x_0)}{x - x_0}$$

Simplifying we get

$$x = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

In general, we can write

$$x_{n+2} = \frac{x_n f(x_{n+1}) - x_{n+1} f(x_n)}{f(x_{n+1}) - f(x_n)}. \quad \text{or} \quad x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

Note: This method fails when $f(x_{n+1}) = f(x_n)$.

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(1) Find a real root of the equation $x^3-5x+1=0$ lies in the interval $(0,1)$. Perform four iterations of the secant method.

Soln: Let $f(x)=x^3-5x+1$

$f(0)=1$

$f(1)=-3$

$x_0=0 \qquad f(x_0)=1$

$x_1=1 \qquad f(x_1)=-3$

$$x_{n+1}=\frac{x_{n-1}\,f(x_n)-x_n\,f(x_{n-1})}{f(x_n)-f(x_{n-1})}$$

Put $n=1$

$$x_2=\frac{x_0\,f(x_1)-x_1\,f(x_0)}{f(x_1)-f(x_0)}$$

$x_0 \qquad x_1$

$f(x_0) \qquad f(x_1)$

$$=\frac{0-1(1)}{-4}=0.25$$

Put $n=2$,

$$x_3=\frac{x_1\,f(x_2)-x_2\,f(x_1)}{f(x_2)-f(x_1)}$$

$f(x_2)=f(0.25)=(0.25)^2-5(0.25)+1$

$=0.234375$

$$=\frac{1(0.234325)-0.25(-3)}{+0.234375+3}$$

$=0.18644.$

$f(x_3)=f(0.18644)=0.07428$

Put $n=3$, we get

$x_4=0.20174$

$f(x_4)=-0.00048$

$\boxed{x_5=0.20081}$

$\therefore$ one of the real root of the equation is $\underline{0.20081}$

(2) Estimate the root of the equation $\cos x - xe^{x} = 0$ using the secant method with initial estimate of $x_0 = 0.5$, $x_1 = 1$.

Soln: $x_0 = 0.5$ $\quad x_1 = 1.$

$$f(x_0) = f(0.5) = \cos(0.5) - 0.5e^{0.5} = 0.0532$$

$$f(x_1) = f(1) = \cos 1 - 1 \cdot e^{1} = -2.17798$$

$$\begin{array}{cc} x_0 & x_1 \\ f(x_0) & f(x_1) \end{array}$$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{(0.5)(-2.17798) - 0.0532)}{-2.17798 - 0.0532} = 0.5119.$$

$$f(x_2) = \cos(0.5119) - (0.5119)e^{0.5119}$$

$$= 0.01773$$

$$\begin{array}{cc} x_1 & x_2 \\ f(x_1) & f(x_2) \end{array}$$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)} = 0.5119$$

$$f(x_3) = 0.01773$$

put $n=2$

$$\begin{array}{cc} x_2 & x_3 \\ f(x_2) & f(x_3) \end{array}$$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)} = 0.5158$$

$$f(x_4) = f(0.5158) = 0.00594$$

Put $n=3$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)} = 0.5178$$

put $n=4$

$$\underline{x_6 = 0.5178}$$

$\therefore$ Root of the given eqn is 0.5178 //