Satellite Monitoring System

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January 2024

All the project's code is avaiable on my github:
github.com/Brusa99/OrbitSimulation

1 Introduction

The goal of this project is to model a network of satellites orbiting an extraterrestrial planet.
The satellites are placed in orbit of a planet from which they gather information and have to transmit it back to the command center. Communication is impeded by other celestial bodies and consumes the satellite's battery. To counter communication blockage, the satellites are able to relay through a connected satellite, at the cost of faster battery depletion, for both the transmitter and the relay.
Due to the mathematical equations involved, the satellites orbits are not closed. If left alone the satellites will stray away from the planet or "fall" into it. [1]
For this reason the satellites are able to change their orbit, increasing *periapsis* (minimum orbit altitude) and decreasing *apoapsis* (maximum orbit altitude) by applying extra force.
In particular a system of three satellites (two with low orbits, one with high orbit) orbiting mars is used in the results provided in this report.

2 System

The system's state is described by the *position* $\mathbf{q}$ of all bodies in the planetary system, along with the satellites battery. The single planet/satellite position is not sufficient to describe its behaviour: for example communication can be

[1] This is accentuated in the simulation due to time discretization

obstructed (or aided) by a third body. For this reason the single satellite acts as an automaton which only observes *altitude, battery* and *connections*.
To better explain how a satellite acts it is convenient to separate the problem.

2.1 Orbit Regulation

Each satellite can find itself in 3 possible orbit states:

Regular Orbit: the satellite is considered in regular orbit when apoapsis and periapsis are within required boundaries.

Boosting Apoapsis: when the satellite finds that its apoapsis is above maximum altitude, and it's position coincides with its periapsis, it enters this state. The satellites stays in this state for a fixed time. While in this state a force of fixed module is applied in the same direction of the satellite's velocity.

Boosting Periapsis: when the satellite finds that its periapsis is below minimum altitude, and it's position coincides with its apoapsis, it enters this state. The satellites stays in this state for a fixed time. While in this state a force of fixed module is applied in the same direction of the satellite's velocity.

The *boost decision model* is summarized in the diagram 1.

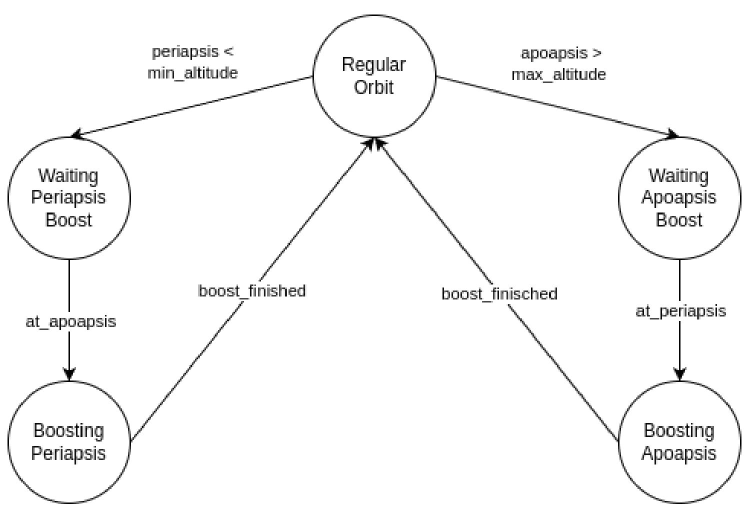

Figure 1: Satellite boost control

2.2 Communication Control

The communication ability of the satellite depends on the battery level. It has three possible states:

Full Operation: the satellite has plenty of battery and transmits, acts as relay and attempts connections to full extent.

Limited Operation: the satellite battery is below safety level. It will not attempt connecting though relay or act as one.

Depleted The satellite has no more battery and will not transmit.

The transition between these states is also described in figure 2.

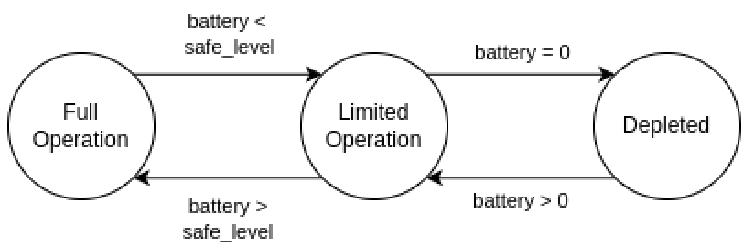

Figure 2: Battery state transition

3 Mathematical Model

In this sections the model equations are described.

3.1 Battery

The battery B of a satellite is described as a percentage. As such it is clipped between 0 and 100, assuming all possible real values in between.
The battery always depletes at a rate b_d, this is to simulate battery self discharge and the power consumption of the satellite's devices, for example spectrometers and thermal cameras.
Satellites have solar panels that charge the battery at rate b_c when the satellite is able to see the sun, which is represented by the boolean c_S.
To connect to the command center the satellite tries a direct connection. If such connection fails, the satellite will try to relay through other satellites. This process increases the integer n_a, which causes the battery to deplete faster.

The number of attempted connections n_a made by the satellite, make the battery deplete by a factor of $n_a \cdot b_a$, where b_a is a constant representing the attempt cost.
Finallly, if the satellite is connected to the command center, represented by the boolean c_m, it will further deplete its battery by a factor of $n_c \cdot b_t$ where b_t is a constant representing transmission cost and n_c is the number of satellites for which this particular satellite is acting as a relay for (including itself).
The final equation is

$$\frac{dB}{dt} = c_s b_c - b_d - n_a b_a - c_m n_c b_t$$

3.2 Position

The coordinate system we use is centered in the starting position of the sun. [2]
Every object in the planetary system is affected by each other following *Newton's law of universal gravitation*:

$$\mathbf{F_{ij}} = \frac{G m_i m_j}{\|\mathbf{q_j} - \mathbf{q_i}\|^2} \cdot \frac{(\mathbf{q_j} - \mathbf{q_i})}{\|\mathbf{q_j} - \mathbf{q_i}\|}$$

where $\mathbf{F_{ij}}$ is the gravitational force acting between two bodies, m_i and m_j are the masses of the bodies, $\mathbf{q_i}, \mathbf{q_j}$ are the positions of their center of masses and G is the gravitational constant.
Summing over all bodies in the planetary system we obtain the *n-bodies equations of motion*: [3]

$$m_i \frac{d^2 \mathbf{q_i}}{dt^2} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{G m_i m_j \, (\mathbf{q_j} - \mathbf{q_i})}{\|\mathbf{q_j} - \mathbf{q_i}\|^3}$$

The resulting dynamical system is chaotic for most initial conditions. [3]
Satellites in particular can increase their velocity by *boosting*. The force applied by the booster is given by $\mathbf{T_i} = T_i \cdot \frac{\mathbf{p_i}}{\|\mathbf{p_i}\|}$ multiplied by $t_d \in \{-1, 0, 1\}$ depending if the boost is lowering apoapsis, not present or raising periapsis, respectively.

4 Requirements

The following natural language requirements are made of the system:

1. A satellite should never completely deplete its battery.

[2] Note that the sun is not in a fixed position
[3] We are not accounting for *general relativity*

2. The battery of a satellite should frequently be above safety levels.
3. A satellite shouldn't stay outside required altitude boundaries for too long.
4. A satellite can't stay disconnected for more than half an hour at the time.

We translated the requirements into the following *LTL formulas*:

1. $\mathbf{G}\,(B > 0)$
2. $\mathbf{F}_{[0,5]}\,(B > B_{sl})$
3. $\mathbf{F}_{[0,5]}\,(a \in [a_{\min}, a_{\max}])$
4. $\neg\mathbf{G}_{[0,30]}(n_c < 1)$

Where B is the satellite's battery and B_{sl} is the safety threshold. a is the altitude, and $a_{\min}, a_{\max}$ are respectively the minimum and maximum required altitude levels. n_c is the number of connections, this includes the satellite itself. The time unit is a minute.

5 Experimental Results

We considered a particular case with 3 satellites: Odissey, Mars Reconnaissance Orbiter and a fictional relay, all orbiting mars. In the reported figures the simulation ran for 10.000 one-minute steps, mainly for plot clarity purpose, but it is not unreasonable to assume the results can be generalized to longer simulation. *Moonlight* [4] was used as a monitoring software.

As we can see in figure 3, the battery is never depleted. This is a *hard* requirement as the satellite's devices wouldn't be able to work without electric power.

The two low orbiting satellites sometimes fail the *soft* safety level requirement. This suggests that the satellite require more powerful solar panels in order to charge the battery faster.

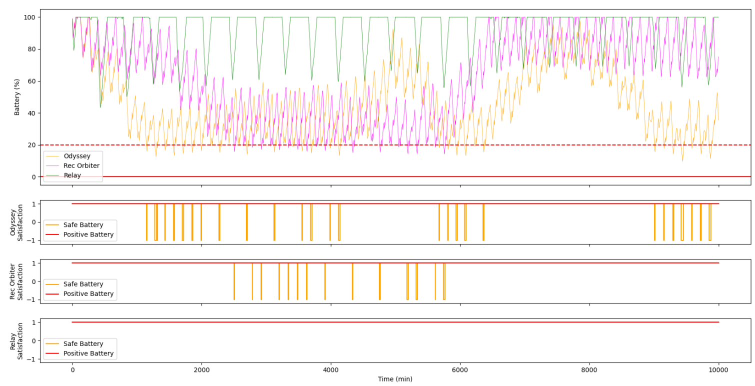

Figure 3: Battery Requirements Satisfaction

In figure 4 we can see that for the low orbit satellites the orbit is unstable. The frequency of the violation can be used to determine the boost strength and duration. Note that the requirement is a soft one, as its violation does not endanger the satellite mission. A more realistic *hard* requirement should have a substantially longer time limit, where the violation would imply that the satellite has left the planet's orbit. [4]

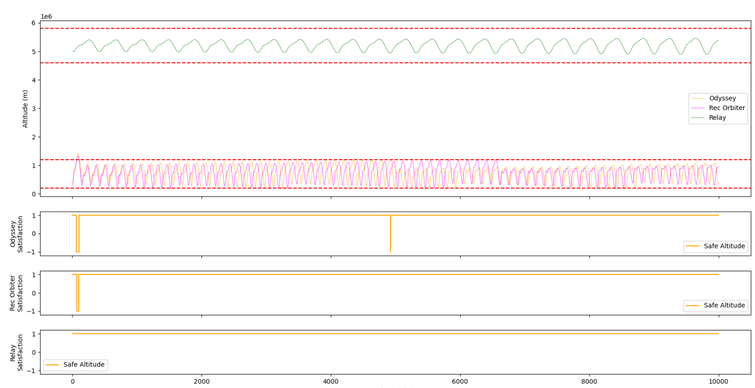

Figure 4: Altitude Requirements Satisfaction

[4] We included the soft one for exposing purposes

Figure 5 shows that the satellites are having troubles remaining connected. This can be partially taken care of along with the battery problem by upgrading the solar panels. An alternative, more expensive, solution could be to add another relay satellite.

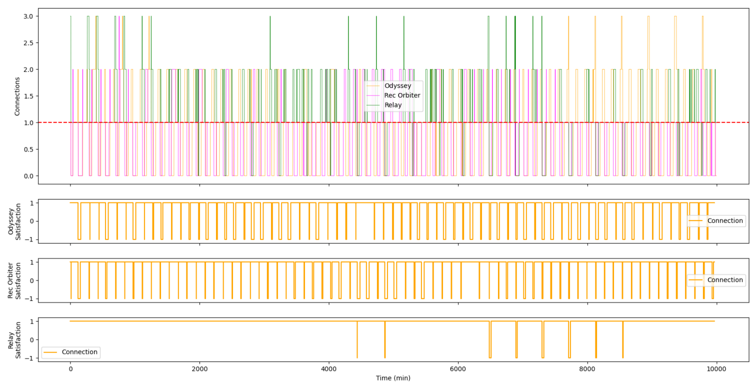

Figure 5: Connection Requirements Satisfaction

Appendix

A Possible Improvements

In this section, possible improvements for the model are discussed.

A.1 Data Transmission

In the model, data is transmitted instantly. In real life condition, data travels at the speed of light c. This can introduce a dynamic where the satellite - command center line of view has to stay continously unobstracted during the duration of the transmission.
Data could be introduced. A distinction could be made by data gathering, low orbit, satellites and data transmitting, high orbit, satellites. *Storage* can be added. So satellites would transmit when possible and store non-transmitted data. A new requirement could be made: storage should never be full.

A.2 Path Prediction

Currently, orbit is adjusted only when it is detected that periapsis or apoapsis are outside of safety bounds. Mathematical or machine learning methods could be used to predict when the satellites apsis will violate the condition, in order to boost before such violation occurs
Due to the nature of the equations involved in the prediction, such methods are probably too expensive to implement and not worth the cost. Especially noticing that the violation does not incur in safety risks for the satellite and restriction of the boundaries a_{min} and a_{max} would have the same effect.

A.3 Noisy Sensors

Currently, the satellites observe the real altitude from the planet. Noise can be introduced to simulate the imperfect sensors that a satellite would actually use.

A.4 Alternative Control

To keep the orbit stable, the satellite boost for constant time at constant force. Control modules could be used to determine both, but they would require a target signal.

The most logical target signal would be the satellite's total energy. But in an n-body system, the satellite's energy is not constant and depends on many variables outside the satellite and the planet.

B Variables Values

In the table 1 the values utilized for the variables and constants are described.

Bibliography

[1] Rajeev Alur. *Principles of Cyber-Physical Systems*. The MIT Press, 2023.

[2] National Aeronautics and Space Administration. *NASA*. For mars' satellites. No date. URL: https://www.nasa.gov/.

[3] S. J. Peale. *Celestial Mechanics*. Encyclopedia Britannica. 2023. URL: https://www.britannica.com/science/celestial-mechanics-physics.

[4] Ennio Visconti. *Moonlight*. 2023. URL: https://github.com/MoonLightSuite/moonlight.

Variable	Description	Value
$\mathbf{q}_i$	position of body i center of mass	$\mathbb{R}^2$
$\mathbf{p}_i$	velocity of body i	$\mathbb{R}^2$
m_i	mass of body i	$\mathbb{R}$
$\mathbf{F}_{ij}$	gravitational force applied by body j on body i	$\mathbb{R}^2$
$\mathbf{T}_i$	force applied by a satellite booster	$\mathbb{R}^2$
t_d	thrust direction	{ -1, 0, 1 }
a	altitude	$\mathbb{R}^+$
B_i	battery of satellite i	[0, 100]
B_{sl}	safe battery level	20
b_d	battery depletion rate	$\mathbb{R}$ (used 0.001)
b_c	battery solar charge rate	$\mathbb{R}$ (used 0.02)
c_s	connected to sun	{0,1}
n_a	number of attempted connections	$\mathbb{N}$
b_a	connection attempt cost	$\mathbb{R}$ (used 0.005)
c_m	connected to command center	{ 0, 1 }
n_c	number of connections	$\mathbb{N}$
b_t	transmission cost	$\mathbb{R}$ (used 0.005)

Table 1: Summary of constant and variables