浙江大學

本科实验报告

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Homework 1

- What's the convolution of two 1-D impulses
  - – $\delta(t)$ and $\delta(t - t_0)$
  - – $u(t)$ and $u(t)$ ($u(t)$ is the step function)

(1) 对单位冲激信号 $\delta(t)$ 与 $\delta(t-t_0)$

$\delta(t) * \delta(t-t_0) = \int_{-\infty}^{\infty} \delta(\tau) \cdot \delta(t-t_0-\tau) d\tau$

由冲激函数的抽样性质，$\int_{-\infty}^{\infty} \delta(\tau-a) f(\tau) d\tau = f(a)$

原式 $= \delta(t-t_0)$

(2) $u(t) * u(t) = \int_{-\infty}^{\infty} u(\tau)\, u(t-\tau)\, d\tau$

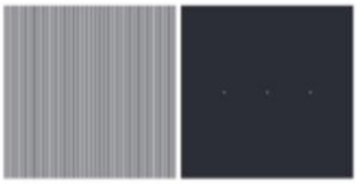
$= \int_0^t 1 \cdot 1\, d\tau$

$= t \cdot u(t)$

Homework 2

- The image on the left in the figure below consists of alternating stripes of black/white, each stripe being two pixels wide. The image on the right is the Fourier spectrum of the image on the left, showing the dc term and the frequency terms corresponding to the stripes. (Remember, the spectrum is symmetric so all components, other than the dc term, appear in two symmetric locations.)

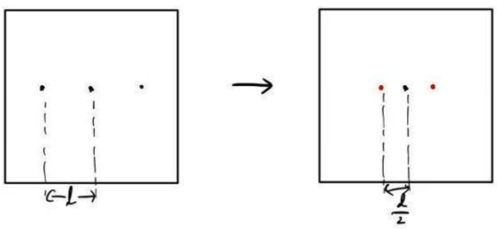
- (a) Why are the components of the spectrum limited to the horizontal axis?
- (b) Suppose that the stripes of an image of the same size are four pixels wide. Sketch what the spectrum of the image would look like.
- (c) What would the spectrum look like for an image of the same size but having stripes that are one pixel wide? Explain the reason for your answer.
- (d) Are the dc terms in (b) and (c) the same, or are they different? Explain.



(a) 因为左图的黑白条纹只在水平方向有像素值变化，垂直方向无频率变化
所以在频谱中只有 horizontal axis 才有成分

(b) 2 pixels → 4 pixels

条纹↑空间频率↓



(c) 2 pixels → 1 pixels

空间频率↑ 故横轴上的频率分量离中心直流项更远，依然保持对称

(d) (b)与(c)的直流项相同
因为两种情况下，两图的平均灰度值都相同

Homework 3

- Consider the images shown.
  - The image on the right was obtained by:
    - (a) multiplying the image on the left by $(-1)^{x+y}$
    - (b) computing the DFT
    - (c) inverse the phase angle
    - (d) computing the inverse DFT
    - (e) multiplying the real part of the result by $(-1)^{x+y}$

  - Explain (mathematically) why the image on the right appears as it does.

设原始图像为 $f(x,y)$，则DFT后 $F(u,v) = \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{N}+\frac{uy}{M}\right)}$

$$f_a(x,y) = f(x,y)(-1)^{x+y} = f(x,y)\, e^{j\pi(x+y)}$$

$$F_b(u,v) = \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} f(x,y)(-1)^{x+y} \cdot e^{-j2\pi\left(\frac{ux}{N}+\frac{uy}{M}\right)} = \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} f(x,y)\, e^{-j2\pi\left(\frac{u-\frac{N}{2}x}{N}+\frac{v-\frac{M}{2}y}{M}\right)}$$

此时 $F_b(u,v)$ 相较 $F(u,v)$ 频谱中心化，低频分量移动到中心

∵ 反转相位角 ⟺ 取共轭

$$\therefore F_c(u,v) = F_b^*(u,v) = \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} \overline{f(x,y)}\, e^{j2\pi\left(\frac{(u-\frac{N}{2})x}{N}+\frac{(v-\frac{M}{2})y}{M}\right)}$$

$$f_d(x,y) = F^{-1}\{F_c(u,v)\} = \frac{1}{MN}\sum_{u=0}^{N-1}\sum_{v=0}^{M-1}\left[\sum_{x=0}^{N-1}\sum_{y=0}^{M-1} \overline{f(x,y)}\, e^{j2\pi\left(\frac{(u-\frac{N}{2})x}{N}+\frac{(v-\frac{M}{2})y}{M}\right)}\right] e^{j2\pi\left(\frac{uX}{N}+\frac{vY}{M}\right)}$$

$$= \frac{1}{MN}\sum_{x=0}^{N-1}\sum_{y=0}^{M-1} \overline{f(x,y)}\, e^{-j\pi(x+y)} \sum_{u=0}^{N-1}\sum_{v=0}^{M-1} e^{j2\pi\left(\frac{u(x+X)}{N}+\frac{v(y+Y)}{M}\right)} \quad \text{(离散整数正交和)}$$

$$= \frac{1}{MN}\sum_{x=0}^{N-1}\sum_{y=0}^{M-1} \overline{f(x,y)}\, e^{-j\pi(x+y)} \cdot MN \cdot \delta_{X,N-x}\,\delta_{Y,M-y} \quad \text{(此时 } X=(N-x) \bmod N,\ Y=(M-y) \bmod M\text{)}$$

综和只在 $\begin{cases} x=N-X \\ y=M-y \end{cases}$ 时筛选为1

$$f_d(x,y) = f(N-X, M-Y)\, e^{-j\pi[(N-X)+(M-Y)]}$$

$$f_e(x,y) = f_d(x,y)\cdot(-1)^{x+y} = f(N-X, M-Y)\, e^{j2\pi(X+Y)} \cdot e^{-j\pi(M+N)} = f(N-X, M-Y)$$

故得到了水平垂直翻转后的图像