浙江大学

本科实验报告

Physical Sensors

课程名称：	生物医学传感与检测
姓名：	
学院：	生物医学工程与仪器科学学院
专业：	生物医学工程
学号：	
指导老师：	唐志峰

2025 年 5 月 25 日

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实验名称：	Physical Sensors	实验类型：		同组学生姓名：	无

一、 Homework 3

Question 1

The Figure 1 shows a direct strain bridge, where $E = 4\,\text{V}$, $R_1 = R_2 = R_3 = R_4 = 120\,\Omega$. Please answer the following questions:

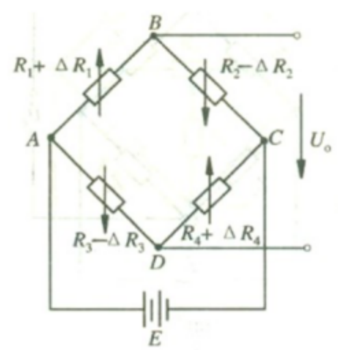


Figure 1 A direct strain bridge

图 1: A direct strain bridge (Question 1)

(1) If R_1 is metal strain gage, the others are external resistors, when increment of R_1 is $\Delta R_1 = 1.2\,\Omega$, what's the output voltage U_0?

The output voltage U_0 is calculated as:

$$U_0 = E\left(\frac{R_1 + \Delta R_1}{R_1 + \Delta R_1 + R_2} - \frac{R_3}{R_3 + R_4}\right) \tag{1}$$

Given $E = 4\,\text{V}$, $R_1 = R_2 = R_3 = R_4 = 120\,\Omega$, $\Delta R_1 = 1.2\,\Omega$:

$$\begin{aligned}
U_0 &= 4\,\text{V}\left(\frac{120 + 1.2}{120 + 1.2 + 120} - \frac{120}{120 + 120}\right) && (2)\\
&= 4\,\text{V}\left(\frac{121.2}{241.2} - \frac{1}{2}\right) && (3)\\
&= 4\,\text{V}(0.50248 - 0.5) = 4\,\text{V} \times 0.00248 = 9.97\,\text{mV} && (4)
\end{aligned}$$

(2) R_1, R_2 are strain gages with the same batch number, which can sense strain with the same direction and size, while the others are external resistors. What's the output voltage U_0?

When R_1 and R_2 change in the same direction and by the same size, the bridge remains balanced. Therefore, the output voltage $U_0 = 0\,\mathrm{V}$.

(3) In question (2), if R_1, R_2 sense strain in opposite direction, and $|\Delta R_1| = |\Delta R_2| = 1.2\,\Omega$, what's the output voltage U_0?

For a differential bridge configuration where R_1 and R_2 sense strain in opposite directions, the output voltage is:

$$U_0 = 4\,\mathrm{V} \times \left(\frac{120 + 1.2}{120 + 120} - 0.5\right) = 4\,\mathrm{V} \times (0.505 - 0.5) = 20\,\mathrm{mV} \tag{5}$$

Question 2

The Figure 2 shows a force-measuring system for equal strength beam. R_1 is resistance strain gage with sensitivity coefficient $K = 2.05$. $R_1 = 120\,\Omega$ when there's no strain. Average strain of strain gage is $\epsilon = 800\,\mu\mathrm{m/m}$ when tested piece is subjected to force F. Then:

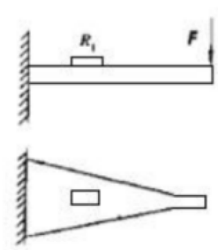


Figure 2 A force-measuring sensor

图 2: A force-measuring sensor (Question 2)

(1) What's the variation ΔR_1 of resistance of strain gage? What's the relative variation of resistance $\Delta R_1/R_1$?

The sensitivity coefficient K is defined as $K = \frac{\Delta R_1/R_1}{\epsilon}$.

The relative variation of resistance is:

$$\frac{\Delta R_1}{R_1} = K\epsilon = 2.05 \times 800 \times 10^{-6} = 1.64 \times 10^{-3} \tag{6}$$

The variation of resistance ΔR_1 is:

$$\Delta R_1 = \left(\frac{\Delta R_1}{R_1}\right) \times R_1 = 1.64 \times 10^{-3} \times 120\,\Omega = 0.1968\,\Omega \tag{7}$$

(2) If resistance strain gage R_1 is added to single arm bridge, and the power voltage is DC 3V, what's the output voltage and nonlinear error of the bridge?

For a single arm bridge, the output voltage U_o is given by:

$$U_o = E\left(\frac{R_1+\Delta R_1}{R_1+\Delta R_1+R_2} - \frac{R_3}{R_3+R_4}\right) \tag{8}$$

Assuming a balanced bridge with $R_1 = R_2 = R_3 = R_4 = 120\,\Omega$ when there is no strain. Given $E = 3\,\text{V}$ and $\Delta R_1 = 0.1968\,\Omega$:

$$U_o = 3\,\text{V}\left(\frac{120+0.1968}{120+0.1968+120} - \frac{120}{120+120}\right) \tag{9}$$

$$= 3\,\text{V}\left(\frac{120.1968}{240.1968} - \frac{1}{2}\right) \tag{10}$$

$$= 3\,\text{V}(0.50041 - 0.5) = 1.23\,\text{mV} \tag{11}$$

The nonlinear error γ is approximately:

$$\gamma \approx \frac{\Delta R_1/R_1}{2} = \frac{1.64\times 10^{-3}}{2} = 0.08\% \tag{12}$$

(3) What's the step to reduce nonlinear error? Analyze the output voltage and nonlinear error of electric bridge.

- **Use a double-arm bridge (differential):** The output voltage is $U_o = E\frac{\Delta R_1}{2R_1}$. Nonlinear error is reduced because the opposite changes in adjacent arms partially cancel the nonlinearity.
- **Use a full-bridge circuit:** The output voltage is $U_o = E\frac{\Delta R_1}{R_1}$. Nonlinear error does not exist in an ideal full bridge circuit because it uses opposite strain changes in adjacent arms and same changes in relative arms, completely eliminating the nonlinearity.

Question 3

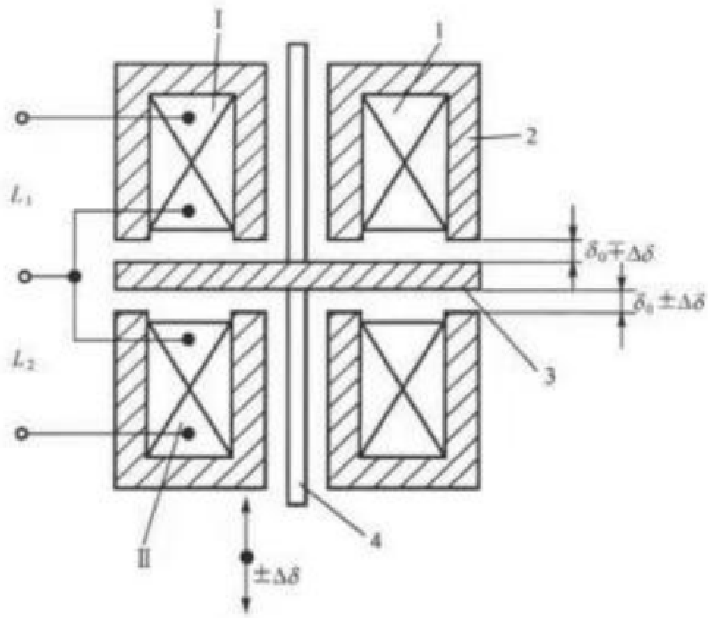
What factors that output characteristics of variable gap inductance transducer (see Figure 3) are related to? How to improve its nonlinearity? How to improve its sensitivity?

(1) The inductance L of a variable gap inductance transducer is given by the formula $L = \frac{W^2\mu S}{2\delta}$. The output characteristics are related to the following factors:

- Air gap thickness: δ
- Magnetic circuit cross-sectional area: S
- Number of coil turns: W
- Magnetic permeability of materials: μ

(2) Improving nonlinearity:

Adopt a differential structure. In a differential inductance transducer, two inductance units work oppositely. When the air gap changes, the inductance changes of the two units counteract part of the nonlinearity.



1. 线圈；2. 铁芯；3. 衔铁；4. 导杆

图 4.2.3 差动变隙式电感传感器

图 3: Variable gap inductance transducer (Question 3)

(3) Improving sensitivity:

Adopt a differential structure. Optimize structural parameters by:

- Reducing δ (within a reasonable range)
- Increasing W
- Increasing S

Question 4

Please briefly describe the work principle of phase-sensitive detection circuit (see Figure 4). What's the condition of guaranteeing its working reliability?

Work Principle of Phase-Sensitive Detection Circuit

The work principle involves an input signal $V_S(t) = V_m \cos(\omega t + \varphi)$ and a reference signal $V_r(t) = V_{ref} \cos(\omega t)$. The output voltage V_{out} is the product of these two signals:

$$V_{out} = V_S(t) \cdot V_r(t) = V_m \cos(\omega t + \varphi) \cdot V_{ref} \cos(\omega t) = \frac{V_m V_{ref}}{2}[\cos(2\omega t + \varphi) + \cos\varphi] \tag{13}$$

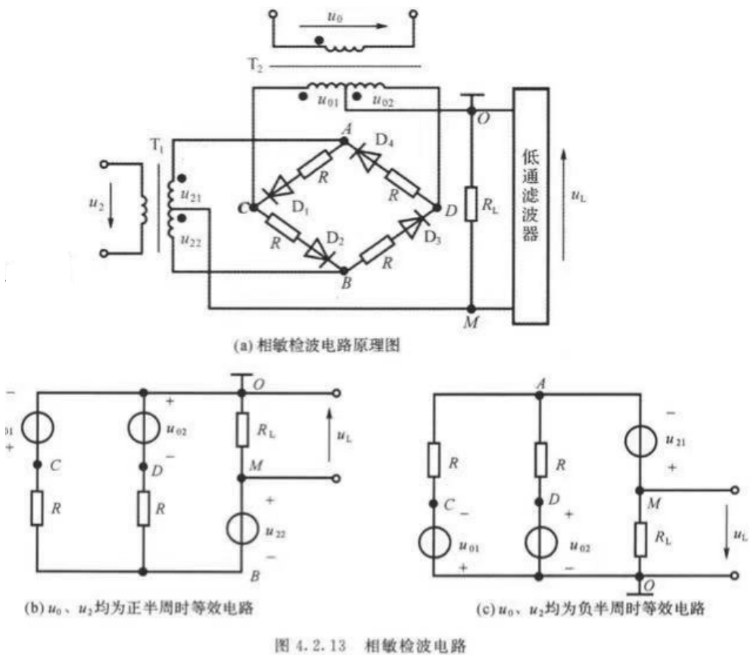


图 4: Phase-sensitive detection circuit (Question 4)

This output is then passed through a low-pass filter, which eliminates the high-frequency component ($2\omega t$), yielding a DC output voltage V_{DC}:

$$V_{DC} = \frac{V_m V_{ref}}{2} \cos\varphi \tag{14}$$

By detecting the phase of V_{DC}, accurate measurement of the signal polarity can be achieved.

Conditions for Guaranteeing Working Reliability

(1) The reference signal must have the same frequency as the input signal and a stable phase relationship.

(2) The amplitude of the input signal should be large enough to ensure that it can be accurately detected in the multiplication operation and subsequent processing.

(3) The cut-off frequency of the low-pass filter should be reasonably selected to effectively filter out the high-frequency components without overly attenuating the useful DC signal.

Question 5

See Fig.3.3.1 in textbook, it is known that the sectional area of variable gap inductance transducer is $S = 1.5\,\text{cm}^2$, the length of magnetic path is $L = 20\,\text{cm}$, the relative magnetic permeability is $\mu_r = 5000$, the air gap is $\delta_0 = 0.5\,\text{cm}$, $\Delta\delta = \pm 0.1\,\text{mm}$, the permeability of vacuum is $\mu_0 = 4\pi \times 10^{-7}\,\text{H/m}$, the coil turn is $w = 3000$. Then what's the sensitivity $\Delta L/\Delta\delta$ of single end transducer? If made into differential form, how the sensitivity of the transducer will change?

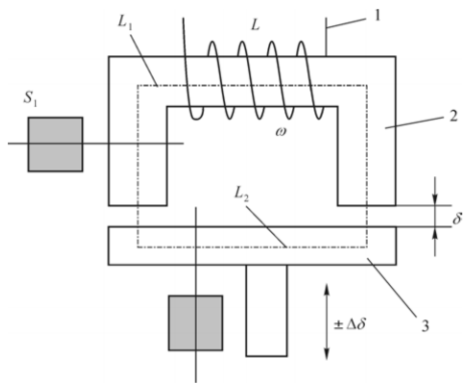


Fig. 3.3.1. Variable reluctance transducer. 1 is coil; 2 is core (fixed core); 3 is armature (movable core)

图 5: Variable gap inductance transducer configuration (Question 5)

Solution

First, calculate the initial inductance L_0:

$$L_0 = \frac{W^2 \mu_0 S}{2\delta_0} \tag{15}$$

Given $W = 3000$, $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H/m}$, $S = 1.5\,\mathrm{cm}^2 = 1.5 \times 10^{-4}\,\mathrm{m}^2$, $\delta_0 = 0.5\,\mathrm{cm} = 0.005\,\mathrm{m}$:

$$L_0 = \frac{3000^2 \times 4\pi \times 10^{-7} \times 1.5 \times 10^{-4}}{2 \times 0.005} = 0.1696\,\mathrm{H} \tag{16}$$

The sensitivity $S_{single} = \left|\frac{\Delta L}{\Delta \delta}\right|$ can be approximated by the derivative $\left|\frac{dL}{d\delta}\right|$.
Since $L = \frac{W^2\mu_0 S}{2\delta}$, then $\frac{dL}{d\delta} = -\frac{W^2\mu_0 S}{2\delta^2}$.
So, the sensitivity of the single end transducer is:

$$S_{single} = \left|-\frac{W^2\mu_0 S}{2\delta_0^2}\right| = \frac{3000^2 \times 4\pi \times 10^{-7} \times 1.5 \times 10^{-4}}{2 \times (0.005)^2} = 33.93\,\mathrm{H/m} \tag{17}$$

If made into differential form, the sensitivity doubles because the changes of two coils add up ($\Delta\delta_1 = +\Delta\delta$, $\Delta\delta_2 = -\Delta\delta$):

$$S_{differential} = 2 \times S_{single} = 2 \times 33.93 = 67.86\,\mathrm{H/m} \tag{18}$$

Question 6

What's eddy current effect? How to use it to measure displacement?

Eddy Current Effect

Definition and Physical Principle Eddy currents are loops of electrical current induced within conductors by a changing magnetic field in the conductor according to Faraday's law of induction. When a metallic conductor is placed in a time-varying magnetic field, or when there is relative motion between a conductor and a magnetic field, circular currents (eddy currents) are generated within the conductor material.

Mathematical Description The fundamental governing equations for eddy current phenomena are:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{(Faraday's law)} \tag{19}$$

$$\mathbf{J} = \sigma \mathbf{E} \quad \text{(Ohm's law)} \tag{20}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad \text{(Ampère's law)} \tag{21}$$

$$\mathbf{B} = \mu \mathbf{H} \quad \text{(Constitutive relation)} \tag{22}$$

where $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic flux density, $\mathbf{J}$ is the current density, σ is the electrical conductivity, $\mathbf{H}$ is the magnetic field intensity, $\mathbf{D}$ is the electric displacement field, and μ is the magnetic permeability.

Characteristics of Eddy Currents

- **Circular flow pattern:** Eddy currents flow in closed loops within the conductor, perpendicular to the magnetic field direction.
- **Energy dissipation:** Due to the electrical resistance of the conductor, eddy currents cause power loss in the form of heat: $P = I^2 R = \frac{J^2}{\sigma}$.
- **Magnetic field opposition:** According to Lenz's law, eddy currents create their own magnetic field that opposes the change in the original magnetic field.
- **Skin effect:** At high frequencies, eddy currents tend to concentrate near the surface of the conductor, with penetration depth $\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$.

Displacement Measurement Using Eddy Current Effect

Basic Measurement Principle Eddy current displacement sensors operate on the principle that the impedance of a coil changes when a conductive target approaches or moves away from it. The sensor consists of a coil energized by a high-frequency alternating current (typically 100 kHz to 10 MHz), which generates an oscillating magnetic field.

Sensor Configuration and Operation

(1) **Excitation coil:** A primary coil is energized with high-frequency AC current, creating an alternating magnetic field.

(2) **Target interaction:** When a conductive target is placed within this magnetic field, eddy currents are induced in the target material.

(3) **Impedance change:** The eddy currents in the target create their own magnetic field that opposes the original field, effectively changing the coil's impedance.

(4) **Distance dependency:** The magnitude of eddy currents (and thus the impedance change) is inversely related to the distance between the coil and the target.

Mathematical Relationship The coil impedance can be expressed as:

$$Z = R + j\omega L = R_0 + \Delta R + j\omega(L_0 + \Delta L) \tag{23}$$

where R_0 and L_0 are the coil resistance and inductance without the target, and ΔR and ΔL are the changes due to eddy current effects.

For small gaps, the relationship between impedance change and distance can be approximated as:

$$\frac{\Delta Z}{Z_0} = K \cdot e^{-\alpha d} \tag{24}$$

where d is the distance, K is a constant depending on target material properties, and α is the decay constant.

Detection Methods

- **Amplitude detection:** Measures the magnitude of impedance change, suitable for displacement measurement of single material targets.
- **Phase detection:** Analyzes both magnitude and phase changes, allowing discrimination between different materials and improved accuracy.
- **Bridge circuits:** Use AC bridge configurations to convert impedance changes to voltage signals.
- **Resonant frequency tracking:** Monitors the shift in resonant frequency of an LC oscillator circuit.
- **Phase-sensitive demodulation:** Employs synchronous detection to extract both in-phase and quadrature components of the signal.

Advantages and Applications **Advantages:**

- Non-contact measurement
- High resolution (sub-micrometer precision possible)
- Fast response time
- Insensitive to dirt, oil, and non-conductive contaminants
- Wide operating temperature range
- No mechanical wear

Limitations and Considerations

- Limited to conductive targets only
- Sensitivity depends on target material conductivity and permeability
- Temperature effects on both sensor and target properties
- Electromagnetic interference from nearby conductors
- Non-linear response for large displacement ranges
- Edge effects when target size is comparable to coil diameter

本科实验报告

物理传感器

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课程名称：生物医学传感与检测　　指导老师：唐志峰　　成绩：
实验名称：物理传感器　　实验类型：　　同组学生姓名：无

一、 作业 3

题目 1

图 1显示了一个直接应变电桥，其中 $E = 4\,\text{V}$，$R_1 = R_2 = R_3 = R_4 = 120\,\Omega$。请回答以下问题：

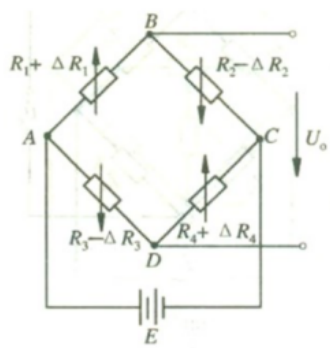


Figure 1 A direct strain bridge

图 1: 直接应变电桥（题目 1）

(1) 如果 R_1 是金属应变片，其他为外部电阻，当 R_1 的增量为 $\Delta R_1 = 1.2\,\Omega$ 时，输出电压 U_0 是多少？
输出电压 U_0 的计算公式为：

$$U_0 = E\left(\frac{R_1 + \Delta R_1}{R_1 + \Delta R_1 + R_2} - \frac{R_3}{R_3 + R_4}\right) \tag{1}$$

已知 $E = 4\,\text{V}$，$R_1 = R_2 = R_3 = R_4 = 120\,\Omega$，$\Delta R_1 = 1.2\,\Omega$：

$$U_0 = 4\,\text{V}\left(\frac{120 + 1.2}{120 + 1.2 + 120} - \frac{120}{120 + 120}\right) \tag{2}$$

$$= 4\,\text{V}\left(\frac{121.2}{241.2} - \frac{1}{2}\right) \tag{3}$$

$$= 4\,\text{V}(0.50248 - 0.5) = 4\,\text{V} \times 0.00248 = 9.97\,\text{mV} \tag{4}$$

(2) R_1、R_2 是同一批次的应变片，能够感受相同方向和大小的应变，其他为外部电阻。输出电压 U_0 是多少？

当 R_1 和 R_2 在相同方向上发生相同大小的变化时，电桥保持平衡。因此，输出电压 $U_0 = 0\,\mathrm{V}$。

(3) 在问题 (2) 中，如果 R_1、R_2 感受相反方向的应变，且 $|\Delta R_1| = |\Delta R_2| = 1.2\,\Omega$，输出电压 U_0 是多少？

对于 R_1 和 R_2 感受相反方向应变的差分电桥配置，输出电压为：

$$U_0 = 4\,\mathrm{V} \times \left(\frac{120+1.2}{120+120} - 0.5\right) = 4\,\mathrm{V} \times (0.505 - 0.5) = 20\,\mathrm{mV} \tag{5}$$

题目 2

图 2显示了等强度梁的力测量系统。R_1 是灵敏度系数为 $K = 2.05$ 的电阻应变片。无应变时 $R_1 = 120\,\Omega$。当测试件受到力 F 作用时，应变片的平均应变为 $\epsilon = 800\,\mu\mathrm{m/m}$。求：

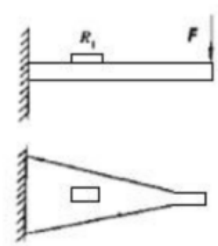


Figure 2 A force-measuring sensor

图 2: 力测量传感器（题目 2）

(1) 应变片电阻的变化量 ΔR_1 是多少？电阻的相对变化量 $\Delta R_1/R_1$ 是多少？

灵敏度系数 K 定义为 $K = \frac{\Delta R_1/R_1}{\epsilon}$。

电阻的相对变化量为：

$$\frac{\Delta R_1}{R_1} = K\epsilon = 2.05 \times 800 \times 10^{-6} = 1.64 \times 10^{-3} \tag{6}$$

电阻变化量 ΔR_1 为：

$$\Delta R_1 = \left(\frac{\Delta R_1}{R_1}\right) \times R_1 = 1.64 \times 10^{-3} \times 120\,\Omega = 0.1968\,\Omega \tag{7}$$

(2) 如果将电阻应变片 R_1 接入单臂电桥，电源电压为直流 3V，电桥的输出电压和非线性误差是多少？

对于单臂电桥，输出电压 U_o 为：

$$U_o = E\left(\frac{R_1 + \Delta R_1}{R_1 + \Delta R_1 + R_2} - \frac{R_3}{R_3 + R_4}\right) \tag{8}$$

假设无应变时电桥平衡，$R_1 = R_2 = R_3 = R_4 = 120\,\Omega$。已知 $E = 3\,\mathrm{V}$，$\Delta R_1 = 0.1968\,\Omega$：

$$U_o = 3\,\mathrm{V}\left(\frac{120+0.1968}{120+0.1968+120} - \frac{120}{120+120}\right) \tag{9}$$

$$= 3\,\mathrm{V}\left(\frac{120.1968}{240.1968} - \frac{1}{2}\right) \tag{10}$$

$$= 3\,\mathrm{V}(0.50041 - 0.5) = 1.23\,\mathrm{mV} \tag{11}$$

非线性误差 γ 约为：

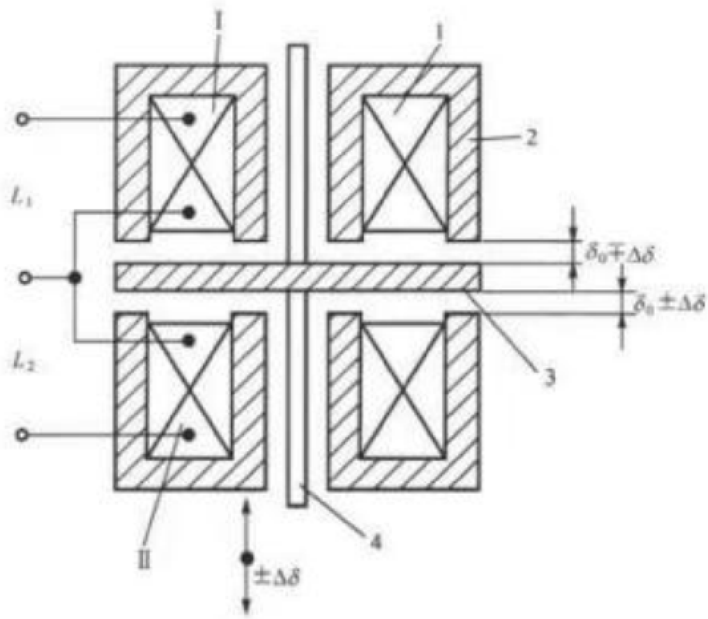
$$\gamma \approx \frac{\Delta R_1/R_1}{2} = \frac{1.64\times 10^{-3}}{2} = 0.08\% \tag{12}$$

(3) 减少非线性误差的措施是什么？分析电桥的输出电压和非线性误差。

- **使用双臂电桥（差分）**：输出电压为 $U_o = E\frac{\Delta R_1}{2R_1}$。非线性误差减小，因为相邻臂的相反变化部分抵消了非线性。
- **使用全桥电路**：输出电压为 $U_o = E\frac{\Delta R_1}{R_1}$。理想全桥电路中不存在非线性误差，因为它利用相邻臂的相反应变变化和相对臂的相同变化，完全消除了非线性。

题目 3

变气隙电感式传感器（见图 3）的输出特性与哪些因素有关？如何改善其非线性？如何提高其灵敏度？



1. 线圈；2. 铁芯；3. 衔铁；4. 导杆

图 4.2.3 差动变隙式电感传感器

图 3: 变气隙电感式传感器（题目 3）

(1) 变气隙电感式传感器的电感 L 由公式 $L = \frac{W^2\mu S}{2\delta}$ 给出。输出特性与以下因素有关：

- 气隙厚度：δ
- 磁路截面积：S
- 线圈匝数：W

- 材料磁导率：μ

(2) 改善非线性：

采用差分结构。在差分电感式传感器中，两个电感单元工作方向相反。当气隙变化时，两个单元的电感变化抵消了部分非线性。

(3) 提高灵敏度：

采用差分结构。通过优化结构参数：

- 减小 δ（在合理范围内）
- 增加 W
- 增加 S

题目 4

请简述相敏检波电路（见图 4）的工作原理。保证其工作可靠性的条件是什么？

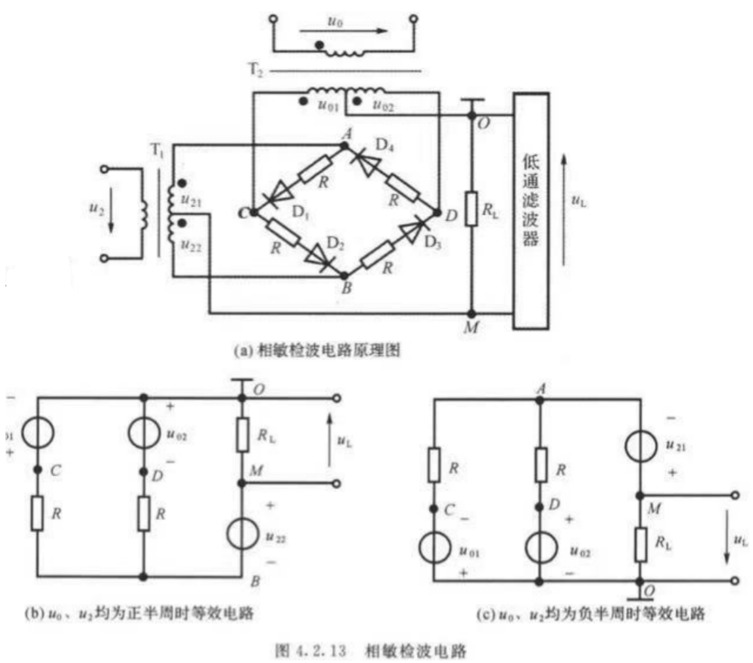


图 4: 相敏检波电路（题目 4）

相敏检波电路的工作原理

工作原理涉及输入信号 $V_S(t) = V_m \cos(\omega t + \varphi)$ 和参考信号 $V_r(t) = V_{ref} \cos(\omega t)$。输出电压 V_{out} 是这两个信号的乘积：

$$V_{out} = V_S(t) \cdot V_r(t) = V_m \cos(\omega t + \varphi) \cdot V_{ref} \cos(\omega t) = \frac{V_m V_{ref}}{2}[\cos(2\omega t + \varphi) + \cos\varphi] \tag{13}$$

该输出然后通过低通滤波器，消除高频分量（$2\omega t$），产生直流输出电压 V_{DC}：

$$V_{DC} = \frac{V_m V_{ref}}{2} \cos\varphi \tag{14}$$

通过检测 V_{DC} 的相位，可以实现信号极性的精确测量。

保证工作可靠性的条件

(1) 参考信号必须与输入信号具有相同的频率和稳定的相位关系。

(2) 输入信号的幅度应足够大，以确保在乘法运算和后续处理中能够准确检测。

(3) 低通滤波器的截止频率应合理选择，以有效滤除高频分量而不过度衰减有用的直流信号。

题目 5

参见教材图 3.3.1，已知变气隙电感式传感器的截面积 $S = 1.5\,\mathrm{cm}^2$，磁路长度 $L = 20\,\mathrm{cm}$，相对磁导率 $\mu_r = 5000$，气隙 $\delta_0 = 0.5\,\mathrm{cm}$，$\Delta\delta = \pm 0.1\,\mathrm{mm}$，真空磁导率 $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H/m}$，线圈匝数 $w = 3000$。求单端传感器的灵敏度 $\Delta L/\Delta\delta$？如果做成差分形式，传感器的灵敏度如何变化？

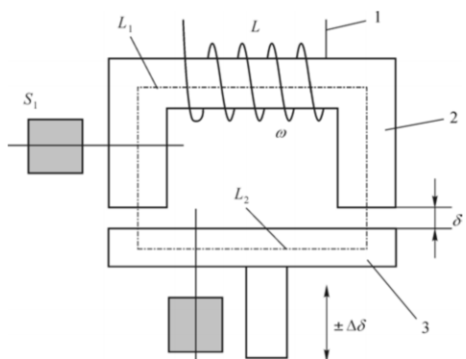


Fig. 3.3.1. Variable reluctance transducer. 1 is coil; 2 is core (fixed core); 3 is armature (movable core)

图 5: 变气隙电感式传感器配置（题目 5）

解答

首先计算初始电感 L_0：

$$L_0 = \frac{W^2 \mu_0 S}{2\delta_0} \tag{15}$$

已知 $W = 3000$，$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H/m}$，$S = 1.5\,\mathrm{cm}^2 = 1.5 \times 10^{-4}\,\mathrm{m}^2$，$\delta_0 = 0.5\,\mathrm{cm} = 0.005\,\mathrm{m}$：

$$L_0 = \frac{3000^2 \times 4\pi \times 10^{-7} \times 1.5 \times 10^{-4}}{2 \times 0.005} = 0.1696\,\mathrm{H} \tag{16}$$

灵敏度 $S_{single} = \left|\frac{\Delta L}{\Delta\delta}\right|$ 可以用导数 $\left|\frac{dL}{d\delta}\right|$ 近似。
由于 $L = \frac{W^2\mu_0 S}{2\delta}$，则 $\frac{dL}{d\delta} = -\frac{W^2\mu_0 S}{2\delta^2}$。

因此，单端传感器的灵敏度为：

$$S_{single} = \left|-\frac{W^2\mu_0 S}{2\delta_0^2}\right| = \frac{3000^2 \times 4\pi \times 10^{-7} \times 1.5 \times 10^{-4}}{2 \times (0.005)^2} = 33.93\,\mathrm{H/m} \tag{17}$$

如果做成差分形式，灵敏度加倍，因为两个线圈的变化相加（$\Delta\delta_1 = +\Delta\delta$，$\Delta\delta_2 = -\Delta\delta$）：

$$S_{differential} = 2 \times S_{single} = 2 \times 33.93 = 67.86\,\mathrm{H/m} \tag{18}$$

题目 6

什么是涡流效应？如何利用它来测量位移？

涡流效应

定义和物理原理 涡流是根据法拉第电磁感应定律，在导体中由变化磁场感应产生的环形电流。当金属导体置于时变磁场中，或者当导体与磁场之间存在相对运动时，导体材料内部会产生环形电流（涡流）。

数学描述 涡流现象的基本控制方程为：

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{（法拉第定律）} \tag{19}$$

$$\mathbf{J} = \sigma \mathbf{E} \quad \text{（欧姆定律）} \tag{20}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad \text{（安培定律）} \tag{21}$$

$$\mathbf{B} = \mu \mathbf{H} \quad \text{（本构关系）} \tag{22}$$

其中 $\mathbf{E}$ 是电场强度，$\mathbf{B}$ 是磁感应强度，$\mathbf{J}$ 是电流密度，σ 是电导率，$\mathbf{H}$ 是磁场强度，$\mathbf{D}$ 是电位移矢量，μ 是磁导率。

涡流的特性

- **环形流动模式**：涡流在导体内部以闭合环路形式流动，垂直于磁场方向。
- **能量耗散**：由于导体的电阻，涡流会以热的形式造成功率损耗：$P = I^2R = \frac{J^2}{\sigma}$。
- **磁场对抗**：根据楞次定律，涡流产生自己的磁场来对抗原磁场的变化。
- **趋肤效应**：在高频下，涡流趋向于集中在导体表面附近，穿透深度为 $\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$。

利用涡流效应进行位移测量

基本测量原理 涡流位移传感器基于这样的原理：当导电目标接近或远离线圈时，线圈的阻抗会发生变化。传感器由高频交流电（通常为 100 kHz 到 10 MHz）激励的线圈组成，产生振荡磁场。

传感器配置和工作原理

(1) **激励线圈**：主线圈由高频交流电激励，产生交变磁场。

(2) **目标相互作用**：当导电目标置于该磁场中时，目标材料中会感应出涡流。

(3) **阻抗变化**：目标中的涡流产生自己的磁场来对抗原磁场，有效地改变了线圈的阻抗。

(4) **距离依赖性**：涡流的大小（因此阻抗变化）与线圈和目标之间的距离成反比关系。

数学关系 线圈阻抗可以表示为：

$$Z = R + j\omega L = R_0 + \Delta R + j\omega(L_0 + \Delta L) \tag{23}$$

其中 R_0 和 L_0 是无目标时的线圈电阻和电感，ΔR 和 ΔL 是由涡流效应引起的变化。

对于小间隙，阻抗变化与距离的关系可近似为：

$$\frac{\Delta Z}{Z_0} = K \cdot e^{-\alpha d} \tag{24}$$

其中 d 是距离，K 是取决于目标材料特性的常数，α 是衰减常数。

检测方法

- **幅度检测**：测量阻抗变化的幅值，适用于单一材料目标的位移测量。
- **相位检测**：分析幅值和相位变化，允许区分不同材料并提高精度。
- **电桥电路**：使用交流电桥配置将阻抗变化转换为电压信号。
- **谐振频率跟踪**：监测 LC 振荡器电路谐振频率的偏移。
- **相敏解调**：采用同步检测提取信号的同相和正交分量。

优点和应用 优点：

- 非接触测量
- 高分辨率（可达亚微米精度）
- 快速响应时间
- 对污垢、油污和非导电污染物不敏感
- 宽工作温度范围
- 无机械磨损

局限性和注意事项

- 仅限于导电目标
- 灵敏度取决于目标材料的电导率和磁导率
- 传感器和目标特性的温度效应
- 来自附近导体的电磁干扰
- 大位移范围内的非线性响应
- 当目标尺寸与线圈直径相当时的边缘效应