浙江大学

本科实验报告

姓名：

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系：生物医学工程

专业：生物医学工程

学号：

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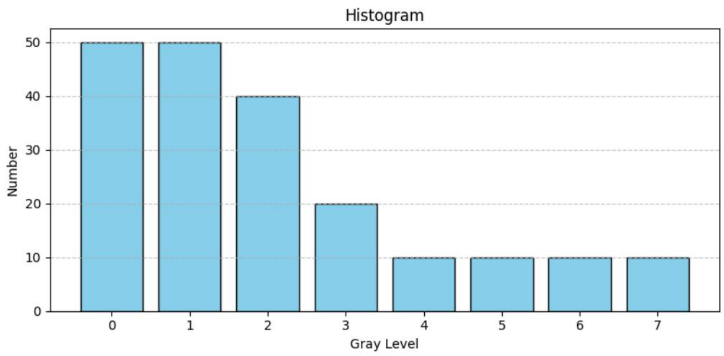
2025 年 4 月 20 日

Homework1

- A grayscale image has the following histogram:



- Use **Otsu's method** to compute the optimal threshold that maximizes the between-class variance.

Otsu 方法是一种无监督的阈值选取算法，它基于类间方差最大化的原则。通过选定一个灰度值作为阈值，大津法将像素分为两类：

前景（灰度值大于/等于阈值）

背景（灰度值小于阈值）

类间方差的计算方法为：

$$\sigma_b^2(T) = w_1(T) \cdot w_2(T) \cdot [\mu_1(T) - \mu_2(T)]^2$$

其中：

前景均值 μ1(T)：前景类中所有灰度级的加权平均值

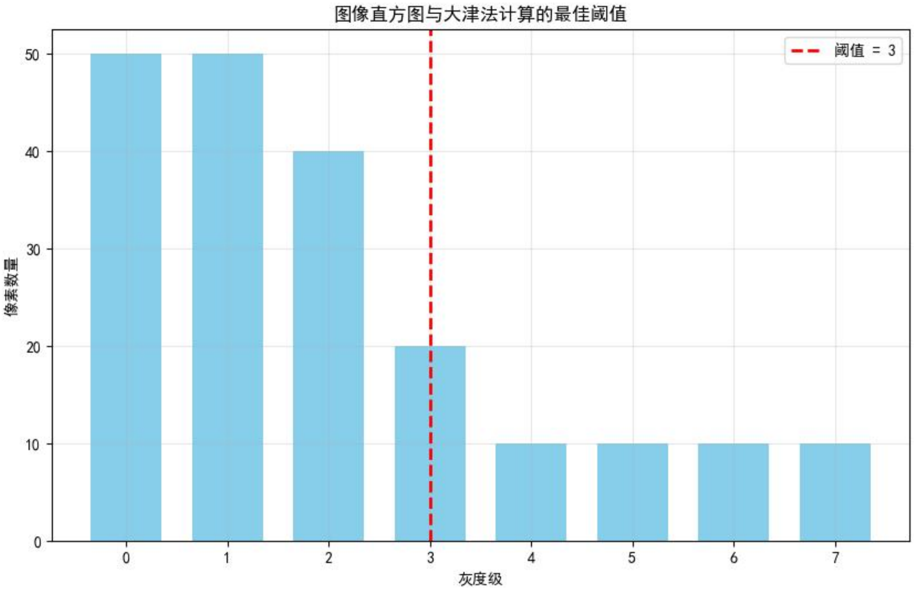
背景均值 μ2(T)：背景类中所有灰度级的加权平均值

前景概率 w1(T)：前景类的总概率

背景概率 w2(T)：背景类的总概率

阈值	ω1	μ1	ω2	μ2	类间方差
0	0.25	0	0.75	2.7333	1.400833
1	0.5	0.5	0.5	3.6	2.4025
2	0.7	0.9286	0.3	4.6667	2.934405
3	0.8	1.1875	0.2	5.5	2.975625

4	0.85	1.3529	0.15	6	2.753382
5	0.9	1.5556	0.1	6.5	2.200278
6	0.95	1.7895	0.05	7	1.289605



由计算得，Otsu 最优阈值为 3。

Homework2

- Modify the Sobel and Prewitt kernels to give the strongest gradient response for edges oriented at ±45° :

−1	−1	−1	−1	0	1
0	0	0	−1	0	1
1	1	1	−1	0	1

Prewitt

−1	−2	−1	−1	0	1
0	0	0	−2	0	2
1	2	1	−1	0	1

Sobel

- Show that the Sobel and Prewitt kernels above, and in (a) above, give iso- tropic results only for horizontal and vertical edges, and for edges oriented at ±45°, respectively.

（a）标准 Sobel/Prewitt 算子的水平（列向导数）和垂直（行向导数）模板为

$$G_x^{(\mathrm{Sobel})} = \begin{pmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{pmatrix}, \quad G_y^{(\mathrm{Sobel})} = \begin{pmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{pmatrix}$$

$$G_x^{(\mathrm{Prewitt})} = \begin{pmatrix} -1 & 0 & +1 \\ -1 & 0 & +1 \\ -1 & 0 & +1 \end{pmatrix}, \quad G_y^{(\mathrm{Prewitt})} = \begin{pmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ +1 & +1 & +1 \end{pmatrix}$$

Sobel/Prewitt 算子本质上是图像梯度的离散近似，对于任意方向 θ 的梯度分量可表示为：

$$G_\theta = G_x \cos\theta + G_y \sin\theta,$$

代入 θ =+45° 时，

$$G_{45^\circ} = \frac{1}{\sqrt{2}}(G_x + G_y).$$

$$G_{45^\circ}^{(\mathrm{Sobel})} = \frac{1}{\sqrt{2}}\left(\begin{pmatrix}-1 & 0 & +1\\ -2 & 0 & +2\\ -1 & 0 & +1\end{pmatrix} + \begin{pmatrix}-1 & -2 & -1\\ 0 & 0 & 0\\ +1 & +2 & +1\end{pmatrix}\right) = \frac{1}{\sqrt{2}}\begin{pmatrix}-2 & -2 & 0\\ -2 & 0 & +2\\ 0 & +2 & +2\end{pmatrix}$$

$$G_{-45^\circ}^{(\mathrm{Sobel})} = \frac{1}{\sqrt{2}}\left(\begin{pmatrix}-1 & 0 & +1\\ -2 & 0 & +2\\ -1 & 0 & +1\end{pmatrix} - \begin{pmatrix}-1 & -2 & -1\\ 0 & 0 & 0\\ +1 & +2 & +1\end{pmatrix}\right) = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & +2 & +2\\ -2 & 0 & +2\\ -2 & -2 & 0\end{pmatrix}$$

$$G_{45^\circ}^{(\mathrm{Prewitt})} = \frac{1}{\sqrt{2}}\left(\begin{pmatrix}-1 & 0 & +1\\ -1 & 0 & +1\\ -1 & 0 & +1\end{pmatrix} + \begin{pmatrix}-1 & -1 & -1\\ 0 & 0 & 0\\ +1 & +1 & +1\end{pmatrix}\right) = \frac{1}{\sqrt{2}}\begin{pmatrix}-2 & -1 & 0\\ -1 & 0 & +1\\ 0 & +1 & +2\end{pmatrix}$$

$$G_{-45^\circ}^{(\mathrm{Prewitt})} = \frac{1}{\sqrt{2}}\left(\begin{pmatrix}-1 & 0 & +1\\ -1 & 0 & +1\\ -1 & 0 & +1\end{pmatrix} - \begin{pmatrix}-1 & -1 & -1\\ 0 & 0 & 0\\ +1 & +1 & +1\end{pmatrix}\right) = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & +1 & +2\\ -1 & 0 & +1\\ -2 & -1 & 0\end{pmatrix}$$

（b）为证明各向同性，首先定义理想阶跃边缘：

水平边缘

$$f(x,y) = \begin{cases} a, & y < 0, \\ b, & y \geq 0. \end{cases}$$

垂直边缘

$$f(x,y) = \begin{cases} a, & x < 0, \\ b, & x \geq 0. \end{cases}$$

+45°边缘

$$f(x,y) = \begin{cases} a, & y - x < 0, \\ b, & y - x \geq 0, \end{cases}$$

–45°边缘

$$f(x,y) = \begin{cases} a, & y + x < 0, \\ b, & y + x \geq 0. \end{cases}$$

我们只需在 3×3 邻域内取 $(x,y)\in\{(-1,1),(0,1),(1,1),\cdots,(1,-1)\}$ 进行卷积。

对于 Sobel 算子，水平边缘凡 y>=0 区域取 b，y<=0 区域取 a，则 3×3 窗口内，第一行全是 a，第三行全是 b，卷积和为

$$G_h = \sum_{i,j} S_h(i,j)\, f(i,j) = [-1-2-1]\cdot a + [+1+2+1]\cdot b = 4\,(b-a).$$

垂直同理，也得 $G_v = 4(b-a)$

而对于 Prewitt 算子，在水平或垂直边缘 Gv = Gh = 3(b-a)

在 θ=±45° 时，省略掉归一化因子 1/√2。

令 f(x,y)=a 当 y-x<0，b 当 y-x>0（45°），矩阵形式也即

y\x:	-1	0	+1
1	a	a	b
0	a	b	b
-1	b	b	b

做卷积和：

$$G_{45} = \sum_{i,j} (S_h + S_v)(i,j)\ f(i,j)$$

展开后，所有含 a 的项系数之和与所有含 b 的项系数之和，都会恰好给出 6(b-a)。

－45° 同理，使用 y+x 判定 a/b，带入卷积，也得到 6(b-a)。

综上,原核在 0°/90° 两个方向各向同性;新核在 ±45° 两个方向各向同性。

Homework3

Given a 5 x 5 grayscale image I as follows:

$$I = \begin{bmatrix} 50 & 55 & 60 & 65 & 70 \\ 55 & 60 & 70 & 75 & 80 \\ 60 & 70 & 150 & 160 & 90 \\ 65 & 75 & 160 & 170 & 100 \\ 70 & 80 & 90 & 100 & 110 \end{bmatrix}$$

Please apply the main steps of the edge detection algorithm to extract edges from the image. The specific requirements are as follows:

- Compute Gradients: Use the 3 x 3 Sobel operator to compute the gradients G_x (horizontal) and G_y (vertical) of the image I. Apply zero-padding to handle the boundaries.
- Compute Gradient Magnitude and Direction: Based on G_x and G_y, calculate the gradient magnitude $\sqrt{G_x^2 + G_y^2}$ and the gradient direction $\theta = \arctan(G_y/G_x)$ for each pixel.

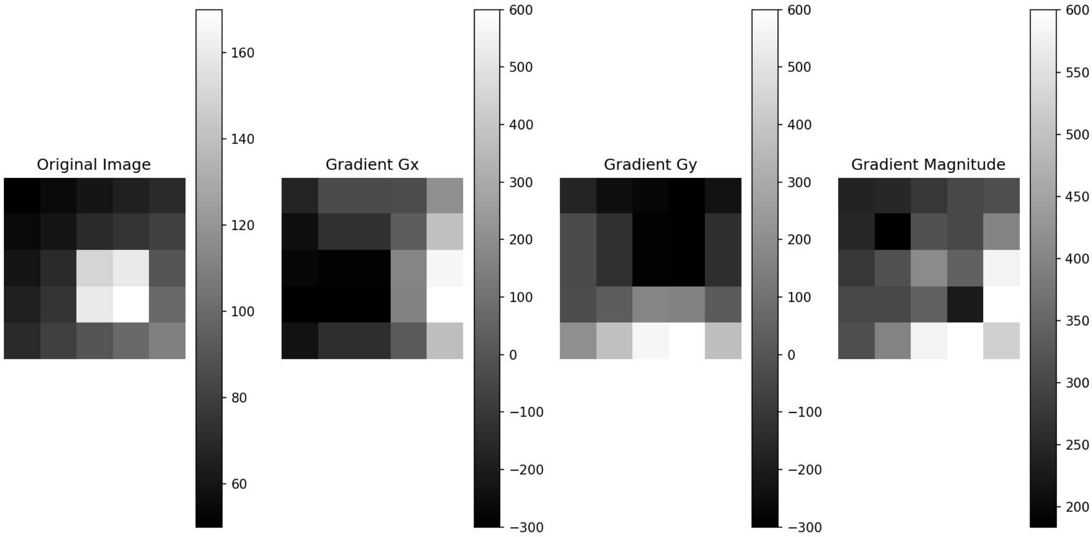
详细计算见 hw3.py

```
Gradient Gx:
[[-170  -35  -35  -30  205]
 [-245 -130 -130   30  375]
 [-275 -290 -290  170  565]
 [-300 -300 -300  160  600]
 [-235 -135 -135   20  370]]

Gradient Gy:
[[-170 -245 -275 -300 -235]
 [ -35 -130 -290 -300 -135]
 [ -35 -130 -290 -300 -135]
 [ -30   30  170  160   20]
 [ 205  375  565  600  370]]

Gradient Magnitude:
[[240.4163056  247.48737342 277.21832551 301.49626863 311.84932259]
 [247.48737342 183.84776311 317.80497164 301.49626863 398.55990767]
```

[277.21832551 317.80497164 410.12193309 344.81879299 580.90446719]

[301.49626863 301.49626863 344.81879299 226.27416998 600.33324079]

[311.84932259 398.55990767 580.90446719 600.33324079 523.25901808]]



Gradient Direction:

[[-2.35619449 -1.71269338 -1.69738845 -1.67046498 -0.85347462]

[-2.9996956 -2.35619449 -1.99221553 -1.47112767 -0.34555558]

[-3.01500053 -2.72017345 -2.35619449 -1.05524732 -0.23454063]

[-3.041924 3.041924 2.62604365 0.78539816 0.033321]

[2.42427095 1.91635191 1.80533696 1.53747533 0.78539816]]