浙江大學

本科实验报告

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学院：生物医学工程与仪器科学学院

系：生物医学工程

专业：生物医学工程

学号：

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2025 年 4 月 5 日

Homework1

$$\begin{matrix} 1 & 3 & 1 \\ 1 & 3 & 1 \\ 1 & 3 & 1 \end{matrix} \qquad \begin{matrix} 1 & 2 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 0 \end{matrix}$$

A B

$$H(A,B) = -\sum_a \sum_b p_{AB}(a,b) log p_{AB}(a,b)$$

- Calculate the entropy: H(A) ,H(B)
- Calculate the joint entropy between A and B
- Calculate the Mutual Information between A and B

- Calculate the grey distribution of A and B (grey level from 0 to 3), then calculate the Kullback-Leibler Distance K(A||B)

(1) 概率分布: $P_A(1) = \frac{6}{9} = \frac{2}{3}$ $P_B(1) = P_B(2) = P_B(0) = \frac{1}{3}$

$P_A(3) = \frac{3}{9} = \frac{1}{3}$

$H(A) = -\sum_i P_A(i) \log_2 P_A(i) = -(\frac{2}{3}\log_2\frac{2}{3} + \frac{1}{3}\log_2\frac{1}{3}) = 0.918$ bits

$H(B) = -\sum_i P_B(i) \log_2 P_B(i) = -3 \times \frac{1}{3}\log_2\frac{1}{3} = 1.585$ bits

(2) $P_{AB}(1,1) = \frac{3}{9} = \frac{1}{3}$

$P_{AB}(3,2) = \frac{3}{9} = \frac{1}{3}$

$P_{AB}(1,0) = \frac{3}{9} = \frac{1}{3}$

联合熵 $H(A,B) = -\sum_a \sum_b P_{AB}(a,b) \log_2 P_{AB}(a,b) = -9 \times \frac{1}{3} \times \log_2 \frac{1}{3} = 1.585$ bits

(3) 互信息 $I(A,B) = H(A) + H(B) - H(A,B) = \sum_a \sum_b P_{AB}(a,b) \log_2 \frac{P_{AB}(a,b)}{P_A(a) P_B(b)}$

$= 0.918$ bits

(4) 灰度分布:

灰度级	0	1	2	3
P_A	0	$\frac{2}{3}$	0	$\frac{1}{3}$
P_B	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0

KL散度: $K(A \| B) = \sum_x P_A(x) \log_2 \frac{P_A(x)}{P_B(x)} = P_A(0) \log_2 \frac{P_A(0)}{P_B(0)} + P_A(1) \log_2 \frac{P_A(1)}{P_B(1)} + P_A(2) \log_2 \frac{P_A(2)}{P_B(2)} + P_A(3) \log_2 \frac{P_A(3)}{P_B(3)}$

$= \frac{2}{3} \log_2 2 + \frac{1}{3} \log_2 \frac{1}{0} \to \infty$

表明分布A对于分布B是不绝对连续的, B无法解释A灰度值为3的这种可能性

Homework2

	2	80	90	100
y	1	40	50	60
	0	10	20	30
		0	1	2
			x	

Suppose a 3*3 matrix, use Bilinear interpolation calculate the grey value of:

$P_1(0.5,0.5)$

$P_2(1.3,1.7)$

$P_3(1.8,0.2)$

先x后y进行双线性插值：$f(x, y_1) = f(x_1, y_1) + \frac{x-x_1}{x_2-x_1}[f(x_2, y_1) - f(x_1, y_1)]$

$f(x, y_2) = f(x_1, y_2) + \frac{x-x_1}{x_2-x_1}[f(x_2, y_2) - f(x_1, y_2)]$

$f(x, y) = f(x, y_1) + \frac{y-y_1}{y_2-y_1}[f(x, y_2) - f(x, y_1)]$

(1) $P_1(0.5, 0) = 15$

$P(0.5, 1) = 45$

$P(0.5, 0.5) = 30$

(2) $P(1.3, 1) = 53$

$P(1.3, 2) = 93$

$P(1.3, 1.7) = 53 + 40 \times 0.7 = 81$

(3) $P(1.8, 0) = 18$

$P(1.8, 1) = 48$

$P(1.8, 0.2) = 18 + 30 \times 0.2 = 24$

Homework3

- Suppose the spatial transformation is represented as:

$$\begin{cases} x' = \mathrm{e}^{(x+3y)/4} \\ y' = \mathrm{e}^{(3x+y)/4} \end{cases}$$

- Calculate the Jabocian matrix;
- Calculate the Jabocian determinant;
- Calculate the inverse transformation's Jabocian matrix;
- Calculate the inverse transformation's Jabocian determinant;

对于二维空间变换 $(x, y) \to (x', y')$, Jacobian矩阵定义为 $J = \begin{pmatrix} \frac{\partial x'}{\partial x} & \frac{\partial x'}{\partial y} \\ \frac{\partial y'}{\partial x} & \frac{\partial y'}{\partial y} \end{pmatrix}$

(1) $\frac{\partial x'}{\partial x} = \frac{1}{4} e^{\frac{x+3y}{4}}$ $\quad \frac{\partial y'}{\partial x} = \frac{3}{4} e^{\frac{3x+y}{4}}$

$\frac{\partial x'}{\partial y} = \frac{3}{4} e^{\frac{x+3y}{4}}$ $\quad \frac{\partial y'}{\partial y} = \frac{1}{4} e^{\frac{3x+y}{4}}$

$$J = \begin{pmatrix} \frac{1}{4} e^{\frac{x+3y}{4}} & \frac{3}{4} e^{\frac{x+3y}{4}} \\ \frac{3}{4} e^{\frac{3x+y}{4}} & \frac{1}{4} e^{\frac{3x+y}{4}} \end{pmatrix}$$

(2) Jacobian行列式 $= \frac{\partial x'}{\partial x}\frac{\partial y'}{\partial y} - \frac{\partial x'}{\partial y}\frac{\partial y'}{\partial x} = \frac{1}{16} e^{\frac{4x+4y}{4}} - \frac{9}{16} e^{x+y} = -\frac{1}{2} e^{x+y}$

(3) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$$J^{-1} = -\frac{2}{e^{x+y}} \begin{pmatrix} \frac{1}{4} e^{\frac{3x+y}{4}} & -\frac{3}{4} e^{\frac{x+3y}{4}} \\ -\frac{3}{4} e^{\frac{3x+y}{4}} & \frac{1}{4} e^{\frac{x+3y}{4}} \end{pmatrix}$$

(4) $\det(J^{-1}) = \frac{1}{\det(J)} = -\frac{2}{e^{x+y}}$