Core-Maths-C3 - 2012-June

Question 1

Express

$$\frac{2(3x+2)}{9x^2-4} - \frac{2}{3x+1}$$

as a single fraction in its simplest form.

(4)

Question 2

$$f(x) = x^3 + 3x^2 + 4x - 12$$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{4(3-x)}{(3+x)}\right)}, \qquad x \neq -3$$

(3)

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{(3+x_n)}\right)}, n \geqslant 0$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1, x_2 and x_3.

(3)

The root of $f(x) = 0$ is α.

(c) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places.

(3)

Question 3

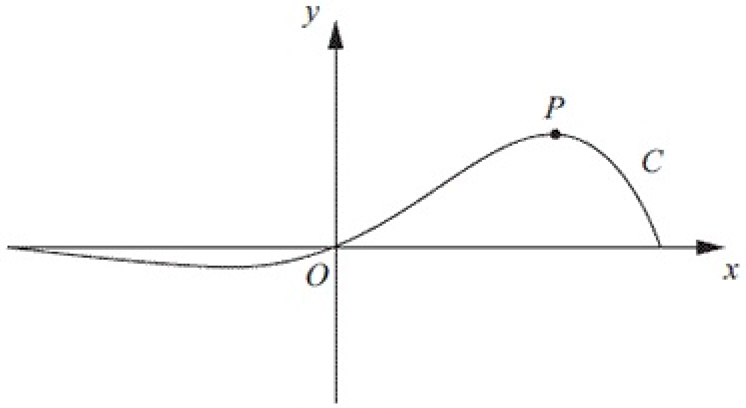

Figure 1

Figure 1 shows a sketch of the curve C which has equation

$$y = e^{x\sqrt{3}} \sin 3x, \quad -\frac{\pi}{3} \leqslant x \leqslant \frac{\pi}{3}$$

(a) Find the x coordinate of the turning point P on C, for which $x > 0$
Give your answer as a multiple of π.

(6)

(b) Find an equation of the normal to C at the point where $x = 0$

(3)

Question 4

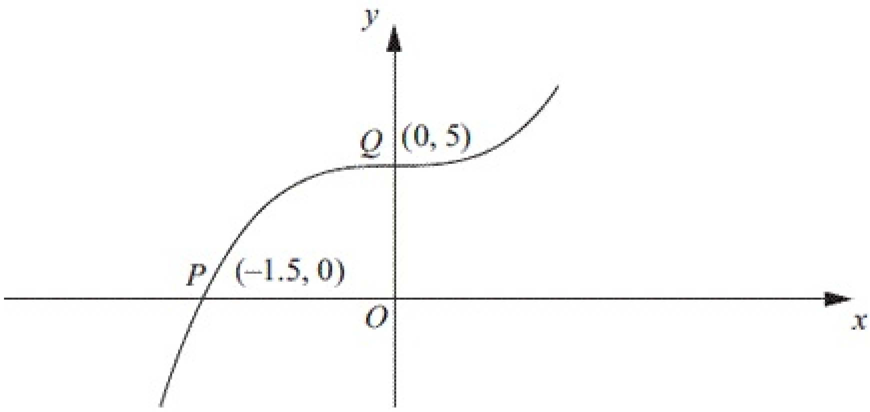

Figure 2

Figure 2 shows part of the curve with equation $y = \mathrm{f}(x)$
The curve passes through the points $P(-1.5,\ 0)$ and $Q(0,\ 5)$ as shown.

On separate diagrams, sketch the curve with equation

(a) $y = |\mathrm{f}(x)|$

(2)

(b) $y = \mathrm{f}(|x|)$

(2)

(c) $y = 2\mathrm{f}(3x)$

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

Question 5

(a) Express $4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta$ in terms of $\sin\theta$ and $\cos\theta$.

(2)

(b) Hence show that

$$4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta = \sec^2\theta$$

(4)

(c) Hence or otherwise solve, for $0 < \theta < \pi$,

$$4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta = 4$$

giving your answers in terms of π.

(3)

Question 6

The functions f and g are defined by

$$\mathrm{f}: x \mapsto \mathrm{e}^x + 2, \qquad x \in \mathbb{R}$$

$$\mathrm{g}: x \mapsto \ln x, \qquad x > 0$$

(a) State the range of f.

(1)

(b) Find $\mathrm{fg}(x)$, giving your answer in its simplest form.

(2)

(c) Find the exact value of x for which $\mathrm{f}(2x+3) = 6$

(4)

(d) Find f^{-1}, the inverse function of f, stating its domain.

(3)

(e) On the same axes sketch the curves with equation $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, giving the coordinates of all the points where the curves cross the axes.

(4)

Question 7

(a) Differentiate with respect to x,

(i) $x^{\frac{1}{2}}\ln(3x)$

(ii) $\dfrac{1-10x}{(2x-1)^5}$, giving your answer in its simplest form.

(6)

(b) Given that $x = 3\tan 2y$ find $\dfrac{dy}{dx}$ in terms of x.

(5)

Question 8

$$f(x) = 7\cos 2x - 24\sin 2x$$

Given that $f(x) = R\cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$,

(a) find the value of R and the value of α.

(3)

(b) Hence solve the equation

$$7\cos 2x - 24\sin 2x = 12.5$$

for $0 \leqslant x < 180^\circ$, giving your answers to 1 decimal place.

(5)

(c) Express $14\cos^2 x - 48\sin x\cos x$ in the form $a\cos 2x + b\sin 2x + c$, where a, b, and c are constants to be found.

(2)

(d) Hence, using your answers to parts (a) and (c), deduce the maximum value of

$$14\cos^2 x - 48\sin x\cos x$$

(2)