Core-Maths-C2 - 2006-June

Question 1

Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(2+x)^6$, giving each term in its simplest form.

(4)

Question 2

Use calculus to find the exact value of $\int_1^2 \left(3x^2 + 5 + \frac{4}{x^2}\right) \mathrm{d}x$.

(5)

Question 3

(i) Write down the value of $\log_6 36$.

(1)

(ii) Express $2 \log_a 3 + \log_a 11$ as a single logarithm to base a.

(3)

Question 4

$$f(x) = 2x^3 + 3x^2 - 29x - 60.$$

(a) Find the remainder when $f(x)$ is divided by $(x + 2)$.

(2)

(b) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$.

(2)

(c) Factorise $f(x)$ completely.

(4)

Question 5

(a) In the space provided, sketch the graph of $y = 3^x$, $x \in \mathbb{R}$, showing the coordinates of the point at which the graph meets the y-axis.

(2)

(b) Complete the table, giving the values of 3^x to 3 decimal places.

x	0	0.2	0.4	0.6	0.8	1
3^x		1.246	1.552			3

(2)

(c) Use the trapezium rule, with all the values from your table, to find an approximation for the value of $\int_0^1 3^x \, dx$.

(4)

Question 6

(a) Given that $\sin\theta = 5\cos\theta$, find the value of $\tan\theta$.

(1)

(b) Hence, or otherwise, find the values of θ in the interval $0 \leqslant \theta < 360°$ for which

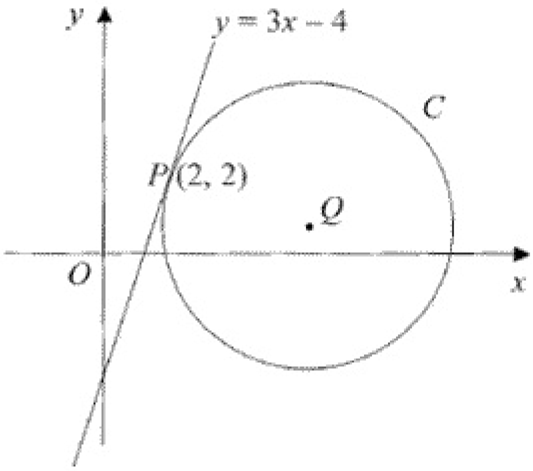
$$\sin\theta = 5\cos\theta,$$

giving your answers to 1 decimal place.

(3)

Question 7

Figure 1



The line $y = 3x - 4$ is a tangent to the circle C, touching C at the point $P(2, 2)$, as shown in Figure 1.

The point Q is the centre of C.

(a) Find an equation of the straight line through P and Q.

(3)

Given that Q lies on the line $y = 1$,

(b) show that the x-coordinate of Q is 5,

(1)

(c) find an equation for C.

(4)

Question 8

Figure 2

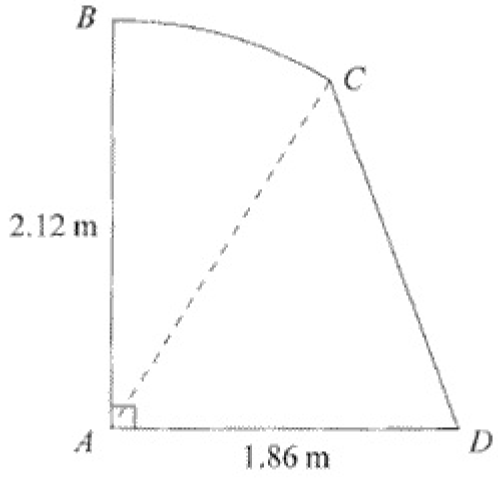


Figure 2 shows the cross section $ABCD$ of a small shed.
The straight line AB is vertical and has length 2.12 m.
The straight line AD is horizontal and has length 1.86 m.
The curve BC is an arc of a circle with centre A, and CD is a straight line.
Given that the size of $\angle BAC$ is 0.65 radians, find

(a) the length of the arc BC, in m, to 2 decimal places,

(2)

(b) the area of the sector BAC, in m^2, to 2 decimal places,

(2)

(c) the size of $\angle CAD$, in radians, to 2 decimal places,

(2)

(d) the area of the cross section $ABCD$ of the shed, in m^2, to 2 decimal places.

(3)

Question 9

A geometric series has first term a and common ratio r.
The second term of the series is 4 and the sum to infinity of the series is 25.

(a) Show that $25r^2 - 25r + 4 = 0$.

(4)

(b) Find the two possible values of r.

(2)

(c) Find the corresponding two possible values of a.

(2)

(d) Show that the sum, S_n, of the first n terms of the series is given by

$$S_n = 25(1 - r^n).$$

(1)

Given that r takes the larger of its two possible values,

(e) find the smallest value of n for which S_n exceeds 24.

(2)

Question 10

Figure 3

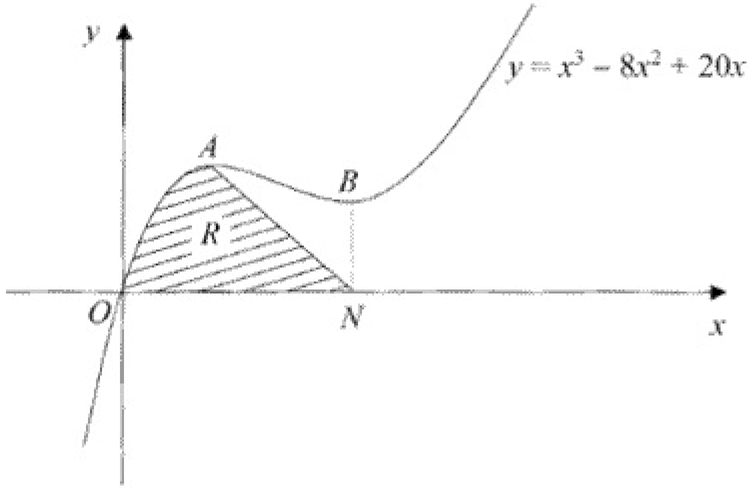


Figure 3 shows a sketch of part of the curve with equation $y = x^3 - 8x^2 + 20x$.
The curve has stationary points A and B.

(a) Use calculus to find the x-coordinates of A and B.

(4)

(b) Find the value of $\frac{d^2y}{dx^2}$ at A, and hence verify that A is a maximum.

(2)

The line through B parallel to the y-axis meets the x-axis at the point N.
The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line from A to N.

(c) Find $\int (x^3 - 8x^2 + 20x)\,dx$.

(3)

(d) Hence calculate the exact area of R.

(5)