Core-Maths-C1 - 2008-January

Question 1

1. Find $\int (3x^2 + 4x^5 - 7)\,dx$.

(4)

Question 2

2. (a) Write down the value of $16^{\frac{1}{4}}$.

(1)

(b) Simplify $(16x^{12})^{\frac{3}{4}}$.

(2)

Question 3

3. Simplify

$$\frac{5 - \sqrt{3}}{2 + \sqrt{3}} ,$$

giving your answer in the form $a + b\sqrt{3}$, where a and b are integers.

(4)

Question 4

4. The point A (−6, 4) and the point B (8, −3) lie on the line L.

(a) Find an equation for L in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

(b) Find the distance AB, giving your answer in the form $k\sqrt{5}$, where k is an integer.

(3)

Question 5

5. (a) Write $\dfrac{2\sqrt{x}+3}{x}$ in the form $2x^p+3x^q$ where p and q are constants.

(2)

Given that $y = 5x - 7 + \dfrac{2\sqrt{x}+3}{x}$, $\qquad x > 0$,

(b) find $\dfrac{dy}{dx}$, simplifying the coefficient of each term.
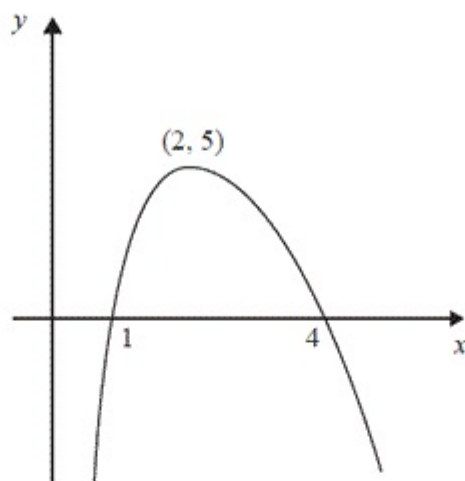
(4)

Question 6

6.

Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$. The curve crosses the x-axis at the points $(1, 0)$ and $(4, 0)$. The maximum point on the curve is $(2, 5)$.
In separate diagrams sketch the curves with the following equations.
On each diagram show clearly the coordinates of the maximum point and of each point at which the curve crosses the x-axis.

(a) $y = 2f(x)$,

(3)

(b) $y = f(-x)$.

(3)

The maximum point on the curve with equation $y = f(x + a)$ is on the y-axis.

(c) Write down the value of the constant a.

(1)

Question 7

7. A sequence is given by:

$$x_1 = 1,$$
$$x_{n+1} = x_n(p + x_n),$$

where p is a constant $(p \neq 0)$.

(a) Find x_2 in terms of p.

(1)

(b) Show that $x_3 = 1 + 3p + 2p^2$.

(2)

Given that $x_3 = 1$,

(c) find the value of p,

(3)

(d) write down the value of x_{2008}.

(2)

Question 8

8. The equation

$$x^2 + kx + 8 = k$$

has no real solutions for x.

(a) Show that k satisfies $k^2 + 4k - 32 < 0$.

(3)

(b) Hence find the set of possible values of k.

(4)

Question 9

9. The curve C has equation $y = f(x)$, $x > 0$, and $f'(x) = 4x - 6\sqrt{x} + \dfrac{8}{x^2}$.

Given that the point $P(4, 1)$ lies on C,

(a) find $f(x)$ and simplify your answer.

(6)

(b) Find an equation of the normal to C at the point $P(4, 1)$.

(4)

Question 10

10. The curve C has equation

$$y = (x+3)(x-1)^2.$$

(a) Sketch C showing clearly the coordinates of the points where the curve meets the coordinate axes.

(4)

(b) Show that the equation of C can be written in the form

$$y = x^3 + x^2 - 5x + k,$$

where k is a positive integer, and state the value of k.

(2)

There are two points on C where the gradient of the tangent to C is equal to 3.

(c) Find the x-coordinates of these two points.

(6)

Question 11

11. The first term of an arithmetic sequence is 30 and the common difference is -1.5

(a) Find the value of the 25th term.

(2)

The rth term of the sequence is 0.

(b) Find the value of r.

(2)

The sum of the first n terms of the sequence is S_n.

(c) Find the largest positive value of S_n.

(3)