Mechanics-M2 - 2012-January

Question 1

A tennis ball of mass 0.1 kg is hit by a racquet. Immediately before being hit, the ball has velocity $30\mathbf{i}$ m s^{-1}. The racquet exerts an impulse of $(-2\mathbf{i} - 4\mathbf{j})$ N s on the ball. By modelling the ball as a particle, find the velocity of the ball immediately after being hit.

(4)

Question 2

A particle P is moving in a plane. At time t seconds, P is moving with velocity $\mathbf{v}$ m s^{-1}, where $\mathbf{v} = 2t\mathbf{i} - 3t^2\mathbf{j}$.

Find

(a) the speed of P when $t = 4$

(2)

(b) the acceleration of P when $t = 4$

(3)

Given that P is at the point with position vector $(-4\mathbf{i} + \mathbf{j})$ m when $t = 1$,

(c) find the position vector of P when $t = 4$

(5)

Question 3

A cyclist and her cycle have a combined mass of 75 kg. The cyclist is cycling up a straight road inclined at 5° to the horizontal. The resistance to the motion of the cyclist from non-gravitational forces is modelled as a constant force of magnitude 20 N. At the instant when the cyclist has a speed of 12 m s^{-1}, she is decelerating at 0.2 m s^{-2}.

(a) Find the rate at which the cyclist is working at this instant.

(5)

When the cyclist passes the point A her speed is 8 m s^{-1}. At A she stops working but does not apply the brakes. She comes to rest at the point B.
The resistance to motion from non-gravitational forces is again modelled as a constant force of magnitude 20 N.

(b) Use the work-energy principle to find the distance AB.

(5)

Question 4

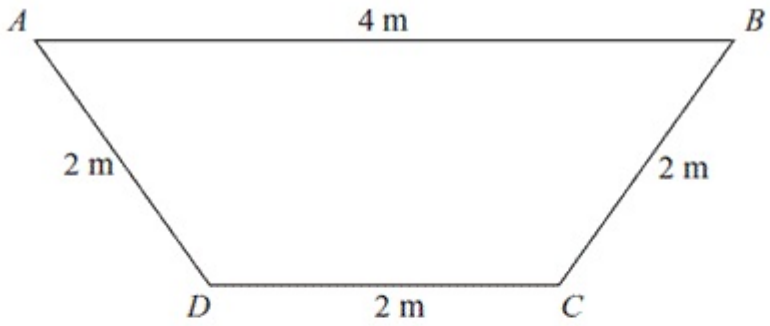

Figure 1

The trapezium $ABCD$ is a uniform lamina with $AB = 4$ m and $BC = CD = DA = 2$ m, as shown in Figure 1.

(a) Show that the centre of mass of the lamina is $\frac{4\sqrt{3}}{9}$ m from AB.

(5)

The lamina is freely suspended from D and hangs in equilibrium.

(b) Find the angle between DC and the vertical through D.

(5)

Question 5

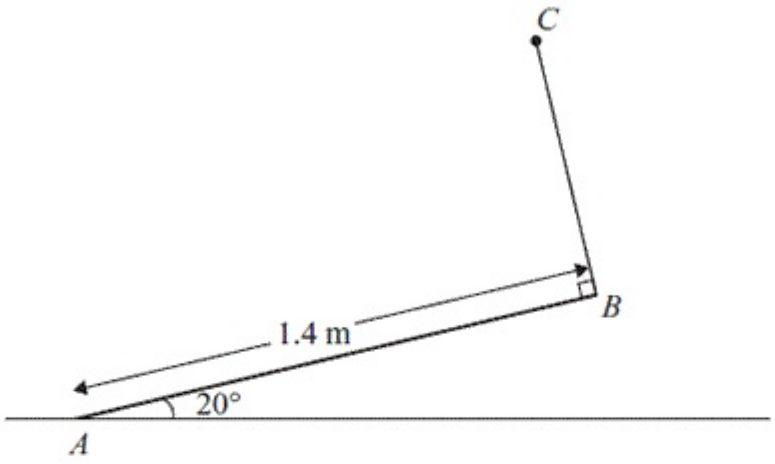

Figure 2

A uniform rod AB has mass 4 kg and length 1.4 m. The end A is resting on rough horizontal ground. A light string BC has one end attached to B and the other end attached to a fixed point C. The string is perpendicular to the rod and lies in the same vertical plane as the rod. The rod is in equilibrium, inclined at 20° to the ground, as shown in Figure 2.

(a) Find the tension in the string.

(4)

Given that the rod is about to slip,

(b) find the coefficient of friction between the rod and the ground.

(7)

Question 6

Three identical particles, A, B and C, lie at rest in a straight line on a smooth horizontal table with B between A and C. The mass of each particle is m. Particle A is projected towards B with speed u and collides directly with B. The coefficient of restitution between each pair of particles is $\frac{2}{3}$.

(a) Find, in terms of u,

(i) the speed of A after this collision,

(ii) the speed of B after this collision.

(7)

(b) Show that the kinetic energy lost in this collision is $\frac{5}{36}mu^2$

(4)

After the collision between A and B, particle B collides directly with C.

(c) Find, in terms of u, the speed of C immediately after this collision between B and C.

(4)

Question 7

[*In this question, the unit vectors* **i** *and* **j** *are horizontal and vertical respectively.*]

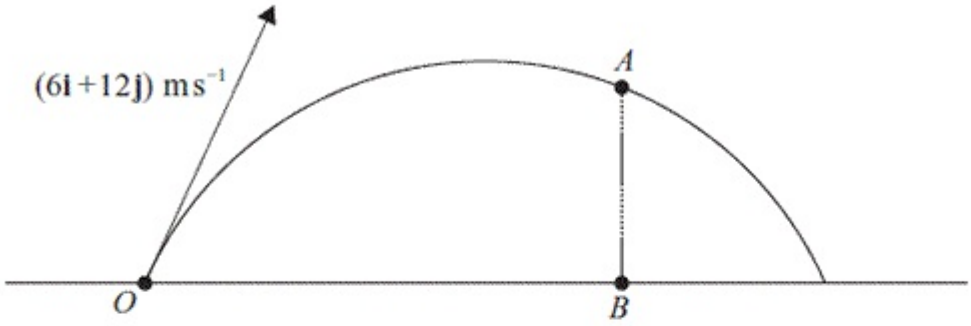

Figure 3

The point O is a fixed point on a horizontal plane. A ball is projected from O with velocity $(6\mathbf{i}+12\mathbf{j})$ m s^{-1}, and passes through the point A at time t seconds after projection. The point B is on the horizontal plane vertically below A, as shown in Figure 3. It is given that $OB = 2AB$.

Find

(a) the value of t,

(7)

(b) the speed, V m s^{-1}, of the ball at the instant when it passes through A.

(5)

At another point C on the path the speed of the ball is also V m s^{-1}.

(c) Find the time taken for the ball to travel from O to C.

(3)