Core-Maths-C1 - 2011-June

Question 1

Find the value of

(a) $25^{\frac{1}{2}}$

(1)

(b) $25^{-\frac{3}{2}}$

(2)

Question 2

Given that $y = 2x^5 + 7 + \frac{1}{x^3}$, $x \neq 0$, find, in their simplest form,

(a) $\frac{dy}{dx}$,

(3)

(b) $\int y \, dx$.

(4)

Question 3

The points P and Q have coordinates $(-1, 6)$ and $(9, 0)$ respectively.

The line l is perpendicular to PQ and passes through the mid-point of PQ.

Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

Question 4

Solve the simultaneous equations

$$x + y = 2$$
$$4y^2 - x^2 = 11$$

(7)

Question 5

A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k,$$
$$a_{n+1} = 5a_n + 3, \quad n \geqslant 1,$$

where k is a positive integer.

(a) Write down an expression for a_2 in terms of k.

(1)

(b) Show that $a_3 = 25k + 18$.

(2)

(c) (i) Find $\sum_{r=1}^{4} a_r$ in terms of k, in its simplest form.

(ii) Show that $\sum_{r=1}^{4} a_r$ is divisible by 6.

(4)

Question 6

Given that $\dfrac{6x + 3x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $6x^p + 3x^q$,

(a) write down the value of p and the value of q.

(2)

Given that $\dfrac{dy}{dx} = \dfrac{6x + 3x^{\frac{5}{2}}}{\sqrt{x}}$, and that $y = 90$ when $x = 4$,

(b) find y in terms of x, simplifying the coefficient of each term.

(5)

Question 7

$$f(x) = x^2 + (k+3)x + k$$

where k is a real constant.

(a) Find the discriminant of $f(x)$ in terms of k.

(2)

(b) Show that the discriminant of $f(x)$ can be expressed in the form $(k+a)^2 + b$, where a and b are integers to be found.

(2)

(c) Show that, for all values of k, the equation $f(x) = 0$ has real roots.

(2)

Question 8

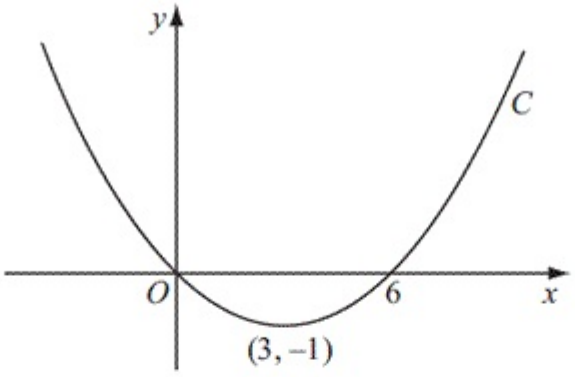

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$.
The curve C passes through the origin and through $(6, 0)$.
The curve C has a minimum at the point $(3, -1)$.

On separate diagrams, sketch the curve with equation

(a) $y = f(2x)$,

(3)

(b) $y = -f(x)$,

(3)

(c) $y = f(x + p)$, where p is a constant and $0 < p < 3$.

(4)

On each diagram show the coordinates of any points where the curve intersects the x-axis and of any minimum or maximum points.

Question 9

(a) Calculate the sum of all the even numbers from 2 to 100 inclusive,

$$2 + 4 + 6 + ... + 100$$

(3)

(b) In the arithmetic series

$$k + 2k + 3k + ... + 100$$

k is a positive integer and k is a factor of 100.

(i) Find, in terms of k, an expression for the number of terms in this series.

(ii) Show that the sum of this series is

$$50 + \frac{5000}{k}$$

(4)

(c) Find, in terms of k, the 50th term of the arithmetic sequence

$$(2k + 1), (4k + 4), (6k + 7), ... ,$$

giving your answer in its simplest form.

(2)

Question 10

The curve C has equation

$$y = (x+1)(x+3)^2$$

(a) Sketch C, showing the coordinates of the points at which C meets the axes.

(4)

(b) Show that $\frac{dy}{dx} = 3x^2 + 14x + 15$.

(3)

The point A, with x-coordinate -5, lies on C.

(c) Find the equation of the tangent to C at A, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

Another point B also lies on C. The tangents to C at A and B are parallel.

(d) Find the x-coordinate of B.

(3)