

# Mechanics-M2 - 2013-January

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## Question 1

Two uniform rods  $AB$  and  $BC$  are rigidly joined at  $B$  so that  $\angle ABC = 90^\circ$ . Rod  $AB$  has length 0.5 m and mass 2 kg. Rod  $BC$  has length 2 m and mass 3 kg. The centre of mass of the framework of the two rods is at  $G$ .

- (a) Find the distance of  $G$  from  $BC$ .

(2)

The distance of  $G$  from  $AB$  is 0.6 m.

The framework is suspended from  $A$  and hangs freely in equilibrium.

- (b) Find the angle between  $AB$  and the downward vertical at  $A$ .

(3)

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## Question 2

A lorry of mass 1800 kg travels along a straight horizontal road. The lorry's engine is working at a constant rate of 30 kW. When the lorry's speed is  $20 \text{ m s}^{-1}$ , its acceleration is  $0.4 \text{ m s}^{-2}$ . The magnitude of the resistance to the motion of the lorry is  $R$  newtons.

- (a) Find the value of  $R$ .

(4)

The lorry now travels up a straight road which is inclined at an angle  $\alpha$  to the horizontal,

where  $\sin \alpha = \frac{1}{12}$ . The magnitude of the non-gravitational resistance to motion is

$R$  newtons. The lorry travels at a constant speed of  $20 \text{ m s}^{-1}$ .

- (b) Find the new rate of working of the lorry's engine.

(5)

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### Question 3

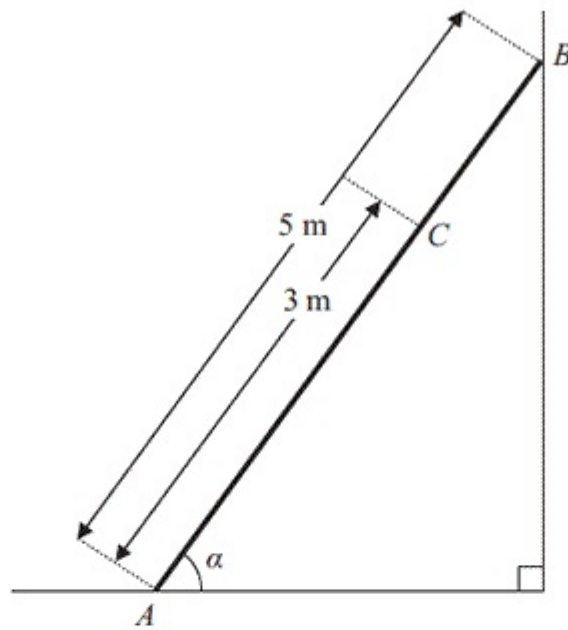


Figure 1

A ladder, of length 5 m and mass 18 kg, has one end  $A$  resting on rough horizontal ground and its other end  $B$  resting against a smooth vertical wall. The ladder lies in a vertical plane perpendicular to the wall and makes an angle  $\alpha$  with the horizontal ground, where  $\tan \alpha = \frac{4}{3}$ , as shown in Figure 1. The coefficient of friction between the ladder and the ground is  $\mu$ . A woman of mass 60 kg stands on the ladder at the point  $C$ , where  $AC = 3$  m. The ladder is on the point of slipping. The ladder is modelled as a uniform rod and the woman as a particle.

Find the value of  $\mu$ .

(9)

## Question 4

At time  $t$  seconds the velocity of a particle  $P$  is  $[(4t-5)\mathbf{i} + 3\mathbf{j}] \text{ m s}^{-1}$ . When  $t = 0$ , the position vector of  $P$  is  $(2\mathbf{i} + 5\mathbf{j}) \text{ m}$ , relative to a fixed origin  $O$ .

(a) Find the value of  $t$  when the velocity of  $P$  is parallel to the vector  $\mathbf{j}$ . (1)

(b) Find an expression for the position vector of  $P$  at time  $t$  seconds. (4)

A second particle  $Q$  moves with constant velocity  $(-2\mathbf{i} + c\mathbf{j}) \text{ m s}^{-1}$ . When  $t = 0$ , the position vector of  $Q$  is  $(11\mathbf{i} + 2\mathbf{j}) \text{ m}$ . The particles  $P$  and  $Q$  collide at the point with position vector  $(d\mathbf{i} + 14\mathbf{j}) \text{ m}$ .

(c) Find (5)

(i) the value of  $c$ ,

(ii) the value of  $d$ .

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## Question 5

The point  $A$  lies on a rough plane inclined at an angle  $\theta$  to the horizontal, where  $\sin \theta = \frac{24}{25}$ .

A particle  $P$  is projected from  $A$ , up a line of greatest slope of the plane, with speed  $U \text{ m s}^{-1}$ . The mass of  $P$  is  $2 \text{ kg}$  and the coefficient of friction between  $P$  and the plane is  $\frac{5}{12}$ . The particle comes to instantaneous rest at the point  $B$  on the plane, where  $AB = 1.5 \text{ m}$ . It then moves back down the plane to  $A$ .

(a) Find the work done against friction as  $P$  moves from  $A$  to  $B$ . (4)

(b) Use the work-energy principle to find the value of  $U$ . (4)

(c) Find the speed of  $P$  when it returns to  $A$ . (3)

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## Question 6

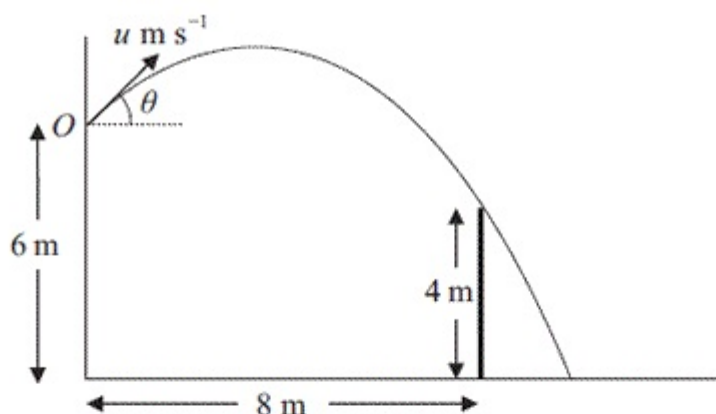


Figure 2

A ball is thrown from a point  $O$ , which is 6 m above horizontal ground. The ball is projected with speed  $u \text{ m s}^{-1}$  at an angle  $\theta$  above the horizontal. There is a thin vertical post which is 4 m high and 8 m horizontally away from the vertical through  $O$ , as shown in Figure 2. The ball passes just above the top of the post 2 s after projection. The ball is modelled as a particle.

(a) Show that  $\tan \theta = 2.2$  (5)

(b) Find the value of  $u$ . (2)

The ball hits the ground  $T$  seconds after projection.

(c) Find the value of  $T$ . (3)

Immediately before the ball hits the ground the direction of motion of the ball makes an angle  $\alpha$  with the horizontal.

(d) Find  $\alpha$ . (5)

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## Question 7

A particle  $A$  of mass  $m$  is moving with speed  $u$  on a smooth horizontal floor when it collides directly with another particle  $B$ , of mass  $3m$ , which is at rest on the floor. The coefficient of restitution between the particles is  $e$ . The direction of motion of  $A$  is reversed by the collision.

(a) Find, in terms of  $e$  and  $u$ ,

(i) the speed of  $A$  immediately after the collision,

(ii) the speed of  $B$  immediately after the collision.

(7)

After being struck by  $A$  the particle  $B$  collides directly with another particle  $C$ , of mass  $4m$ , which is at rest on the floor. The coefficient of restitution between  $B$  and  $C$  is  $2e$ . Given that the direction of motion of  $B$  is reversed by this collision,

(b) find the range of possible values of  $e$ ,

(6)

(c) determine whether there will be a second collision between  $A$  and  $B$ .

(3)

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