Core-Maths-C2 - 2009-January

Question 1

Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(3-2x)^5$, giving each term in its simplest form.

(4)

Question 2

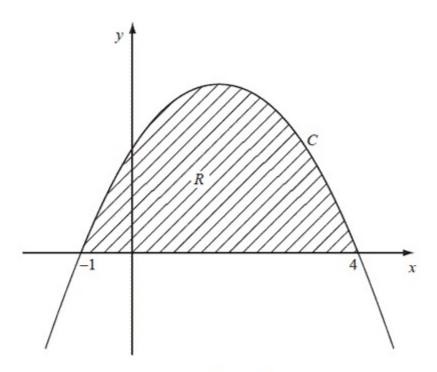


Figure 1

Figure 1 shows part of the curve C with equation y = (1+x)(4-x).

The curve intersects the x-axis at x = -1 and x = 4. The region R, shown shaded in Figure 1, is bounded by C and the x-axis.

Use calculus to find the exact area of R.

(5)

$$y = \sqrt{10x - x^2}.$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	1	1.4	1.8	2.2	2.6	3
y	3	3.47			4.39	

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximation for the value of $\int_{1}^{3} \sqrt{(10x-x^2)} dx$.

(4)

Question 4

Given that $0 \le x \le 4$ and

$$\log_5(4-x) - 2\log_5 x = 1$$
,

find the value of x.

(6)

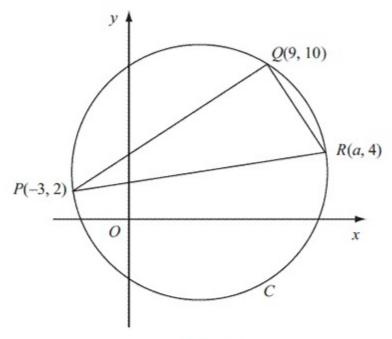


Figure 2

The points P(-3, 2), Q(9, 10) and R(a, 4) lie on the circle C, as shown in Figure 2. Given that PR is a diameter of C,

(a) show that a = 13,

(3)

(b) find an equation for C.

(5)

Question 6

$$f(x) = x^4 + 5x^3 + ax + b$$
,

where a and b are constants.

The remainder when f(x) is divided by (x - 2) is equal to the remainder when f(x) is divided by (x + 1).

(a) Find the value of a.

(5)

Given that (x + 3) is a factor of f(x),

(b) find the value of b.

(3)

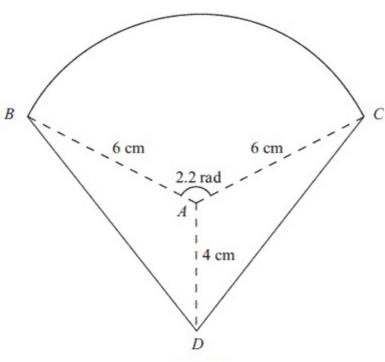


Figure 3

The shape BCD shown in Figure 3 is a design for a logo.

The straight lines DB and DC are equal in length. The curve BC is an arc of a circle with centre A and radius 6 cm. The size of $\angle BAC$ is 2.2 radians and AD = 4 cm.

Find

- (a) the area of the sector BAC, in cm², (2)
- (b) the size of ∠DAC, in radians to 3 significant figures,(2)
- (c) the complete area of the logo design, to the nearest cm².

 (4)

(a) Show that the equation

$$4\sin^2 x + 9\cos x - 6 = 0$$

can be written as

$$4\cos^2 x - 9\cos x + 2 = 0$$
.

(2)

(b) Hence solve, for $0 \le x < 720^\circ$,

$$4\sin^2 x + 9\cos x - 6 = 0$$

giving your answers to 1 decimal place.

(6)

Question 9

The first three terms of a geometric series are (k + 4), k and (2k - 15) respectively, where k is a positive constant.

(a) Show that $k^2 - 7k - 60 = 0$.

(4)

(b) Hence show that k = 12.

(2)

(c) Find the common ratio of this series.

(2)

(d) Find the sum to infinity of this series.

(2)

A solid right circular cylinder has radius r cm and height h cm.

The total surface area of the cylinder is 800 cm².

(a) Show that the volume, $V \text{ cm}^3$, of the cylinder is given by

$$V = 400r - \pi r^3. (4)$$

Given that r varies,

(b) use calculus to find the maximum value of V, to the nearest cm³.

(6)

(c) Justify that the value of V you have found is a maximum.

(2)