

Core-Maths-C3 - 2008-June

Question 1

The point P lies on the curve with equation

$$y = 4e^{2x+1}.$$

The y -coordinate of P is 8.

- (a) Find, in terms of $\ln 2$, the x -coordinate of P . (2)
- (b) Find the equation of the tangent to the curve at the point P in the form $y = ax + b$, where a and b are exact constants to be found. (4)
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Question 2

$$f(x) = 5\cos x + 12\sin x$$

Given that $f(x) = R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$,

- (a) find the value of R and the value of α to 3 decimal places. (4)
- (b) Hence solve the equation
- $$5\cos x + 12\sin x = 6$$
- for $0 \leq x < 2\pi$. (5)
- (c) (i) Write down the maximum value of $5\cos x + 12\sin x$. (1)
- (ii) Find the smallest positive value of x for which this maximum value occurs. (2)
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Question 3

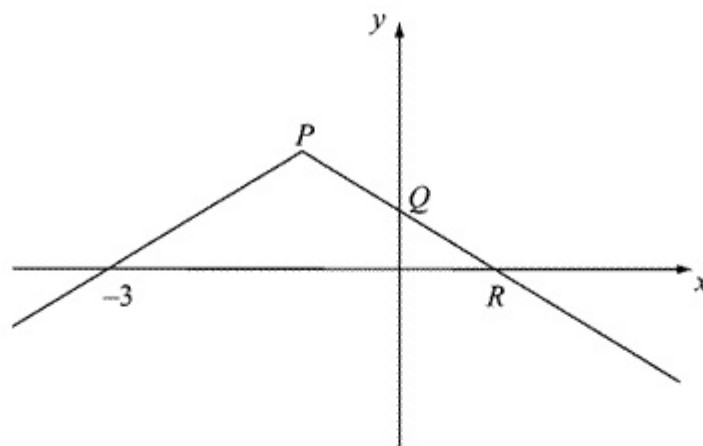


Figure 1

Figure 1 shows the graph of $y = f(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point P .

The graph cuts the y -axis at the point Q and the x -axis at the points $(-3, 0)$ and R .

Sketch, on separate diagrams, the graphs of

(a) $y = |f(x)|$. (2)

(b) $y = f(-x)$. (2)

Given that $f(x) = 2 - |x + 1|$,

(c) find the coordinates of the points P , Q and R , (3)

(d) solve $f(x) = \frac{1}{2}x$. (5)

Question 4

The function f is defined by

$$f : x \mapsto \frac{2(x-1)}{x^2-2x-3} - \frac{1}{x-3}, \quad x > 3.$$

(a) Show that $f(x) = \frac{1}{x+1}$, $x > 3$. (4)

(b) Find the range of f . (2)

(c) Find $f^{-1}(x)$. State the domain of this inverse function. (3)

The function g is defined by

$$g : x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(d) Solve $fg(x) = \frac{1}{8}$. (3)

Question 5

(a) Given that $\sin^2 \theta + \cos^2 \theta \equiv 1$, show that $1 + \cot^2 \theta \equiv \operatorname{cosec}^2 \theta$. (2)

(b) Solve, for $0 \leq \theta < 180^\circ$, the equation

$$2 \cot^2 \theta - 9 \operatorname{cosec} \theta = 3,$$

giving your answers to 1 decimal place. (6)

Question 6

(a) Differentiate with respect to x ,

(i) $e^{3x}(\sin x + 2 \cos x)$, (3)

(ii) $x^3 \ln(5x + 2)$. (3)

Given that $y = \frac{3x^2 + 6x - 7}{(x+1)^2}$, $x \neq -1$,

(b) show that $\frac{dy}{dx} = \frac{20}{(x+1)^3}$. (5)

(c) Hence find $\frac{d^2y}{dx^2}$ and the real values of x for which $\frac{d^2y}{dx^2} = -\frac{15}{4}$. (3)

Question 7

$$f(x) = 3x^3 - 2x - 6$$

(a) Show that $f(x) = 0$ has a root, α , between $x = 1.4$ and $x = 1.45$ (2)

(b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \quad x \neq 0. \quad (3)$$

(c) Starting with $x_0 = 1.43$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of x_1 , x_2 and x_3 , giving your answers to 4 decimal places. (3)

(d) By choosing a suitable interval, show that $\alpha = 1.435$ is correct to 3 decimal places. (3)
