Statistics-S2 - 2009-January

Question 1

A botanist is studying the distribution of daisies in a field. The field is divided into a number of equal sized squares. The mean number of daisies per square is assumed to be 3. The daisies are distributed randomly throughout the field.

Find the probability that, in a randomly chosen square there will be

(a) more than 2 daisies,

(3)

(b) either 5 or 6 daisies.

(2)

The botanist decides to count the number of daisies, x, in each of 80 randomly selected squares within the field. The results are summarised below

$$\sum x = 295 \qquad \sum x^2 = 1386$$

(c) Calculate the mean and the variance of the number of daisies per square for the 80 squares. Give your answers to 2 decimal places.

(3)

(d) Explain how the answers from part (c) support the choice of a Poisson distribution as a model.

(1)

(e) Using your mean from part (c), estimate the probability that exactly 4 daisies will be found in a randomly selected square.

(2)

Question 2

The continuous random variable X is uniformly distributed over the interval $[-2, 7]$.

(a) Write down fully the probability density function $f(x)$ of X.

(2)

(b) Sketch the probability density function $f(x)$ of X.

(2)

Find

(c) $E(X^2)$,

(3)

(d) $P(-0.2 < X < 0.6)$.

(2)

Question 3

A single observation x is to be taken from a Binomial distribution $B(20, p)$.

This observation is used to test $H_0 : p = 0.3$ against $H_1 : p \neq 0.3$

(a) Using a 5% level of significance, find the critical region for this test.
The probability of rejecting either tail should be as close as possible to 2.5%.

(3)

(b) State the actual significance level of this test.

(2)

The actual value of x obtained is 3.

(c) State a conclusion that can be drawn based on this value giving a reason for your answer.

(2)

Question 4

The length of a telephone call made to a company is denoted by the continuous random variable T. It is modelled by the probability density function

$$\mathrm{f}(t)=\begin{cases} kt & 0 \leqslant t \leqslant 10 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the value of k is $\frac{1}{50}$.

(3)

(b) Find $\mathrm{P}(T > 6)$.

(2)

(c) Calculate an exact value for $\mathrm{E}(T)$ and for $\mathrm{Var}(T)$.

(5)

(d) Write down the mode of the distribution of T.

(1)

It is suggested that the probability density function, $\mathrm{f}(t)$, is not a good model for T.

(e) Sketch the graph of a more suitable probability density function for T.

(1)

Question 5

A factory produces components of which 1% are defective. The components are packed in boxes of 10. A box is selected at random.

(a) Find the probability that the box contains exactly one defective component.

(2)

(b) Find the probability that there are at least 2 defective components in the box.

(3)

(c) Using a suitable approximation, find the probability that a batch of 250 components contains between 1 and 4 (inclusive) defective components.

(4)

Question 6

A web server is visited on weekdays, at a rate of 7 visits per minute. In a random one minute on a Saturday the web server is visited 10 times.

(a) (i) Test, at the 10% level of significance, whether or not there is evidence that the rate of visits is greater on a Saturday than on weekdays. State your hypotheses clearly.

(ii) State the minimum number of visits required to obtain a significant result.

(7)

(b) State an assumption that has been made about the visits to the server.

(1)

In a random two minute period on a Saturday the web server is visited 20 times.

(c) Using a suitable approximation, test at the 10% level of significance, whether or not the rate of visits is greater on a Saturday.

(6)

Question 7

A random variable X has probability density function given by

$$f(x) = \begin{cases} -\frac{2}{9}x + \frac{8}{9} & 1 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the cumulative distribution function $F(x)$ can be written in the form $ax^2 + bx + c$, for $1 \leqslant x \leqslant 4$ where a, b and c are constants.

(3)

(b) Define fully the cumulative distribution function $F(x)$.

(2)

(c) Show that the upper quartile of X is 2.5 and find the lower quartile.

(6)

Given that the median of X is 1.88

(d) describe the skewness of the distribution. Give a reason for your answer.

(2)