

Core-Maths-C4 - 2011-January

Question 1

Use integration to find the exact value of

$$\int_0^{\frac{\pi}{2}} x \sin 2x \, dx$$

(6)

Question 2

The current, I amps, in an electric circuit at time t seconds is given by

$$I = 16 - 16(0.5)^t, \quad t \geq 0$$

Use differentiation to find the value of $\frac{dI}{dt}$ when $t = 3$.

Give your answer in the form $\ln a$, where a is a constant.

(5)

Question 3

(a) Express $\frac{5}{(x-1)(3x+2)}$ in partial fractions. (3)

(b) Hence find $\int \frac{5}{(x-1)(3x+2)} dx$, where $x > 1$. (3)

(c) Find the particular solution of the differential equation

$$(x-1)(3x+2)\frac{dy}{dx} = 5y, \quad x > 1,$$

for which $y = 8$ at $x = 2$. Give your answer in the form $y = f(x)$. (6)

Question 4

Relative to a fixed origin O , the point A has position vector $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and the point B has position vector $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$. The points A and B lie on a straight line l .

(a) Find \overrightarrow{AB} . (2)

(b) Find a vector equation of l . (2)

The point C has position vector $2\mathbf{i} + p\mathbf{j} - 4\mathbf{k}$ with respect to O , where p is a constant. Given that AC is perpendicular to l , find

(c) the value of p , (4)

(d) the distance AC . (2)

Question 5

- (a) Use the binomial theorem to expand

$$(2-3x)^{-2}, \quad |x| < \frac{2}{3},$$

in ascending powers of x , up to and including the term in x^3 . Give each coefficient as a simplified fraction.

(5)

$$f(x) = \frac{a+bx}{(2-3x)^2}, \quad |x| < \frac{2}{3}, \quad \text{where } a \text{ and } b \text{ are constants.}$$

In the binomial expansion of $f(x)$, in ascending powers of x , the coefficient of x is 0 and the coefficient of x^2 is $\frac{9}{16}$. Find

- (b) the value of a and the value of b ,

(5)

- (c) the coefficient of x^3 , giving your answer as a simplified fraction.

(3)

Question 6

The curve C has parametric equations

$$x = \ln t, \quad y = t^2 - 2, \quad t > 0$$

Find

(a) an equation of the normal to C at the point where $t = 3$, (6)

(b) a cartesian equation of C . (3)

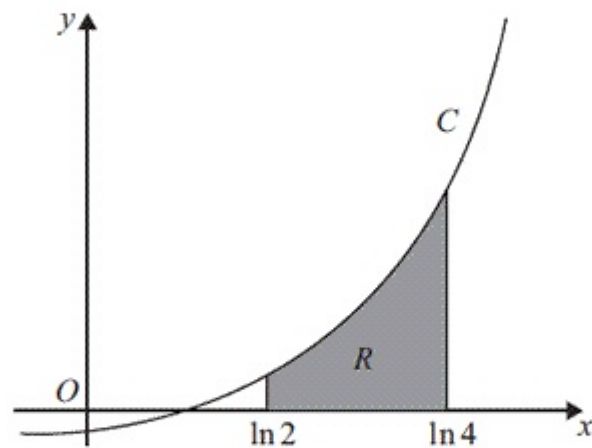


Figure 1

The finite area R , shown in Figure 1, is bounded by C , the x -axis, the line $x = \ln 2$ and the line $x = \ln 4$. The area R is rotated through 360° about the x -axis.

(c) Use calculus to find the exact volume of the solid generated. (6)

Question 7

$$I = \int_2^5 \frac{1}{4 + \sqrt{x-1}} dx$$

- (a) Given that $y = \frac{1}{4 + \sqrt{x-1}}$, complete the table below with values of y corresponding to $x = 3$ and $x = 5$. Give your values to 4 decimal places.

x	2	3	4	5
y	0.2		0.1745	

(2)

- (b) Use the trapezium rule, with all of the values of y in the completed table, to obtain an estimate of I , giving your answer to 3 decimal places.

(4)

- (c) Using the substitution $x = (u-4)^2 + 1$, or otherwise, and integrating, find the exact value of I .

(8)
