

Statistics-S1 - 2011-June

Question 1

On a particular day the height above sea level, x metres, and the mid-day temperature, $y^{\circ}\text{C}$, were recorded in 8 north European towns. These data are summarised below

$$S_{xx} = 3\,535\,237.5 \quad \sum y = 181 \quad \sum y^2 = 4305 \quad S_{xy} = -23\,726.25$$

- (a) Find S_{yy} . (2)
- (b) Calculate, to 3 significant figures, the product moment correlation coefficient for these data. (2)
- (c) Give an interpretation of your coefficient. (1)

A student thought that the calculations would be simpler if the height above sea level, h , was measured in kilometres and used the variable $h = \frac{x}{1000}$ instead of x .

- (d) Write down the value of S_{hh} . (1)
- (e) Write down the value of the correlation coefficient between h and y . (1)

Question 2

The random variable $X \sim N(\mu, 5^2)$ and $P(X < 23) = 0.9192$

- (a) Find the value of μ . (4)
 - (b) Write down the value of $P(\mu < X < 23)$. (1)
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Question 3

The discrete random variable Y has probability distribution

y	1	2	3	4
$P(Y=y)$	a	b	0.3	c

where a , b and c are constants.

The cumulative distribution function $F(y)$ of Y is given in the following table

y	1	2	3	4
$F(y)$	0.1	0.5	d	1.0

where d is a constant.

(a) Find the value of a , the value of b , the value of c and the value of d .

(5)

(b) Find $P(3Y + 2 \geq 8)$.

(2)

Question 4

Past records show that the times, in seconds, taken to run 100 m by children at a school can be modelled by a normal distribution with a mean of 16.12 and a standard deviation of 1.60

A child from the school is selected at random.

(a) Find the probability that this child runs 100 m in less than 15 s.

(3)

On sports day the school awards certificates to the fastest 30% of the children in the 100 m race.

(b) Estimate, to 2 decimal places, the slowest time taken to run 100 m for which a child will be awarded a certificate.

(4)

Question 5

A class of students had a sudoku competition. The time taken for each student to complete the sudoku was recorded to the nearest minute and the results are summarised in the table below.

Time	Mid-point, x	Frequency, f
2 - 8	5	2
9 - 12		7
13 - 15	14	5
16 - 18	17	8
19 - 22	20.5	4
23 - 30	26.5	4

(You may use $\sum fx^2 = 8603.75$)

(a) Write down the mid-point for the 9 - 12 interval. (1)

(b) Use linear interpolation to estimate the median time taken by the students. (2)

(c) Estimate the mean and standard deviation of the times taken by the students. (5)

The teacher suggested that a normal distribution could be used to model the times taken by the students to complete the sudoku.

(d) Give a reason to support the use of a normal distribution in this case. (1)

On another occasion the teacher calculated the quartiles for the times taken by the students to complete a different sudoku and found

$$Q_1 = 8.5 \quad Q_2 = 13.0 \quad Q_3 = 21.0$$

(e) Describe, giving a reason, the skewness of the times on this occasion. (2)

Question 6

Jake and Kamil are sometimes late for school.

The events J and K are defined as follows

J = the event that Jake is late for school

K = the event that Kamil is late for school

$$P(J) = 0.25, \quad P(J \cap K) = 0.15 \quad \text{and} \quad P(J' \cap K') = 0.7$$

On a randomly selected day, find the probability that

(a) at least one of Jake or Kamil are late for school, (1)

(b) Kamil is late for school. (2)

Given that Jake is late for school,

(c) find the probability that Kamil is late. (3)

The teacher suspects that Jake being late for school and Kamil being late for school are linked in some way.

(d) Determine whether or not J and K are statistically independent. (2)

(e) Comment on the teacher's suspicion in the light of your calculation in (d). (1)

Question 7

A teacher took a random sample of 8 children from a class. For each child the teacher recorded the length of their left foot, f cm, and their height, h cm. The results are given in the table below.

f	23	26	23	22	27	24	20	21
h	135	144	134	136	140	134	130	132

(You may use $\sum f = 186$ $\sum h = 1085$ $S_{ff} = 39.5$ $S_{hh} = 139.875$ $\sum fh = 25291$)

- (a) Calculate S_{fh} (2)
- (b) Find the equation of the regression line of h on f in the form $h = a + bf$.
Give the value of a and the value of b correct to 3 significant figures. (5)
- (c) Use your equation to estimate the height of a child with a left foot length of 25 cm. (2)
- (d) Comment on the reliability of your estimate in (c), giving a reason for your answer. (2)

The left foot length of the teacher is 25 cm.

- (e) Give a reason why the equation in (b) should not be used to estimate the teacher's height. (1)

Question 8

A spinner is designed so that the score S is given by the following probability distribution.

s	0	1	2	4	5
$P(S = s)$	p	0.25	0.25	0.20	0.20

(a) Find the value of p . (2)

(b) Find $E(S)$. (2)

(c) Show that $E(S^2) = 9.45$ (2)

(d) Find $\text{Var}(S)$. (2)

Tom and Jess play a game with this spinner. The spinner is spun repeatedly and S counters are awarded on the outcome of each spin. If S is even then Tom receives the counters and if S is odd then Jess receives them. The first player to collect 10 or more counters is the winner.

(e) Find the probability that Jess wins after 2 spins. (2)

(f) Find the probability that Tom wins after exactly 3 spins. (4)

(g) Find the probability that Jess wins after exactly 3 spins. (3)
