Mechanics-M1 - 2012-January

Question 1

A railway truck P, of mass m kg, is moving along a straight horizontal track with speed $15\,\text{m s}^{-1}$. Truck P collides with a truck Q of mass 3000 kg, which is at rest on the same track. Immediately after the collision the speed of P is $3\,\text{m s}^{-1}$ and the speed of Q is $9\,\text{m s}^{-1}$. The direction of motion of P is reversed by the collision.

Modelling the trucks as particles, find

(a) the magnitude of the impulse exerted by P on Q,

(2)

(b) the value of m.

(3)

Question 2

A car of mass 1000 kg is towing a caravan of mass 750 kg along a straight horizontal road. The caravan is connected to the car by a tow-bar which is parallel to the direction of motion of the car and the caravan. The tow-bar is modelled as a light rod. The engine of the car provides a constant driving force of 3200 N. The resistances to the motion of the car and the caravan are modelled as constant forces of magnitude 800 newtons and R newtons respectively.

Given that the acceleration of the car and the caravan is $0.88\,\text{m s}^{-2}$,

(a) show that $R = 860$,

(3)

(b) find the tension in the tow-bar.

(3)

Question 3

Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ acting on a particle P are given by

$$\mathbf{F}_1 = (7\mathbf{i} - 9\mathbf{j})\ \text{N}$$
$$\mathbf{F}_2 = (5\mathbf{i} + 6\mathbf{j})\ \text{N}$$
$$\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j})\ \text{N}$$

where p and q are constants.

Given that P is in equilibrium,

(a) find the value of p and the value of q.

(3)

The force $\mathbf{F}_3$ is now removed. The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$.
Find

(b) the magnitude of $\mathbf{R}$,

(2)

(c) the angle, to the nearest degree, that the direction of $\mathbf{R}$ makes with $\mathbf{j}$.

(3)

Question 4

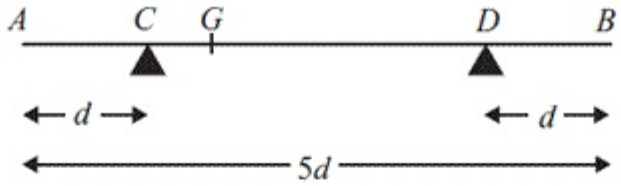

Figure 1

A non-uniform rod AB, of mass m and length $5d$, rests horizontally in equilibrium on two supports at C and D, where $AC = DB = d$, as shown in Figure 1. The centre of mass of the rod is at the point G. A particle of mass $\frac{5}{2}m$ is placed on the rod at B and the rod is on the point of tipping about D.

(a) Show that $GD = \frac{5}{2}d$.

(4)

The particle is moved from B to the mid-point of the rod and the rod remains in equilibrium.

(b) Find the magnitude of the normal reaction between the support at D and the rod.

(5)

Question 5

A stone is projected vertically upwards from a point A with speed $u\,\text{m s}^{-1}$. After projection the stone moves freely under gravity until it returns to A. The time between the instant that the stone is projected and the instant that it returns to A is $3\frac{4}{7}$ seconds.

Modelling the stone as a particle,

(a) show that $u = 17\frac{1}{2}$,

(3)

(b) find the greatest height above A reached by the stone,

(2)

(c) find the length of time for which the stone is at least $6\frac{3}{5}$ m above A.

(6)

Question 6

A car moves along a straight horizontal road from a point A to a point B, where $AB = 885$ m. The car accelerates from rest at A to a speed of $15\,\text{m s}^{-1}$ at a constant rate $a\,\text{m s}^{-2}$. The time for which the car accelerates is $\frac{1}{3}T$ seconds. The car maintains the speed of $15\,\text{m s}^{-1}$ for T seconds. The car then decelerates at a constant rate of $2.5\,\text{m s}^{-2}$ stopping at B.

(a) Find the time for which the car decelerates.

(2)

(b) Sketch a speed-time graph for the motion of the car.

(2)

(c) Find the value of T.

(4)

(d) Find the value of a.

(2)

(e) Sketch an acceleration-time graph for the motion of the car.

(3)

Question 7

[*In this question, the unit vectors* **i** *and* **j** *are due east and due north respectively. Position vectors are relative to a fixed origin O.*]

A boat P is moving with constant velocity $(-4\mathbf{i}+8\mathbf{j})$ km h^{-1}.

(a) Calculate the speed of P.

(2)

When $t = 0$, the boat P has position vector $(2\mathbf{i}-8\mathbf{j})$ km. At time t hours, the position vector of P is $\mathbf{p}$ km.

(b) Write down $\mathbf{p}$ in terms of t.

(1)

A second boat Q is also moving with constant velocity. At time t hours, the position vector of Q is $\mathbf{q}$ km, where

$$\mathbf{q} = 18\mathbf{i} + 12\mathbf{j} - t(6\mathbf{i}+8\mathbf{j})$$

Find

(c) the value of t when P is due west of Q,

(3)

(d) the distance between P and Q when P is due west of Q.

(3)

Question 8

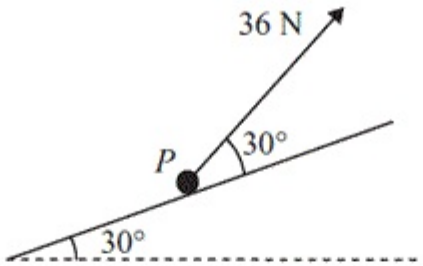

Figure 2

A particle P of mass 4 kg is moving up a fixed rough plane at a constant speed of $16\,\text{m s}^{-1}$ under the action of a force of magnitude 36 N. The plane is inclined at 30° to the horizontal. The force acts in the vertical plane containing the line of greatest slope of the plane through P, and acts at 30° to the inclined plane, as shown in Figure 2. The coefficient of friction between P and the plane is μ. Find

(a) the magnitude of the normal reaction between P and the plane,

(4)

(b) the value of μ.

(5)

The force of magnitude 36 N is removed.

(c) Find the distance that P travels between the instant when the force is removed and the instant when it comes to rest.

(5)