Core-Maths-C1 - 2007-January

Question 1

Given that

$$y = 4x^3 - 1 + 2x^{\frac{1}{2}}, \quad x > 0,$$

find $\dfrac{dy}{dx}$.

(4)

Question 2

(a) Express $\sqrt{108}$ in the form $a\sqrt{3}$, where a is an integer.

(1)

(b) Express $(2 - \sqrt{3})^2$ in the form $b + c\sqrt{3}$, where b and c are integers to be found.

(3)

Question 3

Given that
$$f(x) = \frac{1}{x}, \quad x \neq 0,$$

(a) sketch the graph of $y = f(x) + 3$ and state the equations of the asymptotes.

(4)

(b) Find the coordinates of the point where $y = f(x) + 3$ crosses a coordinate axis.

(2)

Question 4

Solve the simultaneous equations

$$y = x - 2,$$

$$y^2 + x^2 = 10.$$

(7)

Question 5

The equation $2x^2 - 3x - (k + 1) = 0$, where k is a constant, has no real roots.

Find the set of possible values of k.

(4)

Question 6

(a) Show that $(4 + 3\sqrt{x})^2$ can be written as $16 + k\sqrt{x} + 9x$, where k is a constant to be found.

(2)

(b) Find $\int (4 + 3\sqrt{x})^2 dx$.

(3)

Question 7

The curve C has equation $y = f(x)$, $x \neq 0$, and the point $P(2, 1)$ lies on C. Given that

$$f'(x) = 3x^2 - 6 - \frac{8}{x^2},$$

(a) find $f(x)$.

(5)

(b) Find an equation for the tangent to C at the point P, giving your answer in the form $y = mx + c$, where m and c are integers.

(4)

Question 8

The curve C has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2$, $x > 0$.

(a) Find an expression for $\dfrac{dy}{dx}$.

(3)

(b) Show that the point P (4, 8) lies on C.

(1)

(c) Show that an equation of the normal to C at the point P is

$$3y = x + 20.$$

(4)

The normal to C at P cuts the x-axis at the point Q.

(d) Find the length PQ, giving your answer in a simplified surd form.

(3)

Question 9

Ann has some sticks that are all of the same length. She arranges them in squares and has made the following 3 rows of patterns:

Row 1   □

Row 2   ⊏□

Row 3   ⊏⊏□

She notices that 4 sticks are required to make the single square in the first row, 7 sticks to make 2 squares in the second row and in the third row she needs 10 sticks to make 3 squares.

(a) Find an expression, in terms of n, for the number of sticks required to make a similar arrangement of n squares in the nth row.

(3)

Ann continues to make squares following the same pattern. She makes 4 squares in the 4th row and so on until she has completed 10 rows.

(b) Find the total number of sticks Ann uses in making these 10 rows.

(3)

Ann started with 1750 sticks. Given that Ann continues the pattern to complete k rows but does not have sufficient sticks to complete the $(k+1)$th row,

(c) show that k satisfies $(3k-100)(k+35) < 0$.

(4)

(d) Find the value of k.

(2)

Question 10

(a) On the same axes sketch the graphs of the curves with equations

    (i) $y = x^2(x-2)$,

                                               (3)

    (ii) $y = x(6-x)$,

                                               (3)

and indicate on your sketches the coordinates of all the points where the curves cross the x-axis.

(b) Use algebra to find the coordinates of the points where the graphs intersect.

                                               (7)