Core-Maths-C2 - 2008-June

Question 1

$$f(x) = 2x^3 - 3x^2 - 39x + 20$$

(a) Use the factor theorem to show that (x + 4) is a factor of f(x).

(2)

(b) Factorise f(x) completely.

(4)

Question 2

$$y = \sqrt{(5^x + 2)}$$

(a) Complete the table below, giving the values of y to 3 decimal places.

x	0	0.5	1	1.5	2
у			2.646	3.630	

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximation for the value of $\int_0^2 \sqrt{(5^x + 2)} dx$.

(4)

Question 3

(a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1 + ax)^{10}$, where a is a non-zero constant. Give each term in its simplest form.

(4)

Given that, in this expansion, the coefficient of x^3 is double the coefficient of x^2 ,

(b) find the value of a.

(2)

Question 4

- (a) Find, to 3 significant figures, the value of x for which $5^x = 7$.
- (b) Solve the equation $5^{2s} 12(5^x) + 35 = 0$. (4)

Question 5

The circle C has centre (3, 1) and passes through the point P(8, 3).

(a) Find an equation for C.

(4)

(b) Find an equation for the tangent to C at P, giving your answer in the form ax + by + c = 0, where a, b and c are integers.

(5)

Question 6

A geometric series has first term 5 and common ratio $\frac{4}{5}$.

Calculate

(a) the 20th term of the series, to 3 decimal places,

(2)

(b) the sum to infinity of the series.

(2)

Given that the sum to k terms of the series is greater than 24.95,

(c) show that
$$k > \frac{\log 0.002}{\log 0.8}$$
,

(4)

(d) find the smallest possible value of k.

(1)

Question 7

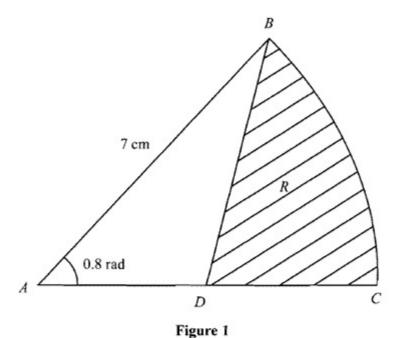


Figure 1 shows ABC, a sector of a circle with centre A and radius 7 cm.

Given that the size of $\angle BAC$ is exactly 0.8 radians, find

(a) the length of the arc BC,

(2)

(b) the area of the sector ABC.

(2)

The point D is the mid-point of AC. The region R, shown shaded in Figure 1, is bounded by CD, DB and the arc BC.

Find

(c) the perimeter of R, giving your answer to 3 significant figures,

(4)

(d) the area of R, giving your answer to 3 significant figures.

(4)

Question 8

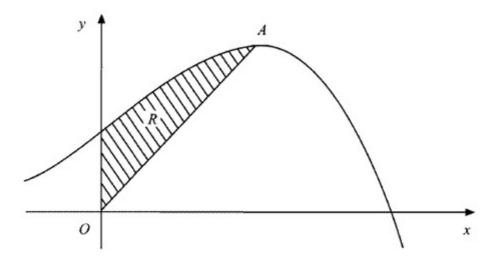


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = 10 + 8x + x^2 - x^3$.

The curve has a maximum turning point A.

(a) Using calculus, show that the x-coordinate of A is 2.

(3)

The region R, shown shaded in Figure 2, is bounded by the curve, the y-axis and the line from O to A, where O is the origin.

(b) Using calculus, find the exact area of R.

(8)

Question 9

Solve, for $0 \le x < 360^{\circ}$,

(a)
$$\sin(x-20^\circ) = \frac{1}{\sqrt{2}}$$

(b)
$$\cos 3x = -\frac{1}{2}$$
 (6)