# Core-Maths-C4 - 2013-January

## Question 1

Given

$$f(x) = (2 + 3x)^{-3}, \qquad |x| < \frac{2}{3}$$

find the binomial expansion of $f(x)$, in ascending powers of $x$, up to and including the term in $x^3$.

Give each coefficient as a simplified fraction.

**(5)**

## Question 2

(a) Use integration to find

$$\int \frac{1}{x^3} \ln x \, dx$$

**(5)**

(b) Hence calculate

$$\int_1^2 \frac{1}{x^3} \ln x \, dx$$

**(2)**

## Question 3

Express $\dfrac{9x^2 + 20x - 10}{(x + 2)(3x - 1)}$ in partial fractions.

**(4)**

# Question 4

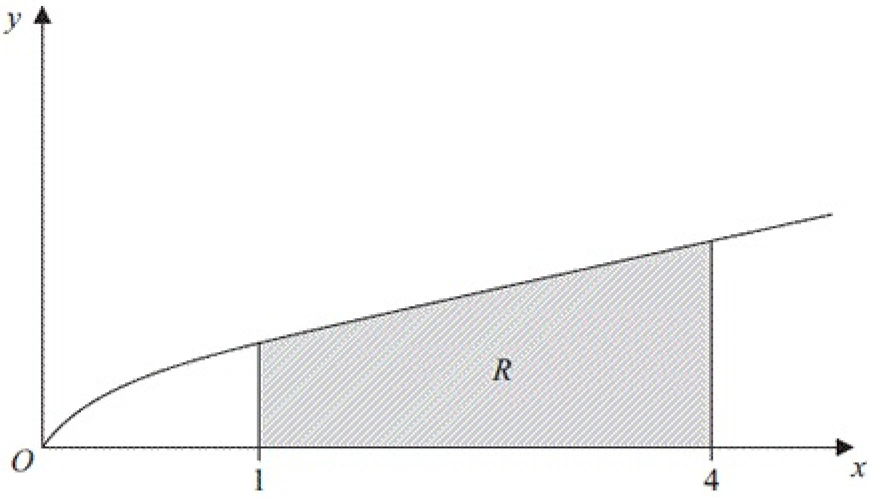


**Figure 1**

Figure 1 shows a sketch of part of the curve with equation $y = \frac{x}{1 + \sqrt{x}}$. The finite region $R$, shown shaded in Figure 1, is bounded by the curve, the $x$-axis, the line with equation $x = 1$ and the line with equation $x = 4$.

(a) Complete the table with the value of $y$ corresponding to $x = 3$, giving your answer to 4 decimal places.

**(1)**

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $y$ | 0.5 | 0.8284 | | 1.3333 |

(b) Use the trapezium rule, with all the values of $y$ in the completed table, to obtain an estimate of the area of the region $R$, giving your answer to 3 decimal places.

**(3)**

(c) Use the substitution $u = 1 + \sqrt{x}$, to find, by integrating, the exact area of $R$.

**(8)**

# Question 5

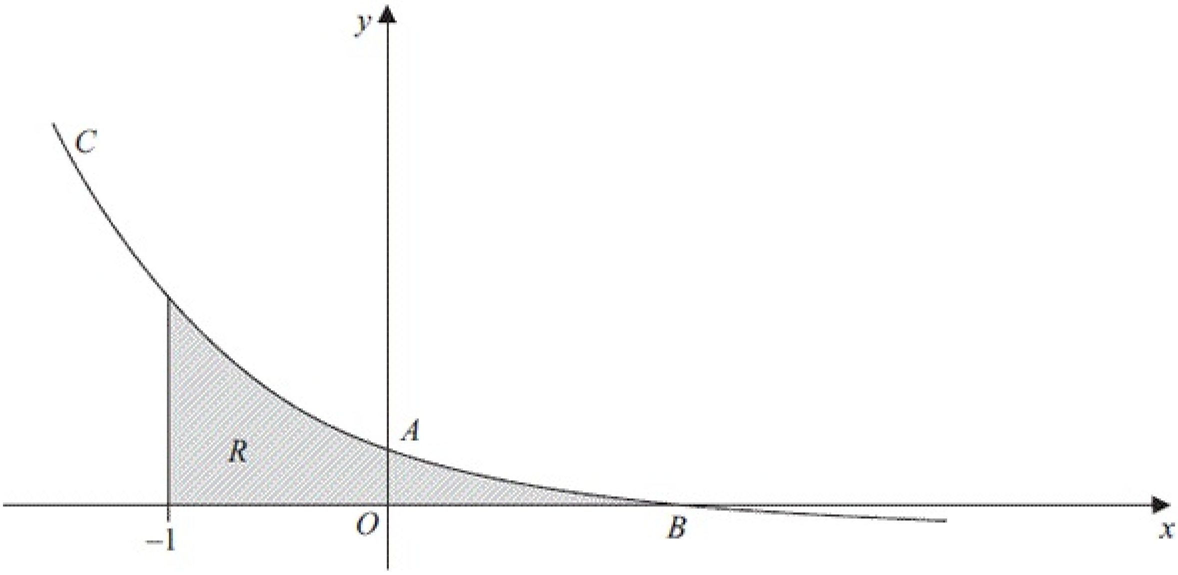


**Figure 2**

Figure 2 shows a sketch of part of the curve $C$ with parametric equations

$$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$$

The curve crosses the $y$-axis at the point $A$ and crosses the $x$-axis at the point $B$.

(a) Show that $A$ has coordinates (0, 3).

**(2)**

(b) Find the $x$ coordinate of the point $B$.

**(2)**

(c) Find an equation of the normal to $C$ at the point $A$.

**(5)**

The region $R$, as shown shaded in Figure 2, is bounded by the curve $C$, the line $x = -1$ and the $x$-axis.

(d) Use integration to find the exact area of $R$.

**(6)**

# Question 6

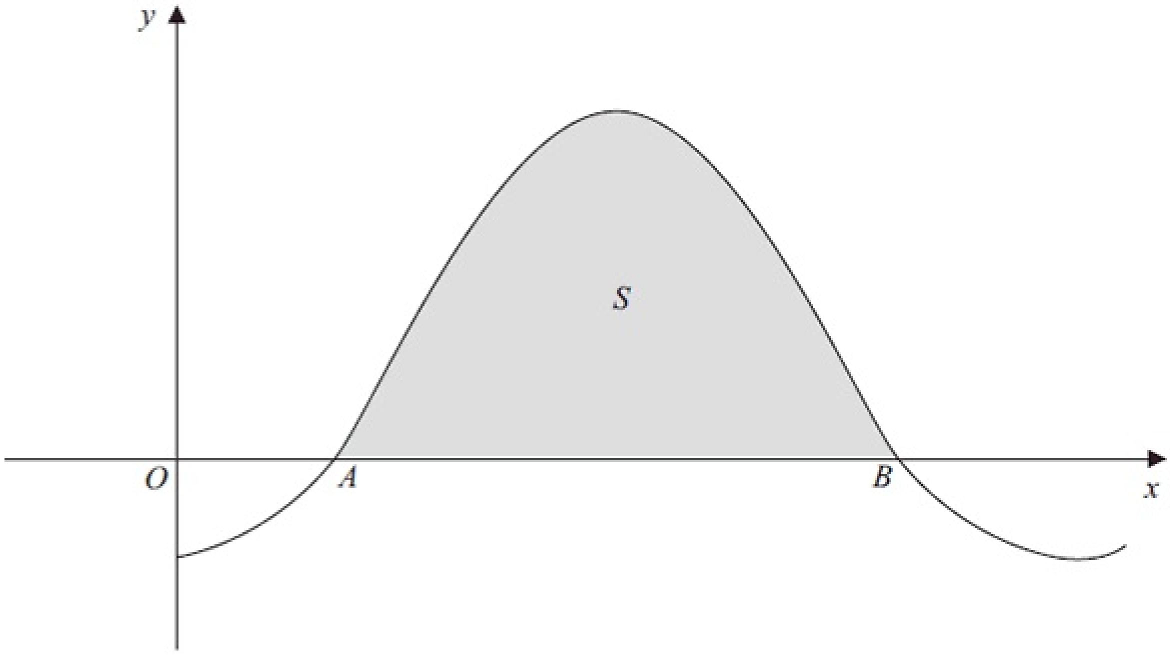


**Figure 3**

Figure 3 shows a sketch of part of the curve with equation $y = 1 - 2\cos x$, where $x$ is measured in radians. The curve crosses the $x$-axis at the point $A$ and at the point $B$.

(a) Find, in terms of $\pi$, the $x$ coordinate of the point $A$ and the $x$ coordinate of the point $B$.
**(3)**

The finite region $S$ enclosed by the curve and the $x$-axis is shown shaded in Figure 3. The region $S$ is rotated through $2\pi$ radians about the $x$-axis.

(b) Find, by integration, the exact value of the volume of the solid generated.
**(6)**

# Question 7

With respect to a fixed origin $O$, the lines $l_1$ and $l_2$ are given by the equations

$$l_1 : \mathbf{r} = (9\mathbf{i} + 13\mathbf{j} - 3\mathbf{k}) + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$$

$$l_2 : \mathbf{r} = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} + \mathbf{k})$$

where $\lambda$ and $\mu$ are scalar parameters.

(a) Given that $l_1$ and $l_2$ meet, find the position vector of their point of intersection.

**(5)**

(b) Find the acute angle between $l_1$ and $l_2$, giving your answer in degrees to 1 decimal place.

**(3)**

Given that the point $A$ has position vector $4\mathbf{i} + 16\mathbf{j} - 3\mathbf{k}$ and that the point $P$ lies on $l_1$ such that $AP$ is perpendicular to $l_1$,

(c) find the exact coordinates of $P$.

**(6)**