

Core-Maths-C2 - 2012-January

Question 1

A geometric series has first term $a = 360$ and common ratio $r = \frac{7}{8}$

Giving your answers to 3 significant figures where appropriate, find

- (a) the 20th term of the series, (2)
 - (b) the sum of the first 20 terms of the series, (2)
 - (c) the sum to infinity of the series. (2)
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Question 2

A circle C has centre $(-1, 7)$ and passes through the point $(0, 0)$. Find an equation for C .

(4)

Question 3

- (a) Find the first 4 terms of the binomial expansion, in ascending powers of x , of

$$\left(1 + \frac{x}{4}\right)^8$$

giving each term in its simplest form.

(4)

- (b) Use your expansion to estimate the value of $(1.025)^8$, giving your answer to 4 decimal places.

(3)

Question 4

Given that $y = 3x^2$,

(a) show that $\log_3 y = 1 + 2 \log_3 x$

(3)

(b) Hence, or otherwise, solve the equation

$$1 + 2 \log_3 x = \log_3 (28x - 9)$$

(3)

Question 5

$f(x) = x^3 + ax^2 + bx + 3$, where a and b are constants.

Given that when $f(x)$ is divided by $(x + 2)$ the remainder is 7,

(a) show that $2a - b = 6$

(2)

Given also that when $f(x)$ is divided by $(x - 1)$ the remainder is 4,

(b) find the value of a and the value of b .

(4)

Question 6

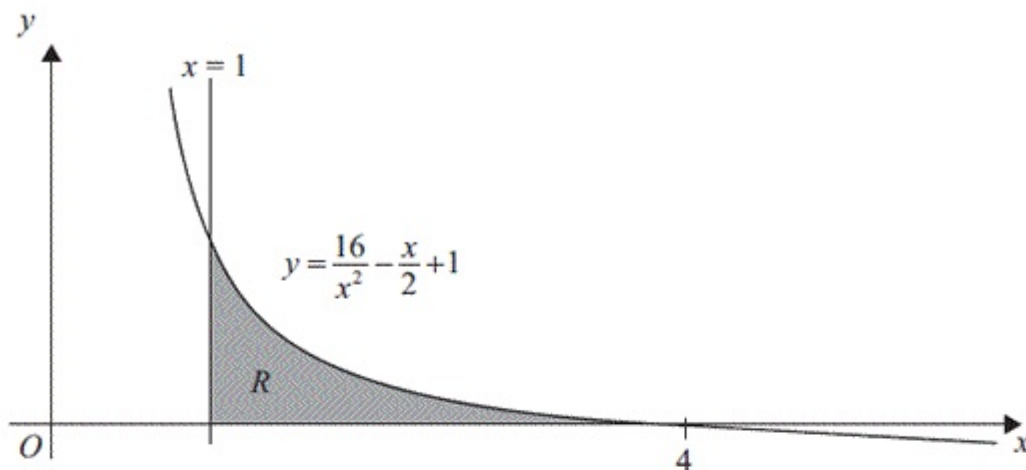


Figure 1

Figure 1 shows the graph of the curve with equation

$$y = \frac{16}{x^2} - \frac{x}{2} + 1, \quad x > 0$$

The finite region R , bounded by the lines $x = 1$, the x -axis and the curve, is shown shaded in Figure 1. The curve crosses the x -axis at the point $(4, 0)$.

- (a) Complete the table with the values of y corresponding to $x = 2$ and 2.5

x	1	1.5	2	2.5	3	3.5	4
y	16.5	7.361			1.278	0.556	0

(2)

- (b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of R , giving your answer to 2 decimal places.

(4)

- (c) Use integration to find the exact value for the area of R .

(5)

Question 7

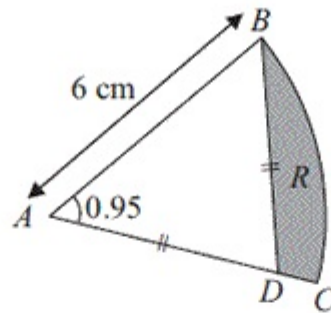


Figure 2

Figure 2 shows ABC , a sector of a circle of radius 6 cm with centre A . Given that the size of angle BAC is 0.95 radians, find

(a) the length of the arc BC , (2)

(b) the area of the sector ABC . (2)

The point D lies on the line AC and is such that $AD = BD$. The region R , shown shaded in Figure 2, is bounded by the lines CD , DB and the arc BC .

(c) Show that the length of AD is 5.16 cm to 3 significant figures. (2)

Find

(d) the perimeter of R , (2)

(e) the area of R , giving your answer to 2 significant figures. (4)

Question 8

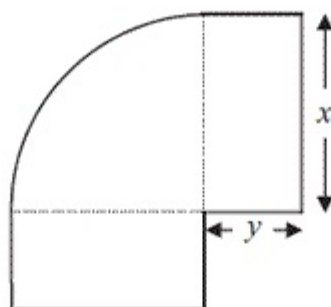


Figure 3

Figure 3 shows a flowerbed. Its shape is a quarter of a circle of radius x metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to x metres and width equal to y metres.

Given that the area of the flowerbed is 4 m^2 ,

(a) show that

$$y = \frac{16 - \pi x^2}{8x} \quad (3)$$

(b) Hence show that the perimeter P metres of the flowerbed is given by the equation

$$P = \frac{8}{x} + 2x \quad (3)$$

(c) Use calculus to find the minimum value of P .

(5)

(d) Find the width of each rectangle when the perimeter is a minimum.
Give your answer to the nearest centimetre.

(2)

Question 9

- (i) Find the solutions of the equation $\sin(3x - 15^\circ) = \frac{1}{2}$, for which $0 \leq x \leq 180^\circ$

(6)

(ii)

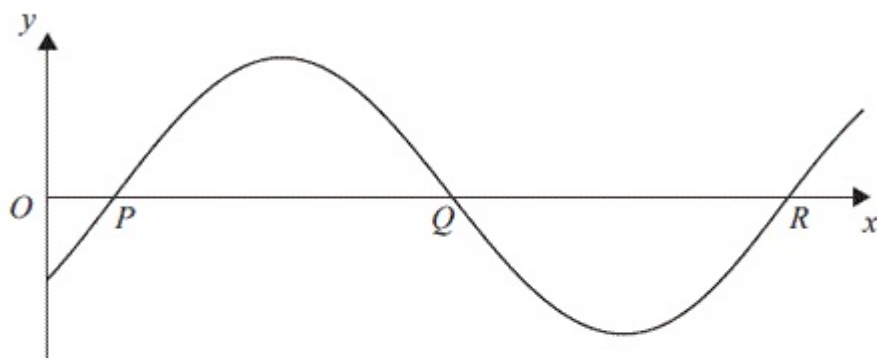


Figure 4

Figure 4 shows part of the curve with equation

$$y = \sin(ax - b), \text{ where } a > 0, 0 < b < \pi$$

The curve cuts the x -axis at the points P , Q and R as shown.

Given that the coordinates of P , Q and R are $\left(\frac{\pi}{10}, 0\right)$, $\left(\frac{3\pi}{5}, 0\right)$ and $\left(\frac{11\pi}{10}, 0\right)$ respectively, find the values of a and b .

(4)
