Core-Maths-C3 - 2007-January

Question 1

(a) By writing $\sin 3\theta$ as $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(5)

(b) Given that $\sin\theta = \frac{\sqrt{3}}{4}$, find the exact value of $\sin 3\theta$.

(2)

Question 2

$$f(x) = 1 - \frac{3}{x+2} + \frac{3}{(x+2)^2}, \quad x \neq -2.$$

(a) Show that $f(x) = \frac{x^2 + x + 1}{(x+2)^2}$, $x \neq -2$.

(4)

(b) Show that $x^2 + x + 1 > 0$ for all values of x.

(3)

(c) Show that $f(x) > 0$ for all values of x, $x \neq -2$.

(1)

Question 3

The curve C has equation

$$x = 2\sin y.$$

(a) Show that the point $P\left(\sqrt{2}, \frac{\pi}{4}\right)$ lies on C.

(1)

(b) Show that $\frac{dy}{dx} = \frac{1}{\sqrt{2}}$ at P.

(4)

(c) Find an equation of the normal to C at P. Give your answer in the form $y = mx + c$, where m and c are exact constants.

(4)

Question 4

(i) The curve C has equation

$$y = \frac{x}{9 + x^2}.$$

Use calculus to find the coordinates of the turning points of C.

(6)

(ii) Given that

$$y = (1 + e^{2x})^{\frac{3}{2}},$$

find the value of $\frac{dy}{dx}$ at $x = \frac{1}{2} \ln 3$.

(5)

Question 5

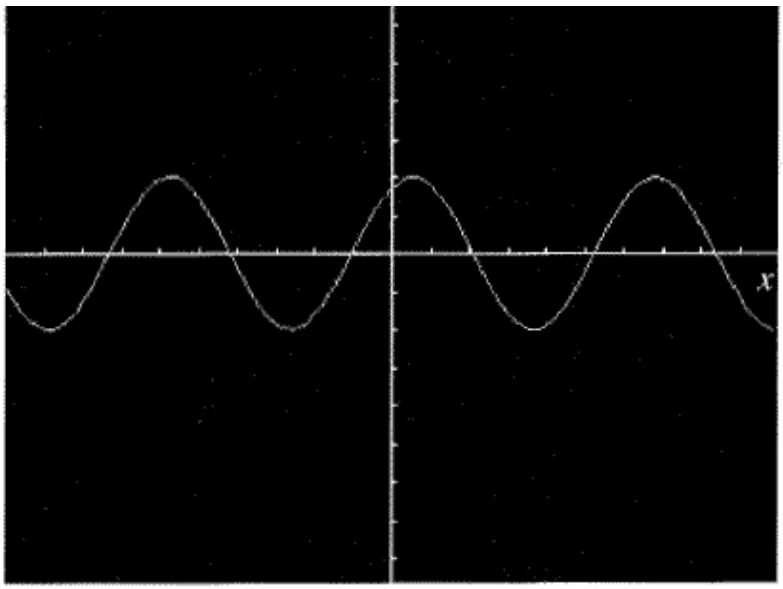

Figure 1 shows an oscilloscope screen.

The curve shown on the screen satisfies the equation

$$y = \sqrt{3}\cos x + \sin x.$$

(a) Express the equation of the curve in the form $y = R\sin(x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(4)

(b) Find the values of x, $0 \leqslant x < 2\pi$, for which $y = 1$.

(4)

Question 6

The function f is defined by

$$f : x \mapsto \ln(4-2x), \quad x < 2 \quad \text{and} \quad x \in \mathbb{R}.$$

(a) Show that the inverse function of f is defined by

$$f^{-1} : x \mapsto 2 - \frac{1}{2}e^{x}$$

and write down the domain of f^{-1}.

(4)

(b) Write down the range of f^{-1}.

(1)

(c) In the space provided on page 16, sketch the graph of $y = f^{-1}(x)$. State the coordinates of the points of intersection with the x and y axes.

(4)

The graph of $y = x + 2$ crosses the graph of $y = f^{-1}(x)$ at $x = k$.

The iterative formula

$$x_{n+1} = -\frac{1}{2}e^{x_n}, \quad x_0 = -0.3$$

is used to find an approximate value for k.

(d) Calculate the values of x_1 and x_2, giving your answers to 4 decimal places.

(2)

(e) Find the value of k to 3 decimal places.

(2)

Question 7

$$f(x) \equiv x^4 - 4x - 8.$$

(a) Show that there is a root of $f(x) = 0$ in the interval $[-2, -1]$.

(3)

(b) Find the coordinates of the turning point on the graph of $y = f(x)$.

(3)

(c) Given that $f(x) = (x - 2)(x^3 + ax^2 + bx + c)$, find the values of the constants, a, b and c.

(3)

(d) In the space provided on page 21, sketch the graph of $y = f(x)$.

(3)

(e) Hence sketch the graph of $y = |f(x)|$.

(1)

Question 8

(i) Prove that

$$\sec^2 x - \operatorname{cosec}^2 x \equiv \tan^2 x - \cot^2 x.$$

(3)

(ii) Given that

$$y = \arccos x, \quad -1 \leqslant x \leqslant 1 \text{ and } 0 \leqslant y \leqslant \pi,$$

(a) express $\arcsin x$ in terms of y.

(2)

(b) Hence evaluate $\arccos x + \arcsin x$. Give your answer in terms of π.

(1)