Mechanics-M1 - 2008-January

Question 1

Two particles A and B have masses 4 kg and m kg respectively. They are moving towards each other in opposite directions on a smooth horizontal table when they collide directly. Immediately before the collision, the speed of A is 5 m s^{-1} and the speed of B is 3 m s^{-1}. Immediately after the collision, the direction of motion of A is unchanged and the speed of A is 1 m s^{-1}.

(a) Find the magnitude of the impulse exerted on A in the collision.

(2)

Immediately after the collision, the speed of B is 2 m s^{-1}.

(b) Find the value of m.

(4)

Question 2

A firework rocket starts from rest at ground level and moves vertically. In the first 3 s of its motion, the rocket rises 27 m. The rocket is modelled as a particle moving with constant acceleration a m s^{-2}. Find

(a) the value of a,

(2)

(b) the speed of the rocket 3 s after it has left the ground.

(2)

After 3 s, the rocket burns out. The motion of the rocket is now modelled as that of a particle moving freely under gravity.

(c) Find the height of the rocket above the ground 5 s after it has left the ground.

(4)

Question 3

A car moves along a horizontal straight road, passing two points A and B. At A the speed of the car is 15 m s^{-1}. When the driver passes A, he sees a warning sign W ahead of him, 120 m away. He immediately applies the brakes and the car decelerates with uniform deceleration, reaching W with speed 5 m s^{-1}. At W, the driver sees that the road is clear. He then immediately accelerates the car with uniform acceleration for 16 s to reach a speed of V m s^{-1} ($V > 15$). He then maintains the car at a constant speed of V m s^{-1}. Moving at this constant speed, the car passes B after a further 22 s.

(a) Sketch, in the space below, a speed-time graph to illustrate the motion of the car as it moves from A to B.

(3)

(b) Find the time taken for the car to move from A to B.

(3)

The distance from A to B is 1 km.

(c) Find the value of V.

(5)

Question 4

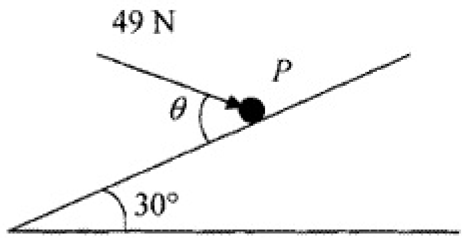

Figure 1

A particle P of mass 6 kg lies on the surface of a smooth plane. The plane is inclined at an angle of 30° to the horizontal. The particle is held in equilibrium by a force of magnitude 49 N, acting at an angle θ to the plane, as shown in Figure 1. The force acts in a vertical plane through a line of greatest slope of the plane.

(a) Show that $\cos \theta = \frac{3}{5}$.

(3)

(b) Find the normal reaction between P and the plane.

(4)

The direction of the force of magnitude 49 N is now changed. It is now applied horizontally to P so that P moves up the plane. The force again acts in a vertical plane through a line of greatest slope of the plane.

(c) Find the initial acceleration of P.

(4)

Question 5

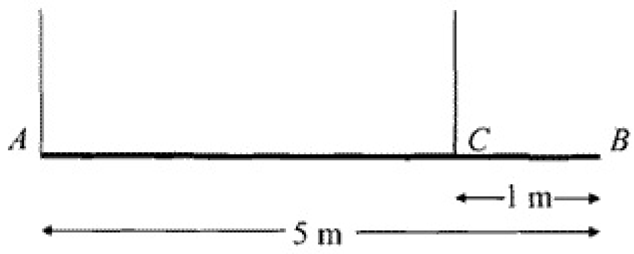

Figure 2

A beam AB has mass 12 kg and length 5 m. It is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. One rope is attached to A, the other to the point C on the beam, where $BC = 1$ m, as shown in Figure 2. The beam is modelled as a uniform rod, and the ropes as light strings.

(a) Find

(i) the tension in the rope at C,

(ii) the tension in the rope at A.

(5)

A small load of mass 16 kg is attached to the beam at a point which is y metres from A. The load is modelled as a particle. Given that the beam remains in equilibrium in a horizontal position,

(b) find, in terms of y, an expression for the tension in the rope at C.

(3)

The rope at C will break if its tension exceeds 98 N. The rope at A cannot break.

(c) Find the range of possible positions on the beam where the load can be attached without the rope at C breaking.

(3)

Question 6

[*In this question, the unit vectors* **i** *and* **j** *are due east and due north respectively.*]

A particle P is moving with constant velocity $(-5\mathbf{i} + 8\mathbf{j})$ m s^{-1}. Find

(a) the speed of P,

(2)

(b) the direction of motion of P, giving your answer as a bearing.

(3)

At time $t = 0$, P is at the point A with position vector $(7\mathbf{i} - 10\mathbf{j})$ m relative to a fixed origin O. When $t = 3$ s, the velocity of P changes and it moves with velocity $(u\mathbf{i} + v\mathbf{j})$ m s^{-1}, where u and v are constants. After a further 4 s, it passes through O and continues to move with velocity $(u\mathbf{i} + v\mathbf{j})$ m s^{-1}.

(c) Find the values of u and v.

(5)

(d) Find the total time taken for P to move from A to a position which is due south of A.

(3)

Question 7

Figure 3

Two particles A and B, of mass m and $2m$ respectively, are attached to the ends of a light inextensible string. The particle A lies on a rough horizontal table. The string passes over a small smooth pulley P fixed on the edge of the table. The particle B hangs freely below the pulley, as shown in Figure 3. The coefficient of friction between A and the table is μ. The particles are released from rest with the string taut. Immediately after release, the magnitude of the acceleration of A and B is $\frac{4}{9}g$. By writing down separate equations of motion for A and B,

(a) find the tension in the string immediately after the particles begin to move,

(3)

(b) show that $\mu = \frac{2}{3}$.

(5)

When B has fallen a distance h, it hits the ground and does not rebound. Particle A is then a distance $\frac{1}{3}h$ from P.

(c) Find the speed of A as it reaches P.

(6)

(d) State how you have used the information that the string is light.

(1)