Mechanics-M1 - 2006-June

Question 1

Figure 1

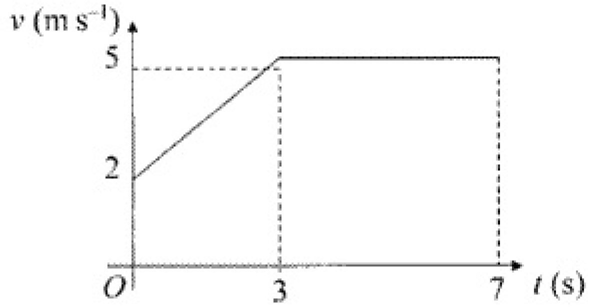


Figure 1 shows the speed-time graph of a cyclist moving on a straight road over a 7 s period. The sections of the graph from $t = 0$ to $t = 3$, and from $t = 3$ to $t = 7$, are straight lines. The section from $t = 3$ to $t = 7$ is parallel to the t-axis.

State what can be deduced about the motion of the cyclist from the fact that

(a) the graph from $t = 0$ to $t = 3$ is a straight line,

(1)

(b) the graph from $t = 3$ to $t = 7$ is parallel to the t-axis.

(1)

(c) Find the distance travelled by the cyclist during this 7 s period.

(4)

Question 2

Two particles A and B have mass 0.4 kg and 0.3 kg respectively. They are moving in opposite directions on a smooth horizontal table and collide directly. Immediately before the collision, the speed of A is 6 m s^{-1} and the speed of B is 2 m s^{-1}. As a result of the collision, the direction of motion of B is reversed and its speed immediately after the collision is 3 m s^{-1}. Find

(a) the speed of A immediately after the collision, stating clearly whether the direction of motion of A is changed by the collision,

(4)

(b) the magnitude of the impulse exerted on B in the collision, stating clearly the units in which your answer is given.

(3)

Question 3

A train moves along a straight track with constant acceleration. Three telegraph poles are set at equal intervals beside the track at points A, B and C, where $AB = 50$ m and $BC = 50$ m. The front of the train passes A with speed 22.5 m s^{-1}, and 2 s later it passes B. Find

(a) the acceleration of the train,

(3)

(b) the speed of the front of the train when it passes C,

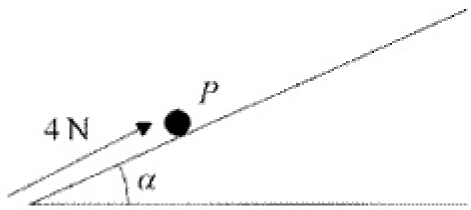
(3)

(c) the time that elapses from the instant the front of the train passes B to the instant it passes C.

(4)

Question 4

Figure 2



A particle P of mass 0.5 kg is on a rough plane inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The particle is held at rest on the plane by the action of a force of magnitude 4 N acting up the plane in a direction parallel to a line of greatest slope of the plane, as shown in Figure 2. The particle is on the point of slipping up the plane.

(a) Find the coefficient of friction between P and the plane.

(7)

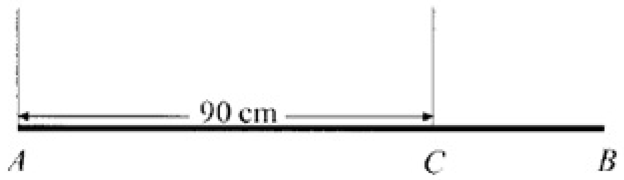
The force of magnitude 4 N is removed.

(b) Find the acceleration of P down the plane.

(4)

Question 5

Figure 3



A steel girder AB has weight 210 N. It is held in equilibrium in a horizontal position by two vertical cables. One cable is attached to the end A. The other cable is attached to the point C on the girder, where $AC = 90$ cm, as shown in Figure 3. The girder is modelled as a uniform rod, and the cables as light inextensible strings.

Given that the tension in the cable at C is twice the tension in the cable at A, find

(a) the tension in the cable at A,

(2)

(b) show that $AB = 120$ cm.

(4)

A small load of weight W newtons is attached to the girder at B. The load is modelled as a particle. The girder remains in equilibrium in a horizontal position. The tension in the cable at C is now three times the tension in the cable at A.

(c) Find the value of W.

(7)

Question 6

A car is towing a trailer along a straight horizontal road by means of a horizontal tow-rope. The mass of the car is 1400 kg. The mass of the trailer is 700 kg. The car and the trailer are modelled as particles and the tow-rope as a light inextensible string. The resistances to motion of the car and the trailer are assumed to be constant and of magnitude 630 N and 280 N respectively. The driving force on the car, due to its engine, is 2380 N. Find

(a) the acceleration of the car,

(3)

(b) the tension in the tow-rope.

(3)

When the car and trailer are moving at 12 m s^{-1}, the tow-rope breaks. Assuming that the driving force on the car and the resistances to motion are unchanged,

(c) find the distance moved by the car in the first 4 s after the tow-rope breaks.

(6)

(d) State how you have used the modelling assumption that the tow-rope is inextensible.

(1)

Question 7

[*In this question the unit vectors* **i** *and* **j** *are due east and north respectively.*]

A ship S is moving with constant velocity $(-2.5\mathbf{i} + 6\mathbf{j})$ km h^{-1}. At time 1200, the position vector of S relative to a fixed origin O is $(16\mathbf{i} + 5\mathbf{j})$ km. Find

(a) the speed of S,

(2)

(b) the bearing on which S is moving.

(2)

The ship is heading directly towards a submerged rock R. A radar tracking station calculates that, if S continues on the same course with the same speed, it will hit R at the time 1500.

(c) Find the position vector of R.

(2)

The tracking station warns the ship's captain of the situation. The captain maintains S on its course with the same speed until the time is 1400. He then changes course so that S moves due north at a constant speed of 5 km h^{-1}. Assuming that S continues to move with this new constant velocity, find

(d) an expression for the position vector of the ship t hours after 1400,

(4)

(e) the time when S will be due east of R,

(2)

(f) the distance of S from R at the time 1600.

(3)