Core-Maths-C2 - 2010-June

Question 1

$$y = 3^x + 2x$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	0	0.2	0.4	0.6	0.8	1
y	1	1.65				5

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate

value for $\displaystyle\int_0^1 (3^x + 2x)\, dx$.

(4)

Question 2

$$f(x) = 3x^3 - 5x^2 - 58x + 40$$

(a) Find the remainder when $f(x)$ is divided by $(x-3)$.

(2)

Given that $(x-5)$ is a factor of $f(x)$,

(b) find all the solutions of $f(x) = 0$.

(5)

Question 3

$$y = x^2 - k\sqrt{x}, \quad \text{where } k \text{ is a constant.}$$

(a) Find $\dfrac{dy}{dx}$.

(2)

(b) Given that y is decreasing at $x = 4$, find the set of possible values of k.

(2)

Question 4

(a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1 + ax)^7$, where a is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 in this expansion is 525,

(b) find the possible values of a.

(2)

Question 5

(a) Given that $5\sin\theta = 2\cos\theta$, find the value of $\tan\theta$.

(1)

(b) Solve, for $0 \leqslant x < 360°$,

$$5\sin 2x = 2\cos 2x,$$

giving your answers to 1 decimal place.
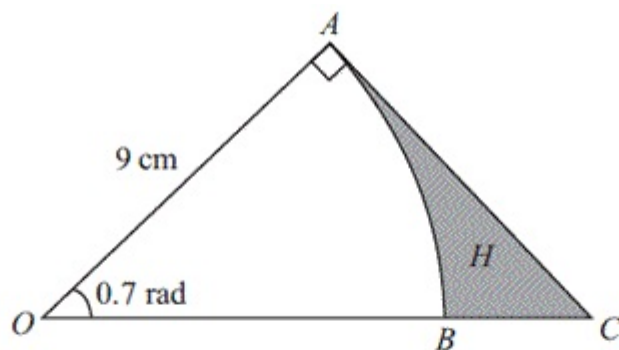
(5)

Question 6

Figure 1

Figure 1 shows the sector OAB of a circle with centre O, radius 9 cm and angle 0.7 radians.

(a) Find the length of the arc AB.

(2)

(b) Find the area of the sector OAB.

(2)

The line AC shown in Figure 1 is perpendicular to OA, and OBC is a straight line.

(c) Find the length of AC, giving your answer to 2 decimal places.

(2)

The region H is bounded by the arc AB and the lines AC and CB.

(d) Find the area of H, giving your answer to 2 decimal places.

(3)

Question 7

(a) Given that

$$2\log_3(x-5) - \log_3(2x-13) = 1,$$

show that $x^2 - 16x + 64 = 0$.

(5)

(b) Hence, or otherwise, solve $2\log_3(x-5) - \log_3(2x-13) = 1$.
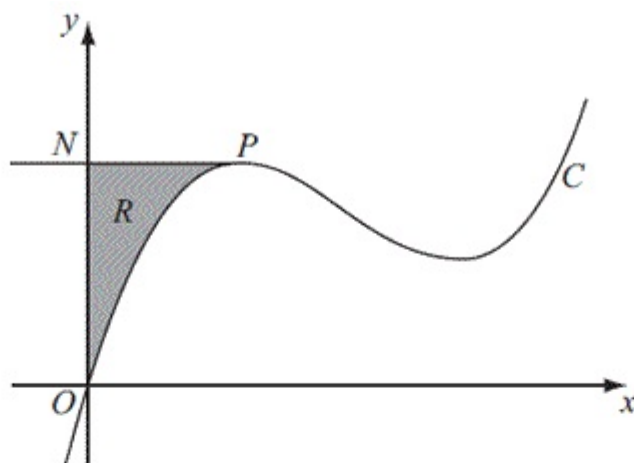
(2)

Question 8

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx,$$

where k is a constant.

The point P on C is the maximum turning point.

Given that the x-coordinate of P is 2,

(a) show that $k = 28$.

(3)

The line through P parallel to the x-axis cuts the y-axis at the point N.
The region R is bounded by C, the y-axis and PN, as shown shaded in Figure 2.

(b) Use calculus to find the exact area of R.

(6)

Question 9

The adult population of a town is 25 000 at the end of Year 1.

A model predicts that the adult population of the town will increase by 3% each year, forming a geometric sequence.

(a) Show that the predicted adult population at the end of Year 2 is 25 750.

(1)

(b) Write down the common ratio of the geometric sequence.

(1)

The model predicts that Year N will be the first year in which the adult population of the town exceeds 40 000.

(c) Show that

$$(N-1)\log 1.03 > \log 1.6$$

(3)

(d) Find the value of N.

(2)

At the end of each year, each member of the adult population of the town will give £1 to a charity fund.

Assuming the population model,

(e) find the total amount that will be given to the charity fund for the 10 years from the end of Year 1 to the end of Year 10, giving your answer to the nearest £1000.

(3)

Question 10

The circle C has centre $A(2,1)$ and passes through the point $B(10,7)$.

(a) Find an equation for C.

(4)

The line l_1 is the tangent to C at the point B.

(b) Find an equation for l_1.

(4)

The line l_2 is parallel to l_1 and passes through the mid-point of AB.

Given that l_2 intersects C at the points P and Q,

(c) find the length of PQ, giving your answer in its simplest surd form.

(3)