Core-Maths-C4 - 2011-June

Question 1

$$\frac{9x^2}{(x-1)^2(2x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(2x+1)}$$

Find the values of the constants A, B and C.

(4)

Question 2

$$f(x) = \frac{1}{\sqrt{(9+4x^2)}}, \qquad |x| < \frac{3}{2}$$

Find the first three non-zero terms of the binomial expansion of $f(x)$ in ascending powers of x. Give each coefficient as a simplified fraction.

(6)

Question 3

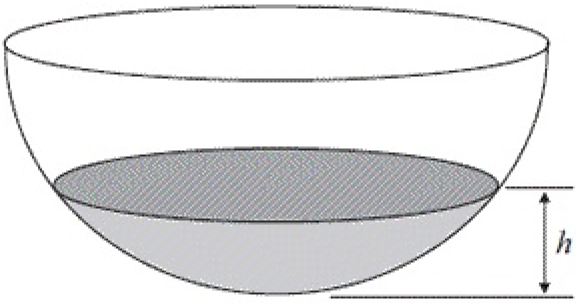

Figure 1

A hollow hemispherical bowl is shown in Figure 1. Water is flowing into the bowl. When the depth of the water is h m, the volume V m^3 is given by

$$V = \frac{1}{12}\pi h^2(3-4h), \qquad 0 \leqslant h \leqslant 0.25$$

(a) Find, in terms of π, $\frac{dV}{dh}$ when $h = 0.1$

(4)

Water flows into the bowl at a rate of $\frac{\pi}{800}$ m^{3}s^{-1}.

(b) Find the rate of change of h, in m s^{-1}, when $h = 0.1$

(2)

Question 4

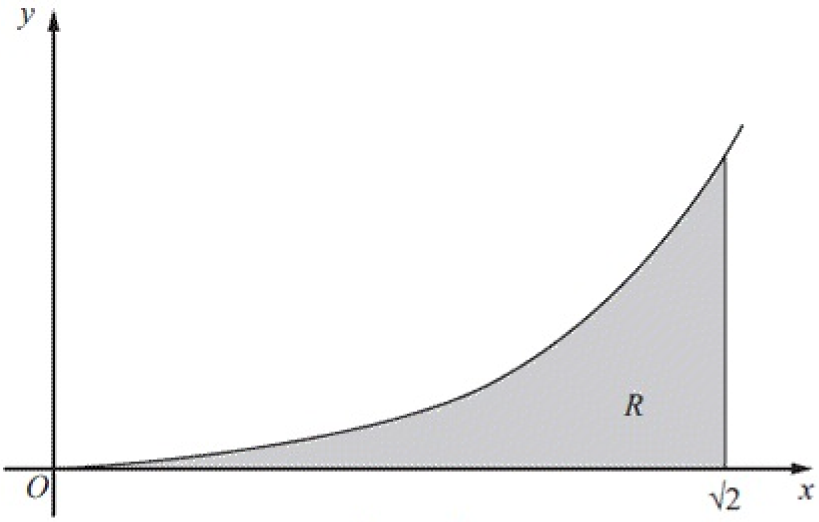

Figure 2

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln(x^2 + 2)$, $x \geqslant 0$.
The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line $x = \sqrt{2}$.

The table below shows corresponding values of x and y for $y = x^3 \ln(x^2 + 2)$.

x	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{4}$	$\sqrt{2}$
y	0		0.3240		3.9210

(a) Complete the table above giving the missing values of y to 4 decimal places.

(2)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.

(3)

(c) Use the substitution $u = x^2 + 2$ to show that the area of R is

$$\frac{1}{2}\int_2^4 (u - 2)\ln u \, \mathrm{d}u$$

(4)

(d) Hence, or otherwise, find the exact area of R.

(6)

Question 5

Find the gradient of the curve with equation

$$\ln y = 2x \ln x, \quad x > 0, \ y > 0$$

at the point on the curve where $x = 2$. Give your answer as an exact value.

(7)

Question 6

With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \ \mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \qquad l_2: \ \mathbf{r} = \begin{pmatrix} -5 \\ 15 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix},$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection A.

(6)

(b) Find, to the nearest $0.1°$, the acute angle between l_1 and l_2.

(3)

The point B has position vector $\begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}$.

(c) Show that B lies on l_1.

(1)

(d) Find the shortest distance from B to the line l_2, giving your answer to 3 significant figures.

(4)

Question 7

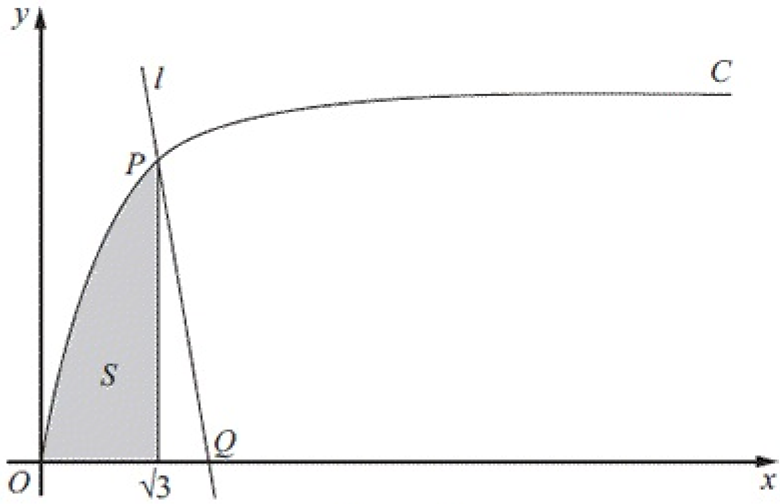

Figure 3

Figure 3 shows part of the curve C with parametric equations

$$x = \tan\theta, \qquad y = \sin\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(\sqrt{3}, \frac{1}{2}\sqrt{3}\right)$.

(a) Find the value of θ at the point P.

(2)

The line l is a normal to C at P. The normal cuts the x-axis at the point Q.

(b) Show that Q has coordinates $(k\sqrt{3}, 0)$, giving the value of the constant k.

(6)

The finite shaded region S shown in Figure 3 is bounded by the curve C, the line $x = \sqrt{3}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(c) Find the volume of the solid of revolution, giving your answer in the form $p\pi\sqrt{3} + q\pi^2$, where p and q are constants.

(7)

Question 8

(a) Find $\int (4y+3)^{-\frac{1}{2}}\, dy$

(2)

(b) Given that $y = 1.5$ at $x = -2$, solve the differential equation

$$\frac{dy}{dx} = \frac{\sqrt{(4y+3)}}{x^2}$$

giving your answer in the form $y = f(x)$.

(6)