

Core-Maths-C2 - 2007-June

Question 1

Evaluate $\int_1^8 \frac{1}{\sqrt{x}} dx$, giving your answer in the form $a + b\sqrt{2}$, where a and b are integers.

(4)

Question 2

$$f(x) = 3x^3 - 5x^2 - 16x + 12.$$

(a) Find the remainder when $f(x)$ is divided by $(x - 2)$.

(2)

Given that $(x + 2)$ is a factor of $f(x)$,

(b) factorise $f(x)$ completely.

(4)

Question 3

(a) Find the first four terms, in ascending powers of x , in the binomial expansion of $(1 + kx)^6$, where k is a non-zero constant.

(3)

Given that, in this expansion, the coefficients of x and x^2 are equal, find

(b) the value of k ,

(2)

(c) the coefficient of x^3 .

(1)

Question 4

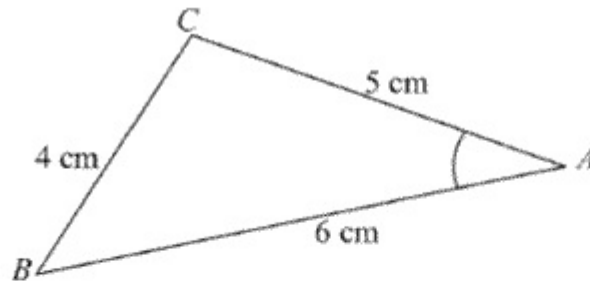


Figure 1

Figure 1 shows the triangle ABC , with $AB = 6$ cm, $BC = 4$ cm and $CA = 5$ cm.

- (a) Show that $\cos A = \frac{3}{4}$.

(3)

- (b) Hence, or otherwise, find the exact value of $\sin A$.

(2)

Question 5

The curve C has equation

$$y = x\sqrt[3]{(x^3+1)}, \quad 0 \leq x \leq 2.$$

- (a) Complete the table below, giving the values of y to 3 decimal places at $x = 1$ and $x = 1.5$.

x	0	0.5	1	1.5	2
y	0	0.530			6

(2)

- (b) Use the trapezium rule, with all the y values from your table, to find an approximation for the value of $\int_0^2 x\sqrt[3]{(x^3+1)}dx$, giving your answer to 3 significant figures.

(4)

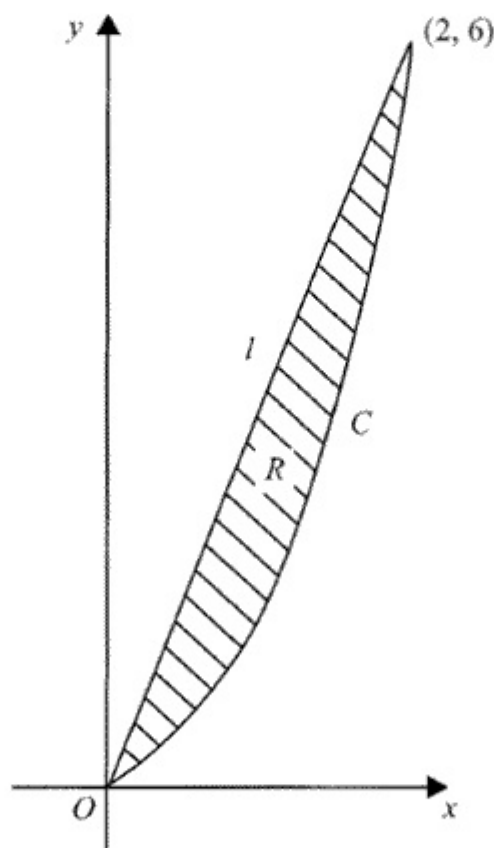


Figure 2

Figure 2 shows the curve C with equation $y = x\sqrt[3]{(x^3+1)}$, $0 \leq x \leq 2$, and the straight line segment l , which joins the origin and the point $(2, 6)$. The finite region R is bounded by C and l .

- (c) Use your answer to part (b) to find an approximation for the area of R , giving your answer to 3 significant figures.

(3)

Question 6

(a) Find, to 3 significant figures, the value of x for which $8^x = 0.8$.

(2)

(b) Solve the equation

$$2\log_3 x - \log_3 7x = 1.$$

(4)

Question 7

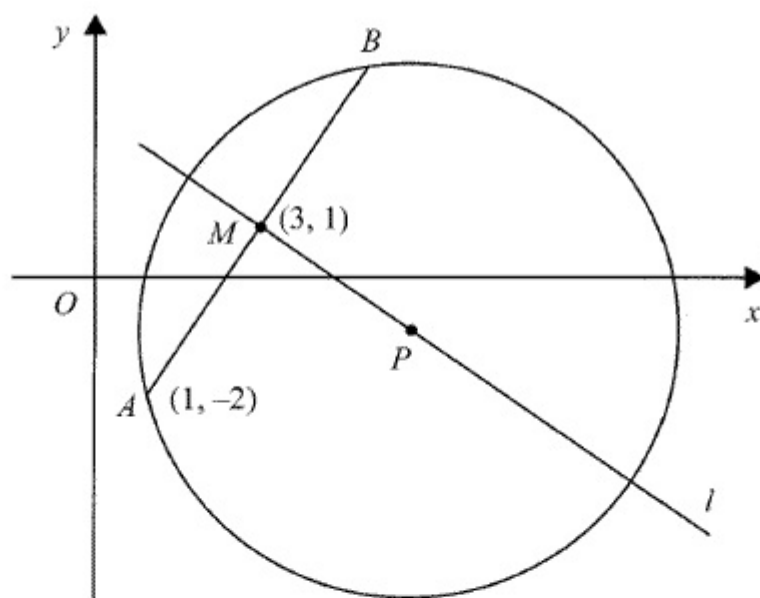


Figure 3

The points A and B lie on a circle with centre P , as shown in Figure 3.

The point A has coordinates $(1, -2)$ and the mid-point M of AB has coordinates $(3, 1)$.

The line l passes through the points M and P .

(a) Find an equation for l .

(4)

Given that the x -coordinate of P is 6,

(b) use your answer to part (a) to show that the y -coordinate of P is -1 ,

(1)

(c) find an equation for the circle.

(4)

Question 8

A trading company made a profit of £50 000 in 2006 (Year 1).

A model for future trading predicts that profits will increase year by year in a geometric sequence with common ratio r , $r > 1$.

The model therefore predicts that in 2007 (Year 2) a profit of £50 000 r will be made.

- (a) Write down an expression for the predicted profit in Year n . (1)

The model predicts that in Year n , the profit made will exceed £200 000.

- (b) Show that $n > \frac{\log 4}{\log r} + 1$. (3)

Using the model with $r = 1.09$,

- (c) find the year in which the profit made will first exceed £200 000, (2)
- (d) find the total of the profits that will be made by the company over the 10 years from 2006 to 2015 inclusive, giving your answer to the nearest £10 000. (3)
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Question 9

- (a) Sketch, for $0 \leq x \leq 2\pi$, the graph of $y = \sin\left(x + \frac{\pi}{6}\right)$. (2)

- (b) Write down the exact coordinates of the points where the graph meets the coordinate axes. (3)

- (c) Solve, for $0 \leq x \leq 2\pi$, the equation

$$\sin\left(x + \frac{\pi}{6}\right) = 0.65,$$

giving your answers in radians to 2 decimal places. (5)

Question 10

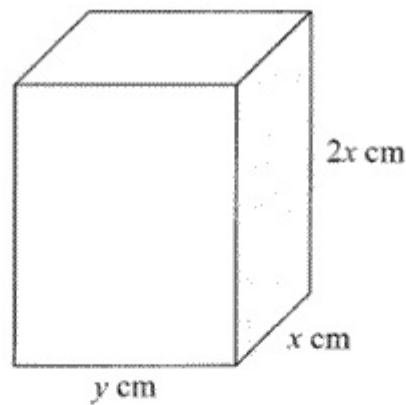


Figure 4

Figure 4 shows a solid brick in the shape of a cuboid measuring $2x$ cm by x cm by y cm.

The total surface area of the brick is 600 cm^2 .

- (a) Show that the volume, $V \text{ cm}^3$, of the brick is given by

$$V = 200x - \frac{4x^3}{3}. \quad (4)$$

Given that x can vary,

- (b) use calculus to find the maximum value of V , giving your answer to the nearest cm^3 . (5)

- (c) Justify that the value of V you have found is a maximum. (2)
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