Core-Maths-C4 - 2006-June

Question 1

A curve C is described by the equation

$$3x^2 - 2y^2 + 2x - 3y + 5 = 0.$$

Find an equation of the normal to C at the point $(0, 1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

Question 2

$$f(x) = \frac{3x-1}{(1-2x)^2}, \qquad |x| < \tfrac{1}{2}.$$

Given that, for $x \neq \tfrac{1}{2}$, $\quad \dfrac{3x-1}{(1-2x)^2} = \dfrac{A}{(1-2x)} + \dfrac{B}{(1-2x)^2}$, $\quad$ where A and B are constants,
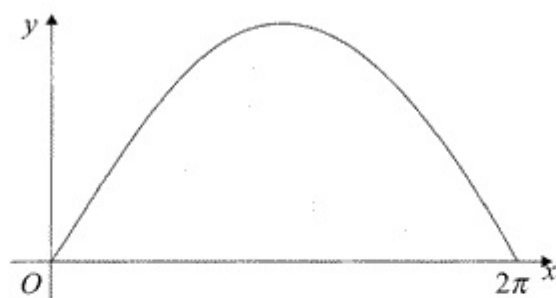
(a) find the values of A and B.

(3)

(b) Hence, or otherwise, find the series expansion of $f(x)$, in ascending powers of x, up to and including the term in x^3, simplifying each term.

(6)

Question 3

Figure 1

The curve with equation $y = 3\sin\dfrac{x}{2}$, $0 \leqslant x \leqslant 2\pi$, is shown in Figure 1. The finite region enclosed by the curve and the x-axis is shaded.

(a) Find, by integration, the area of the shaded region.

(3)

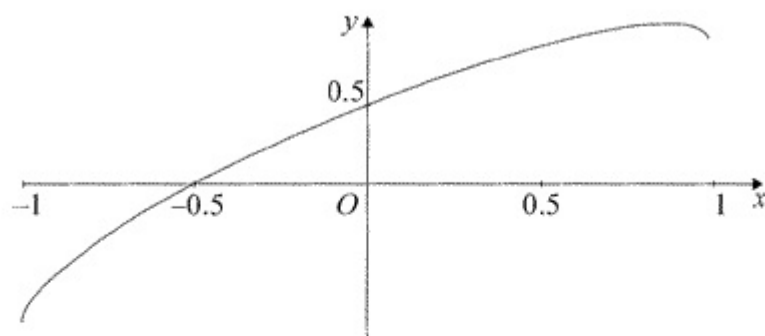This region is rotated through 2π radians about the x-axis.

(b) Find the volume of the solid generated.

(6)

Question 4

Figure 2

The curve shown in Figure 2 has parametric equations

$$x = \sin t, \quad y = \sin\left(t + \tfrac{\pi}{6}\right), \qquad -\tfrac{\pi}{2} < t < \tfrac{\pi}{2}.$$

(a) Find an equation of the tangent to the curve at the point where $t = \tfrac{\pi}{6}$.

(6)

(b) Show that a cartesian equation of the curve is

$$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1-x^2)}, \qquad -1 < x < 1.$$

(3)

Question 5

The point A, with coordinates $(0, a, b)$ lies on the line l_1, which has equation

$$\mathbf{r} = 6\mathbf{i} + 19\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}).$$

(a) Find the values of a and b.

(3)

The point P lies on l_1 and is such that OP is perpendicular to l_1, where O is the origin.

(b) Find the position vector of point P.

(6)

Given that B has coordinates $(5, 15, 1)$,

(c) show that the points A, P and B are collinear and find the ratio $AP:PB$.
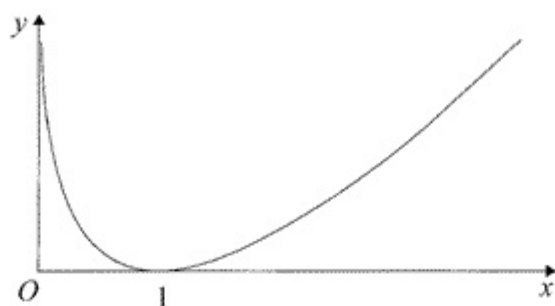
(4)

Question 6

Figure 3

Figure 3 shows a sketch of the curve with equation $y = (x-1)\ln x, \quad x > 0$.

(a) Complete the table with the values of y corresponding to $x = 1.5$ and $x = 2.5$.

x	1	1.5	2	2.5	3
y	0		$\ln 2$		$2\ln 3$

(1)

Given that $I = \displaystyle\int_1^3 (x-1)\ln x \, dx$,

(b) use the trapezium rule

    (i) with values of y at $x = 1$, 2 and 3 to find an approximate value for I to 4 significant figures,

    (ii) with values of y at $x = 1$, 1.5, 2, 2.5 and 3 to find another approximate value for I to 4 significant figures.
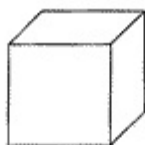
(5)

(c) Explain, with reference to Figure 3, why an increase in the number of values improves the accuracy of the approximation.

(1)

(d) Show, by integration, that the exact value of $\displaystyle\int_1^3 (x-1)\ln x \, dx$ is $\frac{3}{2}\ln 3$.

(6)

Question 7

At time t seconds the length of the side of a cube is x cm, the surface area of the cube is S cm^2, and the volume of the cube is V cm^3.

The surface area of the cube is increasing at a constant rate of 8 cm^{2}s^{-1}.

Show that

(a) $\dfrac{dx}{dt} = \dfrac{k}{x}$, where k is a constant to be found,

(4)

(b) $\dfrac{dV}{dt} = 2V^{\frac{1}{3}}$.

(4)

Given that $V = 8$ when $t = 0$,

(c) solve the differential equation in part (b), and find the value of t when $V = 16\sqrt{2}$.

(7)