Core-Maths-C1 - 2012-January

Question 1

Given that $y = x^4 + 6x^{\frac{1}{2}}$, find in their simplest form

(a) $\dfrac{dy}{dx}$

(3)

(b) $\int y \, dx$

(3)

Question 2

(a) Simplify

$$\sqrt{32} + \sqrt{18}$$

giving your answer in the form $a\sqrt{2}$, where a is an integer.

(2)

(b) Simplify

$$\frac{\sqrt{32} + \sqrt{18}}{3 + \sqrt{2}}$$

giving your answer in the form $b\sqrt{2} + c$, where b and c are integers.

(4)

Question 3

Find the set of values of x for which

(a) $4x - 5 > 15 - x$

(2)

(b) $x(x - 4) > 12$

(4)

Question 4

A sequence $x_1, x_2, x_3, \ldots$ is defined by

$$x_1 = 1$$

$$x_{n+1} = ax_n + 5, \qquad n \geqslant 1$$

where a is a constant.

(a) Write down an expression for x_2 in terms of a.

(1)

(b) Show that $x_3 = a^2 + 5a + 5$

(2)

Given that $x_3 = 41$

(c) find the possible values of a.

(3)

Question 5

The curve C has equation $y = x(5-x)$ and the line L has equation $2y = 5x + 4$

(a) Use algebra to show that C and L do not intersect.

(4)

(b) In the space on page 11, sketch C and L on the same diagram, showing the coordinates of the points at which C and L meet the axes.
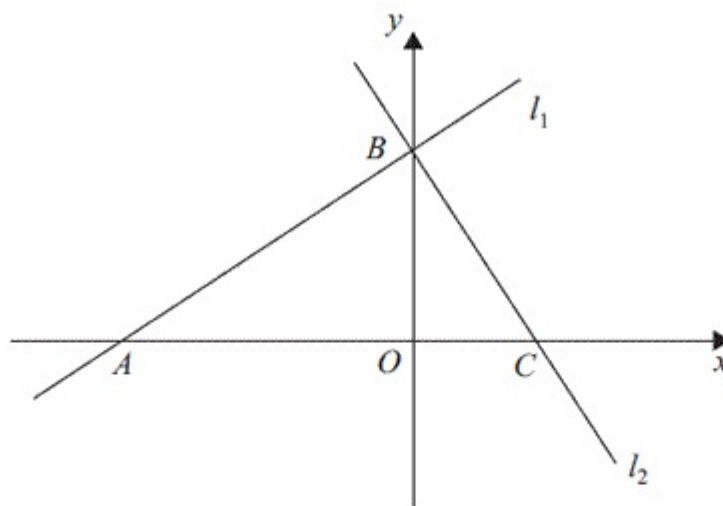
(4)

Question 6

Figure 1

The line l_1 has equation $2x - 3y + 12 = 0$

(a) Find the gradient of l_1.

(1)

The line l_1 crosses the x-axis at the point A and the y-axis at the point B, as shown in Figure 1.

The line l_2 is perpendicular to l_1 and passes through B.

(b) Find an equation of l_2.

(3)

The line l_2 crosses the x-axis at the point C.

(c) Find the area of triangle ABC.

(4)

Question 7

A curve with equation $y = f(x)$ passes through the point $(2, 10)$. Given that

$$f'(x) = 3x^2 - 3x + 5$$

find the value of $f(1)$.

(5)

Question 8

The curve C_1 has equation

$$y = x^2(x+2)$$

(a) Find $\dfrac{dy}{dx}$

(2)

(b) Sketch C_1, showing the coordinates of the points where C_1 meets the x-axis.

(3)

(c) Find the gradient of C_1 at each point where C_1 meets the x-axis.

(2)

The curve C_2 has equation

$$y = (x-k)^2(x-k+2)$$

where k is a constant and $k > 2$

(d) Sketch C_2, showing the coordinates of the points where C_2 meets the x and y axes.

(3)

Question 9

A company offers two salary schemes for a 10-year period, Year 1 to Year 10 inclusive.

Scheme 1: Salary in Year 1 is £P.
Salary increases by £$(2T)$ each year, forming an arithmetic sequence.

Scheme 2: Salary in Year 1 is £$(P + 1800)$.
Salary increases by £T each year, forming an arithmetic sequence.

(a) Show that the **total** earned under Salary Scheme 1 for the 10-year period is

$$£(10P + 90T)$$

(2)

For the 10-year period, the **total** earned is the same for both salary schemes.

(b) Find the value of T.

(4)

For this value of T, the salary in Year 10 under Salary Scheme 2 is £29 850

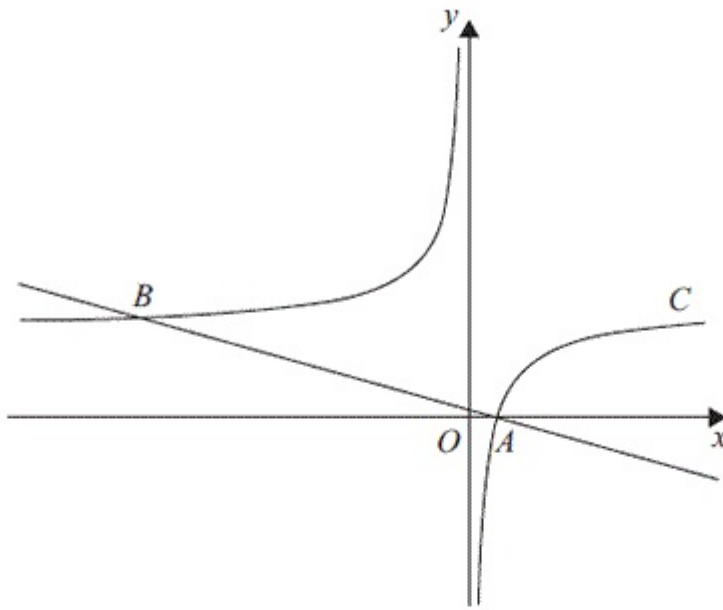(c) Find the value of P.

(3)

Question 10

Figure 2

Figure 2 shows a sketch of the curve C with equation

$$y = 2 - \frac{1}{x}, \quad x \neq 0$$

The curve crosses the x-axis at the point A.

(a) Find the coordinates of A.

(1)

(b) Show that the equation of the normal to C at A can be written as

$$2x + 8y - 1 = 0$$

(6)

The normal to C at A meets C again at the point B, as shown in Figure 2.

(c) Find the coordinates of B.

(4)