

# Core-Maths-C3 - 2007-June

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## Question 1

Find the exact solutions to the equations

(a)  $\ln x + \ln 3 = \ln 6$ , (2)

(b)  $e^x + 3e^{-x} = 4$ . (4)

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## Question 2

$$f(x) = \frac{2x+3}{x+2} - \frac{9+2x}{2x^2+3x-2}, \quad x > \frac{1}{2}.$$

(a) Show that  $f(x) = \frac{4x-6}{2x-1}$ . (7)

(b) Hence, or otherwise, find  $f'(x)$  in its simplest form. (3)

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### Question 3

A curve  $C$  has equation

$$y = x^2 e^x.$$

- (a) Find  $\frac{dy}{dx}$ , using the product rule for differentiation. (3)
- (b) Hence find the coordinates of the turning points of  $C$ . (3)
- (c) Find  $\frac{d^2y}{dx^2}$ . (2)
- (d) Determine the nature of each turning point of the curve  $C$ . (2)
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### Question 4

$$f(x) = -x^3 + 3x^2 - 1.$$

- (a) Show that the equation  $f(x) = 0$  can be rewritten as

$$x = \sqrt{\left(\frac{1}{3-x}\right)}. \quad (2)$$

- (b) Starting with  $x_1 = 0.6$ , use the iteration

$$x_{n+1} = \sqrt{\left(\frac{1}{3-x_n}\right)}$$

to calculate the values of  $x_2$ ,  $x_3$  and  $x_4$ , giving all your answers to 4 decimal places. (2)

- (c) Show that  $x = 0.653$  is a root of  $f(x) = 0$  correct to 3 decimal places. (3)
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## Question 5

The functions  $f$  and  $g$  are defined by

$$f : x \mapsto \ln(2x-1), \quad x \in \mathbb{R}, x > \frac{1}{2},$$

$$g : x \mapsto \frac{2}{x-3}, \quad x \in \mathbb{R}, x \neq 3.$$

- (a) Find the exact value of  $fg(4)$ . (2)
- (b) Find the inverse function  $f^{-1}(x)$ , stating its domain. (4)
- (c) Sketch the graph of  $y = |g(x)|$ . Indicate clearly the equation of the vertical asymptote and the coordinates of the point at which the graph crosses the  $y$ -axis. (3)
- (d) Find the exact values of  $x$  for which  $\left| \frac{2}{x-3} \right| = 3$ . (3)
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## Question 6

- (a) Express  $3 \sin x + 2 \cos x$  in the form  $R \sin(x + \alpha)$  where  $R > 0$  and  $0 < \alpha < \frac{\pi}{2}$ . (4)
- (b) Hence find the greatest value of  $(3 \sin x + 2 \cos x)^4$ . (2)
- (c) Solve, for  $0 < x < 2\pi$ , the equation
- $$3 \sin x + 2 \cos x = 1,$$
- giving your answers to 3 decimal places. (5)
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## Question 7

(a) Prove that

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = 2 \operatorname{cosec} 2\theta, \quad \theta \neq 90n^\circ. \quad (4)$$

(b) On the axes on page 20, sketch the graph of  $y = 2 \operatorname{cosec} 2\theta$  for  $0^\circ < \theta < 360^\circ$ . (2)

(c) Solve, for  $0^\circ < \theta < 360^\circ$ , the equation

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = 3,$$

giving your answers to 1 decimal place. (6)

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## Question 8

The amount of a certain type of drug in the bloodstream  $t$  hours after it has been taken is given by the formula

$$x = De^{-\frac{1}{8}t},$$

where  $x$  is the amount of the drug in the bloodstream in milligrams and  $D$  is the dose given in milligrams.

A dose of 10 mg of the drug is given.

(a) Find the amount of the drug in the bloodstream 5 hours after the dose is given. Give your answer in mg to 3 decimal places. (2)

A second dose of 10 mg is given after 5 hours.

(b) Show that the amount of the drug in the bloodstream 1 hour after the second dose is 13.549 mg to 3 decimal places. (2)

No more doses of the drug are given. At time  $T$  hours after the second dose is given, the amount of the drug in the bloodstream is 3 mg.

(c) Find the value of  $T$ . (3)

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