Core-Maths-C3 - 2011-June

Question 1

Differentiate with respect to x

(a) $\ln\left(x^2+3x+5\right)$

(2)

(b) $\dfrac{\cos x}{x^2}$

(3)

Question 2

$$f(x)=2\sin(x^2)+x-2, \qquad 0\leqslant x<2\pi$$

(a) Show that $f(x)=0$ has a root α between $x=0.75$ and $x=0.85$

(2)

The equation $f(x)=0$ can be written as $x=\left[\arcsin(1-0.5x)\right]^{\frac{1}{2}}$.

(b) Use the iterative formula

$$x_{n+1}=\left[\arcsin(1-0.5x_n)\right]^{\frac{1}{2}}, \quad x_0=0.8$$

to find the values of x_1, x_2 and x_3, giving your answers to 5 decimal places.

(3)

(c) Show that $\alpha=0.80157$ is correct to 5 decimal places.

(3)

Question 3

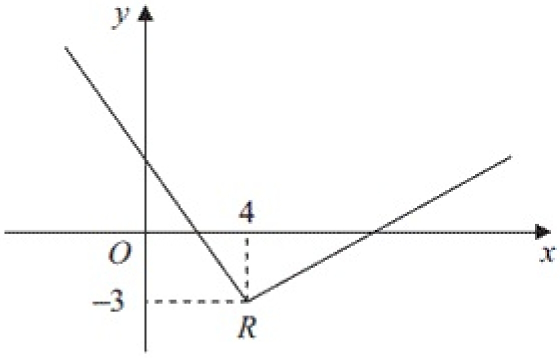

Figure 1

Figure 1 shows part of the graph of $y = \mathrm{f}(x),\ x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $R(4, -3)$, as shown in Figure 1.

Sketch, on separate diagrams, the graphs of

(a) $y = 2\mathrm{f}(x+4)$,

(3)

(b) $y = |\mathrm{f}(-x)|$.

(3)

On each diagram, show the coordinates of the point corresponding to R.

Question 4

The function f is defined by

$$f : x \mapsto 4 - \ln(x+2), \quad x \in \mathbb{R}, \ x \geqslant -1$$

(a) Find $f^{-1}(x)$.

(3)

(b) Find the domain of f^{-1}.

(1)

The function g is defined by

$$g : x \mapsto e^{x^2} - 2, \quad x \in \mathbb{R}$$

(c) Find $fg(x)$, giving your answer in its simplest form.

(3)

(d) Find the range of fg.

(1)

Question 5

The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$$m = p\,e^{-kt}$$

where k and p are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(a) Write down the value of p.

(1)

(b) Show that $k = \frac{1}{4}\ln 3$.

(4)

(c) Find the value of t when $\frac{dm}{dt} = -0.6\ln 3$.

(6)

Question 6

(a) Prove that

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \tan\theta, \quad \theta \neq 90n°, \ n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise,

(i) show that $\tan 15° = 2 - \sqrt{3}$,

(3)

(ii) solve, for $0 < x < 360°$,

$$\operatorname{cosec} 4x - \cot 4x = 1$$

(5)

Question 7

$$f(x) = \frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{x^2-9}, \qquad x \neq \pm 3, \ x \neq -\frac{1}{2}$$

(a) Show that

$$f(x) = \frac{5}{(2x+1)(x+3)}$$

(5)

The curve C has equation $y = f(x)$. The point $P\left(-1, -\frac{5}{2}\right)$ lies on C.

(b) Find an equation of the normal to C at P.

(8)

Question 8

(a) Express $2\cos 3x - 3\sin 3x$ in the form $R\cos(3x+\alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your answers to 3 significant figures.

(4)

$$f(x) = e^{2x} \cos 3x$$

(b) Show that $f'(x)$ can be written in the form

$$f'(x) = Re^{2x} \cos(3x+\alpha)$$

where R and α are the constants found in part (a).

(5)

(c) Hence, or otherwise, find the smallest positive value of x for which the curve with equation $y = f(x)$ has a turning point.

(3)