# Core-Maths-C3 - 2009-June

## Question 1

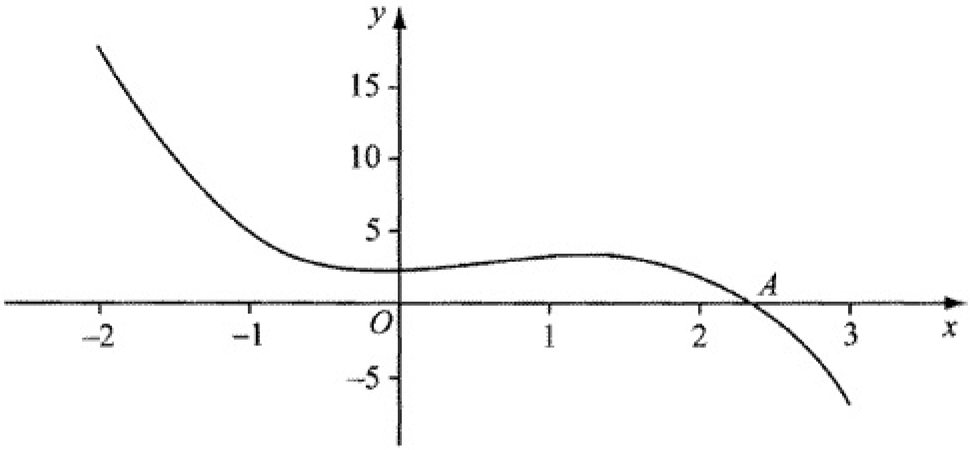


**Figure 1**

Figure 1 shows part of the curve with equation $y = -x^3 + 2x^2 + 2$, which intersects the $x$-axis at the point $A$ where $x = \alpha$.

To find an approximation to $\alpha$, the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2$$

is used.

(a) Taking $x_0 = 2.5$, find the values of $x_1$, $x_2$, $x_3$ and $x_4$.
Give your answers to 3 decimal places where appropriate.

**(3)**

(b) Show that $\alpha = 2.359$ correct to 3 decimal places.

**(3)**

## Question 2

(a) Use the identity $\cos^2\theta + \sin^2\theta = 1$ to prove that $\tan^2\theta = \sec^2\theta - 1$.

**(2)**

(b) Solve, for $0 \leqslant \theta < 360°$, the equation

$$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2$$

**(6)**

## Question 3

Rabbits were introduced onto an island. The number of rabbits, $P$, $t$ years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}, \qquad t \in \mathbb{R}, t \geqslant 0$$

(a) Write down the number of rabbits that were introduced to the island.

**(1)**

(b) Find the number of years it would take for the number of rabbits to first exceed 1000.

**(2)**

(c) Find $\dfrac{dP}{dt}$.

**(2)**

(d) Find $P$ when $\dfrac{dP}{dt} = 50$.

**(3)**

# Question 4

(i) Differentiate with respect to $x$

(a) $x^2 \cos 3x$

**(3)**

(b) $\dfrac{\ln(x^2+1)}{x^2+1}$

**(4)**

(ii) A curve $C$ has the equation

$$y = \sqrt{(4x+1)}, \quad x > -\tfrac{1}{4}, \; y > 0$$

The point $P$ on the curve has $x$-coordinate 2. Find an equation of the tangent to $C$ at $P$ in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**(6)**

# Question 5

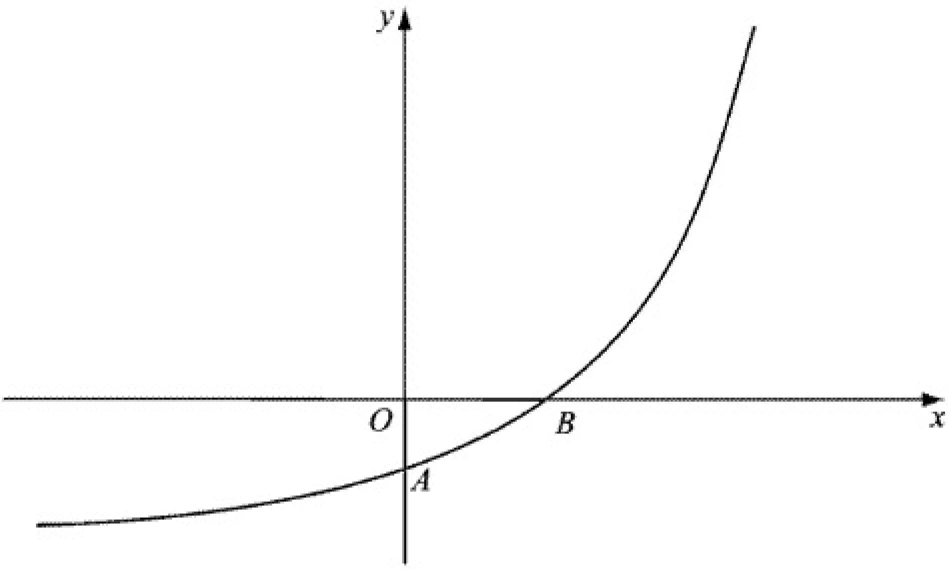


**Figure 2**

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.
The curve meets the coordinate axes at the points $A(0, 1-k)$ and $B(\frac{1}{2}\ln k, 0)$, where $k$ is a constant and $k > 1$, as shown in Figure 2.

On separate diagrams, sketch the curve with equation

(a) $y = |\mathrm{f}(x)|$,

**(3)**

(b) $y = \mathrm{f}^{-1}(x)$.

**(2)**

Show on each sketch the coordinates, in terms of $k$, of each point at which the curve meets or cuts the axes.

Given that $\mathrm{f}(x) = \mathrm{e}^{2x} - k$,

(c) state the range of f ,

**(1)**

(d) find $\mathrm{f}^{-1}(x)$,

**(3)**

(e) write down the domain of $\mathrm{f}^{-1}$.

**(1)**

# Question 6

(a) Use the identity $\cos(A+B) = \cos A\cos B - \sin A\sin B$, to show that

$$\cos 2A = 1 - 2\sin^2 A$$

**(2)**

The curves $C_1$ and $C_2$ have equations

$$C_1: \quad y = 3\sin 2x$$

$$C_2: \quad y = 4\sin^2 x - 2\cos 2x$$

(b) Show that the $x$-coordinates of the points where $C_1$ and $C_2$ intersect satisfy the equation

$$4\cos 2x + 3\sin 2x = 2$$

**(3)**

(c) Express $4\cos 2x + 3\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$, giving the value of $\alpha$ to 2 decimal places.

**(3)**

(d) Hence find, for $0 \leqslant x < 180°$, all the solutions of

$$4\cos 2x + 3\sin 2x = 2$$

giving your answers to 1 decimal place.

**(4)**

# Question 7

The function f is defined by

$$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R}, x \neq -4, x \neq 2$$

(a) Show that $f(x) = \dfrac{x-3}{x-2}$ **(5)**

The function g is defined by

$$g(x) = \frac{e^x - 3}{e^x - 2}, \quad x \in \mathbb{R}, x \neq \ln 2$$

(b) Differentiate $g(x)$ to show that $g'(x) = \dfrac{e^x}{(e^x - 2)^2}$ **(3)**

(c) Find the exact values of $x$ for which $g'(x) = 1$ **(4)**

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# Question 8

(a) Write down $\sin 2x$ in terms of $\sin x$ and $\cos x$. **(1)**

(b) Find, for $0 < x < \pi$, all the solutions of the equation

$$\operatorname{cosec} x - 8\cos x = 0$$

giving your answers to 2 decimal places. **(5)**