Statistics-S2 - 2010-January

Question 1

A manufacturer supplies DVD players to retailers in batches of 20. It has 5% of the players returned because they are faulty.

(a) Write down a suitable model for the distribution of the number of faulty DVD players in a batch.

(2)

Find the probability that a batch contains

(b) no faulty DVD players,

(2)

(c) more than 4 faulty DVD players.

(2)

(d) Find the mean and variance of the number of faulty DVD players in a batch.

(2)

Question 2

A continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0, & x < -2 \\ \frac{x+2}{6}, & -2 \leqslant x \leqslant 4 \\ 1, & x > 4 \end{cases}$$

(a) Find $P(X<0)$.

(2)

(b) Find the probability density function $f(x)$ of X.

(3)

(c) Write down the name of the distribution of X.

(1)

(d) Find the mean and the variance of X.

(3)

(e) Write down the value of $P(X=1)$.

(1)

Question 3

A robot is programmed to build cars on a production line. The robot breaks down at random at a rate of once every 20 hours.

(a) Find the probability that it will work continuously for 5 hours without a breakdown.

(3)

Find the probability that, in an 8 hour period,

(b) the robot will break down at least once,

(3)

(c) there are exactly 2 breakdowns.

(2)

In a particular 8 hour period, the robot broke down twice.

(d) Write down the probability that the robot will break down in the following 8 hour period. Give a reason for your answer.

(2)

Question 4

The continuous random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} k(x^2 - 2x + 2) & 0 < x \leqslant 3 \\ 3k & 3 < x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{1}{9}$

(4)

(b) Find the cumulative distribution function $F(x)$.

(6)

(c) Find the mean of X.

(3)

(d) Show that the median of X lies between $x = 2.6$ and $x = 2.7$

(4)

Question 5

A café serves breakfast every morning. Customers arrive for breakfast at random at a rate of 1 every 6 minutes.

Find the probability that

(a) fewer than 9 customers arrive for breakfast on a Monday morning between 10 am and 11 am.

(3)

The café serves breakfast every day between 8 am and 12 noon.

(b) Using a suitable approximation, estimate the probability that more than 50 customers arrive for breakfast next Tuesday.

(6)

Question 6

(a) Define the critical region of a test statistic.

(2)

A discrete random variable X has a Binomial distribution B(30, p). A single observation is used to test $H_0 : p = 0.3$ against $H_1 : p \neq 0.3$

(b) Using a 1% level of significance find the critical region of this test. You should state the probability of rejection in each tail which should be as close as possible to 0.005

(5)

(c) Write down the actual significance level of the test.

(1)

The value of the observation was found to be 15.

(d) Comment on this finding in light of your critical region.

(2)

Question 7

A bag contains a large number of coins. It contains only 1p and 2p coins in the ratio 1:3

(a) Find the mean μ and the variance σ^2 of the values of this population of coins.

(3)

A random sample of size 3 is taken from the bag.

(b) List all the possible samples.

(2)

(c) Find the sampling distribution of the mean value of the samples.

(6)