Core-Maths-C2 - 2009-June

Question 1

Use calculus to find the value of

$$\int_1^4 \left(2x + 3\sqrt{x}\right) \mathrm{d}x .$$

(5)

Question 2

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 + kx\right)^7$$

where k is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 is 6 times the coefficient of x,

(b) find the value of k.

(2)

Question 3

$$f(x) = (3x - 2)(x - k) - 8$$

where k is a constant.

(a) Write down the value of $f(k)$.

(1)

When $f(x)$ is divided by $(x - 2)$ the remainder is 4

(b) Find the value of k.

(2)

(c) Factorise $f(x)$ completely.

(3)

Question 4

(a) Complete the table below, giving values of $\sqrt{(2^x + 1)}$ to 3 decimal places.

x	0	0.5	1	1.5	2	2.5	3
$\sqrt{(2^x + 1)}$	1.414	1.554	1.732	1.957			3

(2)

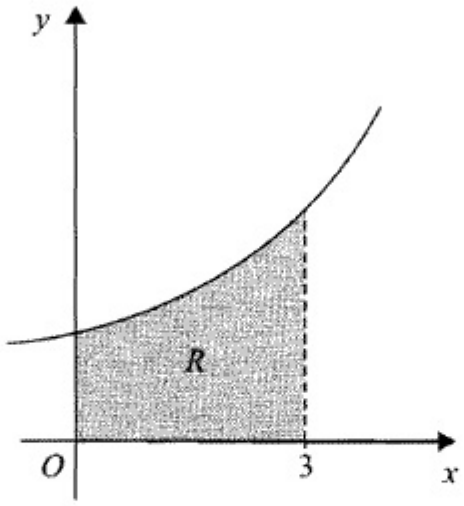

Figure 1

Figure 1 shows the region R which is bounded by the curve with equation $y = \sqrt{(2^x + 1)}$, the x-axis and the lines $x = 0$ and $x = 3$

(b) Use the trapezium rule, with all the values from your table, to find an approximation for the area of R.

(4)

(c) By reference to the curve in Figure 1 state, giving a reason, whether your approximation in part (b) is an overestimate or an underestimate for the area of R.

(2)

Question 5

The third term of a geometric sequence is 324 and the sixth term is 96

(a) Show that the common ratio of the sequence is $\frac{2}{3}$

(2)

(b) Find the first term of the sequence.

(2)

(c) Find the sum of the first 15 terms of the sequence.

(3)

(d) Find the sum to infinity of the sequence.

(2)

Question 6

The circle C has equation

$$x^2 + y^2 - 6x + 4y = 12$$

(a) Find the centre and the radius of C.

(5)

The point $P(-1, 1)$ and the point $Q(7, -5)$ both lie on C.

(b) Show that PQ is a diameter of C.

(2)

The point R lies on the positive y-axis and the angle $PRQ = 90°$.

(c) Find the coordinates of R.

(4)

Question 7

(i) Solve, for $-180° \leqslant \theta < 180°$,

$$(1 + \tan\theta)(5\sin\theta - 2) = 0.$$

(4)

(ii) Solve, for $0 \leqslant x < 360°$,

$$4\sin x = 3\tan x.$$

(6)

Question 8

(a) Find the value of y such that

$$\log_2 y = -3$$

(2)

(b) Find the values of x such that

$$\frac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x$$

(5)

Question 9

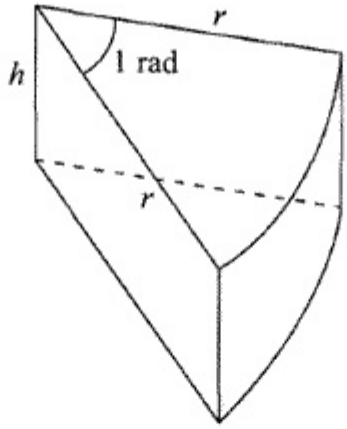

Figure 2

Figure 2 shows a closed box used by a shop for packing pieces of cake. The box is a right prism of height h cm. The cross section is a sector of a circle. The sector has radius r cm and angle 1 radian.

The volume of the box is 300 cm^3.

(a) Show that the surface area of the box, S cm^2, is given by

$$S = r^2 + \frac{1800}{r}$$

(5)

(b) Use calculus to find the value of r for which S is stationary.

(4)

(c) Prove that this value of r gives a minimum value of S.

(2)

(d) Find, to the nearest cm^2, this minimum value of S.

(2)