Core-Maths-C3 - 2013-January

Question 1

The curve C has equation

$$y = (2x-3)^5$$

The point P lies on C and has coordinates $(w, -32)$.

Find

(a) the value of w,

(2)

(b) the equation of the tangent to C at the point P in the form $y = mx + c$, where m and c are constants.

(5)

Question 2

$$g(x) = e^{x-1} + x - 6$$

(a) Show that the equation $g(x) = 0$ can be written as

$$x = \ln(6-x) + 1, \qquad x < 6$$

(2)

The root of $g(x) = 0$ is α.

The iterative formula

$$x_{n+1} = \ln(6-x_n) + 1, \qquad x_0 = 2$$

is used to find an approximate value for α.

(b) Calculate the values of x_1, x_2 and x_3 to 4 decimal places.

(3)

(c) By choosing a suitable interval, show that $\alpha = 2.307$ correct to 3 decimal places.

(3)

Question 3

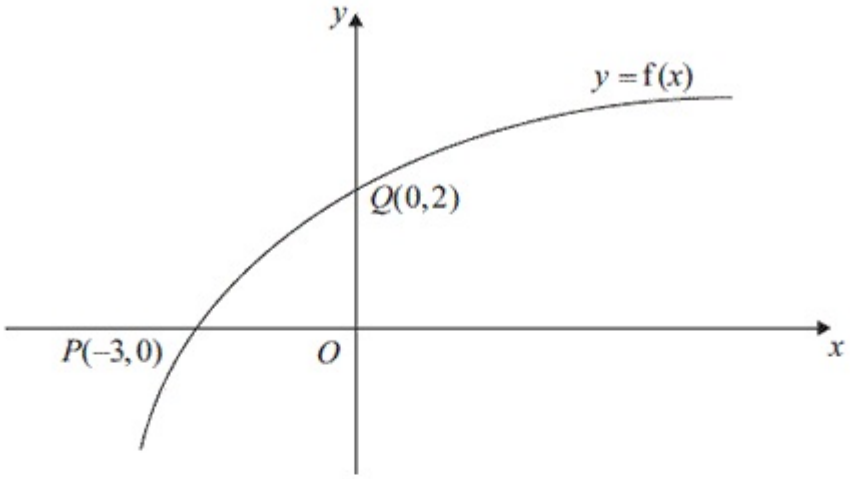

Figure 1

Figure 1 shows part of the curve with equation $y = \mathrm{f}(x), x \in \mathbb{R}$.

The curve passes through the points $Q(0,2)$ and $P(-3,0)$ as shown.

(a) Find the value of $\mathrm{ff}(-3)$.

(2)

On separate diagrams, sketch the curve with equation

(b) $y = \mathrm{f}^{-1}(x)$,

(2)

(c) $y = \mathrm{f}(|x|) - 2$,

(2)

(d) $y = 2\mathrm{f}\left(\frac{1}{2}x\right)$.

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

Question 4

(a) Express $6\cos\theta+8\sin\theta$ in the form $R\cos(\theta-\alpha)$, where $R>0$ and $0<\alpha<\frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(4)

(b) $$p(\theta)=\frac{4}{12+6\cos\theta+8\sin\theta}, \qquad 0\leqslant\theta\leqslant 2\pi$$

Calculate

(i) the maximum value of $p(\theta)$,

(ii) the value of θ at which the maximum occurs.

(4)

Question 5

(i) Differentiate with respect to x

(a) $y=x^3\ln 2x$

(b) $y=(x+\sin 2x)^3$

(6)

Given that $x=\cot y$,

(ii) show that $\frac{dy}{dx}=\frac{-1}{1+x^2}$

(5)

Question 6

(i) Without using a calculator, find the exact value of

$$(\sin 22.5° + \cos 22.5°)^2$$

You must show each stage of your working.

(5)

(ii) (a) Show that $\cos 2\theta + \sin \theta = 1$ may be written in the form

$k \sin^2 \theta - \sin \theta = 0$, stating the value of k.

(2)

(b) Hence solve, for $0 \leqslant \theta < 360°$, the equation

$$\cos 2\theta + \sin \theta = 1$$

(4)

Question 7

$$\mathrm{h}(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \qquad x \geqslant 0$$

(a) Show that $\mathrm{h}(x) = \dfrac{2x}{x^2+5}$

(4)

(b) Hence, or otherwise, find $\mathrm{h}'(x)$ in its simplest form.

(3)

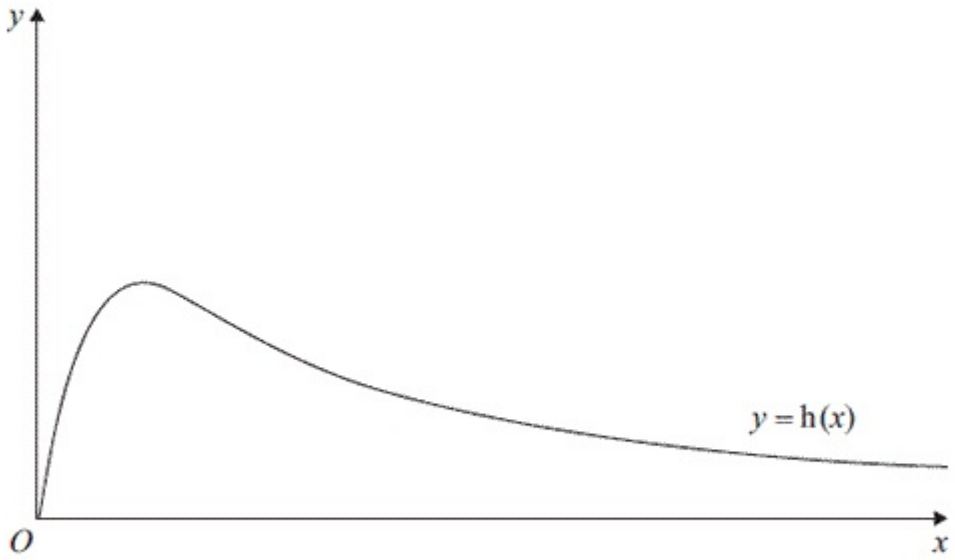

Figure 2

Figure 2 shows a graph of the curve with equation $y = \mathrm{h}(x)$.

(c) Calculate the range of $\mathrm{h}(x)$.

(5)

Question 8

The value of Bob's car can be calculated from the formula

$$V = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$$

where V is the value of the car in pounds (£) and t is the age in years.

(a) Find the value of the car when $t = 0$

(1)

(b) Calculate the exact value of t when $V = 9500$

(4)

(c) Find the rate at which the value of the car is decreasing at the instant when $t = 8$. Give your answer in pounds per year to the nearest pound.

(4)