

Core-Maths-C3 - 2006-January

Question 1

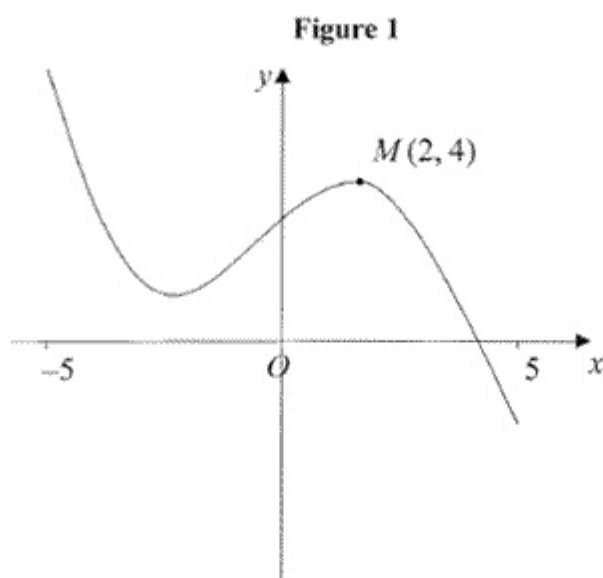


Figure 1 shows the graph of $y = f(x)$, $-5 \leq x \leq 5$.

The point $M(2, 4)$ is the maximum turning point of the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = f(x) + 3$, (2)

(b) $y = |f(x)|$, (2)

(c) $y = f(|x|)$. (3)

Show on each graph the coordinates of any maximum turning points.

Question 2

Express

$$\frac{2x^2 + 3x}{(2x + 3)(x - 2)} - \frac{6}{x^2 - x - 2}$$

as a single fraction in its simplest form.

(7)

Question 3

The point P lies on the curve with equation $y = \ln\left(\frac{1}{3}x\right)$. The x -coordinate of P is 3.

Find an equation of the normal to the curve at the point P in the form $y = ax + b$, where a and b are constants.

(5)

Question 4

(a) Differentiate with respect to x

(i) $x^2 e^{3x+2}$,

(4)

(ii) $\frac{\cos(2x^3)}{3x}$.

(4)

(b) Given that $x = 4 \sin(2y + 6)$, find $\frac{dy}{dx}$ in terms of x .

(5)

Question 5

$$f(x) = 2x^3 - x - 4.$$

- (a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}.$$

(3)

The equation $2x^3 - x - 4 = 0$ has a root between 1.35 and 1.4.

- (b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{1}{2}\right)},$$

with $x_0 = 1.35$, to find, to 2 decimal places, the values of x_1 , x_2 and x_3 .

(3)

The only real root of $f(x) = 0$ is α .

- (c) By choosing a suitable interval, prove that $\alpha = 1.392$, to 3 decimal places.
- (3)**
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Question 6

$$f(x) = 12 \cos x - 4 \sin x.$$

Given that $f(x) = R \cos(x + \alpha)$, where $R \geq 0$ and $0 \leq \alpha \leq 90^\circ$,

(a) find the value of R and the value of α .

(4)

(b) Hence solve the equation

$$12 \cos x - 4 \sin x = 7$$

for $0 \leq x < 360^\circ$, giving your answers to one decimal place.

(5)

(c) (i) Write down the minimum value of $12 \cos x - 4 \sin x$.

(1)

(ii) Find, to 2 decimal places, the smallest positive value of x for which this minimum value occurs.

(2)

Question 7

(a) Show that

$$(i) \quad \frac{\cos 2x}{\cos x + \sin x} \equiv \cos x - \sin x, \quad x \neq (n - \tfrac{1}{4})\pi, n \in \mathbb{Z}, \quad (2)$$

$$(ii) \quad \tfrac{1}{2}(\cos 2x - \sin 2x) \equiv \cos^2 x - \cos x \sin x - \tfrac{1}{2}. \quad (3)$$

(b) Hence, or otherwise, show that the equation

$$\cos \theta \left(\frac{\cos 2\theta}{\cos \theta + \sin \theta} \right) = \frac{1}{2}$$

can be written as

$$\sin 2\theta = \cos 2\theta, \quad (3)$$

(c) Solve, for $0 \leq \theta < 2\pi$,

$$\sin 2\theta = \cos 2\theta,$$

giving your answers in terms of π .

(4)

Question 8

The functions f and g are defined by

$$f: x \rightarrow 2x + \ln 2, \quad x \in \mathbb{R},$$

$$g: x \rightarrow e^{2x}, \quad x \in \mathbb{R}.$$

- (a) Prove that the composite function gf is

$$gf: x \rightarrow 4e^{4x}, \quad x \in \mathbb{R}. \quad (4)$$

- (b) In the space provided on page 19, sketch the curve with equation $y = gf(x)$, and show the coordinates of the point where the curve cuts the y -axis. (1)

- (c) Write down the range of gf . (1)

- (d) Find the value of x for which $\frac{d}{dx}[gf(x)] = 3$, giving your answer to 3 significant figures. (4)
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