

Statistics-S2 - 2011-January

Question 1

A disease occurs in 3% of a population.

- (a) State any assumptions that are required to model the number of people with the disease in a random sample of size n as a binomial distribution. (2)
- (b) Using this model, find the probability of exactly 2 people having the disease in a random sample of 10 people. (3)
- (c) Find the mean and variance of the number of people with the disease in a random sample of 100 people. (2)

A doctor tests a random sample of 100 patients for the disease. He decides to offer all patients a vaccination to protect them from the disease if more than 5 of the sample have the disease.

- (d) Using a suitable approximation, find the probability that the doctor will offer all patients a vaccination. (3)

Question 2

A student takes a multiple choice test. The test is made up of 10 questions each with 5 possible answers. The student gets 4 questions correct. Her teacher claims she was guessing the answers. Using a one tailed test, at the 5% level of significance, test whether or not there is evidence to reject the teacher's claim.
State your hypotheses clearly.

(6)

Question 3

The continuous random variable X is uniformly distributed over the interval $[-1,3]$. Find

(a) $E(X)$ (1)

(b) $\text{Var}(X)$ (2)

(c) $E(X^2)$ (2)

(d) $P(X < 1.4)$ (1)

A total of 40 observations of X are made.

(e) Find the probability that at least 10 of these observations are negative. (5)

Question 4

Richard regularly travels to work on a ferry. Over a long period of time, Richard has found that the ferry is late on average 2 times every week. The company buys a new ferry to improve the service. In the 4-week period after the new ferry is launched, Richard finds the ferry is late 3 times and claims the service has improved. Assuming that the number of times the ferry is late has a Poisson distribution, test Richard's claim at the 5% level of significance. State your hypotheses clearly.

(6)

Question 5

A continuous random variable X has the probability density function $f(x)$ shown in Figure 1.

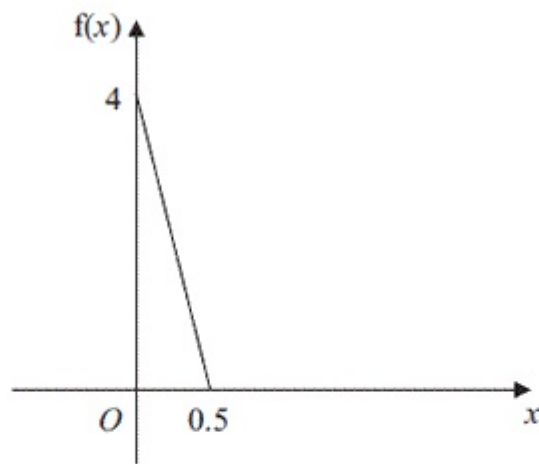


Figure 1

- (a) Show that $f(x) = 4 - 8x$ for $0 \leq x \leq 0.5$ and specify $f(x)$ for all real values of x . (4)
 - (b) Find the cumulative distribution function $F(x)$. (4)
 - (c) Find the median of X . (3)
 - (d) Write down the mode of X . (1)
 - (e) State, with a reason, the skewness of X . (1)
-

Question 6

Cars arrive at a motorway toll booth at an average rate of 150 per hour.

- (a) Suggest a suitable distribution to model the number of cars arriving at the toll booth, X , per minute.

(2)

- (b) State clearly any assumptions you have made by suggesting this model.

(2)

Using your model,

- (c) find the probability that in any given minute

(i) no cars arrive,

(ii) more than 3 cars arrive.

(3)

- (d) In any given 4 minute period, find m such that $P(X > m) = 0.0487$

(3)

- (e) Using a suitable approximation find the probability that fewer than 15 cars arrive in any given 10 minute period.

(6)

Question 7

The queuing time in minutes, X , of a customer at a post office is modelled by the probability density function

$$f(x) = \begin{cases} kx(81 - x^2) & 0 \leq x \leq 9 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Show that $k = \frac{4}{6561}$. (3)

Using integration, find

- (b) the mean queuing time of a customer, (4)
- (c) the probability that a customer will queue for more than 5 minutes. (3)

Three independent customers shop at the post office.

- (d) Find the probability that at least 2 of the customers queue for more than 5 minutes. (3)
-