Core-Maths-C1 - 2009-January

Question 1

(a) Write down the value of $125^{\frac{1}{3}}$.

(1)

(b) Find the value of $125^{-\frac{2}{3}}$.

(2)

Question 2

Find $\int \left(12x^5 - 8x^3 + 3\right) dx$, giving each term in its simplest form.

(4)

Question 3

Expand and simplify $(\sqrt{7} + 2)(\sqrt{7} - 2)$.

(2)

Question 4

A curve has equation $y = f(x)$ and passes through the point (4, 22).

Given that

$$f'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7,$$

use integration to find $f(x)$, giving each term in its simplest form.

(5)

Question 5

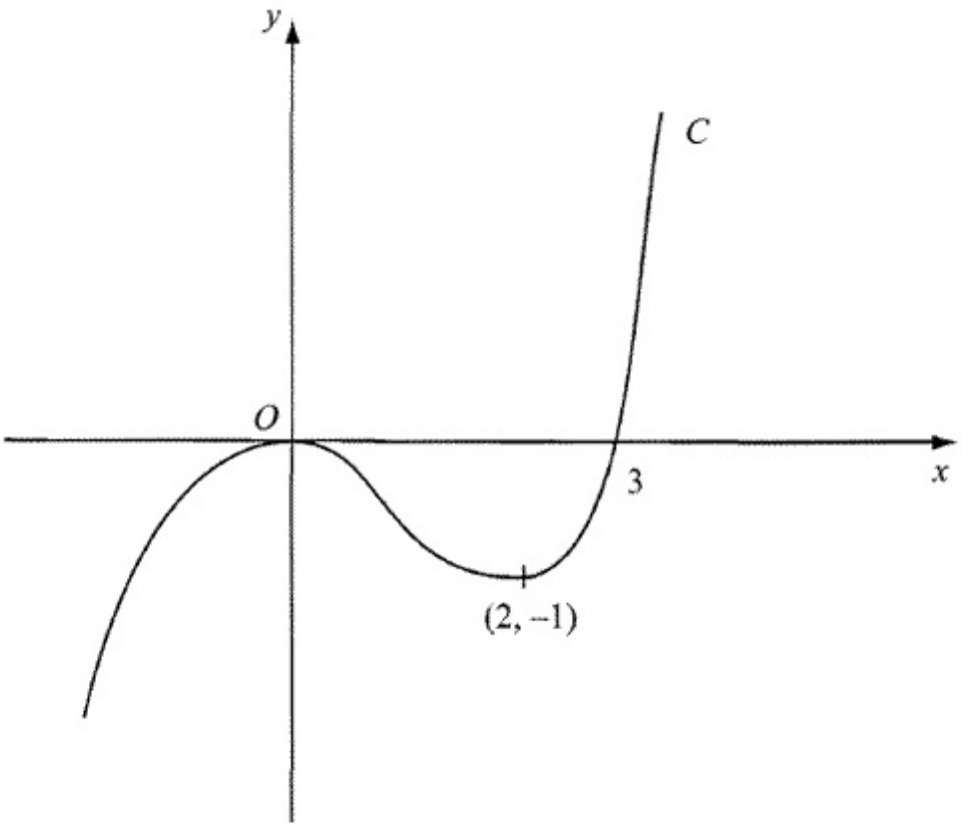

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$. There is a maximum at $(0, 0)$, a minimum at $(2, -1)$ and C passes through $(3, 0)$.

On separate diagrams sketch the curve with equation

(a) $y = f(x + 3)$,

(3)

(b) $y = f(-x)$.

(3)

On each diagram show clearly the coordinates of the maximum point, the minimum point and any points of intersection with the x-axis.

Question 6

Given that $\frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $2x^p - x^q$,

(a) write down the value of p and the value of q.

(2)

Given that $y = 5x^4 - 3 + \frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$,

(b) find $\frac{dy}{dx}$, simplifying the coefficient of each term.

(4)

Question 7

The equation $kx^2 + 4x + (5 - k) = 0$, where k is a constant, has 2 different real solutions for x.

(a) Show that k satisfies

$$k^2 - 5k + 4 > 0.$$

(3)

(b) Hence find the set of possible values of k.

(4)

Question 8

The point $P\ (1, a)$ lies on the curve with equation $y = (x + 1)^2(2 - x)$.

(a) Find the value of a.

(1)

(b) On the axes below sketch the curves with the following equations:

(i) $y = (x + 1)^2(2 - x)$,

(ii) $y = \frac{2}{x}$.

On your diagram show clearly the coordinates of any points at which the curves meet the axes.

(5)

(c) With reference to your diagram in part (b) state the number of real solutions to the equation

$$(x+1)^2(2-x) = \frac{2}{x}.$$

(1)

Question 9

The first term of an arithmetic series is a and the common difference is d.

The 18th term of the series is 25 and the 21st term of the series is $32\frac{1}{2}$.

(a) Use this information to write down two equations for a and d.

(2)

(b) Show that $a = -17.5$ and find the value of d.

(2)

The sum of the first n terms of the series is 2750.

(c) Show that n is given by

$$n^2 - 15n = 55 \times 40.$$

(4)

(d) Hence find the value of n.

(3)

Question 10

The line l_1 passes through the point A (2, 5) and has gradient $-\frac{1}{2}$.

(a) Find an equation of l_1, giving your answer in the form $y = mx + c$.

(3)

The point B has coordinates (–2, 7).

(b) Show that B lies on l_1.

(1)

(c) Find the length of AB, giving your answer in the form $k\sqrt{5}$, where k is an integer.

(3)

The point C lies on l_1 and has x-coordinate equal to p.

The length of AC is 5 units.

(d) Show that p satisfies

$$p^2 - 4p - 16 = 0.$$

(4)

Question 11

The curve C has equation

$$y = 9 - 4x - \frac{8}{x}, \qquad x > 0.$$

The point P on C has x-coordinate equal to 2.

(a) Show that the equation of the tangent to C at the point P is $y = 1 - 2x$.

(6)

(b) Find an equation of the normal to C at the point P.

(3)

The tangent at P meets the x-axis at A and the normal at P meets the x-axis at B.

(c) Find the area of triangle APB.

(4)