Core-Maths-C4 - 2012-January

Question 1

The curve C has the equation $2x + 3y^2 + 3x^2y = 4x^2$.
The point P on the curve has coordinates $(-1, 1)$.

(a) Find the gradient of the curve at P.

(5)

(b) Hence find the equation of the normal to C at P, giving your answer in the form $ax+by+c=0$, where a, b and c are integers.

(3)

Question 2

(a) Use integration by parts to find $\int x \sin 3x \, dx$.

(3)

(b) Using your answer to part (a), find $\int x^2 \cos 3x \, dx$.

(3)

Question 3

(a) Expand

$$\frac{1}{(2-5x)^2}, \quad |x| < \frac{2}{5}$$

in ascending powers of x, up to and including the term in x^2, giving each term as a simplified fraction.

(5)

Given that the binomial expansion of $\dfrac{2+kx}{(2-5x)^2}$, $|x| < \dfrac{2}{5}$, is

$$\frac{1}{2} + \frac{7}{4}x + Ax^2 + \dots$$

(b) find the value of the constant k,

(2)

(c) find the value of the constant A.

(2)

Question 4

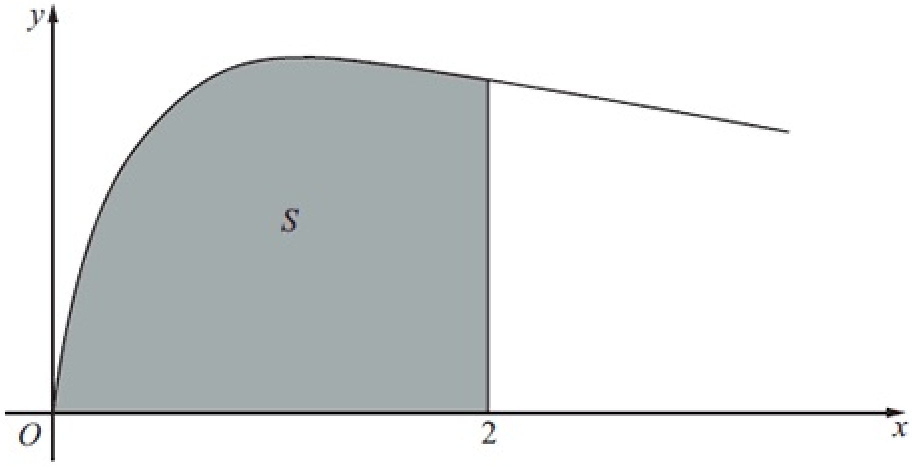

Figure 1

Figure 1 shows the curve with equation

$$y = \sqrt{\left(\frac{2x}{3x^2 + 4}\right)}, \quad x \geqslant 0$$

The finite region S, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 2$

The region S is rotated 360° about the x-axis.

Use integration to find the exact value of the volume of the solid generated, giving your answer in the form $k \ln a$, where k and a are constants.

(5)

Question 5

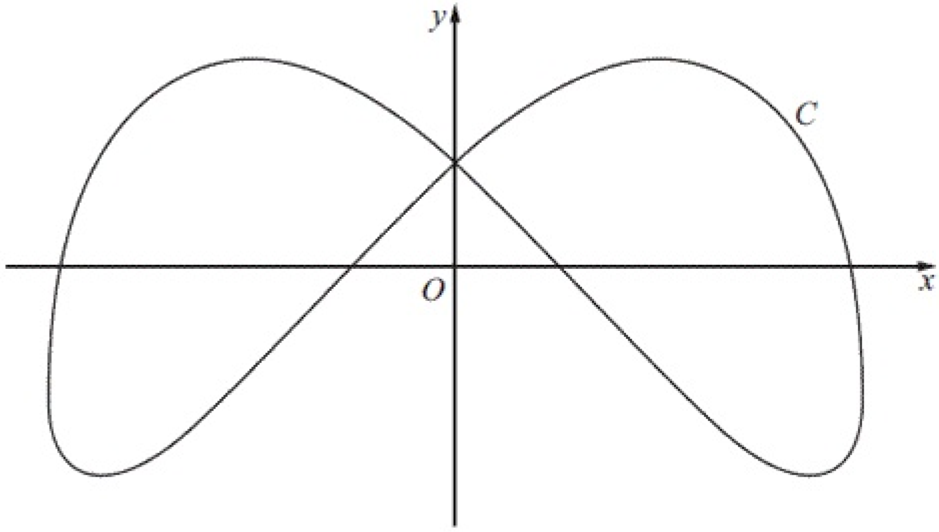

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\sin\left(t + \frac{\pi}{6}\right), \qquad y = 3\cos 2t, \qquad 0 \leqslant t < 2\pi$$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.

(3)

(b) Find the coordinates of all the points on C where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

(5)

Question 6

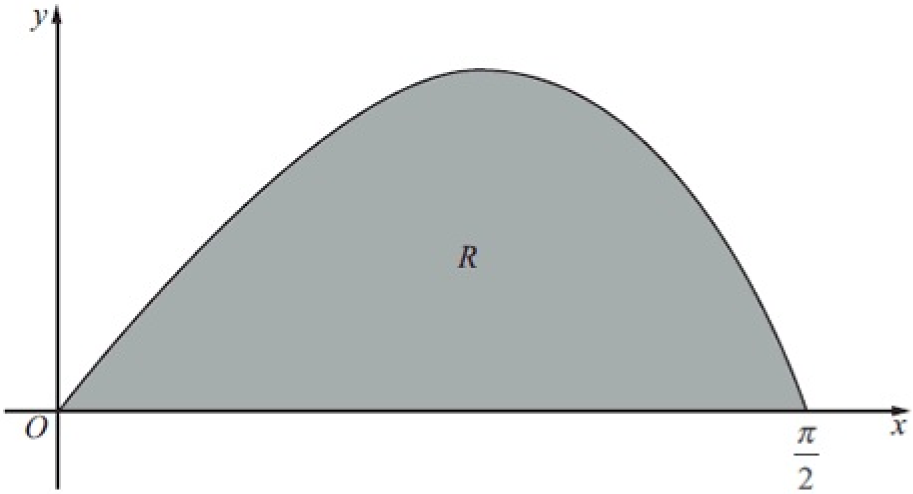

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \dfrac{2\sin 2x}{(1+\cos x)}$, $0 \leqslant x \leqslant \dfrac{\pi}{2}$.

The finite region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

The table below shows corresponding values of x and y for $y = \dfrac{2\sin 2x}{(1+\cos x)}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

(a) Complete the table above giving the missing value of y to 5 decimal places.

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 4 decimal places.

(3)

(c) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2\sin 2x}{(1+\cos x)}\,dx = 4\ln(1+\cos x) - 4\cos x + k$$

where k is a constant.

(5)

(d) Hence calculate the error of the estimate in part (b), giving your answer to 2 significant figures.

(3)

Question 7

Relative to a fixed origin O, the point A has position vector $(2\mathbf{i} - \mathbf{j} + 5\mathbf{k})$,
the point B has position vector $(5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k})$,
and the point D has position vector $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Find a vector equation for the line l.

(2)

(c) Show that the size of the angle BAD is $109°$, to the nearest degree.

(4)

The points A, B and D, together with a point C, are the vertices of the parallelogram $ABCD$, where $\overrightarrow{AB} = \overrightarrow{DC}$.

(d) Find the position vector of C.

(2)

(e) Find the area of the parallelogram $ABCD$, giving your answer to 3 significant figures.

(3)

(f) Find the shortest distance from the point D to the line l, giving your answer to 3 significant figures.

(2)