Core-Maths-C2 - 2013-January

Question 1

Find the first 3 terms, in ascending powers of x, in the binomial expansion of

$$(2-5x)^6$$

Give each term in its simplest form.

(4)

Question 2

$f(x) = ax^3 + bx^2 - 4x - 3$, where a and b are constants.

Given that $(x - 1)$ is a factor of f(x),

(a) show that
$$a + b = 7$$

(2)

Given also that, when f(x) is divided by $(x + 2)$, the remainder is 9,

(b) find the value of a and the value of b, showing each step in your working.

(4)

Question 3

A company predicts a yearly profit of £120 000 in the year 2013. The company predicts that the yearly profit will rise each year by 5%. The predicted yearly profit forms a geometric sequence with common ratio 1.05

(a) Show that the predicted profit in the year 2016 is £138 915

(1)

(b) Find the first year in which the yearly predicted profit exceeds £200 000

(5)

(c) Find the total predicted profit for the years 2013 to 2023 inclusive, giving your answer to the nearest pound.

(3)

Question 4

Solve, for $0 \leqslant x < 180°$,

$$\cos(3x - 10°) = -0.4$$

giving your answers to 1 decimal place. You should show each step in your working.

(7)

Question 5

The circle C has equation

$$x^2 + y^2 - 20x - 24y + 195 = 0$$

The centre of C is at the point M.

(a) Find

    (i) the coordinates of the point M,

    (ii) the radius of the circle C.

(5)

N is the point with coordinates $(25, 32)$.

(b) Find the length of the line MN.

(2)

The tangent to C at a point P on the circle passes through point N.

(c) Find the length of the line NP.

(2)

Question 6

Given that

$$2\log_2(x+15) - \log_2 x = 6$$

(a) Show that

$$x^2 - 34x + 225 = 0$$

(5)

(b) Hence, or otherwise, solve the equation

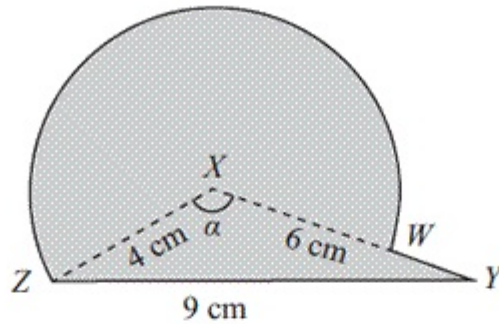$$2\log_2(x+15) - \log_2 x = 6$$

(2)

Question 7

Figure 1

The triangle *XYZ* in Figure 1 has *XY* = 6 cm, *YZ* = 9 cm, *ZX* = 4 cm and angle *ZXY* = *α*. The point *W* lies on the line *XY*.

The circular arc *ZW*, in Figure 1 is a major arc of the circle with centre *X* and radius 4 cm.

(a) Show that, to 3 significant figures, *α* = 2.22 radians.

(2)

(b) Find the area, in cm², of the major sector *XZWX*.

(3)

The region enclosed by the major arc *ZW* of the circle and the lines *WY* and *YZ* is shown shaded in Figure 1.

Calculate

(c) the area of this shaded region,

(3)

(d) the perimeter *ZWYZ* of this shaded region.

(4)

Question 8

The curve C has equation $y = 6 - 3x - \dfrac{4}{x^3}$, $x \neq 0$

(a) Use calculus to show that the curve has a turning point P when $x = \sqrt{2}$

(4)

(b) Find the x-coordinate of the other turning point Q on the curve.

(1)

(c) Find $\dfrac{d^2 y}{dx^2}$.

(1)

(d) Hence or otherwise, state with justification, the nature of each of these turning points P and Q.
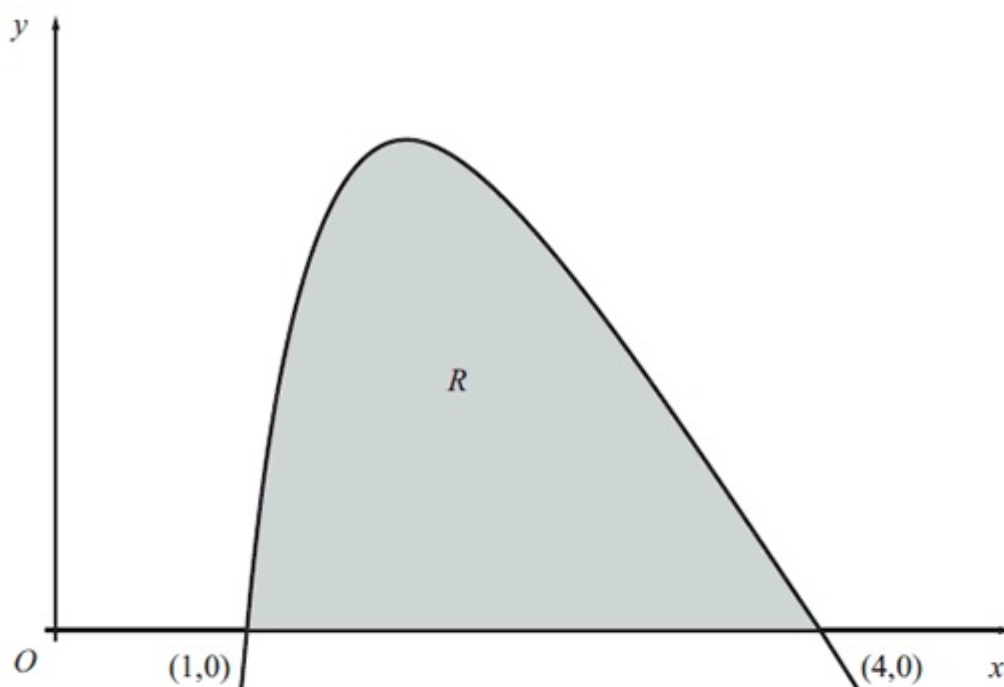
(3)

Question 9

Figure 2

The finite region R, as shown in Figure 2, is bounded by the x-axis and the curve with equation

$$y = 27 - 2x - 9\sqrt{x} - \frac{16}{x^2}, \qquad x > 0$$

The curve crosses the x-axis at the points $(1, 0)$ and $(4, 0)$.

(a) Complete the table below, by giving your values of y to 3 decimal places.

x	1	1.5	2	2.5	3	3.5	4
y	0	5.866		5.210		1.856	0

(2)

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

(c) Use integration to find the exact value for the area of R.

(6)