Mechanics-M1 - 2009-January

Question 1

A particle P moves with constant acceleration $(2\mathbf{i} - 5\mathbf{j})$ m s^{-2}. At time $t = 0$, P has speed u m s^{-1}. At time $t = 3$ s, P has velocity $(-6\mathbf{i} + \mathbf{j})$ m s^{-1}.

Find the value of u.

(5)

Question 2

A small ball is projected vertically upwards from ground level with speed u m s^{-1}. The ball takes 4 s to return to ground level.

(a) Draw, in the space below, a velocity-time graph to represent the motion of the ball during the first 4 s.

(2)

(b) The maximum height of the ball above the ground during the first 4 s is 19.6 m. Find the value of u.

(3)

Question 3

Two particles A and B are moving on a smooth horizontal plane. The mass of A is km, where $2 < k < 3$, and the mass of B is m. The particles are moving along the same straight line, but in opposite directions, and they collide directly. Immediately before they collide the speed of A is $2u$ and the speed of B is $4u$. As a result of the collision the speed of A is halved and its direction of motion is reversed.

(a) Find, in terms of k and u, the speed of B immediately after the collision.

(3)

(b) State whether the direction of motion of B changes as a result of the collision, explaining your answer.

(3)

Given that $k = \frac{7}{3}$,

(c) find, in terms of m and u, the magnitude of the impulse that A exerts on B in the collision.

(3)

Question 4

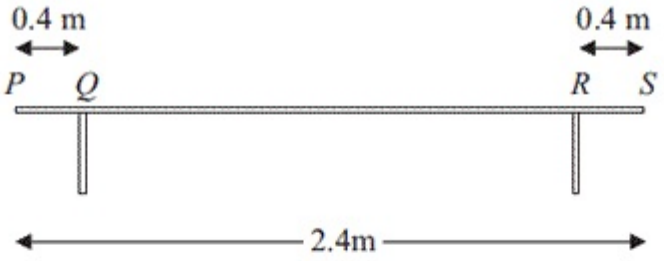

Figure 1

A bench consists of a plank which is resting in a horizontal position on two thin vertical legs. The plank is modelled as a uniform rod PS of length 2.4 m and mass 20 kg. The legs at Q and R are 0.4 m from each end of the plank, as shown in Figure 1.

Two pupils, Arthur and Beatrice, sit on the plank. Arthur has mass 60 kg and sits at the middle of the plank and Beatrice has mass 40 kg and sits at the end P. The plank remains horizontal and in equilibrium. By modelling the pupils as particles, find

(a) the magnitude of the normal reaction between the plank and the leg at Q and the magnitude of the normal reaction between the plank and the leg at R.

(7)

Beatrice stays sitting at P but Arthur now moves and sits on the plank at the point X. Given that the plank remains horizontal and in equilibrium, and that the magnitude of the normal reaction between the plank and the leg at Q is now twice the magnitude of the normal reaction between the plank and the leg at R,

(b) find the distance QX.

(6)

Question 5

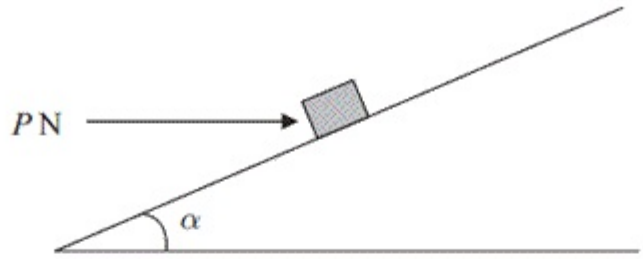

Figure 2

A small package of mass 1.1 kg is held in equilibrium on a rough plane by a horizontal force. The plane is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The force acts in a vertical plane containing a line of greatest slope of the plane and has magnitude P newtons, as shown in Figure 2.

The coefficient of friction between the package and the plane is 0.5 and the package is modelled as a particle. The package is in equilibrium and on the point of slipping down the plane.

(a) Draw, on Figure 2, all the forces acting on the package, showing their directions clearly.

(2)

(b) (i) Find the magnitude of the normal reaction between the package and the plane.

(ii) Find the value of P.

(11)

Question 6

Two forces, $(4\mathbf{i} - 5\mathbf{j})$ N and $(p\mathbf{i} + q\mathbf{j})$ N, act on a particle P of mass m kg. The resultant of the two forces is $\mathbf{R}$. Given that $\mathbf{R}$ acts in a direction which is parallel to the vector $(\mathbf{i} - 2\mathbf{j})$,

(a) find the angle between $\mathbf{R}$ and the vector $\mathbf{j}$,

(3)

(b) show that $2p + q + 3 = 0$.

(4)

Given also that $q = 1$ and that P moves with an acceleration of magnitude $8\sqrt{5}$ m s^{-2},

(c) find the value of m.

(7)

Question 7

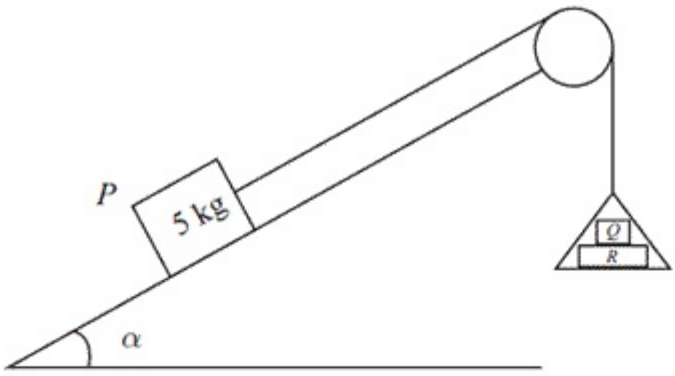

Figure 3

One end of a light inextensible string is attached to a block P of mass 5 kg. The block P is held at rest on a smooth fixed plane which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{3}{5}$. The string lies along a line of greatest slope of the plane and passes over a smooth light pulley which is fixed at the top of the plane. The other end of the string is attached to a light scale pan which carries two blocks Q and R, with block Q on top of block R, as shown in Figure 3. The mass of block Q is 5 kg and the mass of block R is 10 kg. The scale pan hangs at rest and the system is released from rest. By modelling the blocks as particles, ignoring air resistance and assuming the motion is uninterrupted, find

(a) (i) the acceleration of the scale pan,

(ii) the tension in the string,

(8)

(b) the magnitude of the force exerted on block Q by block R,

(3)

(c) the magnitude of the force exerted on the pulley by the string.

(5)