Mechanics-M2 - 2011-January

Question 1

A cyclist starts from rest and moves along a straight horizontal road. The combined mass of the cyclist and his cycle is 120 kg. The resistance to motion is modelled as a constant force of magnitude 32 N. The rate at which the cyclist works is 384 W. The cyclist accelerates until he reaches a constant speed of v m s^{-1}.

Find

(a) the value of v,

(3)

(b) the acceleration of the cyclist at the instant when the speed is 9 m s^{-1}.

(3)

Question 2

A particle of mass 2 kg is moving with velocity $(5\mathbf{i}+\mathbf{j})$ m s^{-1} when it receives an impulse of $(-6\mathbf{i}+8\mathbf{j})$ N s. Find the kinetic energy of the particle immediately after receiving the impulse.

(5)

Question 3

A particle moves along the x-axis. At time $t = 0$ the particle passes through the origin with speed 8 m s^{-1} in the positive x-direction. The acceleration of the particle at time t seconds, $t \geqslant 0$, is $(4t^3 - 12t)$ m s^{-2} in the positive x-direction.

Find

(a) the velocity of the particle at time t seconds,

(3)

(b) the displacement of the particle from the origin at time t seconds,

(2)

(c) the values of t at which the particle is instantaneously at rest.

(3)

Question 4

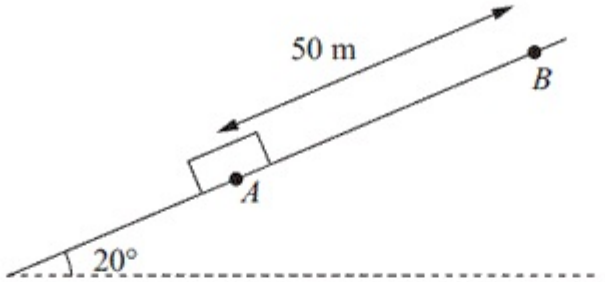

Figure 1

A box of mass 30 kg is held at rest at point A on a rough inclined plane. The plane is inclined at 20° to the horizontal. Point B is 50 m from A up a line of greatest slope of the plane, as shown in Figure 1. The box is dragged from A to B by a force acting parallel to AB and then held at rest at B. The coefficient of friction between the box and the plane is $\frac{1}{4}$. Friction is the only non-gravitational resistive force acting on the box. Modelling the box as a particle,

(a) find the work done in dragging the box from A to B.

(6)

The box is released from rest at the point B and slides down the slope. Using the work-energy principle, or otherwise,

(b) find the speed of the box as it reaches A.

(5)

Question 5

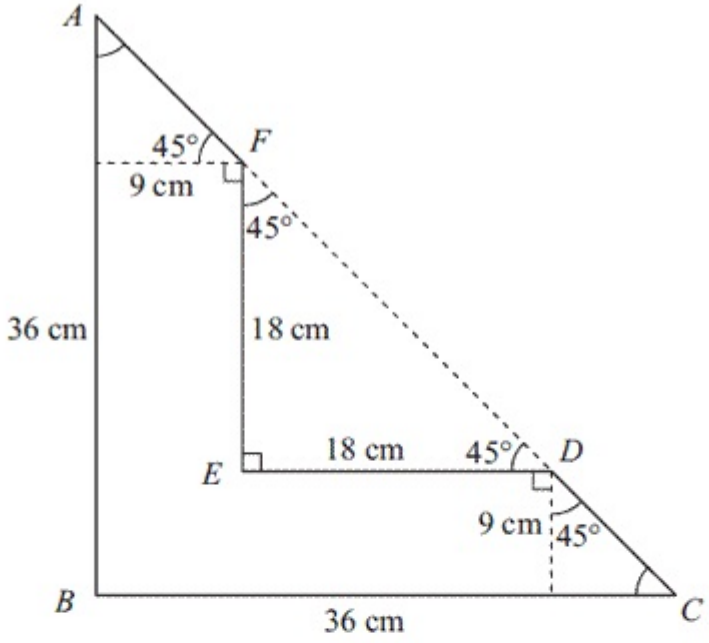

Figure 2

The uniform L-shaped lamina $ABCDEF$, shown in Figure 2, has sides AB and FE parallel, and sides BC and ED parallel. The pairs of parallel sides are 9 cm apart. The points A, F, D and C lie on a straight line.

$AB = BC = 36$ cm, $FE = ED = 18$ cm. $\angle ABC = \angle FED = 90°$, and $\angle BCD = \angle EDF = \angle EFD = \angle BAC = 45°$.

(a) Find the distance of the centre of mass of the lamina from

(i) side AB,

(ii) side BC.

(7)

The lamina is freely suspended from A and hangs in equilibrium.

(b) Find, to the nearest degree, the size of the angle between AB and the vertical.

(3)

Question 6

[*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upwards.*]

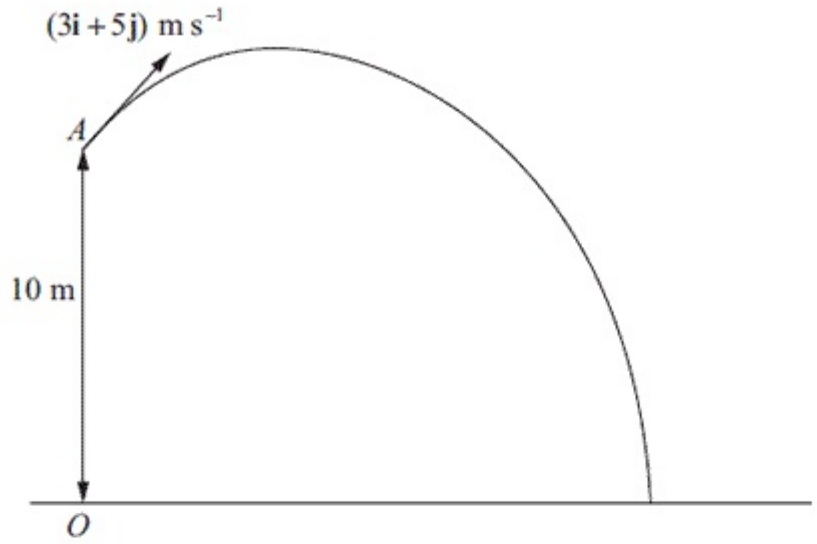

Figure 3

At time $t = 0$, a particle P is projected from the point A which has position vector $10\mathbf{j}$ metres with respect to a fixed origin O at ground level. The ground is horizontal. The velocity of projection of P is $(3\mathbf{i}+5\mathbf{j})$ m s^{-1}, as shown in Figure 3. The particle moves freely under gravity and reaches the ground after T seconds.

(a) For $0 \leqslant t \leqslant T$, show that, with respect to O, the position vector, $\mathbf{r}$ metres, of P at time t seconds is given by

$$\mathbf{r} = 3t\mathbf{i} + (10 + 5t - 4.9t^2)\mathbf{j}$$

(3)

(b) Find the value of T.

(3)

(c) Find the velocity of P at time t seconds $(0 \leqslant t \leqslant T)$.

(2)

When P is at the point B, the direction of motion of P is $45°$ below the horizontal.

(d) Find the time taken for P to move from A to B.

(2)

(e) Find the speed of P as it passes through B.

(2)

Question 7

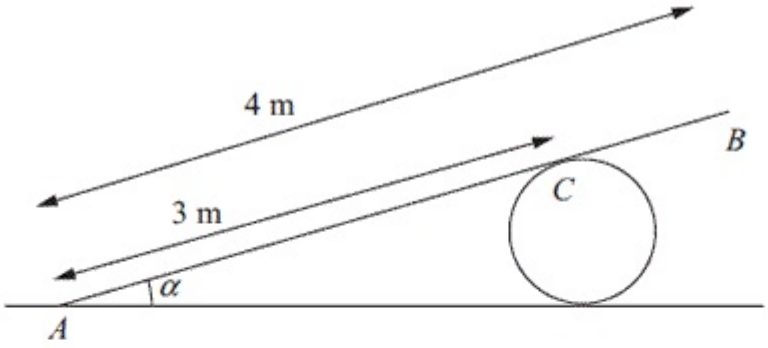

Figure 4

A uniform plank AB, of weight 100 N and length 4 m, rests in equilibrium with the end A on rough horizontal ground. The plank rests on a smooth cylindrical drum. The drum is fixed to the ground and cannot move. The point of contact between the plank and the drum is C, where $AC = 3$ m, as shown in Figure 4. The plank is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle α to the horizontal, where $\sin\alpha = \frac{1}{3}$. The coefficient of friction between the plank and the ground is μ. Modelling the plank as a rod, find the least possible value of μ.

(10)

Question 8

A particle P of mass m kg is moving with speed 6 m s^{-1} in a straight line on a smooth horizontal floor. The particle strikes a fixed smooth vertical wall at right angles and rebounds. The kinetic energy lost in the impact is 64 J. The coefficient of restitution between P and the wall is $\frac{1}{3}$.

(a) Show that $m = 4$.

(6)

After rebounding from the wall, P collides directly with a particle Q which is moving towards P with speed 3 m s^{-1}. The mass of Q is 2 kg and the coefficient of restitution between P and Q is $\frac{1}{3}$.

(b) Show that there will be a second collision between P and the wall.

(7)