

Core-Maths-C3 - 2012-January

Question 1

Differentiate with respect to x , giving your answer in its simplest form,

(a) $x^2 \ln(3x)$ (4)

(b) $\frac{\sin 4x}{x^3}$ (5)

Question 2

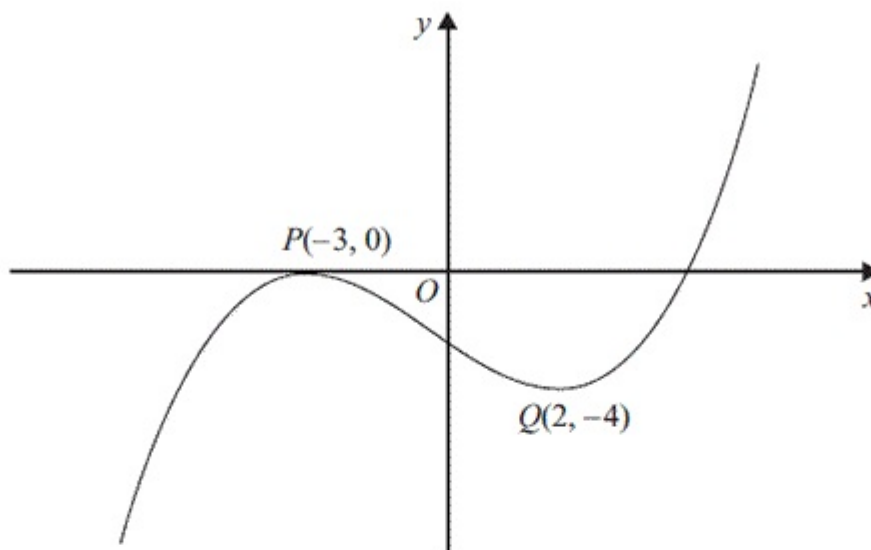


Figure 1

Figure 1 shows the graph of equation $y = f(x)$.

The points $P(-3, 0)$ and $Q(2, -4)$ are stationary points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 3f(x + 2)$ (3)

(b) $y = |f(x)|$ (3)

On each diagram, show the coordinates of any stationary points.

Question 3

The area, $A \text{ mm}^2$, of a bacterial culture growing in milk, t hours after midday, is given by

$$A = 20e^{1.5t}, \quad t \geq 0$$

(a) Write down the area of the culture at midday. (1)

(b) Find the time at which the area of the culture is twice its area at midday. Give your answer to the nearest minute. (5)

Question 4

The point P is the point on the curve $x = 2 \tan\left(y + \frac{\pi}{12}\right)$ with y -coordinate $\frac{\pi}{4}$.

Find an equation of the normal to the curve at P .

(7)

Question 5

Solve, for $0 \leq \theta \leq 180^\circ$,

$$2\cot^2 3\theta = 7\operatorname{cosec} 3\theta - 5$$

Give your answers in degrees to 1 decimal place.

(10)

Question 6

$$f(x) = x^2 - 3x + 2\cos\left(\frac{1}{2}x\right), \quad 0 \leq x \leq \pi$$

(a) Show that the equation $f(x) = 0$ has a solution in the interval $0.8 < x < 0.9$

(2)

The curve with equation $y = f(x)$ has a minimum point P .

(b) Show that the x -coordinate of P is the solution of the equation

$$x = \frac{3 + \sin\left(\frac{1}{2}x\right)}{2}$$

(4)

(c) Using the iteration formula

$$x_{n+1} = \frac{3 + \sin\left(\frac{1}{2}x_n\right)}{2}, \quad x_0 = 2$$

find the values of x_1 , x_2 and x_3 , giving your answers to 3 decimal places.

(3)

(d) By choosing a suitable interval, show that the x -coordinate of P is 1.9078 correct to 4 decimal places.

(3)

Question 7

The function f is defined by

$$f : x \mapsto \frac{3(x+1)}{2x^2 + 7x - 4} - \frac{1}{x+4}, \quad x \in \mathbb{R}, x > \frac{1}{2}$$

(a) Show that $f(x) = \frac{1}{2x-1}$ (4)

(b) Find $f^{-1}(x)$ (3)

(c) Find the domain of f^{-1} (1)

$$g(x) = \ln(x+1)$$

(d) Find the solution of $fg(x) = \frac{1}{7}$, giving your answer in terms of e . (4)

Question 8

(a) Starting from the formulae for $\sin(A+B)$ and $\cos(A+B)$, prove that

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$
 (4)

(b) Deduce that

$$\tan\left(\theta + \frac{\pi}{6}\right) = \frac{1 + \sqrt{3} \tan \theta}{\sqrt{3} - \tan \theta}$$
 (3)

(c) Hence, or otherwise, solve, for $0 \leq \theta \leq \pi$,

$$1 + \sqrt{3} \tan \theta = (\sqrt{3} - \tan \theta) \tan(\pi - \theta)$$

Give your answers as multiples of π . (6)
