Core-Maths-C2 - 2013-June

Question 1

The first three terms of a geometric series are

$$18, 12 \text{ and } p$$

respectively, where p is a constant.

Find

(a) the value of the common ratio of the series,

(1)

(b) the value of p,

(1)

(c) the sum of the first 15 terms of the series, giving your answer to 3 decimal places.

(2)

Question 2

(a) Use the binomial theorem to find all the terms of the expansion of

$$(2 + 3x)^4$$

Give each term in its simplest form.

(4)

(b) Write down the expansion of

$$(2 - 3x)^4$$

in ascending powers of x, giving each term in its simplest form.

(1)

Question 3

$$f(x) = 2x^3 - 5x^2 + ax + 18$$

where a is a constant.

Given that $(x - 3)$ is a factor of $f(x)$,

(a) show that $a = -9$

(2)

(b) factorise $f(x)$ completely.

(4)

Given that

$$g(y) = 2(3^{3y}) - 5(3^{2y}) - 9(3^{y}) + 18$$

(c) find the values of y that satisfy $g(y) = 0$, giving your answers to 2 decimal places where appropriate.

(3)

Question 4

$$y = \frac{5}{(x^2 + 1)}$$

(a) Complete the table below, giving the missing value of y to 3 decimal places.

x	0	0.5	1	1.5	2	2.5	3
y	5	4	2.5		1	0.690	0.5

(1)

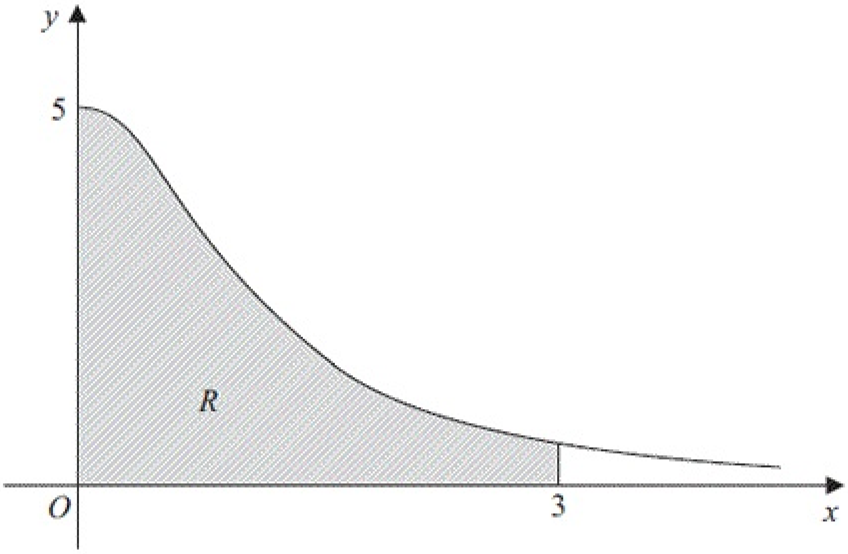

Figure 1

Figure 1 shows the region R which is bounded by the curve with equation $y = \frac{5}{(x^2 + 1)}$, the x-axis and the lines $x = 0$ and $x = 3$

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for the area of R.

(4)

(c) Use your answer to part (b) to find an approximate value for

$$\int_0^3 \left(4 + \frac{5}{(x^2 + 1)}\right) dx$$

giving your answer to 2 decimal places.

(2)

Question 5

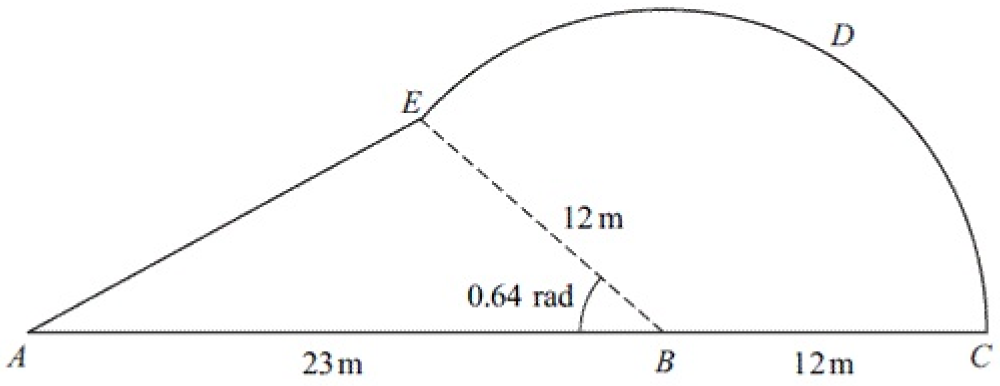

Figure 2

Figure 2 shows a plan view of a garden.
The plan of the garden $ABCDEA$ consists of a triangle ABE joined to a sector $BCDE$ of a circle with radius 12 m and centre B.
The points A, B and C lie on a straight line with $AB = 23$ m and $BC = 12$ m.

Given that the size of angle ABE is exactly 0.64 radians, find

(a) the area of the garden, giving your answer in m^2, to 1 decimal place,

(4)

(b) the perimeter of the garden, giving your answer in metres, to 1 decimal place.

(5)

Question 6

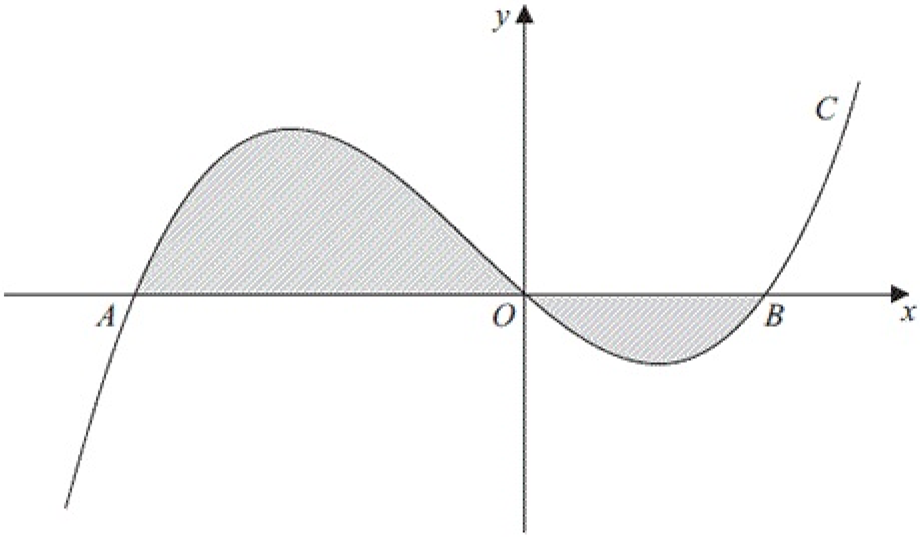

Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = x(x + 4)(x - 2)$$

The curve C crosses the x-axis at the origin O and at the points A and B.

(a) Write down the x-coordinates of the points A and B.

(1)

The finite region, shown shaded in Figure 3, is bounded by the curve C and the x-axis.

(b) Use integration to find the total area of the finite region shown shaded in Figure 3.

(7)

Question 7

(i) Find the exact value of x for which

$$\log_2(2x) = \log_2(5x + 4) - 3$$

(4)

(ii) Given that

$$\log_a y + 3\log_a 2 = 5$$

express y in terms of a.
Give your answer in its simplest form.

(3)

Question 8

(i) Solve, for $-180° \leqslant x < 180°$,

$$\tan(x - 40°) = 1.5$$

giving your answers to 1 decimal place.

(3)

(ii) (a) Show that the equation

$$\sin\theta \tan\theta = 3\cos\theta + 2$$

can be written in the form

$$4\cos^2\theta + 2\cos\theta - 1 = 0$$

(3)

(b) Hence solve, for $0 \leqslant \theta < 360°$,

$$\sin\theta \tan\theta = 3\cos\theta + 2$$

showing each stage of your working.

(5)

Question 9

The curve with equation

$$y = x^2 - 32\sqrt{(x)} + 20, \qquad x > 0$$

has a stationary point P.

Use calculus

(a) to find the coordinates of P,

(6)

(b) to determine the nature of the stationary point P.

(3)

Question 10

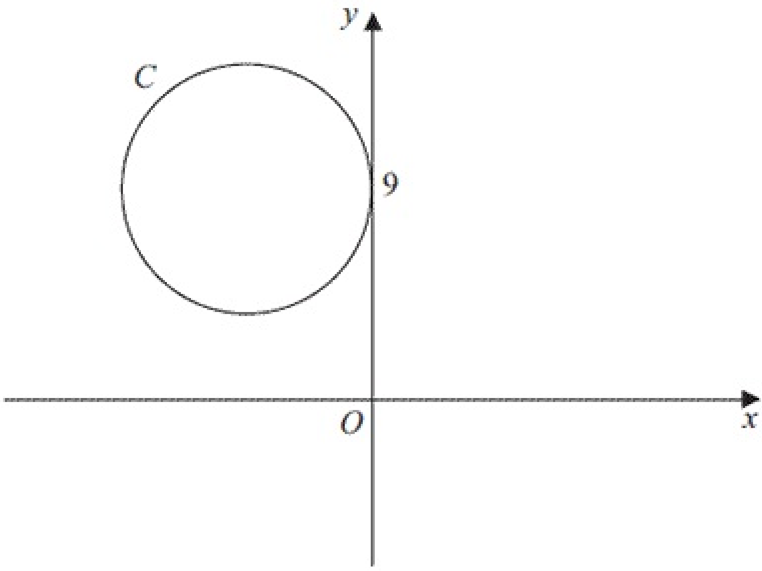

Figure 4

The circle C has radius 5 and touches the y-axis at the point (0, 9), as shown in Figure 4.

(a) Write down an equation for the circle C, that is shown in Figure 4.

(3)

A line through the point $P(8, -7)$ is a tangent to the circle C at the point T.

(b) Find the length of PT.

(3)