Statistics-S2 - 2011-June

Question 1

A factory produces components. Each component has a unique identity number and it is assumed that 2% of the components are faulty. On a particular day, a quality control manager wishes to take a random sample of 50 components.

(a) Identify a sampling frame.

(1)

The statistic F represents the number of faulty components in the random sample of size 50.

(b) Specify the sampling distribution of F.

(2)

Question 2

A traffic officer monitors the rate at which vehicles pass a fixed point on a motorway. When the rate exceeds 36 vehicles per minute he must switch on some speed restrictions to improve traffic flow.

(a) Suggest a suitable model to describe the number of vehicles passing the fixed point in a 15 s interval.

(1)

The traffic officer records 12 vehicles passing the fixed point in a 15 s interval.

(b) Stating your hypotheses clearly, and using a 5% level of significance, test whether or not the traffic officer has sufficient evidence to switch on the speed restrictions.

(6)

(c) Using a 5% level of significance, determine the smallest number of vehicles the traffic officer must observe in a 10 s interval in order to have sufficient evidence to switch on the speed restrictions.

(3)

Question 3

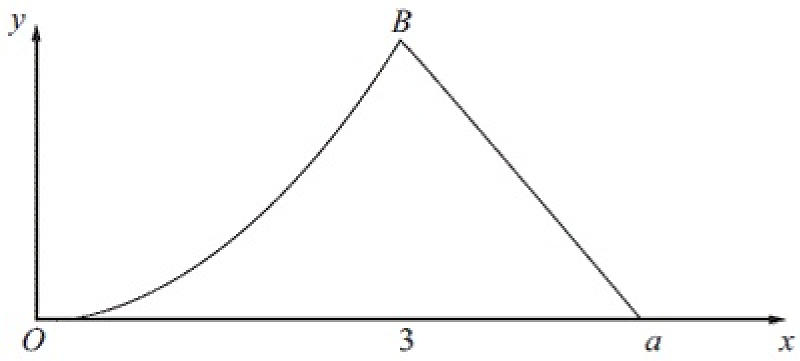

Figure 1

Figure 1 shows a sketch of the probability density function $f(x)$ of the random variable X.

For $0 \leqslant x \leqslant 3$, $f(x)$ is represented by a curve OB with equation $f(x) = kx^2$, where k is a constant.

For $3 \leqslant x \leqslant a$, where a is a constant, $f(x)$ is represented by a straight line passing through B and the point $(a, 0)$.

For all other values of x, $f(x) = 0$.

Given that the mode of X = the median of X, find

(a) the mode,

(1)

(b) the value of k,

(4)

(c) the value of a.

(3)

Without calculating $E(X)$ and with reference to the skewness of the distribution

(d) state, giving your reason, whether $E(X) < 3$, $E(X) = 3$ or $E(X) > 3$.

(2)

Question 4

In a game, players select sticks at random from a box containing a large number of sticks of different lengths. The length, in cm, of a randomly chosen stick has a continuous uniform distribution over the interval [7, 10].

A stick is selected at random from the box.

(a) Find the probability that the stick is shorter than 9.5 cm.

(2)

To win a bag of sweets, a player must select 3 sticks and wins if the length of the longest stick is more than 9.5 cm.

(b) Find the probability of winning a bag of sweets.

(2)

To win a soft toy, a player must select 6 sticks and wins the toy if more than four of the sticks are shorter than 7.6 cm.

(c) Find the probability of winning a soft toy.

(4)

Question 5

Defects occur at random in planks of wood with a constant rate of 0.5 per 10 cm length. Jim buys a plank of length 100 cm.

(a) Find the probability that Jim's plank contains at most 3 defects.

(2)

Shivani buys 6 planks each of length 100 cm.

(b) Find the probability that fewer than 2 of Shivani's planks contain at most 3 defects.

(5)

(c) Using a suitable approximation, estimate the probability that the total number of defects on Shivani's 6 planks is less than 18.

(6)

Question 6

A shopkeeper knows, from past records, that 15% of customers buy an item from the display next to the till. After a refurbishment of the shop, he takes a random sample of 30 customers and finds that only 1 customer has bought an item from the display next to the till.

(a) Stating your hypotheses clearly, and using a 5% level of significance, test whether or not there has been a change in the proportion of customers buying an item from the display next to the till.

(6)

During the refurbishment a new sandwich display was installed. Before the refurbishment 20% of customers bought sandwiches. The shopkeeper claims that the proportion of customers buying sandwiches has now increased. He selects a random sample of 120 customers and finds that 31 of them have bought sandwiches.

(b) Using a suitable approximation and stating your hypotheses clearly, test the shopkeeper's claim. Use a 10% level of significance.

(8)

Question 7

The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{3}{32}(x-1)(5-x) & 1 \leqslant x \leqslant 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch $f(x)$ showing clearly the points where it meets the x-axis.

(2)

(b) Write down the value of the mean, μ, of X.

(1)

(c) Show that $E(X^2) = 9.8$

(4)

(d) Find the standard deviation, σ, of X.

(2)

The cumulative distribution function of X is given by

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{32}\left(a - 15x + 9x^2 - x^3\right) & 1 \leqslant x \leqslant 5 \\ 1 & x > 5 \end{cases}$$

where a is a constant.

(e) Find the value of a.

(2)

(f) Show that the lower quartile of X, q_1, lies between 2.29 and 2.31

(3)

(g) Hence find the upper quartile of X, giving your answer to 1 decimal place.

(1)

(h) Find, to 2 decimal places, the value of k so that

$$P(\mu - k\sigma < X < \mu + k\sigma) = 0.5$$

(2)