

Statistics-S2 - 2009-June

Question 1

A bag contains a large number of counters of which 15% are coloured red. A random sample of 30 counters is selected and the number of red counters is recorded.

- (a) Find the probability of no more than 6 red counters in this sample. (2)

A second random sample of 30 counters is selected and the number of red counters is recorded.

- (b) Using a Poisson approximation, estimate the probability that the total number of red counters in the combined sample of size 60 is less than 13. (3)
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Question 2

An effect of a certain disease is that a small number of the red blood cells are deformed. Emily has this disease and the deformed blood cells occur randomly at a rate of 2.5 per ml of her blood. Following a course of treatment, a random sample of 2 ml of Emily's blood is found to contain only 1 deformed red blood cell.

Stating your hypotheses clearly and using a 5% level of significance, test whether or not there has been a decrease in the number of deformed red blood cells in Emily's blood. (6)

Question 3

A random sample X_1, X_2, \dots, X_n is taken from a population with unknown mean μ and unknown variance σ^2 . A statistic Y is based on this sample.

(a) Explain what you understand by the statistic Y . (2)

(b) Explain what you understand by the sampling distribution of Y . (1)

(c) State, giving a reason which of the following is **not** a statistic based on this sample.

$$(i) \sum_{i=1}^n \frac{(X_i - \bar{X})^2}{n} \quad (ii) \sum_{i=1}^n \left(\frac{X_i - \mu}{\sigma} \right)^2 \quad (iii) \sum_{i=1}^n X_i^2 \quad (2)$$

Question 4

Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager questions whether or not there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

(a) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager's question. You should state the probability of rejection in each tail which should be less than 0.05. (5)

(b) Write down the actual significance level of a test based on your critical region from part (a). (1)

The manager found that 11 customers from the sample of 20 had bought baked beans in single tins.

(c) Comment on this finding in the light of your critical region found in part (a). (2)

Question 5

An administrator makes errors in her typing randomly at a rate of 3 errors every 1000 words.

- (a) In a document of 2000 words find the probability that the administrator makes 4 or more errors.

(3)

The administrator is given an 8000 word report to type and she is told that the report will only be accepted if there are 20 or fewer errors.

- (b) Use a suitable approximation to calculate the probability that the report is accepted.

(7)

Question 6

The three independent random variables A , B and C each has a continuous uniform distribution over the interval $[0, 5]$.

(a) Find $P(A > 3)$. (1)

(b) Find the probability that A , B and C are all greater than 3. (2)

The random variable Y represents the maximum value of A , B and C .

The cumulative distribution function of Y is

$$F(y) = \begin{cases} 0 & y < 0 \\ \frac{y^3}{125} & 0 \leq y \leq 5 \\ 1 & y > 5 \end{cases}$$

(c) Find the probability density function of Y . (2)

(d) Sketch the probability density function of Y . (2)

(e) Write down the mode of Y . (1)

(f) Find $E(Y)$. (3)

(g) Find $P(Y > 3)$. (2)

Question 7

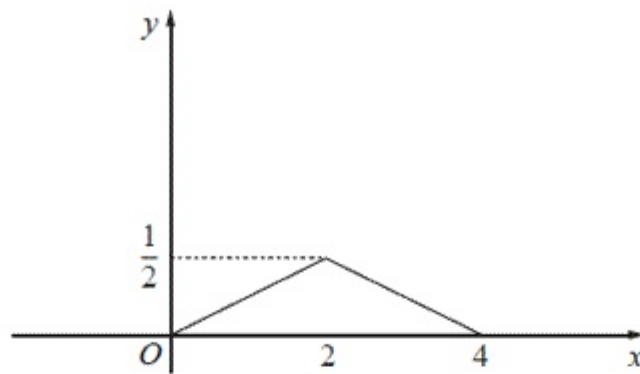


Figure 1

Figure 1 shows a sketch of the probability density function $f(x)$ of the random variable X . The part of the sketch from $x = 0$ to $x = 4$ consists of an isosceles triangle with maximum at $(2, 0.5)$.

(a) Write down $E(X)$.

(1)

The probability density function $f(x)$ can be written in the following form.

$$f(x) = \begin{cases} ax & 0 \leq x < 2 \\ b - ax & 2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

(b) Find the values of the constants a and b .

(2)

(c) Show that σ , the standard deviation of X , is 0.816 to 3 decimal places.

(7)

(d) Find the lower quartile of X .

(3)

(e) State, giving a reason, whether $P(2 - \sigma < X < 2 + \sigma)$ is more or less than 0.5

(2)

Question 8

A cloth manufacturer knows that faults occur randomly in the production process at a rate of 2 every 15 metres.

(a) Find the probability of exactly 4 faults in a 15 metre length of cloth. (2)

(b) Find the probability of more than 10 faults in 60 metres of cloth. (3)

A retailer buys a large amount of this cloth and sells it in pieces of length x metres. He chooses x so that the probability of no faults in a piece is 0.80

(c) Write down an equation for x and show that $x = 1.7$ to 2 significant figures. (4)

The retailer sells 1200 of these pieces of cloth. He makes a profit of 60p on each piece of cloth that does not contain a fault but a loss of £1.50 on any pieces that do contain faults.

(d) Find the retailer's expected profit. (4)
