Core-Maths-C2 - 2011-June

Question 1

$$f(x) = 2x^3 - 7x^2 - 5x + 4$$

(a) Find the remainder when $f(x)$ is divided by $(x-1)$.

(2)

(b) Use the factor theorem to show that $(x+1)$ is a factor of $f(x)$.

(2)

(c) Factorise $f(x)$ completely.

(4)

Question 2

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(3 + bx)^5$$

where b is a non-zero constant. Give each term in its simplest form.

(4)

Given that, in this expansion, the coefficient of x^2 is twice the coefficient of x,

(b) find the value of b.

(2)

Question 3

Find, giving your answer to 3 significant figures where appropriate, the value of x for which

(a) $5^x = 10$,

(2)

(b) $\log_3(x-2) = -1$.

(2)

Question 4

The circle C has equation

$$x^2 + y^2 + 4x - 2y - 11 = 0$$

Find

(a) the coordinates of the centre of C,

(2)

(b) the radius of C,

(2)

(c) the coordinates of the points where C crosses the y-axis, giving your answers as simplified surds.
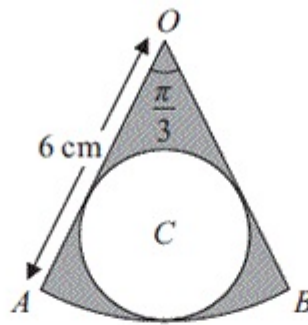
(4)

Question 5

Figure 1

The shape shown in Figure 1 is a pattern for a pendant. It consists of a sector OAB of a circle centre O, of radius 6 cm, and angle $AOB = \frac{\pi}{3}$. The circle C, inside the sector, touches the two straight edges, OA and OB, and the arc AB as shown.

Find

(a) the area of the sector OAB,

(2)

(b) the radius of the circle C.

(3)

The region outside the circle C and inside the sector OAB is shown shaded in Figure 1.

(c) Find the area of the shaded region.

(2)

Question 6

The second and third terms of a geometric series are 192 and 144 respectively.

For this series, find

(a) the common ratio,

(2)

(b) the first term,

(2)

(c) the sum to infinity,

(2)

(d) the smallest value of n for which the sum of the first n terms of the series exceeds 1000.

(4)

Question 7

(a) Solve for $0 \leqslant x < 360°$, giving your answers in degrees to 1 decimal place,

$$3 \sin(x + 45°) = 2$$

(4)

(b) Find, for $0 \leqslant x < 2\pi$, all the solutions of

$$2 \sin^2 x + 2 = 7 \cos x$$

giving your answers in radians.

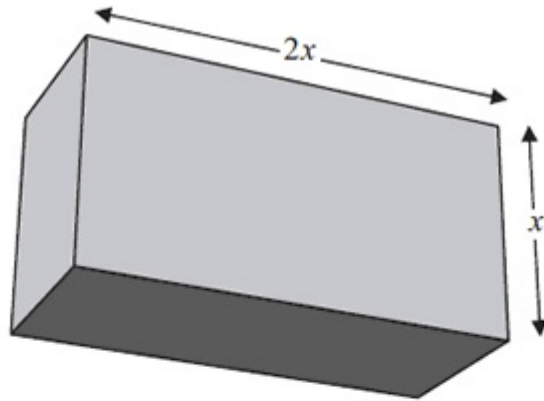You must show clearly how you obtained your answers.

(6)

Question 8

Figure 2

A cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cm, as shown in Figure 2.

The volume of the cuboid is 81 cubic centimetres.

(a) Show that the total length, L cm, of the twelve edges of the cuboid is given by

$$L = 12x + \frac{162}{x^2}$$

(3)

(b) Use calculus to find the minimum value of L.

(6)

(c) Justify, by further differentiation, that the value of L that you have found is a minimum.
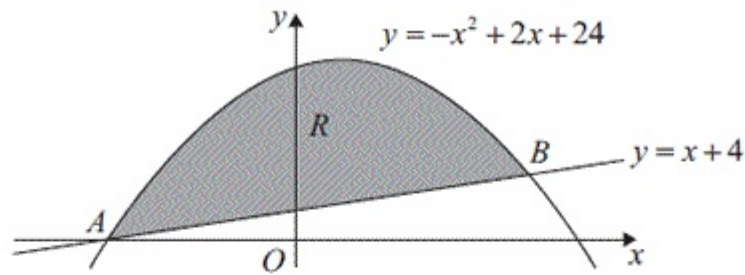
(2)

Question 9

Figure 3

The straight line with equation $y = x + 4$ cuts the curve with equation $y = -x^2 + 2x + 24$ at the points A and B, as shown in Figure 3.

(a) Use algebra to find the coordinates of the points A and B.

(4)

The finite region R is bounded by the straight line and the curve and is shown shaded in Figure 3.

(b) Use calculus to find the exact area of R.

(7)