

Core-Maths-C1 - 2011-January

Question 1

(a) Find the value of $16^{-\frac{1}{4}}$ (2)

(b) Simplify $x(2x^{-\frac{1}{4}})^4$ (2)

Question 2

Find

$$\int (12x^5 - 3x^2 + 4x^{\frac{1}{3}}) \, dx$$

giving each term in its simplest form.

(5)

Question 3

Simplify

$$\frac{5 - 2\sqrt{3}}{\sqrt{3} - 1}$$

giving your answer in the form $p + q\sqrt{3}$, where p and q are rational numbers.

(4)

Question 4

A sequence a_1, a_2, a_3, \dots is defined by

$$a_1 = 2$$

$$a_{n+1} = 3a_n - c$$

where c is a constant.

(a) Find an expression for a_2 in terms of c .

(1)

Given that $\sum_{i=1}^3 a_i = 0$

(b) find the value of c .

(4)

Question 5

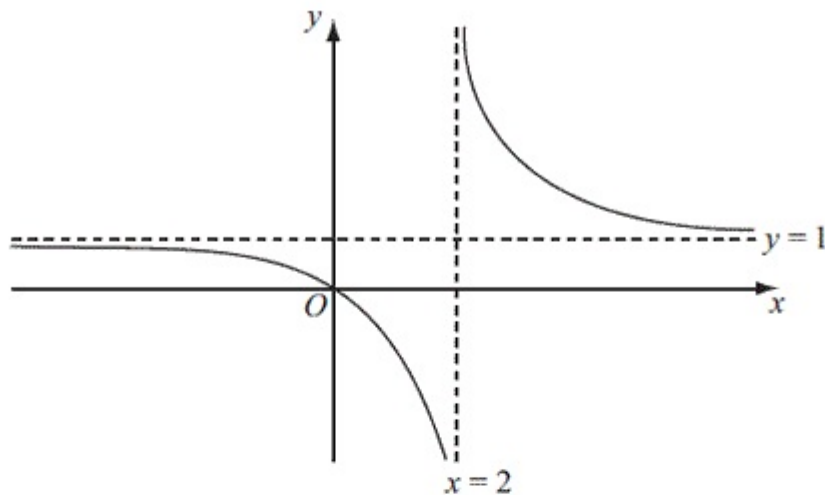


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = \frac{x}{x-2}, \quad x \neq 2$$

The curve passes through the origin and has two asymptotes, with equations $y = 1$ and $x = 2$, as shown in Figure 1.

- (a) In the space below, sketch the curve with equation $y = f(x-1)$ and state the equations of the asymptotes of this curve. (3)
- (b) Find the coordinates of the points where the curve with equation $y = f(x-1)$ crosses the coordinate axes. (4)
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Question 6

An arithmetic sequence has first term a and common difference d . The sum of the first 10 terms of the sequence is 162.

(a) Show that $10a + 45d = 162$ (2)

Given also that the sixth term of the sequence is 17,

(b) write down a second equation in a and d , (1)

(c) find the value of a and the value of d . (4)

Question 7

The curve with equation $y = f(x)$ passes through the point $(-1, 0)$.

Given that

$$f'(x) = 12x^2 - 8x + 1$$

find $f(x)$. (5)

Question 8

The equation $x^2 + (k-3)x + (3-2k) = 0$, where k is a constant, has two distinct real roots.

(a) Show that k satisfies

$$k^2 + 2k - 3 > 0$$
 (3)

(b) Find the set of possible values of k . (4)

Question 9

The line L_1 has equation $2y - 3x - k = 0$, where k is a constant.

Given that the point $A(1, 4)$ lies on L_1 , find

(a) the value of k , (1)

(b) the gradient of L_1 . (2)

The line L_2 passes through A and is perpendicular to L_1 .

(c) Find an equation of L_2 giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. (4)

The line L_2 crosses the x -axis at the point B .

(d) Find the coordinates of B . (2)

(e) Find the exact length of AB . (2)

Question 10

(a) On the axes below, sketch the graphs of

(i) $y = x(x + 2)(3 - x)$

(ii) $y = -\frac{2}{x}$

showing clearly the coordinates of all the points where the curves cross the coordinate axes.

(6)

(b) Using your sketch state, giving a reason, the number of real solutions to the equation

$$x(x + 2)(3 - x) + \frac{2}{x} = 0$$

(2)

Question 11

The curve C has equation

$$y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30, \quad x > 0$$

- (a) Find $\frac{dy}{dx}$. (4)
- (b) Show that the point $P(4, -8)$ lies on C . (2)
- (c) Find an equation of the normal to C at the point P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. (6)
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