Mechanics-M1 - 2011-June

Question 1

At time $t = 0$ a ball is projected vertically upwards from a point O and rises to a maximum height of 40 m above O. The ball is modelled as a particle moving freely under gravity.

(a) Show that the speed of projection is 28 m s^{-1}.

(3)

(b) Find the times, in seconds, when the ball is 33.6 m above O.

(5)

Question 2

Particle P has mass 3 kg and particle Q has mass 2 kg. The particles are moving in opposite directions on a smooth horizontal plane when they collide directly. Immediately before the collision, P has speed 3 m s^{-1} and Q has speed 2 m s^{-1}. Immediately after the collision, both particles move in the same direction and the difference in their speeds is 1 m s^{-1}.

(a) Find the speed of each particle after the collision.

(5)

(b) Find the magnitude of the impulse exerted on P by Q.

(3)

Question 3

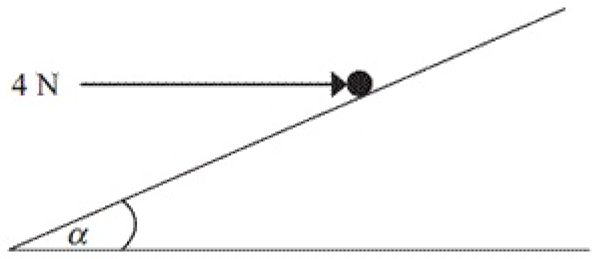

Figure 1

A particle of weight W newtons is held in equilibrium on a rough inclined plane by a horizontal force of magnitude 4 N. The force acts in a vertical plane containing a line of greatest slope of the inclined plane. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$, as shown in Figure 1.

The coefficient of friction between the particle and the plane is $\frac{1}{2}$.

Given that the particle is on the point of sliding down the plane,

(i) show that the magnitude of the normal reaction between the particle and the plane is 20 N,

(ii) find the value of W.

(9)

Question 4

A girl runs a 400 m race in a time of 84 s. In a model of this race, it is assumed that, starting from rest, she moves with constant acceleration for 4 s, reaching a speed of 5 m s^{-1}. She maintains this speed for 60 s and then moves with constant deceleration for 20 s, crossing the finishing line with a speed of V m s^{-1}.

(a) Sketch, in the space below, a speed-time graph for the motion of the girl during the whole race.

(2)

(b) Find the distance run by the girl in the first 64 s of the race.

(3)

(c) Find the value of V.

(5)

(d) Find the deceleration of the girl in the final 20 s of her race.

(2)

Question 5

A plank PQR, of length 8 m and mass 20 kg, is in equilibrium in a horizontal position on two supports at P and Q, where $PQ = 6$ m.

A child of mass 40 kg stands on the plank at a distance of 2 m from P and a block of mass M kg is placed on the plank at the end R. The plank remains horizontal and in equilibrium. The force exerted on the plank by the support at P is equal to the force exerted on the plank by the support at Q.

By modelling the plank as a uniform rod, and the child and the block as particles,

(a) (i) find the magnitude of the force exerted on the plank by the support at P,

(ii) find the value of M.

(10)

(b) State how, in your calculations, you have used the fact that the child and the block can be modelled as particles.

(1)

Question 6

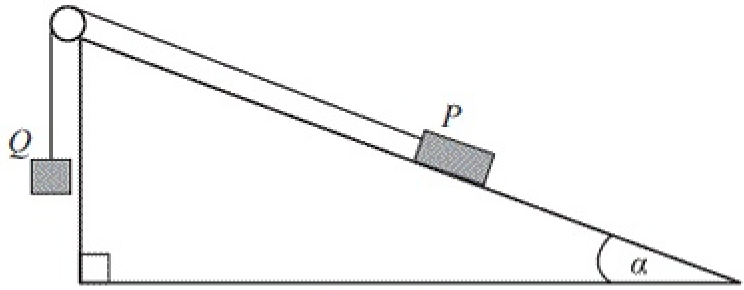

Figure 2

Two particles P and Q have masses 0.3 kg and m kg respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a fixed rough plane. The plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between P and the plane is $\frac{1}{2}$.

The string lies in a vertical plane through a line of greatest slope of the inclined plane. The particle P is held at rest on the inclined plane and the particle Q hangs freely below the pulley with the string taut, as shown in Figure 2.

The system is released from rest and Q accelerates vertically downwards at 1.4 m s^{-2}. Find

(a) the magnitude of the normal reaction of the inclined plane on P,

(2)

(b) the value of m.

(8)

When the particles have been moving for 0.5 s, the string breaks. Assuming that P does not reach the pulley,

(c) find the further time that elapses until P comes to instantaneous rest.

(6)

Question 7

[In this question **i** *and* **j** *are unit vectors due east and due north respectively. Position vectors are given relative to a fixed origin O.]*

Two ships P and Q are moving with constant velocities. Ship P moves with velocity $(2\mathbf{i} - 3\mathbf{j})$ km h^{-1} and ship Q moves with velocity $(3\mathbf{i} + 4\mathbf{j})$ km h^{-1}.

(a) Find, to the nearest degree, the bearing on which Q is moving.

(2)

At 2 pm, ship P is at the point with position vector $(\mathbf{i} + \mathbf{j})$ km and ship Q is at the point with position vector $(-2\mathbf{j})$ km.

At time t hours after 2 pm, the position vector of P is $\mathbf{p}$ km and the position vector of Q is $\mathbf{q}$ km.

(b) Write down expressions, in terms of t, for

(i) $\mathbf{p}$,

(ii) $\mathbf{q}$,

(iii) $\overrightarrow{PQ}$.

(5)

(c) Find the time when

(i) Q is due north of P,

(ii) Q is north-west of P.

(4)