Core-Maths-C3 - 2006-June

Question 1

(a) Simplify $\frac{3x^2 - x - 2}{x^2 - 1}$.

(3)

(b) Hence, or otherwise, express $\frac{3x^2 - x - 2}{x^2 - 1} - \frac{1}{x(x+1)}$ as a single fraction in its simplest form.

(3)

Question 2

Differentiate, with respect to x,

(a) $e^{3x} + \ln 2x$,

(3)

(b) $(5 + x^2)^{\frac{3}{2}}$.

(3)

Question 3

Figure 1

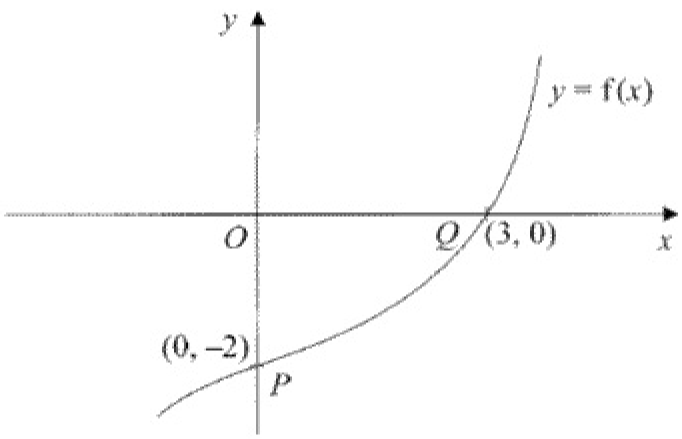

Figure 1 shows part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$, where f is an increasing function of x. The curve passes through the points $P(0, -2)$ and $Q(3, 0)$ as shown.

In separate diagrams, sketch the curve with equation

(a) $y = |f(x)|$,

(3)

(b) $y = f^{-1}(x)$,

(3)

(c) $y = \frac{1}{2}f(3x)$.

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

Question 4

A heated metal ball is dropped into a liquid. As the ball cools, its temperature, T°C, t minutes after it enters the liquid, is given by

$$T = 400\,e^{-0.05t} + 25, \quad t \geqslant 0.$$

(a) Find the temperature of the ball as it enters the liquid.

(1)

(b) Find the value of t for which $T = 300$, giving your answer to 3 significant figures.

(4)

(c) Find the rate at which the temperature of the ball is decreasing at the instant when $t = 50$. Give your answer in °C per minute to 3 significant figures.

(3)

(d) From the equation for temperature T in terms of t, given above, explain why the temperature of the ball can never fall to 20 °C.

(1)

Question 5

Figure 2

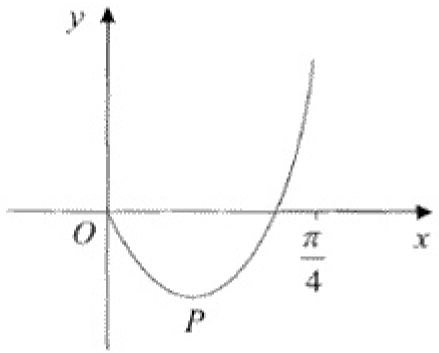

Figure 2 shows part of the curve with equation

$$y = (2x - 1)\tan 2x, \quad 0 \leqslant x < \frac{\pi}{4}.$$

The curve has a minimum at the point P. The x-coordinate of P is k.

(a) Show that k satisfies the equation

$$4k + \sin 4k - 2 = 0.$$

(6)

The iterative formula

$$x_{n+1} = \frac{1}{4}(2 - \sin 4x_n), \quad x_0 = 0.3,$$

is used to find an approximate value for k.

(b) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places.

(3)

(c) Show that $k = 0.277$, correct to 3 significant figures.

(2)

Question 6

(a) Using $\sin^2\theta + \cos^2\theta \equiv 1$, show that $\text{cosec}^2\theta - \cot^2\theta \equiv 1$.

(2)

(b) Hence, or otherwise, prove that

$$\text{cosec}^4\theta - \cot^4\theta \equiv \text{cosec}^2\theta + \cot^2\theta.$$

(2)

(c) Solve, for $90° < \theta < 180°$,

$$\text{cosec}^4\theta - \cot^4\theta = 2 - \cot\theta.$$

(6)

Question 7

For the constant k, where $k > 1$, the functions f and g are defined by

$$\text{f}: x \mapsto \ln(x + k), \quad x > -k,$$
$$\text{g}: x \mapsto |2x - k|, \quad x \in \mathbb{R}.$$

(a) On separate axes, sketch the graph of f and the graph of g.

On each sketch state, in terms of k, the coordinates of points where the graph meets the coordinate axes.

(5)

(b) Write down the range of f.

(1)

(c) Find $\text{fg}\left(\dfrac{k}{4}\right)$ in terms of k, giving your answer in its simplest form.

(2)

The curve C has equation $y = \text{f}(x)$. The tangent to C at the point with x-coordinate 3 is parallel to the line with equation $9y = 2x + 1$.

(d) Find the value of k.

(4)

Question 8

(a) Given that $\cos A = \frac{3}{4}$, where $270° < A < 360°$, find the exact value of $\sin 2A$.

(5)

(b) (i) Show that $\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) \equiv \cos 2x$.

(3)

Given that

$$y = 3\sin^2 x + \cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right),$$

(ii) show that $\frac{dy}{dx} = \sin 2x$.

(4)