

Mechanics-M1 - 2013-January

Question 1

Two particles P and Q have masses $4m$ and m respectively. The particles are moving towards each other on a smooth horizontal plane and collide directly. The speeds of P and Q immediately before the collision are $2u$ and $5u$ respectively. Immediately after the collision, the speed of P is $\frac{1}{2}u$ and its direction of motion is reversed.

- (a) Find the speed and direction of motion of Q after the collision. (4)
- (b) Find the magnitude of the impulse exerted on P by Q in the collision. (3)
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Question 2

A steel girder AB , of mass 200 kg and length 12 m, rests horizontally in equilibrium on two smooth supports at C and at D , where $AC = 2$ m and $DB = 2$ m. A man of mass 80 kg stands on the girder at the point P , where $AP = 4$ m, as shown in Figure 1.

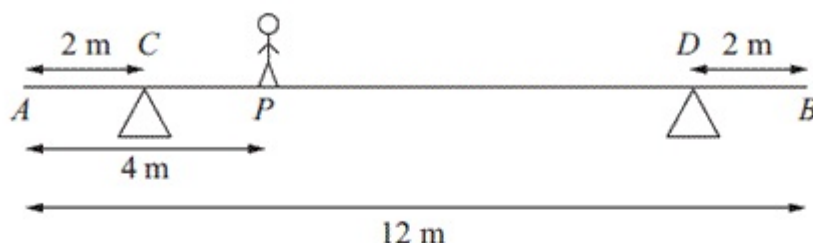


Figure 1

The man is modelled as a particle and the girder is modelled as a uniform rod.

- (a) Find the magnitude of the reaction on the girder at the support at C .

(3)

The support at D is now moved to the point X on the girder, where $XB = x$ metres. The man remains on the girder at P , as shown in Figure 2.

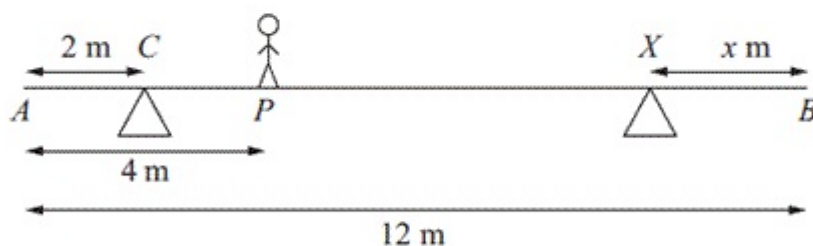


Figure 2

Given that the magnitudes of the reactions at the two supports are now equal and that the girder again rests horizontally in equilibrium, find

- (b) the magnitude of the reaction at the support at X ,

(2)

- (c) the value of x .

(4)

Question 3

A particle P of mass 2 kg is attached to one end of a light string, the other end of which is attached to a fixed point O . The particle is held in equilibrium, with OP at 30° to the downward vertical, by a force of magnitude F newtons. The force acts in the same vertical plane as the string and acts at an angle of 30° to the horizontal, as shown in Figure 3.

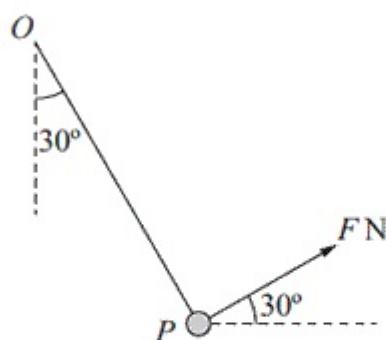


Figure 3

Find

- (i) the value of F ,
- (ii) the tension in the string.

(8)

Question 4

A lifeboat slides down a straight ramp inclined at an angle of 15° to the horizontal. The lifeboat has mass 800 kg and the length of the ramp is 50 m . The lifeboat is released from rest at the top of the ramp and is moving with a speed of 12.6 m s^{-1} when it reaches the end of the ramp. By modelling the lifeboat as a particle and the ramp as a rough inclined plane, find the coefficient of friction between the lifeboat and the ramp.

(9)

Question 5

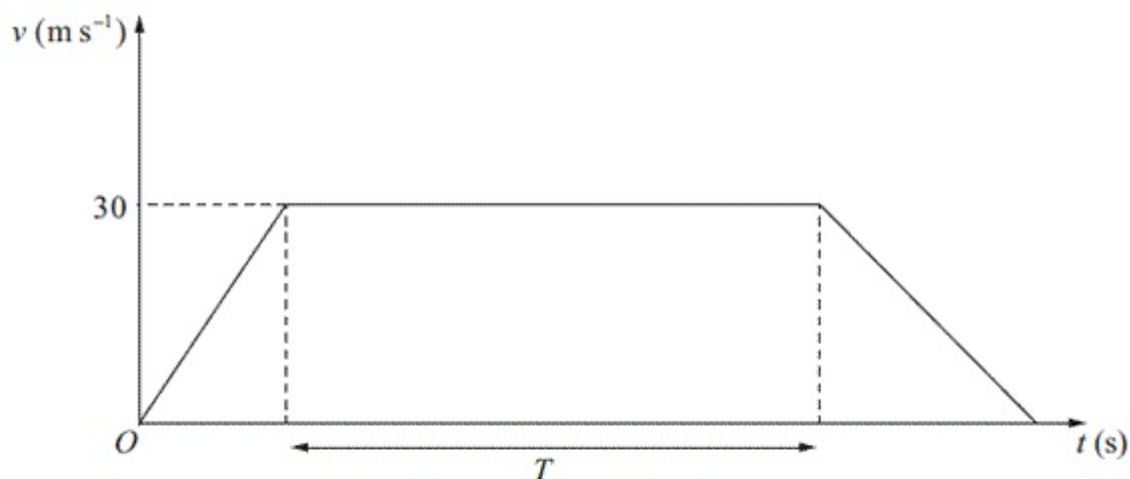


Figure 4

The velocity-time graph in Figure 4 represents the journey of a train P travelling along a straight horizontal track between two stations which are 1.5 km apart. The train P leaves the first station, accelerating uniformly from rest for 300 m until it reaches a speed of 30 m s^{-1} . The train then maintains this speed for T seconds before decelerating uniformly at 1.25 m s^{-2} , coming to rest at the next station.

(a) Find the acceleration of P during the first 300 m of its journey. (2)

(b) Find the value of T . (5)

A second train Q completes the same journey in the same total time. The train leaves the first station, accelerating uniformly from rest until it reaches a speed of $V \text{ m s}^{-1}$ and then immediately decelerates uniformly until it comes to rest at the next station.

(c) Sketch on the diagram above, a velocity-time graph which represents the journey of train Q . (2)

(d) Find the value of V . (6)

Question 6

[In this question, \mathbf{i} and \mathbf{j} are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.]

A ship sets sail at 9 am from a port P and moves with constant velocity. The position vector of P is $(4\mathbf{i} - 8\mathbf{j})$ km. At 9.30 am the ship is at the point with position vector $(\mathbf{i} - 4\mathbf{j})$ km.

(a) Find the speed of the ship in km h^{-1} . (4)

(b) Show that the position vector \mathbf{r} km of the ship, t hours after 9 am, is given by $\mathbf{r} = (4 - 6t)\mathbf{i} + (8t - 8)\mathbf{j}$. (2)

At 10 am, a passenger on the ship observes that a lighthouse L is due west of the ship. At 10.30 am, the passenger observes that L is now south-west of the ship.

(c) Find the position vector of L . (5)

Question 7

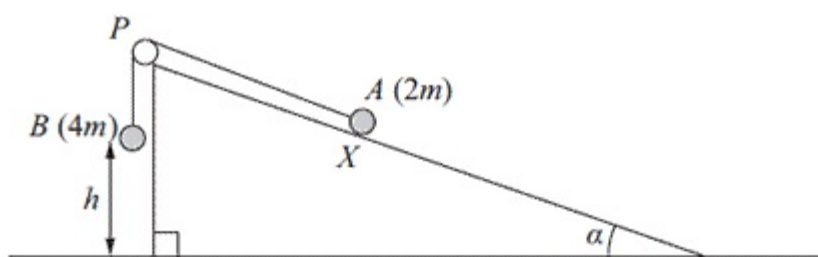


Figure 5

Figure 5 shows two particles A and B , of mass $2m$ and $4m$ respectively, connected by a light inextensible string. Initially A is held at rest on a rough inclined plane which is fixed to horizontal ground. The plane is inclined to the horizontal at an angle α , where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between A and the plane is $\frac{1}{4}$. The string passes over a small smooth pulley P which is fixed at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs vertically below P . The system is released from rest with the string taut, with A at the point X and with B at a height h above the ground.

For the motion until B hits the ground,

- (a) give a reason why the magnitudes of the accelerations of the two particles are the same, (1)
- (b) write down an equation of motion for each particle, (4)
- (c) find the acceleration of each particle. (5)

Particle B does not rebound when it hits the ground and A continues moving up the plane towards P . Given that A comes to rest at the point Y , without reaching P ,

- (d) find the distance XY in terms of h . (6)