Core-Maths-C2 - 2011-January

Question 1

$$f(x) = x^4 + x^3 + 2x^2 + ax + b$$

where a and b are constants.

When $f(x)$ is divided by $(x - 1)$, the remainder is 7.

(a) Show that $a + b = 3$.

(2)

When $f(x)$ is divided by $(x + 2)$, the remainder is -8.

(b) Find the value of a and the value of b.

(5)

Question 2

In the triangle ABC, $AB = 11$ cm, $BC = 7$ cm and $CA = 8$ cm.

(a) Find the size of angle C, giving your answer in radians to 3 significant figures.

(3)

(b) Find the area of triangle ABC, giving your answer in cm^2 to 3 significant figures.

(3)

Question 3

The second and fifth terms of a geometric series are 750 and –6 respectively.

Find

(a) the common ratio of the series,

(3)

(b) the first term of the series,

(2)

(c) the sum to infinity of the series.

(2)

Question 4

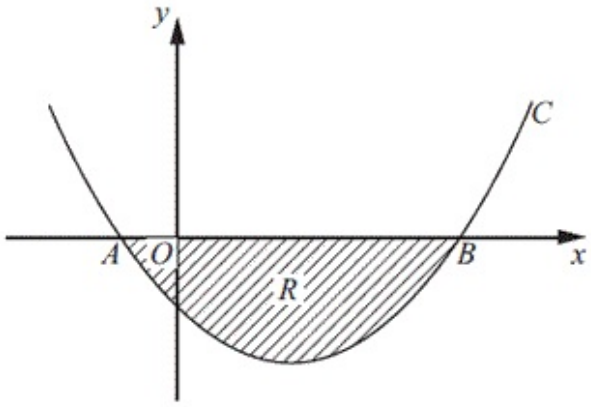

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = (x+1)(x-5)$$

The curve crosses the x-axis at the points A and B.

(a) Write down the x-coordinates of A and B.

(1)

The finite region R, shown shaded in Figure 1, is bounded by C and the x-axis.

(b) Use integration to find the area of R.

(6)

Question 5

Given that $\binom{40}{4} = \frac{40!}{4!b!}$,

(a) write down the value of b.

(1)

In the binomial expansion of $(1+x)^{40}$, the coefficients of x^4 and x^5 are p and q respectively.

(b) Find the value of $\frac{q}{p}$.

(3)

Question 6

$$y = \frac{5}{3x^2 - 2}$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	2	2.25	2.5	2.75	3
y	0.5	0.38			0.2

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for $\int_2^3 \frac{5}{3x^2 - 2}\,dx$.

(4)

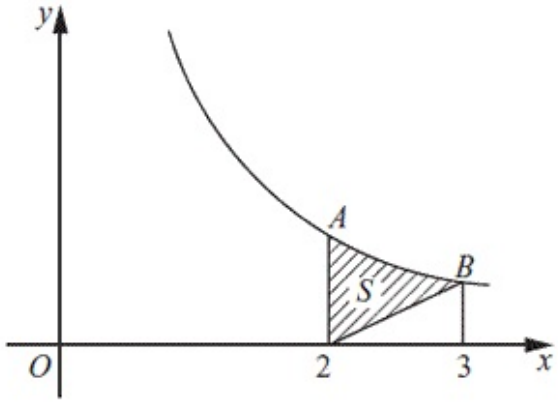

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \frac{5}{3x^2 - 2}$, $x > 1$.

At the points A and B on the curve, $x = 2$ and $x = 3$ respectively.

The region S is bounded by the curve, the straight line through B and (2, 0), and the line through A parallel to the y-axis. The region S is shown shaded in Figure 2.

(c) Use your answer to part (b) to find an approximate value for the area of S.

(3)

Question 7

(a) Show that the equation

$$3\sin^2 x + 7\sin x = \cos^2 x - 4$$

can be written in the form

$$4\sin^2 x + 7\sin x + 3 = 0$$

(2)

(b) Hence solve, for $0 \leqslant x < 360°$,

$$3\sin^2 x + 7\sin x = \cos^2 x - 4$$

giving your answers to 1 decimal place where appropriate.

(5)

Question 8

(a) Sketch the graph of $y = 7^x$, $x \in \mathbb{R}$, showing the coordinates of any points at which the graph crosses the axes.

(2)

(b) Solve the equation

$$7^{2x} - 4(7^x) + 3 = 0$$

giving your answers to 2 decimal places where appropriate.

(6)

Question 9

The points A and B have coordinates $(-2, 11)$ and $(8, 1)$ respectively.

Given that AB is a diameter of the circle C,

(a) show that the centre of C has coordinates $(3, 6)$,

(1)

(b) find an equation for C.

(4)

(c) Verify that the point $(10, 7)$ lies on C.

(1)

(d) Find an equation of the tangent to C at the point $(10, 7)$, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

Question 10

The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5-x)^2, \quad 0 < x < 5$$

(a) Find $\frac{dV}{dx}$.

(4)

(b) Hence find the maximum volume of the box.

(4)

(c) Use calculus to justify that the volume that you found in part (b) is a maximum.

(2)