

Core-Maths-C4 - 2010-January

Question 1

- (a) Find the binomial expansion of

$$\sqrt{1-8x}, \quad |x| < \frac{1}{8},$$

in ascending powers of x up to and including the term in x^3 , simplifying each term. (4)

- (b) Show that, when $x = \frac{1}{100}$, the exact value of $\sqrt{1-8x}$ is $\frac{\sqrt{23}}{5}$. (2)

- (c) Substitute $x = \frac{1}{100}$ into the binomial expansion in part (a) and hence obtain an approximation to $\sqrt{23}$. Give your answer to 5 decimal places. (3)
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Question 2

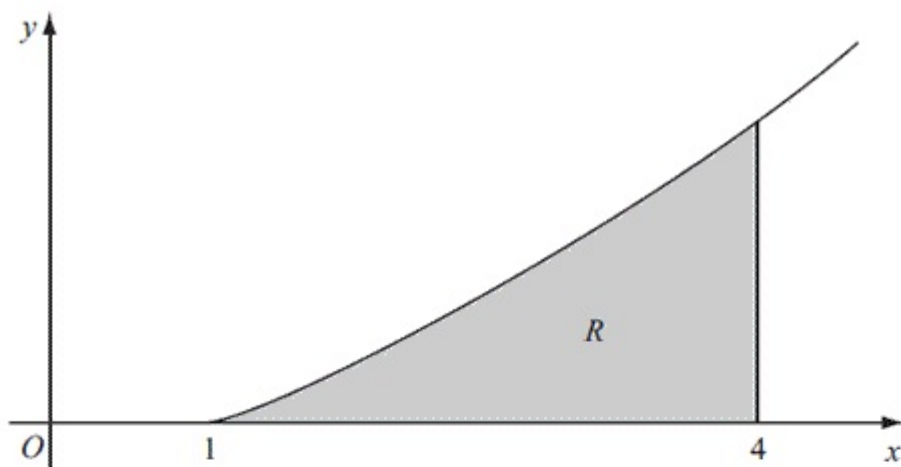


Figure 1

Figure 1 shows a sketch of the curve with equation $y = x \ln x$, $x \geq 1$. The finite region R , shown shaded in Figure 1, is bounded by the curve, the x -axis and the line $x = 4$.

The table shows corresponding values of x and y for $y = x \ln x$.

x	1	1.5	2	2.5	3	3.5	4
y	0	0.608			3.296	4.385	5.545

- (a) Complete the table with the values of y corresponding to $x = 2$ and $x = 2.5$, giving your answers to 3 decimal places. (2)
- (b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R , giving your answer to 2 decimal places. (4)
- (c) (i) Use integration by parts to find $\int x \ln x \, dx$.
- (ii) Hence find the exact area of R , giving your answer in the form $\frac{1}{4}(a \ln 2 + b)$, where a and b are integers. (7)
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Question 3

The curve C has the equation

$$\cos 2x + \cos 3y = 1, \quad -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}, \quad 0 \leq y \leq \frac{\pi}{6}$$

- (a) Find $\frac{dy}{dx}$ in terms of x and y . (3)

The point P lies on C where $x = \frac{\pi}{6}$.

- (b) Find the value of y at P . (3)
- (c) Find the equation of the tangent to C at P , giving your answer in the form $ax + by + c\pi = 0$, where a , b and c are integers. (3)
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Question 4

The line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$$

and the line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point A and the acute angle between l_1 and l_2 is θ .

(a) Write down the coordinates of A .

(1)

(b) Find the value of $\cos \theta$.

(3)

The point X lies on l_1 where $\lambda = 4$.

(c) Find the coordinates of X .

(1)

(d) Find the vector \overrightarrow{AX} .

(2)

(e) Hence, or otherwise, show that $|\overrightarrow{AX}| = 4\sqrt{26}$.

(2)

The point Y lies on l_2 . Given that the vector \overrightarrow{YX} is perpendicular to l_1 ,

(f) find the length of AY , giving your answer to 3 significant figures.

(3)

Question 5

(a) Find $\int \frac{9x+6}{x} dx$, $x > 0$. (2)

(b) Given that $y = 8$ at $x = 1$, solve the differential equation

$$\frac{dy}{dx} = \frac{(9x+6)y^{\frac{1}{3}}}{x}$$

giving your answer in the form $y^2 = g(x)$. (6)

Question 6

The area A of a circle is increasing at a constant rate of $1.5 \text{ cm}^2 \text{ s}^{-1}$. Find, to 3 significant figures, the rate at which the radius r of the circle is increasing when the area of the circle is 2 cm^2 .

(5)

Question 7

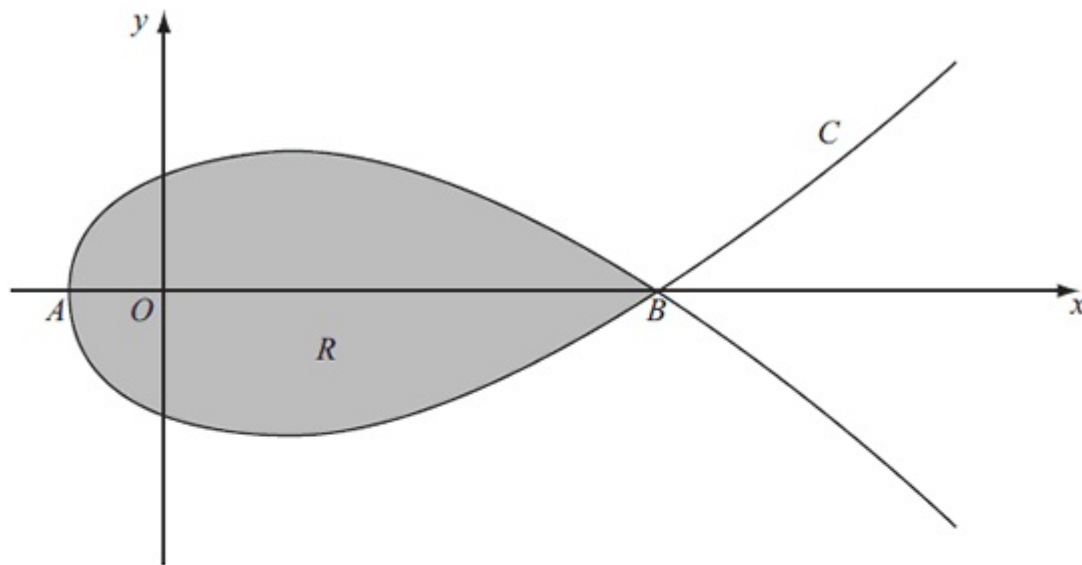


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 5t^2 - 4, \quad y = t(9 - t^2)$$

The curve C cuts the x -axis at the points A and B .

- (a) Find the x -coordinate at the point A and the x -coordinate at the point B . (3)

The region R , as shown shaded in Figure 2, is enclosed by the loop of the curve.

- (b) Use integration to find the area of R . (6)
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Question 8

- (a) Using the substitution $x = 2 \cos u$, or otherwise, find the exact value of

$$\int_1^{\sqrt{2}} \frac{1}{x^2 \sqrt{4-x^2}} dx \quad (7)$$

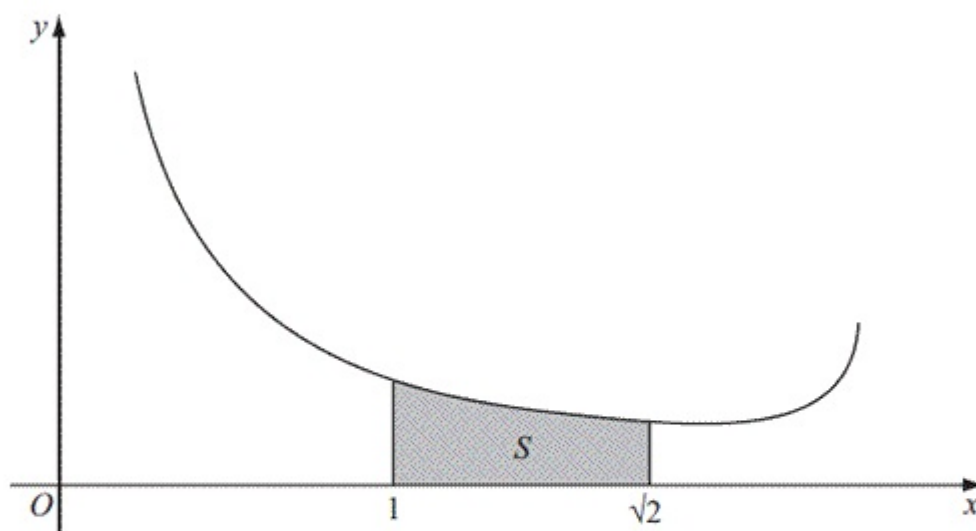


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \frac{4}{x(4-x^2)^{\frac{1}{2}}}$, $0 < x < 2$.

The shaded region S , shown in Figure 3, is bounded by the curve, the x -axis and the lines with equations $x = 1$ and $x = \sqrt{2}$. The shaded region S is rotated through 2π radians about the x -axis to form a solid of revolution.

- (b) Using your answer to part (a), find the exact volume of the solid of revolution formed.

(3)