Core-Maths-C1 - 2013-January

Question 1

Factorise completely $x - 4x^3$

(3)

Question 2

Express 8^{2x+3} in the form 2^y, stating y in terms of x.

(2)

Question 3

(i) Express

$$\left(5 - \sqrt{8}\right)\left(1 + \sqrt{2}\right)$$

in the form $a + b\sqrt{2}$, where a and b are integers.

(3)

(ii) Express

$$\sqrt{80} + \frac{30}{\sqrt{5}}$$

in the form $c\sqrt{5}$, where c is an integer.

(3)

Question 4

A sequence $u_1, u_2, u_3, \ldots$ satisfies

$$u_{n+1} = 2u_n - 1, \quad n \geqslant 1$$

Given that $u_2 = 9$,

(a) find the value of u_3 and the value of u_4,

(2)

(b) evaluate $\displaystyle\sum_{r=1}^{4} u_r$.

(3)

Question 5

The line l_1 has equation $y = -2x + 3$

The line l_2 is perpendicular to l_1 and passes through the point $(5, 6)$.

(a) Find an equation for l_2 in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

The line l_2 crosses the x-axis at the point A and the y-axis at the point B.

(b) Find the x-coordinate of A and the y-coordinate of B.

(2)

Given that O is the origin,

(c) find the area of the triangle OAB.
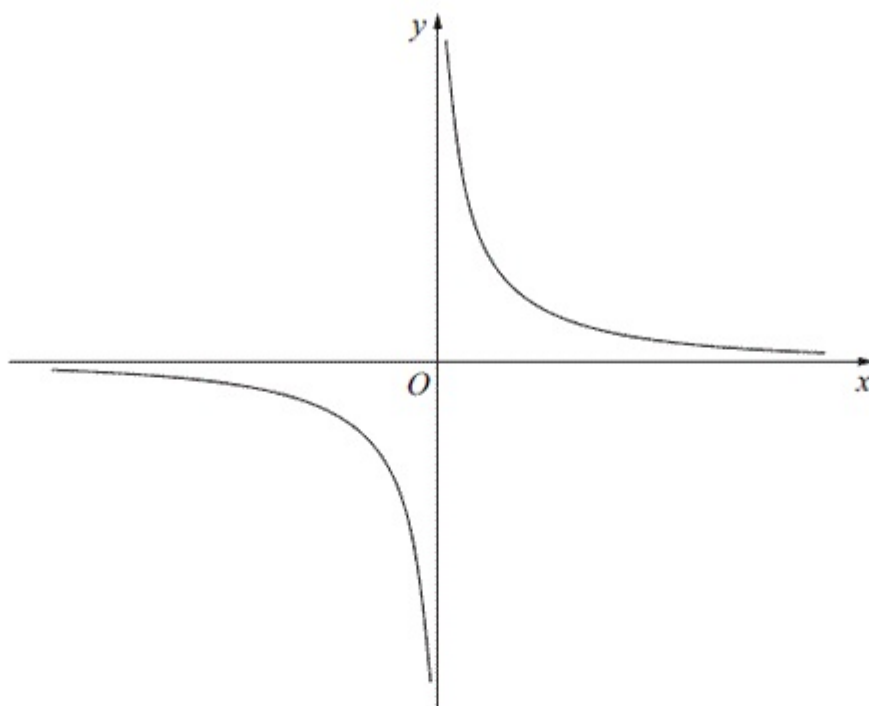
(2)

Question 6

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \dfrac{2}{x}$, $x \neq 0$

The curve C has equation $y = \dfrac{2}{x} - 5$, $x \neq 0$, and the line l has equation $y = 4x + 2$

(a) Sketch and clearly label the graphs of C and l on a single diagram.

On your diagram, show clearly the coordinates of the points where C and l cross the coordinate axes.

(5)

(b) Write down the equations of the asymptotes of the curve C.

(2)

(c) Find the coordinates of the points of intersection of $y = \dfrac{2}{x} - 5$ and $y = 4x + 2$

(5)

Question 7

Lewis played a game of space invaders. He scored points for each spaceship that he captured.

Lewis scored 140 points for capturing his first spaceship.

He scored 160 points for capturing his second spaceship, 180 points for capturing his third spaceship, and so on.

The number of points scored for capturing each successive spaceship formed an arithmetic sequence.

(a) Find the number of points that Lewis scored for capturing his 20th spaceship.

(2)

(b) Find the total number of points Lewis scored for capturing his first 20 spaceships.

(3)

Sian played an adventure game. She scored points for each dragon that she captured. The number of points that Sian scored for capturing each successive dragon formed an arithmetic sequence.

Sian captured n dragons and the total number of points that she scored for capturing all n dragons was 8500.

Given that Sian scored 300 points for capturing her first dragon and then 700 points for capturing her nth dragon,

(c) find the value of n.

(3)

Question 8

$$\frac{dy}{dx} = -x^3 + \frac{4x - 5}{2x^3}, \qquad x \neq 0$$

Given that $y = 7$ at $x = 1$, find y in terms of x, giving each term in its simplest form.

(6)

Question 9

The equation

$$(k + 3)x^2 + 6x + k = 5, \quad \text{where } k \text{ is a constant,}$$

has two distinct real solutions for x.

(a) Show that k satisfies

$$k^2 - 2k - 24 < 0$$

(4)

(b) Hence find the set of possible values of k.

(3)

Question 10

$$4x^2 + 8x + 3 \equiv a(x + b)^2 + c$$

(a) Find the values of the constants a, b and c.

(3)

(b) On the axes on page 27, sketch the curve with equation $y = 4x^2 + 8x + 3$, showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(4)

Question 11

The curve C has equation

$$y = 2x - 8\sqrt{x} + 5, \quad x \geq 0$$

(a) Find $\dfrac{dy}{dx}$, giving each term in its simplest form.

(3)

The point P on C has x-coordinate equal to $\dfrac{1}{4}$

(b) Find the equation of the tangent to C at the point P, giving your answer in the form $y = ax + b$, where a and b are constants.

(4)

The tangent to C at the point Q is parallel to the line with equation $2x - 3y + 18 = 0$

(c) Find the coordinates of Q.

(5)