Statistics-S1 - 2008-June

Question 1

A disease is known to be present in 2% of a population. A test is developed to help determine whether or not someone has the disease.

Given that a person has the disease, the test is positive with probability 0.95

Given that a person does not have the disease, the test is positive with probability 0.03

(a) Draw a tree diagram to represent this information.

(3)

A person is selected at random from the population and tested for this disease.

(b) Find the probability that the test is positive.

(3)

A doctor randomly selects a person from the population and tests him for the disease. Given that the test is positive,

(c) find the probability that he does not have the disease.

(2)

(d) Comment on the usefulness of this test.

(1)

Question 2

The age in years of the residents of two hotels are shown in the back to back stem and leaf diagram below.

Abbey Hotel 8|5|0 means 58 years in Abbey hotel and 50 years in Balmoral hotel Balmoral Hotel

(1)	2	0		
(4)	9751	1		
(4)	9831	2	6	(1)
(11)	99997665332	3	447	(3)
(6)	987750	4	005569	(6)
(1)	8	5	000013667	(9)
		6	233457	(6)
		7	015	(3)

For the Balmoral Hotel,

(a) write down the mode of the age of the residents,

(1)

(b) find the values of the lower quartile, the median and the upper quartile.

(3)

(c) (i) Find the mean, $\bar{x}$, of the age of the residents.

(ii) Given that $\sum x^2 = 81\,213$ find the standard deviation of the age of the residents.

(4)

One measure of skewness is found using

$$\frac{\text{mean} - \text{mode}}{\text{standard deviation}}$$

(d) Evaluate this measure for the Balmoral Hotel.

(2)

For the Abbey Hotel, the mode is 39, the mean is 33.2, the standard deviation is 12.7 and the measure of skewness is – 0.454

(e) Compare the two age distributions of the residents of each hotel.

(3)

Question 3

The random variable X has probability distribution given in the table below.

x	-1	0	1	2	3
$P(X=x)$	p	q	0.2	0.15	0.15

Given that $E(X) = 0.55$, find

(a) the value of p and the value of q,

(5)

(b) $\text{Var}(X)$,

(4)

(c) $E(2X - 4)$.

(2)

Question 4

Crickets make a noise. The pitch, v kHz, of the noise made by a cricket was recorded at 15 different temperatures, t °C. These data are summarised below.

$$\sum t^2 = 10\,922.81, \sum v^2 = 42.3356, \sum tv = 677.971, \sum t = 401.3, \sum v = 25.08$$

(a) Find S_{tt}, S_{vv} and S_{tv} for these data.

(4)

(b) Find the product moment correlation coefficient between t and v.

(3)

(c) State, with a reason, which variable is the explanatory variable.

(2)

(d) Give a reason to support fitting a regression model of the form $v = a + bt$ to these data.

(1)

(e) Find the value of a and the value of b. Give your answers to 3 significant figures.

(4)

(f) Using this model, predict the pitch of the noise at 19 °C.

(1)

Question 5

A person's blood group is determined by whether or not it contains any of 3 substances A, B and C.

A doctor surveyed 300 patients' blood and produced the table below.

Blood contains	No. of Patients
only C	100
A and C but not B	100
only A	30
B and C but not A	25
only B	12
A, B and C	10
A and B but not C	3

(a) Draw a Venn diagram to represent this information.

(4)

(b) Find the probability that a randomly chosen patient's blood contains substance C.

(2)

Harry is one of the patients. Given that his blood contains substance A,

(c) find the probability that his blood contains all 3 substances.

(2)

Patients whose blood contains none of these substances are called universal blood donors.

(d) Find the probability that a randomly chosen patient is a universal blood donor.

(2)

Question 6

The discrete random variable X can take only the values 2, 3 or 4. For these values the cumulative distribution function is defined by

$$F(x) = \frac{(x+k)^2}{25} \text{ for } x = 2, 3, 4$$

where k is a positive integer.

(a) Find k.

(2)

(b) Find the probability distribution of X.

(3)

Question 7

A packing plant fills bags with cement. The weight X kg of a bag of cement can be modelled by a normal distribution with mean 50 kg and standard deviation 2 kg.

(a) Find $P(X>53)$.

(3)

(b) Find the weight that is exceeded by 99% of the bags.

(5)

Three bags are selected at random.

(c) Find the probability that two weigh more than 53 kg and one weighs less than 53 kg.

(4)