Core-Maths-C2 - 2008-January

Question 1

(a) Find the remainder when

$$x^3 - 2x^2 - 4x + 8$$

is divided by

(i) $x - 3$,

(ii) $x + 2$.

(3)

(b) Hence, or otherwise, find all the solutions to the equation

$$x^3 - 2x^2 - 4x + 8 = 0.$$

(4)

Question 2

The fourth term of a geometric series is 10 and the seventh term of the series is 80.

For this series, find

(a) the common ratio,

(2)

(b) the first term,

(2)

(c) the sum of the first 20 terms, giving your answer to the nearest whole number.

(2)

Question 3

(a) Find the first 4 terms of the expansion of $\left(1+\dfrac{x}{2}\right)^{10}$ in ascending powers of x, giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places.

(3)

Question 4

(a) Show that the equation

$$3 \sin^2 \theta - 2 \cos^2 \theta = 1$$

can be written as

$$5 \sin^2 \theta = 3.$$

(2)

(b) Hence solve, for $0° \leqslant \theta < 360°$, the equation

$$3 \sin^2 \theta - 2 \cos^2 \theta = 1,$$

giving your answers to 1 decimal place.

(7)

Question 5

Given that a and b are positive constants, solve the simultaneous equations

$$a = 3b,$$

$$\log_3 a + \log_3 b = 2.$$

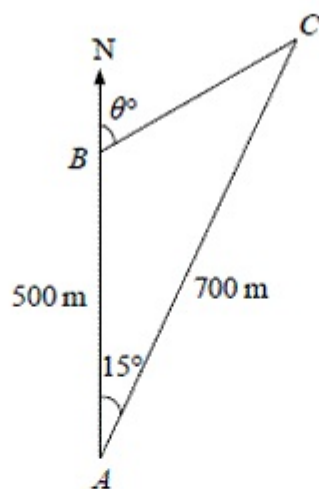Give your answers as exact numbers.

(6)

Question 6

Figure 1

Figure 1 shows 3 yachts A, B and C which are assumed to be in the same horizontal plane. Yacht B is 500 m due north of yacht A and yacht C is 700 m from A. The bearing of C from A is 015°.

(a) Calculate the distance between yacht B and yacht C, in metres to 3 significant figures.
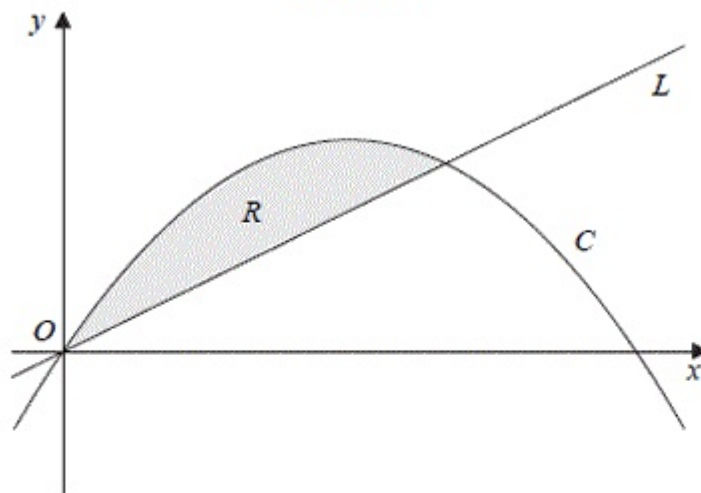
(3)

The bearing of yacht C from yacht B is $\theta°$, as shown in Figure 1.

(b) Calculate the value of θ.

(4)

Question 7

Figure 2

In Figure 2 the curve C has equation $y = 6x - x^2$ and the line L has equation $y = 2x$.

(a) Show that the curve C intersects the x-axis at $x = 0$ and $x = 6$.

(1)

(b) Show that the line L intersects the curve C at the points $(0, 0)$ and $(4, 8)$.

(3)

The region R, bounded by the curve C and the line L, is shown shaded in Figure 2.

(c) Use calculus to find the area of R.

(6)

Question 8

A circle C has centre M (6, 4) and radius 3.

(a) Write down the equation of the circle in the form
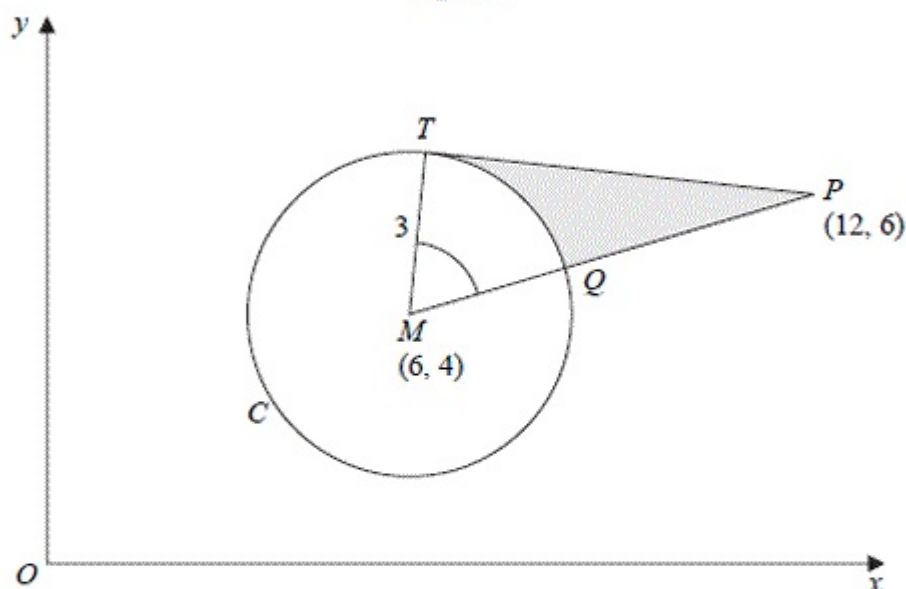
$$(x - a)^2 + (y - b)^2 = r^2.$$

(2)

Figure 3

Figure 3 shows the circle C. The point T lies on the circle and the tangent at T passes through the point P (12, 6). The line MP cuts the circle at Q.

(b) Show that the angle TMQ is 1.0766 radians to 4 decimal places.

(4)

The shaded region TPQ is bounded by the straight lines TP, QP and the arc TQ, as shown in Figure 3.

(c) Find the area of the shaded region TPQ. Give your answer to 3 decimal places.
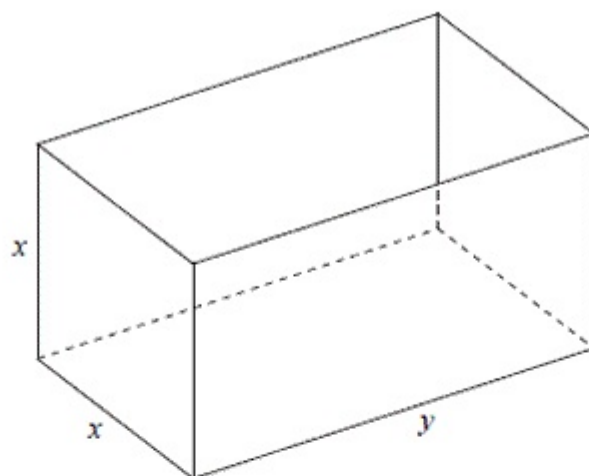
(5)

Question 9

Figure 4

Figure 4 shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is x metres.

The capacity of the tank is 100 m^3.

(a) Show that the area $A \text{ m}^2$ of the sheet metal used to make the tank is given by

$$A = \frac{300}{x} + 2x^2.$$

(4)

(b) Use calculus to find the value of x for which A is stationary.

(4)

(c) Prove that this value of x gives a minimum value of A.

(2)

(d) Calculate the minimum area of sheet metal needed to make the tank.

(2)