Core-Maths-C2 - 2012-June

Question 1

Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 3x)^5$$

giving each term in its simplest form.

(4)

Question 2

Find the values of x such that

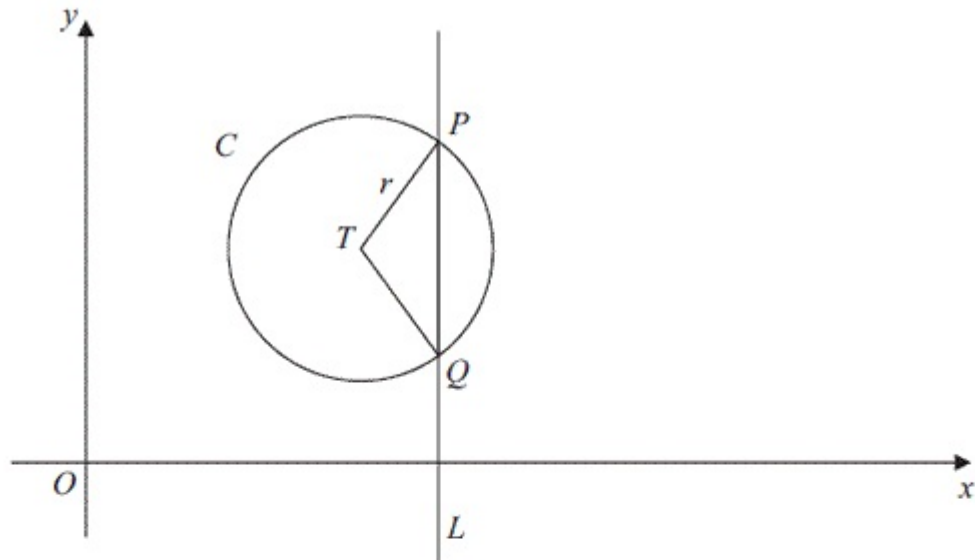$$2\log_3 x - \log_3(x - 2) = 2$$

(5)

Question 3

Figure 1

The circle C with centre T and radius r has equation

$$x^2 + y^2 - 20x - 16y + 139 = 0$$

(a) Find the coordinates of the centre of C.

(3)

(b) Show that $r = 5$

(2)

The line L has equation $x = 13$ and crosses C at the points P and Q as shown in Figure 1.

(c) Find the y coordinate of P and the y coordinate of Q.

(3)

Given that, to 3 decimal places, the angle PTQ is 1.855 radians,

(d) find the perimeter of the sector PTQ.

(3)

Question 4

$$f(x) = 2x^3 - 7x^2 - 10x + 24$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of f(x).

(2)

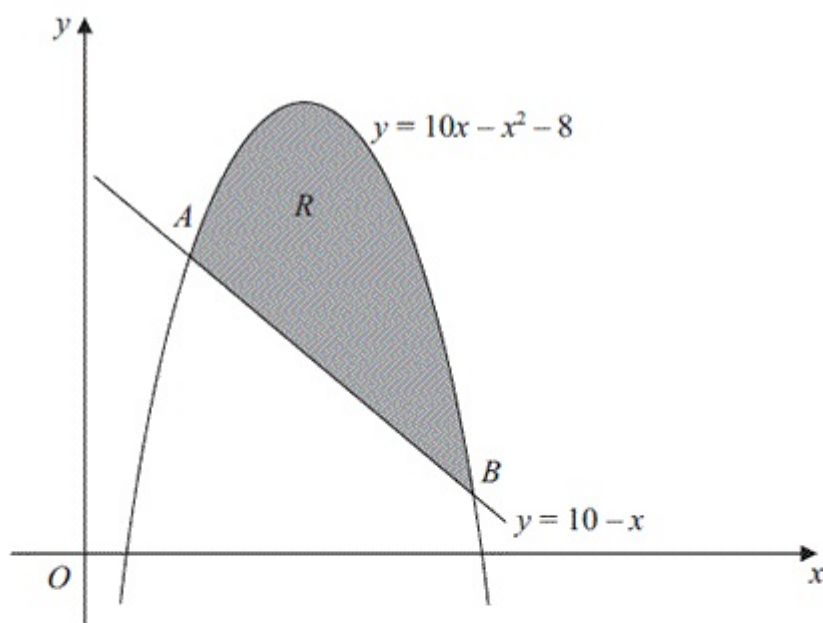(b) Factorise f(x) completely.

(4)

Question 5

Figure 2

Figure 2 shows the line with equation $y = 10 - x$ and the curve with equation $y = 10x - x^2 - 8$

The line and the curve intersect at the points A and B, and O is the origin.

(a) Calculate the coordinates of A and the coordinates of B.

(5)

The shaded area R is bounded by the line and the curve, as shown in Figure 2.

(b) Calculate the exact area of R.

(7)

Question 6

(a) Show that the equation

$$\tan 2x = 5 \sin 2x$$

can be written in the form

$$(1 - 5 \cos 2x) \sin 2x = 0$$

(2)

(b) Hence solve, for $0 \leqslant x \leqslant 180°$,

$$\tan 2x = 5 \sin 2x$$

giving your answers to 1 decimal place where appropriate.
You must show clearly how you obtained your answers.

(5)

Question 7

$$y = \sqrt{(3^x + x)}$$

(a) Complete the table below, giving the values of y to 3 decimal places.

x	0	0.25	0.5	0.75	1
y	1	1.251			2

(2)

(b) Use the trapezium rule with all the values of y from your table to find an approximation
for the value of $\displaystyle\int_0^1 \sqrt{(3^x + x)}\, dx$

You must show clearly how you obtained your answer.
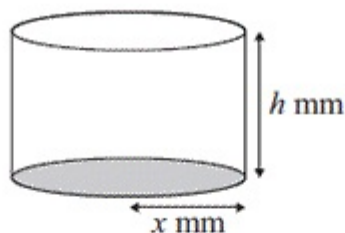
(4)

Question 8

Figure 3

A manufacturer produces pain relieving tablets. Each tablet is in the shape of a solid circular cylinder with base radius x mm and height h mm, as shown in Figure 3.

Given that the volume of each tablet has to be 60 mm³,

(a) express h in terms of x,

(1)

(b) show that the surface area, A mm², of a tablet is given by $A = 2\pi x^2 + \dfrac{120}{x}$

(3)

The manufacturer needs to minimise the surface area A mm², of a tablet.

(c) Use calculus to find the value of x for which A is a minimum.

(5)

(d) Calculate the minimum value of A, giving your answer to the nearest integer.

(2)

(e) Show that this value of A is a minimum.

(2)

Question 9

A geometric series is $a + ar + ar^2 + \ldots$

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r}$$

(4)

The third and fifth terms of a geometric series are 5.4 and 1.944 respectively and all the terms in the series are positive.

For this series find,

(b) the common ratio,

(2)

(c) the first term,

(2)

(d) the sum to infinity.

(3)