Core-Maths-C4 - 2012-June

Question 1

$$f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$$

(a) Find the values of the constants A, B and C.

(4)

(b) (i) Hence find $\int f(x)\,dx$.

(ii) Find $\int_1^2 f(x)\,dx$, leaving your answer in the form $a + \ln b$, where a and b are constants.

(6)

Question 2

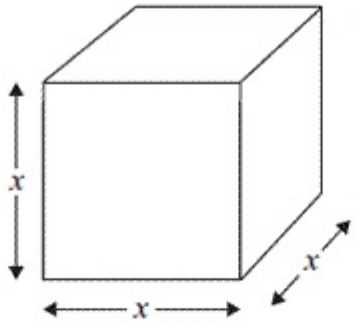

Figure 1

Figure 1 shows a metal cube which is expanding uniformly as it is heated.
At time t seconds, the length of each edge of the cube is x cm,
and the volume of the cube is V cm^3.

(a) Show that $\frac{dV}{dx} = 3x^2$

(1)

Given that the volume, V cm^3, increases at a constant rate of 0.048 cm^{3}s^{-1},

(b) find $\frac{dx}{dt}$, when $x = 8$

(2)

(c) find the rate of increase of the total surface area of the cube, in cm^{2}s^{-1}, when $x = 8$

(3)

Question 3

$$f(x) = \frac{6}{\sqrt{(9-4x)}}, \quad |x| < \frac{9}{4}$$

(a) Find the binomial expansion of $f(x)$ in ascending powers of x, up to and including the term in x^3. Give each coefficient in its simplest form.

(6)

Use your answer to part (a) to find the binomial expansion in ascending powers of x, up to and including the term in x^3, of

(b) $g(x) = \frac{6}{\sqrt{(9+4x)}}, \quad |x| < \frac{9}{4}$

(1)

(c) $h(x) = \frac{6}{\sqrt{(9-8x)}}, \quad |x| < \frac{9}{8}$

(2)

Question 4

Given that $y = 2$ at $x = \frac{\pi}{4}$, solve the differential equation

$$\frac{dy}{dx} = \frac{3}{y\cos^2 x}$$

(5)

Question 5

The curve C has equation

$$16y^3 + 9x^2y - 54x = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Find the coordinates of the points on C where $\frac{dy}{dx} = 0$.

(7)

Question 6

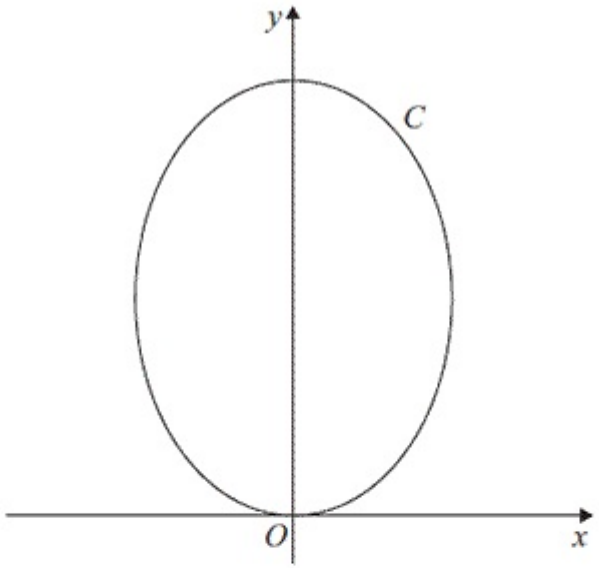

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = (\sqrt{3})\sin 2t, \quad y = 4\cos^2 t, \quad 0 \leqslant t \leqslant \pi$$

(a) Show that $\frac{dy}{dx} = k(\sqrt{3})\tan 2t$, where k is a constant to be determined.

(5)

(b) Find an equation of the tangent to C at the point where $t = \frac{\pi}{3}$.

Give your answer in the form $y = ax + b$, where a and b are constants.

(4)

(c) Find a cartesian equation of C.

(3)

Question 7

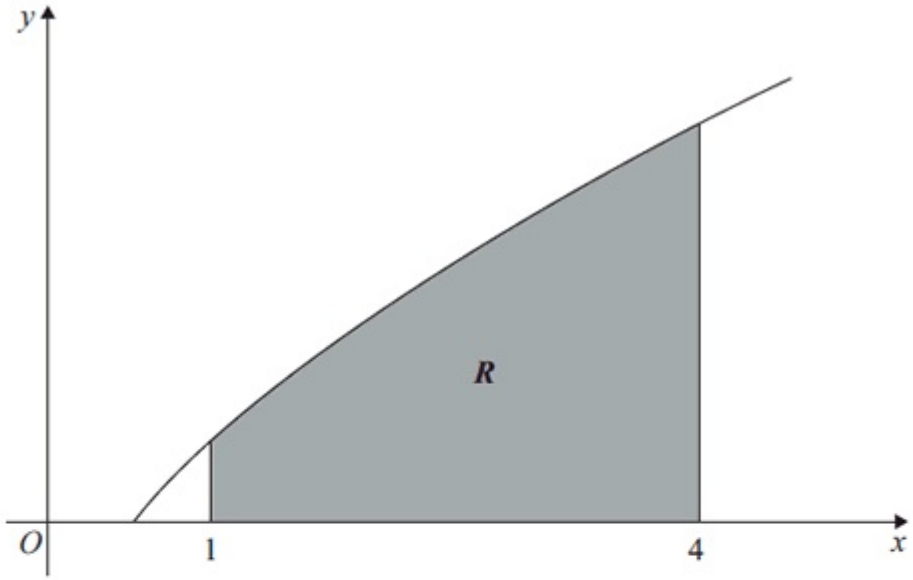

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = x^{\frac{1}{2}} \ln 2x$.

The finite region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$

(a) Use the trapezium rule, with 3 strips of equal width, to find an estimate for the area of R, giving your answer to 2 decimal places.

(4)

(b) Find $\int x^{\frac{1}{2}} \ln 2x \, dx$.

(4)

(c) Hence find the exact area of R, giving your answer in the form $a \ln 2 + b$, where a and b are exact constants.

(3)