

Statistics-S1 - 2009-June

Question 1

The volume of a sample of gas is kept constant. The gas is heated and the pressure, p , is measured at 10 different temperatures, t . The results are summarised below.

$$\sum p = 445 \quad \sum p^2 = 38\,125 \quad \sum t = 240 \quad \sum t^2 = 27\,520 \quad \sum pt = 26\,830$$

- (a) Find S_{pp} and S_{pt} . (3)

Given that $S_{tt} = 21\,760$,

- (b) calculate the product moment correlation coefficient. (2)

- (c) Give an interpretation of your answer to part (b). (1)
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Question 2

On a randomly chosen day the probability that Bill travels to school by car, by bicycle or on foot is $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$ respectively. The probability of being late when using these methods of travel is $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{1}{10}$ respectively.

- (a) Draw a tree diagram to represent this information. (3)

- (b) Find the probability that on a randomly chosen day

- (i) Bill travels by foot and is late,
- (ii) Bill is not late. (4)

- (c) Given that Bill is late, find the probability that he did not travel on foot. (4)
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Question 3

The variable x was measured to the nearest whole number. Forty observations are given in the table below.

x	10 – 15	16 – 18	19 –
Frequency	15	9	16

A histogram was drawn and the bar representing the 10 – 15 class has a width of 2 cm and a height of 5 cm. For the 16 – 18 class find

(a) the width, (1)

(b) the height (2)

of the bar representing this class.

Question 4

A researcher measured the foot lengths of a random sample of 120 ten-year-old children. The lengths are summarised in the table below.

Foot length, l , (cm)	Number of children
$10 \leq l < 12$	5
$12 \leq l < 17$	53
$17 \leq l < 19$	29
$19 \leq l < 21$	15
$21 \leq l < 23$	11
$23 \leq l < 25$	7

- (a) Use interpolation to estimate the median of this distribution. (2)
- (b) Calculate estimates for the mean and the standard deviation of these data. (6)

One measure of skewness is given by

$$\text{Coefficient of skewness} = \frac{3(\text{mean} - \text{median})}{\text{standard deviation}}$$

- (c) Evaluate this coefficient and comment on the skewness of these data. (3)

Greg suggests that a normal distribution is a suitable model for the foot lengths of ten-year-old children.

- (d) Using the value found in part (c), comment on Greg's suggestion, giving a reason for your answer. (2)
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Question 5

The weight, w grams, and the length, l mm, of 10 randomly selected newborn turtles are given in the table below.

l	49.0	52.0	53.0	54.5	54.1	53.4	50.0	51.6	49.5	51.2
w	29	32	34	39	38	35	30	31	29	30

(You may use $S_{ll} = 33.381$ $S_{wl} = 59.99$ $S_{ww} = 120.1$)

- (a) Find the equation of the regression line of w on l in the form $w = a + bl$. (5)
- (b) Use your regression line to estimate the weight of a newborn turtle of length 60 mm. (2)
- (c) Comment on the reliability of your estimate giving a reason for your answer. (2)
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Question 6

The discrete random variable X has probability function

$$P(X = x) = \begin{cases} a(3 - x) & x = 0, 1, 2 \\ b & x = 3 \end{cases}$$

- (a) Find $P(X = 2)$ and complete the table below.

x	0	1	2	3
$P(X = x)$	$3a$	$2a$		b

(1)

Given that $E(X) = 1.6$

- (b) Find the value of a and the value of b .

(5)

Find

- (c) $P(0.5 < X < 3)$,

(2)

- (d) $E(3X - 2)$.

(2)

- (e) Show that the $\text{Var}(X) = 1.64$

(3)

- (f) Calculate $\text{Var}(3X - 2)$.

(2)

Question 7

(a) Given that $P(A) = a$ and $P(B) = b$ express $P(A \cup B)$ in terms of a and b when

- (i) A and B are mutually exclusive,
- (ii) A and B are independent.

(2)

Two events R and Q are such that

$$P(R \cap Q') = 0.15, \quad P(Q) = 0.35 \text{ and } P(R|Q) = 0.1$$

Find the value of

(b) $P(R \cup Q)$,

(1)

(c) $P(R \cap Q)$,

(2)

(d) $P(R)$.

(2)

Question 8

The lifetimes of bulbs used in a lamp are normally distributed.

A company X sells bulbs with a mean lifetime of 850 hours and a standard deviation of 50 hours.

(a) Find the probability of a bulb, from company X , having a lifetime of less than 830 hours.

(3)

(b) In a box of 500 bulbs, from company X , find the expected number having a lifetime of less than 830 hours.

(2)

A rival company Y sells bulbs with a mean lifetime of 860 hours and 20% of these bulbs have a lifetime of less than 818 hours.

(c) Find the standard deviation of the lifetimes of bulbs from company Y .

(4)

Both companies sell the bulbs for the same price.

(d) State which company you would recommend. Give reasons for your answer.

(2)
