Core-Maths-C4 - 2008-June

Question 1

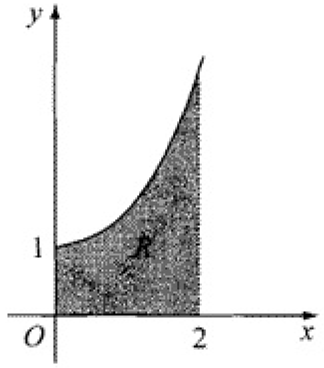

Figure 1

Figure 1 shows part of the curve with equation $y = e^{0.5x^2}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.

(a) Complete the table with the values of y corresponding to $x = 0.8$ and $x = 1.6$.

x	0	0.4	0.8	1.2	1.6	2
y	e^0	$e^{0.08}$		$e^{0.72}$		e^2

(1)

(b) Use the trapezium rule with all the values in the table to find an approximate value for the area of R, giving your answer to 4 significant figures.

(3)

Question 2

(a) Use integration by parts to find $\int x e^x \, dx$.

(3)

(b) Hence find $\int x^2 e^x \, dx$.

(3)

Question 3

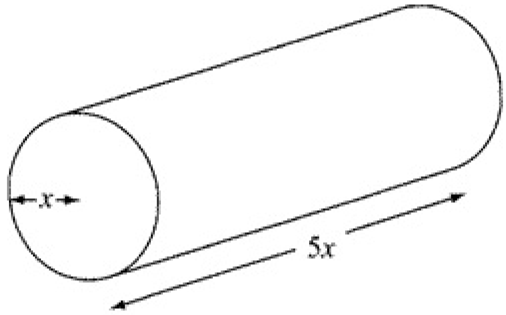

Figure 2

Figure 2 shows a right circular cylindrical metal rod which is expanding as it is heated. After t seconds the radius of the rod is x cm and the length of the rod is $5x$ cm. The cross-sectional area of the rod is increasing at the constant rate of $0.032\ \text{cm}^2\ \text{s}^{-1}$.

(a) Find $\frac{\mathrm{d}x}{\mathrm{d}t}$ when the radius of the rod is 2 cm, giving your answer to 3 significant figures.

(4)

(b) Find the rate of increase of the volume of the rod when $x = 2$.

(4)

Question 4

A curve has equation $3x^2 - y^2 + xy = 4$. The points P and Q lie on the curve. The gradient of the tangent to the curve is $\frac{8}{3}$ at P and at Q.

(a) Use implicit differentiation to show that $y - 2x = 0$ at P and at Q.

(6)

(b) Find the coordinates of P and Q.

(3)

Question 5

(a) Expand $\frac{1}{\sqrt{(4-3x)}}$, where $|x| < \frac{4}{3}$, in ascending powers of x up to and including the term in x^2. Simplify each term.

(5)

(b) Hence, or otherwise, find the first 3 terms in the expansion of $\frac{x+8}{\sqrt{(4-3x)}}$ as a series in ascending powers of x.

(4)

Question 6

With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = (-9\mathbf{i} + 10\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$$

$$l_2: \quad \mathbf{r} = (3\mathbf{i} + \mathbf{j} + 17\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection.

(6)

(b) Show that l_1 and l_2 are perpendicular to each other.

(2)

The point A has position vector $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$.

(c) Show that A lies on l_1.

(1)

The point B is the image of A after reflection in the line l_2.

(d) Find the position vector of B.

(3)

Question 7

(a) Express $\frac{2}{4-y^2}$ in partial fractions.

(3)

(b) Hence obtain the solution of

$$2\cot x \frac{dy}{dx} = (4 - y^2)$$

for which $y = 0$ at $x = \frac{\pi}{3}$, giving your answer in the form $\sec^2 x = g(y)$.

(8)

Question 8

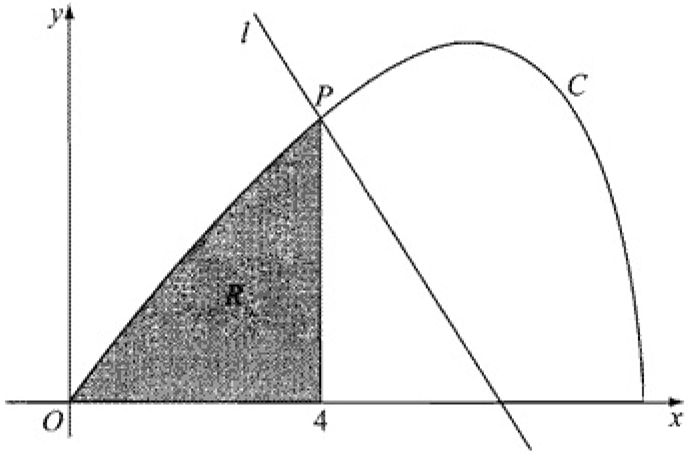

Figure 3

Figure 3 shows the curve C with parametric equations

$$x = 8\cos t, \qquad y = 4\sin 2t, \qquad 0 \leqslant t \leqslant \frac{\pi}{2}.$$

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of t at the point P.

(2)

The line l is a normal to C at P.

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

(6)

The finite region R is enclosed by the curve C, the x-axis and the line $x = 4$, as shown shaded in Figure 3.

(c) Show that the area of R is given by the integral $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64\sin^2 t\cos t \, dt$.

(4)

(d) Use this integral to find the area of R, giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(4)