Computer Vision homework 1

Pan Changxun

March 2025

1 Short Answer Questions

1. (using 0-padding)
   The Answer for full mode is:

$$\begin{bmatrix} 3 & 11 & 26 & 35 & 38 & 33 & 25 & 15 \\ 17 & 50 & 96 & 105 & 100 & 102 & 85 & 48 \\ 49 & 127 & 209 & 198 & 180 & 202 & 172 & 90 \\ 67 & 151 & 186 & 130 & 101 & 184 & 186 & 90 \\ 57 & 149 & 189 & 178 & 156 & 207 & 168 & 81 \\ 11 & 90 & 158 & 248 & 248 & 287 & 200 & 111 \\ 35 & 85 & 164 & 194 & 190 & 194 & 134 & 96 \\ 49 & 105 & 154 & 124 & 118 & 132 & 119 & 63 \end{bmatrix}$$

   The Answer for same mode is:

$$\begin{bmatrix} 50 & 96 & 105 & 100 & 102 & 85 \\ 127 & 209 & 198 & 180 & 202 & 172 \\ 151 & 186 & 130 & 101 & 184 & 186 \\ 149 & 189 & 178 & 156 & 207 & 168 \\ 90 & 158 & 248 & 248 & 287 & 200 \\ 85 & 164 & 194 & 190 & 194 & 134 \end{bmatrix}$$

   The Answer for valid mode is:

$$\begin{bmatrix} 209 & 198 & 180 & 202 \\ 186 & 130 & 101 & 184 \\ 189 & 178 & 156 & 207 \\ 158 & 248 & 248 & 287 \end{bmatrix}$$

2. The second derivative filter f” is $(0.25, 0, -0.5, 0, 0.25)$

3. We just use the ”same” mode padded image to prove the theorem.
   Suppose the image is I, f is $w \times 1$, G is $1 \times h$, then we have:

$$(I \star f)_{i,j} = \sum_{l=0}^{h} I_{i+k,j} f_{w-k}$$

$$\begin{aligned} ((I \star f) \star G)_{i,j} &= \sum_{l=0}^{h} (I \star f)_{i,j+l} G_{h-l} \\ &= \sum_{l=0}^{h} \sum_{k=0}^{w} I_{i+k,j+l} f_{w-k} G_{h-l} \end{aligned}$$

And meanwhile, we have:

$$(I \star (fG))_{i,j} = \sum_{l=0}^{h} \sum_{k=0}^{w} I_{i+k,j+l}(fG)_{w-k,h-l}$$

Since fG is a $w \times h$ matrix, we have:

$$(fG)_{w-k,h-l} = \sum_{m=0}^{w} \sum_{n=0}^{h} f_{w-m}G_{h-n}\delta_{m,k}\delta_{n,l} = f_{w-k}G_{h-l}$$

Thus, we have:

$$(I \star (fG))_{i,j} = \sum_{l=0}^{h} \sum_{k=0}^{w} I_{i+k,j+l} f_{w-k} G_{h-l}$$

Thus, we have $(I \star f) \star G = (I \star (fG))$