№1

1) $$\int \ln^2 x\,dx = \left\{ \begin{array}{ll} t=\ln x & dt=\frac{dx}{x} \quad dx=dt\cdot x \\ x=e^{\ln x}=e^t & dx=e^t dt \end{array} \right\} = \int t^2 e^t dt = \left[\int u\,dv\right] =$$

$$= \left\{ \begin{array}{ll} u=t^2 & du=2t\,dt \\ dv=e^t dt & v=e^t \end{array} \right\} = t^2\cdot e^t - \int e^t\cdot 2t\,dt = t^2 e^t - 2\int e^t t\,dt =$$

$$= \left\{ \begin{array}{ll} u=t & du=dt \\ dv=e^t dt & v=e^t \end{array} \right\} = t^2 e^t - 2\left(t\cdot e^t - \int e^t dt\right) = t^2 e^t - 2te^t + 2e^t + C =$$

$$= \ln^2 x\cdot x - 2\ln x\cdot x + 2x + C$$

2) $$\int \cos 2x \cdot e^{-x} dx = -\int \cos 2x\, d(e^{-x}) = -\left(e^{-x}\cdot \cos 2x - \int e^{-x} d(\cos 2x)\right) =$$

$$= -\left(e^{-x}\cos 2x + 2\int e^{-x}\sin 2x\,dx\right) = -\left(e^{-x}\cos 2x - 2\int \sin 2x\, d(e^{-x})\right) =$$

$$= -\left(e^{-x}\cos 2x - 2\left(e^{-x}\sin 2x - \int e^{-x} d(\sin 2x)\right)\right) = -\left(e^{-x}\cos 2x - 2\left(e^{-x}\sin 2x -\right.\right.$$
$$\left.\left. - 2\int e^{-x}\cos 2x\,dx\right)\right)$$

$$\exists \int e^{-x}\cos 2x\,dx = A$$

$$A = -\left(e^{-x}\cos 2x - 2\left(e^{-x}\sin 2x - 2A\right)\right) = -e^{-x}\cos 2x + 2e^{-x}\sin 2x - 4A$$

$$5A = -e^{-x}\cos 2x + 2e^{-x}\sin 2x$$

$$A = \frac{-e^{-x}\cos 2x + 2e^{-x}\sin 2x}{5}$$

$$\int \cos 2x \cdot e^{-x} dx = \frac{-e^{-x}\cos 2x + 2e^{-x}\sin 2x}{5} + C$$

3) $\int \sin(\ln x)\,dx = \left\{ \begin{matrix} t = \ln x & x = e^t \\ dt = \frac{dx}{x} & dx = x \cdot dt = e^t dt \end{matrix} \right\} = \int \sin t\, e^t\, dt =$

$= \int \sin t\, d(e^t) = e^t \sin t - \int e^t d(\sin t) = e^t \sin t - \int e^t \cos x\, dx =$

$= e^t \cdot \sin t - \int \cos x\, d(e^t) = e^t \sin t - (e^t \cos t - \int e^t d(\cos t)) =$

$= e^t \sin t - (e^t \cos t + \int e^t \sin t\, dt)$

$] \int e^t \sin t\, dt = A$

$A = e^t \sin t - (e^t \cos t + A) = e^t \sin t - e^t \cos t - A$

$2A = e^t \sin t - e^t \cos t$

$A = \frac{e^t \sin t - e^t \cos t}{2}$

$\int e^t \sin t\, dt = \frac{e^t \sin t - e^t \cos t}{2} + C$

$\int \sin(\ln x)\, dx = \frac{x \cdot \sin(\ln x) - x \cdot \cos(\ln x)}{2} + C$

4) $\int \frac{\ln x\, dx}{x\sqrt{1+\ln x}} = \left\{ \begin{matrix} t = \ln x & x = e^t \\ dt = \frac{dx}{x} & dx = x dt = e^t dt \end{matrix} \right\} = \int \frac{t e^t dt}{e^t \sqrt{1+t}} = \int \frac{t\, dt}{\sqrt{1+t}} =$

$= \left\{ \begin{matrix} k = t+1 \\ t = k-1 \\ dk = dt \end{matrix} \right\} = \int \frac{(k-1)\,dk}{\sqrt{k}} = \int \frac{k\,dk}{\sqrt{k}} - \int \frac{dk}{\sqrt{k}} = \int \sqrt{k}\,dk - \int \frac{dk}{\sqrt{k}} =$

$= \int k^{\frac{1}{2}} dk - \int k^{-\frac{1}{2}} dk = \frac{2k^{\frac{3}{2}}}{3} - 2k^{\frac{1}{2}} + C = \frac{(2k-6)\sqrt{k}}{3} + C$

$\frac{(2k-6)\sqrt{k}}{3} = \frac{(2t-4)\sqrt{t+1}}{3} = \frac{(2\ln x - 4)\sqrt{\ln x + 1}}{3}$

$\int \frac{\ln x\, dx}{x\sqrt{1+\ln x}} = \frac{2(\ln x - 2)\sqrt{\ln x + 1}}{3} + C$

$$5)\quad \int \frac{dx}{x\sqrt{x^2+x+1}} = \left\{ \begin{array}{ll} t = \frac{1}{x} & x = \frac{1}{t} \\ dt = -\frac{1}{x^2}dx = -t^2 dx & dx = -\frac{dt}{t^2} \end{array} \right\} = -\int \frac{dt}{t\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}} =$$

$$= -\int \frac{dt}{\sqrt{t^2+t+1}} = -\int \frac{dt}{\sqrt{\left(t^2+t+\frac{1}{4}\right)+\frac{3}{4}}} = -\int \frac{dt}{\left(t+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \left\{ \begin{array}{l} k = t+\frac{1}{2} \\ t = k-\frac{1}{2} \\ dk = dt \end{array} \right\} =$$

$$= -\int \frac{dk}{\sqrt{k^2+\left(\frac{\sqrt{3}}{2}\right)^2}} = -\ln\left|k+\sqrt{k^2+\frac{3}{4}}\right| + C$$

$$k+\sqrt{k^2+\frac{3}{4}} = t+\frac{1}{2}+\sqrt{t^2+t+1} = \frac{1}{x}+\frac{1}{2}+\sqrt{\frac{1}{x^2}+\frac{1}{x}+1}$$

$$\int \frac{dx}{x\sqrt{x^2+x+1}} = -\ln\left|\frac{1}{x}+\frac{1}{2}+\sqrt{\frac{1}{x^2}+\frac{1}{x}+1}\right| + C$$