№3

1) $$\int \frac{dx}{1+3\cos^2 x} = \left\{ 1+3\cos^2 x = 1 + \frac{3}{tg^2 x+1} = \frac{tg^2 x+4}{tg^2 x+1} \right.$$

$$\left. \begin{aligned} &t = tg\, x \\ &dt = \frac{dx}{\cos^2 x} \\ &dx = \cos^2 x\, dt = \frac{dt}{t^2+1} \end{aligned} \right\} =$$

$$= \int \frac{t^2+1}{t^2+4} \cdot \frac{dt}{t^2+1} = \int \frac{dt}{t^2+4} = \frac{1}{2} arctg\, \frac{t}{2} + C = \frac{arctg\left(\frac{tg x}{2}\right)}{2} + C$$

2) $$\int \frac{dx}{\sin^2 x - 5\sin x\cos x} = \left\{ \begin{aligned} &t = tg\, x \\ &dt = \frac{dx}{\cos^2 x} \\ &dx = dt\cos^2 x \end{aligned} \right\} = \int \frac{dt\cos^2 x}{t^2\cos^2 x - 5t\cos^2 x} = \int \frac{dt}{t^2-5t} = \int \frac{dt}{t(t-5)}$$

$$\frac{A}{t} + \frac{B}{t-5} = \frac{A(t-5)+Bt}{t(t-5)}$$

$$\left\{ \begin{aligned} &A+B=0 \\ &-5A=1 \end{aligned} \right. \qquad \left. \begin{aligned} &A = -\frac{1}{5} \\ &B = \frac{1}{5} \end{aligned} \right\}$$

$$\int \left(-\frac{dt}{5t} + \frac{dt}{5(t-5)}\right) = -\frac{1}{5}\int \frac{dt}{t} + \frac{1}{5}\int \frac{dt}{t-5} = -\frac{1}{5}\ln|t| + \frac{1}{5}\ln|t-5| + C =$$

$$= \frac{\ln\left|\frac{t-5}{t}\right|}{5} + C = \frac{\ln\left|\frac{tg x-5}{tg x}\right|}{5} + C$$

3) $\int \frac{dx}{8-4\sin x+7\cos x} = \int \frac{dx}{8-8\sin\frac{x}{2}\cos\frac{x}{2}+7(1-2\sin^2\frac{x}{2})} = \left\{ t=tg\frac{x}{2} \quad dx=2\cos^2\frac{x}{2}\,dt \right\} =$

$= 2\int \frac{\cos^2\frac{x}{2}\,dt}{15-8t\cos^2\frac{x}{2}-14t^2\cos^2 x} = -2\int \frac{dt}{14t^2+8t-\frac{15}{\cos^2\frac{x}{2}}} = \left\{ \frac{15}{\cos^2\frac{x}{2}} = 15\,tg^2 x+15 \right\} =$

$= -2\int \frac{dt}{-t^2+8t-15} = 2\int \frac{dt}{t^2-8t+15} = 2\int \frac{dt}{(t-3)(t-5)}$

$\frac{A}{(t-3)} + \frac{B}{(t-5)} = \frac{A(t-5)+B(t-3)}{(t-3)(t-5)} \qquad \left\{ \begin{array}{ll} A+B=0 & A=-\frac{1}{2} \\ -5A-3B=1 & B=\frac{1}{2} \end{array} \right\}$

$2\int\left(-\frac{dt}{2(t-3)} + \frac{dt}{2(t-5)}\right) = -\int\frac{dt}{t-3} + \int\frac{dt}{t-5} = -\ln|t-3|+\ln|t-5|+C =$

$= \ln\left|\frac{t-5}{t-3}\right| + C = \ln\left|\frac{tg\frac{x}{2}-5}{tg\frac{x}{2}-3}\right| + C$

4) $\int\sqrt{4-x^2}\,dx = \{x=2\sin t\} = \int\sqrt{4-4\sin^2 t}\,d(2\sin t) = \int 2\cos t\,d(2\sin t) =$

$= \int 4\cos^2 t\,dt = \int(2(2\cos^2 t-1)+2)\,dt = \int(2\cos 2t+2)\,dt = 2\int(\cos 2t+1)\,dt =$

$= 2\int\cos 2t\,dt + 2\int dt = \sin 2t+2t+C = 2\sin t\cos t+2t+C =$

$= x\cdot\sqrt{1-\sin^2 t} + 2\arcsin\frac{x}{2} + C = x\cdot\sqrt{1-\frac{x^2}{4}} + 2\arcsin\frac{x}{2} + C$

5) $\int\frac{x^2-4}{x}\,dx = \int x\,dx - 4\int\frac{1}{x}\,dx = \frac{x^2}{2} - 4\ln|x| + C$