Mathematics 3 **Honours Analysis** **2017/18**

These notes provide only the briefest structural outline of the material on uniform convergence, and should be read in conjunction with Wade's book, the relevant worksheets, and the amplifications given during the lectures themselves. The material is in Wade's book, Sections 7.1 and 7.2 up to and including the Weierstrass M-test and not beyond.

1. **Uniform convergence of sequences of functions.**

   - Definition of pointwise and uniform convergence of a sequence of functions. Examples.
   - Useful **Proposition**: The following are equivalent concerning a sequence of functions $f_n : E \to \mathbb{R}$ and $f : E \to \mathbb{R}$:
     (i) $f_n \to f$ uniformly on E
     (ii) $\sup_{x \in E} |f_n(x) - f(x)| \to 0$ as $n \to \infty$
     (iii) there exists a sequence $a_n > 0$ such that $|f_n(x) - f(x)| \leq a_n$ for all $x \in E$.
   - Uniform convergence of sequences of **continuous** functions: continuity of the limit (including proof). Examples.
   - Uniform convergence and integration – theorems on interchanging limits and integrals (including proofs from integration section). Applications to derivatives. Examples.
   - Uniform Cauchy criterion: $f_n \to f$ uniformly if and only if (f_n) is uniformly Cauchy (including proof).

2. **Uniform convergence of series of functions.**

   - Definition of pointwise and uniform convergence of a series of functions. Examples.
   - Weierstrass M-test (including proof) and applications.
   - Uniform convergence of series of **continuous** functions: continuity of the sum. Examples.
   - Uniform convergence and integration – theorems on interchanging limits and integrals. Applications to derivatives. Examples.