Mathematics 3 **Honours Analysis**

Informal course overview

This course is a second course in real analysis, following on directly from the analysis component of Fundamentals of Pure Mathematics. A good understanding of the analysis component of Fundamentals of Pure Mathematics is therefore essential for Honours Analysis, and we will frequently make use of notions and results from it. You are therefore advised to thoroughly re-familiarise yourself with its highlights at an early stage.

The rudiments of analysis of the real line, sequences and series of real numbers, and functions $f : \mathbb{R} \to \mathbb{R}$ were established in Fundamentals of Pure Mathematics. In this course we seek to broaden our point of view to encompass the analysis of n-dimensional euclidean space $\mathbb{R}^n$, sequences of points in $\mathbb{R}^n$ and functions $f : \mathbb{R}^m \to \mathbb{R}^m$. A key notion in this development is that of the **distance** between two points of $\mathbb{R}^n$. Surprisingly, we shall see that a large amount of the theory for euclidean spaces extends in a more or less routine manner to a much more abstract setting in which we have some set X upon which a suitable distance function or **metric** can be defined. This leads to far-reaching generalisations of the theory which are also extremely useful in a variety of settings, including the study of differential equations, coding theory, information networks, and which provide the foundations for much of modern mathematical analysis, topology and geometry.

Before doing this, however, we shall first prepare for the journey by considering **sequences and series of functions** (as opposed to sequences and series of *real numbers* which were studied in Fundamentals of Pure Mathematics). We shall then develop the theory of **integration of functions** which, aside from its obvious practical value, will allow us to construct very interesting examples of **metrics defined on spaces of functions**. Then we shall embark on the general analysis of euclidean space in particular and more generally of **metric spaces**. The notion of **completeness** will play an essential role. Finally, we shall draw these notions together in studying **Fourier Series**.

Course plan:

1. Course overview and revision of topics from Fundamentals of Pure Mathematics [3 lectures]
2. Sequences and series of functions: uniform convergence and power series [6 lectures]
3. Integration of functions $f : \mathbb{R} \to \mathbb{R}$ [6 lectures]
4. Metric topology of euclidean spaces [3 lectures]
5. Metric spaces and examples [3 lectures]
6. Completeness and contraction mappings [2 lectures]
7. Compactness in metric spaces [4 lectures]
8. Fourier series [6 lectures]

Course textbook: The course textbook is

An Introduction to Analysis, (4th edition) by William Wade (published by Pearson).