

HONOURS ANALYSIS – READING AND EXERCISE ASSIGNMENTS

Week 1

Thursday lecture: Revise Chapters 1 and 2 of Wade.

Friday lecture: Revise Chapters 3,4 and 6 of Wade.

Week 2

Monday lecture: *Pointwise vs. uniform convergence; continuity of uniform limits of continuous functions.*

Wade Chapter 7, pp. 222 – 226 up to but not including Theorem 7.10.

Exercises 7.1.1, 7.1.3, 7.1.5.

Thursday lecture: *Integration; Cauchy criterion; differentiation.*

Wade Chapter 7, pp.226 (statement – but not proof – of Theorem 7.10 and then from Lemma 7.11) – 229.

Exercises 7.1.2, 7.1.8, 7.1.9, 7.1.10.

Friday lecture: *Catch-up*

Wade Chapter 7, pp.230 – 232 (up to but not including Theorem 7.16).

Exercises 7.2.1, 7.2.2, 7.2.3.

Week 3

Monday lecture: *Uniform convergence of series of functions, differentiation & integration; M-test.*

Exercises 7.2.4, 7.2.5, 7.2.6

Thursday lecture: *Power series: radius of convergence, interval of convergence; uniform and absolute convergence.*

Notes on Power Series up to the end of the proof of Theorem 2 on p.3.

Exercises 7.3.1 (a),(c), 7.3.2 (a), (b),(c).

Friday lecture: *Continuity, differentiability, integrability of power series.*

Rest of Notes on Power series.

Exercises 7.3.3, 7.3.4, 7.3.5.

Week 4

Monday lecture: *Integration of step functions*

Notes on Riemann Integration, Sections 1 and 2.

Thursday lecture: *The Riemann integral*

Notes on Riemann Integration, Section 3, up to but not including Theorem 3.

Exercises 5.1.0 (a), (b), 5.1.3.

Friday lecture: Leeway.

Week 5

Monday lecture: *Properties of the Riemann integral*

Notes on Riemann Integration up to the end of Section 3.

Exercises 5.1.5, 5.2.0(b), 5.2.2, 5.2.6, 5.2.8.

Thursday lecture: *Fundamental Theorem of Calculus; uniform limits*

Notes on Riemann Integration up to the end of Section 5.

Exercise 5.3.1.

Friday lecture: *Some odds and ends*

Notes on Riemann Integration up to the end of Section 6.

Exercises 5.4.1 (a),(d), 5.4.2 (a), (b), (c), 5.4.3, 5.4.4 (a), (b), (e).

Week 6

Monday lecture: *Introduction to metric spaces*

Wade Chapter 10 pp.342 – 344.

Thursday lecture: *Open sets*

Wade Chapter 10 pp.345 – 348 incl.

Exercises 10.1.4, 10.1.5, 10.1.6, 10.1.8.

Friday lecture: *More on open sets*

Exercises 8.3.1, 8.3.2, 10.1.9.

Week 7

Monday lecture: *Closed sets*

Exercises 8.3.1, 8.3.2, 10.1.9.

Thursday lecture: *Properties of open & closed sets, interior, closure*

Wade Chapter 10 pp.355 – 359.

Exercises 10.3.1, 10.3.2, 10.3.4, 10.3.5, 10.3.7, 10.3.11.

Friday lecture: *Convergence, Cauchy sequences and completeness*

Wade Chapter 10 pp.345 – 348 incl.

Exercises 10.1.4, 10.1.5, 10.1.6.

Week 8

Monday lecture: *Closedness, limit points, cluster points and completeness*

Wade Chapter 10 pp.350 – 353, Exercises 10.2.1, 10.2.2, 10.2.3, 10.2.4

Thursday lecture: *Limits and continuity; introduction to compactness*

Wade Chapter 10 pp.350 – 353, p. 361. Examples on pp. 315 – 318.

Exercise 10.3.9.

Friday lecture: *Compactness*

Wade Chapter 10 pp.361 – end of Remark 10.47 on p.363.

Exercises 10.4.1, 10.4.2, 10.4.3.

Week 9

Monday lecture: *More on compactness*

Read some of the problems at the end of Workshop 8 Exercises 10.4.10 (a), 10.4.7, 10.4.8, 10.4.9.

Thursday lecture: *Connectedness*

Wade Chapter 10 pp.367 – 370 up to but not including Theorem 10.57.

Exercises 10.5.1, 10.5.3, 10.5.4, 10.5.5, 10.5.11.

Friday lecture: *Continuous functions, open sets, compactness & connectedness*

Wade Chapter 10 pp.372 – 375.

Exercises 10.6.1, 10.6.2, 10.6.3, 10.6.5, 10.6.9.

Week 10

Monday lecture: *Contraction mapping theorem*

Notes on contraction mappings, first 2 pages.

Thursday lecture: *Ordinary differential equations*

Notes on ODEs.

Friday lecture: *Catch-up*

Week 11

Optional reading on Fourier Series:

Wade, Chapter 14, Section 1

Exercises 14.1.2, 14.1.4, 14.1.5, 14.1.6.

Wade, Chapter 14, Sections 2,3 and Theorem 14.29

Exercise 14.3.7.

Supplementary Reading

I've compiled the following guide to supplementary reading, broken down by topic. The books I suggest are:

T. Tao, Analysis I

T. Tao, Analysis II

T. Apostol, Mathematical Analysis (2nd or later edition)

W. A. Sutherland, An Introduction to Metric and Topological Spaces

G. F. Simmons, An Introduction to Topology and Modern Analysis

P. L. Walker, The Theory of Fourier Series and Integrals

H. Dym and H. P. McKean, Fourier Series and Integrals

W. Rudin, Principles of Mathematical Analysis

R. Seeley, An Introduction to Fourier Series and Integrals

D. J. H. Garling, Mathematical Analysis Volume I

D. J. H. Garling, Mathematical Analysis Volume II

Neither the order above nor the order below is indicative of any level of preference. You can understand the course entirely by following the lectures, the lecture notes and the reading from Wade; these suggestions are merely to present possible alternative angles on the same material.

I. Sequences and series of functions and uniform convergence; Power Series

- Tao II, Ch. 15, pp. 474–509
- Rudin, Ch. 7, pp. 143–153, Ch. 8 Sections 1–3, pp. 172–183
- Apostol, Ch 9, pp. 218–251
- Sutherland, Ch 8, pp. 114–122

II. Uniform continuity

- Sutherland, p. 87 ff.
- Apostol, p. 90 ff.
- Simmons, p. 77 ff.

III. The Riemann integral

There is no treatment in the literature exactly corresponding to ours. The closest are

- Tao I, Ch. 11, pp. 306–348
- Garling I, Ch. 8, pp. 209–239

IV. Metric spaces

- Sutherland, Ch. 2, pp. 19-44, Ch 6, pp 93–102, Ch 9, pp. 123–128
- Simmons, Ch. 2, pp.49–91; Ch 6, pp. 142–146
- Apostol, Ch. 3,4 pp.47–89
- Tao II, Ch. 12,13 pp. 389–439
- Rudin, Ch. 2, pp. 24–46
- Garling II, Ch. 11, 12, 15, 16

V. Compactness in metric spaces

- Apostol, Ch. 3, pp 59–65
- Sutherland Ch. 5, pp. 75–92, Ch. 7 pp. 108–113, Ch 10 Sect. 1

VI. The Banach contraction mapping theorem

- Tao II, Ch. 17, Sect. 17.6
- Sutherland, Ch. 9, pp. 123–133.
- Apostol, p.92

VII. Fourier Series

All of the following go rather deeper into the theory than we have, (especially Dym and McKean).

- P. L. Walker, The Theory of Fourier Series and Integrals
- H. Dym and H. P. McKean, Fourier Series and Integrals
- R. Seeley, An Introduction to Fourier Series and Integrals