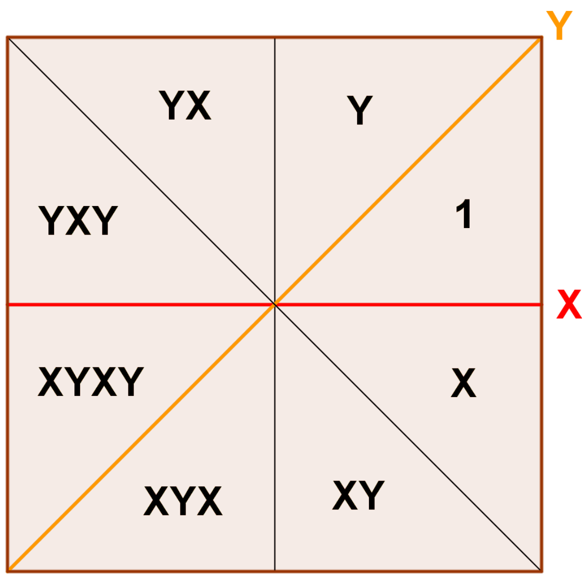
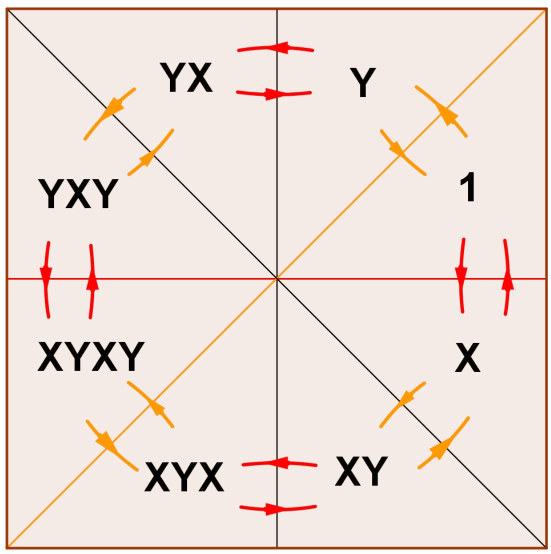
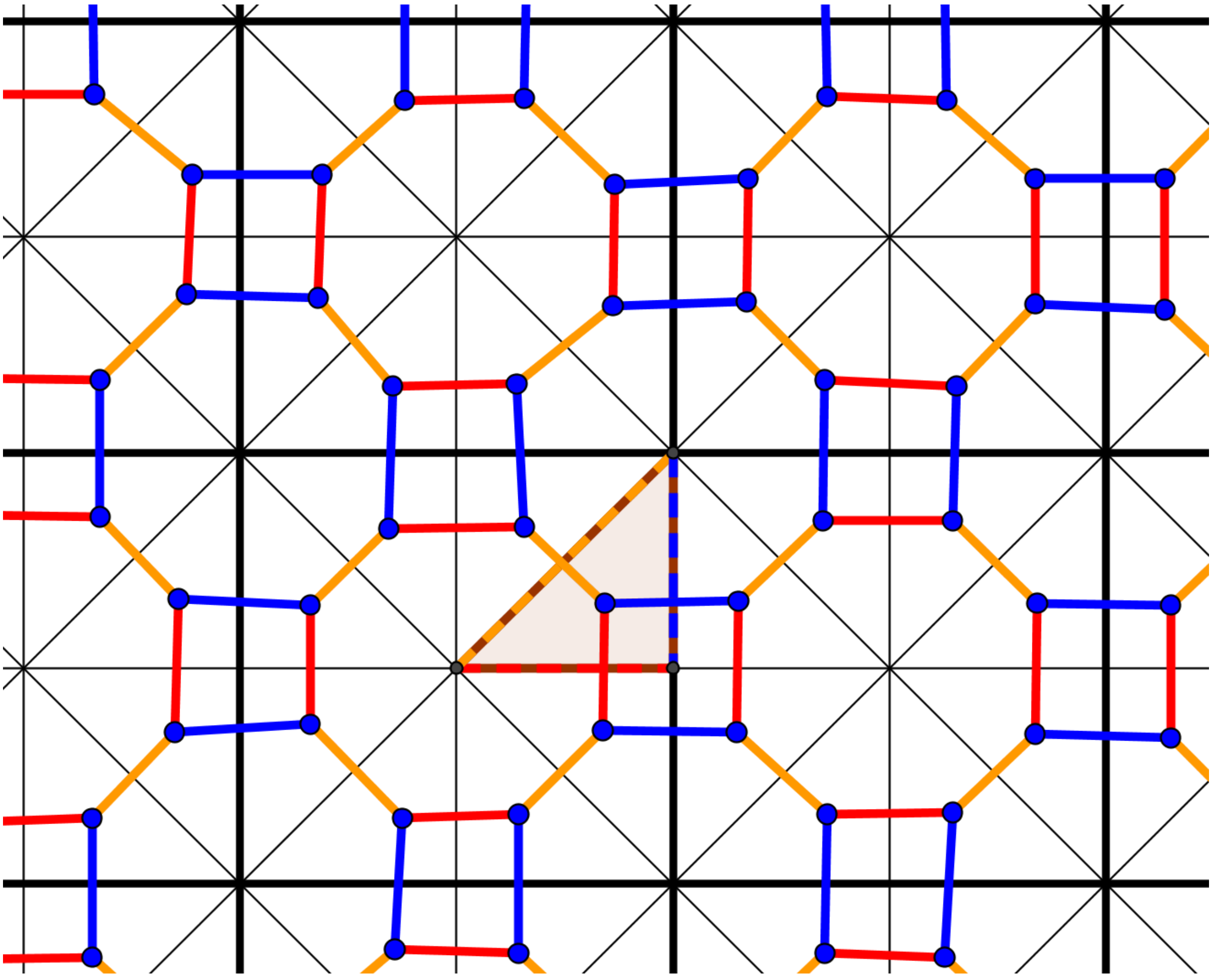




Claim: D4 = < X, Y | $X^2 = Y^2 = (XY)^4 = 1$ >

1. In the **first picture**, two particular reflections X,Y that generate D4 have been chosen.
2. Our reference fundamental domain is the one bounded by X,Y and is identified with the identity "1" in D4.
3. Each other fundamental domain is labelled by the group element that maps the reference fundamental domain to it.
    - So XYXY maps the reference fundamental domain to the one diametrically opposite. Check this - the route goes all over the place. Note that your first move is to apply a Y.
4. In the **second picture,** suppose a fundamental domain is labelled with the group element g. Then the red arrow points to the domain labelled by gX and the yellow one points to gY. *The great thing is that these are both adjacent domains.*
5. So an alternative way to see that the domain opposite 1 is XYXY is to walk round the bottom of the square writing down X for each red arrow and Y for each yellow one!
6. The picture "proves" the claim. X,Y generate D4 because there is a path following arrows from the reference fundamental domain to any other. Now we need to show that given a "word" in the generators X,Y that equals 1 in the group it can be simplified to 1 using the relations given.
    - Such a word is a path starting and finishing at 1.
    - We can remove any "doubling back" using $X^2 = Y^2 = 1$.
    - So now the path just winds n times round the circle and so can be reduced to nothing using $(XY)^4 = 1$.
7. In future for generators of order two we will just join fundamental domains with a line rather than two opposing arrows.



- Understand the whole argument given by this picture for *442 (from the notes)

- Consider the 333 example (available in class) or consider one of your own. Eg consider *2222 or ** using the squared grid or 4*2 using the “isosceles right-angle” grid.