Point groups, lattices, theorems

Q1 Every Euclidean symmetry of the plane can be written in the form $x \mapsto Ax + b$ as follows.

- Translation by b: set $A = \mathbb{I}$.
- Rotation by θ about a: see notes §1.3.4
- Reflection in a the line: set A to be reflection about a corresponding line through the origin and $b/2$ is the position vector of the closest point on the line to the origin.
- Glide in a line: construct the reflection in the line as above and then add to b the correct vector parallelto the direction of the line.

Construct explicitly a representative example of each sort.

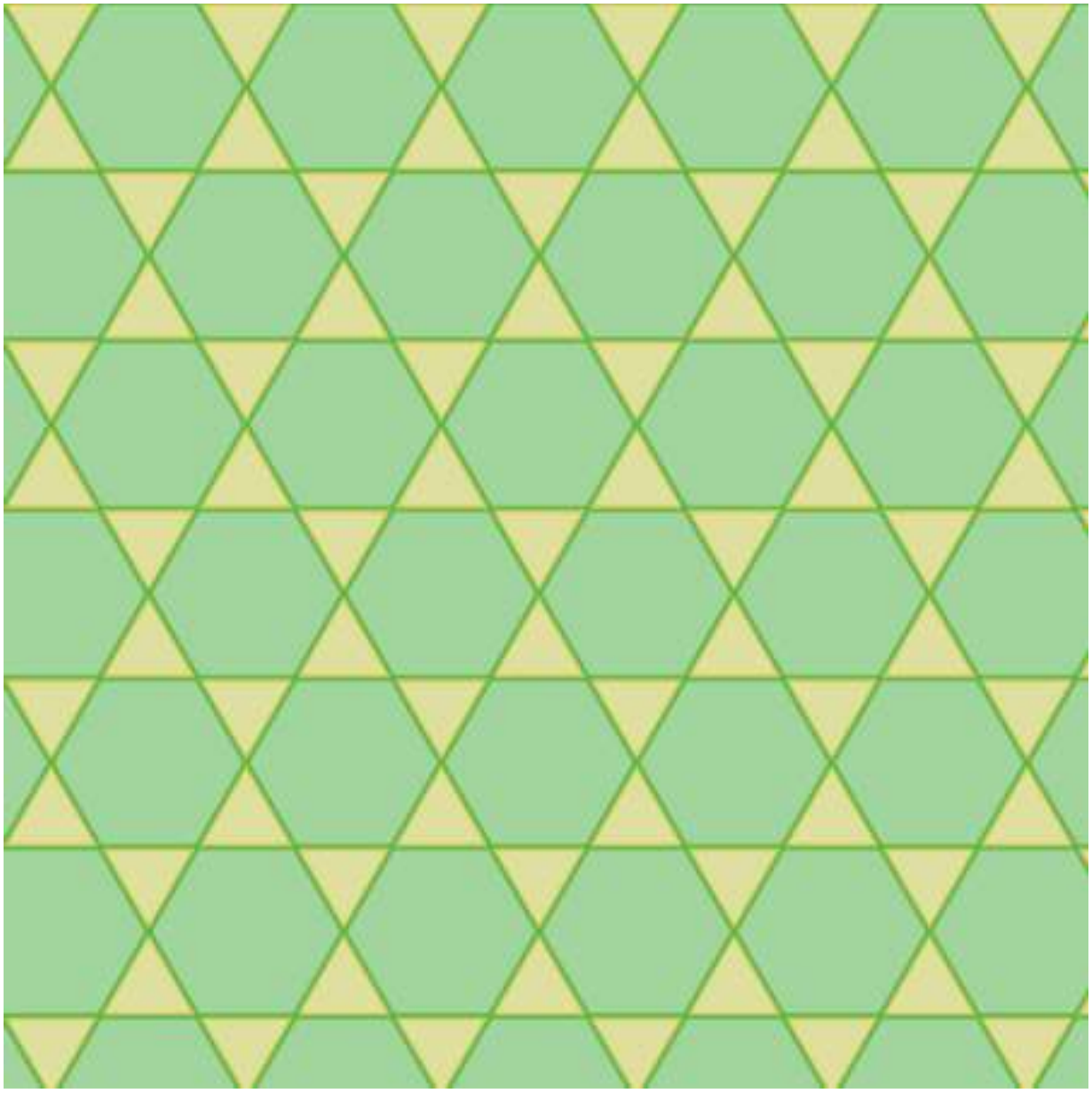
Q2 The *point group* of a wallpaper group is the set of all the A's that arise in all the symmetries of the wallpaper group. It is an "abstract" construction that does not relate to any particular place in the pattern. (You might think of it as the view from infinity: you can see there are half turns in the pattern but you are too far away to make out exactly where the centres of rotation are.)
What is the point group of each wallpaper group? (The only possibilities are the cyclic groups C_2, C_3, C_4, C_6, dihedral groups D_1, D_2, D_3, D_4, D_6 and the trivial group.) See §1.3.24 for help.

Q3 The point group of a wallpaper group has to be a subgroup (not necessarily proper) of the symmetry group of its lattice of translations. What does this tell us about the possible lattice of translation lattices of $*632$, $4*2$, 2222?

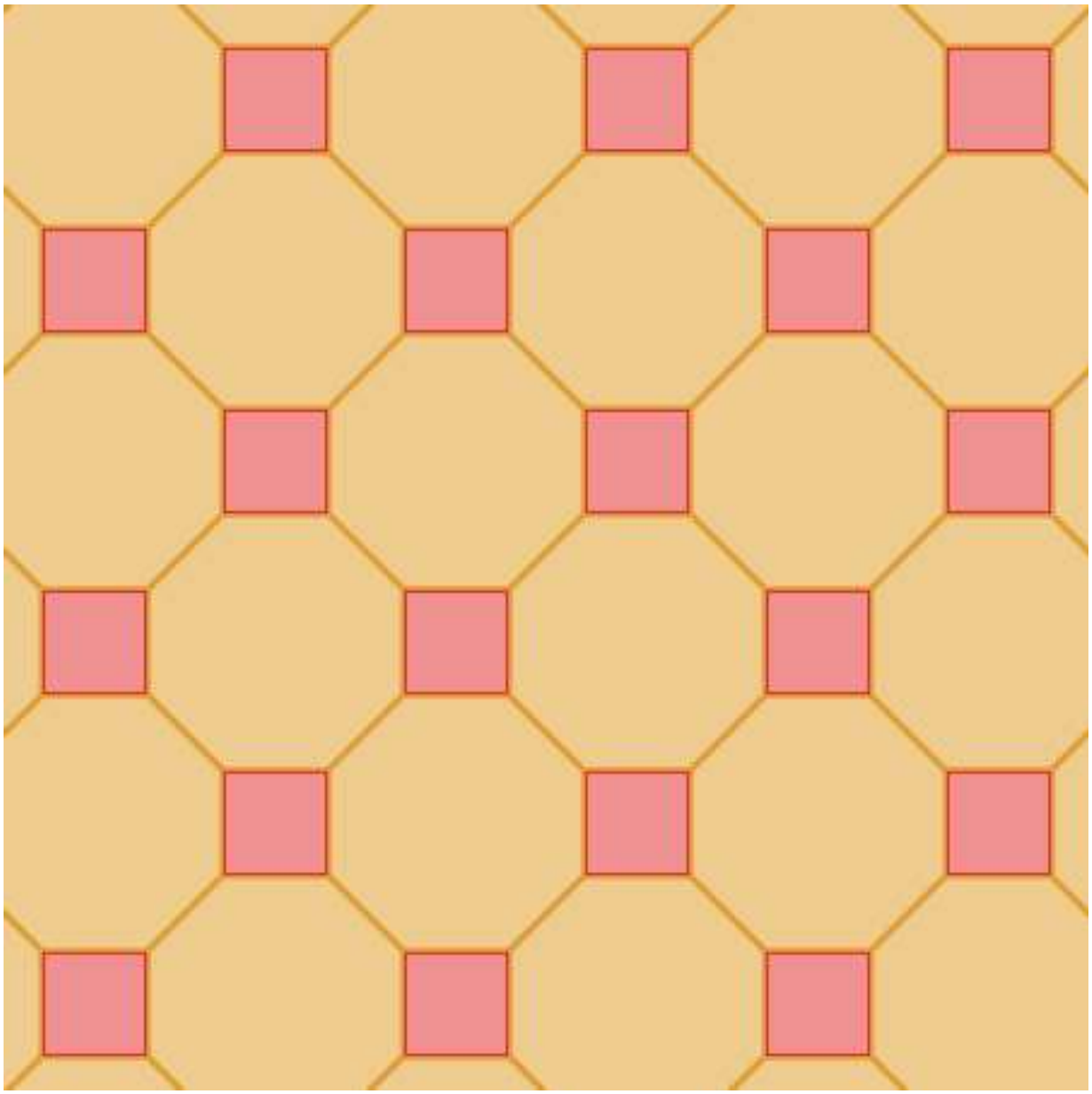
Q4 Use the point group to explain why $22*$ must have glide symmetries perpendicular to its mirrors.

Q5 Looking forward to Chapter 3, study the tilings of the plane by regular polygons provided separately. In each case, calculate the wallpaper group symmetry of the tiling. Convince yourself for each that while there are different sorts of tiles, the vertices are all the same.
For the hexadeltille and the truncated quadrille, find a fundamental domain for the wallpaper group in the tiling. Observe how the vertices and edges of the tiling appear in the fundamental domain.
Construct the "Catalan tiling" for each pattern as follows: the vertices of the Catalan tiling are at the face centres of the original; two vertices are then joined by an edge if and only if the they correspond to tiles meeting at an edge in the original. What do you notice about these new tilings?

The hexadeltille



The truncated quadrille



The snub quadrille