Honours Analysis – Summary of Prerequisites

2017/18

This course is a second course in real analysis, following on directly from the analysis component of Fundamentals of Pure Mathematics. A good understanding of the analysis component of Fundamentals of Pure Mathematics is therefore essential for Honours Analysis, and we will frequently make use of notions and results from it. You are therefore advised to thoroughly re-familiarise yourself with its highlights at an early stage.

To make this a little easier, you can use the following guide:

1. **Basics**
    - De Morgan's laws for unions and intersections of sets
    - Maps between sets: domain, range, image, injectivity, surjectivity, inverses, compositions
    - Countability and uncountability
2. **The real numbers**
    - Least upper bound or supremum axiom
    - Density of rationals and irrationals in the real numbers
3. **Sequences of real numbers**
    - Notion of convergence of a sequence
    - An increasing sequence which is bounded above converges
    - Notion of a Cauchy sequence

- Notion of a subsequence
- Bolzano–Weierstrass theorem: every bounded sequence has a convergent subsequence
- Nested interval theorem
- Understanding that a sequence of real numbers is convergent if and only if it is Cauchy
- Squeeze theorem and limits of sums, products etc. of sequences

4. **Series**

    - Notion of convergence and divergence of a series
    - $\sum a_n$ convergent implies $a_n \to 0$ – but not conversely!
    - Geometric series
    - Harmonic series and p-series
    - Cauchy criterion
    - Absolute convergence and its relation with convergence
    - Comparison tests
    - Ratio test and root test

5. **Functions of a real variable**

    - Limits
    - Squeeze theorem and limits of sums, products etc. of functions
    - Limits via sequences
    - Continuity
    - Compositions of continuous functions
    - Intermediate Value Theorem
    - Extreme Value Theorem

6. **Differentiation theory**

    - Mean value theorem
    - Taylor's theorem