# Complex Variables Skills - Hand In One

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## 1 Communication Exercise

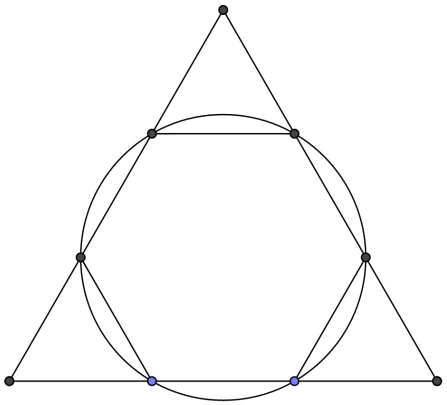

Figure 1: Try and draw this shape given only the instructions below!

- **Step 1:** Draw a circle then mark six equally-spaced points (left uncoloured so far) so that you get the vertices of a hexagon; the base of the hexagon should be horizontal.
- **Step 2:** Now we'll colour the vertices. Starting from the leftmost vertex and going in a clockwise direction colour the vertices with the following permutation (black, black, black, black, blue, blue), connect the vertices to make a regular hexagon.
- **Step 3:** Finally we wish to add three equilateral triangles whose edges have the same length as one of the hexagon's edges. Draw the first two so that they have a single black vertex and a single blue one, they should be pointing outwards away from the origin of the circle. The third triangle should have no vertices in common with the other two

and also point outwards. Colour the outermost vertices black, the final result should appear as a hexagon contained in both a circle and a large equilateral triangle.

# 2 Holomorphic

**Definition 2.1.** For any complex differentiable function $f : \mathbb{C} \to \mathbb{C}$ and $z = x + iy \in \mathbb{C}$ we can write $f(z) = u(x, y) + iv(x, y)$, then the Cauchy-Riemann equations are

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x},$$

which give us one criterion for differentiablility; if $f$ is holomorphic then these equations hold.

**Theorem 2.2.** *The Cauchy-Riemann equations allow us to see that if a function $f(z)$ is holomorphic on $\mathbb{C}$ and takes only real values then it must be constant.*

*Proof.* If a function $f : \mathbb{C} \to \mathbb{C}$ takes only real values (and hence can be restricted to $f : \mathbb{R} \to \mathbb{C}$) then we can re-write $f$ in terms of its real and imaginary parts so that $f(z) = u(x, y)$ for $z = x + iy \in \mathbb{C}$ where we've taken $v(x, y) = 0$. By the Cauchy-Riemann equations in definition 2.1 we have that $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 0$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} = 0$ since $v(x, y) = 0$, then integrating both equations we just get $u(x, y) = c_y + \varphi_x(x) = c_x + \varphi_y(y)$, and so $u(x, y) = c_x + c_y$ which is a constant. □

# 3 Micro-writing

**Definition 3.1** (Plagiarism and Collusion)**.** Plagiarism is defined by the Oxford English dictionary as "The practice of taking someone else's work or ideas and passing them off as one's own [1]" though in terms of university work, unintentionally doing so is still considered plagiarism. Should there be no intentions of cheating, and if it's the first offence that the student has made, then it is most common that the student will not receive a penalty on their grade [2]. However if there was a deliberate intention of cheating, the student may lose between 10% and 30% of their marks or, if it is not their first offence, they may face a 100% reduction of marks and possibly - depending on the severity of the case - will be refereed to the Student Discipline Officers for further punishment. In the case of two students colluding the penalty mark may be shared between them.

# References

[1] Angus Stevenson. *Oxford Dictionary of English*. Oxford University Press, 2010

[2] Charlotte Matheson, *Academic Misconduct Investigation Procedures* Edinburgh University, [ed.ac.uk/academic-services/students/conduct/academic-misconduct/academic-misconduct-procedure]