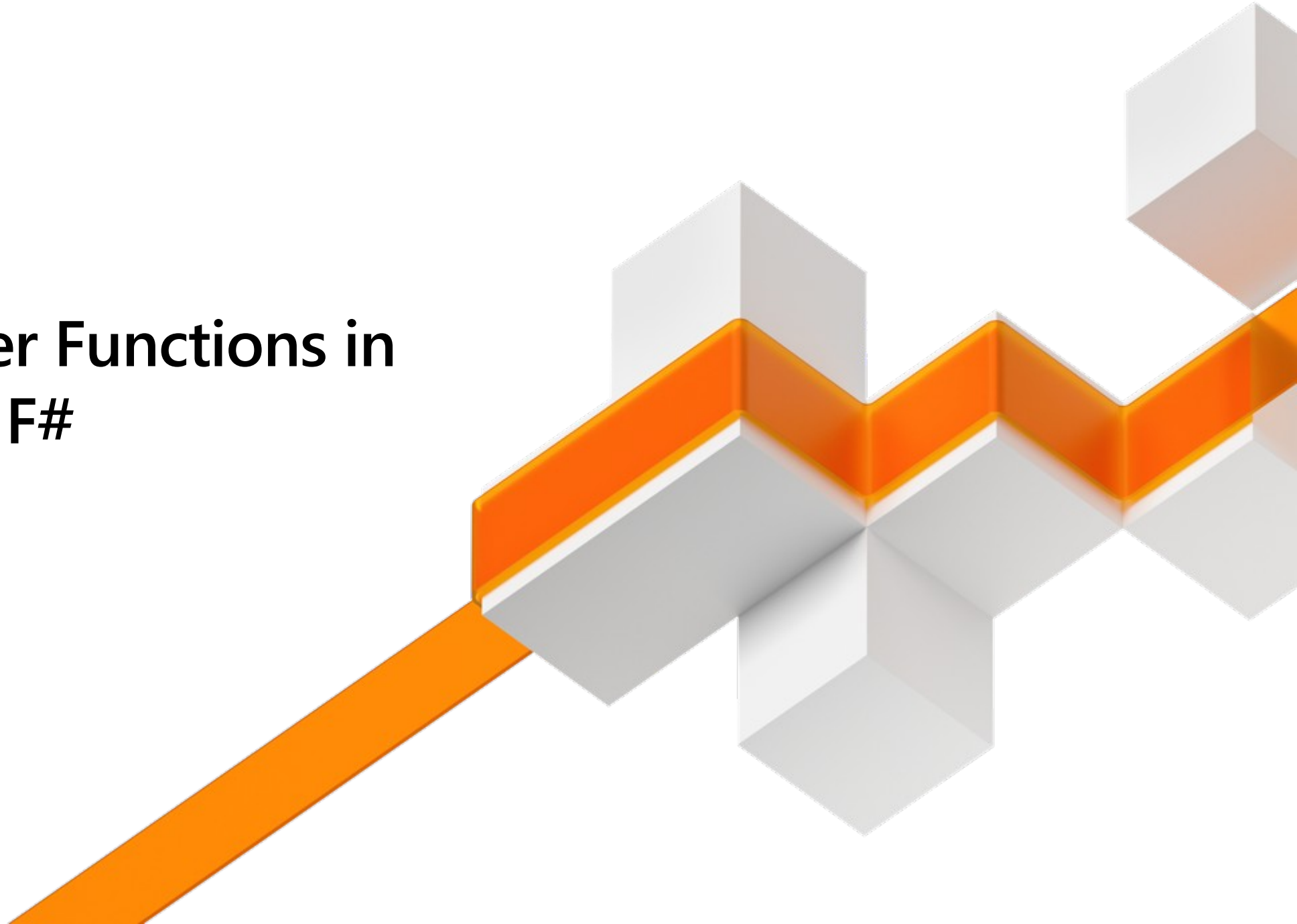
Microsoft

Microsoft Ignite
The Tour

Microsoft

Higher Order Functions in Python and F#

Chang Hai Bin
Data Engineer



Remark

Techniques from F#

About speaker

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Data Engineer (Hedge Fund)

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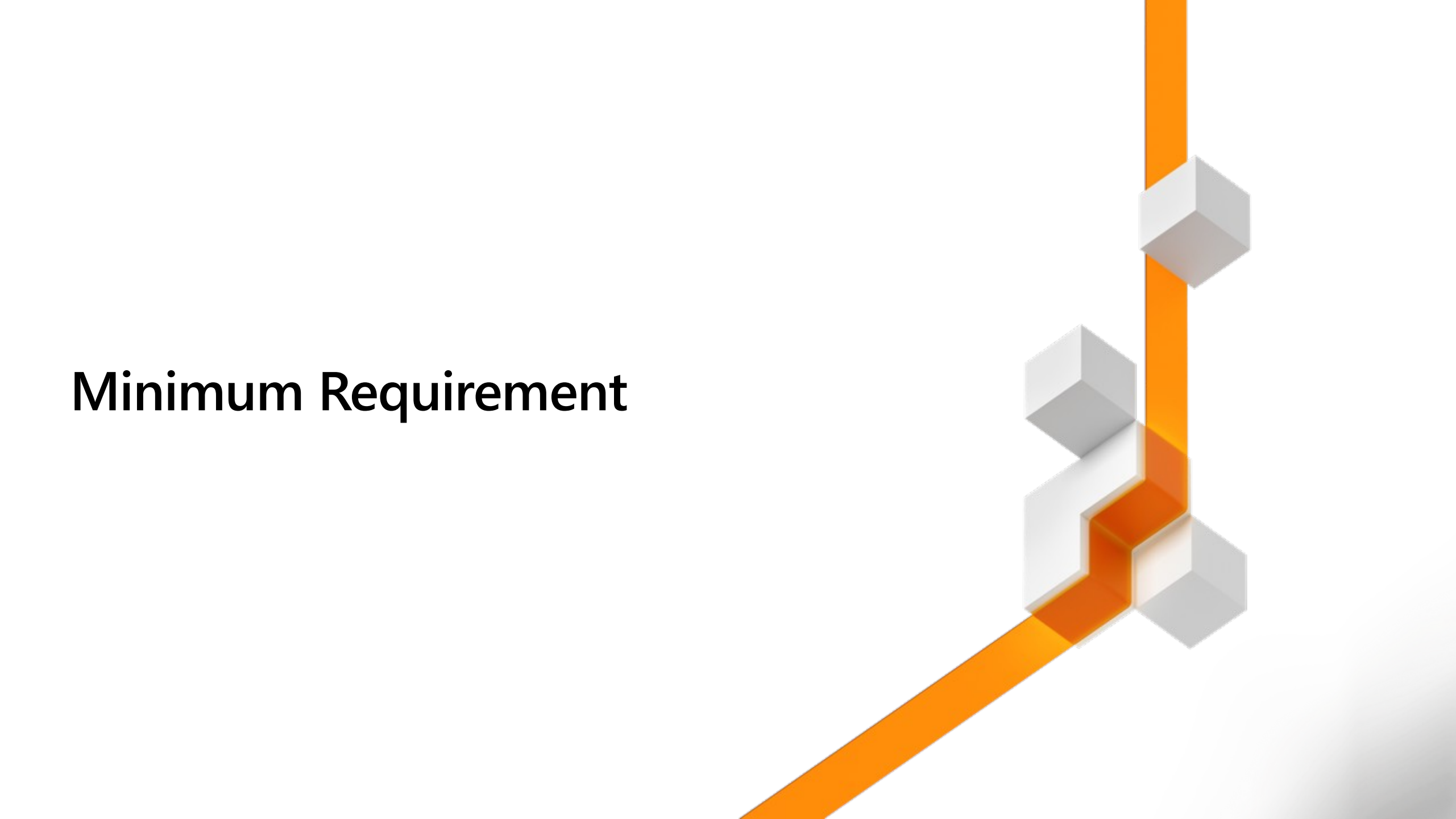
Business Analyst (U.S. Mattress)

Minimum Requirement



Function

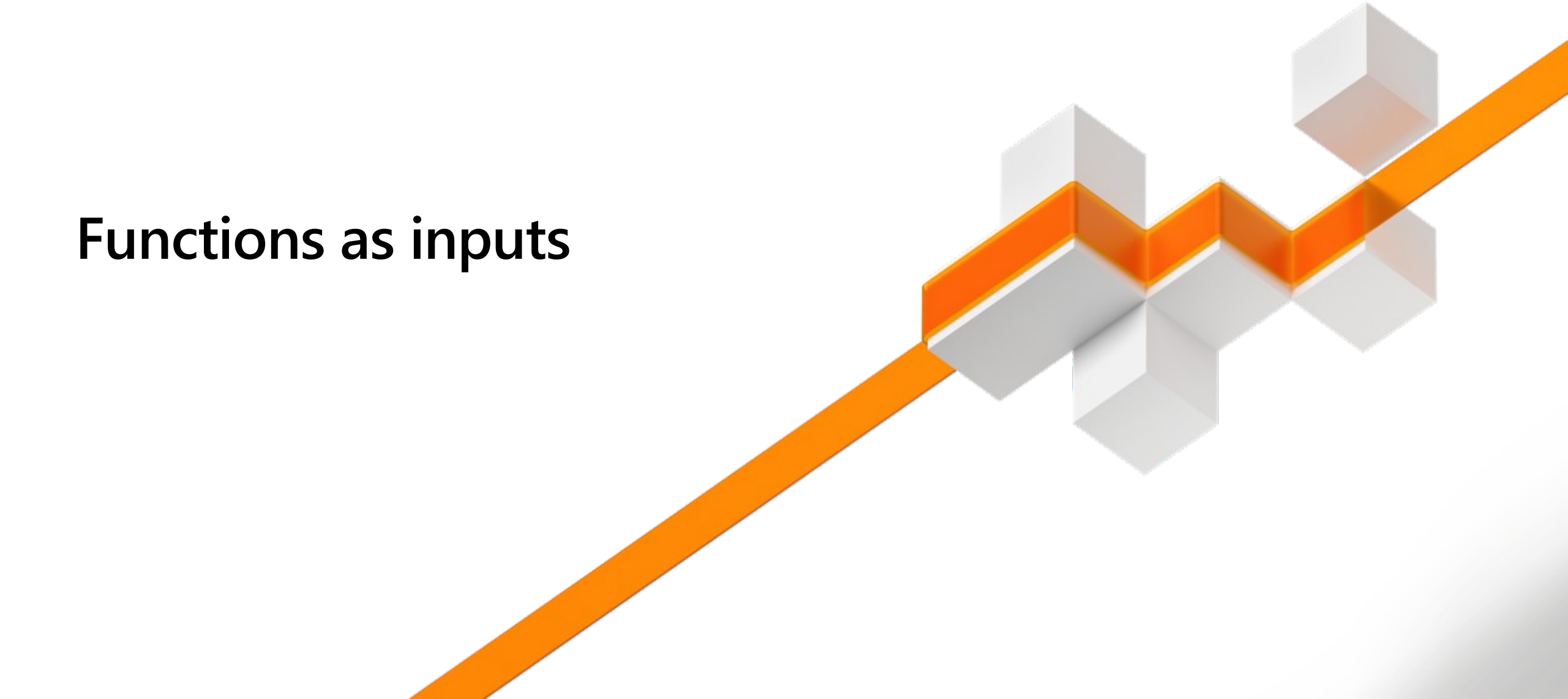
Function is a machine that **takes input(s)**, and **returns an output**

3 Concepts

3 concepts

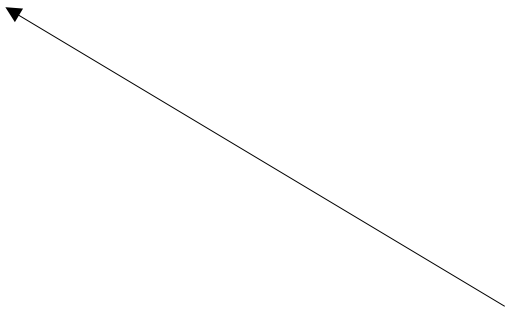
- Functions as inputs
- Functions as output
- Partial Application

Functions as inputs



```
def f(x,y,z):
    return x + y + z
```

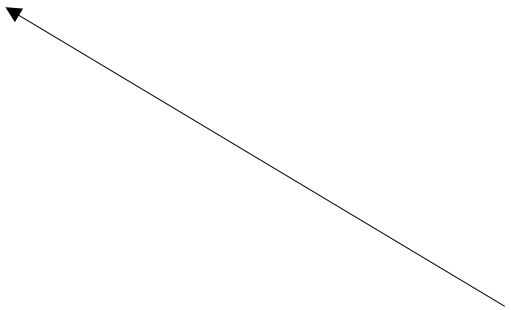
```
def f(x,y,z):
```



Most of the time, x, y, z are simple data types

e.g. string, int, float, date, List, Set, Dictionary, etc.

```
def f(x,y,z):
```



Most of the time, x, y, z are simple data types

But they can actually be functions as well!

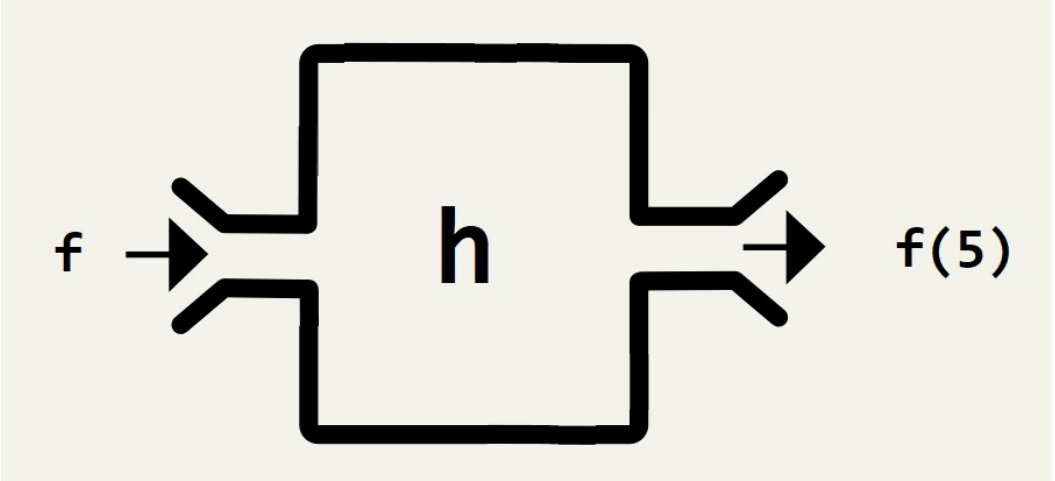

Cöffëë

Example

```
def h(f):
    return f(5)
```

Example

```
def h(f):
    return f(5)
```



h is a bigger function that:

- Accepts a smaller function f
- Returns a value f(5)

Example 1

```
def h(f):
    return f(5)

def g(x):
    return x + 1
```

Example 1

```
def h(f):
    return f(5)

def g(x):
    return x + 1
```

Then h(g) = 6

Example 2

```
def h(f):
    return f(5)

def k(x):
    return x * 100
```

Example 2

```
def h(f):
    return f(5)

def k(x):
    return x * 100
```

Then h(k) = 500

How is this useful?

Newton's Method

Newton’s method

Newton’s method helps you find the (approximate) solution of a function.

It is available in “scipy” library.

Newton's method

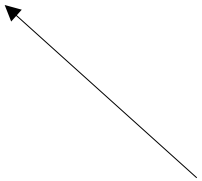
- $f(x) = x^2 - 3$
- $f(x) = 0$ when $x = \sqrt{3} \approx 1.732$

Newton's method

```
from scipy import optimize

def f(x):
    return x * x - 3

solution = optimize.newton(f,5)


print(solution)

# 1.7320508075688772
```

Newton's method

```
from scipy import optimize

def f(x):
    return x * x - 3

solution = optimize.newton(f,5)


print(solution)

# 1.7320508075688772
```



Smaller function "f" accepted by a bigger function!

Designing Insurance Product

Pricing Insurance Product

How much should I charge for insurance?

```
def price():
    ....
```

Pricing Insurance Product

```
e.g. Depends on age


def price(age):
    ....



age: int
```

Pricing Insurance Product

e.g. Depends on probability of injury

```
def price(age, prob):
    ....
```

```
age: int
prob: float
```

Pricing Insurance Product

What if prob depends on time?

```
def price(age, prob):
    ....
```

age: int

prob: ???

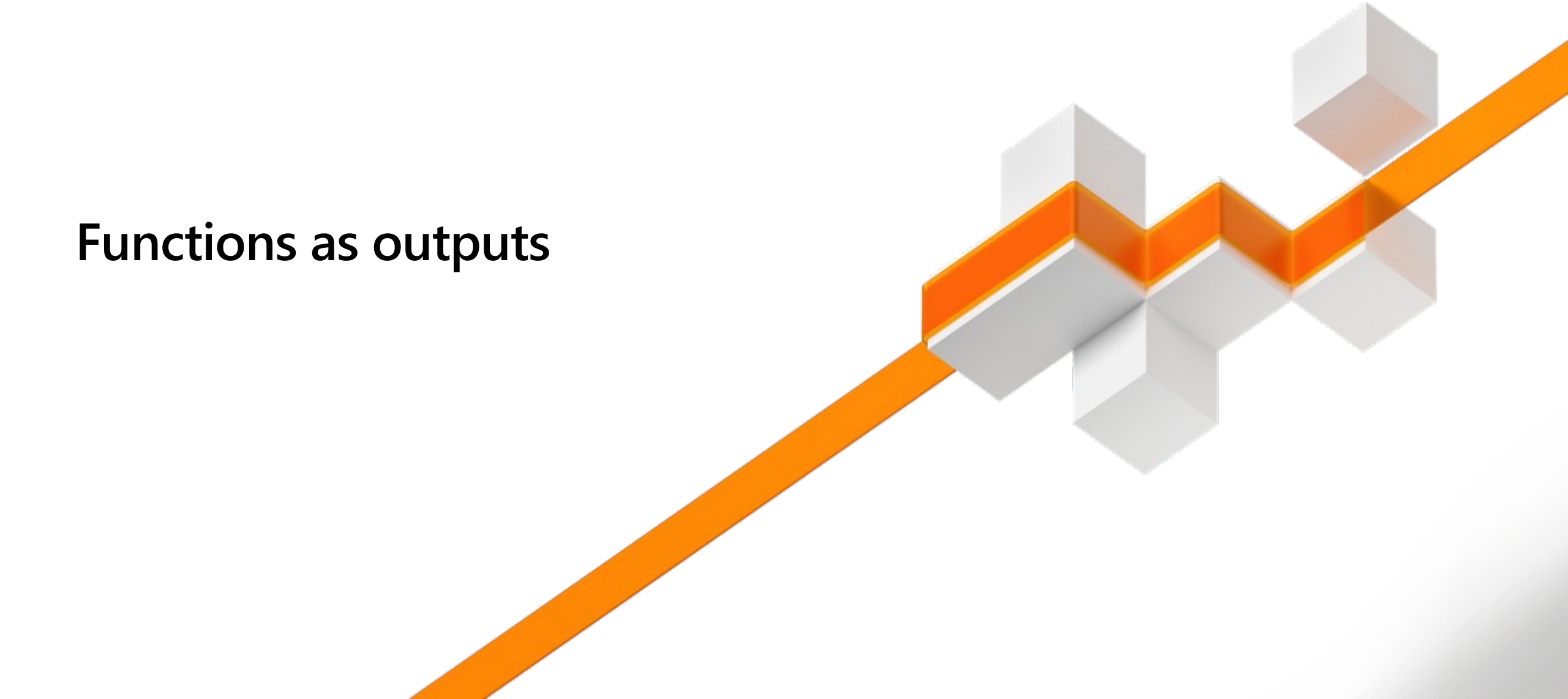
Pricing Insurance Product

Input a function instead!

```
def price(age, probFunc):
    ....
```

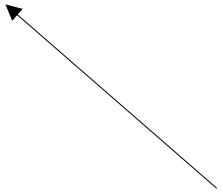
```
age: int
probFunc: datetime -> float
```

Functions as outputs



```
def f(x,y,z):
    return X + y + z
```

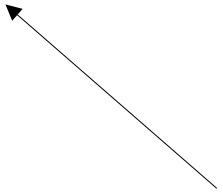
```
def f(x,y,z):
    return ......
```



Most of the time, we also return simple data types

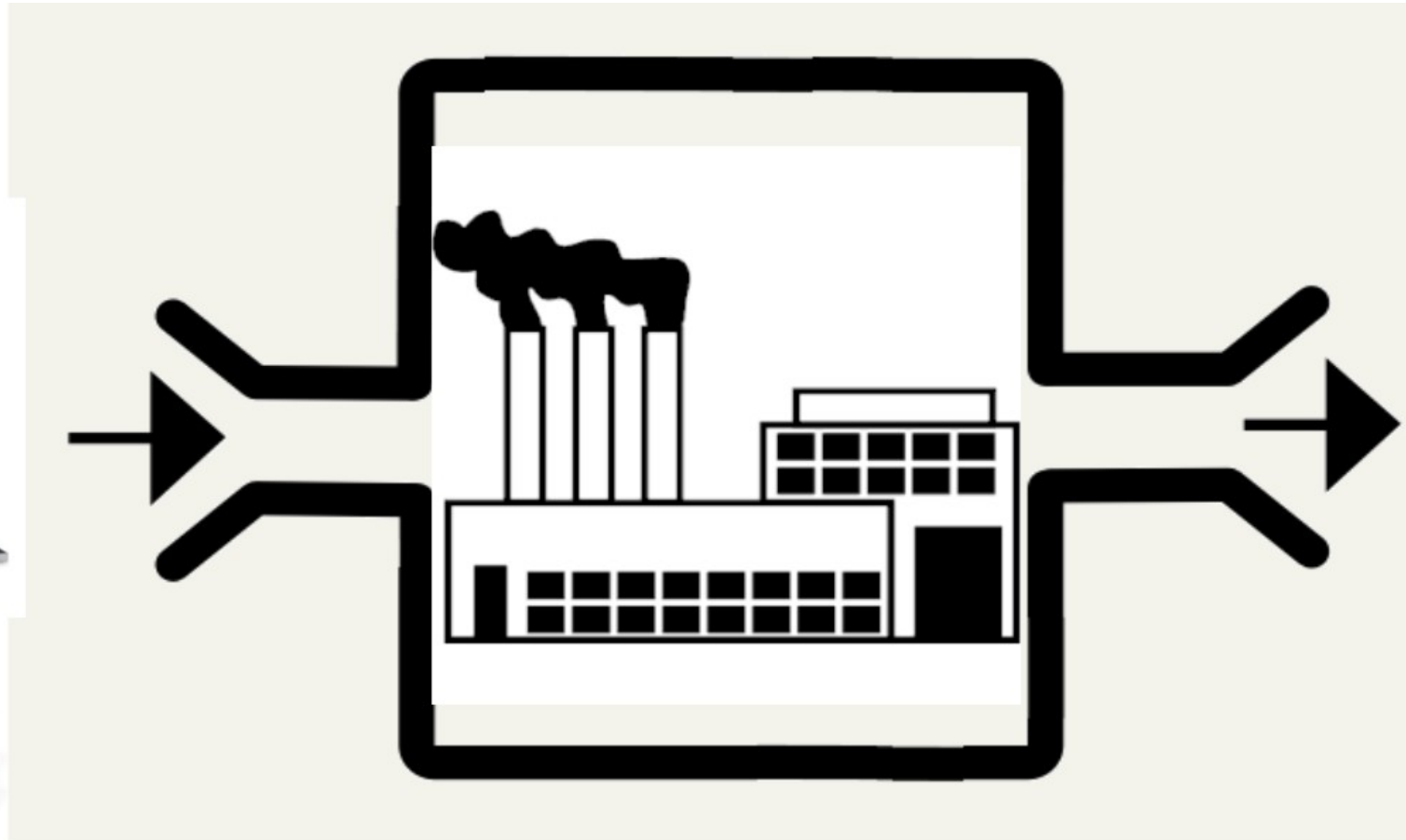
e.g. string, int, float, date, List, Set, Dictionary, etc.

```
def f(x,y,z):
    return ......
```

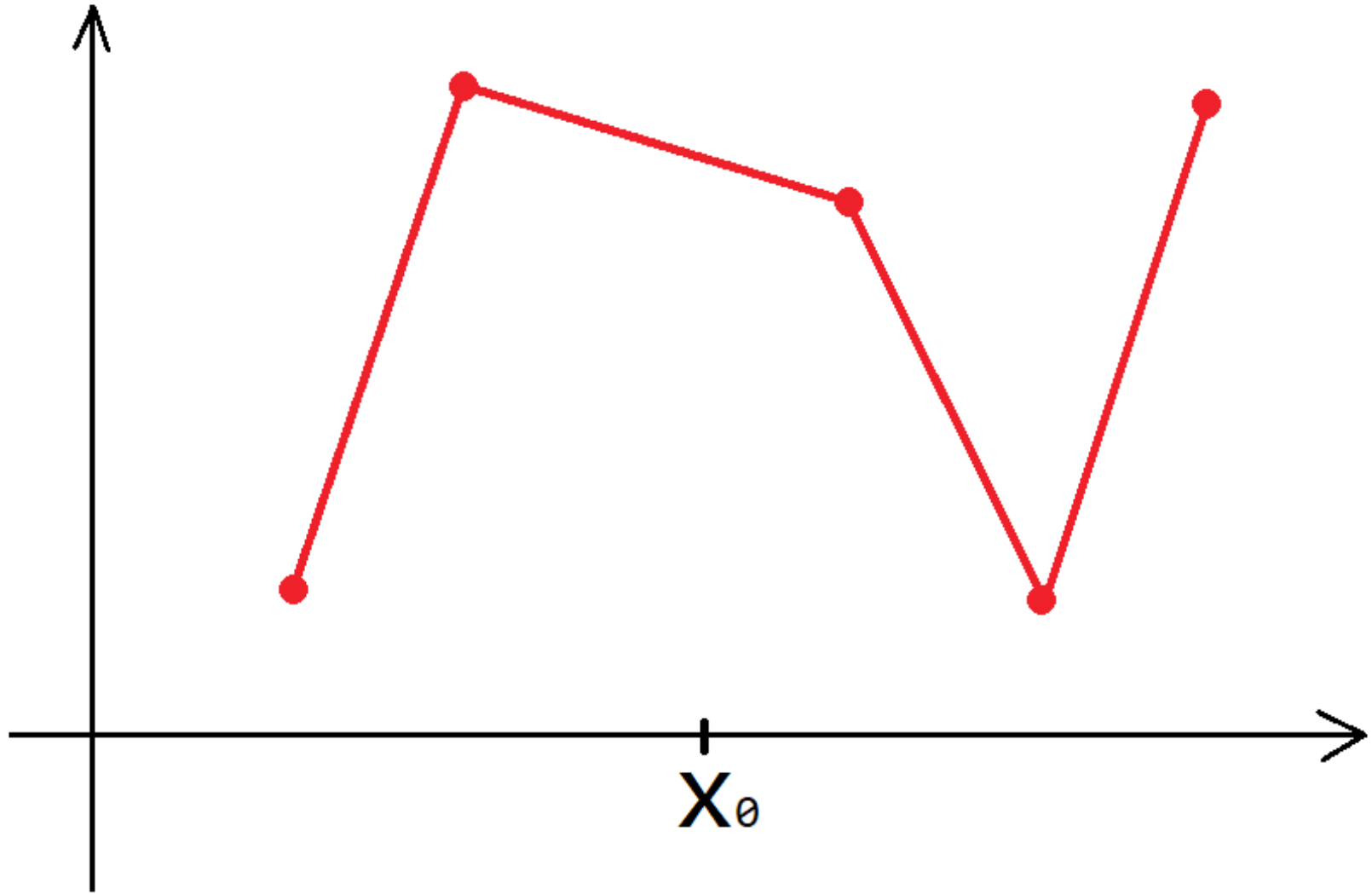


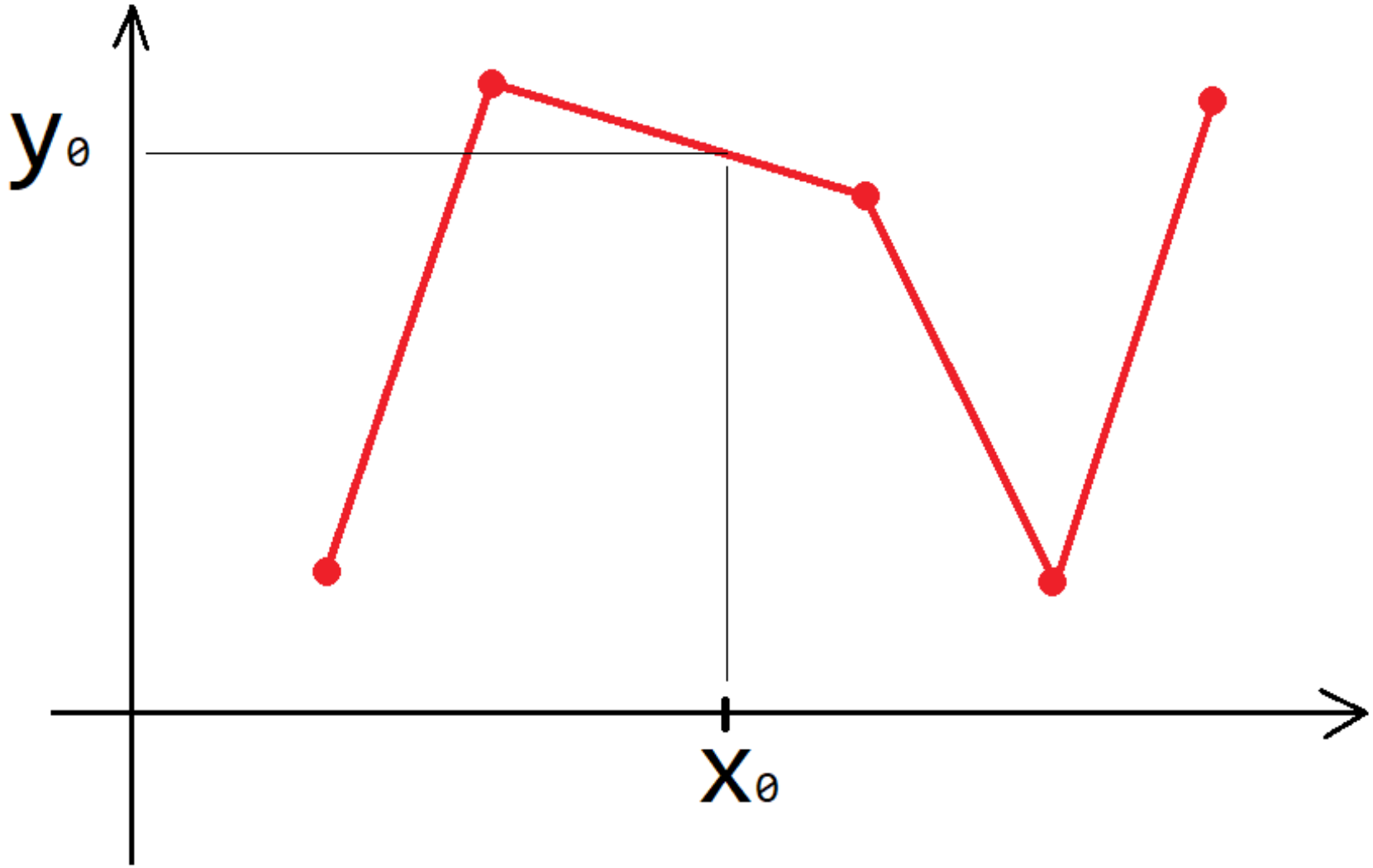
Most of the time, we also return simple data types

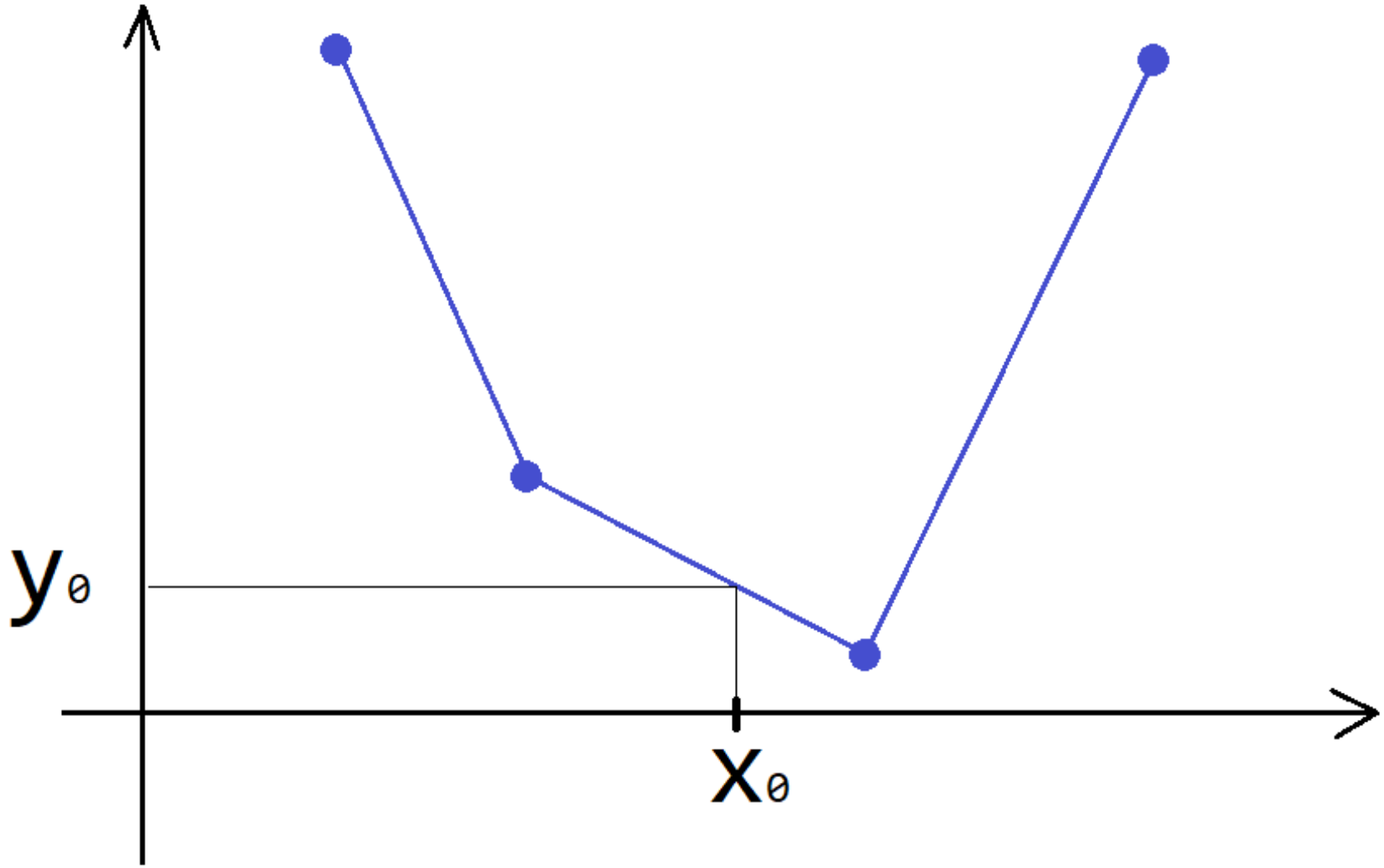
Again, you can also return a function!



Interpolation


X_0


y0
x0


y0
x0

Interpolate

Interpolate function depends on:

- Original Dataset
- Value being queried x_0

Implementation 1

```
def interpolate(dataset, x0):
    .......
```

```
dataset = [(-1,4),(1,7),(5,3)]
```

Implementation 2

```
class Interpolate:
    def __init__(self,dataset):
        ......

    def get_value(self,x0):
        .......
```

Implementation 2

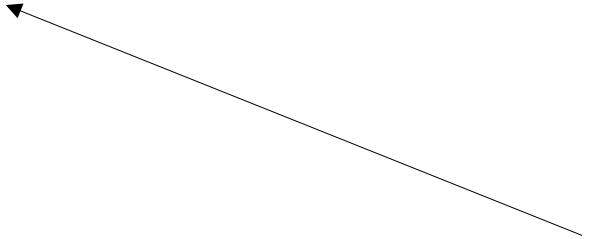
```
class Interpolate:
    def __init__(self,dataset):
    def get_value(self,x0):


dataset = [(-1,4),(1,7),(5,3)]


inter_obj = Interpolate(dataset)
result = inter_obj.get_value(2)
```

Implementation 3

```
def interpolate(dataset):
    def get_value(x0):
        ......
    return get_value
```

Implementation 3

```
def interpolate(dataset):
    def get_value(x0):
        ......
    return get_value
```



“get_value” is a smaller function that is returned by a bigger function “interpolate” !

Implementation 3

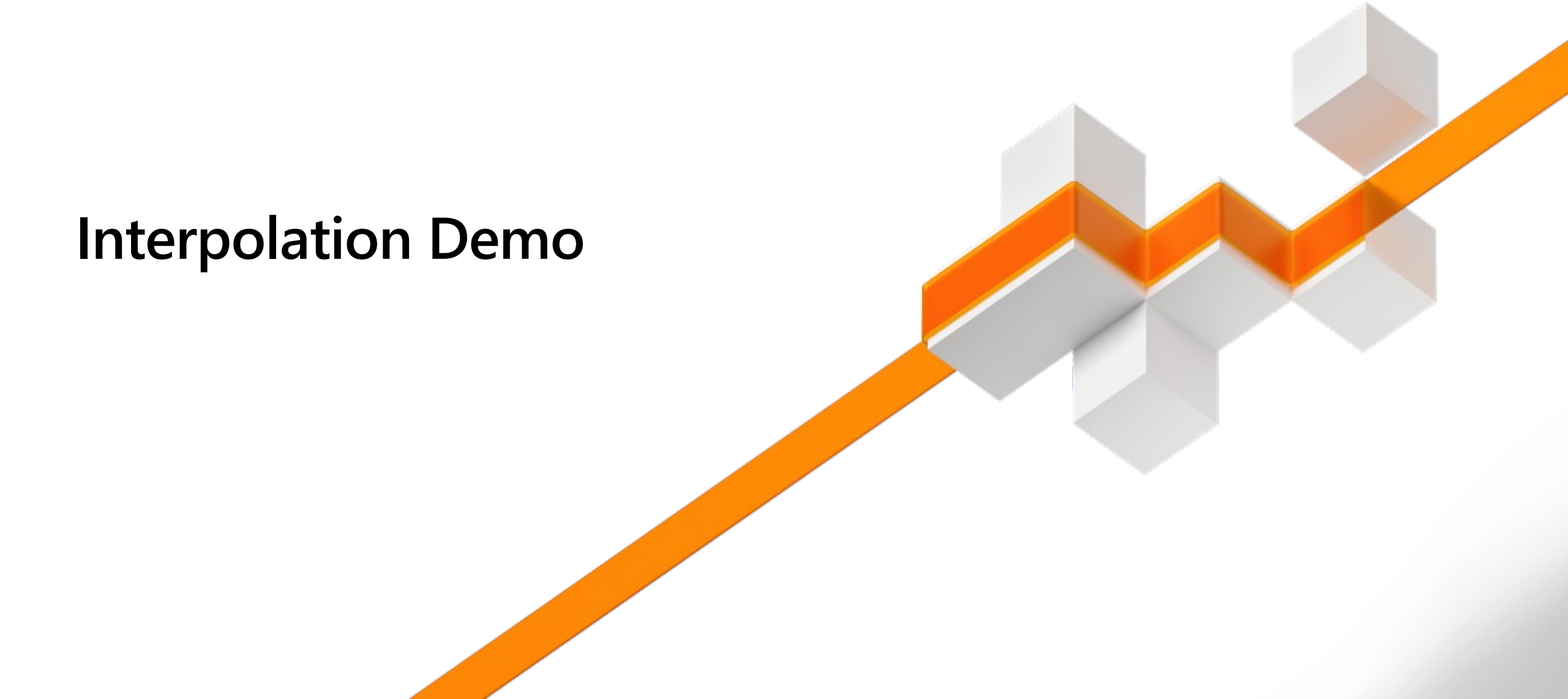
```
def interpolate(dataset):
    def get_value(x0):
        ......
    return get_value


dataset = [(-1,4),(1,7),(5,3)]


inner_func = interpolate(dataset)
result = inner_func(2)
```

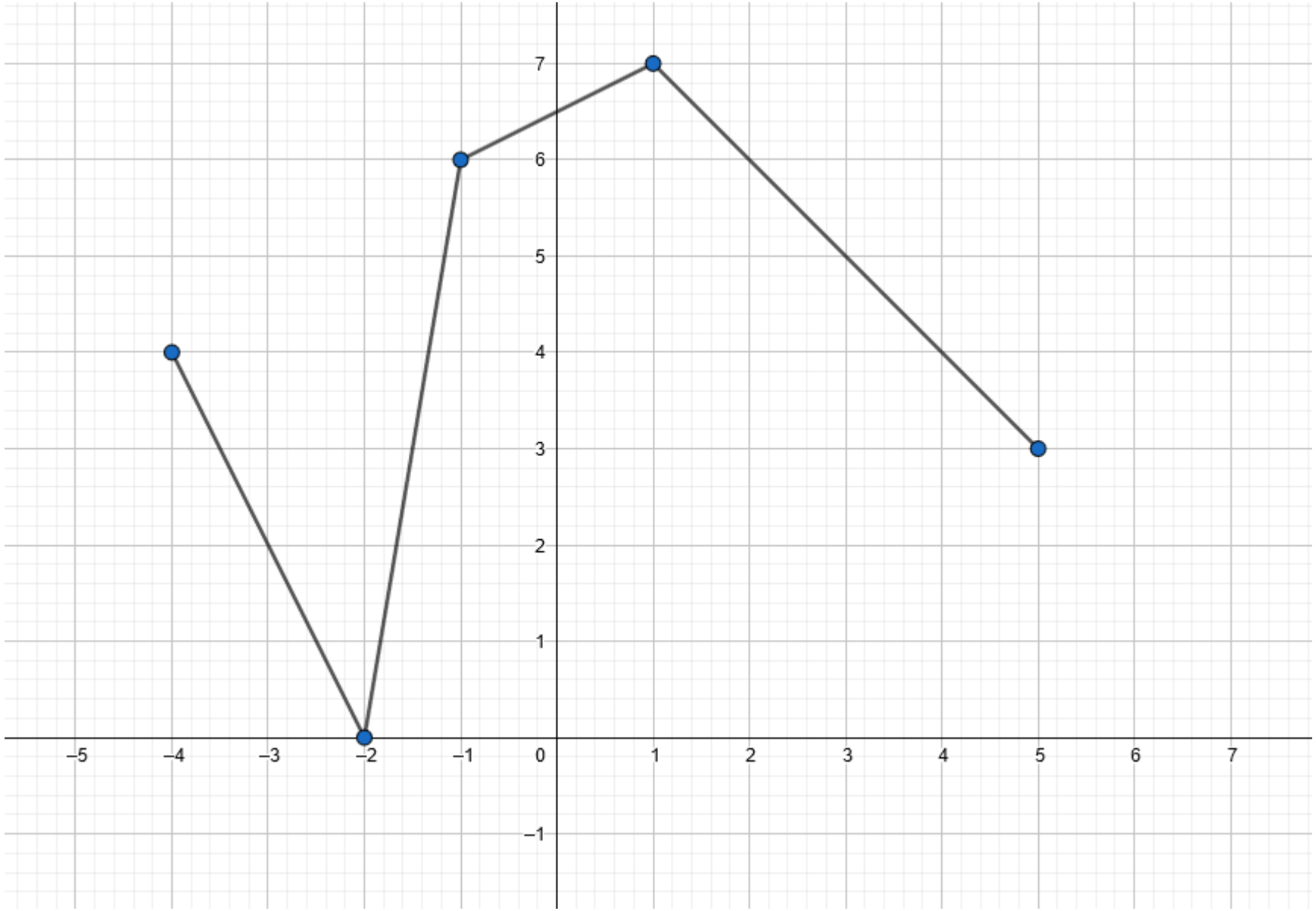
Directly use inner_func! No need to find the method hidden inside an object.

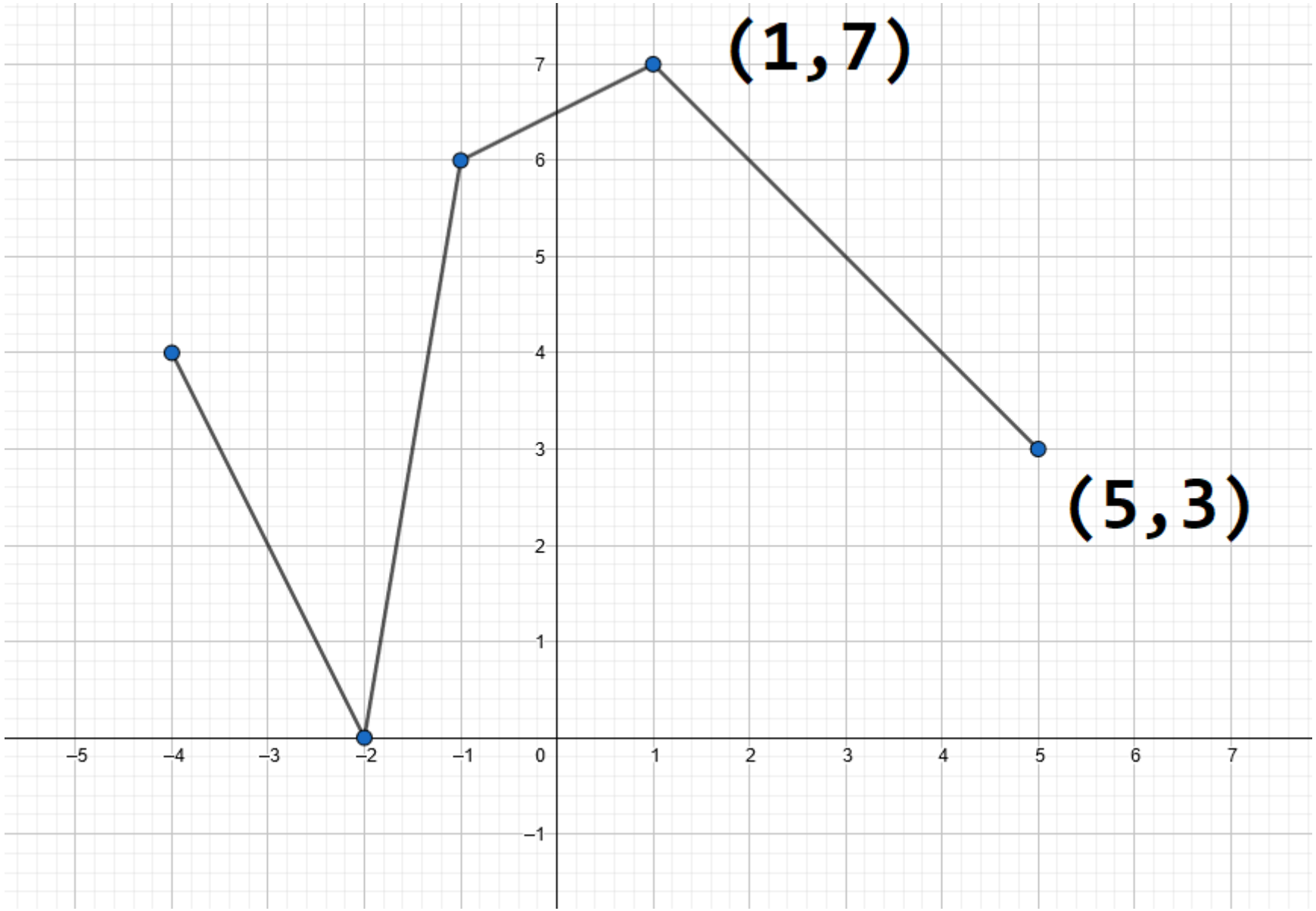
Implementation 3

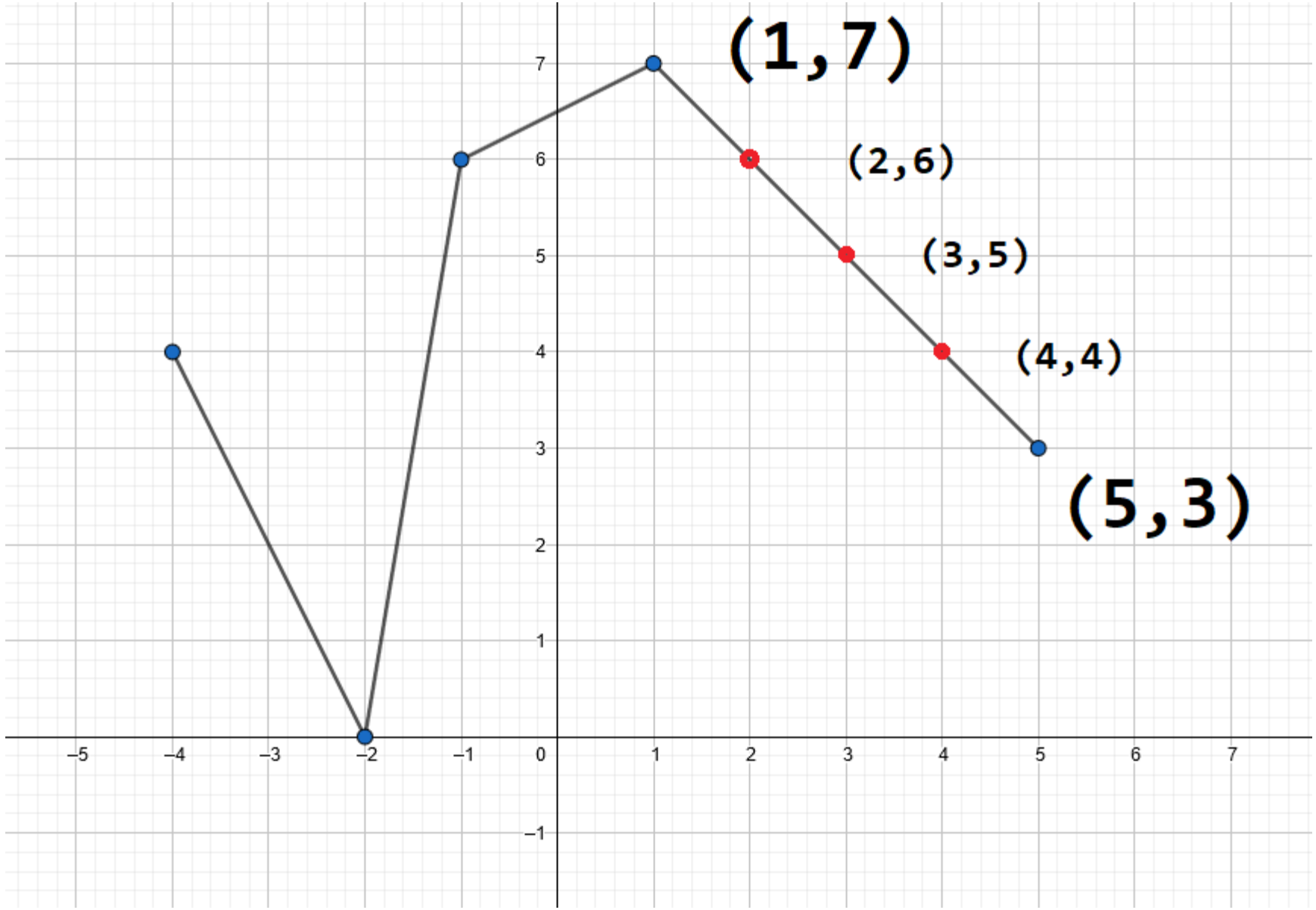
```
def interpolate(dataset):
    def get_value(x0):
        ......
    return get_value

dataset = [(-1,4),(1,7),(5,3)]

inner_func = interpolate(dataset)
result = inner_func(2)
```

Interpolation Demo




7
6
5
4
3
2
1
−1
−5
−4
−3
−2
−1
0
1
2
3
4
5
6
7


(1,7)
(5,3)
7
6
5
4
3
2
1
0
−1
−5
−4
−3
−2
−1
1
2
3
4
5
6
7


(1,7)
(2,6)
(3,5)
(4,4)
(5,3)
7
6
5
4
3
2
1
0
−1
−5
−4
−3
−2
−1
1
2
3
4
5
6
7

Partial Application

Analogy

- If a function/ machine

  - Needs 3 inputs
  - But only 2 inputs provided

  - Still needs additional 1 inputs.

Analogy

If a function/ machine

- Needs 3 inputs
- But only 2 inputs provided

- Becomes a brand new function/machine that needs 1 inputs.

Example

```
def f(x,y,z):
    return x + y + z
```

Example

```
def f(x,y,z):
    return x + y + z


result = f(1,2,3)



# result = 6
```

Missing Variable

```
def f(x,y,z):
    return x + y + z


result = f(1,2)



# TypeError: f() missing 1 required positional argument:
'z'
```

Lambda Definition

```
f = lambda x: lambda y: lambda z: \
        x + y + z
```

Lambda Definition

```
f = lambda x: lambda y: lambda z: \
        x + y + z

result = f(1)(2)(3)



# result = 6
```

Lambda Definition

```
f = lambda x: lambda y: lambda z: \
        x + y + z


result = f(1)(2)
```

```
# <function <lambda>.<locals>.<lambda>.<locals>.<lambda>
at 0x013737C8>
```

```
Valid code!
```

Lambda Definition

```
f = lambda x: lambda y: lambda z: \
        x + y + z


f : X -> Y -> Z -> result
```

Lambda Definition

```
f = lambda x: lambda y: lambda z: \
        x + y + z


f : X -> Y -> Z -> result
f(x)  -> Y -> Z -> result
```

Lambda Definition

```
f = lambda x: lambda y: lambda z: \
        x + y + z


f : X -> Y -> Z -> result
f(x)  -> Y -> Z -> result
f(x)(y)    -> Z -> result
```

Lambda Definition

```
f = lambda x: lambda y: lambda z: \
        x + y + z


f : X -> Y -> Z -> result
f(x)  -> Y -> Z -> result
f(x)(y)    -> Z -> result
f(x)(y)(z)      -> result
```

Lambda Definition

```
f = lambda x: lambda y: lambda z: \
        x + y + z


def f(x):
    def inner_1(y):




    return inner_1
```

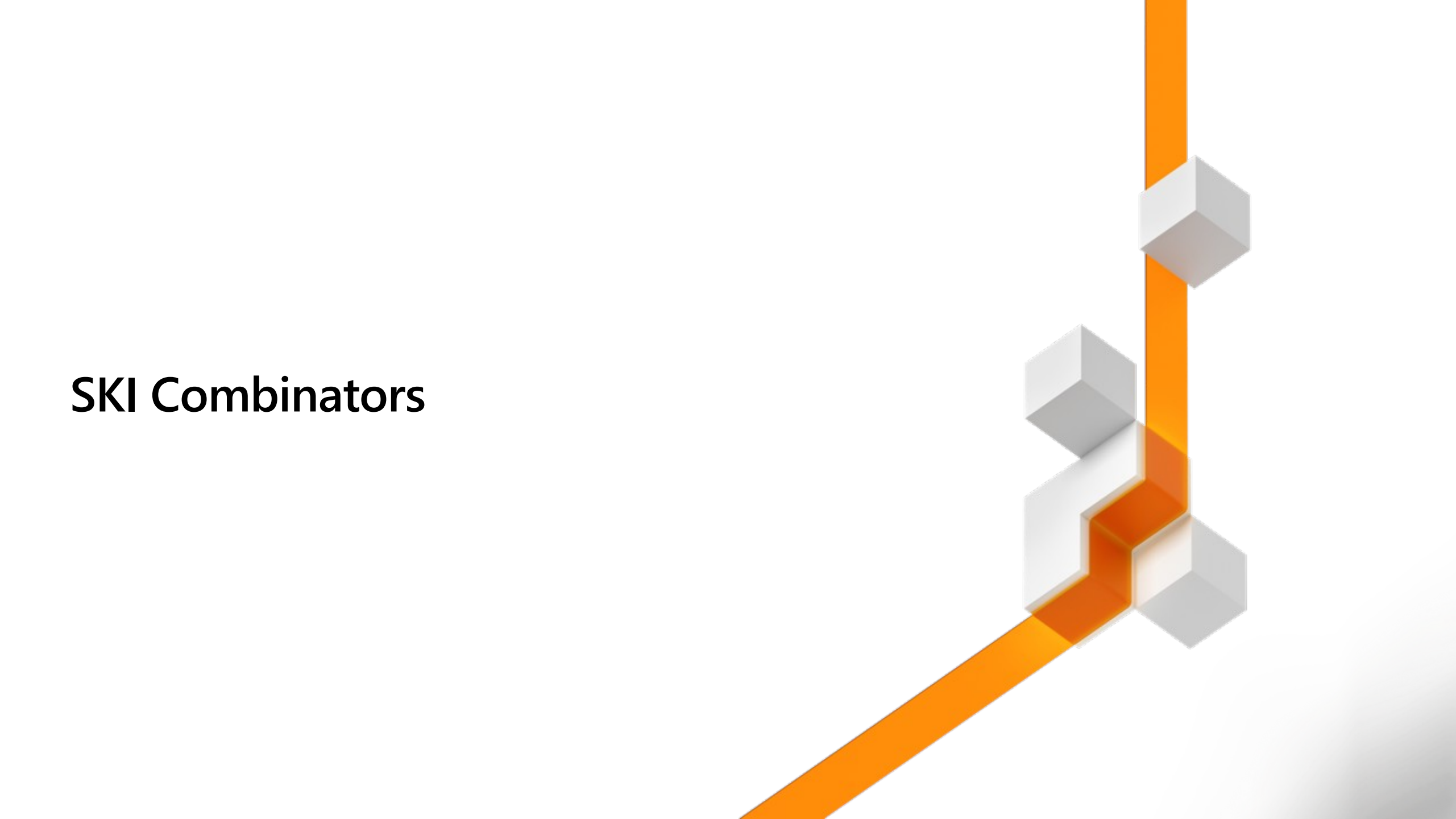
Lambda Definition

```
f = lambda x: lambda y: lambda z: \
        x + y + z


def f(x):
    def inner_1(y):
        def inner_2(z):

            return inner_2
    return inner_1
```

Lambda Definition

```
f = lambda x: lambda y: lambda z: \
        x + y + z


def f(x):
    def inner_1(y):
        def inner_2(z):
            return x + y + z
        return inner_2
    return inner_1
```

SKI Combinators



Lambda Calculus

```
def B(x,y,z):
    return x(y(z))


def D(w,x,y,z):
    return w(x,y(z))


def O(x,y):
    return y(x(y))
```

Lambda Calculus

```
def B(x,y,z):
    return x(y(z))


def D(w,x,y,z):
    return w(x,y(z))


def O(x,y):
    return y(x(y))
```

Are these valid expressions?

What are the dependencies between w,x,y,z?

SKI

```
def S(x,y,z):
    return x(z,y(z))

def K(x,y):
    return x

def I(x):
    return x
```

SKI

```
S = lambda x: lambda y: lambda z: \
    x(z, y(z))          **

K = lambda x: lambda y:
    x

I = lambda x:
    x
```

**Written in easier form

SKI

You can express any lambda expression using only S, K, I.

```
S = lambda x: lambda y: lambda z: \
    x(z, y(z))                **
K = lambda x: lambda y:
    x
I = lambda x:
    x
```

**Written in easier form

Number Game Analogy

Given a number:

1. Take the first digit
2. Multiply by the rest of the digits
3. Repeat

927583 -> **9** x **27583** -> 248247

Number Game Analogy

927583 -> 9 x 27583 -> 248247

Number Game Analogy

927583 -> 9 x 27583 -> 248247

248247 -> 2 x 48247 -> 96494

Number Game Analogy

927583 -> 9 x 27583 -> 248247

248247 -> 2 x 48247 -> 96494

96494 -> 9 x 6494 -> 58446

Number Game Analogy

927583 -> 9 x 27583 -> 248247

248247 -> 2 x 48247 -> 96494

96494 -> 9 x 6494 -> 58446

58446 -> 5 x 8446 -> 42230

Example

```
T(x,y) = y(x)
```

Example

```
T(x,y) = y(x)

Goal: T = S(K(S(I)))(K)
```

Example

```
T(x,y) = y(x)
```

```
Goal: T = S(K(S(I)))(K)
```

```
T(x) = S(K(S(I)))(K)(x)
```

Example

```
T(x,y) = y(x)
```

```
Goal: T = S(K(S(I)))(K)
```

```
T(x) = S(K(S(I)))(K)(x)
     = K(S(I))(x)[K(x)]
```

```
Because S(x,y,z) = x(z)(y(z))
```

Example

```
T(x,y) = y(x)
Goal: T = S(K(S(I)))(K)


T(x) = S(K(S(I)))(K)(x)
     = K(S(I))(x)[K(x)]
     = S(I)[K(x)]


Because K(x,y) = x
```

Example

```
T(x,y) = y(x)
```

```
Goal: T = S(K(S(I)))(K)
```

```
T(x) = ... = S(I)[K(x)]
```

```
T(x)(y) = S(I)[K(x)](y)
```

Example

```
T(x,y) = y(x)

Goal: T = S(K(S(I)))(K)


T(x) = ... = S(I)[K(x)]

T(x)(y) = S(I)[K(x)](y)
        = I(y)[K(x)(y)]


Because S(x,y,z) = x(z)(y(z))
```

Example

```
T(x,y) = y(x)
Goal: T = S(K(S(I)))(K)


T(x) = ... = S(I)[K(x)]
T(x)(y) = S(I)[K(x)](y)
        = I(y)[K(x)(y)]
        = y[K(x)(y)]
Because I(x) = x
```

Example

```
T(x,y) = y(x)

Goal: T = S(K(S(I)))(K)


T(x) = ... = S(I)[K(x)]

T(x)(y) = S(I)[K(x)](y)
        = I(y)[K(x)(y)]
        = y[K(x)(y)] = y[x]

Because K(x,y) = x
```

Another example

Can do you this?

```
def g(y):
    return y(y)
```

Can do you this?

```
def g(y):
    return y(y)


def I(x):
    return x


result = g(I)
print(result)
```

Can do you this?

```
def g(y):
    return y(y)


def I(x):
    return x


result = g(I)
print(result)
# <function I at 0x033209C0>
```

Take away

- Functions accepting/returning functions can be a powerful tool
- Implementation at the language level is messy
- It encourages you to think in terms of dependencies.

Summary

3 concepts:

Functions as input

Functions as output

Partial Application

Q&A