Combinators and Partial Application in Python

Remark

- Techniques from F#.

About speaker

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Minimum requirement

Function

- Function is a machine that **take input(s)**, and **returns an output**

3 concepts

3 concepts:

- Functions as input
- Functions as output
- Partial Application

Functions as inputs

- ```python
  def f(x,y,z):
      return x + y + z
  ```
  ```

- def f(x,y,z):

Most of the time, x, y, z are simple data types

- e.g. string, int, float, date, List, Set, Dictionary, etc.

- `def f(x,y,z):`

Most of the time, x, y, z are simple data types

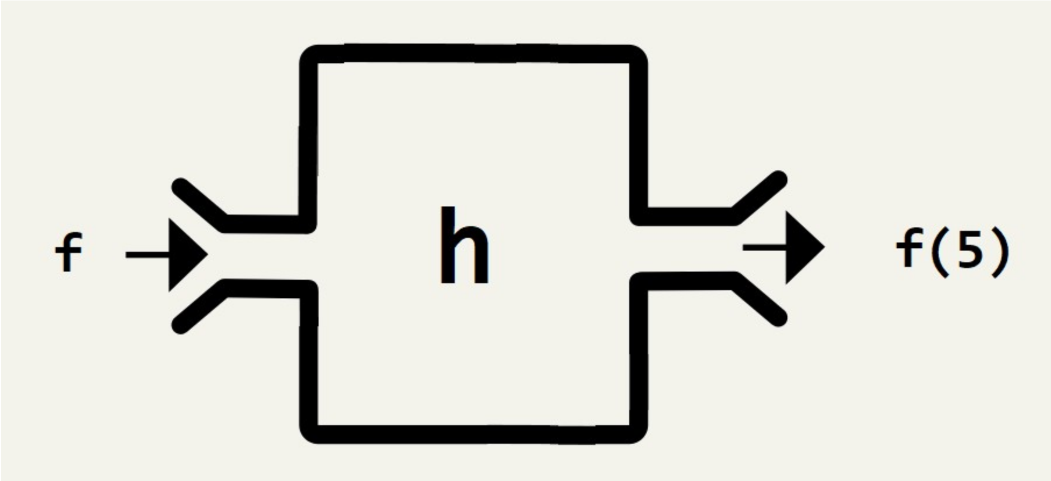
- But they can actually be functions as well!


Cöffee
MENU

Example

- ```
  def h(f):
      return f(5)
  ```
  ```

Example



- `def h(f):`
  `    return f(5)`

- h is a bigger function that:
  - Accepts a smaller function f
  - Returns a value f(5)

Example 1

- ```
  def h(f):
      return f(5)
  ```

- ```
  def g(x):
      return x + 1
  ```

Example 1

- ```
  def h(f):
      return f(5)
  ```

- ```
  def g(x):
      return x + 1
  ```

- Then `h(g) = 6`

Example 2

- ```
  def h(f):
      return f(5)
  ```

- ```
  def k(x):
      return x * 100
  ```

Example 2

- `def h(f):`
  ```
  return f(5)
  ```

- `def k(x):`
  ```
  return x * 100
  ```

- Then h(k) = 500

How is this useful?

Newton's Method

Newton’s method

- Newton’s method helps you find the (approximate) solution of a function.

- It is available in “scipy” library.

Newton's method

- $f(x) = x^2 - 3$
- $f(x) = 0 \text{ when } x = \sqrt{3} \approx 1.732$

Newton’s method

```
from scipy import optimize

def f(x):
    return x * x - 3

solution = optimize.newton(f,5)




print(solution)
# 1.7320508075688772
```

Newton’s method

```
from scipy import optimize

def f(x):
    return x * x - 3

solution = optimize.newton(f,5)
```

Smaller function “f”
accepted by a bigger
function!

```
print(solution)

# 1.7320508075688772
```

Designing Insurance Product

Pricing Insurance Product

- How much should I charge for insurance?

- def price():

```
    ....
```

Pricing Insurance Product

- e.g. Depends on age
- ```
  def price(age):
      ....
  ```
- age: int
  ```

Pricing Insurance Product

- e.g. Depends on probability of injury
- def price(age, prob):

  ```
      ....
  ```

- age: int
- prob: float

Pricing Insurance Product

- `What if prob depends on time?`

- `def price(age, prob):`

  `....`

- `age: int`
- **`prob`**`: ???`

Pricing Insurance Product

- `Input a function instead!`
- `def price(age, probFunc):`

```
    ....
```

- `age: int`
- `probFunc: datetime -> float`

Functions as outputs

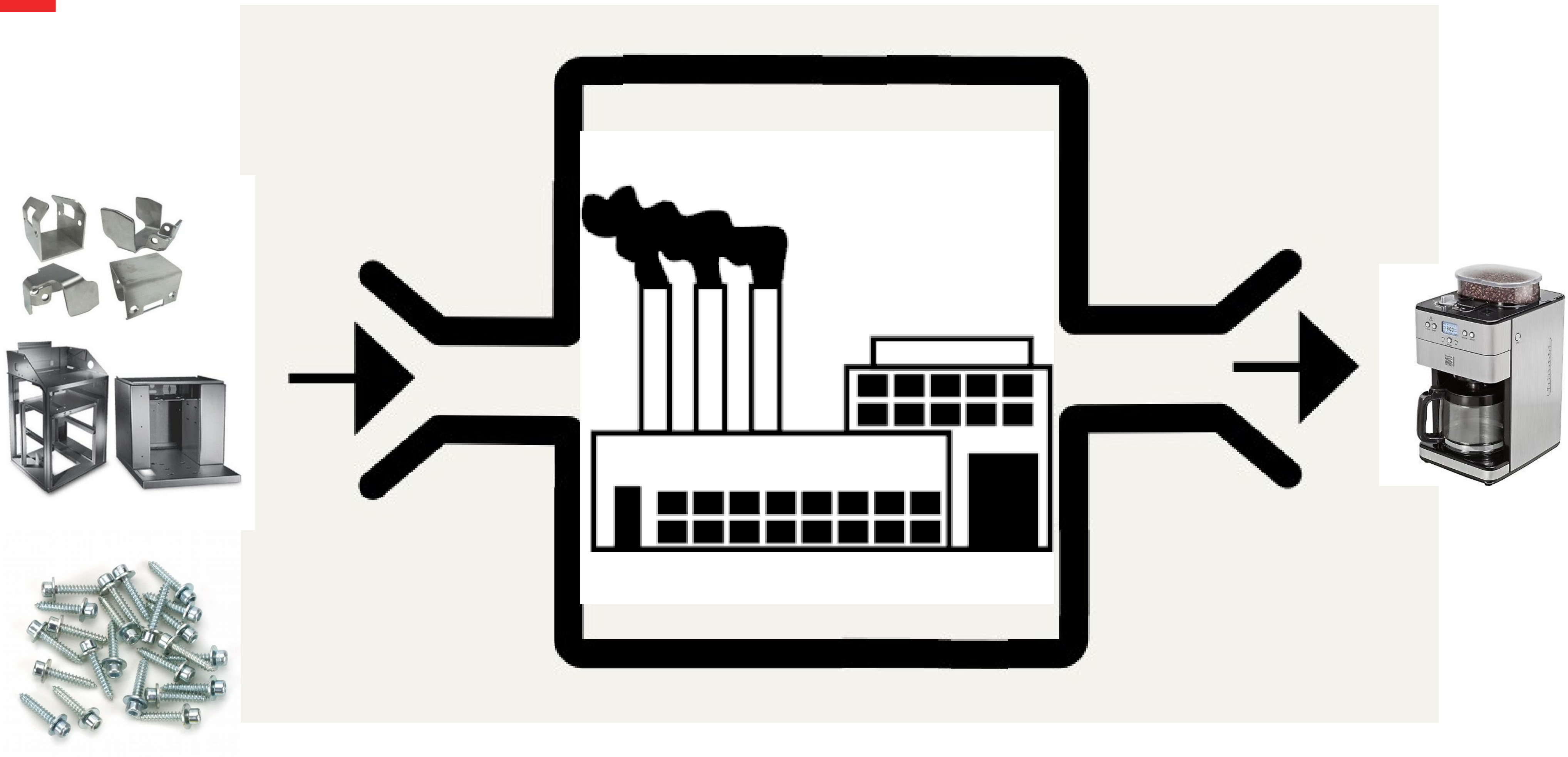
- ```python
  def f(x,y,z):
      return x + y + z
  ```
  ```

- ```
  def f(x,y,z):
      return ......
  ```

Most of the time, we also return simple data types

- e.g. string, int, float, date, List, Set, Dictionary, etc.
  ```

- def f(x,y,z):

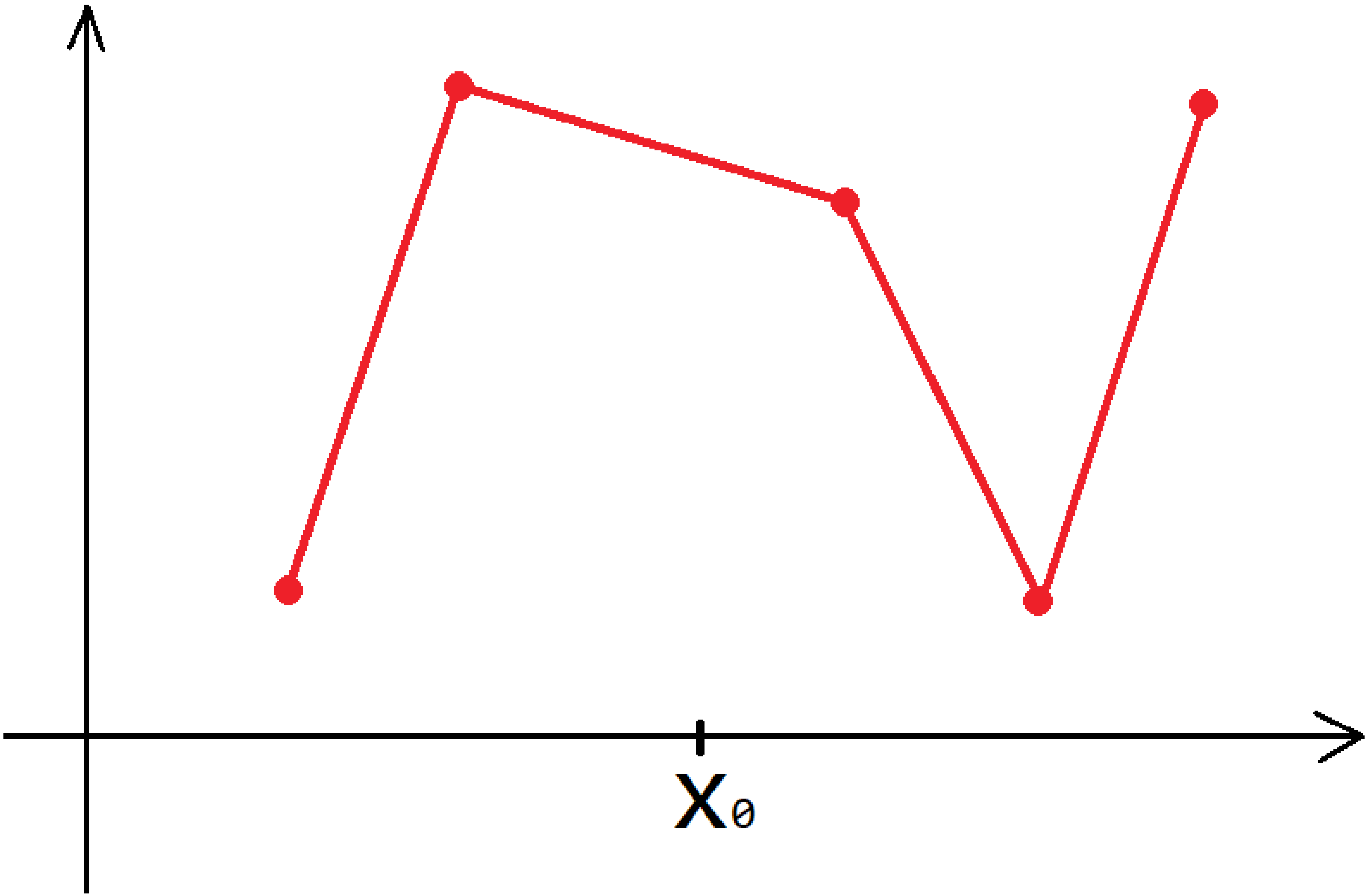
      return ......

Most of the time, we also return simple data types

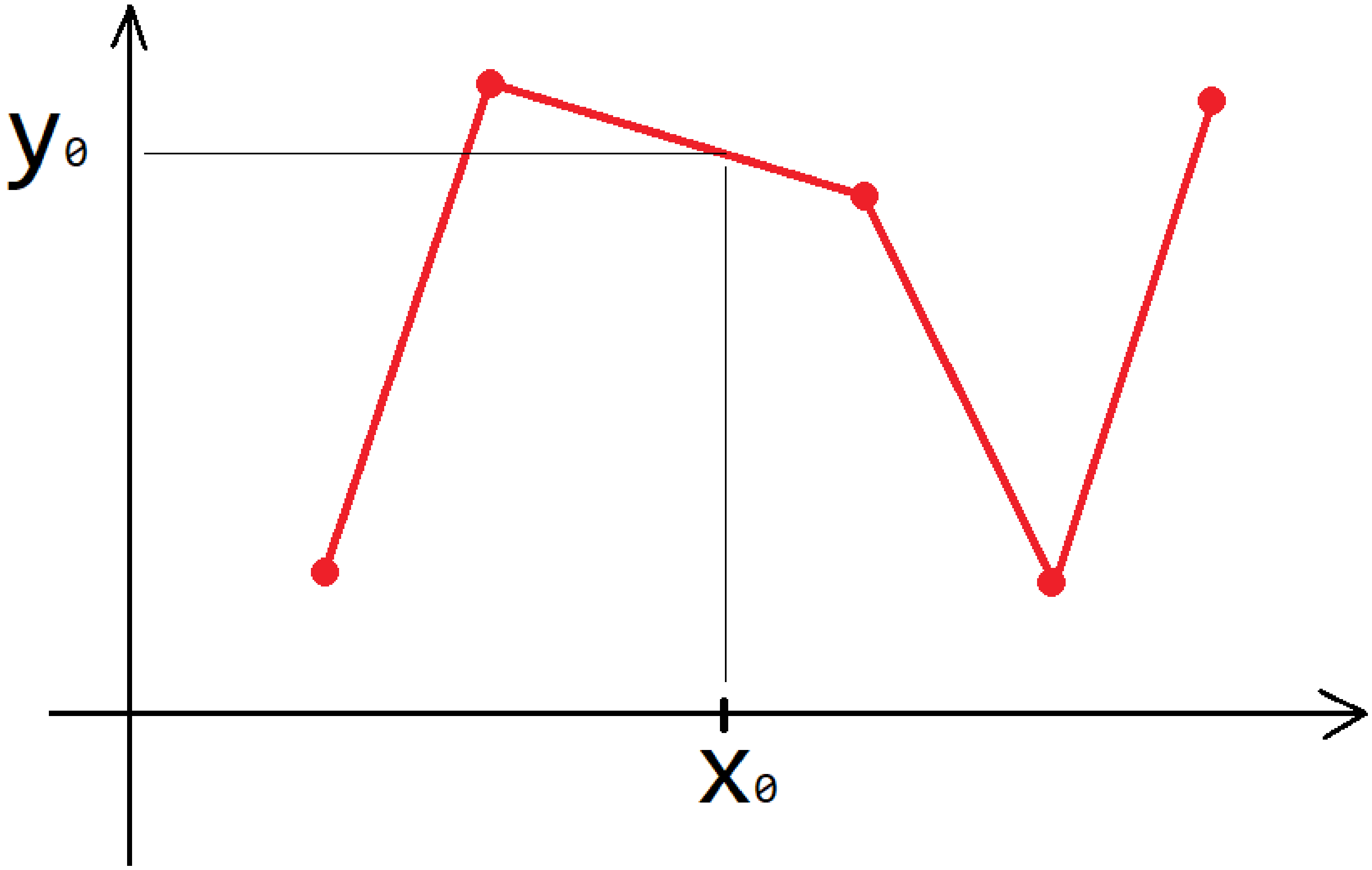
- Again, you can also return a function!

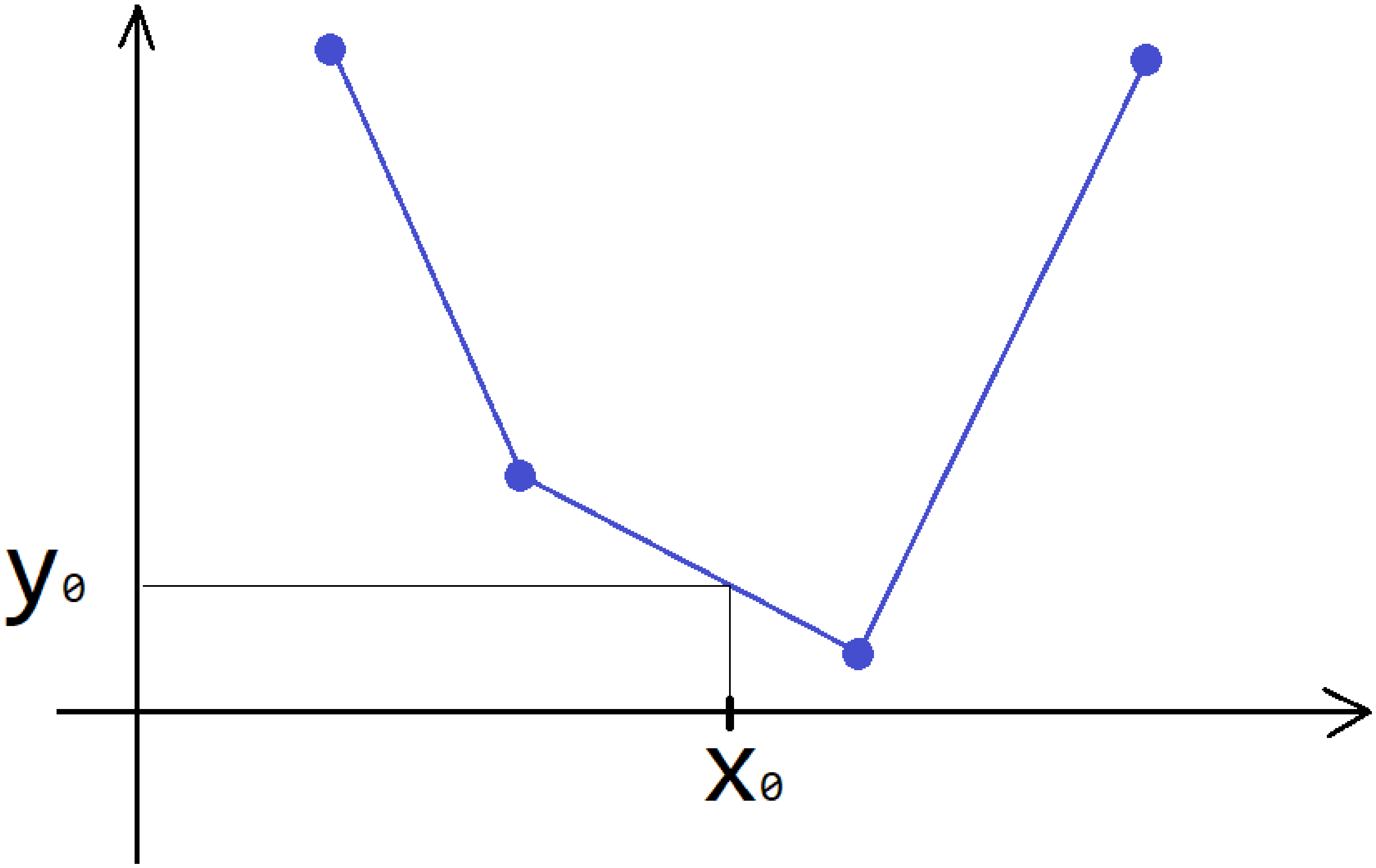


Interpolation



x_0


y0
x0


y0
x0

Interpolate

- Interpolate function depends on:
  - Original Dataset
  - Value being queried x_0

Implementation 1

- `def interpolate(dataset, x0):`

  `........`

- `dataset = [(-1,4),(1,7),(5,3)]`

Implementation 2

- ```
  class Interpolate:
      def __init__(self,dataset):
          ......

      def get_value(self,x0):
          .......
  ```
  ```

Implementation 2

- `class Interpolate:`
  ```
      def __init__(self,dataset):
      def get_value(self,x0):
  ```

- `dataset = [(-1,4),(1,7),(5,3)]`

- `inter_obj = Interpolate(dataset)`
- `result = inter_obj.get_value(2)`

Implementation 3

- ```
  def interpolate(dataset):
      def get_value(x0):
          ......
      return get_value
  ```
  ```

Implementation 3

- ```
  def interpolate(dataset):
      def get_value(x0):
          ......
      return get_value
  ```

“get_value” is a smaller function that is returned by a bigger function “interpolate” !
  ```

Implementation 3

- ```
  def interpolate(dataset):
      def get_value(x0):
          ......
      return get_value
  ```

- `dataset = [(-1,4),(1,7),(5,3)]`

- `inner_func = interpolate(dataset)`
- `result = inner_func(2)`
  ```

Implementation 3

- `def interpolate(dataset):`
  ```
  def get_value(x0):
      ......
  return get_value
  ```
- `dataset = [(-1,4),(1,7),(5,3)]`
- `inner_func = interpolate(dataset)`
- `result = inner_func(2)`

Directly use inner_func!
No need to find the method hidden inside an object.

Partial Application

Analogy

- If a function/ machine
  - Needs 3 inputs
  - But only 2 inputs provided
  - Still needs additional 1 inputs.

Analogy

- If a function/ machine
  - Needs 3 inputs
  - But only 2 inputs provided
  - Becomes a brand new function/machine that needs 1 inputs.

Example

- ```
  def f(x,y,z):
      return x + y + z
  ```
  ```

Example

- ```
  def f(x,y,z):
      return x + y + z
  ```

- `result = f(1,2,3)`

- `# result = 6`
  ```

Missing Variable

- `def f(x,y,z):`

  ```
      return x + y + z
  ```

- `result = f(1,2)`

- `# TypeError: f() missing 1 required positional argument: 'z'`

Lambda Definition

- ```
  f = lambda x: lambda y: lambda z: \
          x + y + z
  ```
  ```

Lambda Definition

- `f = lambda x: lambda y: lambda z: \`
  `        x + y + z`
- `result = f(1)(2)(3)`
- `# result = 6`

Lambda Definition

- `f = lambda x: lambda y: lambda z: \`

  `        x + y + z`

- `result = f(1)(2)`

- `# <function <lambda>.<locals>.<lambda>.<locals>.<lambda> at 0x013737C8>`
- Valid code!

Lambda Definition

- `f = lambda x: lambda y: lambda z: \`
  `x + y + z`

- `f : X -> Y -> Z -> result`

Lambda Definition

- `f = lambda x: lambda y: lambda z: \`
  `x + y + z`

- `f : X -> Y -> Z -> result`
- `f(x)  -> Y -> Z -> result`

Lambda Definition

- ```
  f = lambda x: lambda y: lambda z: \
          x + y + z
  ```

- `f : X -> Y -> Z -> result`
- `f(x)  -> Y -> Z -> result`
- `f(x)(y)    -> Z -> result`
  ```

Lambda Definition

- `f = lambda x: lambda y: lambda z: \`
  `        x + y + z`

- `f : X -> Y -> Z -> result`
- `f(x)  -> Y -> Z -> result`
- `f(x)(y)     -> Z -> result`
- `f(x)(y)(z)        -> result`

Lambda Definition

- ```
  f = lambda x: lambda y: lambda z: \
          x + y + z
  ```

- ```
  def f(x):
      def inner_1(y):




      return inner_1
  ```

Lambda Definition

- `f = lambda x: lambda y: lambda z: \`
  `x + y + z`

- ```
  def f(x):
      def inner_1(y):
          def inner_2(z):

          return inner_2
      return inner_1
  ```
  ```

Lambda Definition

- ```
  f = lambda x: lambda y: lambda z: \
          x + y + z
  ```

- ```
  def f(x):
      def inner_1(y):
          def inner_2(z):
              return x + y + z
          return inner_2
      return inner_1
  ```

SKI Combinators

SKI

- ```
  def S(x,y,z):
      return x(z)(y(z))
  ```

- ```
  def K(x,y):
      return x
  ```

- ```
  def I(x):
      return x
  ```
  ```

SKI

- `S = lambda x: lambda y: lambda z: \`
  `x(z)(y(z))`

- `K = lambda x: lambda y:`
  `x`

- `I = lambda x:`
  `x`

SKI

```
You can express any lambda expression
using only S, K, I.
```

```
S = lambda x: lambda y: lambda z: \
    x(z)(y(z))
K = lambda x: lambda y:
    x
I = lambda x:
    x
```

Example

```
T(x,y) = y(x)
```

Example

```
T(x,y) = y(x)
Goal: T = S(K(S(I)))(K)
```

Example

```
T(x,y) = y(x)
Goal: T = S(K(S(I)))(K)
```

- `T(x) = S(K(S(I)))(K)(x)`

Example

```
T(x,y) = y(x)
Goal: T = S(K(S(I)))(K)
```

- `T(x) = S(K(S(I)))(K)(x)`
  `     = K(S(I))(x)[K(x)]`
- `Because S(x,y,z) = x(z)(y(z))`

Example

```
T(x,y) = y(x)
Goal: T = S(K(S(I)))(K)
```

- `T(x) = S(K(S(I)))(K)(x)`
  `     = K(S(I))(x)[K(x)]`
  `     = S(I)[K(x)]`

- `Because K(x,y) = x`

Example

```
T(x,y) = y(x)
Goal: T = S(K(S(I)))(K)
```

- `T(x) = S(K(S(I)))(K)(x)`
  `     = K(S(I))(x)[K(x)]`
  `     = S(I)[K(x)]`
- `T(x)(y) = S(I)[K(x)](y)`

Example

```
T(x,y) = y(x)
Goal: T = S(K(S(I)))(K)
```

- `T(x) = S(K(S(I)))(K)(x)`
  `     = K(S(I))(x)[K(x)]`
  `     = S(I)[K(x)]`
- `T(x)(y) = S(I)[K(x)](y)`
  `        = I(y)[K(x)(y)]`
- `Because S(x,y,z) = x(z)(y(z))`

Example

```
T(x,y) = y(x)
Goal: T = S(K(S(I)))(K)
```

- `T(x) = S(K(S(I)))(K)(x)`
  `      = K(S(I))(x)[K(x)]`
  `      = S(I)[K(x)]`
- `T(x)(y) = S(I)[K(x)](y)`
  `        = I(y)[K(x)(y)]`
  `        = y[K(x)(y)]`
- `Because I(x) = x`

Example

```
T(x,y) = y(x)
Goal: T = S(K(S(I)))(K)
```

- `T(x) = S(K(S(I)))(K)(x)`
  `     = K(S(I))(x)[K(x)]`
  `     = S(I)[K(x)]`
- `T(x)(y) = S(I)[K(x)](y)`
  `        = I(y)[K(x)(y)]`
  `        = y[K(x)(y)] = y[x]`
- `Because K(x,y) = x`

More slides to be added......

- Work in progress......

Q&A