

MA 350 Number Theory – Spring 2024

Homework 4

Due: March 29, 2024

Submit your written work in Canvas as a single PDF file, and be sure to show your work. Answers without accompanying work will receive zero credit.

1. Using the Chinese remainder theorem, find a multiple of 13 that leaves a remainder of 1 when divided by each of the integers 2, 7 and 11.
2. Find the solutions of the following system of linear congruences.

$$2x + 3y \equiv 5 \pmod{7}$$

$$x + 5y \equiv 6 \pmod{7}$$

3. (a) A repunit is an integer with expansion containing all 1s. Determine which repunits are divisible by 3, which are divisible 9, and which are divisible by 11.
(b) A palindrome integer is an integer whose decimal representation reads the same forward and backward. For example, 3569653 is a palindrome integer. Prove that a palindrome integer with an even number of digits is divisible by 11.
4. What is the remainder when $20!$ is divided by 23?
5. What is the remainder when 6^{325} is divided by 11?
6. Prove that $2^{p-1} + 3^{p-1} + \cdots + (p-1)^{p-1} \equiv -2 \pmod{p}$ where p is a prime.
7. Let p be a prime and $a, b, c \in \mathbb{Z}^+$. Show that, if $a^p + b^p = c^p$ then p divides $a + b - c$.
8. Prove that $66 \mid (n^{11} - n)$ for any positive integer n .
9. Show that 45 is a pseudoprime to the base 17.
10. Find $\varphi(11)$, $\varphi(15)$ and $\varphi(25)$.