even # 1's in the error vector → single parity check

$$FUE(L=5) = \frac{\text{total \# undetectable } L=5}{\text{total \# } L=5} = \frac{\left[\binom{3}{0}+\binom{3}{2}\right] * 1}{2^3 * 1} = \frac{4}{8} = 50\%$$

$e = [1 \underbrace{? ? ?}_{0,2} 1] \Rightarrow \binom{3}{0}+\binom{3}{2} = 1 + 3 = 4$

$$FUE(L=2) = \frac{(5-2+1) * 1}{(5-2+1) * 2^{2-2}} = 100\%$$

$e = [1 \; ? \; ? \; ? \; 1]_{1\times 5}$

$$\begin{cases} n-L+1 = 5-5+1 = 1 \\ L-2 = 3 \Rightarrow 2^{L-2} = 2^3 \end{cases}$$

$e = [0\,0\,0\,1\,1]$

$0\,0\,1\,1\,0$

$0\,1\,0\,0$

$1\,1\,0\,0\,0$

5. (30 points) Suppose that two check bits are added to five information bits (i_4, i_3, i_2, i_1, i_0). The first check bit c_1 is the even parity check of the first two information bits (i_4, i_3), and the second check bit c_0 is the even parity check of the final three information bits (i_2, i_1, i_0). The codeword is (i_4, i_3, i_2, i_1, i_0, c_1, c_0).
   a. (15 points) What fraction of errors is undetectable? Justify your answer.
   b. (15 points) What fraction of 2-bit errors is undetectable? Justify your answer.

G1 G2

$[\underbrace{i_4 \; i_3}_{c_1} \; \underbrace{i_2 \; i_1 \; i_0}_{c_0} \; c_1 \; c_0]_{1\times 7}$

$[\underbrace{1 \; 1}_{0} \; \underbrace{0 \; 1 \; 0}_{1} \; 0 \; 1]_{1\times 7}$

$e = [1\;1\;0\;0\;0\;0\;1]$ detectable

$e = [1\;1\;1\;1\;0\;0\;0]$ undetectable

$e = [1\;0\;1\;0\;0\;0\;0]$ detectable

$e = [\_\ \_\ \_\ \_\ \_\ \_\ \_]_{1\times 7}$

Pattern: even # 1's in G1, and even # 1's in G2

		G1: 3 bits # error bits in G1	G2: 4 bits # error bits in G2	# such errors
$m=2^*$	Case 1:	0	2	$\binom{3}{0}*\binom{4}{2} = 1*6 = 6$ ✓
4	Case 2:	0	4	$\binom{3}{0}*\binom{4}{4} = 1$
4	Case 3:	2	2	$\binom{3}{2}*\binom{4}{2} = 18$
6	Case 4:	2	4	$\binom{3}{2}*\binom{4}{4} = 3$
2^*	Case 5	2	0	$\binom{3}{2}*\binom{4}{0} = 3$ ✓
		(0	0)	no error

$FUE(M=2) = \frac{6+3}{\binom{7}{2}} = \frac{9}{21} = \frac{3}{7} \approx 42.9\%$

$FUE = \frac{6+1+18+3+3}{2^7 - 1} = \frac{31}{127} \approx 24.4\%$

$FUE = \frac{2^k - 1}{2^n - 1} = \frac{2^5 - 1}{2^7 - 1} = \frac{31}{127}$

LDPC

Low
Density
Parity
Check

Wi-Fi 6 (11ax)

10 Gbps Ethernet

CRC (Cyclic Redundancy Check) code

Polynomial code

$$\text{info bits} \longrightarrow \text{Encoder} \xrightarrow[\substack{\text{code word bits} \\ (b)}]{} \text{Channel} \xrightarrow[\substack{\text{received bits} \\ R = b + e}]{} \text{Decoder} \longrightarrow$$

error (e) ↓ Channel

$$\begin{cases} 1+1=0 \\ 1+0=1 \\ 0+1=1 \\ 0+0=0 \end{cases}$$

"6 steps"

① k info bits $\Rightarrow$ info polynomial

$i_{k-1}, i_{k-2}, \cdots, i_0 \Rightarrow i(x) = \underline{i_{k-1}} * X^{k-1} + i_{k-2} * X^{k-2} + \cdots + i_1 * X^1 + i_0 * \underbrace{X^0}_{1}$

(binary polynomial)

② $\underline{(n-k) \text{ check bits}}$ $\Rightarrow$ generator poly $\underline{g(x)}$ of $\underline{\text{degree } (n-k)}$ $\stackrel{*}{=}$ # check bits

k : # info bits

n : # codeword bits

$g(x) = \underline{g_{n-k}} * x^{n-k} + \cdots + \underline{g_0} * x^0$

$$\begin{cases} \rightarrow g_{n-k} = 1 \\ \rightarrow g_0 = 1 \end{cases}$$

E.g. $\underline{n-k=3}$

$g(x) = x^3 + x^2 + x$ → not valid

$g(x) = x^3 + 1$ ✓

$g(x) = x^3 + x + 1$ ✓

$g(x) = x^3 + x^2 + 1$ ✓

③ dividend poly

$a(x) = \underbrace{i(x)}_{\text{info}} * x^{\underline{n-k}}$ # check bits

⊛ ④ use $g(x)$ as divisor poly

find the remainder poly : $\underline{r(x)}$

$a(x) = g(x) * \underbrace{q(x)}_{\text{quotient poly}} + r(x)$

⑤ codeword poly

$\underline{b(x)} = \underline{a(x)} + \underline{r(x)}$

⑥ $b(x)$ coefficients ⇒ codeword bits (n)

" binary polynomial arithmetic "

⇒ coefficient calculations follow 2's complement arithmetic

1+1=0	0-1=1	1+0=1	0+0=0
1-1=0	0-0=0	1-0=1	0-0=0