

The Fibonacci Numbers

Fibonacci sequence $(f_n)_{n=1}^{\infty}$ is defined by

$$f_1 = 1, f_2 = 1$$

$$f_n = f_{n-1} + f_{n-2} \text{ for } n \geq 3$$

$$1, 1, 2, 3, 5, 8, 13, 21, 34, \dots$$

ex: $\sum_{k=1}^n f_k = f_{n+2} - 1$

To prove this, we can use

$$\sum_{k=1}^n f_k = \sum_{k=1}^n (f_{k+2} - f_{k+1})$$

$$= (f_3 - f_2) + (f_4 - f_3) + (f_5 - f_4) + \dots + (f_{n+2} - f_{n+1})$$

$$= f_{n+2} - f_2$$