$k = 4, \quad n - k = 1, \quad n = 5$

Pattern:
even # 1's in the codeword

$B = [1 \; 0 \; 1 \; 1 \,\vdots\, 1]$

$\underline{R} = \underline{B} + \underline{e}$ → "XOR" $\begin{cases} 1+1=0 \\ 0+1=1 \\ 1+0=1 \\ 0+0=0 \end{cases}$

$R = [0 \; 0 \; 1 \; 1 \; 1]$ $e = [1 \; 0 \; 0 \; 0 \; 0]$ single-bit error : detectable

$R = [1 \; 0 \; 0 \; 0 \; 1]$ $e = [0 \; 0 \; 1 \; 1 \; 0]$ 2-bit error : undetectable

$R = [0 \; 0 \; 0 \; 1 \; 0]$ $e = [1 \; 0 \; 1 \; 0 \; 1]$ 3-bit error : detectable

$R = [0 \; 1 \; 0 \; 0 \; 1]$ $e = [1 \; 1 \; 1 \; 1 \; 0]$ 4-bit error : undectable

$R = [1 \; 1 \; 1 \; 1 \; 0]$ $e = [0 \; 1 \; 0 \; 0 \; 1]$ undetectable

$R = [0 \; 1 \; 0 \; 0 \; 0]$ $e = [1 \; 1 \; 1 \; 1 \; 1]$ 5-bit error: detectable

$R = [1 \; 0 \; 1 \; 1 \; 1]$ $e = [0 \; 0 \; 0 \; 0 \; 0]$ no error

$$FUE = \frac{\text{total \# undectable errors}}{\text{total \# valid errors}}$$ ← # different error vectors?

$$= \frac{\binom{5}{2} + \binom{5}{4}}{2^5 - 1}$$

$$= \frac{10 + 5}{31} = \frac{15}{31}$$

$$FUE = \frac{2^{(n-1)} - 1}{2^n - 1} = \frac{2^k - 1}{2^n - 1}$$

2^k: # possible codewords

undectable error: convert original codeword to a diff codeword

2-bit errors

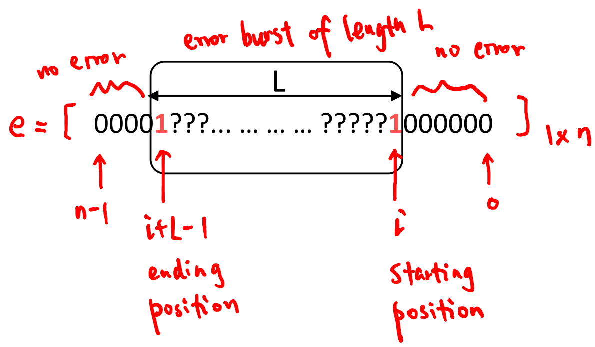
$$FUE(M=2) = \frac{\text{total \# undetectable 2-bit errors}}{\text{total \# 2-bit errors}}$$

\# errors

$$= \frac{\binom{5}{2}}{\binom{5}{2}} = 100\%$$

Error Burst

- Errors can be classified according to:
  - Number of bit error positions: M-bit error
  - Separation of bit error positions: error burst of length L
    - Error starts at bit position i and ends at bit position (i + L - 1)



$e = [\,1 \;\; 0 \;\; 0 \;\; 0 \;\; 0\,]$ $\quad M=1 \equiv L=1$

$e = [\,0 \;\; 0 \;\; 1 \;\; 1 \;\; 0\,]$ $\quad M=2 \overset{?}{=} L=2$

$e = [\,1 \;\; 0 \;\; 0 \;\; 0 \;\; 1\,]$ $\quad M=2 \quad L=5$

$e = [\,1 \;\; 1 \;\; 1 \;\; 1 \;\; 1\,]$ $\quad M=5 \quad L=5$

$e = [\,1 \;\; 0 \;\; 1 \;\; 1 \;\; 0\,]$ $\quad M=3 \quad L=4$

$$FUE(L=4) = \frac{\text{total \# undetectable } L=4}{\text{total \# error bursts of } L=4} \qquad \frac{\left(\binom{2}{0}+\binom{2}{2}\right)*2}{2^2*2} = \frac{1}{2}$$

$e = [\_\;\_\;\_\;\_\;\_\,]_{1\times 5}$

0 1 ? ? 1

1 ? ? 1 0

$$\Rightarrow \left\{ \begin{matrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{matrix} \right\} \qquad 2^{2} \leftarrow \#\text{"?"}$$

$$\frac{\left(\binom{2}{0}+\binom{2}{2}\right)*2}{\left(\binom{L-2}{0}+\binom{L-2}{2}+\cdots\right)*(n-L+1)}$$

$n=5$, $L=4$
$$\left\{ \begin{matrix} \#\text{ starting positions } = 2 \\ \#\text{ "?" }:\; 2^2 \end{matrix} \right\} \begin{matrix} \Rightarrow (n-L+1) \\ \Rightarrow 2^{L-2} \end{matrix} \Bigg\} \; (n-L+1) * 2^{L-2}$$

$\Rightarrow \quad 2^2 * 2 = 8$

5. (30 points) Suppose that two check bits are added to five information bits (i_4, i_3, i_2, i_1, i_0). The first check bit c_1 is the even parity check of the first two information bits (i_4, i_3), and the second check bit c_0 is the even parity check of the final three information bits (i_2, i_1, i_0). The codeword is (i_4, i_3, i_2, i_1, i_0, c_1, c_0).
   a. (15 points) What fraction of errors is undetectable? Justify your answer.
   b. (15 points) What fraction of 2-bit errors is undetectable? Justify your answer.

$$[\underbrace{i_4 \quad i_3}_{c_1} \quad \underbrace{i_2 \quad i_1 \quad i_0}_{c_0} \quad c_1 \quad c_0]_{1\times 7}$$

$$[\underbrace{1 \quad 1}_{0} \quad \underbrace{0 \quad 1 \quad 0}_{1} \quad 0 \quad 1]_{1\times 7}$$

$e = [1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1]$ detectable

$e = [1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0]$ undetectable

$e = [1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0]$ detectable