HW 2 Joseph Schmidt

7. $\underbrace{i_4\ i_3}_{c_1 \text{ even}}\ \underbrace{i_2\ i_1\ i_0}_{c_0 \text{ even}}\ c_1\ c_0$

a) What fraction of error bursts length 2 undetectable?

Both c_1 & c_0 are even so if two bits flip in their group it is undetectable.

- $i_4\ i_3$ undetectable for c_1

- $i_2\ i_1$ undetectable for c_0
- $i_1\ i_0$

$$FUE(L=2) = \frac{\text{total \# undetectable } L=2}{\text{total \# error bursts of } L=2}$$

$$= \frac{3}{6} = \frac{1}{2}$$

b) $i_4\ i_3\ i_2\ i_1$ undetectable

$$FUE(L=4) = \frac{2}{16} = \frac{1}{8}$$

undetectable when $i_4\ i_3\ i_2\ i_1$: 11 11 or 1 ? ? 1 i_1 and c_0: 1001

can move 2 places

\# starting pos $= 4 \Rightarrow (n-L+1)$

\# ? : 2^2

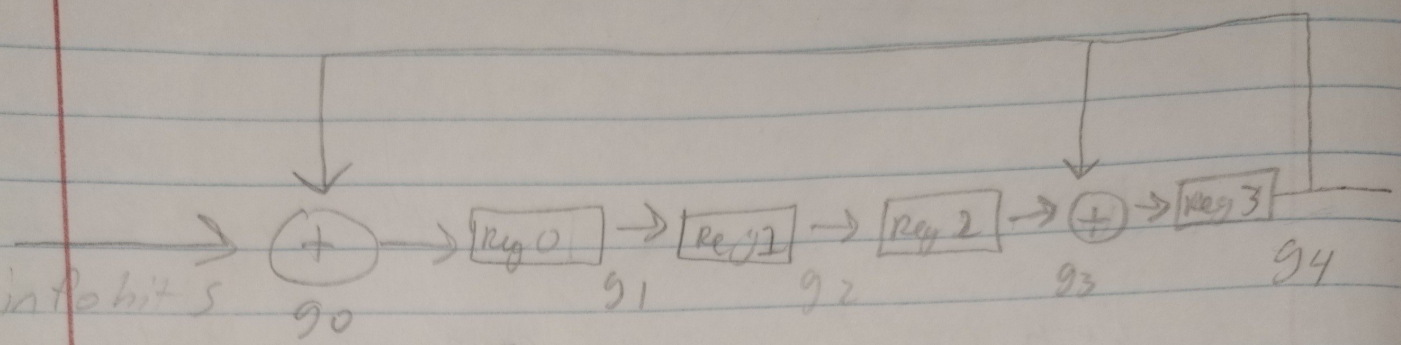
$\Rightarrow 2^2 \cdot 4 = 16$ total errors

1 ? ? 1 -2^2
1 ? ? 1 -2^2
1 ? ? 1 -2^2
1 ? ? 1 -2^2

16

2. CRC code with $g(x) = x^4 + x^3 + 1$

a)

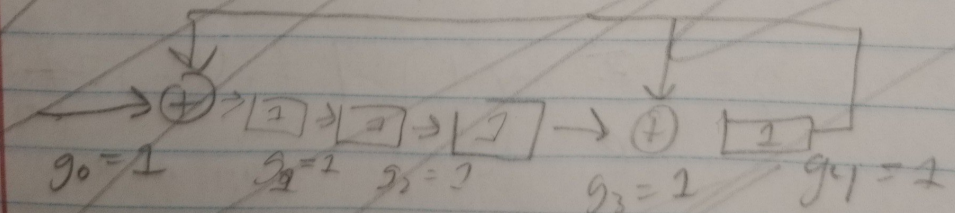


0 0 0 i_0 i_1 i_2 i_3

(1110)

cycle	input	Reg 0	Reg 1	Reg 2	Reg 3
0	–	0	0	0	0
1	$1 = i_3$	1	0	0	0
2	$1 = i_2$	1	1	0	0
3	$1 = i_1$	1	1	1	0
4	$0 = i_0$	0	1	1	1
5	0	1	0	1	1
6	0	1	1	1	1
7	0	1	1	1	1
8	0	1	1	1	1

9th clock cycle




```
11111 11111
      11110000
       00011000
          11111
          00111
      1110100
```

b) $\underline{11061}\ \overline{)11100000} = c(x) = (x^3+x^2+x)\cdot x^4$

$\Uparrow$ $g(x)$ $\qquad\qquad$ $\Uparrow$ $g(x)$ biggest term

```
11001
-----
01010
11001
-----
10011
11001
 10100
 11001
  11010
  11001
  -----
  00011
```

$\Rightarrow$ codeword: (11010 | 0011)

c) [1 ? ? ? ? ? ? 1]

$e(x) = x^7 + ??? + 1$

$= g(x)\cdot c(x) = (x^4+x^3+1)(c(x))$

$c(x) = x^3 + ??? + 1$

2^2 positions where $0\cdot x^2 + 0\cdot x^0$

$0x^2 + 1\cdot x^1$

$1x^2 + 0\cdot x^1$

$1x^2 + 1\cdot x^1$

so 4 undetectable errors

FUE = total # error patterns − detectable error

$= \dfrac{2^8 - \overbrace{\left(\binom{8}{0}+\binom{8}{1}+\binom{8}{2}+\binom{8}{3}+\binom{8}{4}\right)}^{\text{total}}}{2^8} = \dfrac{94}{256}$

c. $e(x) = g(x) \cdot c(x)$

$= (x^4 + x^3 + 1) \cdot c(x)$

$c(x) = x^3 + ?? + 1 \quad c(x) = x^2 + ? + 1 \quad c(x) = x + 1$

$2^2 + 2(2^1) + 3(2^0)$

2 positions 3 positions

$c(x)$ can be any polynomial: $x^7 + ???? + 1$

$x^7 + \left| x^6 \; x^5 \; x^4 \; x^3 \; x^2 \; x^1 \right| \begin{matrix} x^6 - x \\ + 1 \end{matrix}$

2^6 ways to choose inside of polynomial

$\frac{1}{2^6} = FHE$

$FHE(L) = \frac{1}{2^{n-k}}$ For L 1-8 $= 8\left(\frac{1}{2^{n-k}}\right) = 1$

d. $e = [1 ? ? ? ? 1]$

$e(x) = x^5 + ?? + 1$

$= g(x) \cdot c(x) = (x^4 + x^3 + 1) \cdot c(x)$

$c(x) = x + 1$

$FUE(6) = \frac{1}{2^{n-k}} = \frac{1}{2^{8-5}} = \frac{1}{2^3} = \frac{1}{8}$

$5 - 3 > 0$ "okay"

$FUE(6) = \frac{1}{2^{n-k}}$

error $L=6$

E. i. [1 ? ? ? ? 1]

$e(x) = x^5 + ??? + 1$

$= g(x) \cdot c(x) = (x^4 + x^3 + 1)\, c(x)$

$c(x) = x + 1$

$e(x) = g(x) \cdot c(x)$

$$
\begin{array}{lll}
g(x)\cdot 1 \rightarrow & 11001 & \Rightarrow g(x)\cdot 1 + g(x)\cdot x \\
+\ g(x)\cdot x \rightarrow & \underline{110010} & = g(x)\cdot(x+1) \\
 & 101011 & = g(x)\cdot c(x)
\end{array}
$$

$e = [101011]$

ii. $g(x) = x^4 + x^3 + 1$

$$
\begin{array}{ll}
 & 11001 \leftarrow g(x)\cdot 1 \\
+ & 11001\,00 \leftarrow g(x)\cdot x^2 \\
\hline
e = & [01101 01]
\end{array}
$$

$e(x) = x^6 + x^5 + x^4 + x^2 + 1 = g(x)\cdot(x^2+1)$

iii. [1 ? ? ? ? ? ? 1]

$e(x) = x^7 + ??? + 1$

$= g(x)\cdot c(x) = (x^4 + x^3 + 1)\, c(x)$

$$
\begin{array}{ll}
 & 11001 \leftarrow g(x)\cdot 1 \\
 & 11001 \leftarrow g(x)\cdot x^2 \\
 & 11001000 \leftarrow g(x)\cdot x^3 \\
\hline
 & 10110101
\end{array}
$$

$c(x) = x^3 + ?? + 1$

so $c(x) = x^3 + x^2 + 1$

$e(x) = c(x)\cdot g(x) = x^7 + x^5 + x^4 + x^2 + 1$