HW4

1. a) Temp of downtown Ames throughout day
   continuous-time & discrete space

b) The high temp in Ames for each day in year
   discrete-time & discrete space

c) Num of customers thougout day
   continous-time & discrete space

d) Total number of customers served each day
   discrete-time & discrete space

e) Operational state (binary) at end of each hour
   discrete-time & discrete space

2. b)
$$P = \begin{array}{c} \begin{array}{ccc} \text{full} & \text{part} & \text{broken} \end{array} \\ \begin{pmatrix} .7 & .2 & .1 \\ 0 & .6 & .4 \\ .8 & 0 & .2 \end{pmatrix} \end{array}$$

a) $X = \{\text{"Full"}, \text{"part"}, \text{"broken"}\} = (1,2,3)$

c) $$P_1 = P_0 \cdot P = (0,0,1)\begin{pmatrix} .7 & .2 & .1 \\ 0 & .6 & .4 \\ .8 & 0 & .2 \end{pmatrix}$$

a) $$= (.8, 0, .2)$$

3) $$P = \begin{array}{c} \begin{array}{cc} \text{zero} & \text{one} \end{array} \\ \begin{pmatrix} .96 & .04 \\ .05 & .95 \end{pmatrix} \end{array}$$

b) $$P_1 = P_0 \cdot P = (.8, .2)\begin{pmatrix} .96 & .04 \\ .05 & .95 \end{pmatrix}$$

$$= (.778, .222)$$

4\. a) $P = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & .5 & .5 \\ .5 & 0 & .5 \\ .5 & .5 & 0 \end{bmatrix} \end{array}$ $\Pi = \lim_{n \to \infty} P^n = \begin{pmatrix} \pi_1 & \pi_2 & \cdots & \pi_n \\ \vdots & & & \vdots \\ \pi_1 & \pi_2 & \cdots & \pi_n \end{pmatrix}$

If regular, then it has a steady state

b) $P^{34} = \begin{pmatrix} 0.\overline{333} & 0.\overline{333} & 0.\overline{333} \\ 0.\overline{333} & 0.\overline{333} & 0.\overline{333} \\ 0.\overline{333} & 0.\overline{333} & 0.\overline{333} \end{pmatrix}$ $\therefore$ has steady state $\therefore$ It is regular

can also square and if not all 0 will be regular

c) $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

d) $\frac{1}{2}$

5\. a) $P = \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & 0 \end{pmatrix} \end{array}$

$\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0.75 & 0 & 0.25 \\ 0.5 & 0.25 & 0.25 \\ 0 & 0.5 & 0.5 \end{pmatrix}$

1×3 $\quad 3 \times 3$ $\quad 1 \times 3$

$m \times n$ $\quad n \times p =$ $\quad m \times p$

b) $p_0 \cdot P^2 = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}) \begin{pmatrix} \frac{3}{4} & 0 & \frac{1}{4} \\ \frac{2}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{2}{4} \end{pmatrix} = (\frac{3}{12} + \frac{2}{12} + 0, 0 + \frac{1}{12} + \frac{2}{12}, \frac{1}{12} + \frac{1}{12} + \frac{2}{12}) = (\frac{5}{12}, \frac{3}{12}, \frac{4}{12})$

c) P^4 has all entries non zero making it regular

d) $(0.\overline{4}, 0.\overline{2}, 0.\overline{333})$

P^{30} //

6. Radio blackouts : 2 / year

a) 3 have happened, prob of 3 more ?

$X_t \sim Pois(2t)$ $\lambda = 2/year$
$t = 3$

or

$Pois(t)$ $\lambda = 1/6mths$
$x = 3$

$P(X=3) = \frac{e^{-1} \cdot 1^3}{3!} = \frac{1}{e \cdot 3!} \doteq 0.0613$

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b) $X_t \sim Pois(4)$ with $E(4) = 4$
$\lambda \cdot t = 1 \cdot 4$

c) $\lambda = 1/6months$ when exp(2)
$P(X \le t) = 1 - e^{-\lambda t}$ for all $t \ge 0$
$P(X \le t) = 1 - e^{-\lambda t} = 0.5$
$-e^{-\lambda t} = -0.5$
$e^{-\lambda t} = 0.5$
$-1t = \ln(0.5)$
$t = -\ln(0.5)$
$t = .69314$ of 6 months
$.34657$ of years

d) $T \sim Gamma(4,2)$, $P(T < 2) = P(X \ge 4)$
$Pois(4)$ where $P(X \ge 4) = 1 - P(X \le 3)$
$= 1 - .4335$
$= .5665$

7. pois(1/2) $\lambda = \frac{1}{2}$ per hour .2

$\lambda = 1$ per 2hrs

a) $P(X_{24}=0) = 6.14 \times 10^{-6}$ P(12)

0 events happen

b) $P(X_{24} > 25) = 1 - P(X_{24} \leq 25) = 0.000308$

c) $P(X_2 \leq 5) = P(X_2 - X_0 \leq 4) = 0.9963$

8. $\lambda = 1$ per min

a) $X \sim P(60)$ $E(X) = 60$ $SD = \sqrt{60} = 7.746$

b) P(5) and $P(X > 10) = 1 - P(X \leq 10) = 0.01369$

c) $P(X < \frac{1}{2}) = 1 - P(X \geq \frac{1}{2}) = 1 - .6065 = .3935$

d) $X \sim \text{Gamma}(100, 1)$ $E(X) = 100$ $Var(X) = 100$ $SD = 10$

9. $B \sim Exp(\lambda)$ & $D \sim Exp(\mu) = P(B > t)P(D > t)$

$= e^{-t\lambda} e^{-t\mu} = e^{-t(\lambda+\mu)}$

the cdf of Y is

$$F_Y(t) = P(Y \leq t) = 1 - e^{-t(\lambda+\mu)}$$

This is cdf of exponential distn, but seen with a rate of $\lambda + \mu$

so, $Y \sim Exp(\lambda+\mu)$