Sum and Products

Notation:

$$a_1 + a_2 + \cdots + a_n = \sum_{k=1}^{n} a_k$$

Properties:

1. $$\sum_{j=m}^{n} k a_j = k \sum_{j=m}^{n} a_j$$

2. $$\sum_{j=m}^{n} (a_j + b_j) = \sum_{j=m}^{n} a_j + \sum_{j=m}^{n} b_j$$

3. $$\sum_{i=m}^{n} \sum_{j=p}^{q} a_i \, b_j = \sum_{j=p}^{q} \sum_{i=m}^{n} a_i \, b_j$$

Ex:

$$\sum_{k=1}^{n} k = \frac{1}{2} n (n+1)$$

$$\sum_{k=1}^{n} k^2 = \frac{1}{6} n (n+1)(2n+1)$$

$$\sum_{k=1}^{n} (2k-1) = n^2$$

$$\sum_{k=0}^{n} a r^k = \frac{a(1 - r^{k+1})}{1 - r}$$

$$\sum_{k=0}^{n} 2^k = 2^{n+1} - 1$$

Products: $\displaystyle\prod_{k=1}^{n} a_k = a_1 a_2 \cdots a_n$

A Property: $\displaystyle\prod_{k=1}^{n} a_k b_k = \prod_{k=1}^{n} a_k \prod_{k=1}^{n} b_k$