

even # 1's in the error vector \rightarrow single parity check

$$FUE(L=5) = \frac{\text{total \# undetectable } L=5}{\text{total \# } L=5}$$

$$e = [1 \ ? \ ? \ ? \ 1] \Rightarrow \binom{3}{0} + \binom{3}{2} = 1 + 3 = 4$$

$$= \frac{\binom{3}{0} + \binom{3}{2}}{2^3 * 1} = \frac{4}{8} = 50\%$$

$$FUE(L=2) = \frac{(5-2+1) * 1}{(5-2+1) * 2^{2-2}} = 100\%$$

$$e = [1 \ ? \ ? \ ? \ 1]_{1 \times 5}$$

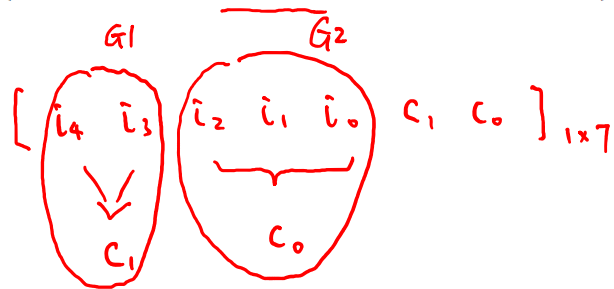
$$\begin{cases} n-L+1 = 5-5+1 = 1 \\ L-2 = 3 \Rightarrow 2^{L-2} = 2^3 \end{cases}$$

$$e = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

\uparrow

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5. (30 points) Suppose that two check bits are added to five information bits (i_4, i_3, i_2, i_1, i_0). The first check bit c_1 is the even parity check of the first two information bits (i_4, i_3), and the second check bit c_0 is the even parity check of the final three information bits (i_2, i_1, i_0). The codeword is ($i_4, i_3, i_2, i_1, i_0, c_1, c_0$).
- (15 points) What fraction of errors is undetectable? Justify your answer.
 - (15 points) What fraction of 2-bit errors is undetectable? Justify your answer.



$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}_{1 \times 7}$$

$\underbrace{\hspace{2cm}}_0 \quad \underbrace{\hspace{2cm}}_1$

$$e = [1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 1] \text{ detectable}$$

$$e = [1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0] \text{ undetectable}$$

$$e = [1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0] \text{ detectable}$$

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$$e = [\text{-----}]_{107}$$

Pattern: even # 1's in $G1$, and even # 1's in $G2$

		$G1: 3 \text{ bits}$ <u># error bits in $G1$</u>	$G2: 4 \text{ bits}$ <u># error bits in $G2$</u>	<u># such errors</u>
$m = 2^*$	Case 1:	0	2	$\binom{3}{0} * \binom{4}{2} = 1 * 6 = 6 \checkmark$
4	Case 2:	0	4	$\binom{3}{0} * \binom{4}{4} = 1$
4	Case 3:	2	2	$\binom{3}{2} * \binom{4}{2} = 18$
6	Case 4:	2	4	$\binom{3}{2} * \binom{4}{4} = 3$
2^*	Case 5	2	0	$\binom{3}{2} * \binom{4}{0} = 3 \checkmark$
		0	0) no error

$$FUE(M=2) = \frac{6+3}{\binom{7}{2}} = \frac{9}{21} = \frac{3}{7} \approx 42.9\%$$

$$FUE = \frac{6+1+18+3+3}{2^7 - 1} = \frac{31}{127} \approx 24.4\%$$

$$FUE = \frac{2^k - 1}{2^n - 1} = \frac{2^5 - 1}{2^7 - 1} = \frac{31}{127}$$

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LDPC

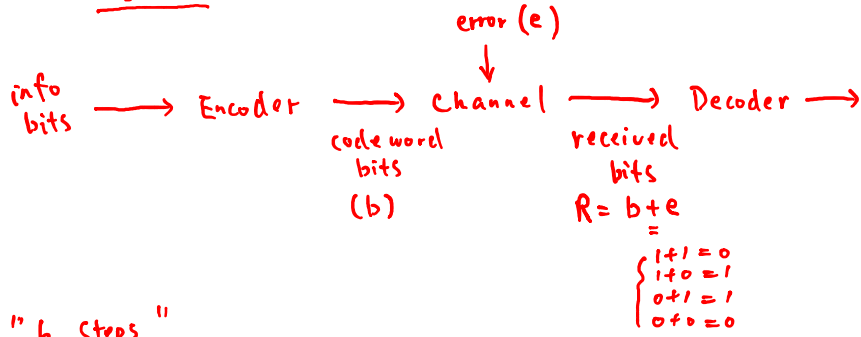
Low
Density
Parity
check

Wi-Fi 6 (11ax)

10 Gbps Ethernet

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{ CRC (Cyclic Redundancy Check) code
 Polynomial code



"6 Steps"

① k info bits \Rightarrow info polynomial

$$i_{k-1}, i_{k-2}, \dots, i_0 \Rightarrow i(x) = \underbrace{i_{k-1}}_{1} * X^{k-1} + i_{k-2} * X^{k-2} + \dots + i_1 * X^1 + \underbrace{i_0}_{1} * X^0$$

(binary polynomial)

② $(n-k)$ check bits \Rightarrow generator poly $g(x)$ of degree $(n-k)$ $\stackrel{*}{=} \#$ check bits

$$g(x) = \underbrace{g_{n-k}}_{=0} \cdot x^{n-k} + \dots + \underbrace{g_0}_{=0} \cdot x^0$$

k : # info bits

n : # codeword bits

$$\{ \rightarrow g_{n-k} = 1$$

$$\rightarrow g_0 = 1$$

E.g. $n-k=3$

$$g(x) = x^3 + x^2 + x \rightarrow \text{not valid}$$

$$g(x) = x^3 + 1 \quad \checkmark$$

$$g(x) = x^3 + x + 1 \quad r$$

$$g(x) = x^3 + x^2 + 1 \quad \checkmark$$

③ dividend poly
 $a(x) = \underbrace{\bar{L}(x)}_{\text{info}} * X^{\underline{n-k}} \# \text{ check bits}$

④ use $g(x)$ as divisor poly
 find the remainder poly : $\underline{r(x)}$
 $a(x) = g(x) * \underbrace{q(x)}_{\text{quotient poly}} + r(x)$

⑤ codeword poly
 $\underline{b(x)} = \underline{a(x)} + \underline{r(x)}$

⑥ $b(x)$ coefficients \Rightarrow codeword bits (n)

"binary polynomial arithmetic"

\Rightarrow coefficient calculations follow 2's complement arithmetic

$$\begin{array}{llll} 1+1=0 & 0-1=1 & 1+0=1 & 0+0=0 \\ 1-1=0 & 0-0=0 & 1-0=1 & 0-0=0 \end{array}$$