Stat 330 HW2

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1 $X = 0$ $\{F, F, F\}$

$x = 1$ $\{S, F, F\}, \{F, S, F\}, \{F, F, S\}$

$x = 2$ $\{S, S, F\}, \{S, F, S\}, \{F, S, S\}$

$x = 3$ $\{S, S, S\}$

2 X = # red marbles chosen

4 red 3 green 2 blue

9 marbles

a) $Im(x) = \{0, 1, 2, 3\}$

& c)

b)

X	0	1	2	3
$p_X(x)$	$\frac{60}{504}$	$\frac{80}{504}$	$\frac{60}{504}$	$\frac{24}{504}$

$\frac{5}{9} \cdot \frac{4}{8} \cdot \frac{3}{7}$ $\frac{4}{9} \cdot \frac{5}{8} \cdot \frac{4}{7}$ $\frac{4}{9} \cdot \frac{3}{8} \cdot \frac{5}{7}$ $\frac{4}{9} \cdot \frac{3}{8} \cdot \frac{2}{7}$

3.

x	0	1	2	3	4	5	6
$p_X(x)$	.10	.15	.2	.25	.2	.06	.04

a) $p(x \leq 3) = p(0) + p(1) + p(2) + p(3)$

$= .10 + .15 + .2 + .25 = .7$

b) $p(2 \leq x \leq 5) = p(2) + p(3) + p(4) + p(5)$

$= .2 + .25 + .2 + .06 = .71$

c)

x	0	1	2	3	4	5	6
$p_X(x)$	.10	.15	.2	.25	.2	.06	.04
$F_X(x)$	.10	.25	.45	.7	.9	.96	1

d) $E(X) = 0 \cdot (.10) + 1(.15) + 2(.2) + 3(.25) + 4(.2) + 5(.06) + 6(.04)$

$E(x) = 2.64$

e) $Var(x) = E(x^2) - [E(x)]^2$

$= 0(.10) + 1^2(.15) + 2^2(.2) + 3^2(.25) + 4^2(.2) + 5^2(.06) + 6^2(.04) - [E(x)]^2$

$Var(x) = 9.34 - (2.64)^2 = 2.3704$

f) $2.64 \pm 2.3704 = 5.01$ higlest # of expected lines

1 supervisor

4. Bernoulli:

a) $X \sim Bern(.51)$

x	0	1
$p_X(x)$	0.49	0.51

b) $E(X) = 0 \cdot (.49) + 1(.51)$
$= .51$

c) $var(x) = E(x^2) - [E(x)]^2$

$= 0 \cdot (.49) + 1^2(.51) - [.51]^2$
$= .51 - .2601 = .2499$

d) $E(x^{20}) = 0 \cdot (.49) + 1^{20}(.51) = .51$

5. a) Yes, $Bin(10, 1/6)$

b) Yes, $Bin(10, 1/4)$

c) No, probability cannot change between trials

d) ~~No, there is no probability for the weight of an apple~~

Yes, $Bin(15, p)$ for some p unknown
probability of apple being more 150 g

6. Poisson $.3 \cdot 5$ iphones $= 1.5 = \lambda$

$X \sim Pois(1.5)$ $\quad p_X(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$

a) $p_X(2) = \frac{e^{-1.5} \cdot 1.5^2}{2!}$

$= 0.25$

b) $p_X(x \geq 2) = 1 - p_X(x=1) - p_X(x=0) = .442$

c) $E(x) = \lambda = 1.5$ iphones

d) $\sigma = \sqrt{\lambda} = \sqrt{1.5} = 1.22$

7. a) $p(x=2) + p(x=3) = .0104$

b) $p(x=3) + p(x=4) + p(x=5) = .00196$

c) $25{,}000 \cdot .06 = \underline{1500}$ error bits without repeats

$25{,}000 \cdot .0104 = 260$ error bits with triplets

8. X = # of games till loss

$X \sim Geo(p)$, F = winning

S = loss

60% to win, so 40% to lose $= p$

$X \sim Geo(.4)$

a) $p_X(x) = (1-p)^{x-1} p$

$p_X(2) = (1-.4)^{2-1} \cdot .4$

$= .6 \cdot .4 = .24$

b) $F_X(t) = P(X \geq 3) = 1 - p(X<3) = 1 - (p(2) + p(1))$

$= 1 - (.24 + 1 \cdot .4)$

$= 1 - .64$

$= .36$

c) $E(X) = \frac{1}{p} = \frac{1}{.4} = 2.5$ games

d) $\sigma = \sqrt{\frac{1-p}{p^2}} = \sqrt{\frac{.6}{(.4^2)}} = 1.5811$

9. a) $X \sim Pois(.8)$

b) $p(x<2) = 0.8088$ via Distribution chart

c) $p(X \geq 1) = 1 - p(x=0) = 1 - \frac{e^{-.8} \cdot .8^0}{0!}$

$= 1 - e^{-.8} = .55$

d) $E(x) = 5\lambda = 4$ goals

e) Y = # of games till goal scored

$Y \sim Geo(.55)$, ~~goals a game expected, so the average of scoring a goal for a game~~

f) $p_x(x>3) = 1 - p_x(x \leq 3)$

$= 1 - (1-(1-.55)^{\lfloor 3. \rfloor}$

$= .09125$

10. a)

X \ Y	0	1	2	$p_X(X)$
0	0.3	0.1	0.1	0.5
1	0.2	0.1	0	0.3
2	0.1	0.1	0	0.2

$p_Y(Y) = 0.6 \quad 0.3 \quad 0.1$

b) $E(Y) = 0 \cdot 0.6 + 1 \cdot 0.3 + 2 \cdot 0.1$
$= 0.3 + 0.2 = 0.5$

$E(X) = 0 \cdot 0.5 + 1 \cdot 0.3 + 2 \cdot 0.2$
$= 0.3 + 0.4 = 0.7$

$var(Y) = 0^2 \cdot 0.6 + 1^2 \cdot 0.3 + 2^2 \cdot 0.1 - [E(Y)]^2$
$= 0.3 + 0.4 - 0.25$
$= 0.45$

$var(X) = 0^2 \cdot 0.5 + 1^2 \cdot 0.3 + 2^2 \cdot 0.2 - [E(X)]^2$
$= 0.3 + 0.8 - 0.49$
$= 0.61$

c) $cov(X,Y) = E(XY) - E(X)E(Y)$
$E(X,Y) = 0 + 0 + 0 + 0 + 0.1 + 0 + 0 + 0.2 + 0$
$= 0.3$
$cov(X,Y) = 0.3 - .35 = -.05$

$corr(X,Y) = \frac{cov(X,Y)}{\sqrt{var(X) \cdot var(Y)}} = \frac{-.05}{\sqrt{0.45 \cdot 0.61}} = .668$

d) $cov(X,Y) \neq 0 \Rightarrow$ not independent

11. a) $P(X=Y)$

$= p_{X,Y}(0,0) + p_{X,Y}(1,1) + p_{X,Y}(2,2)$

$= 0.3 + 0.1 + 0 = 0.4$

b) $P(X<Y) = p_{X,Y}(0,1) + p_{X,Y}(0,2) + p_{X,Y}(1,2)$

$= 0.1 + 0.1 + 0 = 0.2$

c) $P(X>Y) = p_{X,Y}(1,0) + p_{X,Y}(2,0) + p_{X,Y}(2,1)$

$= 0.2 + 0.1 + 0.1 = 0.4$

d) $P(X=2 \mid Y=1) = p_{X,Y}(2,0) + p_{X,Y}(2,1) + p_{X,Y}(2,2)$

$+ p_{X,Y}(0,1) + p_{X,Y}(1,1)$

$= 0.1 + 0.1 + 0 + 0.1 + 0.1 = 0.4$

e) $p_{X,Y}(0,0) = 0.3$

f) $p_{X,Y}(1,2) = 0$