Theorem 1. $\sum_{i=1}^{n} = \frac{n(n+1)}{2}, \forall n \in \mathbf{Z}^{+}.$

Proof.

(1) for n = 1,

$$\sum_{i=1}^{1} = 1 \qquad \cdots\cdots\cdots \qquad (1)$$

$$\frac{n(n+1)}{2} = \frac{1 \times (1+1)}{2} = 1 \qquad \cdots\cdots\cdots \qquad (2)$$

$\because$ (1) is equal to (2)

$\therefore for\ n = 1,\ the\ equation\ holds.$

(2) for n = 2,

$$\sum_{i=1}^{2} = 1 + 2 = 3 \qquad \cdots\cdots\cdots \qquad (1)$$

$$\frac{n(n+1)}{2} = \frac{2 \times (1+2)}{2} = 3 \qquad \cdots\cdots\cdots \qquad (2)$$

$\because$ (1) is equal to (2)

$\therefore for\ n = 2,\ the\ equation\ also\ holds.$

(3) Assume for n = k, the equation holds

$$\sum_{i=1}^{k} = 1 + 2 + 3 + \cdots + k = \frac{k \times (k+1)}{2}$$

(4) let n = k + 1, such that:

$$target : \sum_{i=1}^{k+1} = \frac{(k+1)(k+2)}{2} \qquad \cdots\cdots \qquad L.H.S.$$

$$\sum_{i=1}^{k+1} = \sum_{i=1}^{k} +(k+1)$$

$$= \frac{k \times (k+1)}{2} + (k+1)$$

$$= \frac{k \times (k+1) + 2(k+1)}{2}$$

$$= \frac{(k+2) \times (k+1)}{2} \qquad \cdots\cdots \qquad R.H.S$$

$$\because \; L.H.S. \ is \ equal \ to \ R.H.S.$$

$$Therefore, \ when \ n = k+1, \ the \ equation \ holds.$$

(5) According to definitions of Mathematical Induction, the equation holds.

$\square$