

**Theorem 1.**  $\sum_{i=1}^n = \frac{n(n+1)}{2}, \forall n \in \mathbf{Z}^+.$

*Proof.*

(1) for  $n = 1,$

$$\sum_{i=1}^1 = 1 \quad \dots\dots\dots (1)$$

$$\frac{n(n+1)}{2} = \frac{1 \times (1+1)}{2} = 1 \quad \dots\dots\dots (2)$$

$\therefore$  (1) is equal to (2)

$\therefore$  for  $n = 1,$  the equation holds.

(2) for  $n = 2,$

$$\sum_{i=1}^2 = 1 + 2 = 3 \quad \dots\dots\dots (1)$$

$$\frac{n(n+1)}{2} = \frac{2 \times (1+2)}{2} = 3 \quad \dots\dots\dots (2)$$

$\therefore$  (1) is equal to (2)

$\therefore$  for  $n = 2,$  the equation also holds.

(3) Assume for  $n = k,$  the equation holds

$$\sum_{i=1}^k = 1 + 2 + 3 + \dots + k = \frac{k \times (k+1)}{2}$$

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(4) let  $n = k + 1$ , such that:

$$\text{target : } \sum_{i=1}^{k+1} = \frac{(k+1)(k+2)}{2} \quad \dots\dots \quad L.H.S.$$

$$\sum_{i=1}^{k+1} = \sum_{i=1}^k + (k+1)$$

$$= \frac{k \times (k+1)}{2} + (k+1)$$

$$= \frac{k \times (k+1) + 2(k+1)}{2}$$

$$= \frac{(k+2) \times (k+1)}{2} \quad \dots\dots \quad R.H.S$$

$\therefore$  *L.H.S. is equal to R.H.S.*

*Therefore, when  $n = k + 1$ , the equation holds.*

(5) According to definitions of Mathematical Induction, the equation holds.

□