Theorem 1. $\sum_{i=1}^{n} = \frac{n(n+1)}{2}, \forall n \in \mathbf{Z}^+$.

Proof.

(1) for n = 1,

$$\sum_{i=1}^{1} = 1 \qquad \cdots\cdots\cdots \qquad (1)$$

$$\frac{n(n+1)}{2} = \frac{1 \times (1+1)}{2} = 1 \qquad \cdots\cdots\cdots \qquad (2)$$

$$\because (1) \text{ is equal to } (2)$$

$$\therefore for\ n = 1,\ the\ equation\ holds.$$

(2) for n = 2,

$$\sum_{i=1}^{2} = 1 + 2 = 3 \qquad \cdots\cdots\cdots \qquad (1)$$

$$\frac{n(n+1)}{2} = \frac{2 \times (1+2)}{2} = 3 \qquad \cdots\cdots\cdots \qquad (2)$$

$$\because (1) \text{ is equal to } (2)$$

$$\therefore for\ n = 2,\ the\ equation\ also\ holds.$$

(3) Assume for n = k, the equation holds

$$\sum_{i=1}^{k} = 1 + 2 + 3 + \cdots + k = \frac{k \times (1+k)}{2}$$

□