Restitution Coefficient

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In this experiment, I found an experimental value for the restitution coefficient (e) for poplar wood with respect to a plastic ball. I did this by dropping a plastic ball on a poplar wooden plank from ten different heights and analyzing the max height the ball reached after bouncing off the wood. Also, I determined whether the angle at which the plank is angled influences the restitution coefficient. I did this by dropping the ball from ten different heights at ten different angles. To conclude, I found that the average restitution coefficient for the non-angled wooden plank and angled plank scenario to be e = 0.373042918 with an uncertainty of ± 0.016036501. Some interesting observations I saw while doing the experiment were that the height at which you drop the ball onto the wood has no effect on the restitution coefficient. The angle at which you drop the ball onto the wooden plank has no effect on the restitution coefficient. The height at which you drop the ball onto the angled wooden plank has no correlation to the angle at which the ball bounces off the plank with respect to the perpendicular. My recorded θ_f (angle at which the ball bounces off the plank) values aligned closely with the theoretical values I got from my V-Python program, with an average % error of 1.68% with a standard deviation of ± 2.201888161%. And my recorded h_f (the maximum height the ball reaches after bouncing once) values aligned relatively close to the theoretical values I got from my V-Python program, with an average % error of 6.05% with a standard deviation of ± 2.755548448%.

I. Introduction

The restitution coefficient is defined as the ratio of the final velocity to the initial velocity between two objects after their collision[1]. There are many factors that influence the restitution coefficient such as the material of both objects in the collision, the temperature of the two objects, and the shape of the objects. For instance, if you were to drop a rubber ball on a tile floor, you will expect the ball to bounce back a bit before the original height at where you dropped it. However, if you were to drop a heavy metal ball on the same tile floor, the ball will lose a lot of its speed after the collision, and therefore, barely bounce at all. That said, a restitution coefficient value of 1 signifies that the collision is perfectly elastic, meaning that no velocity was lost between the two objects after colliding. A restitution coefficient value of 0 signifies that the collision is perfectly inelastic, meaning that all velocity was lost between the two objects after colliding.

Pertaining to the collision of two objects, in 1687, Isaac Newton formulated a theory that is known as Newton's law of restitution. In Newton's seminal work, "Philosophiæ Naturalis Principia Mathematica", he described the behavior of colliding bodies in terms of the relative velocities before and after the collision. He then described the ratio between velocity before and after impact to be the "restitution coefficient", where it has a key property that the coefficient must be between 0 and 1, and it can be used to describe the elasticity of the collision[2].

After Newton, many scientists such as August Toepler (1836–1912), Albert A. Michelson (1836–1912), and John Clerk Maxwell (1831–1879) expanded on Newton's findings of collisions and the restitution coefficient. August Toepler, most known for his work on electrostatics, also conducted experiments to gain a deeper understanding of the mechanical properties of materials, including the elasticity of a material[3]. Albert Michelson, although known mostly for his work on the speed of light and optics, conducted experiments on the elasticity of solids. Regarding collisions and elasticity, Michelson was most known for his interferometric methods which allowed for more precise measurements in the world of physics[4]. John Maxwell, more known for his studies of statistical mechanics, laid the foundation for statistical mechanics and the study of particle collisions[5].

To further add to these findings, I performed an experiment to determine the restitution coefficient of a poplar wooden plank with respect to a plastic ball. I aimed to determine whether the angle at which the wooden plank is tilted at influences the restitution coefficient, as well as whether the height at which you drop the plastic ball influences the angle at which the ball bounces from the wood. I also aimed to find how accurate my recorded hf and θ_f values were with respect to the theoretical values I retrieve from my V-Python program.

II. Equations

To describe the restitution coefficient for the collision in my experiment where a plastic ball falls and collides with a stationary object, the wooden plank, the equation $e = \frac{vf}{vi}$, where e represents the restitution coefficient, v_f represents the final velocity of the ball after the collision, and v_i represents the initial velocity of the ball before the collision. To determine v_f, the derived kinematics equation $v_f = \sqrt{(2h_f * g)}$ is used, where h_f represents the maximum height, the ball reaches after one bounce, and g represents the value of gravity in Houston (g=9.79). To determine v_i, the derived kinematics equation $v_i = \sqrt{(2h_i * g)}$ is used, where h_i represents the height at which the ball is dropped at, and g represents the value of gravity as seen in the equation for v_f.

To describe the restitution coefficient for the collision between a stationary angled object, the wooden plank, and a plastic ball falling to the perpendicular of the wooden plank, the equation $e = \frac{\tan(\theta i)}{\tan(\theta f)}$ is used. Where e represents the restitution coefficient. θ_i represents the angle, the ball hits the plank with respect to the perpendicular the board, this is also the angle at which the board is set. θ_f represents the angle at which the ball bounces off the plank with respect to the perpendicular of the plank. To derive this equation, trigonometry is used where you divide the velocity of the ball after bouncing off perpendicular to the angled plank by the velocity of the ball before it bounces off the angled plank. The velocity of the ball after bouncing off perpendicular to the angled plank is $V_i \sin\theta_i / \tan\theta_f$, where the velocity of the ball before it bounces off the angled plank is $V_i \cos\theta_i$. Simplifying, the restitution coefficient equation for angled collisions between a stationary object and a falling object ends up being $e = \frac{\tan(\theta i)}{\tan(\theta f)}$. Interesting thing to note is that the restitution coefficient only depends on the initial angle that the ball before it bounces off the angled plank with respect to the angled plank, and the final angle at which the ball bounces off the perpendicular to the plank.

Although air drag has a minimal effect on the motion of the ball due to the dropping heights being small, it is still important to take air drag into account to retrieve accurate values in the V-Python program simulation. That said, the equation for air drag is $F_{drag} = -(kv^2)\langle v \rangle$, where k represents air force drag coefficient, velocity represents the magnitude of the ball's velocity, and <v> represents the direction of velocity. To determine the air force drag coefficient, the equation k = (1/2)Aρc, where A represents the cross section of the ball (A = 4.91*10-4m2), ρ represents the density of the air (ρ = 1.2 kg/m3), and c represents the aerodynamic drag constant (c = 0.47).

To determine the percent error between experimental and theoretical data, the equation $100\% * \left| \frac{\text{Experimental} - \text{Theoritical}}{\text{Theoritical}} \right|$ is used. Where Experimental represents the experimental values observed from the experiment, and Theoretical represents the theoretical values observed from a computer or a scientist that has accurately determined a precise value.

III. Experimental Setup

To perform this experiment, I used a rectangular poplar wooden plank (can be any size), tape, a plastic ball, a phone with a slow-mo video feature, a meter stick, and a poster board with angles between 0 and 180 sketched out. For the first part of the experiment where I measured the restitution coefficient from a falling plastic ball bouncing off a stationary wooden plank. I laid out the rectangular poplar wooden plank flat on the floor against a wall while setting a meter stick on top of the plank leaning against the wall completely vertical. To have the meter stick not slide or fall, I used tape to keep the meter stick in place. Performing the experiment, I dropped the plastic ball from 10 different heights, hi, (0.10, 0.14, 0.18, 0.22, 0.26, 0.30, 0.34, 0.38, 0.42, and 0.46m) while recording with my phone using the slow-mo feature to then look back at the recording and record the final height (h_f) the ball reaches after bouncing once, and I used the bottom of the ball to record the h_i and h_f values. Below in FIG 1 is a sketch of the experimental setup for the first part of the experiment as well as the V-Python program used to find simulate the experiment.

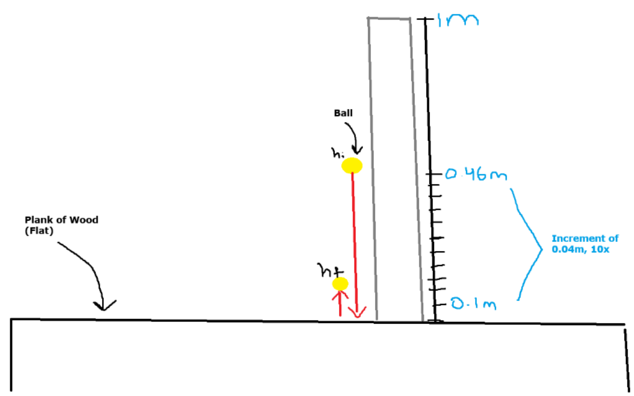


FIG 1. Sketch of the experimental setup of the first part of the experiment. The trinket for the V-Python computer simulation of the first part of the experiment: https://trinket.io/glowscript/02d0f25fa2.

To perform the second part of the experiment, I sketched angles from 0-180 on a white poster board by using a protractor, and I laid the white poster flat along a vertical wall. I then angled the wooden plank along the poster board at angle values (θ_i) of (5°, 10°, 15°, 20°, 25°, 30°, 35°, 40°, 45°, 50°), where I dropped the ball at 10 different heights, h_i, (same heights as the first part of the experiment) for each angle. With my phone, I slo-mo recorded the ball falling at 10 different heights at 10 different angles and observed the angle at which the ball bounced from the plank (θ_f). Below in FIG 2 is a sketch of the experimental setup for the second part of the experiment as well as the V-Python program used to simulate the experiment.

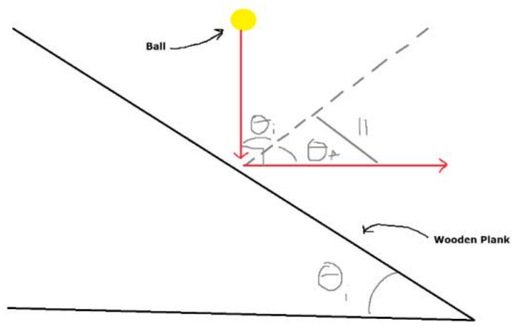


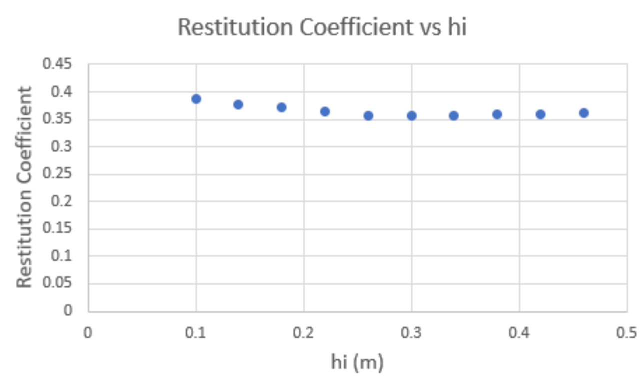
FIG 2. Sketch of the experimental setup of the second part of the experiment. The trinket for the V-Python program for the second part of the experiment: https://trinket.io/glowscript/1f45f16e8f.

IV. Data for Flat Wooden Plank Part

	θi = 0			
hi (m)	hf (m)	e	hf (vpython)	% error (hf and hf(vpython))
0.1	0.015	0.387294	0.0138841	8.037251244
0.14	0.02	0.377955	0.019425	2.96010296
0.18	0.025	0.372686	0.0249586	0.165874688
0.22	0.029	0.363081	0.0304849	4.870936103
0.26	0.033	0.35626	0.0360041	8.343771959
0.3	0.038	0.355908	0.0415158	8.468583045
0.34	0.043	0.355635	0.0470206	8.550720323
0.38	0.049	0.359081	0.0525177	6.698122728
0.42	0.054	0.358566	0.0580081	6.909552287
0.46	0.06	0.361162	0.063491	5.498417098

e avg	e STDEV
0.364762572	0.010878078
% error avg	% error STDEV
6.050333244	2.755548448

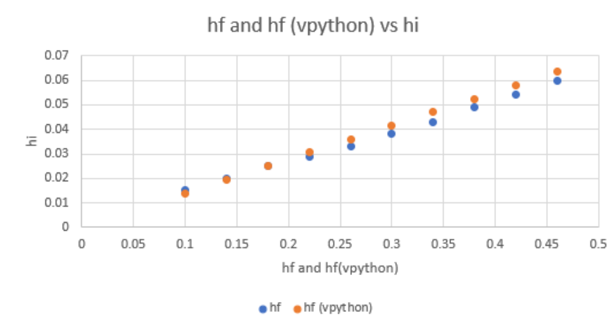
TABLE 1. Initial h_i and h_f values recorded during the experiment. Calculated restitution coefficient values for each of the different dropping heights, the average and standard deviation of all the restitution coefficient values. Also, the theoretical h_f values from the V-Python program simulation as well as the % error, % error average, and % error standard deviation between my recorded h_f and the V-Python h_f values.



GRAPH 1. Graph showing the relationship between the restitution coefficient and h_i values.

As seen in GRAPH 1, there is no correlation between the restitution coefficient and the height at which you drop the ball. This is because as seen TABLE 1, the higher you drop the ball, the higher the maximum height the ball reaches after bouncing once (h_f). That said, in the derived kinematics equations $v_i = \sqrt{(2h_i * g)}$ and $v_f = \sqrt{(2h_i * g)}$, the heights (hi, hf) are directly related to the velocity of the ball before (v_i) and after (v_f)

it bounces. Using the restitution coefficient equation for flat surface collisions, $e = \frac{vf}{vi}$, the restitution coefficient experiences no change since when h_i increases, so does v_f and v_i.



GRAPH 2. Graph showing the relationship between the recorded and theoretical h_f values and h_i values.

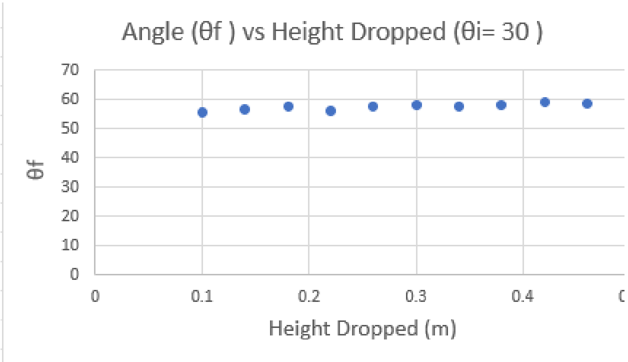
As seen in GRAPH 2, my experimental values h_f seem to fit closely to the theoretical V-Python values h_f (vpython). As seen in TABLE 1, the average % error my experimental values and the V-Python values was relatively small with an average of 6.05% with a standard deviation of ± 2.755548448%. I found this by using the % error equation % error = 100%*|(Experimental-Theoritical)/Theoritical| for each h_f and h_f(vpython) value to the respected h_i value. I then took the average and standard deviation of all the errors, and a deeper look into each of the individual errors can be seen in TABLE 1. Interestingly enough, the error started to increase the longer I did the experiment. This could be attributed to the ball making a small dent on the wood after landing on it many times.

V. Data for Angled Wooden Plank Experiment

θi	θf avg	θf STDEV	e avg	e STDEV	θf (vpython)	% error (θf avg and θf (vpython))
0	0	0	0.364763	0.010878	0	0
5	12.24	1.250955	0.407290686	0.044425569	13.1989	7.264999356
10	25.64	0.562139	0.3675329	0.009165256	25.2988	1.34868057
15	37.21	1.191125	0.353257868	0.015250299	35.6889	4.262109507
20	44.73	0.828721	0.367557493	0.010687261	44.2947	0.982736084
25	51.35	1.156864	0.373139986	0.015458639	51.3404	0.018698725
30	57.6	1.146977	0.366585876	0.016353713	57.1323	0.818626241
35	61.65	1.141393	0.377986007	0.018026219	61.9531	0.489241055
40	67.097	1.069829	0.354630409	0.018368698	66.0313	1.613931575
45	68.48	1.435889	0.394571437	0.02896949	69.5423	1.527559485
50	72.49	1.217876	0.376156438	0.027903743	72.6188	0.177364539

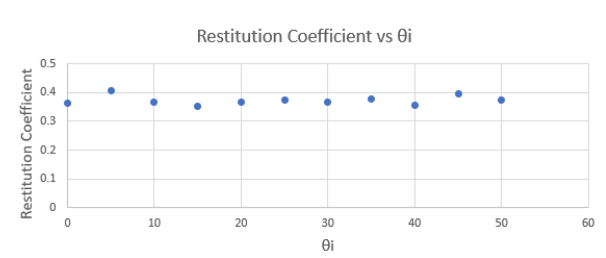
overall e avg	STDEV overall e avg	% error average	% error STDEV
0.373042918	0.016036501	1.682177012	2.201888161

TABLE 2. Recorded θ_i, θ_f avg (average angle at which the ball bounced from the wooden plank for the respective θ_i) values, and the standard deviation of the θ_f values. Calculated restitution coefficient and standard deviation values for all θ_i. Theoretical θ_f values from V-Python, and the % error between the recorded and theoretical θ_f values. The overall restitution coefficient average and standard deviation of all the restitution coefficient values. And the average % error and % error standard deviation of the theoretical and experimental θ_f values.



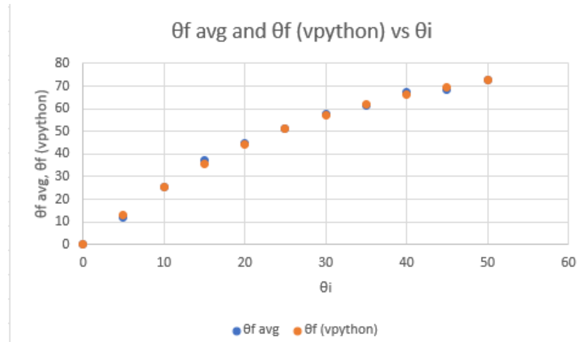
GRAPH 3. Graph showing the relationship between θ_f and h_i at an angle of $\theta_i = 30$.

As seen in GRAPH 3, there is no correlation between the height at which you drop the ball and the angle at which the ball bounces off the wooden plank. This can be explained using the restitution coefficient equation regarding angled surface collisions $e = \frac{\tan(\theta i)}{\tan(\theta f)}$. The restitution coefficient only depends on the angle at which the wooden plank is at (θi), and the angle at which the ball bounces off the wooden plank (θ_f).



GRAPH 4. Graph showing the relationship between the restitution coefficient and θ_i.

As seen in GRAPH 4, there is no correlation between restitution coefficient, and the angle at which the wooden plank is at (θi). Looking at the restitution coefficient equation for angled surface collisions $e = \frac{\tan(\theta i)}{\tan(\theta f)}$, why would the restitution coefficient not increase with an increasing θ_i? Like the hi and hf relationship seen with flat surface collisions, as the angle of the wooden plank increases, so does the angle at which the ball bounces off the wooden plank. This can also be seen in TABLE 2, as θ_i increases, so does θ_f, which means that the initial angle has no effect on the restitution coefficient.



GRAPH 5. Graph showing the relationship between the θ_f avg, θ_f (vpython), and θ_i.

As seen in GRAPH 5, my experimental values of θ_f seem to be close to the theoretical values of θ_f I got from my V-Python program. As seen in TABLE 2, the average % error is 1.68% with a standard deviation of ± 2.201888161%. I calculated this using the % error equation, % error = 100%*|(Experimental-Theoritical)/Theoritical| for each θ_f and θ_f (vpython) values with respect to their respective θ_i value. I then took the average and standard deviation of all the errors, and a deeper look into each of the errors individually can be seen in TABLE 2. The largest errors seem to have occurred in the beginning of the experiment, which is interesting because it is the contrary to the flat wooden plank part of the experiment. This could possibly be attributed to human error where the ball could have hit different parts of the wood in the beginning of the experiment compared to the middle and end of the experiment.

VI. Conclusions

In this experiment I found that the average restitution coefficient for a plastic ball colliding with a poplar wooden plank to be e = 0.373042918 with an uncertainty of ± 0.016036501. I determined that the height at which you dropped the ball onto the wooden plank has no effect on the restitution coefficient. I determined that the angle at which you drop the ball onto the wooden plank (the angle of the inclined wooden plank), has no effect on the restitution coefficient. And I also determined that the height at which you drop the ball does not affect the angle at which the ball bounces off the wooden plank. Overall, I found that my recorded experimental values for h_f and θ_f aligned closely to the theoretical values I got from my V-Python program. For the h_f values, I got an average % error of 6.05% with a standard deviation of ± 2.7555548448%. For the θ_f values, I got an average % error of 1.68% with a standard deviation of ± 2.201888161%.

The errors found in this experiment may be attributed to the ball hitting the same area of the board too much, which in return, leaves a dent in the wood. Also, it could may be attributed to the ball not consistently hitting the same spot of the wooden plank. Although both issues contradict each other, a way to improve from this experiment and eliminate these errors as much as possible is by having multiple wooden planks (about 3-4) with the same dimensions and to mark out the area the ball is going to hit with a marker. This would make it that the ball can consistently hit the same spot, and once a dent starts to appear, that is when the other wooden planks will be of use. To expand from this experiment, I suggest experimenting on the restitution coefficient of objects with different materials and/or shapes. Another interesting experiment could be how does temperature affect the restitution coefficient between two objects since the temperature can determine how stiff and hard a material is.

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Appendix

	θi= 10	
hi	θf	e
0.1	25	0.378134
0.14	25.1	0.376418
0.18	25.3	0.373023
0.22	25.3	0.373023
0.26	25.2	0.374714
0.3	25.5	0.369677
0.34	25.9	0.363131
0.38	26.5	0.353657
0.42	26.4	0.355208
0.46	26.2	0.358344

	θi=20	
hi	θf	e
0.1	43.1	0.388947
0.14	44.2	0.374279
0.18	45.5	0.357673
0.22	44.3	0.372974
0.26	44.6	0.369088
0.3	44.7	0.367802
0.34	44.4	0.371674
0.38	44.9	0.365243
0.42	45.9	0.352712
0.46	45.7	0.355184

	θi=30	
hi	θf	e
0.1	55.5	0.396802
0.14	56.9	0.37637
0.18	57.8	0.363577
0.22	56	0.389428
0.26	57.9	0.362171
0.3	58.2	0.357972
0.34	57.6	0.366398
0.38	58.4	0.355188
0.42	59.1	0.345537
0.46	58.6	0.352416

	θ=40	
hi	θf	e
0.1	65.8	0.377106
0.14	66.1	0.371838
0.18	66.4	0.366594
0.22	66.4	0.366594
0.26	66.7	0.361373
0.3	67.2	0.352725
0.34	67.07	0.354967
0.38	67.7	0.34414
0.42	68.4	0.332223
0.46	69.2	0.318744

	θi=50	
hi	θf	e
0.1	70.5	0.422022
0.14	71	0.410354
0.18	71.7	0.394135
0.22	72	0.387224
0.26	72.4	0.378047
0.3	72.4	0.378047
0.34	73.8	0.346236
0.38	73.4	0.355277
0.42	74	0.34173
0.46	73.7	0.348493

Measurements for θ_i = 10, 20, 30, 40, and 50.

	θi= 5	
hi	θf	e
0.1	10	0.496173
0.14	11.2	0.44185
0.18	11.5	0.430021
0.22	11.5	0.430021
0.26	11.9	0.415163
0.3	12.5	0.394636
0.34	13.1	0.37596
0.38	12.9	0.381996
0.42	13.8	0.35619
0.46	14	0.350898

	θi= 15	
hi	θf	e
0.1	35.5	0.375651
0.14	36	0.3688
0.18	36	0.3688
0.22	36.5	0.362113
0.26	38.5	0.336858
0.3	37.5	0.349198
0.34	38	0.342959
0.38	37	0.355581
0.42	38.1	0.341728
0.46	39	0.33089

	θi= 25	
hi	θf	e
0.1	49.5	0.398264
0.14	50.1	0.389894
0.18	50.5	0.384394
0.22	50.8	0.380311
0.26	51.2	0.374921
0.3	51.5	0.370918
0.34	51.7	0.368268
0.38	52.5	0.35781
0.42	53	0.351388
0.46	52.7	0.355231

	θi= 35	
hi	θf	e
0.1	60	0.404265
0.14	60.5	0.396158
0.18	60.5	0.396158
0.22	60.9	0.389731
0.26	61.5	0.380182
0.3	61.8	0.375448
0.34	63	0.356774
0.38	62.5	0.364505
0.42	62.7	0.361404
0.46	63.1	0.355236

	θi= 45	
hi	θf	e
0.1	66.5	0.434812
0.14	66.8	0.428601
0.18	67	0.424475
0.22	67.9	0.406058
0.26	68.5	0.39391
0.3	68.5	0.39391
0.34	69	0.383864
0.38	70.1	0.361995
0.42	70.5	0.354119
0.46	70	0.36397

Measurements for θ_i = 5, 15, 25, 35, and 45.