



FLAGR

A flexible rank aggregation library

User Manual

Version: 1.0.8

Web Site: <https://flagr.site/>

Github: <https://github.com/lakritidis/FLAGR>

Python Package Index: <https://pypi.org/project/pyflagr/>

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1. Getting started

1.1 Introduction

FLAGR is a high performance, modular, open source library for rank aggregation problems. It implements baseline and recent state-of-the-art aggregation algorithms that accept ranked preference lists and generate a single consensus list of elements.

The core project is developed in C++. The source code is [available on GitHub](#) and can be compiled as a standard application, or as a shared library. In the second case, the library file can be linked or loaded by other programs in other languages. PyFLAGR is an example of such application. In brief, FLAGR:

- employs efficient data structures and algorithms that ensure high performance,
- is cross-platform supporting Windows and Linux. An extension that supports MacOS users is currently under construction,
- is modular, allowing third-party programmers to easily implement their methods within the core library,
- is open-source.

The current version of FLAGR is 1.0.8. It includes implementations of the following algorithms:

- [CombSUM](#) linear combination with 5 different score/rank normalization techniques; namely: Rank, Borda, Simple Borda, Score, and Z-Score normalization (Renda, et al., 2003).
- [CombMNZ](#) linear combination with 5 different score/rank normalization techniques; namely: Rank, Borda, Simple Borda, Score, and Z-Score normalization (Renda, et al., 2003).
- [Borda Count](#) (equivalent to CombSUM with Borda normalization, (Renda, et al., 2003)).
- [Condorcet Winners](#).
- [Copeland Winners](#).
- [Outranking Approach](#) of Farah & Vanderpooten, 2007.
- [Distance-based iterative unsupervised algorithm](#) of Akritidis et al., 2022 (all the above methods can be used as the starting non-weighted aggregator).
- [Robust Rank Aggregation](#) in two variants: the first one employs the Stuart/Ares method for p-value correction, whereas the other one does not.
- [Kemeny optimal aggregation](#) (brute force implementation, not applicable to large, or many input preference lists).
- [Markov Chains \(MC\) methods](#) of Dwork et al., 2001 and DeConde et al., 2006.
- [Weighted agglomerative aggregation method](#) of Chatterjee et al., 2018.
- [Preference relation unsupervised algorithm](#) of Desarkar et al., 2016.

These methods are also supported by PyFLAGR through a set of modules and classes.

1.2 Download

FLAGR is an open-source project, licensed under the [Apache License, version 2](#).

The library can be downloaded from its GitHub repository at <https://github.com/lakritidis/FLAGR>. The repository contains:

- The core C++ components (`/src` directory) and rank aggregation algorithm implementations (`/src/ram` directory).
- The dynamic library references (`cflagr.cpp` and `dllflagr.cpp` files), which allow the compilation of FLAGR as a shared or a dynamic link library.
- A precompiled shared library (`pyflagr/pyflagr/flagr.so`, Linux only) and a dynamic link library (`pyflagr/pyflagr/flagr.dll`, Windows only) with its dependencies.
- The PyFLAGR library, which allows the usage of FLAGR in Python applications (`/pyflagr` directory).
- Documentation (`/docs` directory).
- Code examples for C++ and Python (`/examples` directory).

1.3 Compilation and execution

FLAGR can be compiled as a standard console application, or as a shared/dynamic link library in both Linux and Windows systems.

Building FLAGR in Linux

In Linux platforms, the user must navigate to the root directory of the package through the Terminal and execute the makefile that exists there by typing:

```
make
```

The build script compiles the source code and produces two files within the `/bin/Release` directory:

- the binary executable file `/bin/Release/FLAGR`
- the Linux shared library `/bin/Release/flagr.so` that can be linked by third-party applications to obtain access to the FLAGR algorithm implementations.

Building FLAGR in Windows

Similarly to the previous case, in Windows platforms the user must navigate to the root directory of the package through the Command Prompt and execute the batch file that exists there by typing:

```
makefile.bat
```

The build script compiles the source code and produces two files within the `/bin/Release` directory:

- the binary executable file `/bin/Release/FLAGR.exe`
- the Dynamic Link Library library `/bin/Release/flagr.dll` that can be linked by third-party applications to obtain access to the FLAGR algorithm implementations.

Running the FLAGR executable

The FLAGR binary is executed in an identical manner, regardless of the operating system. The application accepts 4 optional arguments in the following fashion:

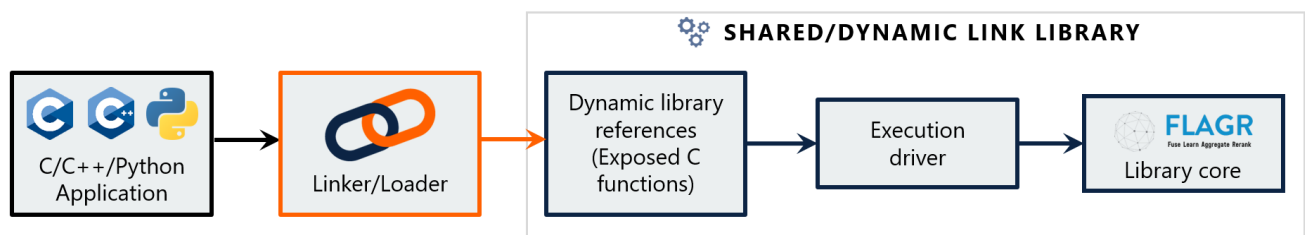
```
FLAGR [cutoff] [input_file] [output_path] [qrels_file]
```

The input arguments are:

- **cutoff**: this is the evaluation cut-off point. That is, the number of items of the aggregate list that will be included in the evaluation process. If nothing is passed, then the value 10 is used.
- **input_file**: The full path to the input file that stores the input lists to be aggregated. This is where the aggregation algorithm/s read data from.
- **output_path**: This is where the program writes the generated aggregate lists and the results of the evaluation process. If nothing is passed, then the default value output is used.
- **qrels_file**: This file stores the relevance judgments of the list elements. It is used by FLAGR to evaluate the employed rank aggregation algorithm/s. If nothing is passed, then no evaluation takes place.

1.4 Dynamic library references (Exposed C functions)

FLAGR exposes a set of C functions through an extern "C" statement, allowing their linkage from other C programs. These are the dynamic library references to which a client C code can link to. At first, FLAGR must be [compiled as a shared/dynamic link library](#). Then, the client C program can link to that shared library and include a typical C header file that contains just the declaration of these functions. The called function is able to access the C++ FLAGR core through the [Execution Driver](#). The following diagram depicts this scenario:



The FLAGR architecture and the possibility of building it as a shared library allows its usage not only in third party C programs, but also in programs written in other languages. PyFLAGR is an example of such case. PyFLAGR is a Python library that enables the execution of the algorithm C++ implementations of FLAGR from standard Python programs. The Python program must be able to successfully import the PyFLAGR modules.

The exposed C functions of FLAGR exist in two files: `cflagr.cpp` and `dllflagr.cpp`. These files are almost identical: they contain the same functions with exactly the same arguments and bodies. What changes is the usage of several special keywords in the function declarations of `dllflagr.cpp` that enable the building of FLAGR as a DLL for Windows-based systems. These functions are:

- **void Linear()**: It executes one of the supported linear combination methods (CombSUM and CombMNZ, each one with 5 variants).

- `void Condorcet()`: It executes the Condorcet Winners method.
- `void Copeland()`: It executes the Copeland Winners method.
- `void OutrankingApproach()`: It executes the Outranking Approach of Farah & Vanderpooten, 2007.
- `void Kemeny()`: It executes the optimal Kemeny optimal aggregation algorithm (brute force implementation).
- `void RobustRA()`: It executes the Robust Rank Aggregation (RRA) method of Kolde et al., 2012.
- `void DIBRA()`: It executes the distance-based iterative rank aggregation method of Akritidis et al., 2022.
- `void PrefRel()`: It executes the preference relations method of Desarkar et al., 2016.
- `void Agglomerative()`: It executes the agglomerative rank aggregation method of Chatterjee et al., 2018.
- `void MC()`: It executes the Markov Chain-based rank aggregation method of Dwork et al., 2001.

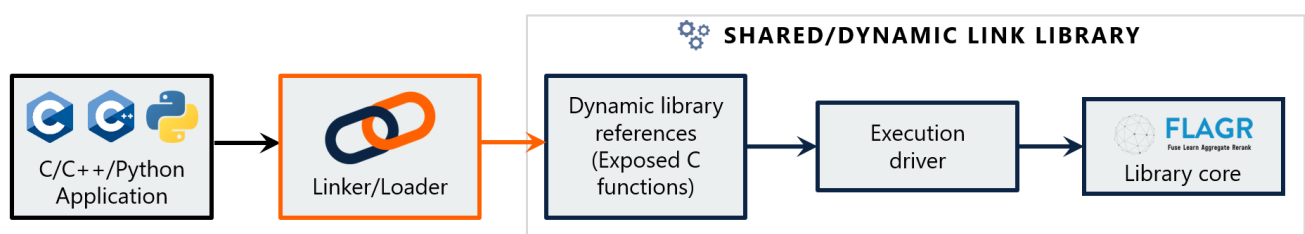
1.5 Execution driver

The Execution Driver is a simple function defined in `driver.cpp`. Its role is to offer a unified manner of executing the `exposed C functions`. It takes as an argument a simple C structure that stores the user-defined input parameters and algorithm hyper-parameters, and orchestrates the execution flow.

More specifically, the Execution driver initially copies the user-defined input parameters from the aforementioned C structure to an `InputParams` C++ object. In the sequel, it creates an `InputData` object which immediately starts reading the provided input file/s. Next, the `aggregate()` method of `InputData` is called to perform rank aggregation. Notice that the `InputParams` object is propagated to all FLAGR components and carries all the required parameters with it. Therefore, the rank aggregation method to be applied is automatically executed without any further checks, since `InputParams` notifies the `Aggregator` about the aggregation method that was selected by the user.

In case the user provided a valid input file with relevance judgments, the Execution Driver proceeds to the evaluation of the generated aggregate list. This is achieved by calling the `evaluate()` method of `InputData`, which in turn triggers the `Evaluator`.

The following block diagram depicts the role of the execution driver as the connector between the dynamic library references and the C++ FLAGR core.



2. Rank Aggregation methods

This section describes the rank aggregation algorithms implemented in FLAGR. The following articles also contain brief descriptions of the respective exposed C functions that make these algorithms accessible from other programs (in case the FLAGR shared library is used).

2.1 Linear Combination methods – The `Linear()` function

In linear combination methods, the score of each element is computed by summing up the partial scores of that element with respect to its rankings in each input preference list. The `Linear()` function triggers the execution of two such combination methods: `CombSUM`, and `CombMNZ`. Both of them are implemented in accordance to the paper of Renda et al., 2003.

Each method is accompanied by an element rank/score normalization technique, as it is described in the aforementioned paper. These techniques are: Rank normalization, Borda normalization, Score normalization, and Z-Score normalization. In FLAGR, there is a fifth normalization technique, called Simple Borda. In contrast to the traditional Borda normalization, Simple Borda assigns zero partial scores in case an element has not been ranked by an input preference list.

In addition, notice that `CombSUM` with Borda normalization is equivalent to the well-known `BordaCount` rank aggregation method.

Function Definitions

```
void Linear(const char inf[], const char relf[], const int evpts, const int ram, const char ranstr[], const char out[])
```

and

```
__declspec(dllexport) void __cdecl Linear(const char inf[], const char relf[], const int evpts, const int ram, const char ranstr[], const char out[])
```

Implementation Files

- `Linear()` function: [cflagr.cpp](#) and [dllflagr.cpp](#).
- `CombSUM`: [src/ram/CombSUM.cpp](#).
- `CombMNZ`: [src/ram/CombMNZ.cpp](#).

Input arguments

- `const char inf[]`: The path of the input file that stores the preference lists to be aggregated.
- `const char relf[]`: The path of the input file that stores the element relevance judgments.
- `const int evpts`: the elements in the aggregate list on which the evaluation measures (i.e. Precision, and nDCG) will be computed.
- `const int ram`: The selected rank aggregation method (see the `aggregation_method` parameter in [this document](#) for a list of possible values).
- `const char ranstr[]`: A string that is embedded in the names of the output files. Used when FLAGR is compiled as a shared library.
- `const char out[]`: The file system location where the output file with the aggregate list will be stored.

Description

The input parameters are parsed and stored in a special C structure called UserParams that is defined in [src/InputParams.h](#). Then, UserParams is passed to the [execution driver](#) and the rank aggregation process starts.

2.2 Condorcet Winners – The Condorcet() function

The Condorcet() function executes the Condorcet Winners method. The score of an element r_i is determined by the number of its "victories" against all the other involved elements. A victory for r_i is achieved if the majority of the voters rank r_i higher than another element r_j . Finally, the elements are sorted in decreasing victory order.

Function Definitions

```
void Condorcet(const char inf[], const char relf[], const int evpts, const char ranstr[], const char out[])
```

and

```
__declspec(dllexport) void __cdecl Condorcet(const char inf[], const char relf[], const int evpts, const char ranstr[], const char out[])
```

Implementation Files

- Condorcet() function: [cflagr.cpp](#) and [dllflagr.cpp](#).
- Condorcet Winners method: [src/ram/CondorcetWinners.cpp](#).

Input arguments

- `const char inf[]`: The path of the input file that stores the preference lists to be aggregated.
- `const char relf[]`: The path of the input file that stores the element relevance judgments.
- `const int evpts`: the elements in the aggregate list on which the evaluation measures (i.e. Precision, and nDCG) will be computed.
- `const char ranstr[]`: A string that is embedded in the names of the output files. Used when FLAGR is compiled as a shared library.
- `const char out[]`: The file system location where the output file with the aggregate list will be stored.

Description

The input parameters are parsed and stored in a special C structure called UserParams that is defined in [src/InputParams.h](#). Then, UserParams is passed to the [execution driver](#) and the rank aggregation process starts.

2.3 Copeland Winners – The Copeland() function

The Copeland() function executes the Copeland Winners method. The score of an element r_i is determined by the number of its "victories" against all the other involved elements. A victory for r_i is achieved if the majority of the voters rank r_i higher than another element r_j . In contrast to the Condorcet method, Copeland Winners assign "half" a victory (i.e., a score of 0.5) to each element of a pair (r_i, r_j) in the case of tie. A tie happens when exactly half of the voters rank r_i higher than r_j and the other half voters rank r_j higher than r_i .

Finally, the elements are sorted in decreasing victory order.

Function Definitions

```
void Copeland(const char inf[], const char relf[], const int evpts, const char ranstr[], const char out[])
```

and

```
__declspec(dllexport) void __cdecl Copeland(const char inf[], const char relf[], const int evpts, const char ranstr[], const char out[])
```

Implementation Files

- Copeland() function: [cflagr.cpp](#) and [dllflagr.cpp](#).
- Copeland Winners method: [src/ram/CopelandWinners.cpp](#).

Input arguments

- `const char inf[]`: The path of the input file that stores the preference lists to be aggregated.
- `const char relf[]`: The path of the input file that stores the element relevance judgments.
- `const int evpts`: the elements in the aggregate list on which the evaluation measures (i.e. Precision, and nDCG) will be computed.
- `const char ranstr[]`: A string that is embedded in the names of the output files. Used when FLAGR is compiled as a shared library.
- `const char out[]`: The file system location where the output file with the aggregate list will be stored.

Description

The input parameters are parsed and stored in a special C structure called UserParams that is defined in [src/InputParams.h](#). Then, UserParams is passed to the [execution driver](#) and the rank aggregation process starts.

2.4 Outranking Approach – OutrankingApproach()

The Outranking Approach of Farah & Vanderpooten is a majoritarian method that identifies the "winning" elements by performing pairwise comparisons of their individual rankings. The method is implemented in accordance to the following paper:

- Farah, M., Vanderpooten, D., "An outranking approach for rank aggregation in information retrieval", In Proceedings of the 30th ACM Conference on Research and Development in Information Retrieval, pp. 591-598, 2007.

The algorithm is based on four threshold values that introduce different perspectives of the majority criterion. These values are the concordance, discordance, preference, and veto thresholds. The user may pass all of them to FLAGR as hyper-parameters, through the input arguments of the `OutrankingApproach()` function (see below).

Function Definitions

```
void OutrankingApproach(const char inf[], const char relf[], const int evpts,
const char ranstr[], const char out[], const float pref_t, const float veto_t,
const float conc_t, const float disc_t)
```

and

```
__declspec(dllexport) void __cdecl OutrankingApproach(const char inf[], const
char relf[], const int evpts, const char ranstr[], const char out[], const float
pref_t, const float veto_t, const float conc_t, const float disc_t)
```

Implementation Files

- The `OutrankingApproach()` C function: [cflagr.cpp](#) and [dllflagr.cpp](#).
- Algorithm Implementation: [src/ram/OutrankingApproach.cpp](#).

Input arguments

- `const char inf[]`: The path of the input file that stores the preference lists to be aggregated.
- `const char relf[]`: The path of the input file that stores the element relevance judgments.
- `const int evpts`: the elements in the aggregate list on which the evaluation measures (i.e. Precision, and nDCG) will be computed.
- `const char ranstr[]`: A string that is embedded in the names of the output files. Used when FLAGR is compiled as a shared library.
- `const char out[]`: The file system location where the output file with the aggregate list will be stored.
- `const float pref_t`: Algorithm hyper-parameter - The value of the preference threshold.
- `const float veto_t`: Algorithm hyper-parameter - The value of the veto threshold.
- `const float conc_t`: Algorithm hyper-parameter - The value of the concordance threshold.
- `const float disc_t`: Algorithm hyper-parameter - The value of the discordance threshold.

Description

The input parameters are parsed and stored in a special C structure called `UserParams` that is defined in [src/InputParams.h](#). Then, `UserParams` is passed to the [execution driver](#) and the rank aggregation process starts.

2.5 Markov Chains methods – The MC() function

The Markov Chains methods constitute a well-established family of rank aggregation methods. Originally proposed by Dwork et al., (2001), they consider an aggregate list as a system that moves from one state to another with respect to a particular criterion. Dwork et al. (2001) introduced four such methods in the following article:

- C. Dwork, R. Kumar, M. Naor, D. Sivakumar, "Rank Aggregation Methods for the Web", In Proceedings of the 10th International Conference on World Wide Web, pp. 613-622, 2001.

In addition, DeConde et al. (2006) introduced MCT, a variant that constructs the transition matrix by considering the proportion of lists which prefer an element over another.

- R.P. DeConde, S. Hawley, S. Falcon, N. Clegg, B. Knudsen, R. Etzioni, "Combining results of microarray experiments: a rank aggregation approach", Statistical Applications in Genetics and Molecular Biology, vol. 5, no. 1, 2006.

The execution of all five methods takes place by passing the appropriate arguments to the MC exposed C function of FLAGR.

Function Definitions

```
void MC(const char inf[], const char relf[], const int evpts, const int ram,
const char ranstr[], const char out[], const float ergodic_number, const float
delta, const int iter)
```

and

```
__declspec(dllexport) void __cdecl MC(const char inf[], const char relf[], const
int evpts, const int ram, const char ranstr[], const char out[], const float
ergodic_number, const float delta, const int iter)
```

Implementation Files

- MC() function: [cflagr.cpp](#) and [dllflagr.cpp](#).
- Markov Chains implementation: [src/ram/MC.cpp](#).

Input arguments

- `const char inf[]`: The path of the input file that stores the preference lists to be aggregated.
- `const char relf[]`: The path of the input file that stores the element relevance judgments.
- `const int evpts`: the elements in the aggregate list on which the evaluation measures (i.e. Precision, and nDCG) will be computed.
- `const int ram`: The selected (Markov Chains-based) rank aggregation method (801, 802, 803, 804, or 805 - see the `aggregation_method` parameter in [this document](#)).
- `const char ranstr[]`: A string that is embedded in the names of the output files. Used when FLAGR is compiled as a shared library.
- `const char out[]`: The file system location where the output file with the aggregate list will be stored.
- `const float ergodic_number`: The ergodic number, used during the computation of the ergodic transition matrix from the normalized transition matrix.
- `const float iter`: Controls the maximum number of iterations before convergence.

Description

The input parameters are parsed and stored in a special C structure called `UserParams` that is defined in [src/InputParams.h](#). Then, `UserParams` is passed to the [execution driver](#) and the rank aggregation process starts.

2.6 Kemeny Optimal Aggregation – The `Kemeny()` function

This function executes Kemeny optimal aggregation. This algorithm identifies the optimal aggregate list as the list that minimizes its distance from all the input preference lists.

Kemeny optimal aggregation is an NP-hard problem, with very high computational complexity. It requires the computation of all permutations of the input items and the calculation of the distances of each permutation from all input lists. *The brute force solution becomes infeasible when the number of elements gets greater than 15-20, or the number of input lists is greater than 4*, so caution is advised.

Function Definitions

```
void Kemeny(const char inf[], const char relf[], const int evpts, const char ranstr[], const char out[])
```

and

```
__declspec(dllexport) void __cdecl Kemeny(const char inf[], const char relf[], const int evpts, const char ranstr[], const char out[])
```

Implementation Files

- `Kemeny()` function: [cflagr.cpp](#) and [dllflagr.cpp](#).
- Kemeny optimal aggregation method: [src/ram/KemenyOptimal.cpp](#).

Input arguments

- `const char inf[]`: The path of the input file that stores the preference lists to be aggregated.
- `const char relf[]`: The path of the input file that stores the element relevance judgments.
- `const int evpts`: the elements in the aggregate list on which the evaluation measures (i.e. Precision, and nDCG) will be computed.
- `const char ranstr[]`: A string that is embedded in the names of the output files. Used when FLAGR is compiled as a shared library.
- `const char out[]`: The file system location where the output file with the aggregate list will be stored.

Description

The input parameters are parsed and stored in a special C structure called `UserParams` that is defined in [src/InputParams.h](#). Then, `UserParams` is passed to the [execution driver](#) and the rank aggregation process starts.

2.7 Robust Rank Aggregation – The RobustRA() function

This function executes the Robust Rank Aggregation (RRA) method of Kolde et al., 2012. The method is implemented in accordance to the following paper:

- R. Kolde, S. Laur, P. Adler, J. Vilo, "Robust rank aggregation for gene list integration and meta-analysis", Bioinformatics, vol. 28, no. 4, pp. 573-580, 2012.

RRA is mostly used in bio-informatics applications to aggregate gene lists. It is based on a probabilistic model (beta distribution) that makes the algorithm parameter free and robust to outliers, noise and errors. The FLAGR C++ implementation of RRA produces the same results as the R implementation of Kolde (see the [RobustRankAggreg R package](#)).

The computation of the incomplete beta function is performed with the [John Burkardt's implementation of ASA063 algorithm](#) (K.L. Majumder and G. Bhattacharjee):

- K.L. Majumder, G.P. Bhattacharjee, "Algorithm AS 63: The incomplete Beta Integral", Applied Statistics, vol. 22, no. 3, pp. 409-411, 1973.

Furthermore, the computation of the inverse of the incomplete beta function is performed with the [John Burkardt's implementation of ASA109 algorithm](#) (GW Cran, KJ Martin and GE Thomas):

- G.W. Cran, M.J. Martin, G.E. Thomas, "Remark AS R19 and Algorithm AS 109: A Remark on Algorithms AS 63: The Incomplete Beta Integral and AS 64: Inverse of the Incomplete Beta Integral", Applied Statistics, Volume 26, Number 1, 1977, pages 111-114.

Function Definitions

```
void RobustRA(const char inf[], const char relf[], const int evpts, const char ranstr[], const char out[], const bool exact)
```

and

```
__declspec(dllexport) void __cdecl RobustRA(const char inf[], const char relf[], const int evpts, const char ranstr[], const char out[], const bool exact)
```

Implementation Files

- RobustRA() function: [cflagr.cpp](#) and [dllflagr.cpp](#).
- RRA implementation: [src/ram/RobustRA.cpp](#).
- Incomplete beta function implementation (ASA 063 algorithm), inverse of the incomplete Beta function implementation (ASA 109 algorithm): [src/ram/tools/BetaDistribution.cpp](#).

Input arguments

- `const char inf[]`: The path of the input file that stores the preference lists to be aggregated.
- `const char relf[]`: The path of the input file that stores the element relevance judgments.
- `const int evpts`: the elements in the aggregate list on which the evaluation measures (i.e. Precision, and nDCG) will be computed.
- `const char ranstr[]`: A string that is embedded in the names of the output files. Used when FLAGR is compiled as a shared library.

- `const char out[]`: The file system location where the output file with the aggregate list will be stored.
- `const bool exact`: If true the Stuart algorithm for p-value correction is applied.

Description

The input parameters are parsed and stored in a special C structure called `UserParams` that is defined in [src/InputParams.h](#). Then, `UserParams` is passed to the [execution driver](#) and the rank aggregation process starts.

2.8 Iterative Distance-Based weighted aggregation – The DIBRA() function

This function executes the iterative, distance-based method (abbreviated DIBRA) of Akritidis et al. 2022. The method is implemented in accordance to the following paper:

- L. Akritidis, A. Fevgas, P. Bozanis, Y. Manolopoulos, "An Unsupervised Distance-Based Model for Weighted Rank Aggregation with List Pruning", *Expert Systems with Applications*, vol. 202, pp. 117435, 2022.

DIBRA belongs to the weighted rank aggregation methods. It employs exploratory analysis to automatically identify the expert voters in an unsupervised fashion. Then, it assigns higher weights to the voters who were identified as experts, thus boosting the scores of their submitted elements.

In particular, DIBRA employs a standard non-weighted method to generate an initial aggregate ranking (see `aggregation_method` in [this document](#) for a list of the supported methods). Then, it repeatedly assigns higher weights to the input lists that have smaller distances from the aggregate lists. The process terminates when the voter weights converge and a stable aggregate list is obtained.

The algorithm also includes an optional list pruning mechanism that arranges the input list lengths according to the respective voter weights.

Function Definitions

```
void DIBRA(const char inf[], const char relf[], const int evpts, const int agg,
const char ranstr[], const char out[], const int wnorm, const int dist, const
bool prune, const float gamma, const float d1, const float d2, const float tol,
const int iter, const float pref_t, const float veto_t, const float conc_t,
const float disc_t)
```

and

```
__declspec(dllexport) void __cdecl DIBRA(const char inf[], const char relf[],
const int evpts, const int agg, const char ranstr[], const char out[], const
int wnorm, const int dist, const bool prune, const float gamma, const float d1,
const float d2, const float tol, const int iter, const float pref_t, const float
veto_t, const float conc_t, const float disc_t)
```

Implementation Files

- DIBRA() function: [cflagr.cpp](#) and [dllflagr.cpp](#).
- DIBRA method implementation: [src/ram/DIBRA.cpp](#).

Input arguments

- `const char inf[]`: The path of the input file that stores the preference lists to be aggregated.
- `const char relf[]`: The path of the input file that stores the element relevance judgments.
- `const int evpts`: the elements in the aggregate list on which the evaluation measures (i.e. Precision, and nDCG) will be computed.
- `const int agg`: The selected non-weighted base rank aggregation method (see the `aggregation_method` parameter in [this document](#) for a list of possible values).
- `const char ranstr[]`: A string that is embedded in the names of the output files. Used when FLAGR is compiled as a shared library.
- `const char out[]`: The file system location where the output file with the aggregate list will be stored.
- `const int wnorm`: The voter weights normalization method (see the `weights_normalization` parameter in [this document](#) for a list of possible values).
- `const int dist`: The correlation method that is used to measure the distance between an input list and the temporary aggregate list (see the `correlation_method` parameter in [this document](#) for a list of possible values).
- `const bool prune`: Triggers a weight-dependent list pruning mechanism.
- `const float gamma`: The γ hyper-parameter.
- `const float d1`: The δ_1 hyper-parameter (applicable when `prune=true`).
- `const float d2`: The δ_2 hyper-parameter (applicable when `prune=true`).
- `const float tol`: Controls the convergence precision. This tolerance threshold represents the minimum precision of the difference between the voter weight in an iteration and the voter weight of the previous iteration.
- `const int iter`: Controls the maximum number of iterations.
- `const float pref_t`: The preference threshold (applicable when the Outranking Approach is selected as the non-weighted base method; namely, `agg=5300`).
- `const float veto_t`: The veto threshold (applicable when the Outranking Approach is selected as the non-weighted base method; namely, `agg=5300`).
- `const float conc_t`: The concordance threshold (applicable when the Outranking Approach is selected as the non-weighted base method; namely, `agg=5300`).
- `const float disc_t`: The discordance threshold (applicable when the Outranking Approach is selected as the non-weighted base method; namely, `agg=5300`).

Description

The input parameters are parsed and stored in a special C structure called `UserParams` that is defined in [src/InputParams.h](#). Then, `UserParams` is passed to the [execution driver](#) and the rank aggregation process starts.

2.9 Preference Relations weighted method

This function executes the Preference Relations weighted rank aggregation method of Desarkar et al., 2016. The method is implemented in accordance to the following paper:

- M.S. Desarkar, S. Sarkar, P. Mitra, "Preference relations based unsupervised rank aggregation for metasearch", Expert Systems with Applications, vol. 49, pp. 86-98, 2016.

The Preference Relations algorithm belongs to the weighted rank aggregation methods. It employs exploratory analysis to automatically identify the expert voters in an unsupervised fashion. Then, it assigns higher weights to the voters who were identified as experts, thus boosting the scores of their submitted elements.

The method constructs a preference relations graph which contains the individual elements as vertices and their weights as edges.

Function Definitions

```
void PrefRel(const char inf[], const char relf[], const int evpts, const char ranstr[], const char out[], const float alpha, const float beta)
```

and

```
__declspec(dllexport) void __cdecl PrefRel(const char inf[], const char relf[], const int evpts, const char ranstr[], const char out[], const float alpha, const float beta)
```

Implementation Files

- PrefRel() function: [cflagr.cpp](#) and [dllflagr.cpp](#).
- Preference Relations method implementation: [src/ram/PrefRel.cpp](#).
- Helper class MergedItemPair implementation: [src/ram/tools/MergedItemPair.cpp](#).

Input arguments

- `const char inf[]`: The path of the input file that stores the preference lists to be aggregated.
- `const char relf[]`: The path of the input file that stores the element relevance judgments.
- `const int evpts`: the elements in the aggregate list on which the evaluation measures (i.e. Precision, and nDCG) will be computed.
- `const char ranstr[]`: A string that is embedded in the names of the output files. Used when FLAGR is compiled as a shared library.
- `const char out[]`: The file system location where the output file with the aggregate list will be stored.
- `const float alpha`: The α hyper-parameter.
- `const float beta`: The β hyper-parameter.

Description

The input parameters are parsed and stored in a special C structure called UserParams that is defined in [src/InputParams.h](#). Then, UserParams is passed to the [execution driver](#) and the rank aggregation process starts.

2.10 Agglomerative weighted aggregation

This function executes the Agglomerative weighted rank aggregation method of Chatterjee et al., 2018. The method is implemented in accordance to the following paper:

- S. Chatterjee, A. Mukhopadhyay, M. Bhattacharyya, "A weighted rank aggregation approach towards crowd opinion analysis", Knowledge-Based Systems, vol. 149, pp. 47-60, 2018.

The Agglomerative Aggregation algorithm belongs to the weighted rank aggregation methods. It employs exploratory analysis to automatically identify the expert voters in an unsupervised fashion. Then, it assigns higher weights to the voters who were identified as experts, thus boosting the scores of their submitted elements.

This method works very similarly to the well-established agglomerative clustering algorithm. Specifically, it repeatedly merges the two most similar input lists into a temporary aggregate list. During list merging, it modifies the weights of the respective voters, thus affecting the future merges.

Function Definitions

```
void Agglomerative(const char inf[], const char relf[], const int evpts, const char ranstr[], const char out[], const float c1, const float c2)
```

and

```
__declspec(dllexport) void __cdecl Agglomerative(const char inf[], const char relf[], const int evpts, const char ranstr[], const char out[], const float c1, const float c2)
```

Implementation Files

- Agglomerative() function: [cflagr.cpp](#) and [dllflagr.cpp](#).
- Agglomerative Aggregation method implementation: [src/ram/Agglomerative.cpp](#).

Input arguments

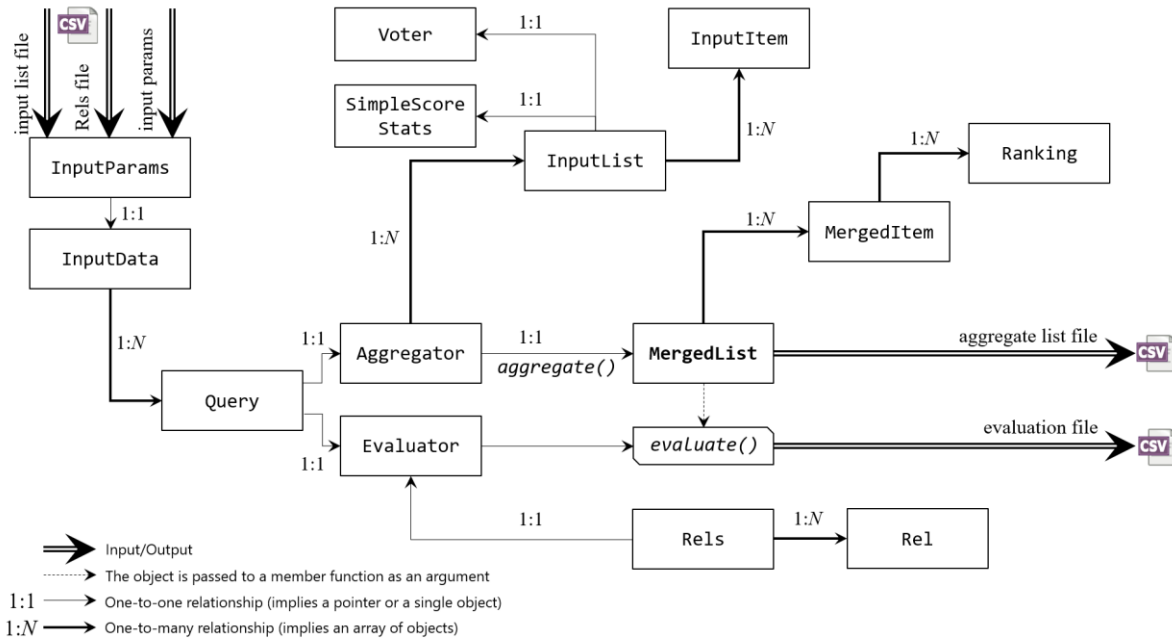
- `const char inf[]`: The path of the input file that stores the preference lists to be aggregated.
- `const char relf[]`: The path of the input file that stores the element relevance judgments.
- `const int evpts`: the elements in the aggregate list on which the evaluation measures (i.e. Precision, and nDCG) will be computed.
- `const char ranstr[]`: A string that is embedded in the names of the output files. Used when FLAGR is compiled as a shared library.
- `const char out[]`: The file system location where the output file with the aggregate list will be stored.
- `const float c1`: The c_1 hyper-parameter.
- `const float c2`: The c_2 hyper-parameter.

Description

The input parameters are parsed and stored in a special C structure called UserParams that is defined in [src/InputParams.h](#). Then, UserParams is passed to the [execution driver](#) and the rank aggregation process starts.

3. FLAGR API Documentation

In this section the FLAGR library core is presented. The following Figure depicts a block diagram of the core's architecture.



In the following subsections, the format of the input and output files is described. Then, the participating classes of FLAGR are presented. The final Subsection contains a detailed guide on how custom algorithm implementations can be integrated to FLAGR.

3.1 Input and Output files

Input 1: The file of the input preference lists

FLAGR requires that the input preference lists to be aggregated are stored in a single CSV file, regardless of the number of the involved topics (queries) or voters (rankers). The columns of this CSV file must be organized in the following manner:

Query/Topic String, Voter Name, Item Code, Item Score, Algorithm/Dataset
--

where:

- **Query/Topic:** the query string or the topic for which the preference list is submitted.
- **Voter:** the name of the voter, or the ranker who submits the preference list for the specified Query/Topic.
- **Item Code:** a unique name that identifies a particular element of the preference list. A voter cannot submit the same element for the same query/topic two or more times. This means that each element appears exactly once in each preference list. However, the same element may appear in lists submitted by other voters.

- **Item Score:** the preference score assigned to an item by a specific voter. It reflects the importance (or the relevance, or the weight) of the element. In many cases (e.g., search engine rankings), the preference scores are unknown. In such cases, the scores can be replaced by the (reverse) ranking of an item in such a manner that the top rankings receive higher scores than the ones that have been assigned lower rankings.
- **Algorithm/Dataset:** A user-defined string that usually represents the origin of a particular preference list. It may receive any non-blank value.

You may find an example of an input list CSV file [here](#). This example file contains the preference lists that were submitted by 50 voters for 20 queries. Each input list contains 30 elements. Therefore, the number of rows in this file is equal to $50 \cdot 20 \cdot 30 = 30000$.

Output 1: The file of the aggregate list/s

In this file FLAGR stores the result (output) of the selected rank aggregation method. Namely, the final lists that derive after the aggregation of the input preference lists. The library creates one aggregate list per input query/topic. So, if there are Q input queries, FLAGR generates Q aggregate lists and stores them in a CSV file. Each row in the file represents an element of the aggregate list stored in decreasing score order. The columns are organized as follows:

Query/Topic String, Voter Name, Item Code, Item Score

Input 2: The file of relevant elements (or, the Rels file)

Optionally, the user may provide a second CSV file (we call it Rels file) that contains relevance judgments for the preference list elements of the primary input file for each query. The Rels file is employed by the FLAGR's evaluation module to evaluate each created aggregate list. Its columns must be formatted as follows:

Query/Topic String, \emptyset , Item Code, Relevance Score
--

where:

- **Query/Topic:** the query string or the topic for which the list is submitted.
- **\emptyset :** unused. This value *must be always* 0.
- **Item Code:** a unique name that identifies a particular element. There cannot be two relevance judgments for the same element for the same query.
- **Relevance Score:** An integer value that represents the relevance of the item with respect to the mentioned Query/Topic. Typically, zero values represent irrelevant and incorrect elements, negative values represent spam elements; and positive values represent relevant, correct and informative elements.

You may find an example of an input Rels file [here](#). This example file contains the relevance judgments for the elements of all preference lists for all queries of the [previous input list file](#). Notice that in case FLAGR does not find a relevance judgment for an element, then it automatically considers it as irrelevant (that is, it sets its Relevance Score equal to 0).

Output 2: The evaluation file

As soon as a valid Rels file is provided, the evaluation process takes place automatically. In this case FLAGR evaluates each aggregate list individually and outputs a second CSV file, where it writes the results of the evaluation.

If there are Q input queries, then Q aggregate lists are generated and the evaluation file contains $Q + 1$ rows. The first Q rows store the evaluation metrics for each aggregate list, whereas the last row contains the average values. On the other hand, the columns of the evaluation file depend on the `eval_pts` parameter that is set by the user. More specifically, the columns are $6 + 4 \cdot \text{eval_pts}$:

```
q, num_ret, num_rel, num_rel_ret, ap, P@1, ..., P@eval_pts, R@1, ..., R@eval_pts, D@1, ..., D@eval_pts, N@1, ..., N@eval_pts, ram
```

where:

- `q` is the query string.
- `num_ret` is the length (i.e. the number of elements in the) aggregate list.
- `num_rel` is the total number of relevant elements for this query.
- `num_rel_ret` is the number of relevant elements included in the aggregate list.
- `ap` is the Average Precision for a specific aggregate list w.r.t `q`.
- `P@X` is the running Precision at the X -th element of the aggregate list.
- `R@X` is the running Recall at the X -th element of the aggregate list..
- `D@X` is the running Discounted Cumulative Gain (DCG) at the X -th item of the aggregate list.
- `N@X` is the running normalized Discounted Cumulative Gain (nDCG) at the X -th item.
- `ram` is the name of the applied rank aggregation method.

3.2 Aggregator class

The Aggregator class triggers the execution of a rank aggregation algorithm on a collection (array) of `InputLists`. Typically, the output of the aggregation process is a single `MergedList` object.

Implementation files

The Aggregator class is defined in the `src/Aggregator.h` header file; its member functions are implemented in `src/Aggregator.cpp`.

Technical Details

The `input_lists` member variable stores the input lists to be aggregated. Technically, it is an array (i.e. double pointer) of `InputList` objects. The allocated size of this array is equal to `num_alloc_lists`, whereas the real number of non-null input lists is `num_lists`. Naturally, it must always hold that `num_alloc_lists` \geq `num_lists`.

The rank aggregation process takes places inside a `Query` object. For this reason, a `Query` object contains a pointer to a single Aggregator object (see the respective block diagram).

The `aggregate()` member function is responsible for executing the rank aggregation procedure. It accepts an `InputParams` object that stores the selected rank aggregation method, the hyper-parameter values and the execution parameters and returns a single `MergedList`.

3.3 Evaluator class

The Evaluator class evaluates the quality of an aggregate list with respect to a given set of judgments that determine the relevance of (some, or all) list elements. The set of relevance judgments is stored in a `ReIs` object. The results of the evaluation (i.e. evaluation measures) are written in a `CSV file`.

Implementation files

The `Evaluator` class is defined in the `src/Evaluator.h` header file; its member functions are implemented in `src/Evaluator.cpp`.

Technical Details

The evaluation process takes place inside a `Query` object, provided that a file of relevance judgments is provided to FLAGR (see more in the article about [Input and Output files](#)). For this reason, a `Query` object contains a pointer to a single `Evaluator` object.

The evaluation process is performed by the `evaluate()` member function. The procedure is based on the `ReIs` object that contains the aforementioned relevance judgments. Finally, the four following arrays are created:

- `precision`: in its i -th position, it stores the value of the Precision measure at the i -th element of the aggregate list.
- `recall`: in its i -th position, it stores the value of the Recall measure at the i -th element of the aggregate list.
- `dcg`: in its i -th position, it stores the value of the Discounted Cumulative Gain (DCG) measure at the i -th element of the aggregate list.
- `ndcg`: in its i -th position, it stores the value of the normalized Discounted Cumulative Gain (nDCG) measure at the i -th element of the aggregate list.

3.4 InputItem class

An `InputItem` is a single element that is read from the `input file`. It is a part of an `InputList` and possesses three properties:

- A unique string identifier that is used by FLAGR to identify the common elements among all `InputLists`,
- Its (integer) ranking in the input preference list, and
- An optional score value that is assigned by the associated voter and justifies its ranking in the input list. The score data type can be either `float`, or `double`; this is determined by the `score_t` data type definition in `driver.cpp`.

Implementation files

The `InputItem` class is defined in the `src/InputItem.h` header file; its member functions are implemented in `src/InputItem.cpp`.

3.5 InputList class

This class is used to store and represent an input preference list submitted by a voter. It consists of an array of `InputItem` objects and it is connected to the voter who submitted it via a pointer that points to a `Voter` object.

The input lists are read from the `input file` by using an `InputData` object. The entire collection of them is handled by an `Aggregator`.

Implementation files

The `InputList` class is defined in the [src/InputList.h](#) header file; its member functions are implemented in [src/InputList.cpp](#).

Technical Details

The actual number of elements in an `InputList` is `num_items`; the allocated memory is `num_alloc_items`. Naturally it derives that `num_alloc_items` must always be greater than, or equal to `num_items`.

3.6 InputParams class

The `InputParams` class stores options and execution parameters that have been passed to FLAGR by the user. These parameters concern input and output file locations, rank aggregation methods, algorithm hyper-parameters, etc. See the table below for a complete list of the supported parameters and their respective valid values.

This object is passed as an argument to multiple functions of FLAGR including the implementations of the rank aggregation methods. This is how these implementations get access to the user-defined hyper-parameters.

Implementation files

The `InputParams` class is defined in the [src/InputParams.h](#) header file; its member functions are implemented in [src/InputParams.cpp](#).

Details

The supported parameters include:

Parameter	Data Type	Description
<code>input_file</code>	String (ASCII)	The path of the input CSV file that contains the preference lists to be aggregated.
<code>rels_file</code>	String (ASCII)	The path of the optional CSV file that contains the relevance judgments of the input list elements. If set, it automatically triggers the evaluation process of the generated aggregate list. Otherwise, no evaluation takes place.
<code>output_file</code>	String (ASCII)	The file system location where the output file with the aggregate list will be stored. If left empty, the default OS temp directory is used.
<code>eval_file</code>	String (ASCII)	The file system location where the output file with the evaluation of the aggregate list will be stored. If left empty, the default OS temp directory is used.
<code>random_string</code>	String (ASCII)	A string that is embedded in the names of the output files. Used when FLAGR is compiled as a shared library.

aggregation_method	Integer	<p>Determines the algorithm that will be used to perform rank aggregation. The valid values are:</p> <p>100: for CombSUM with Borda normalization 101: for CombSUM with Rank normalization 102: for CombSUM with Score normalization 103: for CombSUM with Z-Score normalization 104: for CombSUM with Simple Borda normalization 110: for CombMNZ with Borda normalization 111: for CombMNZ with Rank normalization 112: for CombMNZ with Score normalization 113: for CombMNZ with Z-Score normalization 114: for CombMNZ with Simple Borda normalization 200: for the Condorcet Winners method 201: for the Copeland Winners method 300: for the Outranking Approach 400: for Kemeny optimal aggregation 401: for Robust Rank Aggregation (RRA) 5100: for DIBRA with CombSUM and Borda Normalization 5101: for DIBRA with CombSUM and Rank Normalization 5102: for DIBRA with CombSUM and Score Normalization 5103: for DIBRA with CombSUM and Z-Score Normalization 5104: for DIBRA CombSUM and Simple Borda Normalization 5110: for DIBRA with CombMNZ and Borda Normalization 5111: for DIBRA with CombMNZ and Rank Normalization 5112: for DIBRA with CombMNZ and Score Normalization 5113: for DIBRA with CombMNZ and Z-Score Normalization 5114: for DIBRA CombMNZ and Simple Borda Normalization 5200: for DIBRA with the Condorcet Winners method 5201: for DIBRA with the Copeland Winners method 5300: for DIBRA with the Outranking Approach 600: for the Preference Relations Method 700: for the Agglomerative Aggregation Method 801: for Markov Chains 1 (MC1) 802: for Markov Chains 2 (MC2) 803: for Markov Chains 3 (MC3) 804: for Markov Chains 4 (MC4) 805: for MCT</p>
correlation_method	Integer	<p>The correlation method that is used to measure the distance between an input list and the temporary aggregate list. The valid values are:</p> <p>1: for the Spearman's ρ correlation coefficient. 2: for the scaled variant of Spearman's Footrule distance. 3: for Cosine similarity of the lists' vector representations. 5: for the Kendall's τ correlation coefficient.</p>

weights_normalization	Integer	The voter weights normalization method. Used when the DIBRA algorithm is selected. The valid values are: 1: for no voter weight normalization. 2: for normalizing the voter weights with min-max scaling. 3: for z-normalizing the voter weights.
max_iterations	Integer	This parameter controls the maximum number of iterations. FLAGR will stop the execution of DIBRA if the requested number of iterations have been performed, even if the voter weights have not fully converged.
max_list_items	Integer	Limits the length of the input preference lists.
eval_points	Integer	Determines the elements in the aggregate list on which the evaluation measures (i.e. Precision, and nDCG) will be computed. For example, for eval_pts=10 FLAGR will compute $P@1, P@2, \dots, P@10$ and $N@1, N@2, \dots, N@10$.
list_pruning	Boolean	Triggers a weight-dependent list pruning mechanism. Used in combination with the DIBRA weighted method only.
convergence_precision	Float or Double	Controls the convergence precision. This tolerance threshold represents the minimum precision of the difference between the voter weight in an iteration and the voter weight of the previous iteration. Used in combination with the DIBRA weighted method only.
alpha	Float or Double	The α hyper-parameter of the Preference Relations method.
beta	Float or Double	The β hyper-parameter of the Preference Relations method.
gamma	Float or Double	The γ hyper-parameter of DIBRA.
c1	Float or Double	The c_1 hyper-parameter of the Agglomerative Aggregation method.
c2	Float or Double	The c_2 hyper-parameter of the Agglomerative Aggregation method.
pref_thr	Float or Double	The preference threshold of the Outranking Approach. It takes values in the range [0,1].
veto_thr	Float or Double	The veto threshold of the Outranking Approach. It takes values in the range [0,1].
conc_thr	Float or Double	The concordance threshold of the Outranking Approach. It takes values in the range [0,1].
disc_thr	Float or Double	The discordance threshold of the Outranking Approach. It takes values in the range [0,1].

3.7 InputData class

This class is responsible for reading and parsing the input data file/s. For the moment, FLAGR accepts only CSV-formatted input files.

Implementation files

The InputData class is defined in the [src/input/InputData.h](#) header file; its member functions are implemented in [src/input/InputData.cpp](#) and [src/input/InputDataCSV.cpp](#).

Input list files

Detailed information about how the input files must be formatted can be found [here](#).

Also notice that FLAGR is designed to accept data directly from [RASDaGen](#), a synthetic dataset generator for rank aggregation problems.

Technical Details

The role of InputData is broader and it is not limited to just reading the input files. More specifically, during the input file parsing process, one or more [Query](#) objects are constructed. In the sequel, inside each [Query](#) the corresponding [Aggregator](#) and [Evaluator](#) objects are created to initialize and evaluate the aggregation process.

There is also a pointer called params that connects InputData with [InputParams](#). In this manner, InputData is able to subsequently pass user-defined algorithm hyper-parameters and several other execution parameters to the rest of the application components.

3.8 MergedItem class

A MergedItem is an element of an aggregate [MergedList](#). The class derives from [InputItem](#) and inherits all its member variables and functions.

Implementation files

The MergedItem class is defined in the [src/MergedItem.h](#) header file; its member functions are implemented in [src/MergedItem.cpp](#).

Technical Details

MergedItem maintains an array of [Ranking](#) objects that store the individual rankings (and scores) of the element in each preference [InputList](#). Notice that the number of elements of this array (num_alloc_rankings) is always equal to the number of the input preference lists. In case the associated [InputItem](#) has not been ranked by a preference list (i.e. it is not included in the list), then its corresponding ranking in the [Rankings](#) array is set equal to NOT_RANKED_ITEM_RANK (defined in [driver.cpp](#)). The actual number of the input preference lists that include a particular MergedItem is stored in the num_rankings member variable. This approach consumes more memory, but enables $O(1)$ constant search times of the ranking of a particular item on a particular [InputList](#).

The class also includes a self-class next pointer that points to another MergedItem object. This allows the storage of a collection of MergedItems in dynamic data structures, e.g. linked lists. Such structures are employed in [MergedList](#), where MergedItems are stored in a hash table with linked lists as chains.

3.9 MergedList class

The result of the rank aggregation process is an aggregate list that is stored in a `MergedList` object. `MergedList` contains a collection of `MergedItems`, typically sorted in decreasing score order. The score is assigned by a rank aggregation method.

Regarding the evaluation, the generated aggregate list is fed to the `evaluate()` function of an `Evaluator`. The results are written in a CSV file according to [this document](#).

Implementation files

The `MergedList` class is defined in the `src/MergedList.h` header file; its member functions are implemented in `src/MergedList.cpp`. The implementations of the supported rank aggregation methods are stored in the `/ram` directory. More specifically:

- `src/ram/Agglomerative.cpp` implements the Agglomerative Aggregation method of Chatterjee et al., 2018.
- `src/ram/CombMNZ.cpp` implements the CombMNZ linear combination methods as they are described in the paper of Renda et al., 2003.
- `src/ram/CombSUM.cpp` implements the CombSUM linear combination methods (including Borda Count) as they are described in the paper of Renda et al., 2003.
- `src/ram/CondorcetWinners.cpp` implements one of the oldest approaches to rank aggregation, the Condorcet criterion method.
- `src/ram/CopelandWinners.cpp` implements the method of Copeland Winners which is a variant of the Condorcet Winners method.
- `src/ram/CustomMethods.cpp` contains two sample functions that facilitate the integration of custom rank aggregation methods.
- `src/ram/DIBRA.cpp` implements the Iterative Distance-Based Weighted method of Akritidis et al., 2022.
- `src/ram/KemenyOptimal.cpp` contains the brute force implementation of Kemeny optimal aggregation.
- `src/ram/MC.cpp` implements the four Markov Chains method of Dwork et al., 2001 and the MCT method of DeConde et al., 2006.
- `src/ram/OutrankingApproach.cpp` implements the Outranking Approach of Farah and Vanderpooten, 2007.
- `src/ram/PrefRel.cpp` implements the Preference Relations Weighted method of Desarkar et al., 2016.
- `src/ram/RobustRA.cpp` implements the Robust Rank Aggregation method (RRA) of Kolde et al., 2012.

For more details, please visit the [Publications](#) section, or follow the links in the [Introduction](#) section. A guide on how custom rank aggregation implementations can be integrated to FLAGR is given [here](#).

Technical Details

The elements of `MergedList` are organized in two ways. More specifically:

- A hash table (`hash_table` member variable) with linked lists as chains (for collision resolution) is employed to support fast fusion of the individual [InputLists](#). The contents of this hash table are [MergedItem](#) objects; the search keys are the unique identifiers of the associated [InputItems](#) ([MergedItem](#) inherits the members of [InputItem](#)).
- A typical array of [MergedItem](#) pointers (`item_list` member variable) that is used to sort the objects in decreasing score order.

3.10 Query class

A Query represents a topic for which a set of voters or rankers submit their preference lists. For example, a Query may be a simple question like *"who is the best football player for 2022?"*, or a more complex structure like a sequence of genes.

Implementation files

The Query class is defined in the [src/Query.h](#) header file; its member functions are implemented in [src/Query.cpp](#).

Technical Details

The role of this object is central in the entire rank aggregation process, since it connects the input preference lists and the output aggregate list. In particular, a Query is connected to:

- an [Aggregator](#) that triggers the execution of a rank aggregation algorithm on the collection of the [InputLists](#), and
- an [Evaluator](#) that evaluates the quality of the generated aggregate list with respect to a set of relevance judgments.

3.11 Ranking class

This is a simple class that stores the ranking information of a [MergedItem](#) in a particular input preference list. An array of Ranking objects is maintained inside each [MergedItem](#).

Implementation files

The Ranking class is defined in the [src/Ranking.h](#) header file; its member functions are implemented in [src/Ranking.cpp](#).

Technical Details

A Ranking class comprises three members:

- A pointer to the corresponding preference [InputList](#),
- The (integer) ranking in the preference [InputList](#), and
- The score value that is assigned by the voter to this element. The score data type can be either `float`, or `double`; this is determined by the `score_t` data type definition in [driver.cpp](#).

3.12 Re1 class

A Re1 object contains a relevance judgement about a list element. It is read from a special [input CSV](#) file and it is used by an [Evaluator](#) to compute several evaluation measures about the generated aggregate list.

A Re1 object is a member of a [Re1s](#) collection, which in turn is referenced by an [Evaluator](#).

Implementation files

The Re1 class is defined in the [src/Re1.h](#) header file; its member functions are implemented in [src/Re1.cpp](#).

Technical Details

The Re1 class consists of the following member variables:

- a string that represents the unique identifier of a [MergedItem](#),
- an integer relevance judgment. The higher the value of this variable, the more relevant/important the item is considered.
- a next pointer to another Re1 object that allows the creation of dynamic data structures (e.g. linked lists of Re1 objects, etc.).

3.13 Re1s class

Re1s contains the relevance judgements that are required by the [Evaluator](#) in order to evaluate the quality of the generated aggregate list.

The relevance judgments are provided as an input to the library via a special [CSV](#) file.

Implementation files

The Re1s class is defined in the [src/Re1s.h](#) header file; its member functions are implemented in [src/Re1s.cpp](#).

Technical Details

Notice that the [Evaluator](#) contains a pointer to a Re1s object. In this way, the [Evaluator](#) can quickly access the required relevance judgments during the evaluation procedure.

The Re1s object is implemented as a standard hash table with the string item identifiers being its search key. The hash values are computed by the [hash function of Daniel J. Bernstein](#), whereas the collisions are resolved by using chains in the form of linked lists.

3.14 SimpleScoreStats class

SimpleScoreStats is a very simple class that stores several score statistics. In the current FLAGR implementations, it is used exclusively for storing statistical information about the generated aggregate lists. More specifically, the minimum, maximum, and mean score values are stored there, including their standard deviation.

Implementation files

The SimpleScoreStats class is defined in the [src/SimpleScoreStats.h](#) header file; its member functions are implemented in [src/SimpleScoreStats.cpp](#).

3.15 Voter class

A voter (also called ranker, or source), submits preferences for one or more topics (queries) in the form of a ranked preference list. The preference lists of all voters are subsequently aggregated by a rank aggregation method in order to generate a single consensus ranking. In FLAGR, a Voter submits a single [InputList](#) for each [Query](#).

A Voter object possesses two properties:

- A unique string identifier that represents the voter's name, and
- A weight value that reflects the importance (degree of expertise) of the voter for a particular query. Non-weighted rank aggregation methods consider that all voters are equivalent. Therefore, their lists are processed in an identical manner. In contrast, the weighted methods apply unsupervised learning techniques and exploratory analysis to automatically determine the significance of each voter. The weight data type can be either float, or double; this is determined by the score_t data type definition in [driver.cpp](#).

Implementation files

The Voter class is defined in the [src/Voter.h](#) header file; its member functions are implemented in [src/Voter.cpp](#).

3.16 Integrating custom methods

This step-by-step guide describes how custom rank aggregation methods can be implemented and integrated into FLAGR. If you intend to implement fewer than three methods, then several of the steps below are already implemented in FLAGR. For three or more custom methods, additional actions must be performed.

Implementing your own method/s

Custom methods must be implemented as C++ functions in the [src/ram/CustomMethods.cpp](#) file. This file already contains two such functions: CustomMethod1() and CustomMethod2(). In case you desire to implement more methods (e.g., CustomMethod3(), etc.), then you must implement them similarly in that file.

Typically, a rank aggregation method assigns scores to the list elements and then, it sorts the elements in either increasing, or decreasing score order. For this reason, the last step in your implementation must be the sorting of the elements of [MergedList](#). Observe that CustomMethod1() and CustomMethod2() contain a call to qsort (QuickSort) in order to perform the required sorting.

Each algorithm implementation (including the built-in ones) takes three arguments:

- An array of the input preference lists (`class InputList ** inlists`). Most rank aggregation methods do not require access to this array, since when the function is called, the input lists have already been merged in the `MergedList` object. However, several methods require the computation of list distances (e.g. DIBRA, Agglomerative) and the `inlists` pointer provides access to this array.
- A pointer to a `SimpleScoreStats` object in case you desire to store score statistics (max, min, mean, etc.).
- A pointer to the `InputParams` object that contains the user-defined input parameters.

The following code example demonstrates an iteration through the elements of the aggregate list. For each element q , an iteration through its individual rankings in each input preference list is performed:

```
void MergedList::CustomMethod1 (class InputList ** inlists, class
SimpleScoreStats * s, class InputParams * prms) {
    class MergedItem * q;
    class Ranking * r;

    for (rank_t i = 0; i < this->num_nodes; i++) {
        /// q stores an element of the aggregate list
        q = this->item_list[i];

        /// Iterate through the individual rankings of q
        for (uint32_t j = 0; j < q->get_num_alloc_rankings(); j++) {
            r = q->get_ranking(j);
            /// Do something with q and r

        }
    }
    /// Sort the list elements in decreasing score order
    qsort(this->item_list, this->num_nodes, sizeof(class MergedItem *),
        &MergedList::cmp_score_desc);
}
```

Calling the new method/s

The implementation of the new methods can be immediately used, provided that you have not changed the names of the functions `CustomMethod1()` and `CustomMethod2()`. In this case, the following piece of code inside the `main()` function in `main.cpp` the new implementations:

```
/// Execution of CustomMethod1
Custom1(input_file, qrels_file, 20, "Custom1", output_dir);

/// Execution of CustomMethod2
Custom2(input_file, qrels_file, 20, "Custom2", output_dir);
```


If you change the aforementioned function names, or you create a new function for a new algorithm implementation, then a sequence of actions must be taken so that it becomes available for usage. More specifically:

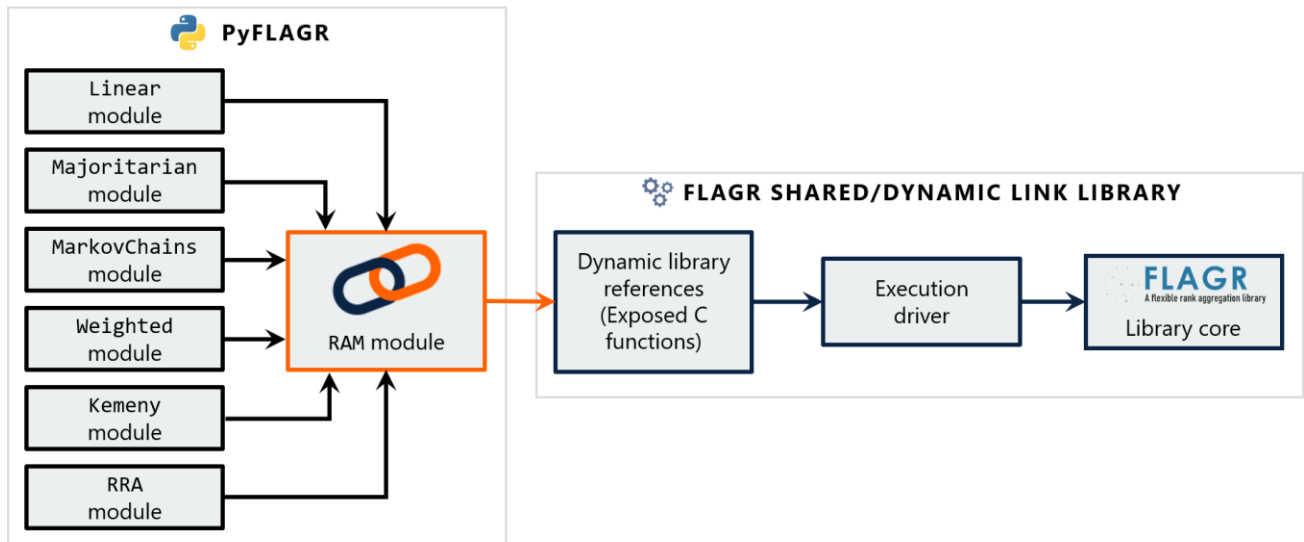
- The user must determine an integer identifier for the algorithm. To avoid conflicts with the built-in methods of FLAGR, it is advised that the leading digit of the identifier is 9 (e.g., 900).
- The user must update the `aggregate()` function of the [Aggregator](#) to allow the execution of the custom method. More specifically, the `if` statement must be appropriately extended.
- An exposed C function must be written in both `cflagr.cpp` and `dllflagr.cpp` with the aim of including the custom implementation in a shared/dynamic link library. The function can be called in the `main()` function of `main.cpp`.

Additional details for custom method implementations

1. A custom rank aggregation method must be declared as a public member function of the [MergedList](#) class. This is performed in the class descriptor, in the `src/MergedList.h` header file. `CustomMethod1()` and `CustomMethod2()` are already public members of [MergedList](#).
2. The required `cpp` file that contains the implementation of the member function must be stored in the `src/ram` directory.
3. Then, the `cpp` file is imported in the project with an include statement in `src/MergedList.cpp`.

4. PyFLAGR API Documentation

PyFLAGR is a Python library built on top of FLAGR. It includes a driver module called RAM that links to the FLAGR shared library and provides access to the algorithm implementations. Then, a set of classes inherit from RAM and allow the end user to execute the selected algorithm.



4.1 Introduction

PyFLAGR is a Python library built on top of FLAGR. It provides easy access to the algorithm implementations of FLAGR from standard Python programs. PyFLAGR has been designed with simplicity in mind: with only a few lines of code the programmer may efficiently execute complex rank aggregation methods and get the results in a Pandas Dataframe.

From a technical perspective, PyFLAGR links to the FLAGR shared/dynamic link library and makes use of its [reference functions \(namely, its exposed C functions\)](#) to pass the user-defined parameters and perform rank aggregation. In the sequel, it simply reads the output files that are produced by FLAGR and stores their contents in Pandas Dataframes. The Dataframes are then returned to the user.

PyFLAGR consists of the following modules:

- **RAM:** It implements the RAM base class that is responsible for several functional procedures like the library I/O, the linkage and loading of the FLAGR shared library, Dataframe handling, and so on. All the other classes of PyFLAGR derive from this class.
- **Linear:** It includes several classes that execute linear combination rank aggregation methods (CombSUM, CombMNZ, Borda Count).
- **Majoritarian:** It includes several classes that execute majoritarian rank aggregation methods (Condorcet Winners, Copeland Winners, Outranking Approach of Farah and Vanderpooten, 2007).
- **MarkovChains:** It executes the algorithms which are based on Markov Chains (MC1, MC2, MC3, MC4, see Dwork et al., 2001).

- **Weighted**: It executes the rank aggregation methods that automatically determine the voter weights in an unsupervised learning fashion (Preference Relations Method of Desarkar et al., 2016, Agglomerative Aggregation method of Chatterjee et al., 2018, Iterative Distance-based method of Akritidis et al., 2022).
- **Kemeny**: It executes Kemeny optimal aggregation (brute force, NP-Hard implementation).
- **RRA**: It executes the Robust Rank Aggregation (RRA) method of Kolde et al., 2012.

Please refer to the [Publications](#) section for more information about the relevant papers.

4.2 Installation

PyFLAGR can be installed directly by using pip:

```
pip install pyflagr
```

Alternatively, PyFLAGR can be installed from the sources by navigating to the directory where `setup.py` resides:

```
pip install /path/to/setup.py
```

4.3 RAM module

The RAM module implements a driver base class also named RAM. The RAM base class performs several important functional procedures, including PyFLAGR I/O, linkage and loading of the FLAGR shared library, I/O Dataframe handling, and so on. The majority of the other classes of PyFLAGR derive from this class and inherit its members and properties.

The class constructor takes as argument the `eval_pts` parameter that determines the elements in the aggregate list on which the evaluation measures (i.e., Precision, and nDCG) will be computed. The most important operation of the constructor is the loading of the FLAGR shared library, according to the underlying operating system. Therefore, if PyFLAGR is executed on a Linux-based system, then [pyflagr/pyflagr/flagr.so](#) is loaded. Similarly, if PyFLAGR is executed on a Windows-based system, then [pyflagr/pyflagr/flagr.dll](#) is loaded. For the time being, FLAGR has not been tested on MacOS-based systems and no pre-compiled shared libraries exist for this platform.

The successful loading of the FLAGR shared/dynamic link library creates the `flagr_lib` connection handler. `flagr_lib` acts as a connector between PyFLAGR and FLAGR, making the [exposed C functions](#) of FLAGR accessible from RAM and its derived classes.

Other member functions include `check_get_input()` and `check_get_rels_input()`. These two functions perform several sanity checks on the provided input files. On the other hand, the role of `get_output()` is to read the output files created by FLAGR and load their content into two Pandas Dataframes. These two Dataframes are eventually returned to the user.

Implementation file

[pyflagr/pyflagr/RAM.py](#)

4.4 Linear module

The Linear module provides access to the implementations of the [linear combination methods of FLAGR](#). In these methods, the score of each element is computed by summing up the partial scores of that element with respect to its rankings in each input preference list. The module includes four classes which are described below: CombSUM, CombMNZ, BordaCount and SimpleBordaCount.

Implementation file

[pyflagr/pyflagr/Linear.py](#)

The CombSUM and CombMNZ Python classes

Both classes derive from RAM, a base class defined in the [RAM](#) module. They inherit the `flagr_lib` connector from RAM, and through it, they obtain access to the FLAGR shared library. Their constructors are identical and determine the data types of the input arguments and the return type of the [Linear\(\) exposed function](#). Observe the similarity between the members of `self.flagr_lib.Linear.argtypes` and the input arguments of the [Linear\(\) exposed function](#).

The arguments of the constructors of CombSUM and CombMNZ include:

Parameter	Type	Default	Description
eval_pts	Integer, Optional. Considered only if rels_file or rels_df is set.	10	Determines the elements in the aggregate list on which the evaluation measures (i.e., Precision, and nDCG) will be computed. For example, for <code>eval_pts=10</code> FLAGR will compute Average Precision, $P@1, P@2, \dots P@10$ and $N@1, N@2, \dots N@10$.
norm	String, Optional.	borda	Rank or score normalization methods. <ul style="list-style-type: none">• borda: The aggregation is performed by normalizing the element rankings according to the Borda normalization method.• rank: The aggregation is performed by normalizing the element rankings according to the Rank normalization method.• score: The aggregation is performed by normalizing the element scores according to the Score normalization method.• z-score: The aggregation is performed by normalizing the element scores according to the Z-Score normalization method.• simple-borda: Similar to borda normalization but no partial score is assigned to an element if it is not ranked by a voter.

CombSUM and CombMNZ also include an `aggregate()` function that receives the user-defined input parameters and passes them to the [Linear\(\) exposed C function](#), that subsequently performs the aggregation of the ranked input preference lists. The arguments of the `aggregate()` function include the following:

Parameter	Type	Default	Description
input_file	String Required, unless input_df is set.	Empty String	A CSV file that contains the input lists to be aggregated.
input_df	Pandas DataFrame - Required, unless input_file is set.	None	A Pandas DataFrame that contains the input lists to be aggregated. Note: If both input_file and input_df are set, only the former is used; the latter is ignored.
rels_file	String, Optional.	Empty String	A CSV file that contains the relevance judgements of the involved list elements. If such a file is passed, FLAGR will evaluate the generated aggregate list(s) by computing several retrieval effectiveness evaluation measures. The results of the evaluation will be stored in the eval_df DataFrame. Otherwise, no evaluation will take place and eval_df will be empty.
rels_df	Pandas DataFrame, Optional.	None	A Pandas DataFrame that contains the relevance judgements of the involved list elements. If such a dataframe is passed, FLAGR will evaluate the generated aggregate list(s) by computing several retrieval effectiveness evaluation measures. The results of the evaluation will be stored in the eval_df DataFrame. Otherwise, no evaluation will take place and eval_df will be empty. Note: If both rels_file and rels_df are set, only the former is used; the latter is ignored.
output_dir	String, Optional.	Temporary directory (OS-specific)	The directory where the output files (aggregate lists and evaluation) will be stored. If it is not set, the default location will be used.

The BordaCount and SimpleBordaCount Python classes

These two classes have been included in PyFLAGR for historical reasons. They are equivalent to the CombSUM linear combination method with norm='borda' and norm='simple-borda' normalization methods, respectively.

BordaCount and SimpleBordaCount derive from CombSUM (which in turn derives from RAM). They do not have an aggregate() method, so they both call the same aggregate() function of CombSUM. Their only difference lies in their constructors: The former initializes CombSUM with norm='borda', whereas the latter with norm='simple-borda'.

4.5 Majoritarian module

The Majoritarian module provides access to the implementations of the majoritarian rank aggregation methods of FLAGR. These methods are based on the majority criterion that, under several circumstances, identify the “winning” elements. The module includes three classes which are described below: `CondorcetWinners`, `CopelandWinners` and `OutrankingApproach`. Each method implements different scenarios for the majority criterion.

Implementation file

[pyflagr/pyflagr/Majoritarian.py](#)

The CondorcetWinners and CopelandWinners Python classes

Both classes derive from `RAM`, a base class defined in the [RAM](#) module. They inherit the `flagr_lib` connector from `RAM`, and through it, they obtain access to the respective exposed functions of FLAGR.

The constructor of `CondorcetWinners` determines the data types of the input arguments and the return type of the [Condorcet\(\) exposed function](#). Similarly, the constructor of `CopelandWinners` determines the data types of the input arguments and the return type of the [Copeland\(\) exposed function](#). Both constructors accept just one argument:

Parameter	Type	Default	Description
<code>eval_pts</code>	Integer, Optional. Considered only if <code>rels_file</code> or <code>rels_df</code> is set.	10	Determines the elements in the aggregate list on which the evaluation measures (i.e., Precision, and nDCG) will be computed. For example, for <code>eval_pts=10</code> FLAGR will compute Average Precision, $P@1, P@2, \dots, P@10$ and $N@1, N@2, \dots, N@10$.

`CondorcetWinners` and `CopelandWinners` also include an `aggregate()` function that receives the user-defined parameters and passes them to the [Condorcet\(\)](#) and [Copeland\(\)](#) exposed functions respectively. The arguments of the `aggregate()` function are identical to those of the `CombSUM` and `CombMNZ` classes (refer to the respective table of [Subsection 4.4](#)).

The OutrankingApproach Python class

This class can be used to execute the [Outranking Approach of Farah and Vanderpooten, 2007](#). Similarly to the other majoritarian methods, `OutrankingApproach` derives from `RAM`, a base class defined in the [RAM](#) module. It also inherits the `flagr_lib` connector from `RAM` and obtains access to the [OutrankingApproach\(\) exposed function](#) of FLAGR.

The constructor of `OutrankingApproach` determines the data types of the input arguments and the return type of the [OutrankingApproach\(\) exposed function](#). Observe the similarity between the members of `self.flagr_lib.OutrankingApproach.argtypes` and the input arguments of the [OutrankingApproach\(\) exposed function](#).

Parameter	Type	Default	Description
eval_pts	Integer, Optional. Considered only if rels_file or rels_df is set.	10	Determines the elements in the aggregate list on which the evaluation measures (i.e., Precision, and nDCG) will be computed. For example, for eval_pts=10 FLAGR will compute Average Precision, $P@1, P@2, \dots P@10$ and $N@1, N@2, \dots N@10$.
preference	Hyper-parameter, Float, Optional.	0.00	The value of the preference threshold.
veto	Hyper-parameter, Float, Optional.	0.75	The value of the veto threshold.
concordance	Hyper-parameter, Float, Optional.	0.00	The value of the concordance threshold.
discordance	Hyper-parameter, Float, Optional.	0.25	The value of the discordance threshold.

OutrankingApproach includes an `aggregate()` function that receives the user-defined parameters and passes them to the [OutrankingApproach\(\) exposed function](#). The arguments of the `aggregate()` function are identical to those of the CombSUM and CombMNZ classes (refer to the respective table of [Subsection 4.4](#)).

4.6 MarkovChains module

The MarkovChains module provides access to the implementations of the [Markov Chains methods](#) of Dwork et al., 2001 and DeConde et al., 2006. It includes the MC driver class, and five small derived classes (namely, MC1, MC2, MC3, MC4 and MCT) that can be employed by the user to execute the corresponding methods.

Implementation file

[pyflagr/pyflagr/MarkovChains.py](#)

The MC base class

This is the driver class of the module. Its direct usage is weakly discouraged. The users should prefer employing the 5 classes that derive from MC (see below) and indirectly trigger the corresponding algorithm implementations of FLAGR.

Similarly to the other PyFLAGR classes, MC derives from RAM, another base class that is defined in the [RAM](#) module. It inherits the `flagr_lib` connector from RAM, and through it, it obtains access to the FLAGR shared library. Its constructor determines the data types of the input arguments and the return type of the FLAGR's [MC\(\) exposed function](#). Observe the similarity between the members of `self.flagr_lib.MC.argtypes` and the input arguments of the [MC\(\) exposed function](#).

The constructor of MC takes the following arguments:

Parameter	Type	Default	Description
eval_pts	Integer, Optional. Considered only if rels_file or rels_df is set.	10	Determines the elements in the aggregate list on which the evaluation measures (i.e., Precision, and nDCG) will be computed. For example, for eval_pts=10 FLAGR will compute Average Precision, $P@1, P@2, \dots P@10$ and $N@1, N@2, \dots N@10$.
ergodic_number	Float, Optional.	0.15	The ergodic number, used during the computation of the ergodic transition matrix from the normalized transition matrix.
max_iterations	Integer, Optional.	100	The maximum number of iterations for the computation of the state matrix.
chain	Integer, Optional.	804	The Markov Chain method to execute. The possible values include: <ul style="list-style-type: none"> • 801: Markov Chains Method 1 (MC1). • 802: Markov Chains Method 2 (MC2). • 803: Markov Chains Method 3 (MC3). • 804: Markov Chains Method 4 (MC4). • 805: Markov Chains Thurstone Method (MCT).

MC includes an `aggregate()` function that receives the user-defined parameters and passes them to the `MC()` exposed function, that subsequently performs the aggregation of the ranked input preference lists. The arguments of the `aggregate()` function are identical to those of the CombSUM and CombMNZ classes (refer to the respective table of [Subsection 4.4](#)).

The MC1, MC2, MC3, MC4 and MCT derived classes

These classes derive from the aforementioned MC base class; as children of MC, they also inherit from the `RAM` class. Each of these classes triggers the execution of a different Markov Chain algorithm, simply by passing different parameters to the constructor MC. Hence, MC1 sets `chain=801`, MC2 sets `chain=802`, and so on.

Also notice that none of these classes have an `aggregate()` function. Consequently, the `aggregate()` of the base class (i.e. MC) is executed when the end user invokes that function.

4.7 Weighted module

The Weighted module provides access to the implementations of the weighted methods of FLAGR. Most rank aggregation approaches treat all input preference lists equally. In contrast, the weighted methods employ exploratory analysis techniques to automatically identify the expert voters in an unsupervised fashion. In the sequel, they assign higher weights to those who were identified as

experts, thus boosting the scores of their submitted elements. The module in question includes three classes: DIBRA, Agglomerative and PreferenceRelationsGraph.

Implementation file

[pyflagr/pyflagr/Weighted.py](#)

Distance-Based Iterative Rank Aggregation: The DIBRA Python class

This class links to the FLAGR implementation of the weighted method of Akritidis et al., 2022. It derives from RAM, a base class defined in the [RAM](#) module. It inherits the `flagr_lib` connector from RAM and through it, it obtains access to the FLAGR shared library. Its constructor determines the data types of the input arguments and the return type of the [DIBRA\(\) exposed function](#). Observe the similarity between the members of `self.flagr_lib.DIBRA.argtypes` and the input arguments of the [DIBRA\(\) exposed function](#).

The algorithm initially employs a standard non-weighted method to generate a starting consensus list. Then, it iteratively assigns converging weights to the voters according to the distances of their submitted lists with this consensus list. Therefore, the DIBRA constructor takes as arguments all the possible hyper-parameters of the supported non-weighted methods. Specifically:

Parameter	Type	Default	Description
<code>eval_pts</code>	Integer, Optional. Considered only if <code>rels_file</code> or <code>rels_df</code> is set.	10	Determines the elements in the aggregate list on which the evaluation measures (i.e., Precision, and nDCG) will be computed. For example, for <code>eval_pts=10</code> FLAGR will compute Average Precision, $P@1, P@2, \dots P@10$ and $N@1, N@2, \dots N@10$.
<code>aggregator</code>	Hyper-parameter, String, Optional.	<code>combsum:borda</code>	The selected non-weighted method that does the initial aggregation. Possible values include: <ul style="list-style-type: none">• <code>combsum:borda</code>: CombSUM with Borda normalization.• <code>combsum:rank</code>: CombSUM with Rank normalization.• <code>combsum:score</code>: CombSUM with minmax score normalization.• <code>combsum:z-score</code>: CombSUM with Z-score normalization.• <code>combsum:simple-borda</code>: CombSUM with simple Borda normalization.• <code>combmnz:borda</code>: CombMNZ with Borda normalization.• <code>combmnz:rank</code>: CombMNZ with Rank normalization.• <code>combmnz:score</code>: CombMNZ with minmax score normalization.• <code>combmnz:z-score</code>: CombMNZ with Z-score normalization.

			<ul style="list-style-type: none"> • <code>combmz:simple-borda</code>: CombMNZ with simple Borda normalization. • <code>condorcet</code>: Condorcet Winners. • <code>copeland</code>: Copeland Winners. • <code>outrank</code>: Outranking Approach.
<code>w_norm</code>	Hyper-parameter, String, Optional.	<code>minmax</code>	<p>The voter weights normalization method. Possible values include:</p> <ul style="list-style-type: none"> • <code>none</code>: no weight normalization takes place. • <code>minmax</code>: minmax weight normalization. • <code>z</code>: z weight normalization.
<code>dist</code>	Hyper-parameter, String, Optional.	<code>cosine</code>	<p>The correlation/distance metric that measures the distance between an input list and the temporary aggregate list. Possible values include:</p> <ul style="list-style-type: none"> • <code>rho</code>: Spearman's ρ. • <code>cosine</code>: a metric based on cosine similarity (see Akritidis et al., 2022). • <code>footrule</code>: Spearman's Footrule distance. • <code>tau</code>: Kendall's τ
<code>prune</code>	Hyper-parameter, Boolean, Optional.	<code>False</code>	Triggers a weight-dependent list pruning mechanism.
<code>gamma</code>	Hyper-parameter, Float, Optional.	<code>1.5</code>	The γ hyper-parameter that determines the steplength of weight learning.
<code>d1</code>	Hyper-parameter, Float, Optional.	<code>0.4</code>	The δ_1 hyper-parameter of the list pruning mechanism. Applies only if <code>prune=True</code> .
<code>d2</code>	Hyper-parameter, Float, Optional.	<code>0.1</code>	The δ_2 hyper-parameter of the list pruning mechanism. Applies only if <code>prune=True</code> .
<code>tol</code>	Hyper-parameter, Float, Optional.	<code>0.01</code>	Controls the convergence precision. This tolerance threshold represents the minimum precision of the difference between the voter weight in an iteration and the voter weight of the previous iteration.
<code>max_iter</code>	Hyper-parameter, Integer, Optional.	<code>50</code>	Controls the maximum number of iterations before the voter weights converge.
<code>pref</code>	Hyper-parameter, Float, Optional.	<code>0.0</code>	The preference threshold. Applies only if <code>aggregator=outrank</code> .
<code>veto</code>	Hyper-parameter, Float, Optional.	<code>0.75</code>	The veto threshold. Applies only if <code>aggregator=outrank</code> .

conc	Hyper-parameter, Float, Optional.	0.0	The concordance threshold. Applies only if aggregator=outrank.
disc	Hyper-parameter, Float, Optional.	0.25	The discordance threshold. Applies only if aggregator=outrank.

DIBRA also includes an `aggregate()` function that receives the user-defined parameters and passes them to the [DIBRA\(\) exposed function](#), that subsequently performs the aggregation of the ranked input preference lists. The arguments of the `aggregate()` function are identical to those of the CombSUM and CombMNZ classes (refer to the respective table of [Subsection 4.4](#)).

Agglomerative weighted aggregation: The Agglomerative Python class

This class employs the implementation of the [Agglomerative weighted method of Chatterjee et al., 2018](#). Similarly to all weighted methods, it derives from RAM, a base class defined in the [RAM](#) module. It inherits the `flagr_lib` connector from RAM, and, through it, they obtain access to the FLAGR shared library. Its constructor determines the data types of the input arguments and the return type of the [Agglomerative\(\) exposed function](#). Observe the similarity between the members of `self.flagr_lib.Aglomerative.argtypes` and the input arguments of the [Agglomerative\(\) exposed function](#).

The constructor's arguments include:

Parameter	Type	Default	Description
eval_pts	Integer, Optional. Considered only if rels_file or rels_df is set.	10	Determines the elements in the aggregate list on which the evaluation measures (i.e., Precision, and nDCG) will be computed. For example, for <code>eval_pts=10</code> FLAGR will compute Average Precision, $P@1, P@2, \dots P@10$ and $N@1, N@2, \dots N@10$.
c1	Hyper-parameter, Float, Optional.	0.1	The c_1 hyper-parameter of the algorithm.
c2	Hyper-parameter, Float, Optional.	0.5	The c_2 hyper-parameter of the algorithm.

Agglomerative includes an `aggregate()` function that receives the user-defined parameters and passes them to the [Agglomerative\(\) exposed function](#), that subsequently performs the aggregation of the ranked input preference lists. The arguments of the `aggregate()` function are identical to those of the CombSUM and CombMNZ classes (refer to the respective table of [Subsection 4.4](#)).

Preference relations weighted method: The PreferenceRelationsGraph Python class

This class links to the FLAGR implementation of the [weighted method of Desarkar et al., 2016](#). Similarly to all weighted methods, it derives from RAM, a base class defined in the [RAM](#) module. It inherits the `flagr_lib` connector from RAM, and through it, they obtain access to the FLAGR shared library. Its constructor determines the data types of the input arguments and the return type of the [PrefRel\(\)](#)

[exposed function](#). Observe the similarity between the members of `self.flagr_lib.PrefRel.argtypes` and the input arguments of the [PrefRel\(\) exposed function](#).

The constructor's arguments include:

Parameter	Type	Default	Description
eval_pts	Integer, Optional. Considered only if rels_file or rels_df is set.	10	Determines the elements in the aggregate list on which the evaluation measures (i.e. Precision, and nDCG) will be computed. For example, for eval_pts=10 FLAGR will compute Average Precision, $P@1, P@2, \dots P@10$ and $N@1, N@2, \dots N@10$.
alpha	Hyper-parameter, Float, Optional.	0.1	The α hyper-parameter of the algorithm.
beta	Hyper-parameter, Float, Optional.	0.5	The β hyper-parameter of the algorithm.

PreferenceRelationsGraph also includes an `aggregate()` function that receives the user-defined parameters and passes them to the [PrefRel\(\) exposed function](#), that subsequently performs the aggregation of the ranked input preference lists. The arguments of the `aggregate()` function are identical to those of the CombSUM and CombMNZ classes (refer to the respective table of [Subsection 4.4](#)).

4.8 Kemeny module

The Kemeny module provides access to the implementation of [Kemeny Optimal Aggregation](#) of FLAGR. It is stressed out that due to the method's high complexity, the brute force implementation becomes infeasible even when the number of elements, or the number of input lists receive moderate values.

Implementation file

[pyflagr/pyflagr/Kemeny.py](#)

The KemenyOptimal class

The `KemenyOptimal` class derives from `RAM`, a base class defined in the [RAM](#) module. It inherits the `flagr_lib` connector from `RAM`, and through it, it obtains access to the FLAGR shared library. Its constructor determines the data types of the input arguments and the return type of the FLAGR's [Kemeny\(\) exposed function](#). Observe the similarity between the members of `self.flagr_lib.Kemeny.argtypes` and the input arguments of the [Kemeny\(\) exposed function](#).

The arguments of the constructor includes one parameter:

Parameter	Type	Default	Description
eval_pts	Integer, Optional. Considered only if rels_file or rels_df is set.	10	Determines the elements in the aggregate list on which the evaluation measures (i.e., Precision, and nDCG) will be computed. For example, for eval_pts=10 FLAGR will compute Average Precision, $P@1, P@2, \dots P@10$ and $N@1, N@2, \dots N@10$.

KemenyOptimal also includes an `aggregate()` function which receives the user-defined parameters and passes them to the [Kemeny\(\) exposed function](#), that subsequently performs the aggregation of the ranked input preference lists. The arguments of the `aggregate()` function are identical to those of the CombSUM and CombMNZ classes (refer to the respective table of [Subsection 4.4](#)).

4.9 RRA module

The RRA module provides access to the implementation of [Robust Rank Aggregation](#) of Kolde et al., 2012.

Implementation file

[pyflagr/pyflagr/RRA.py](#)

The RRA class

The RRA class derives from RAM, a base class defined in the [RAM](#) module. It inherits the `flagr_lib` connector from RAM, and through it, it obtains access to the FLAGR shared library. Its constructor determines the data types of the input arguments and the return type of the FLAGR's [RobustRA\(\) exposed function](#). Observe the similarity between the members of `self.flagr_lib.RobustRA.argtypes` and the input arguments of [RobustRA\(\) exposed function](#).

The arguments of the constructor includes two parameters:

Parameter	Type	Default	Description
eval_pts	Integer, Optional. Considered only if rels_file or rels_df is set.	10	Determines the elements in the aggregate list on which the evaluation measures (i.e. Precision, and nDCG) will be computed. For example, for eval_pts=10 FLAGR will compute Average Precision, $P@1, P@2, \dots P@10$ and $N@1, N@2, \dots N@10$.
exact	Boolean, Optional.	False	Determines whether the computed p-Values of the list elements will be corrected with the Stuart-Ares method.

RRA also includes an `aggregate()` function that receives the user-defined parameters and passes them to the [RobustRA\(\) exposed function](#), that subsequently performs the aggregation of the ranked input preference lists. The arguments of the `aggregate()` function are identical to those of the CombSUM and CombMNZ classes (refer to the respective table of [Subsection 4.4](#)).

4.10 Comparator module

The Comparator module includes a class that implements several tools for conducting performance comparisons of rank aggregation algorithms. The input includes a data file with the input preference lists, another file that contains the relevance judgments for the involved list elements, and a group of rank aggregation algorithms to be compared. After running the selected algorithms on the input data, Comparator produces comparison tables in various formats (e.g. CSV, LaTeX, etc.) and plots of multiple evaluation measures. Extended code examples of usage are presented in [this notebook](#).

Implementation file

[pyflagr/pyflagr/Comparator.py](#)

Member variables

The class maintains three member variables:

- **aggregators**: A Python list that contains the objects that handle rank aggregation algorithms, along with a user-defined description.
- **results**: A Pandas Dataframe that stores the results of the evaluation (namely, the values of various evaluation measures).
- **ev_pts**: An integer that represents the cutoff point at which the evaluation measures will be computed.

Member methods

add_aggregator(): This function appends new records into the aggregators list. Each record represents a rank aggregation method that will participate in the comparison tests.

Parameter	Type	Default	Description
name	String, Required.	-	The name of the rank aggregation algorithm that is inserted.
obj	Object, Required.	-	An object that handles the corresponding rank aggregation method.

Here is a quick example that initializes a Comparator object and appends three rank aggregation methods:

```
import pyflagr.Linear as Linear
import pyflagr.Majoritarian as Majoritarian
import pyflagr.Weighted as Weighted
import pyflagr.Comparator as Comparator

EV_PTS = 10
cmp = Comparator.Comparator(EV_PTS)
cmp.add_aggregator("CombSUM-Borda", Linear.CombSUM(norm='borda',
eval_pts=EV_PTS))
cmp.add_aggregator("Copeland", Majoritarian.CopelandWinners(eval_pts=EV_PTS))
cmp.add_aggregator("DIBRA-Prune", Weighted.DIBRA(aggregator='combsum:borda',
gamma=1.2, prune=True, w_norm='minmax', d1=0.3, d2=0.05, eval_pts=EV_PTS))
```

aggregate(): Sequentially invokes the `aggregate()` method of each algorithm included in the `aggregators` array.

This method also requires a file (or a Dataframe) that contains relevance judgments for the individual list elements. The generated aggregate lists of each algorithm are automatically evaluated (by FLAGR) by using these relevance judgments. The class computes the values of multiple well-established evaluation measures including Mean Average Precision (MAP), Precision, Recall, DCG (Discounted Cumulative Gain), and nDCG (normalized DCG). The computed values are written into the `self.results` Dataframe.

`aggregate()` takes four arguments:

Parameter	Type	Default	Description
<code>input_file</code>	String, Required, unless <code>input_df</code> is set.	Empty String	A CSV file that contains the input lists to be aggregated.
<code>input_df</code>	Pandas DataFrame, Required, unless <code>input_file</code> is set.	None	A Pandas DataFrame that contains the input lists to be aggregated. Note: If both <code>input_file</code> and <code>input_df</code> are set, only the former is used; the latter is ignored.
<code>rels_file</code>	String, Required, unless <code>rels_df</code> is set.	Empty String	A CSV file that contains the relevance judgements of the involved list elements. FLAGR will evaluate the generated aggregate list/s by computing the values of multiple performance evaluation measures. The results of the evaluation will be stored in the <code>self.results</code> Dataframe.
<code>rels_df</code>	Pandas DataFrame, Required, unless <code>rels_file</code> is set.	None	A Pandas DataFrame that contains the relevance judgements of the involved list elements. FLAGR will evaluate the generated aggregate list/s by computing the values of multiple performance evaluation measures. The results of the evaluation will be stored in the <code>self.results</code> Dataframe. Note: If both <code>rels_file</code> and <code>rels_df</code> are set, only the former is used; the latter is ignored.

Example:

```
# The input data file with the input lists to be aggregated.
lists = 'testdata.csv'

# The input data file with the relevance judgements.
qrels = 'testdata_qrels.csv'

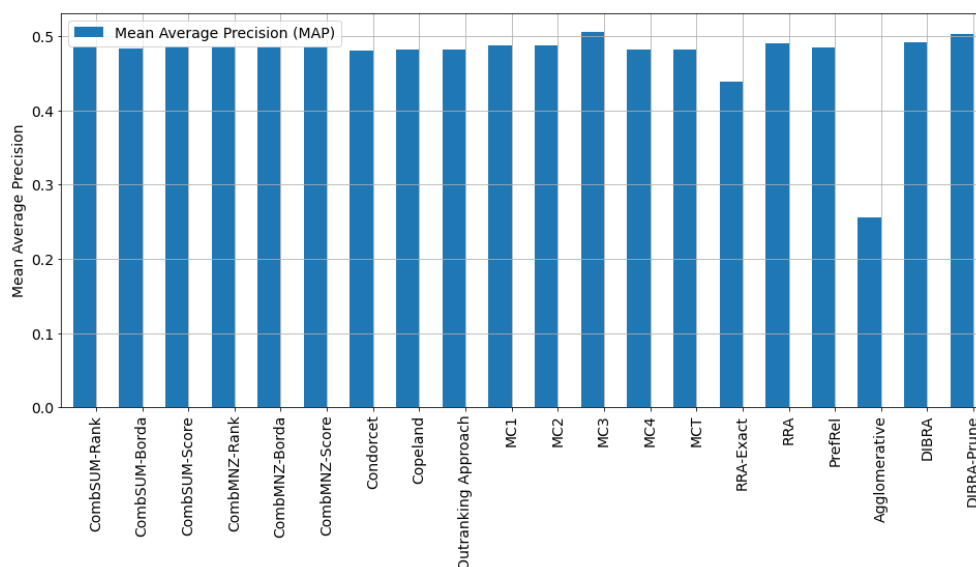
cmp.aggregate(input_file=lists, rels_file=qrels)
```

plot_average_precision(): Creates a comparative bar plot of Mean Average Precision (MAP). The arguments include:

Parameter	Type	Default	Description
dimensions	(x,y) tuple - Optional.	(10.24,7.68)	The plot dimensions (width, height).
show_grid	Boolean - Optional.	True	Determines whether the plot will include grid lines.
query	String - Optional.	'all'	In case the input data file contains preference lists for multiple queries, this parameter determines which query to plot. Notice that 'all' does not mean that all queries will be plotted; instead, it dictates the plotting of the average MAP for all queries.

Example:

```
cmp.plot_average_precision((16, 7), True, query='all')
```



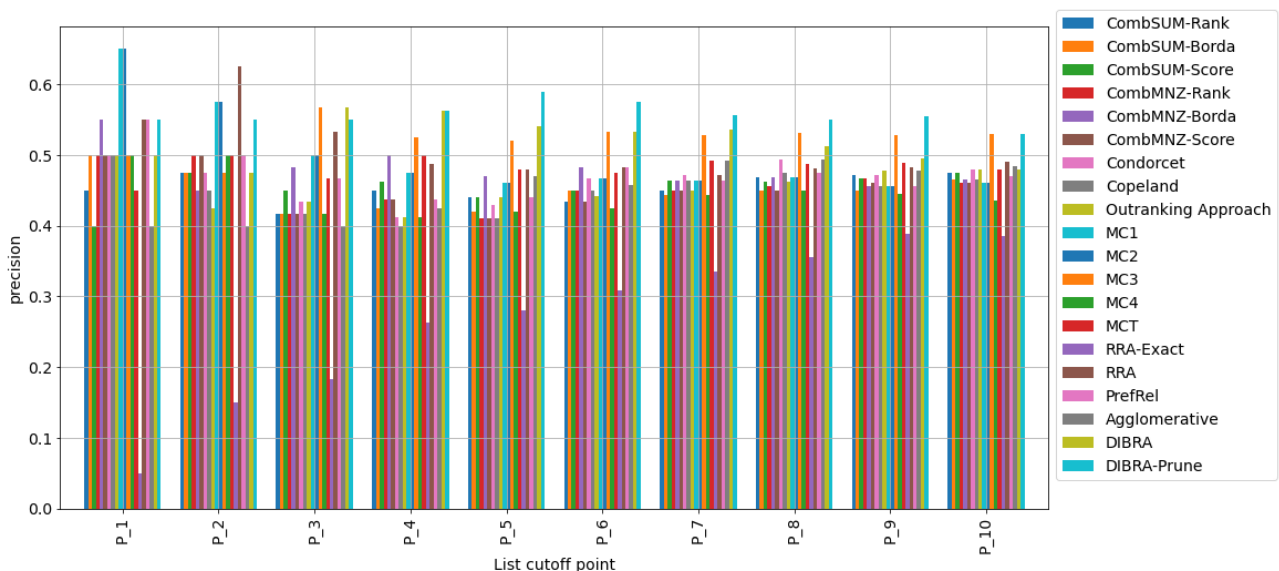
plot_metric(): Creates a plot for a metric at a given cutoff point. The input arguments include:

Parameter	Type	Default	Description
cutoff	Integer - Required.	-	The cutoff point in the aggregate list. The cutoff point must be lower than self.ev_pts.
metric	String - Required.	-	Determines the evaluation measure to be plotted. Acceptable values are 'precision', 'recall', 'dcg', and 'ndcg'.
plot_type	String - Optional.	'bar'	Determines the plot type. Acceptable values are 'bar' and 'lines'.

dimensions	(x,y) tuple - Optional.	(10.24, 7.68)	The plot dimensions (width, height).
show_grid	Boolean - Optional.	True	Determines whether the plot will include grid lines.
query	String - Optional.	'all'	In case the input data file contains preference lists for multiple queries, this parameter determines which query to plot. Notice that 'all' does not mean that all queries will be plotted; instead, it dictates the plotting of the average MAP for all queries.

Example:

```
cmp.plot_metric (EV_PTS, metric='precision', plot_type='bar', dimensions=(16,8),
show_grid=True, query='all')
```



get_results(): Returns slices of `self.results` by setting specific evaluation measures (columns) and queries (rows). The input arguments include:

Parameter	Type	Default	Description
cutoff	Integer - Required.	-	The cutoff point in the aggregate list. The cutoff point must be lower than <code>self.ev_pts</code> .
metric	String - Required.	-	Determines the evaluation measure to be plotted. Acceptable values are 'precision', 'recall', 'dcg', and 'ndcg'.
query	String - Optional.	'all'	In case the input data file contains preference lists for multiple queries, this parameter determines which query to retrieve. Notice that 'all' does not mean that all queries will be plotted; instead, it dictates the plotting of the average MAP for all queries.

convert_to_latex(): Returns the LaTeX code of slices of `self.results`. The input arguments are:

Parameter	Type	Default	Description
cutoff	Integer Required.	-	The cutoff point in the aggregate list. The cutoff point must be lower than <code>self.ev_pts</code> .
metric	String Required.	-	Determines the evaluation measure to be plotted. Acceptable values are 'precision', 'recall', 'dcg', and 'ndcg'.
query	String Optional.	'all'	In case the input data file contains preference lists for multiple queries, this parameter determines which query to retrieve. Notice that 'all' does not mean that all queries will be plotted; instead, it dictates the plotting of the average MAP for all queries.
dec_pts	Integer Optional Maximum value is 6.	6	Sets the precision (i.e. the number of decimal points) of the values of the returned evaluation measures.

5. Appendix

5.1 Evaluation measures

This section provides some brief descriptions of the most popular performance evaluation measures for rank aggregation algorithms. These measures are computed by the evaluation tool of FLAGR in case a valid qrels file is provided as input.

Precision@ k

Precision measures the ability of an algorithm to precisely detect the relevant elements. It is defined as the ratio of the number of relevant elements at the k -th element of a list, divided by the number of retrieved elements (i.e. k):

$$\text{Precision@}k = \frac{\text{true positives@}k}{(\text{true positives@}k) + (\text{false positives@}k)}$$

Recall@ k

Recall measures the ability of an algorithm to detect the relevant elements early. It is defined as the ratio of the number of relevant elements at the k -th element of a list, divided by the number of all relevant elements:

$$\text{Recall@}k = \frac{\text{true positives@}k}{(\text{true positives@}k) + (\text{false negatives@}k)}$$

F1@ k

F1 is a well-established measure that combines Precision and Recall into a single scoring formula:

$$F1@k = \frac{2 \cdot \text{Precision@}k \cdot \text{Recall@}k}{\text{Precision@}k + \text{Recall@}k}$$

Discounted Cumulative Gain $DCG@k$

DCG is another measure for evaluating the performance of an algorithm. In contrast to the previous measures, this one can handle non-binary relevance judgments. In other words, the relevance score of an item may be a real value, and not just a relevant/non-relevant label. It is defined by the following formula:

$$DCG@k = \sum_{i=1}^k \frac{2^{rel_i} - 1}{\log_2(i + 1)}$$

where rel_i is the relevance score of the element at index i . For binary problems, we set $rel_i = 1$ if the i -th list element is relevant and $rel_i = 0$ otherwise.

Normalized Discounted Cumulative Gain ($nDCG@k$)

One disadvantage of DCG is that it is non-decreasing; it either stays the same (if the current element is non-relevant), or it increases (if the current element is relevant). This means that queries that return

larger result sets will probably always have higher DCG scores than queries that return small result sets. The Normalized Discounted Cumulative Gain ($nDCG$) confronts this problem by dividing DCG with the maximum possible DCG at each threshold k :

$$nDCG@k = \frac{DCG@k}{IDCG@k}$$

where $IDCG@k$ is the Ideal $DCG@k$. To compute it, we first create an ideal ranking, where the elements are ranked in decreasing relevance order. Then, $IDCG@k$ is simply equal to $DCG@k$ in that ideal ranking, namely:

$$IDCG@k = \sum_{i=1}^{\text{relevant items @k}} \frac{2^{rel_i} - 1}{\log_2(i + 1)}$$

Average Precision (AP)

AP is another evaluation metric that quantifies the ability of an algorithm to rank the relevant elements in the highest list positions. It is defined by the following equation:

$$AP = \sum_k (\text{Recall}@k - \text{Recall}@k - 1) \cdot \text{Precision}@k$$

Mean Average Precision (MAP)

Average Precision quantifies the quality of a single ranked list compared with the ground truth. In other words, AP examines a single ranking that is generated in response to a single query. In contrast, Mean Average Precision (MAP) evaluates a ranking model for a set of queries Q . MAP is simply defined as the mean of the Average Precisions over all queries. Consequently, its computation is performed by firstly summing up the AP_q value for each query q in the dataset and then, the sum is divided by the number of queries:

$$MAP = \frac{1}{|Q|} \sum_{q=1}^{|Q|} AP_q$$

Example

Consider a ranked list including 8 elements that has been submitted as a response to a query. From these elements, the 1st, 3rd, 4th, and 6th are relevant to the query. The rest of the elements are considered as not relevant. The first two columns of the following table show the list and the relevance of its elements. The rest of the columns contain the running values of Precision, Recall, $F1$, DCG , $IDCG$, $nDCG$ and Average Precision (AP) at each list element.

Rank	Relevant	Precision@k	Recall@k	F1@k	DCG@k	IDCG@k	nDCG@k	AP
1	Yes	1.00	0.25	0.40	1.00	1.00	1.00	1.00
2	No	0.50	0.25	0.33	1.00	1.63	0.61	1.00
3	Yes	0.67	0.50	0.62	1.50	2.13	0.70	0.83
4	Yes	0.75	0.75	0.75	1.93	2.56	0.75	0.81
5	No	0.60	0.75	0.66	1.93	2.56	0.75	0.81
6	Yes	0.67	1.00	0.80	2.29	2.56	0.89	0.77
7	No	0.57	1.00	0.73	2.29	2.56	0.89	0.77
8	No	0.50	1.00	0.66	2.29	2.56	0.89	0.77

Example calculations at the 5th element of the list (@5)

According to the aforementioned definitions, the following calculations are performed:

$$\text{Precision@5} = \frac{\text{relevant elements found up to position 5}}{\text{retrieved elements up to position 5}} = \frac{3}{5} = 0.60$$

$$\text{Recall@5} = \frac{\text{relevant elements found up to position 5}}{\text{all relevant elements}} = \frac{3}{4} = 0.75$$

$$F1@5 = \frac{2 \cdot \text{Precision@5} \cdot \text{Recall@5}}{\text{Precision@5} + \text{Recall@5}} = \frac{2 \cdot 0.60 \cdot 0.75}{0.60 + 0.75} = 0.67$$

$$DCG@5 = \sum_{i=1}^5 \frac{2^{rel_i} - 1}{\log_2(i + 1)} = \frac{2^1 - 1}{\log_2(1 + 1)} + \frac{2^0 - 1}{\log_2(2 + 1)} + \frac{2^1 - 1}{\log_2(3 + 1)} + \frac{2^1 - 1}{\log_2(4 + 1)} + \frac{2^0 - 1}{\log_2(5 + 1)} = 1.9$$

$$IDCG@5 = \sum_{i=1}^4 \frac{2^{rel_i} - 1}{\log_2(i + 1)} = \frac{2^1 - 1}{\log_2(1 + 1)} + \frac{2^1 - 1}{\log_2(2 + 1)} + \frac{2^1 - 1}{\log_2(3 + 1)} + \frac{2^1 - 1}{\log_2(4 + 1)} = 2.56$$

$$nDCG@5 = \frac{DCG@5}{IDCG@5} = \frac{1.93}{2.56} = 0.75$$

$$AP = \frac{1 + 0.67 + 0.75}{3} = 0.81$$

5.2 References

FLAGR utilizes algorithms, methods, and techniques from the following bibliography:

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5.3 Acknowledgments

FLAGR, PyFLAGR, the supporting Web site <https://flagr.site> and this user manual have been created by Leonidas Akritidis. You may contact the author at lakritidis@ihu.gr.

Deep thanks to John Burkardt for kindly permitting the usage of the ASA063 and ASA109 algorithm implementations about the computation of the incomplete beta integrals.

Bugs should be reported through the [FLAGR's GitHub repository](#).

Third-party researchers are strongly encouraged to submit their own algorithm implementations, regardless of the programming language they are using.