FINANCIAL MANAGEMENT

SEMESTER TWO (2)

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TIME VALUE OF MONEY

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INTRODUCTION

Time value of money is a critical consideration in **financial and investment decisions**. For example, compound interest calculations are needed to determine **future sums of money resulting from an investment**. How investors and borrowers interact to value investments and determine interest rates on loans and fixed income securities.

Discounting, or the calculation of present value, which is inversely related to compounding, is used to evaluate future cash flow associated with capital budgeting projects.

Discounting is the process of **determining the present value** of a **payment or a stream of payments** that is to be **received in the future**. Given the time value of money, a cedi is **worth more today** than it would be **worth tomorrow**. There are plenty of applications of time value of money in finance.

Interest is paid by borrowers to lenders for the use of lenders' money. The level of interest charged is typically stated as a percentage of the *principal* (the amount of the loan). When a loan matures, the principal must be repaid along with any unpaid accumulated interest.

In a free market economy, interest rates are determined jointly by the supply of and demand for money. Thus, lenders will usually attempt to impose as high an interest rate as possible on the money they lend; borrowers will attempt to obtain the use of money at the lowest interest rates available to them.

Competition among borrowers and competition among lenders will tend to lead interest rates toward some competitive level

SIMPLE INTEREST

Interest is computed on a *simple* basis if it is **paid only** on the principal of the loan. Simple interest is the interest income that is earned on a principal sum over an investment period.

It normally applies when an investment is for only one investment period. i.e. one year fixed deposit or when the investor collects the interest accruing on the investment at the end of each investment period.

i.e. interest (I) = $\mathbf{P \times k \times n}$

Where P = Principal

k = interest rate

n = No of years or investment period

Example 1

Kojo Mensah borrowed GHS 13,000 from his father to start a business. His father decided to give the loan on soft terms. Mensah was therefore asked to pay only 10% interest on the loan. If the loan is taken for three (3) years.

a. Determine the interest on the loan

b. How much Mensah will pay at the end of the period

SIMPLE INTEREST

Solution

a. Interest (I) = P × k × n where P = GHS 13,000, k = 10%, n = 3

= 13000 × .10 × 3

= **GHS 3,900**

B. Amount = Principal (P) + Interest (I) = GHS 13000 + GHS 3900 = **GHS 16,900**

A = P + (P × k × n) = PV (1 + kn) = A = PV (1 + kn)

Example 2

Josh lends GHS 2500 to his friend at the rate of 21% of 5 years. Determine the amount to be paid at the end of the year.

Solution

Interest (I) = P × k × n where P = GHS 2500, k = 21%, n = 5

= 2500 × .21 × 5 = GHS 2625 + 2500 = GHS 5125

A = PV (1 + kn) where PV = 2500, k = 21%, n = 5

A = 2500 (1 + (.21 × 5)) = 2500 (1 + 1.05) = 2500 × 2.05 = GHS 5125

COMPOUND INTEREST

A cedi in hand today is worth more than a cedi to be received tomorrow because of the **interest** it could **earn** from putting it in a savings account or placing it in an investment account. Compounding interest means that **interest earns interest**. For the discussion of the concepts of compounding and time value, let us define:

Fn = Future value = the amount of money at the end of year *n*

P = Principal

i = Annual interest rate

n = Number of years

Then,

$F1$ = the amount of money at the end of year 1 = Principal and Interest = $P + iP$

$$= \boldsymbol{P(1 + i)}$$

$F2$ = the amount of money at the end of year 2 = $Fi(1 + i) = P(1 + i)(1 + i) = P(1 + i)^2$

The future value of an investment **compounded annually** at rate ***i*** for ***n*** years is

$$\mathbf{Fn = P(1 + i)^n}$$

COMPOUND INTEREST

EXAMPLE 2

George Jackson placed GHS 1,000 in a savings account earning 8 percent interest compounded annually. How much money will he have in the account at the end of 4 years?

$Fn = P(1 + i)^n$ Where P = GHS 1000, i = 8%, n = 4

A = GHS 1000 $(1 + .08)^4$

= GHS 1000 × 1.36048896

= GHS 1360.49.

EXAMPLE 3.

Mr. George Owusu is a businessman in the microfinance business. He gives loans to small scale entrepreneurs. Samuel Walker, an artisan went for a loan of GHS 12000 to expand his business. If the loan is given for five (5) years and the interest rate is 18%, what will be the amount to pay at the end of the period when interest is calculated on

a. Simple interest basis

b. Compound interest basis.

Compounding Once a Year

Future value is the value which a present amount will grow to at a compounding interest. In other words how much a cedi today would be given the interest rate. This amount is arrived at taking into consideration timing difference and risk

Future value is the use of **compounding technique** to find the future value of a known amount at the end of the investment's life.

$$FV = PV\,(1 + K)^n$$

Example 1.

Kwame deposited GHS 10,000 in a fixed deposit with GCB Bank. If the bank pays 15% on such deposit, calculate how much money will grow in the account, if

a. A year's time

b. Two year's time

c. Five year's time

Compounding Once a Year

Solution

a. (1 year) = FV = PV $(1 + k)^n$ where FV=GHS 10,000, k=15% n=1

FV = GHS 10, 000 $(1 +0.15)^1$

= GHS 11,500.00

b. (2 years) = GHS 10,000 $(1 + 0.15)^2$

= GHS 10,000 × 1.3225

= **GHS 13,225.00**

c. (5 years) = GHS 10,000 $(1 + 0.15)^2$

= GHS 10,000 × 2.011357187

= GHS 20, 113.57

Compounding More Frequently than One a Year

The earlier formula assumes that interest is compound annually. However, in practise interest may compound daily, weekly, monthly, quarterly or semi-annually. Eg. Treasury Bills have 90 day, 180 and one year. When interest accrues more than once a year, the general formula is

$$FV = PV\,(1 + K/m)^{n \times m}$$

Example 1.

Ama deposited GHS 1000 at Prudential Bank, which gives interest rate of 9%. How much will she have in 2 years if interest is paid?

a. Semi-annually

b. Quarterly

c. Monthly

Compounding More Frequently than One a Year

Solution

a. (Semi-annually (2)) = FV = PV $(1 + {}^{k}/_{m})^{n\times m}$ where FV=GHS 1,000, k=9% n=2

$$FV = GHS\ 1,000\ (1 + \frac{0.09}{2})^{2\times2}$$

= GHS 1,000.00 × 1.192518601

= **GHS 1,192.52**

b. (Quarterly (4)) = GHS 1,000 $(1 + \frac{0.09}{4})^{2\times4}$

= GHS 1,000 × 1.194831142

= **GHS 1,194.83**

c. (Monthly (12)) = GHS 1,000 $(1 + \frac{0.09}{12})^{2\times12}$

= GHS 1,000 × 1.196413529

= GHS 1,196.41

Continuous Compounding

Continuous compounding involves the calculation and accumulation of interest on an ongoing basis. The general formula is

$$FV = PVe^{kn}$$

Example 1.

Find the value of GHS 100 in one year if the interest rate is 9% compounded continuously.

FV = GHS 100 $e^{0.09\times1}$ Where P = GHS 100, k = 9%, n = 1

= GHS 100 ($2.7183^{(0.09\times1)}$) = GHS 100 (1.094174284)

= **GHS 109.42**

Example 2.

Kojo Boateng deposits huge sums of money in his bank account. On 1st January 2008, he deposited GHS 2,000.00 in his bank account. The bank agreed to pay an interest of 15%. If interest is compounded continuously, how much will the money grow to at the end 2009.

Continuous Compounding

Solution

a. (2 years) = FV = PV e^{kn} where FV=GHS 2,000, k=15% n=2

= GHS 2000 ($2.7183^{(0.15\times2)}$) = GHS 2,000 (1.349858808)

= GHS 2,699.72

Nominal and Effective Interest Rates

The interest rate may be the contractual rate charged by a lender or charged to a borrower. This is known as nominal interest rate. The effective rate is the interest that is **actually paid or earned**. The main difference is that the effective interest rate takes into consideration **timing differences** and the **impact of compounding frequency**. The general formula is

$K_{eff} = (1 + \frac{k}{m})^m - 1$ where k = nominal interest rate, m = period

Or $K_{eff} = (FV/PV) - 1$ Where $FV = PV\,(1 + \frac{k}{m})^m$

Example 1

What is the effective interest rate if you deposit GHS 100 at a bank today and earn interest rate of 8% compounded semi-annually?

Example 2.

Calculate the effective interest rate if GHS 5000 is deposited today at Barclays Bank which earn interest rate of 12% compounded quarterly.

Nominal and Effective Interest Rates

Solution 1

$K_{eff} = (1 + \frac{k}{m})^m - 1$ Where k= 8%, m = 2

$= (1 + \frac{0.08}{2})^2 - 1$ $= (1.04)^2 - 1 = 1.0816 - 1 = 0.0816 =$ **8.16%**

Alternative Solution

$K_{eff} = (\frac{FV}{PV}) - 1$ Where $FV = PV\,(1 + \frac{k}{m})^m$

$K_{eff} = (\frac{FV}{100}) - 1$ Where $FV = 100\,(1 + \frac{0.08}{2})^2$ $= 100\,(1.0816) =$ GHS 108.16

$K_{eff} = (\frac{108.16}{100}) - 1 = 1.0816 - 1 = 0.0816 =$ **8.16%**

Solution 2.

$K_{eff} = (1 + \frac{k}{m})^m - 1$ Where k= 12%, m = 4

$= (1 + \frac{0.12}{4})^4 - 1$ $= (1.03)^4 - 1 = 1.1255 - 1 = 0.1255 =$ **12.55%**

If compounding continuously future values then $K_{eff} = (e^k - 1)$

Future Value of Mixed Streams

The approach to calculating future values of mixed stream of cash flows involves a two step process:

a. Calculate the future value of each cash flow

b. Sup up all the individual future values to determine the total future value of cash.

Example 1.

Calculate the future value of the following yearly cash flows at the end of the fifth year assuming a 10% return annum.

Year	Cash Flow (GHS)
1	500
2	300
3	200
4	100
5	50

CF1 $(1 + k)^5$

Future Value of Mixed Streams

Solution

FV = CF1 $(1 + k)^5$ + CF2 $(1 + k)^4$ + CF3 $(1 + k)^3$ + CF2 $(1 + k)^2$ + CF1 $(1 + k)^1$

FV = $500(1 + .10)^5 + 300(1 + .10)^4 + 200(1 + .10)^3 + 100(1 + .10)^2 + 50(1 + .10)^1$

= 805.255 + 439.23 + 266.2 + 121 + 55

= GHS 1686.685

Future Value of Mixed Streams

1. Calculate the future value of the following cash flows at the end of the fifth year if the interest rate is 20%.

Year	Cash flows
1	5000
2	6000
3	7000
4	8000
5	8500

2. Gifty deposited GHS 10000 in a fixed deposit account for 10 years for a building project. How much will Gifty receive at an interest rate of 18% if interest is compounding
   a. annually
   b. semi-annually
   c. Quarterly

FUTURE VALUE OF ANNUITY

An annuity is a stream of equal periodic cash flow (inflow or outflow) over a fixed period. Equally spaced level stream of cash flows for a limited period of time

Annuity is a fixed payment or receipts each period for a specific number of periods. Eg. If you rent an apartment and promise to make a monthly payment of rent, you have created an annuity.

There are two basic types of annuity, namely **Ordinary annuity** and **Annuity Due**.

An **ordinary annuity** is the one that pays/receives a constant amount at the **end of each period** for a specified number of periods.

An annuity due is the one that pays/receives a constant amount at the **beginning of each period** for a specified number of periods.

FUTURE VALUE OF ORDINARY ANNUITY

The value of annuity looks at how much an equal amounts deposited into an account at the end of each year for *n* years, and if deposited earns interest rate *i* compounded annually will grow to at the end of the *n* years.

$$FVA_n = PMT(1+k)^{n-1} + PMT(1+k)^{n-2} + \ldots + PMT(1+k)^{n-n}$$

$$FVA_n = PMT + PMT(1+k) + PMT(1+k)^2 + \ldots + PMT(1+k)^{n-2} + PMT(1+k)^{n-1}$$

$$FVA_n = PMT\left[1 + (1+k) + (1+k)^2 + \ldots + (1+k)^{n-2} + (1+k)^{n-1}\right]$$

$$FVA_n = PMT\left[1 \times \left[\frac{(1+k)^n - 1}{(1+k) - 1}\right]\right] = PMT\left[\frac{(1+k)^n - 1}{k}\right]$$

FUTURE VALUE OF ORDINARY ANNUITY

Example 1.

Mr. Bosu is planning his child university education. He has therefore decided to make periodic contribution towards that. If he deposits GHS 2,000 at the end of every year into a special account for seven years and the bank pay 15% interest on such deposit, how much will the deposits grow to at the end of the seventh year.

FVA =

$$FVA_n = PMT\left[\frac{(1+k)^n - 1}{k}\right]$$

Where PMT = 2000

k = .15, n= 7

$$FVA_7 = 2000\left[\frac{(1+0.15)^7 - 1}{0.15}\right]$$

$$FVA_7 = 2000\left[\frac{(1+0.15)^7 - 1}{0.15}\right]$$

$= 2000 \times 11.0667992$

= GHS 22,133.60

FUTURE VALUE OF ORDINARY ANNUITY

Example 1.

Mr. Ansah is planning his child university education. He has therefore decided to make periodic contribution towards that. If he deposits GHS 5,500 at the end of every year into a special account for nine years and the bank pay 21% interest on such deposit, how much will the deposits grow to at the end of the ninth year.

FUTURE VALUE OF ANNUITY DUE

An annuity due is the one that pays/receives a constant amount at the **beginning of each period** for a specified number of periods.

$$FVA_n = PMT(1+k)^n + PMT(1+k)^{n-1} + \ldots + PMT(1+k)^1$$

$$FVA_n = PMT(1+k) + PMT(1+k)^2 + \ldots + PMT(1+k)^{n-1} + PMT(1+k)^n$$

$$FVA_n = PMT\left[1 + (1+k) + (1+k)^2 + \ldots + (1+k)^{n-2} + (1+k)^{n-1}\right](1+k)$$

$$FVA_n = PMT\left[1 \times \left[\frac{(1+k)^n - 1}{(1+k) - 1}\right](1+k)\right] = PMT\left[\frac{(1+k)^n - 1}{k}\right](1+k)$$

FUTURE VALUE OF ANNUITY DUE

Example 1

Philip deposits GHS 1000 per year for three years at the beginning of the year in an account earning 6% interest. What is the account at the end of the third year.

$$FVA_n = PMT\left[\frac{(1+k)^n - 1}{k}\right](1+k)$$

where PMT = 1000, k = 0.06, n=3

$$FVA_3 = 1000\left[\frac{(1+0.06)^3 - 1}{0.06}\right](1+0.06)$$

= 1000 × 3.1836 × 1.06 = GHS 3,374.616

FUTURE VALUE FORMULA

1. Ordinary Annuity is given by :

$$FVA_n = PMT\left[\frac{(1+k)^n - 1}{k}\right]$$

2. Annuity Due is given by :

$$FVA_n = PMT\left[\frac{(1+k)^n - 1}{k}\right](1+k)$$

FUTURE VALUE

Example 2

Suppose you deposit GHS 6000 in a bank account at the beginning of each year for four years. If the bank pays interest of 10% what will be the value of your account at the end of the four years.

Example 3

Compute the future value of an annual cash flow of GHS 10,000 for seven years if the prevailing interest rate is 14% and the cash flow occur at the beginning of each year.

PRESENT VALUE OF SINGLE AMOUNT

Present value are the discounted amount of future cash flows. It is use a discounting technique to find the current value of a future amount.

$$\text{Present Value} = \text{PV}$$

$$\text{PV} = \frac{\text{Future Value after t periods}}{(1+\text{r})^{\text{t}}}$$

$$PV = FV \times \frac{1}{(1+r)^t}$$

PRESENT VALUE OF SINGLE AMOUNT

Example 1

Amos won a context as the best financial management student. The prize for the context is GHS 4,500 to be received in four year time. If the current rate is 18%, what is the value of your prize in today's terms?

Solution.

$$PV = FV \times \frac{1}{(1+r)^t}$$

where FV = GHS 4,500, r= .18, n= 4

$$PV = 4500 \times \frac{1}{(1+0.18)^4}$$

= 4500 * 0.515788875

= GHS 2321.05

PRESENT VALUE OF SINGLE AMOUNT

Example 2

John Mensah has invested in Business hoping to receive GHS 5000 in three years time. The current interest rate is 15%. Determine the present value of this investment.

$$PV = FV \times \frac{1}{(1+r)^t}$$

where FV = GHS 5,000, r= .15, n= 3

$$PV = 5000 \times \frac{1}{(1 + 0.15)^3} = 5000 \times 0.657516232 = 3,287.58$$

Example 3. Assignment

You borrowed GHS 5000 from a friend and you are required to pay back GHS 6802 in four years time. If interest is compound annually what is the rate of the loan.

PRESENT VALUE OF MIXED STREAM

- Calculate the present values of each future amount to be received using discounting technique
- Sum up all the individual Present Values to get the total Present Values.

Example 1

You have undertaken a project and expect to receive the following cash flows

Years	GHS
1	5000
2	6500
3	7000
4	8500

If the prevailing interest rate is 20%, what is the present value of these cash flows.

PRESENT VALUE OF MIXED STREAM

Solution

$$PV = FV \times \frac{1}{(1+r)^t} + FV \times \frac{1}{(1+r)^t} + FV \times \frac{1}{(1+r)^t} + FV \times \frac{1}{(1+r)^t}$$

$$PV = 5000 \times \frac{1}{(1+0.2)^1} + 6500 \times \frac{1}{(1+0.2)^2} + 7000 \times \frac{1}{(1+0.2)^3} + 8500 \times \frac{1}{(1+0.2)^4}$$

PV = GHS 4166.67 + GHS 4,513.89 + GHS 4,050.9 + GHS 4,099.15

PV = GHS 16,830.61

PRESENT VALUE OF MIXED STREAM

Example 2

Joana is undertaking an investment with the following cash flows for five years

Years	GHS
1	10,000
2	15,000
3	20,000
4	25,000
5	30,000

You are required to compute the present value of the expected cash flows if Joana requires 25% return on all her investment.

PRESENT VALUE OF ORDINARY ANNUITY

Ordinary Annuity

The present value of an ordinary annuity is the sum of present values of a series of equal period cash flows (payment or receipt).

$$PVA = PMT\left[\frac{1-(1+k)^{-n}}{k}\right]$$

PMT = cash payment

k = interest rate

n = Number of years cash payment is received

Example 1.

Find the present value of GHS 2,000 paid at the end of each year for 4 years with interest rate of 10%.

$$PVA = GHS\ 2000 \times \left\{\frac{[1-(1+.10)^{-4}}{.10}\right\} = GHS\ 6{,}339.73$$

PRESENT VALUE OF ANNUITY

Example 2

Dede took up an insurance policy which promise to pay GHS 4000 at the end of each year for the next four years. Determine the present value of the policy if the current interest rate is 14%.

$$PVA = PMT\left[\frac{1-(1+k)^{-n}}{k}\right]$$

PRESENT VALUE OF ANNUITY DUE

Annuity Due

The present value of an ordinary annuity is the sum of present values of a series of equal period cash flows (payment or receipt).

$$PVA = PMT\left[\frac{1-(1+k)^{-n}}{k}\right](1+k)$$

Example 1.

John has an investment which is supposed to yield GHS 5000 at the beginning of each year for the next 10 years. Calculate the present value of the cash flows if the prevailing bank rate is 10%.

$$PVA = \text{GHS } 5000 \times \left[\frac{[1-(1+.10)^{-10}]}{.10}\right](1+0.10) = \text{GHS } 5000 \times 6.144567106 \times 1.1$$

$$= \text{GHS } 33{,}795.12$$

PRESENT VALUE OF ANNUITY DUE

Example 2.

Your brother is expected to enter the university next year. It is estimated that he will need GHS 10,000 each year for the four year university education. You are planning to deposit a lump sum in an account that will yield 10% annum to cater for his university education. How much should be deposited.

$$PVA = PMT\left[\frac{1-(1+k)^{-n}}{k}\right](1+k)$$

PERPETUITIES

PV of Perpetuity Formula

$$\boxed{PV = \frac{PMT}{k}}$$

PMT = cash payment

k = interest rate

LOAN AMORTIZATION

Loan amortization refers to the repayment of a loan in full plus the interest incurred on the loan over a specific period. The amortization process involves finding future payments whose present value will be equivalent to the loan and the interest.

Types of Loans:

Pure Discount Loans: the borrower repays a single lump sum at some time in the future.

Interest Only Loans: the borrower pays interest and repay the whole principal at the end of the period.

Amortized Loans: A loan repaid in installments over time. It caters of the interest and part of the principal each time it is repaid.

LOAN AMORTIZATION

$$PV = PMT\left[\frac{1-(1+k)^{-n}}{k}\right]$$

$$PMT = \frac{PV}{\left[\frac{1-(1+k)^{-n}}{k}\right]} = \frac{PV}{PVAIFA}$$

PMT = cash payment

k = interest rate

LOAN AMORTIZATION

Asiedu borrowed GHC10,000.00 from his bank at the rate of 18% and agreed to pay equal instalments over five years. Determine the size of the payments and set up an amortization schedule for the loan:

Solution:

$$PV = PMT\left[\frac{1-(1+k)^{-n}}{k}\right]$$

$$10{,}000 = PMT\left[\frac{1-(1+.18)^{-5}}{.18}\right]$$

$$10{,}000 = PMT(3.1272)$$

$$PMT = \left[\frac{10{,}000}{3.1272}\right] = 3{,}197.78$$

LOAN AMORTIZATION

Year	Begining Balance	Total Payment	Interest Paid (18%)	Principal Paid	Ending Balance
1	10,000	3,197.78	1,800.00	1,397.78	8,602.22
2	8,602.22	3,197.78	1,548.40	1,649.38	6,952.84
3	6,952.84	3,197.78	1,251.51	1,946.27	5,006.57
4	5,006.57	3,197.78	901.18	2,296.60	2,709.97
5	2,709.97	3,197.78	487.79	2,709.97	0
Totals		**15,988.90**	**5,988.88**	**10,000.00**	

LOAN AMORTIZATION

An individual borrows GHS 5,000 at 12% and agrees to pay equal instalment over four years.

a. Determine the size of the payments

b. Set up an amortization schedule for the loan.

Solution

$$PV = PMT\left[\frac{1-(1+k)^{-n}}{k}\right]$$

Where PV = GHS 5,000, k= 12%, n = 4

$$5000 = PMT\left[\frac{1-(1+.12)^{-4}}{.12}\right]$$

5,000 = PMT × 3.037349347 =

= GHS 1,646.17

LOAN AMORTIZATION

Year	Begining Balance	Total Payment	Interest Paid (12%)	Principal Paid	Ending Balance
1	5,000.00	1,646.17	600.00	1,046.17	3,953.83
2	3,953.83	1,646.17	474.46	1,172.25	2,781.58
3	2,781.58	1,646.17	333.79	1,312.38	1,469.2
4	1,469.2	1,646.17	176.30	1,469.2	0
Totals		**6,584.68**	**1584.55**	**5,000.00**	

LOAN AMORTIZATION

Illustration 3

You are a member of a credit union at your work place. You contracted a loan of GHS 250,000 at a rate of 16%. If the loan together with the interest is to be paid in seven years in equal instalment.

a. Calculate how much should be paid each year

b. Prepare a loan amortization schedule for the loan.

Thank you