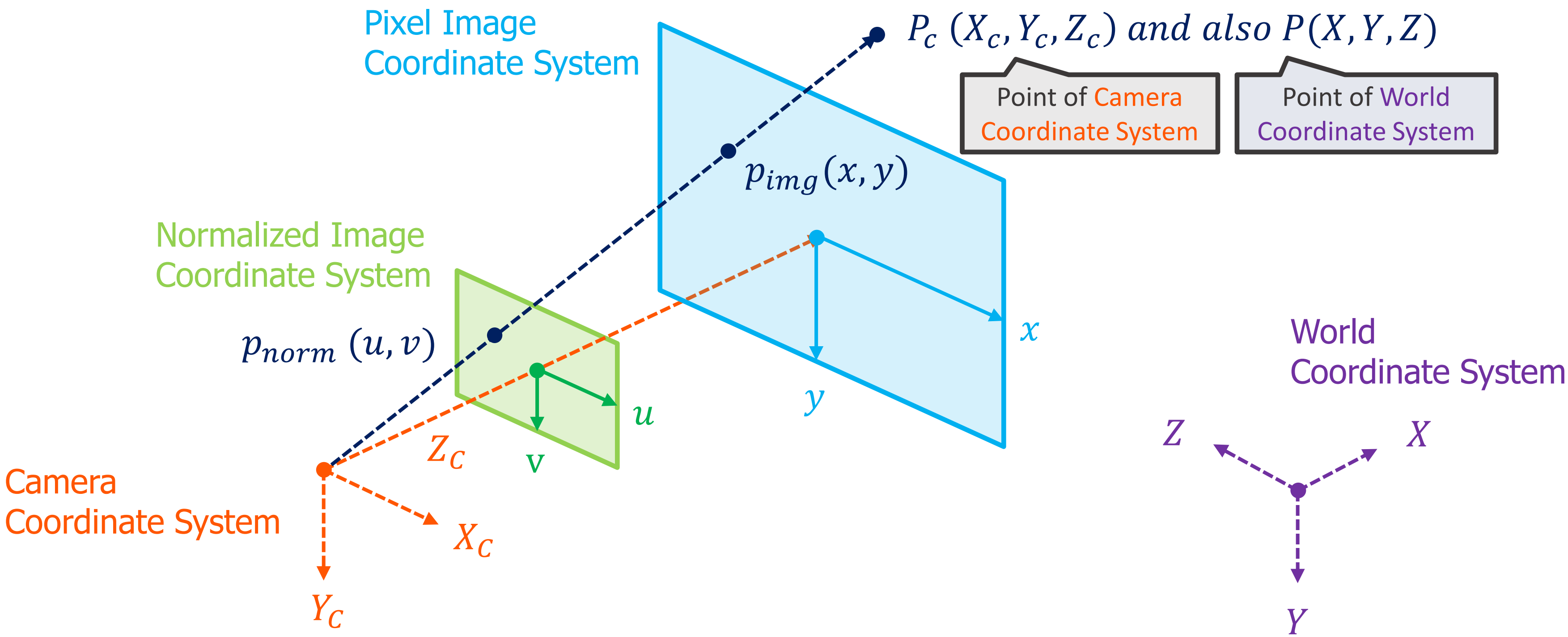
Programming Assignment2

Structure-from-Motion

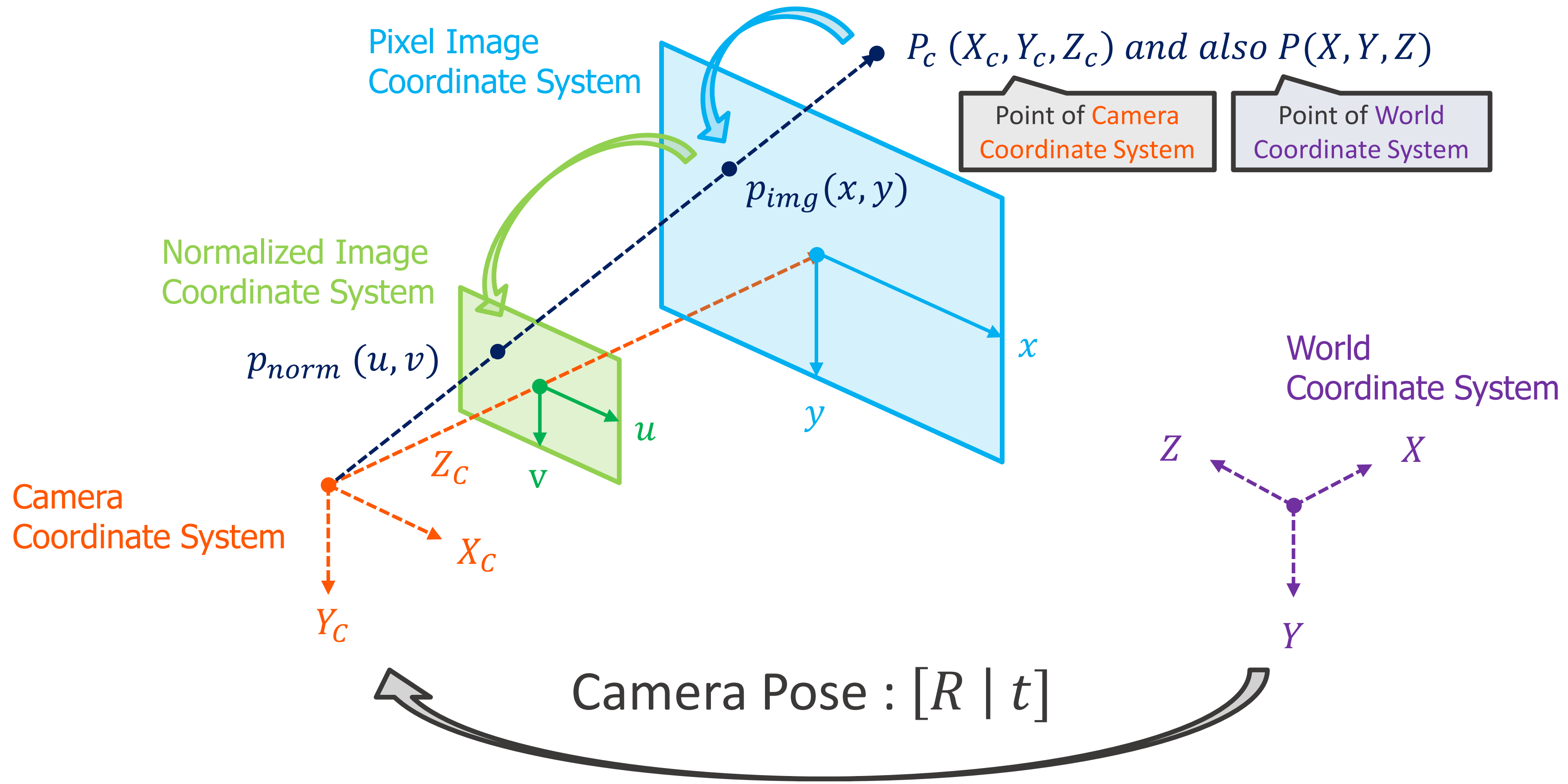
Prof. Hae-Gon Jeon

Recall: Coordinate System



Recall: Coordinate System

$$p_{norm}(u,v) \leftarrow p_{img}(x,y) \leftarrow P_c(X_c,Y_c,Z_c) \leftarrow P(X,Y,Z)$$



Recall: Coordinate System

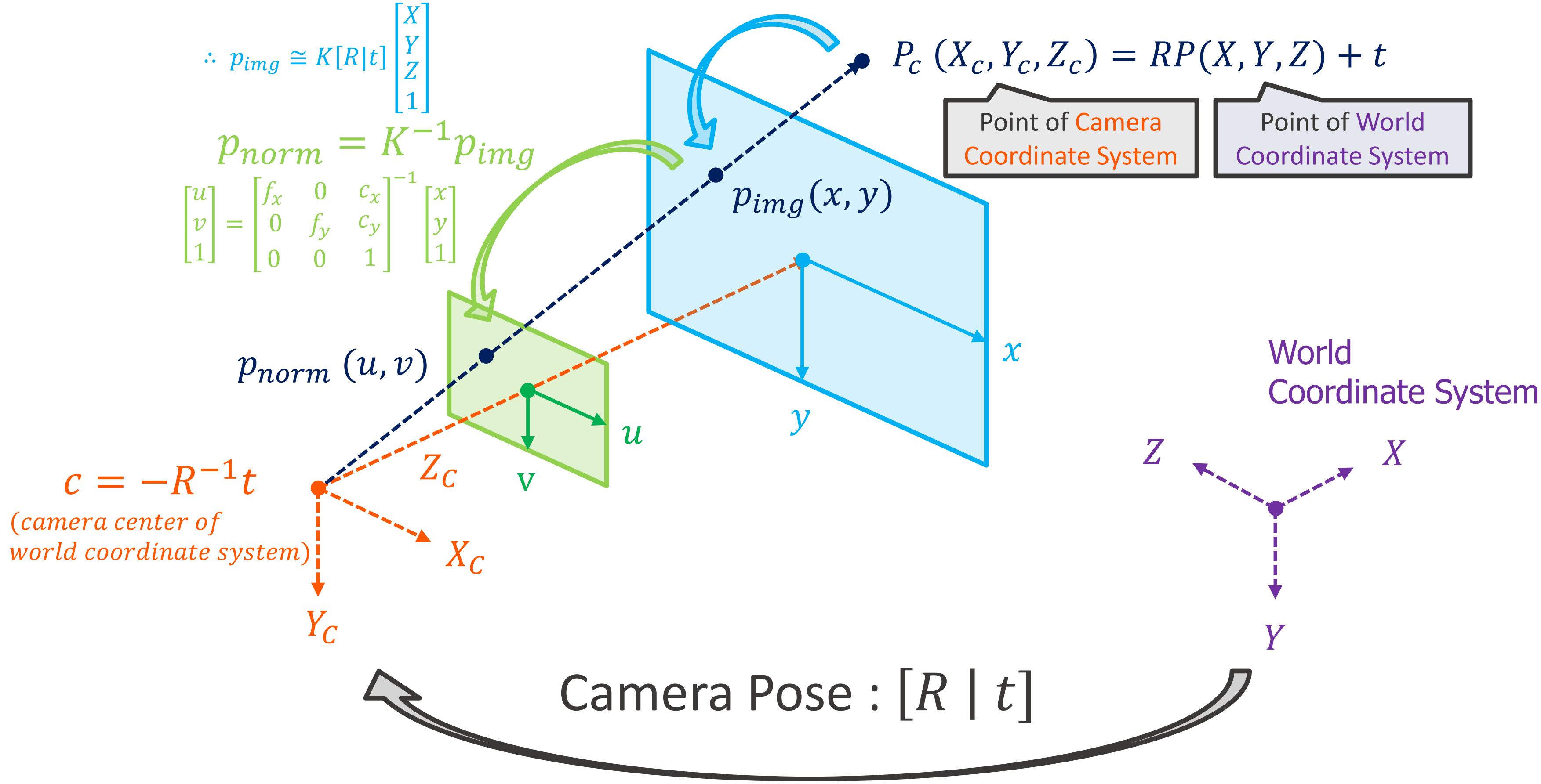
$$p_{norm}(u,v) \leftrightarrow p_{img}(x,y) \leftarrow P_c(X_c,Y_c,Z_c) \leftrightarrow P(X,Y,Z)$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} (f_x X_c + c_x Z_c)/Z_c \\ (f_y Y_c + c_y Z_c)/Z_c \\ 1 \end{bmatrix} \sim \begin{bmatrix} f_x X_c + c_x Z_c \\ f_y Y_c + c_y Z_c \\ Z_c \end{bmatrix} = K \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} \quad w.r.t \;\; K = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

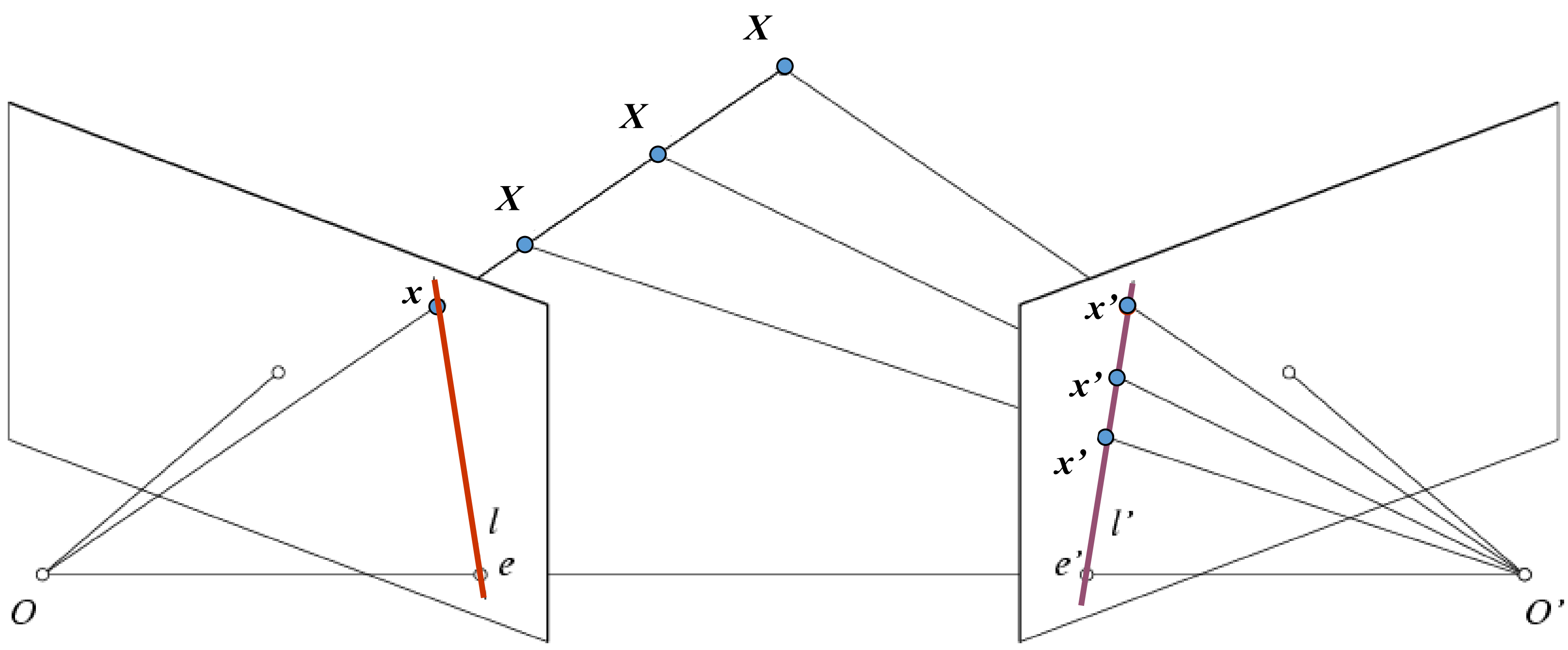
$$\therefore \; p_{img} \cong K[R|t] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$p_{norm} = K^{-1} p_{img}$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



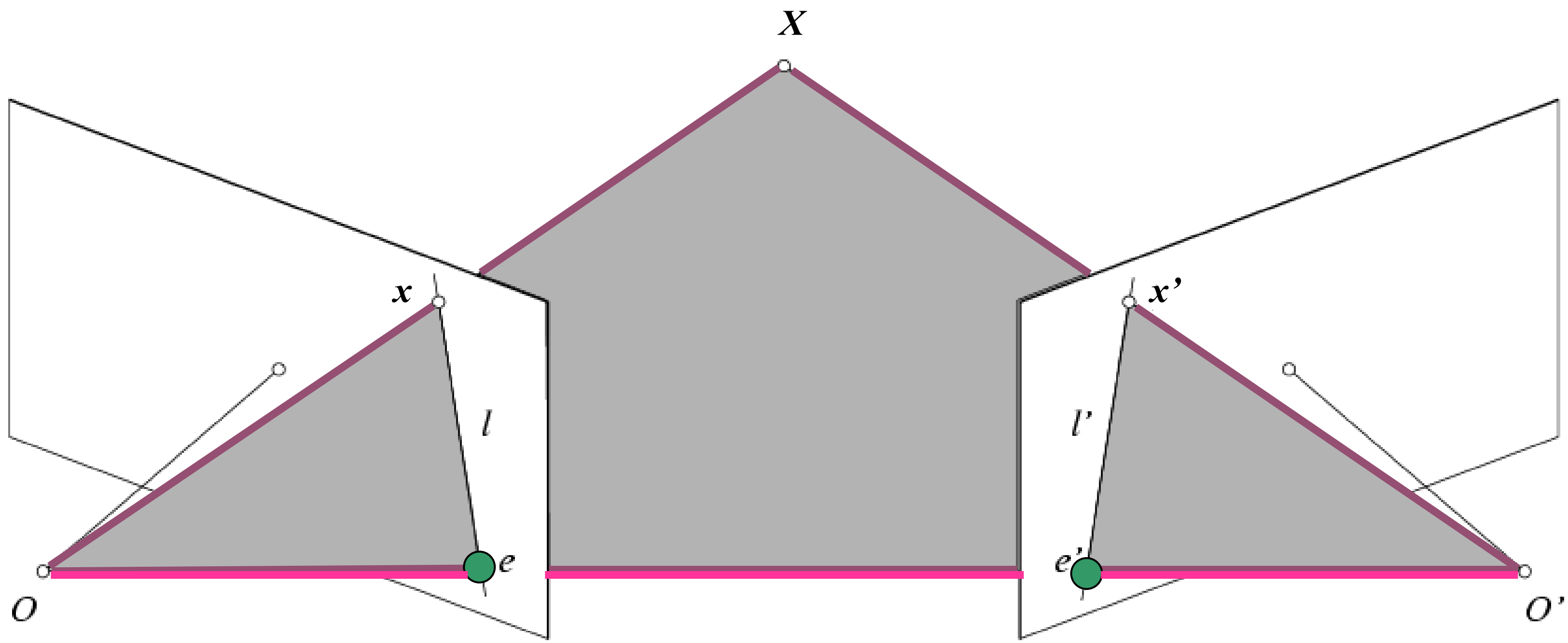
Recall: Epipolar constraint



Potential matches for *x* have to lie on the corresponding line *l'*.

Potential matches for *x'* have to lie on the corresponding line *l*.

Recall: Epipolar geometry notation



- **Baseline** – line connecting the two camera centers

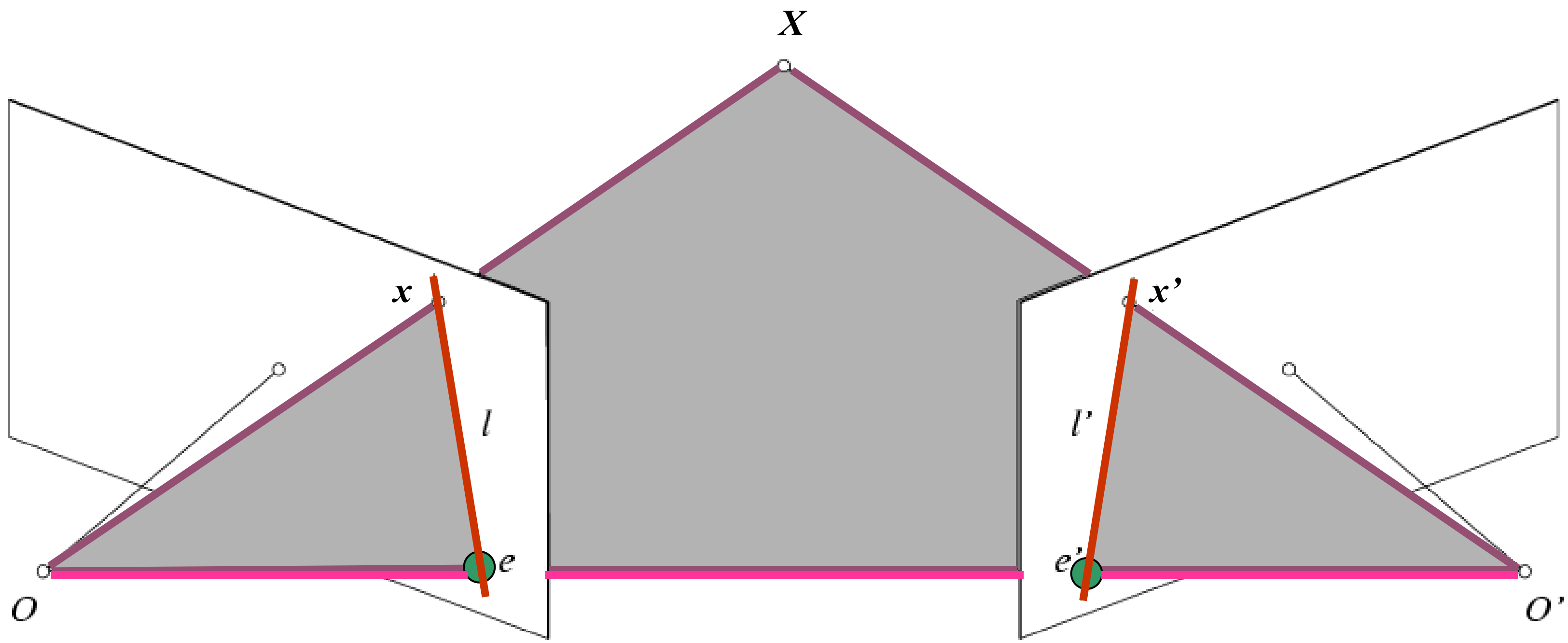
- **Epipoles**

= intersections of baseline with image planes

= projections of the other camera center

- **Epipolar Plane** – plane containing baseline (1D family)

Recall: Epipolar geometry notation



- **Baseline** – line connecting the two camera centers

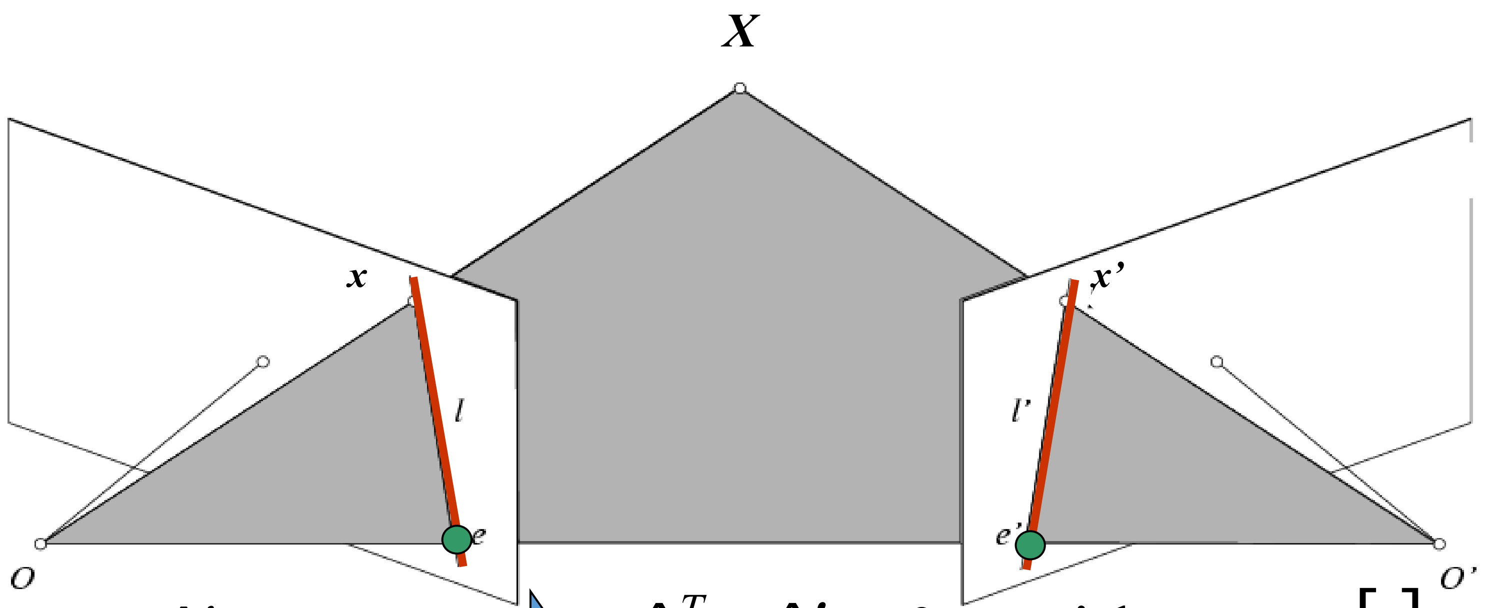
- **Epipoles**

= intersections of baseline with image planes

= projections of the other camera center

- **Epipolar Plane** – plane containing baseline (1D family)
- **Epipolar Lines** - intersections of epipolar plane with image planes (always come in corresponding pairs)

Recall: Properties of the Essential matrix



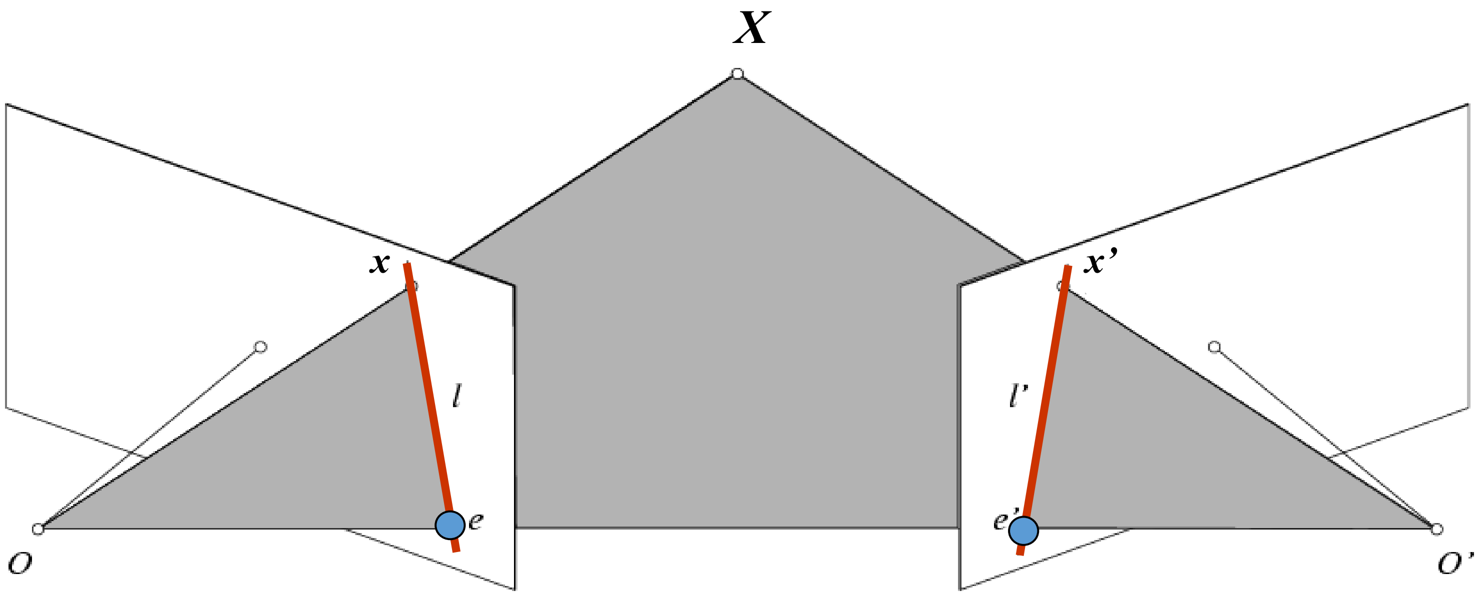
$$\hat{x}\cdot[t\times(R\hat{x}')]=0 \quad \Rightarrow \quad \hat{x}^T E \hat{x}' = 0 \quad \text{with} \quad E = [t]_\times R$$

Drop ^ below to simplify notation

Skew -symmetric matrix

- $E x'$ is the epipolar line associated with x' ($l = E x'$)
- $E^T x$ is the epipolar line associated with x ($l' = E^T x$)
- $E e' = 0$ and $E^T e = 0$
- E is singular (rank two)
- E has five degrees of freedom
  - (3 for R, 2 for t because it's up to a scale)

Recall: Properties of the Fundamental matrix



- $F x'$ is the epipolar line associated with x' $(l = Fx')$
- $F^T x$ is the epipolar line associated with x $(l' = F^T x)$
- $F e' = 0$ and $F^T e = 0$
- F is singular (rank two): det(F)=0
- F has seven degrees of freedom: 9 entries but defined up to scale, det(F)=0

Recall: Estimating the Fundamental Matrix

- 8-point algorithm
  - Least squares solution using SVD on equations from 8 pairs of correspondences
  - Enforce det(F)=0 constraint using SVD on F

- 7-point algorithm
  - Use least squares to solve for null space (two vectors) using SVD and 7 pairs of corres pondences
  - Solve for linear combination of null space vectors that satisfies det(F)=0

- Minimize reprojection error
  - Non-linear least squares

Note: estimation of F (or E) is degenerate for a planar scene.

Recall: 8-point algorithm

- Solve a system of homogeneous linear equations
  1. Write down the system of equations

$$\mathbf{x}^T F \mathbf{x}' = 0$$

$$uu'f_{11} + uv'f_{12} + uf_{13} + vu'f_{21} + vv'f_{22} + vf_{23} + u'f_{31} + v'f_{32} + f_{33} = 0$$

$$\mathrm{A}\boldsymbol{f} = \begin{bmatrix} u_1u_1' & u_1v_1' & u_1 & v_1u_1' & v_1v_1' & v_1 & u_1' & v_1' & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_nu_v' & u_nv_n' & u_n & v_nu_n' & v_nv_n' & v_n & u_n' & v_n' & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ \vdots \\ f_{33} \end{bmatrix} = \mathbf{0}$$

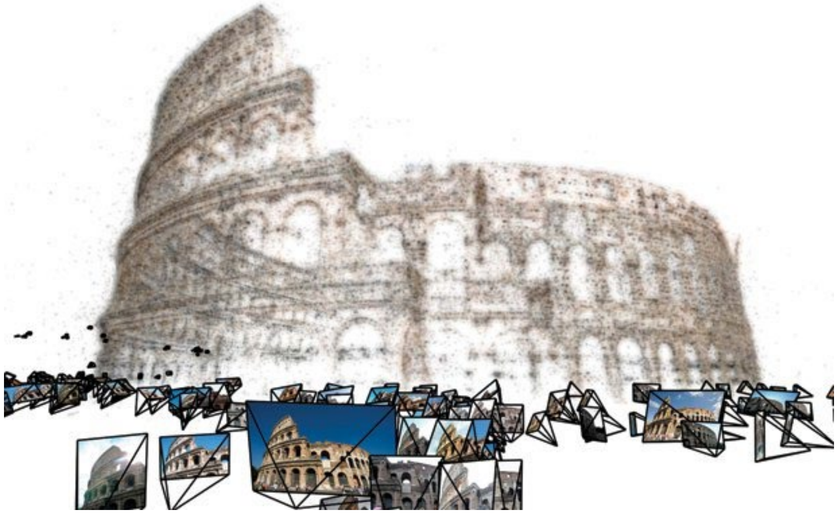
Recall: 8-point algorithm

- Solve a system of homogeneous linear equations
  1. Write down the system of equations
  2. Solve **f** from A**f**=**0** using SVD

     Matlab:
     ```
     [U, S, V] = svd(A);
     f = V(:, end);
     F = reshape(f, [3 3])';
     ```

- Resolve det(F) = 0 constraint using SVD

  Matlab:
  ```
  [U, S, V] = svd(F);
  S(3,3) = 0;
  F = U*S*V';
  ```

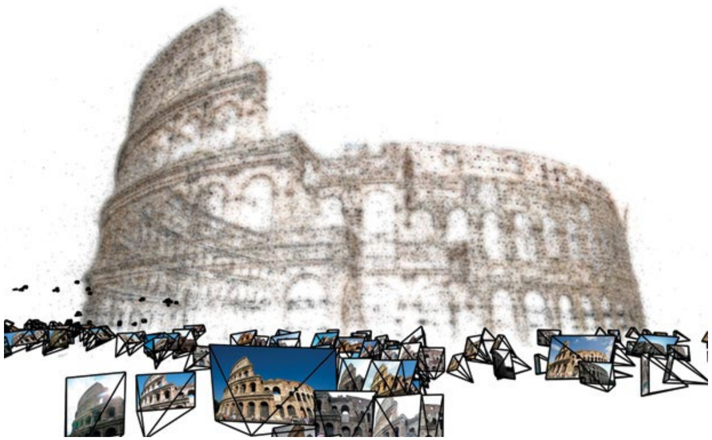


Structure from Motion

Chapter 7 in Szeliski

What is SfM?

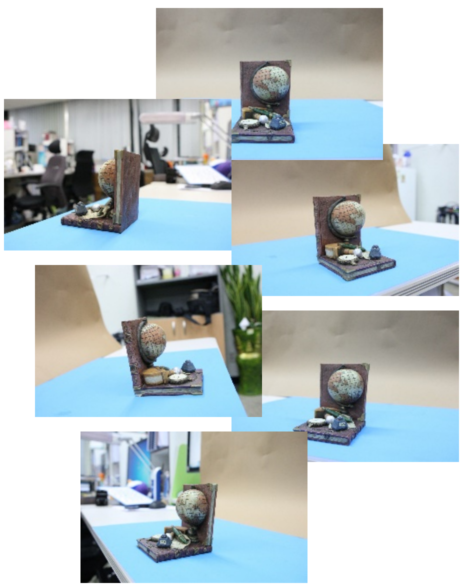
- **Structure from Motion (SfM)**
  - The process of **estimating three-dimensional structures from two-dimensional image sequences** which may be coupled with local motion signals.
  - Input: Freely taken images with overlapped scenery
  - Output: camera pose and 3D structure of the scene
  - Reference
    - http://photosynth.net
    - N.Snavely et al., "Photo Tourism: Exploring photo collections in 3D", SIGGRAPH 2006

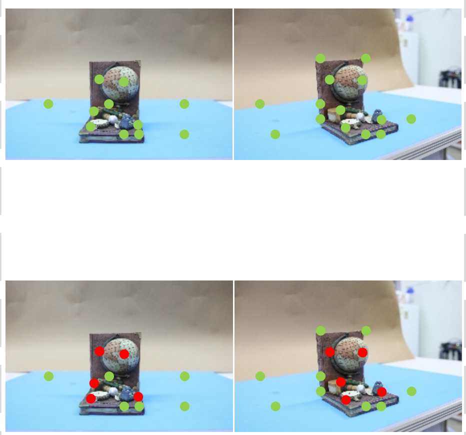


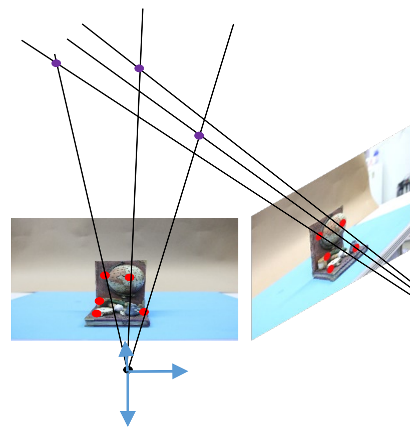
Overall Strategy

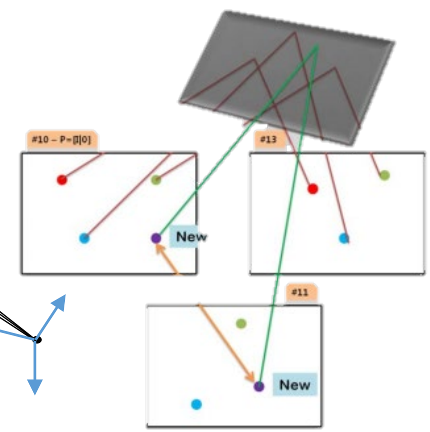
We will only do two-view in this PA

A set of images → Feature Extraction & Matching → (Two-view) Initialization Step → (> Three-view) Growing Step





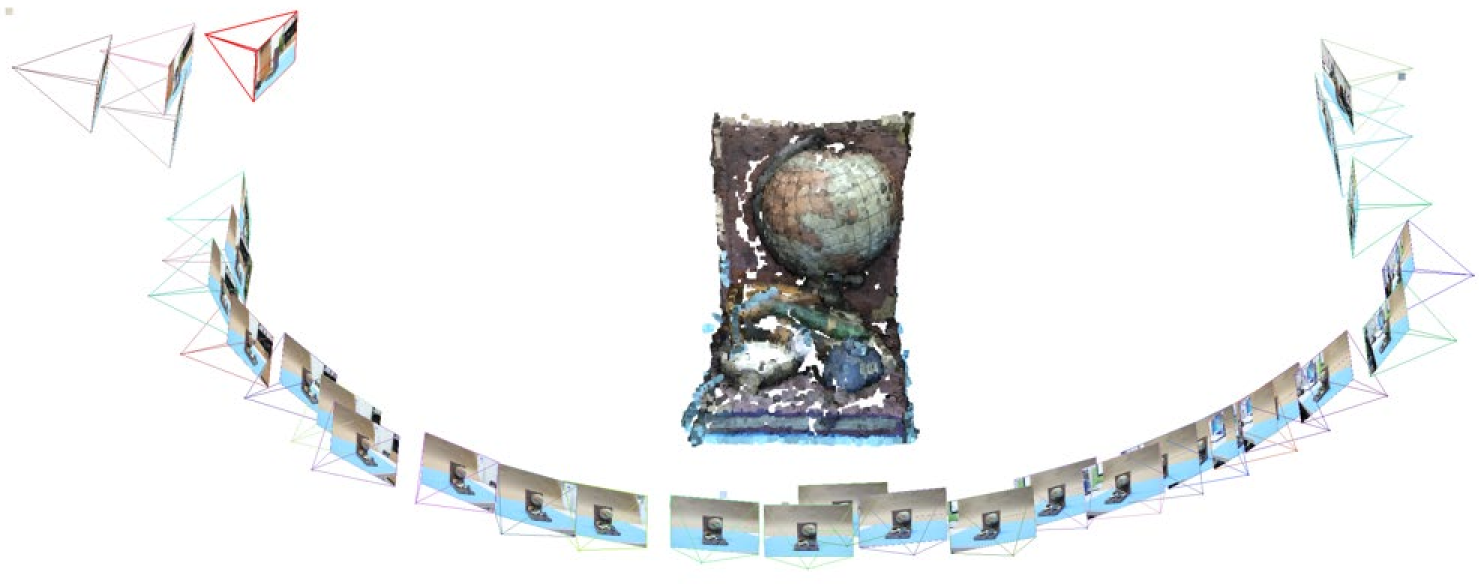




[2] Hartley, Richard, and Andrew Zisserman. *Multiple view geometry in computer vision*. Cambridge university press, 2003.
[3] Szeliski, Richard. *Computer vision: algorithms and applications*. Springer Science & Business Media, 2010.

Output

- Goal: Build a 3D & Estimate camera poses, given the set of images

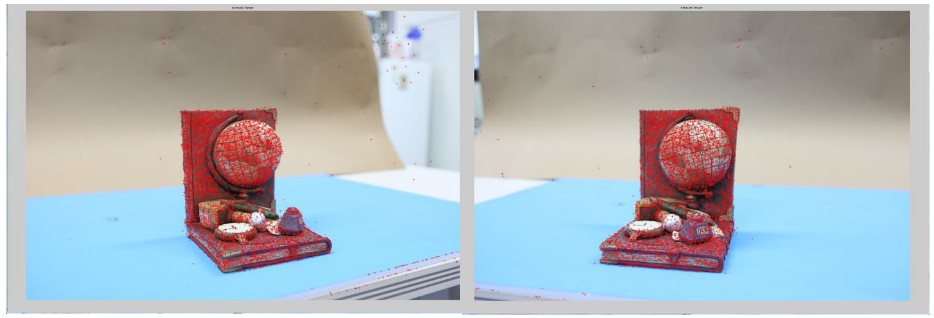


Overall Strategy

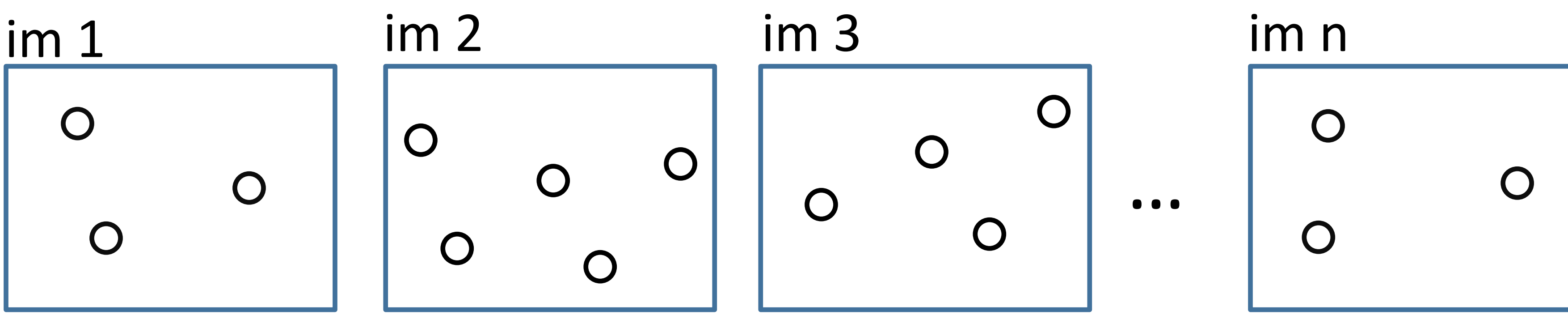
- **Correspondence** search, Relating images
  - 1. Extract SIFT from every image and find putative matches
  - 2. Outliers should be rejected by applying RANSAC
- **Initialization** Step
  - 3. Find the best image pair(simply, has the maximum matches or take the base-line into account)
  - 4. Estimate motion(R and t) and Reconstruct 3D points for the selected image pair. The camera coordinate of one camera is used for the world coordinate.
- **Growing** Step
  - 5. Search images which have enough points seeing the reconstructed 3D point
  - 6. Compute pose(R and t) for those images and reconstruct more 3D points seen from more than two images
  - Bundle Optimization
- **Repeat** the Growing step until every camera is included.

Step I. Feature extraction & matching in general

- Feature types: SIFT, ORB, Hessian-Laplacian, …



Feature Extraction

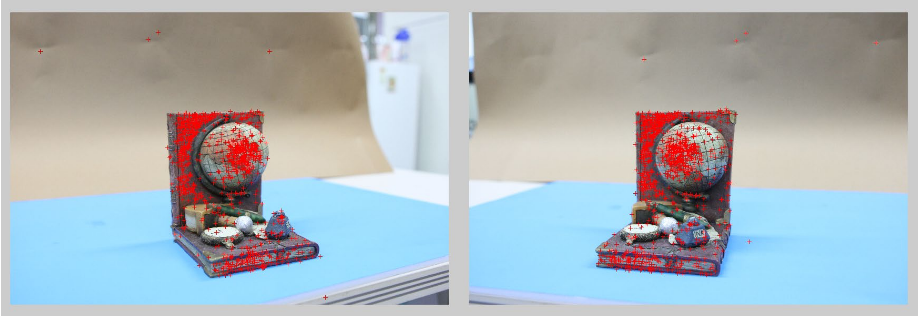


Each circle represents a set of detected features

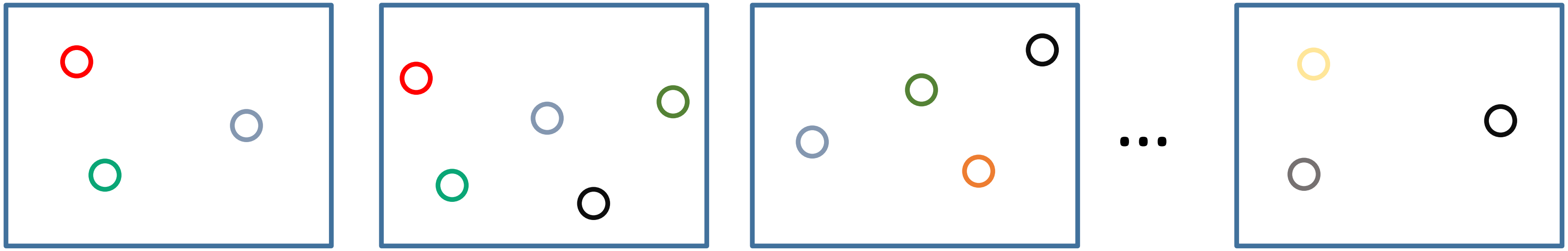
Step I. Feature extraction & matching in general

For each pair of images:

1. Match feature descriptors via approximate nearest neighbor
2. Solve for F and find inlier feature correspondences

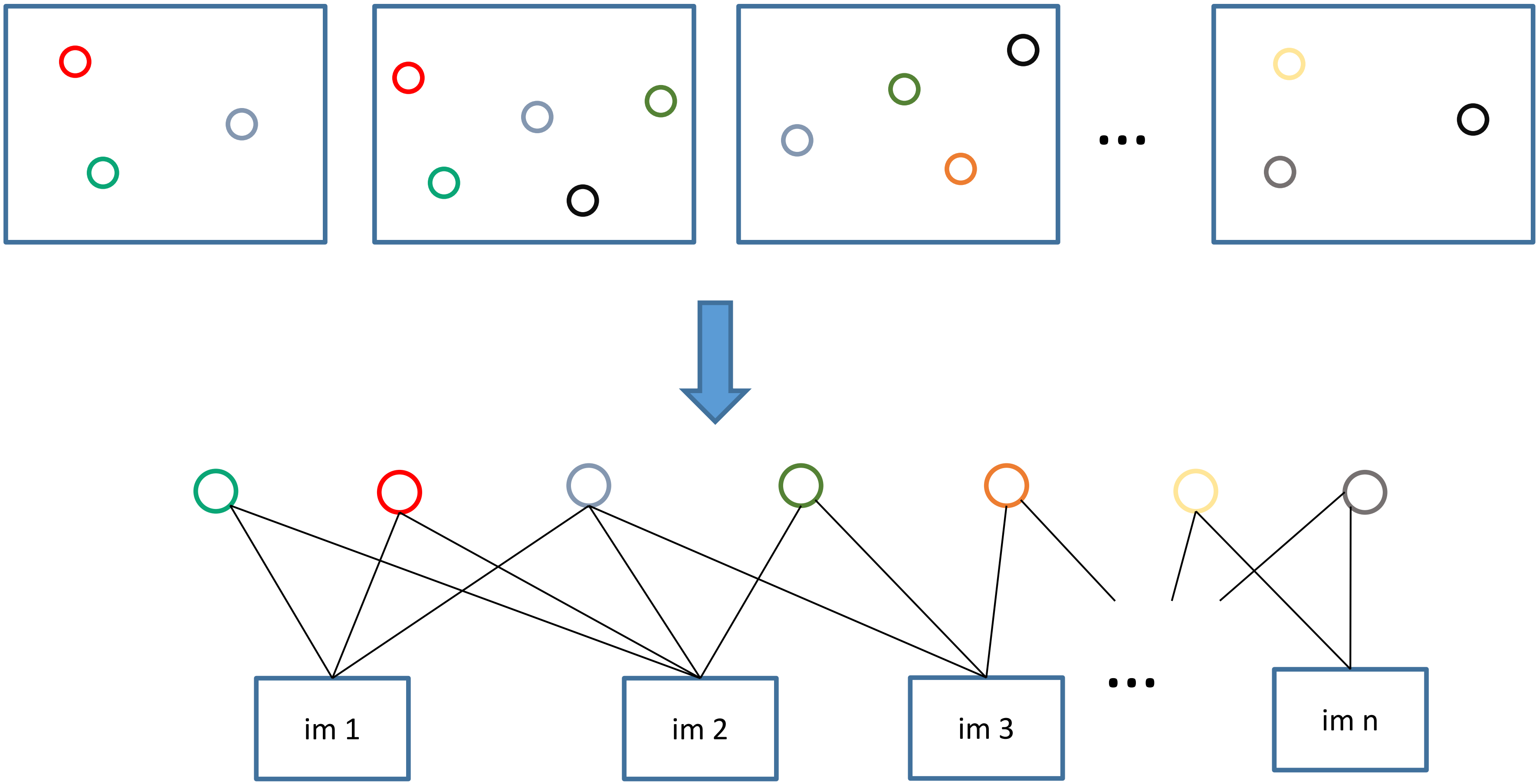


Feature Matching



Points of same color have been matched to each other

Step I. Feature extraction & matching in general

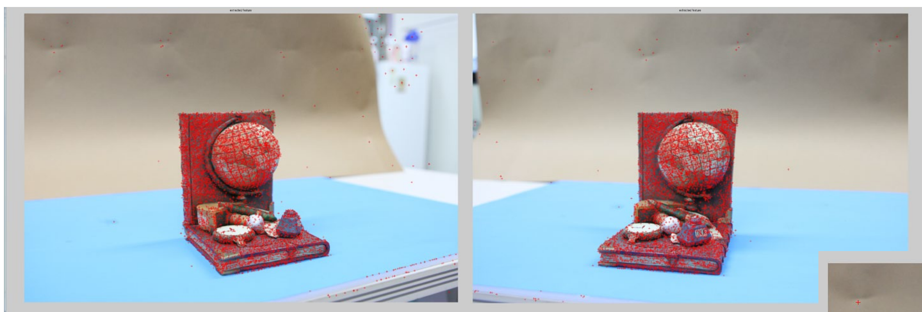


tracks graph: bipartite graph between observed 3D points and images

Step I. Feature extraction & matching

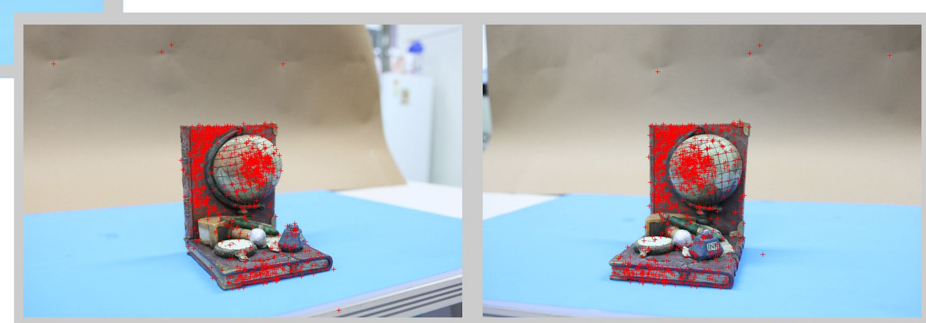
- SIFT (Scale Invariant Feature Transform)
  - Use the functions ‘vl_sift’, ‘vl_ubcmatch’
  - Website : http://www.vlfeat.org, https://docs.opencv.org/4.x/da/df5/tutorial_py_sift_intro.html

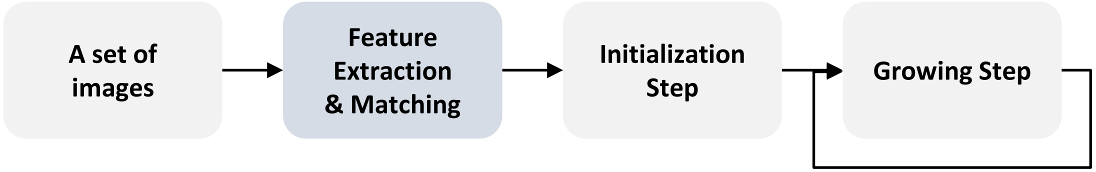
[4] Lowe, David G. "Distinctive image features from scale-invariant keypoints."*International journal of computer vision* 60.2 (2004): 91-110.



Feature Extraction

Feature Matching

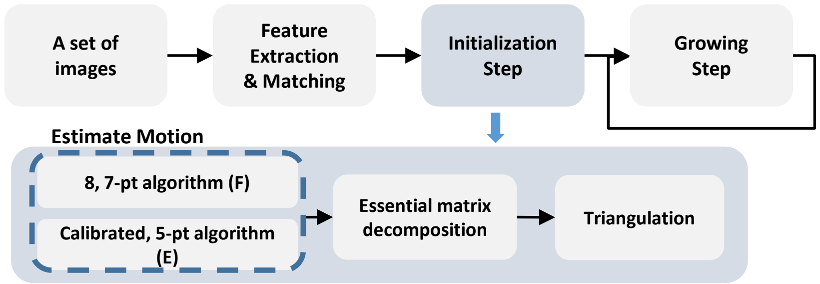




Step II. Essential matrix estimation

[5] Nistér, David. "An efficient solution to the five-point relative pose problem." *TPAMI* 2004

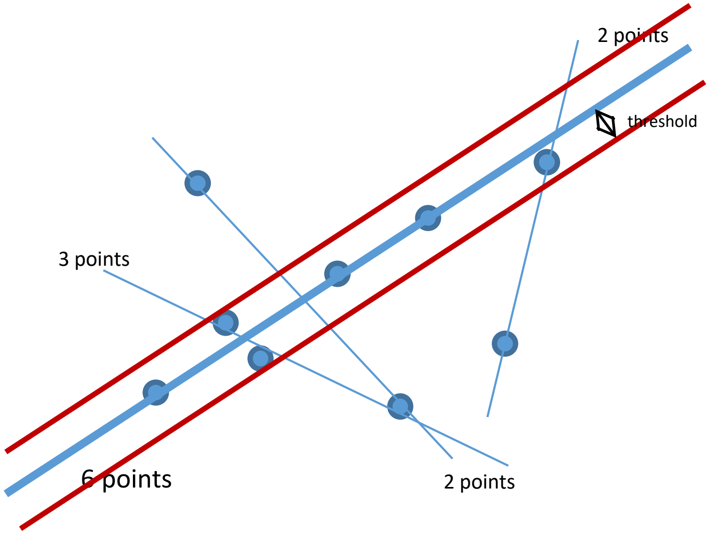
- Estimate Essential matrix 'E' given a set of match points {x, x'}
  - Use the functions 'calibrated_fivepoint' (Matlab)
  - Website : https://github.com/amrollah/Computer_Vision_course/blob/master/5/u5_code/calibrated_fivepoint.m
  - 'load_matlab_file_in_python_example.py' for Python
  - **Tip**
    - 5-point algorithm needs only 5 feature correspondences.
    - Randomly select 5 points repeatedly
    - Find the best essential matrix '**E**' by counting the number of inliers
    - This calls **RANSAC**



Step II. Essential matrix estimation

- 5-pts algorithms with RANSAC

1. Randomly select sets of 5 points
2. Generate E(hypothesis) and evaluate using other points with pre-defined threshold - **epipolar distance**
3. Do this for many times and choose the most supportive hypothesis having the most inliers

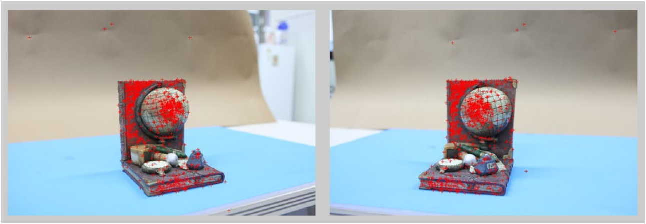


Randomly Selected correspondence

$\{x_{c1}, \ldots, x_{c5}\}$

$\{x'_{c1}, \ldots, x'_{c5}\}$

E using calibrated 5-pt algorithm



Definition 9.16. The defining equation for the essential matrix is

$$\hat{\mathrm{x}}'^{\mathsf{T}} \mathrm{E} \hat{\mathrm{x}} = 0 \qquad (9.11)$$

in terms of the normalized image coordinates for corresponding points $\mathrm{x} \leftrightarrow \mathrm{x}'$.

Step III. Essential matrix decomposition

- Essential Matrix Decomposition to [R|T]
  - Camera matrix to essential matrix

MVG 9.6 (p.257-p.260)
6.2.3 (p.162)

$$E = [t]_\times R = R[R^T t]$$

R, t: Rotation, Translation

  - Essential matrix to camera matrix
    - Make your own code

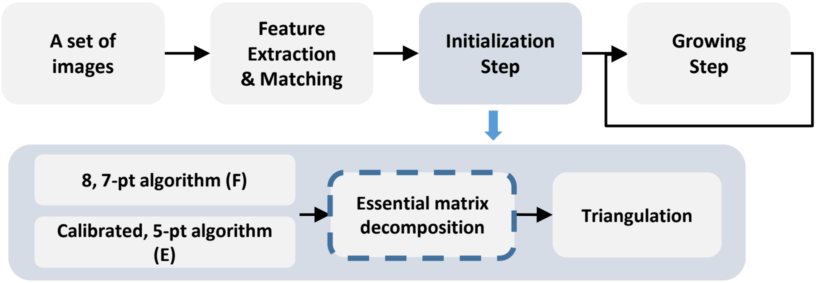
$$P' = [UWV^T | +u_3]$$

$$P' = [UWV^T | -u_3]$$

$$P' = [UW^TV^T | +u_3]$$

$$P' = [UW^TV^T | -u_3]$$

- $SVD(E) = Udiag(1,1,0)V^T$
- $W = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
- $u_3 = U(0,0,1)^T$: The last column vector of U



Proof. This is easily deduced from the decomposition of $\mathtt{E}$ as $[\mathbf{t}]_\times \mathtt{R} = \mathtt{SR}$, where $\mathtt{S}$ is skew-symmetric. We will use the matrices

$$\mathtt{W} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathtt{Z} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{9.13}$$

It may be verified that $\mathtt{W}$ is orthogonal and $\mathtt{Z}$ is skew-symmetric. From Result A4.1-(*p*581), which gives a block decomposition of a general skew-symmetric matrix, the 3×3 skew-symmetric matrix $\mathtt{S}$ may be written as $\mathtt{S} = k\mathtt{UZU}^\mathsf{T}$ where $\mathtt{U}$ is orthogonal. Noting that, up to sign, $\mathtt{Z} = \mathrm{diag}(1,1,0)\mathtt{W}$, then up to scale, $\mathtt{S} = \mathtt{U}\,\mathrm{diag}(1,1,0)\mathtt{WU}^\mathsf{T}$, and $\mathtt{E} = \mathtt{SR} = \mathtt{U}\,\mathrm{diag}(1,1,0)(\mathtt{WU}^\mathsf{T}\mathtt{R})$. This is a singular value decomposition of $\mathtt{E}$ with two equal singular values, as required. Conversely, a matrix with two equal singular values may be factored as $\mathtt{SR}$ in this way. $\square$

Since $\mathtt{E} = \mathtt{U}\,\mathrm{diag}(1,1,0)\mathtt{V}^\mathsf{T}$, it may seem that $\mathtt{E}$ has six degrees of freedom and not five, since both $\mathtt{U}$ and $\mathtt{V}$ have three degrees of freedom. However, because the two singular values are equal, the SVD is not unique – in fact there is a one-parameter family of SVDs for $\mathtt{E}$. Indeed, an alternative SVD is given by $\mathtt{E} = (\mathtt{U}\,\mathrm{diag}(\mathtt{R}_{2\times2}, 1))\,\mathrm{diag}(1,1,0)(\mathrm{diag}(\mathtt{R}^\mathsf{T}_{2\times2}, 1))\mathtt{V}^\mathsf{T}$ for any 2×2 rotation matrix $\mathtt{R}$.

9.6.2 Extraction of cameras from the essential matrix

The essential matrix may be computed directly from (9.11) using normalized image coordinates, or else computed from the fundamental matrix using (9.12). (Methods of computing the fundamental matrix are deferred to chapter 11). Once the essential matrix is known, the camera matrices may be retrieved from $\mathtt{E}$ as will be described next. In contrast with the fundamental matrix case, where there is a projective ambiguity, the camera matrices may be retrieved from the essential matrix up to scale and a four-fold ambiguity. That is there are four possible solutions, except for overall scale, which cannot be determined.

We may assume that the first camera matrix is $\mathtt{P} = [\mathtt{I} \mid \mathbf{0}]$. In order to compute the second camera matrix, $\mathtt{P}'$, it is necessary to factor $\mathtt{E}$ into the product $\mathtt{SR}$ of a skew-symmetric matrix and a rotation matrix.

Result 9.18. *Suppose that the SVD of* $\mathtt{E}$ *is* $\mathtt{U}\,\mathrm{diag}(1,1,0)\mathtt{V}^\mathsf{T}$. *Using the notation of (9.13), there are (ignoring signs) two possible factorizations* $\mathtt{E} = \mathtt{SR}$ *as follows:*

$$\mathtt{S} = \mathtt{UZU}^\mathsf{T} \quad \mathtt{R} = \mathtt{UWV}^\mathsf{T} \ \text{ or } \ \mathtt{UW}^\mathsf{T}\mathtt{V}^\mathsf{T}\,. \tag{9.14}$$

Proof. That the given factorization is valid is true by inspection. That there are no other factorizations is shown as follows. Suppose $\mathtt{E} = \mathtt{SR}$. The form of $\mathtt{S}$ is determined by the fact that its left null-space is the same as that of $\mathtt{E}$. Hence $\mathtt{S} = \mathtt{UZU}^\mathsf{T}$. The rotation $\mathtt{R}$ may be written as $\mathtt{UXV}^\mathsf{T}$, where $\mathtt{X}$ is some rotation matrix. Then

$$\mathtt{U}\,\mathrm{diag}(1,1,0)\mathtt{V}^\mathsf{T} = \mathtt{E} = \mathtt{SR} = (\mathtt{UZU}^\mathsf{T})(\mathtt{UXV}^\mathsf{T}) = \mathtt{U}(\mathtt{ZX})\mathtt{V}^\mathsf{T}$$

from which one deduces that $\mathtt{ZX} = \mathrm{diag}(1,1,0)$. Since $\mathtt{X}$ is a rotation matrix, it follows that $\mathtt{X} = \mathtt{W}$ or $\mathtt{X} = \mathtt{W}^\mathsf{T}$, as required. $\square$

The factorization (9.14) determines the $\mathbf{t}$ part of the camera matrix $\mathtt{P}'$, up to scale, from $\mathtt{S} = [\mathbf{t}]_\times$. However, the Frobenius norm of $\mathtt{S} = \mathtt{UZU}^\mathsf{T}$ is $\sqrt{2}$, which means that if $\mathtt{S} = [\mathbf{t}]_\times$ *including scale* then $\|\mathbf{t}\| = 1$, which is a convenient normalization for the baseline of the two camera matrices. Since $\mathtt{S}\mathbf{t} = \mathbf{0}$, it follows that $\mathbf{t} = \mathtt{U}(0,0,1)^\mathsf{T} = \mathbf{u}_3$, the last column of $\mathtt{U}$. However, the sign of $\mathtt{E}$, and consequently $\mathbf{t}$, cannot be determined. Thus, corresponding to a given essential matrix, there are four possible choices of the camera matrix $\mathtt{P}'$, based on the two possible choices of $\mathtt{R}$ and two possible signs of $\mathbf{t}$. To summarize:

Result 9.19. *For a given essential matrix* $\mathtt{E} = \mathtt{U}\,\mathrm{diag}(1,1,0)\mathtt{V}^\mathsf{T}$, *and first camera matrix* $\mathtt{P} = [\mathtt{I} \mid \mathbf{0}]$, *there are four possible choices for the second camera matrix* $\mathtt{P}'$, *namely*

$$\mathtt{P}' = [\mathtt{UWV}^\mathsf{T} \mid +\mathbf{u}_3] \ \text{ or } \ [\mathtt{UWV}^\mathsf{T} \mid -\mathbf{u}_3] \ \text{ or } \ [\mathtt{UW}^\mathsf{T}\mathtt{V}^\mathsf{T} \mid +\mathbf{u}_3] \ \text{ or } \ [\mathtt{UW}^\mathsf{T}\mathtt{V}^\mathsf{T} \mid -\mathbf{u}_3].$$

9.6.3 Geometrical interpretation of the four solutions

It is clear that the difference between the first two solutions is simply that the direction of the translation vector from the first to the second camera is reversed.

The relationship of the first and third solutions in result 9.19 is a little more complicated. However, it may be verified that

$$[\mathtt{UW}^\mathsf{T}\mathtt{V}^\mathsf{T} \mid \mathbf{u}_3] = [\mathtt{UWV}^\mathsf{T} \mid \mathbf{u}_3] \begin{bmatrix} \mathtt{VW}^\mathsf{T}\mathtt{W}^\mathsf{T}\mathtt{V}^\mathsf{T} & \\ & 1 \end{bmatrix}$$

and $\mathtt{VW}^\mathsf{T}\mathtt{W}^\mathsf{T}\mathtt{V}^\mathsf{T} = \mathtt{V}\,\mathrm{diag}(-1,-1,1)\mathtt{V}^\mathsf{T}$ is a rotation through 180° about the line joining the two camera centres. Two solutions related in this way are known as a "twisted pair".

The four solutions are illustrated in figure 9.12, where it is shown that a reconstructed point $\mathbf{X}$ will be in front of both cameras in one of these four solutions only. Thus, testing with a single point to determine if it is in front of both cameras is sufficient to decide between the four different solutions for the camera matrix $\mathtt{P}'$.

Note. The point of view has been taken here that the essential matrix is a homogeneous quantity. An alternative point of view is that the essential matrix is defined exactly by the equation $\mathtt{E} = [\mathbf{t}]_\times \mathtt{R}$, (i.e. including scale), and is determined only up to indeterminate scale by the equation $\mathbf{x}'^\mathsf{T}\mathtt{E}\mathbf{x} = 0$. The choice of point of view depends on which of these two equations one regards as the defining property of the essential matrix.

9.7 Closure

9.7.1 The literature

The essential matrix was introduced to the computer vision community by Longuet-Higgins [LonguetHiggins-81], with a matrix analogous to $\mathtt{E}$ appearing in the photogrammetry literature, e.g. [VonSanden-08]. Many properties of the essential matrix have been elucidated particularly by Huang and Faugeras [Huang-89], [Maybank-93], and [Horn-90].

The realization that the essential matrix could also be applied in uncalibrated situations, as it represented a projective relation, developed in the early part of the 1990s,

Step III. Essential matrix decomposition

- (Result 9.18) Suppose that the SVD of E is $\mathrm{Udiag}(1,1,0)\mathrm{V}^T$. Using the notation of W and Z, there are (ignoring signs) two possible factorizations $\mathrm{E} = \mathrm{SR}$ as follows:

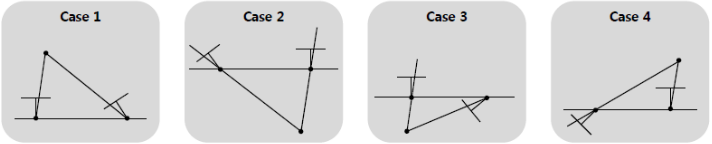
$$\mathrm{S} = \mathrm{UZU}^T, \mathrm{R} = \mathrm{UWV}^T, \mathrm{R} = \mathrm{UW}^T\mathrm{V}^T$$

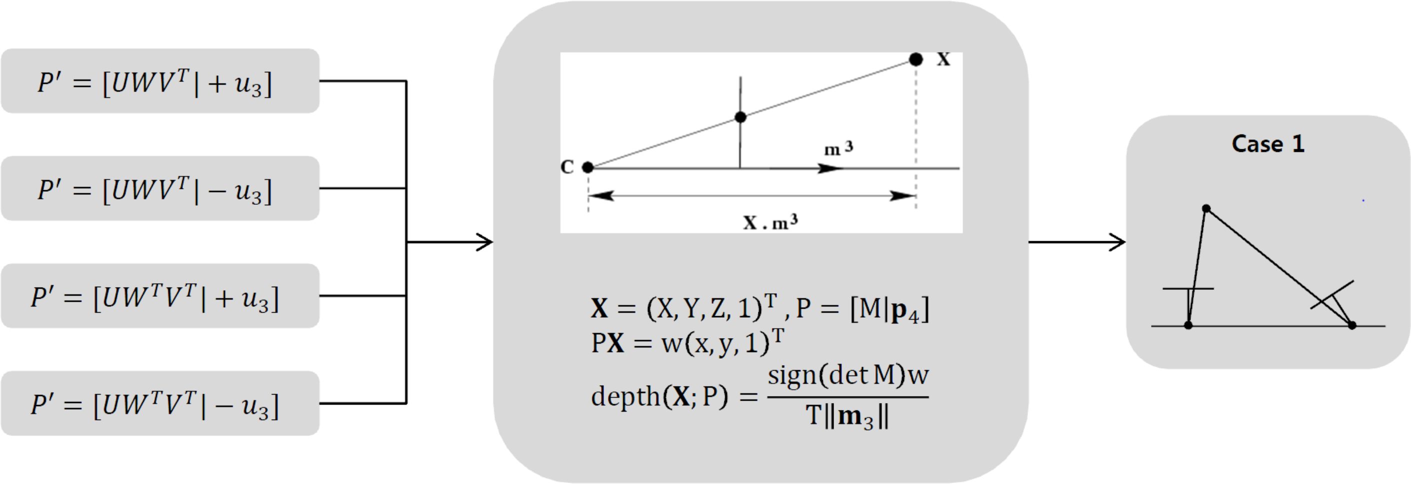
$$\mathrm{W} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } \mathrm{Z} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

  - (Property 1) A 3x3 matrix is an essential matrix if and only if two of its singular values are equal, and the third is zero
  - (Property 2: Block decomposition) 3x3 skew-symmetric matrix S may be written as $\mathrm{S} = k\mathrm{UZU}^T$ where U is orthogonal.

Step III. Essential matrix decomposition

- Essential Matrix Decomposition to [R|T]
  - There are four possible reconstruction
    - Depth for both camera should be positive value (case 1)
    - Not negative value (case 2, 3, 4)
  - You should figure out the optimal camera pose





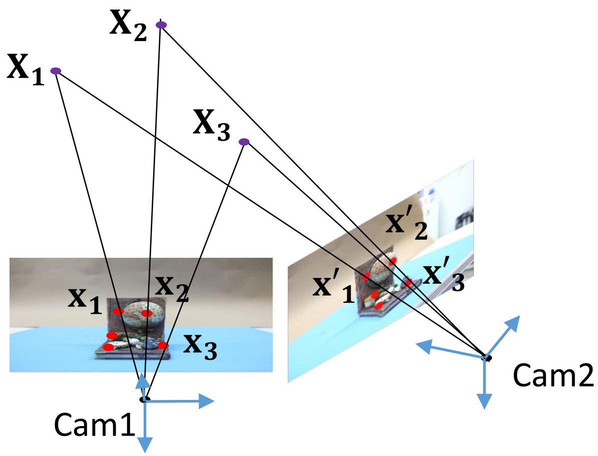
- Triangulation
  - Get 3D points from Camera pose & correspondences

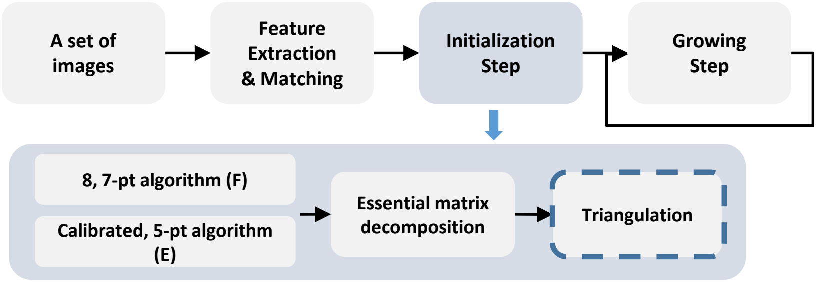
$\mathbf{X}$: 3D point
$\mathbf{x}$: Point on image coordinate
$\mathbf{K}$: Intrinsic matrix
$\mathbf{P}$ $(\mathbf{K}[\boldsymbol{R}|\boldsymbol{t}])$: Extrinsic matrix

$$x_{ci} = P\boldsymbol{X}_i$$
$$[x_{ci}]_\times P\boldsymbol{X}_i = 0$$

$$x(\boldsymbol{p}^{3T}\boldsymbol{X}) - (\boldsymbol{p}^{1T}\boldsymbol{X}) = 0$$
$$y(\boldsymbol{p}^{3T}\boldsymbol{X}) - (\boldsymbol{p}^{2T}\boldsymbol{X}) = 0$$
$$x(\boldsymbol{p}^{3T}\boldsymbol{X}) - y(\boldsymbol{p}^{1T}\boldsymbol{X}) = 0$$

$$A\boldsymbol{X} = 0$$
$$A = \begin{bmatrix} x\boldsymbol{p}^{3T} - \boldsymbol{p}^{1T} \\ y\boldsymbol{p}^{3T} - \boldsymbol{p}^{2T} \\ x'\boldsymbol{p}'^{3T} - \boldsymbol{p}'^{1T} \\ y'\boldsymbol{p}'^{3T} - \boldsymbol{p}'^{2T} \end{bmatrix}$$

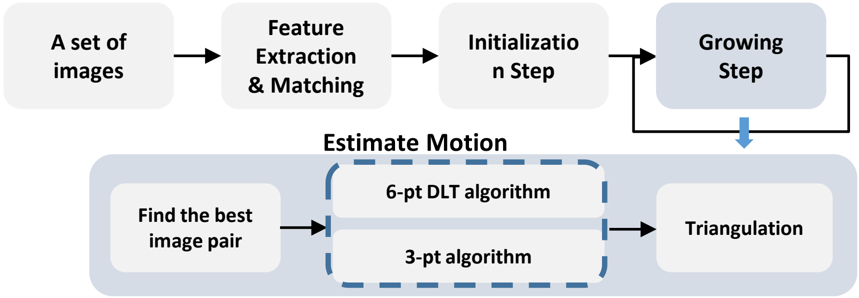




Step V. Growing step

- Estimate Camera matrix ‘P’ given a set of match points and 3D {x, X}
  - Use the functions ‘PerspectiveThreePoint’ or ‘cv2.solvePnP’

- **Tip**
  - 3-point algorithm needs only 3 feature correspondences.
  - Randomly select 3 points repeatedly
  - Find the best camera matrix ‘P’ by counting the number of inliers
  - This is called **RANSAC**



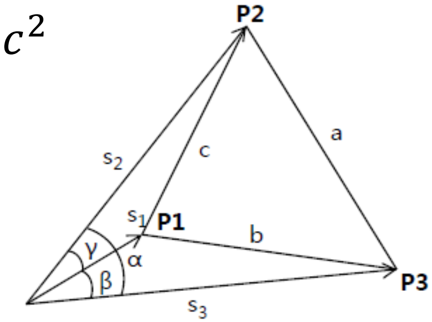
[8] Haralick, Bert M., et al. "Review and analysis of solutions of the three point perspective pose estimation problem." *IJCV* (1994)

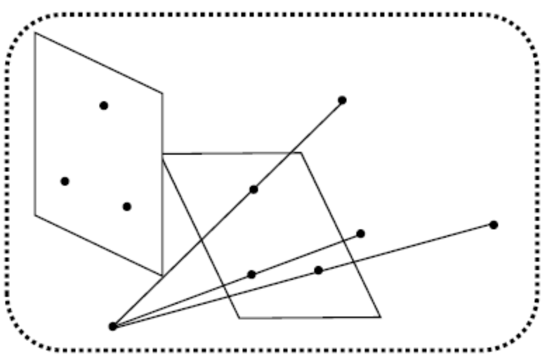
Step V. Growing step

- 3-point algorithm

[8] Haralick, Bert M., et al. "Review and analysis of solutions of the three point perspective pose estimation problem." *IJCV* (1994)

$$s_2{}^2 + s_3{}^2 - 2s_2s_3\cos\alpha = a^2$$
$$s_1{}^2 + s_3{}^2 - 2s_1s_3\cos\beta = b^2$$
$$s_1{}^2 + s_2{}^2 - 2s_1s_2\cos\gamma = c^2$$





Known : a, b, c, and unit vectors j_1,j_2,j_3

The **problem** is to determine the lengths s_1, s_2, s_3 from which the 3D vertex point positions P_1, P_2, and P_3 can be determined.

Six solutions

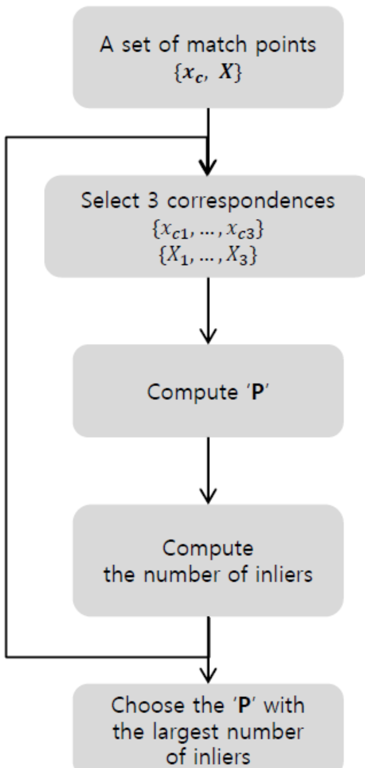
Grunert(1841), Finsterwalder(1937), Merritt(1949), Fischler & Bolles(1981), Linnainmaa et al(1988), Grafarend et al(1989).

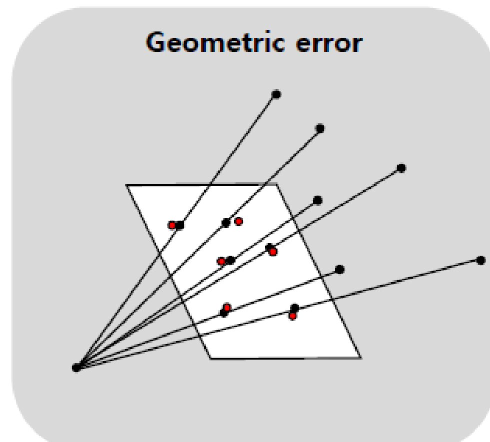
Estimate Pose(P)

Step V. Growing step

[2] Hartley, Richard, and Andrew Zisserman. *Multiple view geometry in computer vision*. Cambridge university press, 2003.

- Geometric error
  - Compute the number of inliers
  - Choose the best **P** with the largest number of inliers





$$d^2 = d(\boldsymbol{x}_1, KP_1\boldsymbol{X})^2 + d(\boldsymbol{x}_2, KP_2\boldsymbol{X})^2$$

$d < t$ pixels

"Review and Analysis of Solutions of the Three Point Perspective Pose Estimation Problem", IJCV 1994

Step V. Growing step

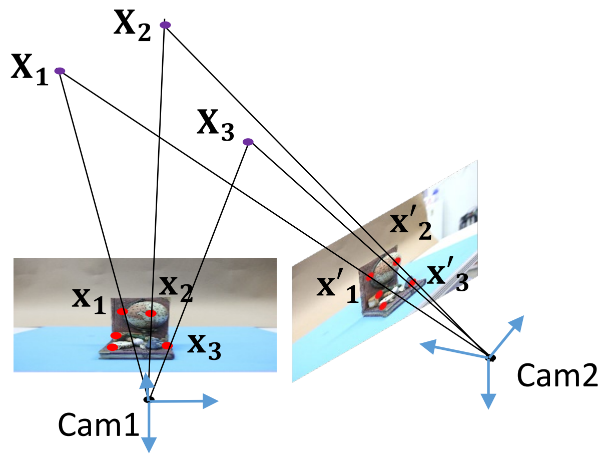
- Triangulation
  - Get 3D points from Camera pose & correspondences

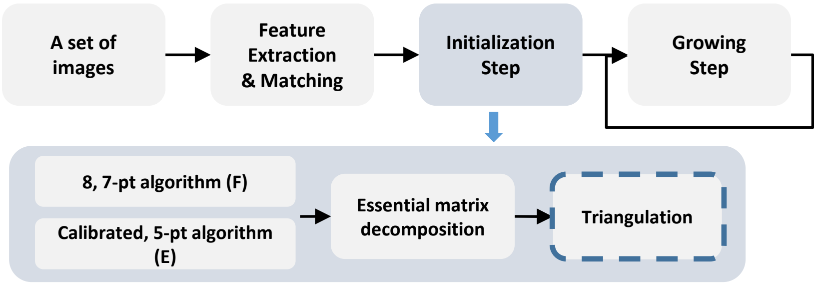
$\mathbf{X}$: 3D point
$\mathbf{x}$: Point on image coordinate
$\mathbf{K}$: Intrinsic matrix
$\mathbf{P}$ $(\mathbf{K}[\boldsymbol{R}|\boldsymbol{t}])$: Extrinsic matrix

$$x_{ci} = P\boldsymbol{X}_i$$
$$[\boldsymbol{x}_{ci}]_\times P\boldsymbol{X}_i = 0$$

$$x(\boldsymbol{p}^{3T}\boldsymbol{X}) - (\boldsymbol{p}^{1T}\boldsymbol{X}) = 0$$
$$y(\boldsymbol{p}^{3T}\boldsymbol{X}) - (\boldsymbol{p}^{2T}\boldsymbol{X}) = 0$$
$$x(\boldsymbol{p}^{3T}\boldsymbol{X}) - y(\boldsymbol{p}^{1T}\boldsymbol{X}) = 0$$

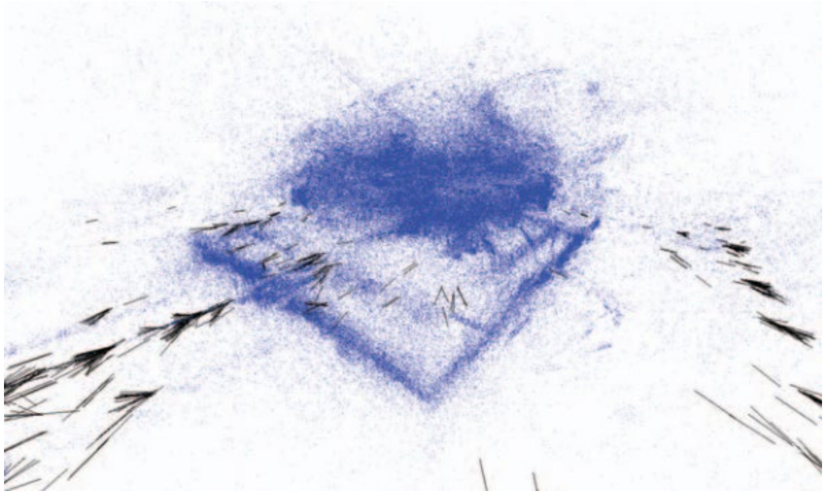
$$A\boldsymbol{X} = 0$$
$$A = \begin{bmatrix} x\boldsymbol{p}^{3T} - \boldsymbol{p}^{1T} \\ y\boldsymbol{p}^{3T} - \boldsymbol{p}^{2T} \\ x'\boldsymbol{p}'^{3T} - \boldsymbol{p}'^{1T} \\ y'\boldsymbol{p}'^{3T} - \boldsymbol{p}'^{2T} \end{bmatrix}$$



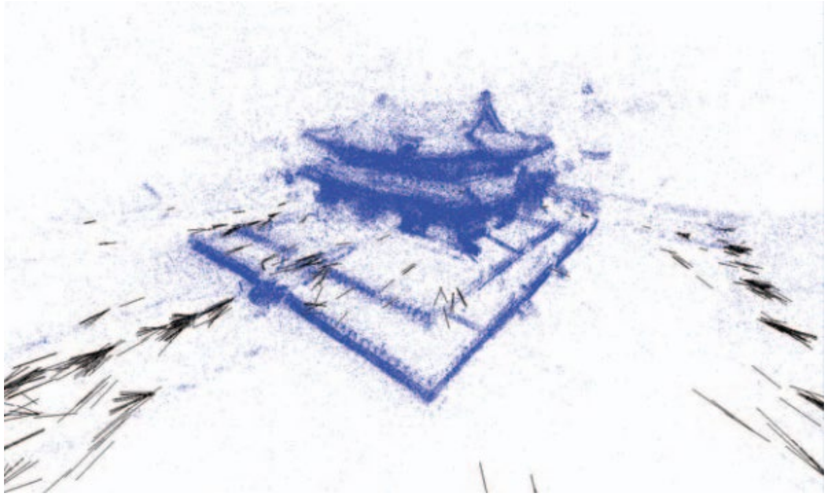


Step VI. Optimization

- Bundle adjustment
  - Refines a visual reconstruction to produce jointly optimal 3D structure and viewing parameters
  - ‘Bundle’ refers to the bundle of light rays leaving each 3D feature and converging on each camera center.



Before Bundle adjustment

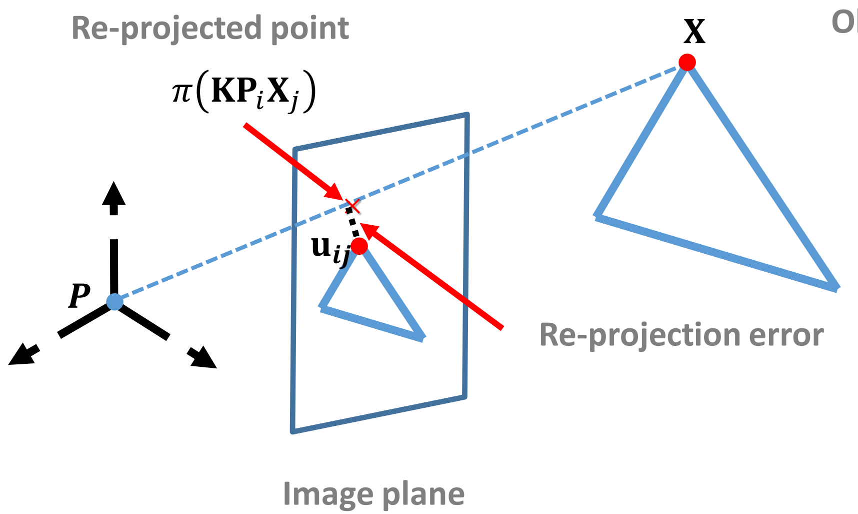


After Bundle adjustment

[6] Triggs, Bill, et al. "Bundle adjustment—a modern synthesis." *International workshop on vision algorithms*. Springer Berlin Heidelberg, 1999.
[7] Jeong, Yekeun, et al. "Pushing the envelope of modern methods for bundle adjustment." *IEEE transactions on pattern analysis and machine intelligence* (2012): 1605-1617.

Step VI. Optimization

- Bundle Adjustment's Mathematical Problem
  - Minimize re-projection error
  - Non-linear Least Square approach
  - Good approximate values are needed



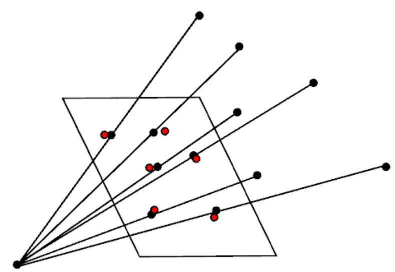
Objective function

$$C(\mathbf{P},\mathbf{X}) = \sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij} \parallel \mathbf{u}_{ij} - \langle \mathbf{KP}_i\mathbf{X}_j \rangle \parallel^2$$

n: The number of cameras
m: The number of features
$\pi(\cdot)$: The projection function ($\mathcal{R}^3 \rightarrow \mathcal{R}^2$)
w_{ij}: indicator variable
1 if visible, 0 other wise

Step VI. Optimization

- LM Optimization
  - Make your own cost function (*fun*)
  - Optimize the function with 'lsqnonlin' function
  - See the error(residual) is decreasing
    - 3D points and camera poses might be barely moved



$$\textbf{CostFuction} = min \sum_i d(x_i, \hat{x}_i)^2$$
$$= min \sum_i d(x_i, KP\boldsymbol{X}_i)^2$$

lsqnonlin

Solve nonlinear least-squares (nonlinear data-fitting) problems

Equation

Solves nonlinear least-squares curve fitting problems of the form

$$\min_x \|f(x)\|_2^2 = \min_x \left(f_1(x)^2 + f_2(x)^2 + ... + f_n(x)^2\right)$$

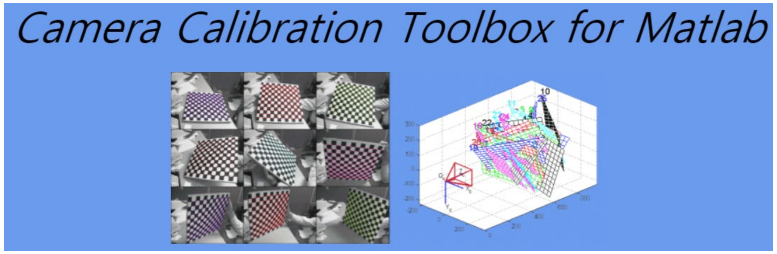
Syntax

```
x = lsqnonlin(fun,x0)
x = lsqnonlin(fun,x0,lb,ub)
x = lsqnonlin(fun,x0,lb,ub,options)
x = lsqnonlin(problem)
[x,resnorm] = lsqnonlin(...)
[x,resnorm,residual] = lsqnonlin(...)
[x,resnorm,residual,exitflag] = lsqnonlin(...)
[x,resnorm,residual,exitflag,output] = lsqnonlin(...)
[x,resnorm,residual,exitflag,output,lambda] = lsqnonlin(...)
[x,resnorm,residual,exitflag,output,lambda,jacobian] = lsqnonlin(...)
```

```
options=optimset('Algorithm',{'levenberg-marquardt' 0.001},'Display','off');
options=optimset('Algorithm',{'levenberg-marquardt' 0.001},'TolFun',1e-8,'TolX',1e-8,'Display','off');
```

Step VII. Camera calibration

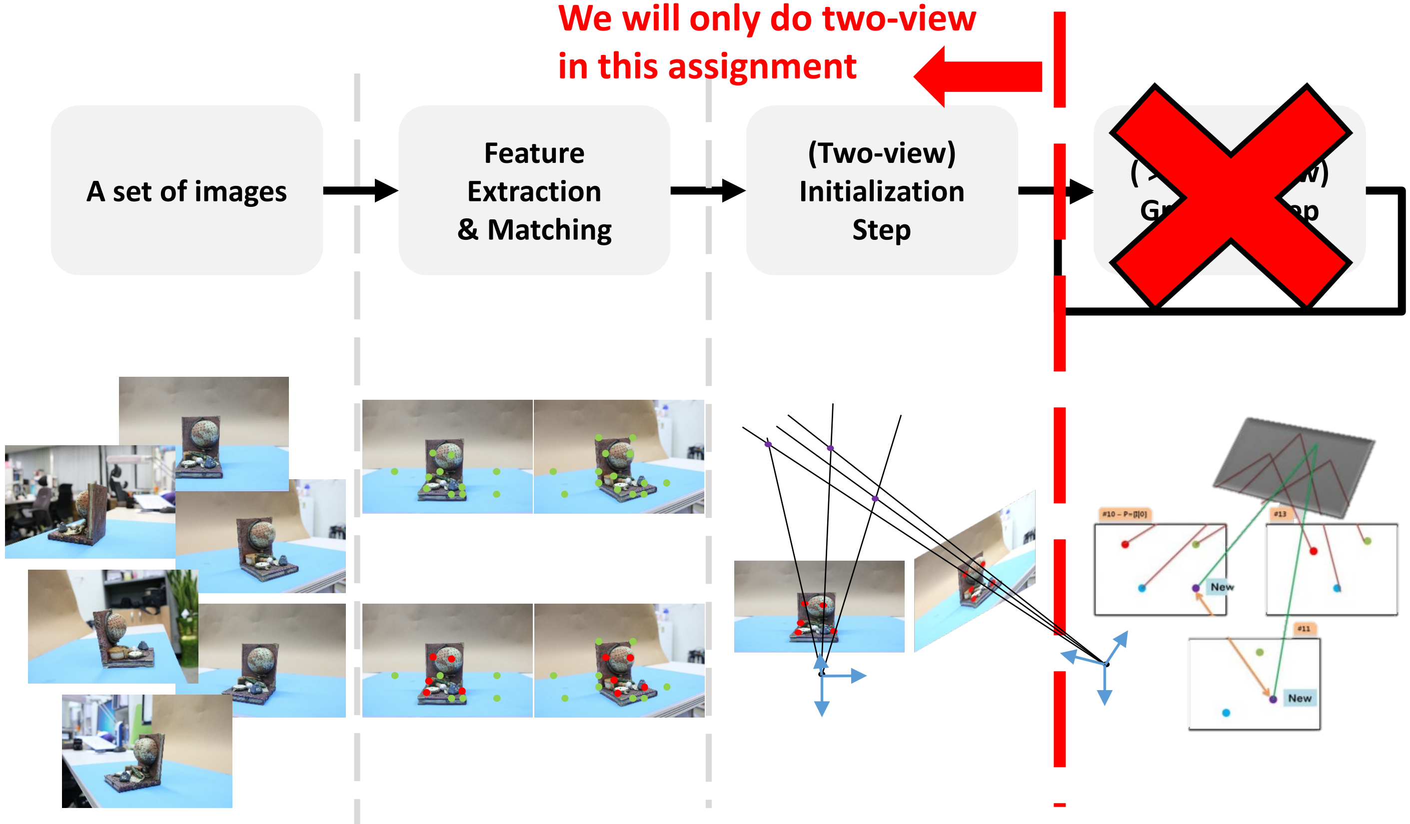
- Procedure
  - Download camera calibration toolbox
  - Print checkerboard
  - Capture multiple images of checkerboard
  - Run the camera calibration toolbox



- Camera Calibration using Toolbox
  - Matlab instruction
    - http://www.vision.caltech.edu/bouguetj/calib_doc/index.html
  - C Library
    - http://docs.opencv.org/doc/tutorials/calib3d/camera_calibration/camera_calibration.html
  - Python
    - https://learnopencv.com/camera-calibration-using-opencv/

To Do List

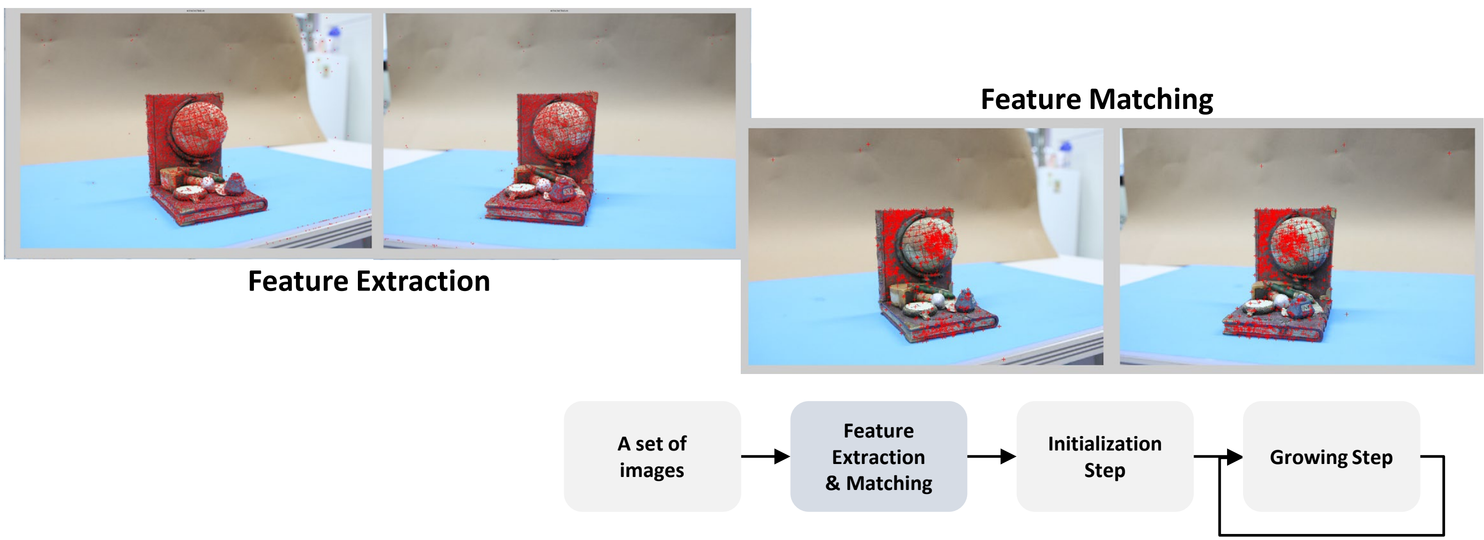
- Overall strategy



[4] Hartley, Richard, and Andrew Zisserman. *Multiple view geometry in computer vision*. Cambridge university press, 2003.
[5] Szeliski, Richard. *Computer vision: algorithms and applications*. Springer Science & Business Media, 2010.

To Do List

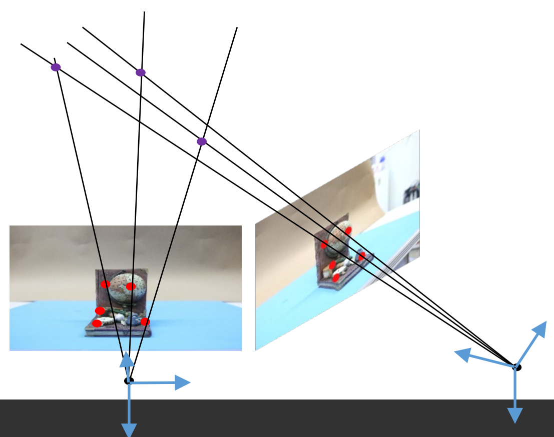
1. Load the input images ('sfm01.jpg', 'sfm02.jpg')
2. **Extract features** from both images using the function 'vl_sift' (2 pts)
3. **Match features** (find correspondence) between two images using the function 'vl_ubcmatch' (3 pts)

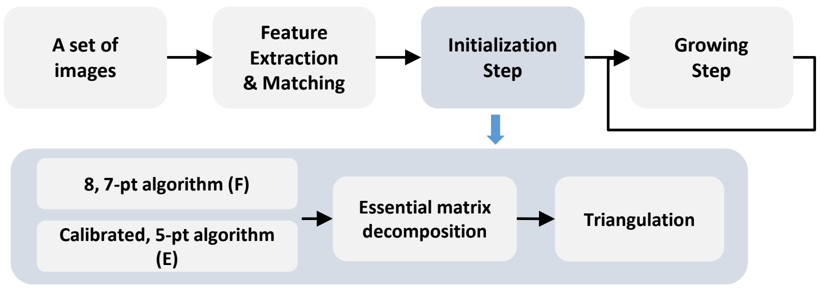


Refer: http://www.vlfeat.org/overview/sift.html

To Do List

4. Estimate **Essential matrix E** with **RANSAC** using ‘calibrated_fivepoint’ (5 pts)
5. Decompose essential matrix $\mathbf{E}$ to camera extrinsic $[\mathbf{R}|\mathbf{T}]$(5 pts)
6. Generate 3D point by implementing **Triangulation** (5 pts)
7. Extract **Intrinsic parameters** through **Camera Calibration** from own images (2 pts, see. Sec VII)
8. If you reconstruct 3D models from multiple view images (more than 3 views), I will give a huge extra credit (up to 5pts, see. Sec V)





For Python Student

- **Not** allow to use OpenCV functions except below :
  - cv2.SIFT_create (Step I)
  - cv2.xfeatures2d.SIFT_create (Step I)
  - cv2.findChessboardCorners (Step VII)
  - cv2.cornerSubPix (Step VII)
  - cv2.calibrateCamera (Step VII)
  - cv2.solvePnP() (Step V)
  - Image I/O function
  - Image visualization function
  - If you think other functions are mandatory, email me

SfM with your own dataset

- Tips for making your own dataset (using your mobile phone)

1) Use a fixed-focus camera.

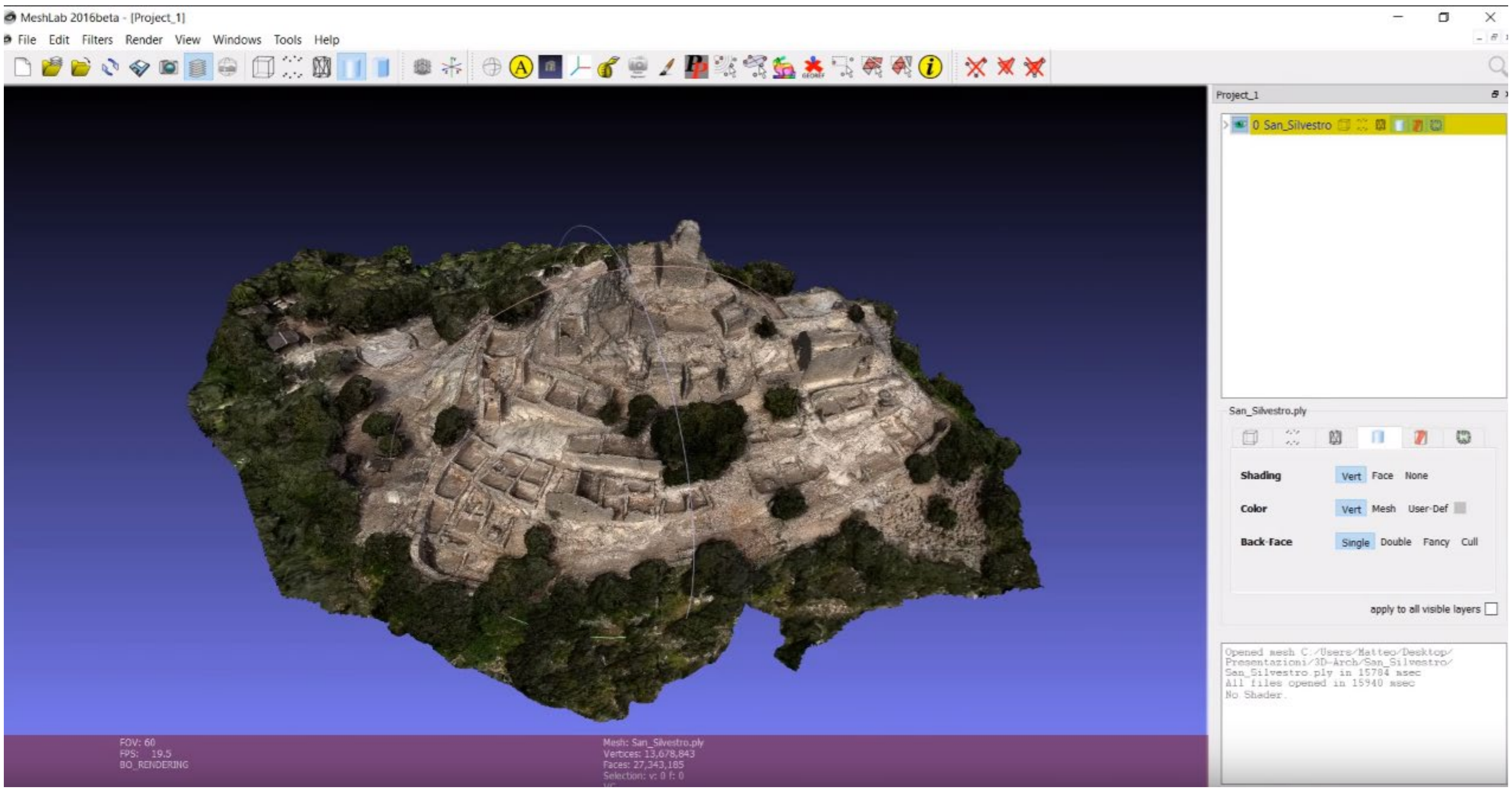
2) Do calibration for camera parameter estimation using toolbox (to get intrinsic).

3) Take pictures of your target scene with the camera of which the intrinsic parameter is known now.

4) Run your SfM program with your dataset by replacing the images and the camera intrinsic matrix K to yours.

Display your 3D results (ply file)

Use Meshlab (download: http://www.meshlab.net)



Submission

- Submission should include…
  - Source code
  - Results (3D point cloud in ply file) of your code
  - Readme file explaining how to execute the program
  - **Report** (3pts)
- Report should include…
  - Your understanding of each steps of algorithms
  - Figures of results from your implementation
- Notice
  - [Delayed submission] Not allowed
  - [Plagiarism] Definitely F grade for copied codes (from friends or internet)
  - [Implementation] No use any open function such as findFundamentalMat() other than the mentioned function

Due : Nov. 5, 11:55PM
To : LMS System

TA session: 11/28 and 11/30
TA: Sang-Hun Han
Office Hour : Tue, Wed, Thu PM 1:30~2:30
(e-mail: yesjames4231@gm.gist.ac.kr)

Auxiliary References

- Multiple-View Geometry in Computer Vision (https://github.com/liulinbo/slam/blob/master/Multiple%20View%20Geometry%20in%20Computer%20Vision.pdf)
  - Basic Projective Geometry (ch. 2,3)
  - Camera Models and Calibration (ch. 6,8)
  - Epipolar Geometry and Implementation (ch. 9, 11)
  - Triangulation (ch. 12)

- Computer Vision: Algorithms and Applications (http://szeliski.org/Book/drafts/SzeliskiBook_20100903_draft.pdf)
  - Structure from Motion (ch. 7)

- Phillp Torr's Structure from Motion toolkit
  - Includes F-matrix estimation, RANSAC, Triangulation and etc.
  - https://kr.mathworks.com/matlabcentral/fileexchange/4576-structure-and-motion-toolkit-in-matlab