Lab 1: One-Dimensional Linear Regression (Batch Gradient Descent)

Abstract

In this lab, we use one-dimensional linear regression as the example and solve for the parameters $\theta = (\theta_0, \theta_1)$ using batch gradient descent. On the given training set (age → height), we perform parameter estimation, draw the fitted line, make two point predictions, and visualize the cost function $J(\theta)$ with a 3D surface and contour plot to build intuition about the behavior and convergence of gradient descent.

1. Data and Preprocessing

The data files are located in `./ex1Data/`. Read age into a matrix $X \in \mathbb{R}^{m\times 1}$ and height into a vector $y \in \mathbb{R}^{m\times 1}$, and standardize both to column-vector shapes to facilitate subsequent matrix operations and vectorized implementation.

In [26]:

```python
import sys,os
from pathlib import Path
import numpy as np
import matplotlib.pyplot as plt

DATA_DIR=Path('ex1Data')
X_Path=DATA_DIR/'ex1x.dat'
y_Path=DATA_DIR/'ex1y.dat'
DATA_DIR.mkdir(parents=True,exist_ok=True)

X=np.loadtxt(X_Path,dtype=float).reshape(-1,1)
y=np.loadtxt(y_Path,dtype=float).reshape(-1,1)
m=X.shape[0]
print(f"样本数 m = {m}")
print("X 预览（前 5 行）：\n",X[:5].ravel())
print("y 预览（前 5 行）：\n",y[:5].ravel())

plt.figure()
plt.scatter(X,y,marker='o',label="data")
plt.xlabel("Age(years)")
plt.ylabel("Height")
plt.title("Training Data")
plt.grid(True,linestyle='--',alpha=0.3)
plt.legend()
plt.show()
```

```
样本数 m = 50
X 预览（前 5 行）：
 [2.0658746 2.3684087 2.5399929 2.5420804 2.549079 ]
y 预览（前 5 行）：
 [0.77918926 0.91596757 0.90538354 0.90566138 0.9389889 ]
```

2. Model and Objective

Under the linear model $h_\theta(x) = \theta_0 + \theta_1 x$, we adopt the mean-squared-error (MSE) cost:

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2,$$

and in implementation we obtain $X_b = [\mathbf{1},\ X]$ by concatenating a constant column to the feature matrix.

In [27]:

```
def add_intercept(X):
    if X.ndim==1:
        X=X.reshape(-1,1)
    ones=np.ones((X.shape[0],1),dtype=X.dtype)
    return np.hstack([ones,X])
def cost(X_b,y,theta):
    m=X_b.shape[0]
    res=X_b@theta-y
    return float((res**2).sum()/(2*m))
def gradient_descent(X,y,alpha=0.07,iters=1500):
    X_b=add_intercept(X)
    m=X_b.shape[0]
    theta=np.zeros((2, 1))
    J_hist=[]
    theta_after_first_iter=None
    for t in range(1,iters+1):
        res=X_b@theta-y
        grad=(X_b.T@res)/m
```

```
        theta=theta-alpha*grad
        J_hist.append(cost(X_b,y,theta))
        if t==1:
            theta_after_first_iter=theta.copy()
    return theta,np.array(J_hist),theta_after_first_iter
def predict(age, theta):
    x_b=np.array([[1.0,float(age)]],dtype=float)
    return (x_b@theta).item()
```

3. Training Method (Batch Gradient Descent)

With learning rate α, we iterate:

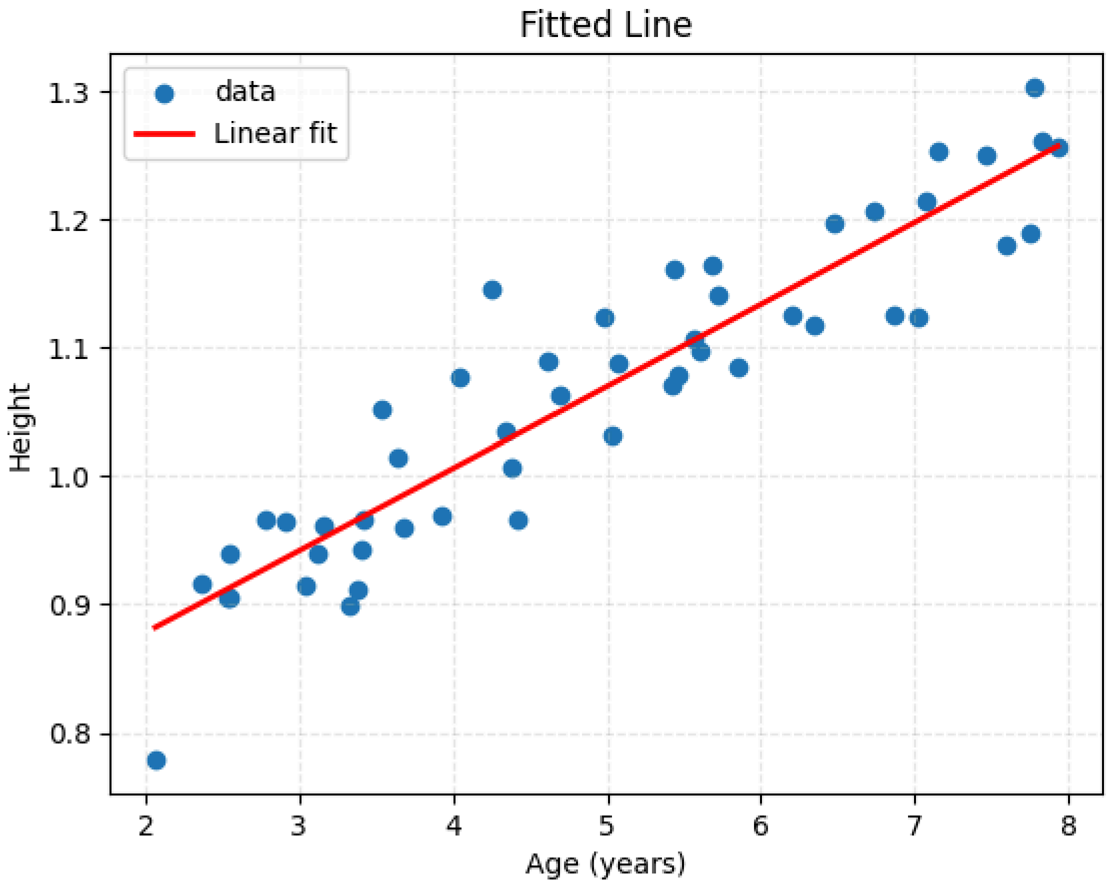
$$\theta := \theta - \alpha \nabla_\theta J(\theta), \quad \nabla_\theta J(\theta) = \frac{1}{m} X_b^\top (X_b \theta - y).$$

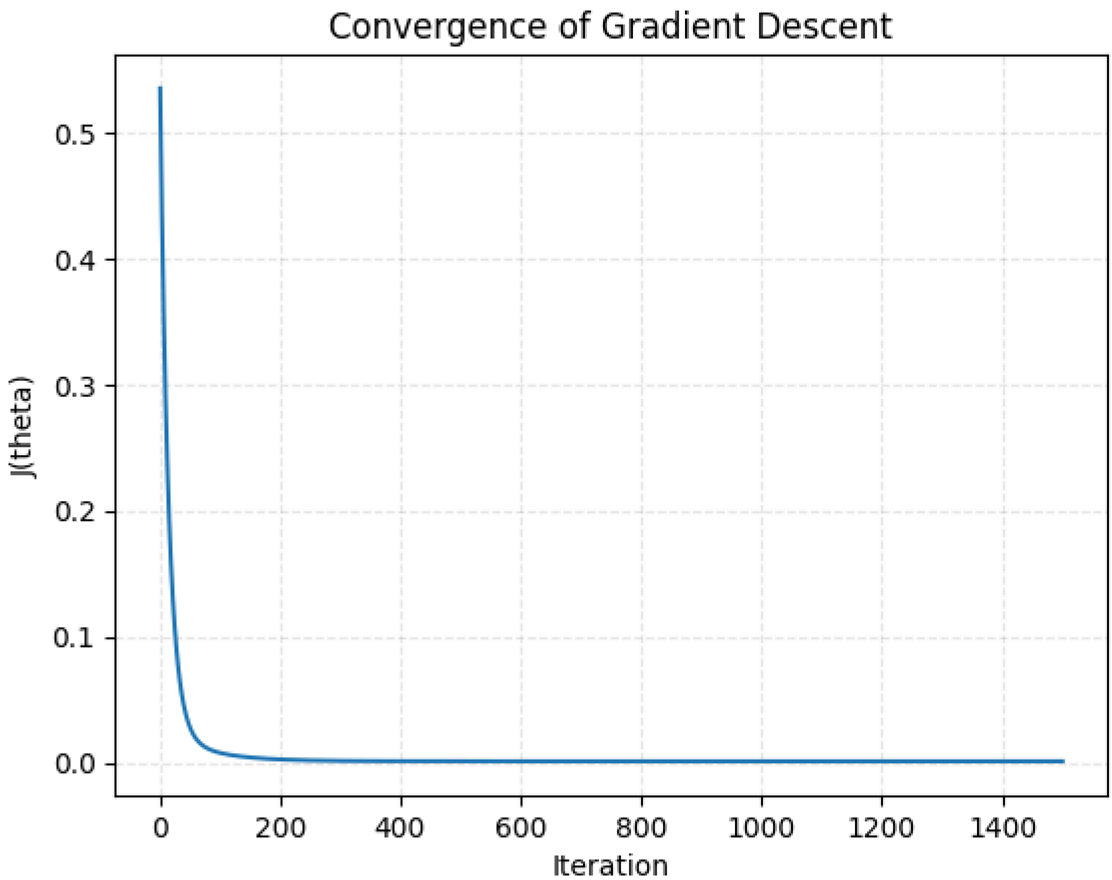
By default we set $\alpha = 0.07$, run 1,500 iterations, and record the parameters after the first update and after convergence.

In [28]:

```
alpha=0.07
iters=1500
theta_final,J_hist,theta_first=gradient_descent(X,y,alpha=alpha,iters=iters)
print(f"第 1 次迭代后的参数: theta0={theta_first[0,0]:.6f}, theta1={theta_first[
print(f"最终参数 ({iters} 次迭代): theta0={theta_final[0,0]:.6f}, theta1={theta_
print(f"最终代价 J(theta) = {J_hist[-1]:.6f}")

X_line=np.linspace(float(X.min()),float(X.max()),100).reshape(-1,1)
y_line=add_intercept(X_line)@theta_final

plt.figure()
plt.scatter(X,y,marker='o',label="data")
plt.plot(X_line,y_line,color='red',linewidth=2.0,label="Linear fit")
plt.xlabel("Age (years)")
plt.ylabel("Height")
plt.title("Fitted Line")
plt.grid(True,linestyle='--',alpha=0.3)
plt.legend()
plt.show()

plt.figure()
plt.plot(np.arange(1,len(J_hist)+1),J_hist)
plt.xlabel("Iteration")
plt.ylabel("J(theta)")
plt.title("Convergence of Gradient Descent")
plt.grid(True,linestyle='--',alpha=0.3)
plt.show()
```

```
第 1 次迭代后的参数: theta0=0.074528, theta1=0.380022
最终参数 (1500 次迭代): theta0=0.750150, theta1=0.063883
最终代价 J(theta) = 0.000987
```





4. Experimental Results (Parameters and Fit)

The learned parameters θ_0, θ_1 are shown above. Together with the scatter plot, the **red** fitted line illustrates how well the linear model captures the trend in the data. The convergence curve, $J(\theta)$ vs. iterations, further demonstrates the monotonic decrease during training and the stability of convergence.

5. Prediction (Ages 3.5 and 7)

Use the trained linear model to complete the prediction tasks given in the handout.

In [29]:
```
pred_3_5=predict(3.5,theta_final)
pred_7_0=predict(7.0,theta_final)

print(f"预测: age=3.5, height≈ {pred_3_5:.4f}")
print(f"预测: age=7.0, height≈ {pred_7_0:.4f}")
```

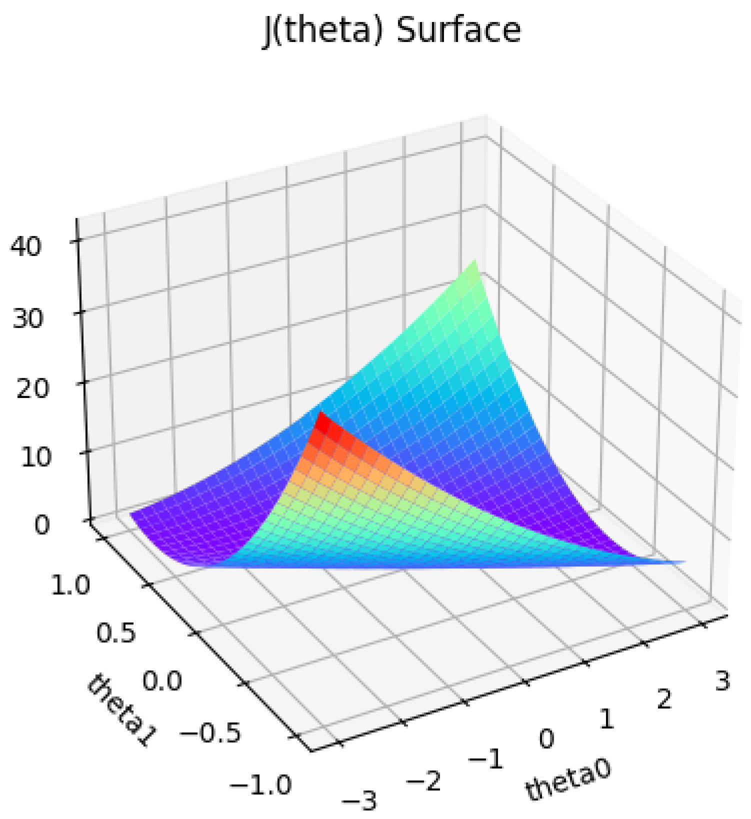
```
预测: age=3.5, height≈ 0.9737
预测: age=7.0, height≈ 1.1973
```

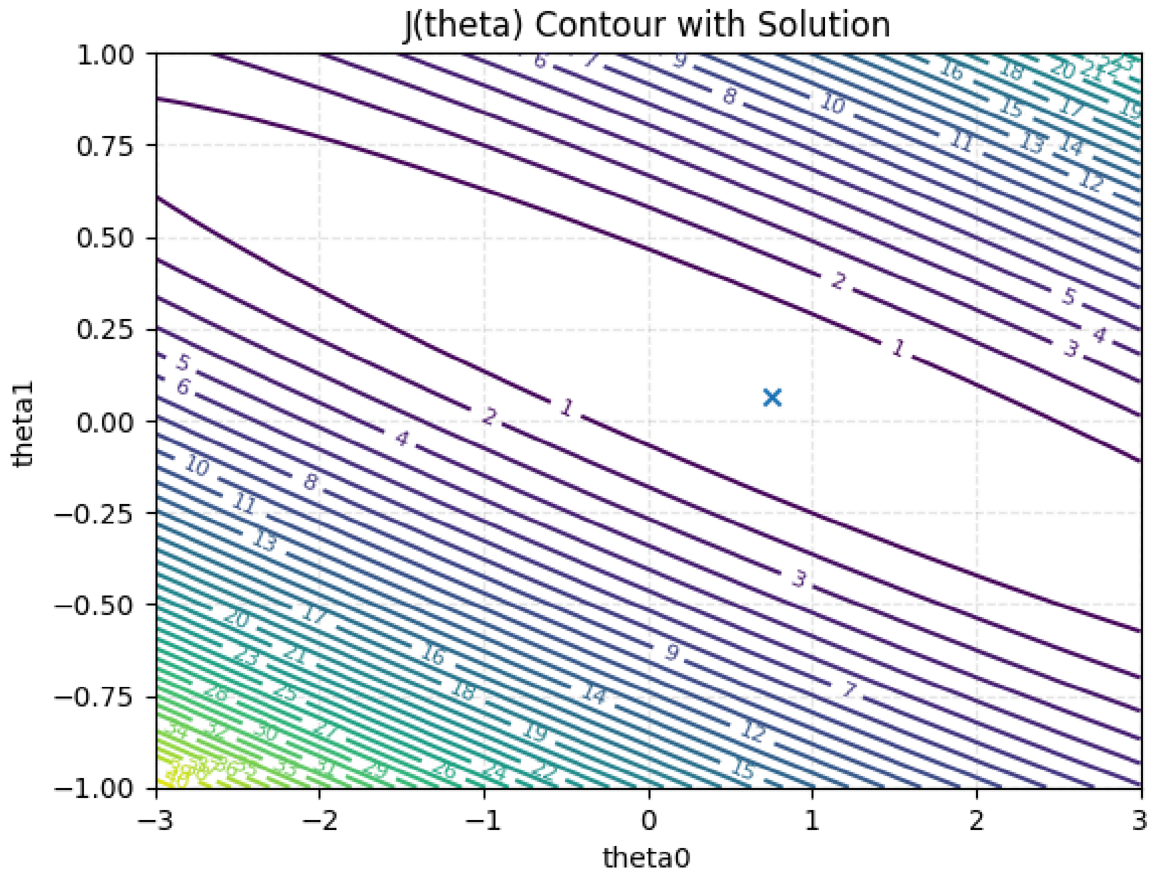
6. 3D Surface and Contours of the Cost $J(\theta)$

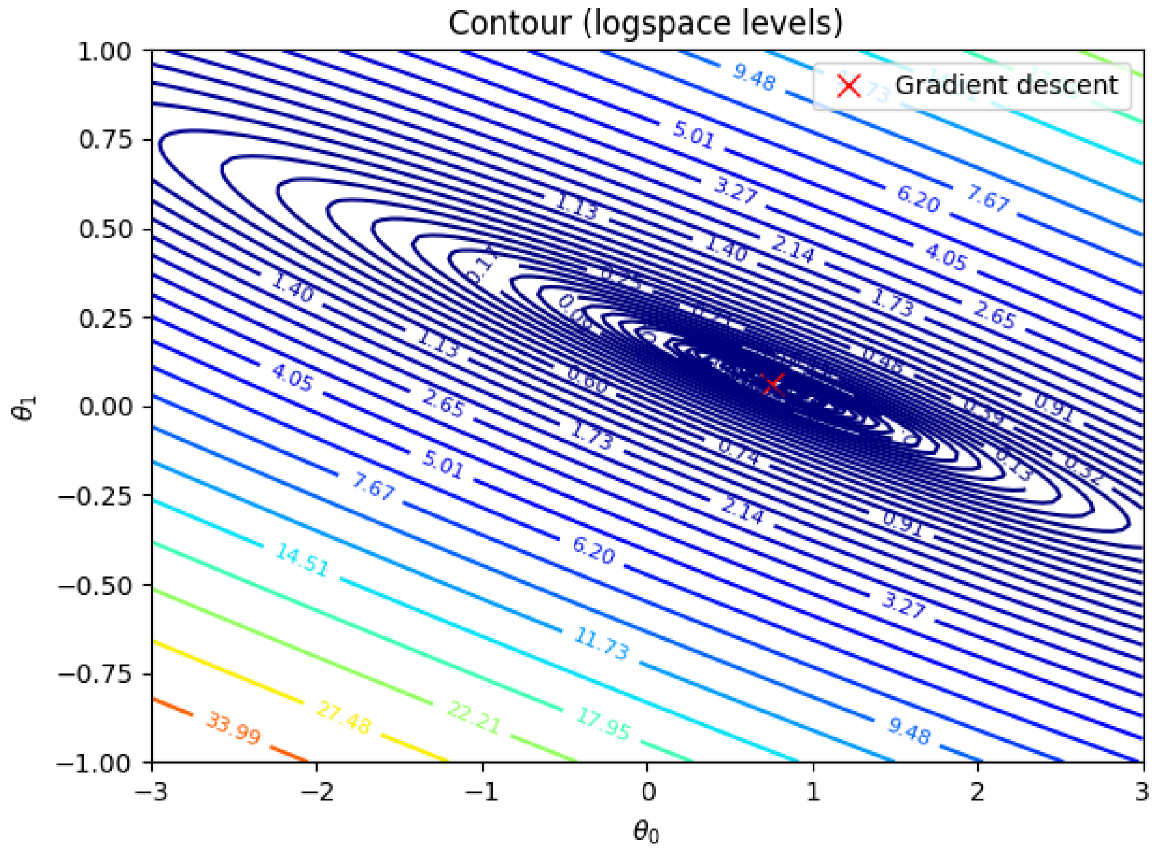
Construct a grid with $\theta_0 \in [-3, 3]$ and $\theta_1 \in [-1, 1]$, compute $J(\theta)$ at each grid point, and mark the location of the obtained optimum on the plots.

In [30]:
```
theta0_vals=np.linspace(-3,3,100)
theta1_vals=np.linspace(-1,1,100)
T0,T1=np.meshgrid(theta0_vals,theta1_vals)
X_b=add_intercept(X)
J_vals=np.zeros_like(T0)
for i in range(T0.shape[0]):
    for j in range(T0.shape[1]):
        th=np.array([[T0[i,j]],[T1[i,j]]])
        J_vals[i,j]=cost(X_b,y,th)

from mpl_toolkits.mplot3d import Axes3D

fig=plt.figure()
ax=fig.add_subplot(111, projection='3d')
ax.plot_surface(T0,T1,J_vals,rstride=3,cstride=3,linewidth=0.2,antialiased=True,
ax.view_init(elev=30,azim=-120)
ax.set_xlabel("theta0")
ax.set_ylabel("theta1")
ax.set_zlabel("J(theta)")
ax.set_title("J(theta) Surface")
plt.show()

plt.figure()
CS=plt.contour(T0, T1, J_vals, levels=50)
plt.clabel(CS,inline=True,fontsize=8)
plt.scatter([theta_final[0,0]],[theta_final[1,0]],marker='x')
plt.xlabel("theta0")
plt.ylabel("theta1")
plt.title("J(theta) Contour with Solution")
plt.grid(True,linestyle='--',alpha=0.3)
```

```
plt.show()

J=np.asarray(J_vals,float)
vmin=J[J>0].min()
vmax=J.max()
levels_log=np.geomspace(vmin, vmax, 50)
Z=np.clip(J,vmin,None)
plt.figure()
CS_log=plt.contour(theta0_vals,theta1_vals,Z,levels=levels_log,cmap='jet')
plt.clabel(CS_log,inline=True,fontsize=8)
plt.plot([theta_final[0,0]],[theta_final[1,0]],'rx',markersize=8,label='Gradient
plt.xlabel(r'$\theta_0$')
plt.ylabel(r'$\theta_1$')
plt.title("Contour (logspace levels)")
plt.legend(loc='upper right',frameon=True,fancybox=True)
plt.tight_layout()
plt.show()
```







7. Visualization and Discussion of the Cost Function

To build intuition about the behavior of gradient descent, construct a grid with $\theta_0 \in [-3, 3]$ and $\theta_1 \in [-1, 1]$ and compute $J(\theta)$:

- The **3D surface** shows the overall landscape of the cost function;
- The **contour plot** marks the gradient-descent solution (red “×”), which lies in the valley of the contours.

In this single-feature setting, the algorithm converges without feature scaling; however, in multi-feature problems, feature scaling can significantly speed up and stabilize convergence.

In addition, to better highlight details near the minimum (as suggested in the course handout), we add a **log-spaced contour plot**: logarithmically spaced contour levels allocate more emphasis to low-value regions, revealing the “valley floor” of $J(\theta)$ more clearly.