Finding $n_0$.

$f(n) = n^2 + n + 3$ where $g(n) = n^2$

So we have:
Equation from Big Oh Definition:
$0 <= f(n) <= cg(n)$ for all $n >= n_0$.

Find $N_0$ and c. They both depend on each other so we can simply choose c in this case.

$n^2 + n + 3 <= c\,(n^2)$

Let us pick c = 2.

f(n):
f(1): 1 + 1 + 3 = 5
f(2): 4 + 2 + 3 = 9
f(3): 9 + 3 + 3 = 15

cg(n):
cg(1): 2(1) = 2
cg(2): 2(4) = 8
cg(3): 2(9) = 18

$n_0 = 3$.

So in this case, there is a constant c (2) where $f(n) <= cg(n)$ for all $n >= n_0(3)$.

If we were to plot both equations we could see that when n = 1 and 2, function f(n) is higher than function cg(n). However, when n >= 3, we can see that cg(n) is higher than f(n). The definition of Big Oh states that this inequality will hold true for every $n >= n_0$. Since $n_0 = 3$, we can expect the behavior to continue for any size n when c is 2.

We could of chosen a different c, for example c = 4.

cg(n):
cg(1): 4(1) = 4
cg(2): 4(4) = 16
cg(3): 4(9) = 36

In this case, $n_0 = 2$ when c = 4.

STEPS:
1. Pick a c.
2. Make a table for f(n) and cg(n).
3. Find where cg(n) >= f(n). This is $n_0$.