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RIVER DEPLETION RESULTING FROM PUMPING A WELL NEAR A RIVER

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Abstract--A well adjacent to a river will take a portion of its supply from the river. A theoretical formula is developed which permits the draft on the river to be computed in terms of the distance of the well from the river, the properties of the aquifer, and time. The formula applies where the river can be considered to flow in a straight course which extends for a considerable distance both upstream and downstream from the well location.

When pumping of a well near a river begins, water is drawn, at first, from the water table in the immediate neighborhood of the well. As the zone of influence widens, however, it begins to draw a part of its flow from the river and, ultimately, the river supplies the entire flow. It is the purpose of this analysis to develop a formula for estimating the amount of flow drawn from the river at any time after pumping begins.

Notation:

- D saturated thickness of the water-bearing stratum or aquifer, feet
- K permeability, ft/sec
- Q flow of the well ft^3/sec
- q_1 flow crossing a straight line at a distance x_1 feet from the well if no river is present, ft^3/sec
- q the flow taken from a river at a distance x_1 feet from the well, ft^3/sec
- r a radius measured from the center of the well, feet
- s draw-down at the radius r at the time t, feet
- t time, from beginning of pumping, seconds
- V volume of water yielded by a horizontal square foot of the aquifer if the pressure is dropped one foot, dimensionless
- x and y rectangular coordinates measured from the center of the well, feet
- x_1 the distance of a well from a river, measured along a normal to the direction of flow. The river is assumed to extend indefinitely upstream and downstream from the well, feet
- $\alpha = KD/V$
- λ a time variable running between zero and t, seconds

The differential draw-down ds at the time t due to removal of the quantity of water $Qd\lambda$ at the time λ is [CARSLAW, 1921]

$$ds = [Q/4\pi KD(t-\lambda)]\, e^{-r^2/4\alpha(t-\lambda)}\, d\lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

This expression satisfies the continuity condition

$$\partial s/\partial t = \alpha\,[(\partial^2 s/\partial r^2) + (1/r)(\partial s/\partial r)] \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

and the condition that the draw-down is zero everywhere when $(t-\lambda) = 0$. Let

$$u = r/\sqrt{4\alpha(t-\lambda)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Then, by substitution and integration

$$s = (Q/2\pi KD)\int_{r/\sqrt{4\alpha t}}^{\infty} (e^{-u^2}/u)\, du \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The relation between the formula for well draw-down presented here with the one used by THEIS [1941] can be established by a simple change of variable. If, in the expression

$$s = (Q/4\pi KD)\int_{r^2/4\alpha t}^{\infty} (\epsilon^{-v}/v)\,dv$$

We make the substitution of variable

$$v = u^2$$

We obtain at once

$$s = (Q/2\pi KD)\int_{r/\sqrt{4\alpha t}}^{\infty} (\epsilon^{-u^2}/u)\,du$$

Eq. (4) is a form of the exponential integral [INGERSOLL and Others, 1948] which can be evaluated from tables [FEDERAL WORKS AGENCY, 1940; JAHNKE and EMDE, 1945] by use of the relation

$$\int_{r/\sqrt{4\alpha t}}^{\infty} (e^{-u^2}/u)\,du = -0.5\,\mathrm{Ei}\,(-r^2/4\,\alpha t) \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The draw-down produced in an aquifer of infinite extent due to a well pumped at the rate Q is given by (4). This expression is valid if s/D is small compared to unity.

We return now to (1) and set $r^2 = x^2 + y^2$. Then the flow of water across the line $x = x_1$ due to the withdrawal $Qd\lambda$ is obtained from the relation

$$\partial q_1/\partial\lambda = -\,KD\int_{-\infty}^{+\infty} (\partial^2 s/\partial x\,\partial\lambda)\,dy$$

$$= [2Qxe^{-x^2/4\alpha(t-\lambda)}/16\pi\alpha(t-\lambda)^2]\int_{-\infty}^{+\infty} e^{-y^2/4\alpha(t-\lambda)}\,dy \quad \ldots\ldots\ldots\ldots (6)$$

The integral in this last expression is a form of the probability integral [FEDERAL WORKS AGENCY, 1941; JAHNKE and EMDE, 1945; PEIRCE, 1929]. This permits an evaluation of this expression in the form

$$\partial q_1/\partial\lambda = 2Q\alpha x e^{-x^2/4\alpha(t-\lambda)}/\sqrt{\pi}\,[4\alpha(t-\lambda)]^{3/2} \quad \ldots\ldots\ldots\ldots (7)$$

To integrate this expression with respect to λ from $\lambda = 0$ to $\lambda = t$ set

$$v = x/\sqrt{4\alpha(t-\lambda)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Then, by substitution

$$q_1 = (Q/\sqrt{\pi})\int_{x/\sqrt{4\alpha t}}^{\infty} e^{-v^2}\,dv \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

This can be expressed in terms of the probability integral

$$P(Z) = (2/\sqrt{\pi})\int_0^{Z} e^{-v^2}\,dv \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

which has been extensively tabulated in terms of the upper limit Z. Then (9) can be put in the form

$$q_1/Q = 0.5\,[1 - P(x_1/\sqrt{4\alpha t})] \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

This formula implies that there will be a drop in the ground water level along the line $x = x_1$. If a river exists at this distance from the well there will be no drop in level but, instead, the river will supply an additional flow to the well. These factors can be accounted for if a recharge well is placed at the point where the pumped well is imaged by the riverbank. A similar form to (11) would apply to the recharge well also, with the result that the flow across the line $x = x_1$ is doubled. Then the total flow across the line if the river maintains the level is

$$q/Q = 1 - P(x_1/\sqrt{4\alpha t}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

These results may be summarized as follows: The draw-down in an aquifer of infinite extent due to a well discharging at the rate Q is given by (4). This formula is valid, as a first approximation, if the draw-down s is small compared to the depth D. A similar well located at the distance x_1 from a river will draw the flow q from the river. The part of the total flow of the well which comes from the river is q/Q. The value of this ratio is given by (12).

Suppose we have three wells at distances of 1000, 5000, and 10,000 ft from a river, respectively, which have been pumped for a period of five years. It is desired to estimate the part of their flow which comes from the river. If the aquifer data are: D = 100 ft, K = 0.001 ft/sec, V = 0.2, $\alpha = KD/V = 0.5$, t = 157,770,000 sec, and $\sqrt{4\alpha t}$ = 17, 763; then the values for the three wells are as shown in Table 1.

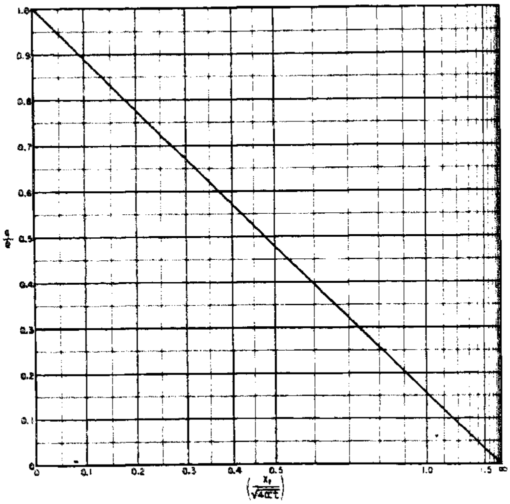

Fig. 1--The part of the well flow taken from the river as function of the parameter $x_1/\sqrt{4\alpha t}$

Table 1--Values for the three wells

Well	x_1	$x_1/\sqrt{4\alpha t}$	$P(x_1/\sqrt{4\alpha t})^a$	q/Q^b
1	1,000	0.0563	0.0635	0.9365
2	5,000	0.2815	0.3094	0.6906
3	10,000	0.5630	0.5741	0.4259

[a]From tables of the probability integral.
[b]From Eq. (12) or from Figure 1 directly.

Then at the end of five years the three wells are drawing, respectively, 93, 69, and 42 pct of their flow from the river.

While English units have been used here, the formulas given are valid when used in any consistent unit system. Such a system permits the use of only one unit of a kind. Since, in our case, only units of length and time are involved, change to another system of units can be accomplished by replacement of the foot and second units, used herein, with the new units.

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