Technical Note

Stream Depletion from Pumping a Semiconfined Aquifer in a Two-Layer Leaky Aquifer System

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Abstract: We consider the case of pumping a semiconfined aquifer overlain by a phreatic aquifer hydraulically connected to an infinitely long and straight stream. The hydraulics of the system are described by a linearized model in which it is assumed that flow in the aquifers is essentially horizontal, and that drawdown in the phreatic aquifer is small enough to allow the water table to be sufficiently well described by the Theis equation. Fourier-Laplace transforms of drawdown are obtained and numerically inverted. Comparisons are made with another well-known analytic model of Hunt, who considered a single aquifer/aquitard system. It is observed that while both models may have excellent agreement during the initial phase of pumping, the longer-term consequences of sustained pumping are quite distinct. **DOI: 10.1061/(ASCE)HE.1943-5584.0000382.** © *2011 American Society of Civil Engineers.*

CE Database subject headings: Groundwater management; Rivers and streams; Aquifers; Leakage.

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Introduction

Hunt (2009) considers the case of pumping an unconfined aquifer next to an infinitely long and straight stream underlain by a leaky aquifer. The intent of this paper was to demonstrate the effect of the underlying aquifer on long-term drawdown predictions and to contrast these predictions with Hantush-Jacob leaky aquifer models where the lower aquifer is assumed to have zero drawdown or infinite storage.

In practice, the case in which a well is screened in the lower aquifer is also frequently encountered, shown in plan and cross-sectional view in Fig. 1. This system is described by the following equations:

$$\mathcal{S}_1 \frac{\partial s_1}{\partial t} = T_1 \nabla^2 s_1 - \frac{K'}{B'}(s_1 - s_2) - \lambda \delta(x) s_1 \tag{1}$$

$$\mathcal{S}_2 \frac{\partial s_2}{\partial t} = T_2 \nabla^2 s_2 + \frac{K'}{B'}(s_1 - s_2) + Q\delta(x - L)\delta(y) \tag{2}$$

in which s_1 and s_2 denote drawdowns in the upper and lower aquifers, respectively; ∇^2 is the two-dimensional Laplacian; L is the distance of the pumped well from the stream and Q the constant well discharge rate; T_1 and $\mathcal{S}_1$ are transmissivity and specific yield, respectively, for aquifer 1; T_2 and $\mathcal{S}_2$ are transmissivity and storativity, respectively, for aquifer 2; the thickness and hydraulic conductivity for the aquitard are denoted by B' and K', respectively; and the streambed conductance λ is defined by

$$\lambda = \frac{K'' b}{B''} \tag{3}$$

where K'' is the hydraulic conductivity of the streambed, B'' is the streambed thickness, and b is the stream width.

Eqs. (1) and (2) are obtained by vertically averaging the three-dimensional groundwater flow equation, assuming that flow in the aquitards is essentially vertical and that drawdown in the phreatic aquifer (which is assumed to be incompressible) is sufficiently small to allow the free surface condition for the water table to be linearized. Therefore, flow in the aquitards becomes source terms for the equations for drawdown in the aquifers. A general discussion of multilayered models can be found in Bear (1979, Chapter 5).

This paper has two aims. The main aim is to deduce a semianalytic solution to Eqs. (1) and (2) over an infinite domain using standard transform techniques. The secondary aim is to compare the model with Boulton's delayed-yield model (Boulton 1954, 1955, 1963, 1973) and Hunt's stream depletion model (Hunt 2003). Several writers, notably Cooley and Case (1973), Boulton (1973), and Hunt (2003), showed that Boulton's semiempirical model described by a delay equation can be interpreted as horizontal flow in an aquifer underlain by an aquiclude and overlain by an aquitard containing a free surface whose equation has been linearized to allow only vertical motion. Using the same notation as above, Boulton's delay equation can be uncoupled and written as

$$\mathcal{S}_1 \frac{\partial s_1}{\partial t} = -\frac{K'}{B'}(s_1 - s_2) \tag{4}$$

$$\mathcal{S}_2 \frac{\partial s_2}{\partial t} = T_2 \nabla^2 s_2 + \frac{K'}{B'}(s_1 - s_2) + Q\delta(x - L)\delta(y) \tag{5}$$

It is not uncommon for s_1 to be used to model drawdown in an overlying water table aquifer separated from the pumped aquifer by an aquitard as in Fig. 1, in which case K'/B' must be interpreted as an effective vertical aquitard conductance taking into account both the aquitard and the surface aquifer. Thus, in the absence of stream depletion, Eqs. (1) and (2) may be viewed as a extension

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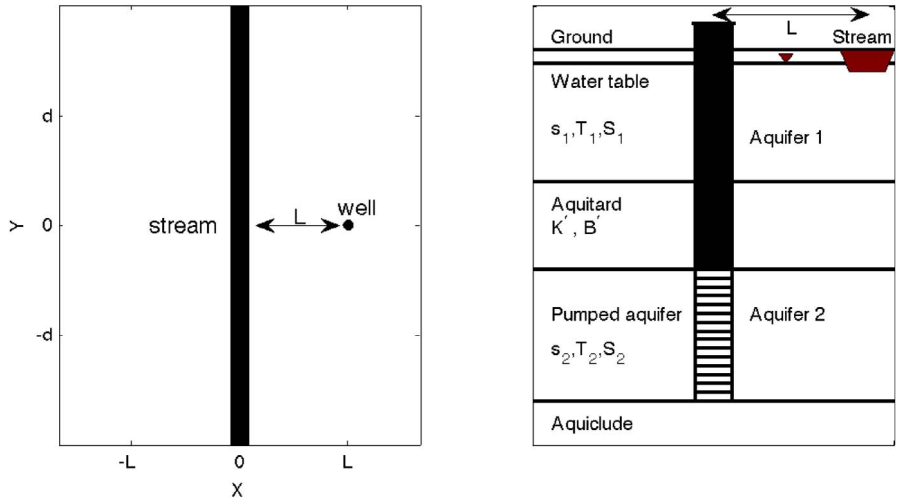

Fig. 1. Plan and cross-sectional views of model described by Eqs. (1) and (2)

of Boulton's model, in which horizontal flow is allowed in the water table aquifer. Such two-layer aquifer systems and generalization to multilayers have been considered by Hemker (1984, 1985, 1999a, 1999b), Hemker and Mas (1987), Meesters et al. (2004), and Hunt and Scott (2007), among others.

Hunt (2003) considered a stream depletion model by adding a stream depletion term to Eq. (5):

$$\mathcal{S}_2 \frac{\partial s_2}{\partial t} = T_2 \nabla^2 s_2 + \frac{K'}{B'}(s_1 - s_2) + Q\delta(x-L)\delta(y) - \lambda\delta(x)s_2 \quad (6)$$

Fig. 2 shows a cross-sectional view of Hunt's model. Note that there is an essential difference with Eqs. (1) and (2) because in Hunt (2003), the stream depletion term is directly connected to the pumped aquifer via Eq. (6), whereas the effects in Eqs. (1) and (2) are transmitted to the pumped and overlying aquifers via the water table [Eq. (1)]. Therefore, λ in Eq. (6) must be interpreted as an effective streambed conductance λ_{eff} taking into account the streambed conductance and the aquitard vertical permeability:

$$\lambda_{\text{eff}} = \frac{b}{\frac{B'}{K'} + \frac{B''}{K''}} \quad (7)$$

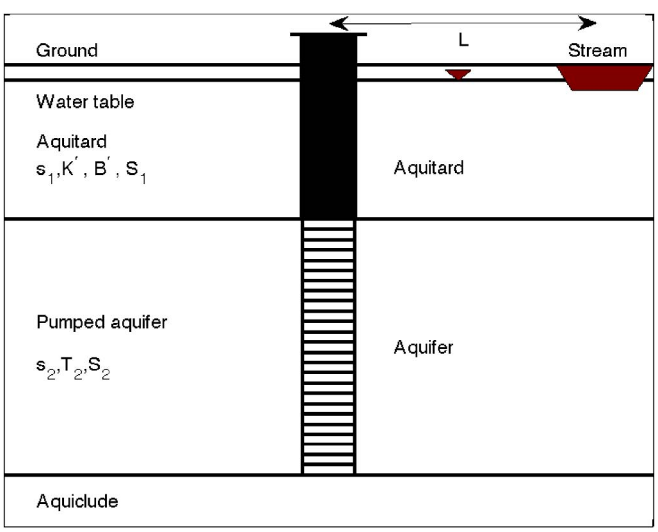

Fig. 2. Cross-sectional view of Hunt's single aquifer/aquitard model (Hunt 2003)

It is helpful to nondimensionalize Eqs. (1) and (2) using the following variables:

$$(s_1^*, s_2^*, x^*, y^*, t^*, T_1^*, S_1^*, K^*, \lambda^*) = \left(\frac{s_1 T_2}{Q}, \frac{s_2 T_2}{Q}, \frac{x}{L}, \frac{y}{L}, \frac{t T_2}{\mathcal{S}_2 L^2}, \frac{T_1}{T_2}, \frac{\mathcal{S}_1}{\mathcal{S}_2}, \frac{(K'/B')L^2}{T_2}, \frac{\lambda L}{T_2}\right)$$

This gives the following nondimensional versions of Eqs. (1) and (2):

$$\mathcal{S}_1 \frac{\partial s_1}{\partial t} = T_1 \nabla^2 s_1 - K(s_1 - s_2) - \lambda\delta(x)s_1 \quad (8)$$

$$\frac{\partial s_2}{\partial t} = \nabla^2 s_2 + K(s_1 - s_2) + \delta(x-1)\delta(y) \quad (9)$$

where we have dropped the primes for the sake of convenience.

Except in the details, the method of solution is identical to Hunt (2009). Eqs. (8) and (9) are solved in Fourier-Laplace space, subject to the usual initial condition of zero drawdown in both aquifers at $t = 0$ and boundary condition of zero drawdown at infinity. The Fourier-Laplace solution may then be inverted numerically using standard quadrature formulas for the inverse Fourier transform and the Stehfest transform for the inverse Laplace transform. To obtain numerically stable solutions with this recipe, the Fourier transform needs to be inverted first. The requisite formulas are given in the appendix.

Results and Discussion

We illustrate the behavior of the model described in this paper for a hydrogeological setting with nondimensional variables of $x = 0.5$, $y = 1.0$, $\mathcal{S}_1 = 1{,}000$, $K = 0.1$, and $\lambda = 0.1$. Fig. 3 shows a typical dimensionless plot of drawdown versus time for varying T_1. The drawdown curves shown for the pumped aquifer exhibit the usual four segments: (1) an initial segment controlled primarily by elastic storage release from and horizontal flow within the semiconfined aquifer and similar to the Theis equation (Theis 1935) for fully confined flow, (2) a second segment of pseudo-steady-state drawdown controlled primarily by leakage from the upper aquifer to the lower aquifer, (3) a departure from the steady-state flow controlled primarily by the specific yield of and horizontal flow within the

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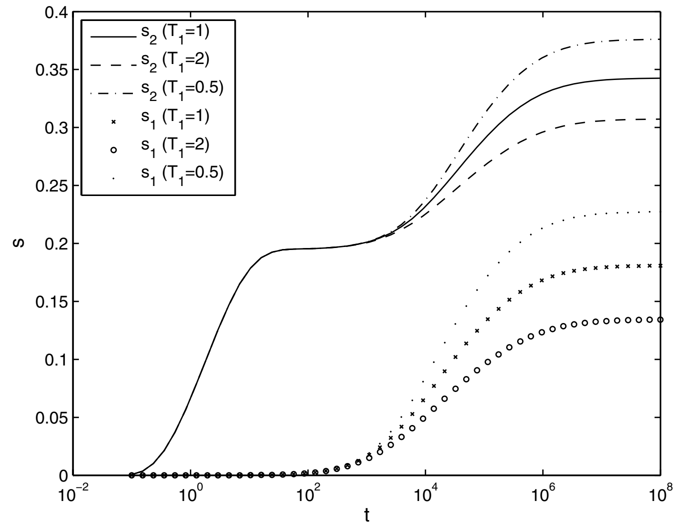

Fig. 3. Illustration of drawdown behavior and dependence on T_1 [with $x = 0.5$, $y = 1$; $S_1 = 1{,}000$, $K = 0.1$, $\lambda = 0.1$, in Eqs. (8) and (9)]

phreatic aquifer, and (4) steady-state drawdown controlled primarily by flow depletion from the stream.

Fig. 3 shows the effect that T_1 has on the behavior of the solution. The slope of the third segment is inversely proportional to T_1. As T_1 decreases, the segment steepens, and the drawdown curves for the pumped aquifer and at the water table remain parallel through this segment, although the difference between them decreases. This behavior is also illustrated in Hunt and Scott (2007).

Fig. 4 provides a comparison with the behavior of the Hunt (2003) stream depletion model for $T_1 = 1$, where $\lambda_{\text{eff}} = 0.001$ and $\epsilon = 1/S_1$ in Hunt (2003). For this example, there is excellent agreement between the two models for $t \leq 10^3$. As T_1 decreases, the time range of agreement between the two models increases, as shown in Fig. 5. However, in general, the longer-term predictions between the two models are quite distinct: the Hunt model (Hunt 2003) will predict larger longer-term drawdowns than the current model, even though both models may give excellent fits to shorter-term pump test data. This divergence is due to an essential difference between the two models because in the model described by Eqs. (1) and (2), the stream depletion is a function of drawdown of the water table in a neighborhood of the stream and the streambed conductance, whereas in the solution of Hunt (2003), stream depletion is a function of drawdown in the pumped semiconfined aquifer beneath the stream and the effective streambed conductance. As a result, the effects of the stream are transmitted directly to drawdown in the pumped aquifer along a line in Hunt (2003), whereas in Eqs. (1) and (2), they are transmitted aerially via the water table. This disparity between the models is further magnified in the total stream depletion predictions and is shown in Fig. 6.

These comparisons show that the Hunt (2003) stream depletion model may give very different predictions of long-term drawdown and stream depletion in situations where horizontal flow in a phreatic aquifer overlying a pumped aquifer is not insignificant. Furthermore, they show that stream depletion effects may develop more rapidly from pumping a semiconfined aquifer for the case

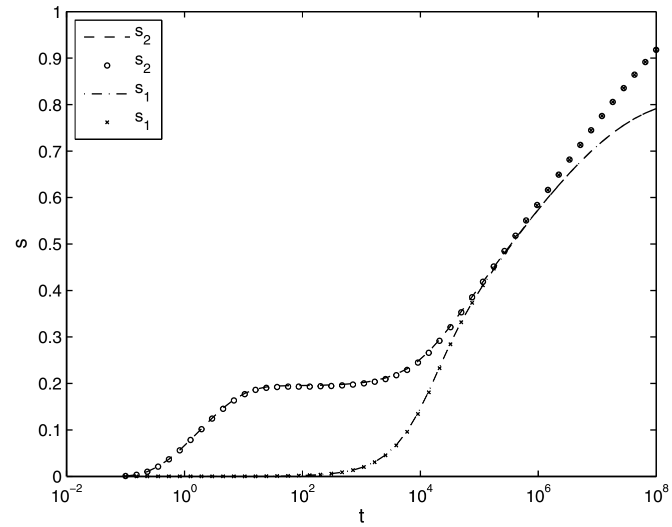

Fig. 5. Comparison of drawdowns with Hunt (2003) for $T_1 = 0.0001$ [with $x = 0.5$, $y = 1$; $S_1 = 1{,}000$, $K = 0.1$, $\lambda = 0.1$, in Eqs. (8) and (9); $\epsilon = 1/S_1$, $K = 0.1$, and $\lambda_{\text{eff}} = 0.001$ in Hunt (2003)]

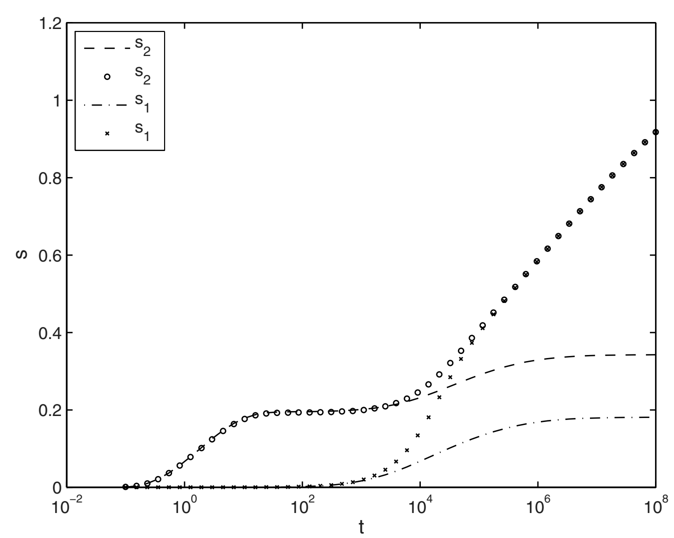

Fig. 4. Comparison with Hunt (2003) for $T_1 = 1$ [with $x = 0.5$, $y = 1$; $S_1 = 1{,}000$, $K = 0.1$, $\lambda = 0.1$, in Eqs. (8) and (9); $\epsilon = 1/S_1$, $K = 0.1$, and $\lambda_{\text{eff}} = 0.001$, in Hunt (2003)]

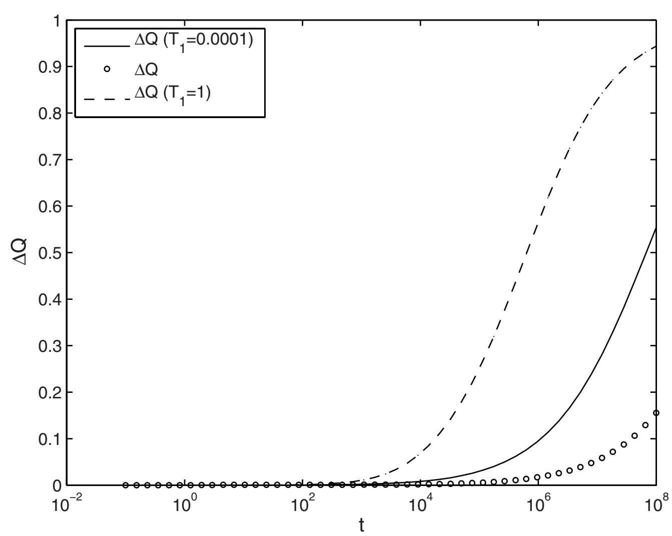

Fig. 6. Comparison of stream depletion with Hunt (2003) for varying T_1 [with $T_1 = 1$ or 0.0001, $S_1 = 1{,}000$, $K = 0.1$, $\lambda = 0.1$, in Eqs. (8) and (9); $\epsilon = 1/S_1$, $K = 0.1$, and $\lambda_{\text{eff}} = 0.001$, in Hunt (2003)]

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where there is significant horizontal flow in an overlying phreatic aquifer than for the case where the aquifer is overlain by an aquitard with a much lower hydraulic conductivity than the pumped aquifer.

Conclusions

The main aim of this paper was to give an account of a two-layer aquifer system in which the upper phreatic aquifer is hydraulically connected to an infinitely long stream and the lower semiconfined aquifer is pumped at a constant discharge rate Q. The model equations are obtained by vertically integrating the general groundwater flow equation under the assumptions that the aquitard connecting the aquifers induces essentially vertical flow between the aquifers, that flow in the aquifers is essentially horizontal, and that drawdown in the overlying aquifer is sufficiently small to allow the water table to be sufficiently well described by the Theis equation (an account of this can be found in Bear 1979).

This work extends a result of Hunt (2009), who considers the same system in which the upper aquifer is pumped. The Fourier-Laplace transform of drawdown formulas for both aquifers are obtained and integrated using standard quadrature formulas (e.g., in the standard Matlab arsenal). Drawdown and stream depletion predictions were compared with Hunt (2003) and show that while both models may give extremely good fits to pump test data, the effect of allowing horizontal flow in the phreatic aquifer may reduce the longer-term drawdown predictions in both the pumped and the overlying aquifer and increase the total stream depletion.

Appendix

To solve Eqs. (8) and (9), we appeal to transform theory and take a Fourier-Laplace transform $\mathcal{FL}:(y,t)\mapsto(\theta,p)$, where

$$\mathcal{FL}(f)(\theta,p)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{0}^{\infty}e^{-pt+i\theta y}f(y,t)dtdy \tag{10}$$

to give

$$\mathcal{S}_1p\bar{\hat{s}}_1=T_1\left(\frac{\partial^2\bar{\hat{s}}_1}{\partial x^2}-\theta^2\bar{\hat{s}}_1\right)-K(\bar{\hat{s}}_1-\bar{\hat{s}}_2)-\lambda\delta(x)\bar{\hat{s}}_1 \tag{11}$$

$$p\bar{\hat{s}}_2=\frac{\partial^2\bar{\hat{s}}_2}{\partial x^2}-\theta^2\bar{\hat{s}}_2+K(\bar{\hat{s}}_1-\bar{\hat{s}}_2)+\frac{\delta(x-1)}{2\pi p} \tag{12}$$

where $\hat{s}$ denotes the Fourier transform and $\bar{s}$ denotes the Laplace transform of s. (For background on transform techniques, see Davis 2002; Duffy 2004; Schiff 1999.) In matrix notation, Eqs. (11) and (12) may be written as

$$A\frac{\partial^2\bar{\hat{\mathbf{s}}}}{\partial x^2}-B\bar{\hat{\mathbf{s}}}=\lambda\delta(x)C\bar{\hat{\mathbf{s}}}-\frac{\delta(x-1)}{2\pi p}D \tag{13}$$

where

$$A=\begin{pmatrix}T_1 & 0\\ 0 & 1\end{pmatrix}\qquad B=\begin{pmatrix}T_1\theta^2+S_1p+K & -K\\ -K & \theta^2+p+K\end{pmatrix}$$

$$C=\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\qquad D=\begin{pmatrix}0\\ 1\end{pmatrix}$$

and

$$\bar{\hat{\mathbf{s}}}=\begin{pmatrix}\bar{\hat{s}}_1\\ \bar{\hat{s}}_2\end{pmatrix}$$

We first consider the homogeneous problem

$$A\frac{\partial^2\bar{\hat{\mathbf{s}}}}{\partial x^2}-B\bar{\hat{\mathbf{s}}}=\mathbf{0} \tag{14}$$

and consider elementary solutions of the form

$$\bar{\hat{\mathbf{s}}}=\begin{pmatrix}\mu\\ \nu\end{pmatrix}e^{\pm\sqrt{\gamma}x}$$

Substituting into Eq. (14) gives the generalized eigenvalue problem

$$(\gamma A-B)\mathbf{e}=\mathbf{0} \tag{15}$$

This has a nontrivial solution, provided that $\det(\gamma A-B)=0$, or, writing

$$\begin{aligned}&b_{11}=T_1\theta^2+\mathcal{S}_1p+K\qquad b_{12}=b_{21}=-K\\ &b_{22}=\theta^2+p+K\end{aligned} \tag{16}$$

gives

$$\gamma_i=\frac{1}{2}\left(\frac{b_{11}}{T_1}+b_{22}\right)\pm\sqrt{\frac{1}{4}\left(\frac{b_{11}}{T_1}+b_{22}\right)^2+\frac{(b_{12}^2-b_{11}b_{22})}{T_1}} \tag{17}$$

with corresponding choice of eigenvectors

$$\mathbf{e}_i=\begin{pmatrix}1\\ \frac{\gamma_iT_1-b_{11}}{b_{12}}\end{pmatrix} \tag{18}$$

Note that $\mathbf{e}_i$ satisfies the orthogonality condition

$$\mathbf{e}_i^TA\mathbf{e}_j=\begin{cases}0, & i\neq j\\ l_j^2, & i=j\end{cases} \tag{19}$$

where

$$l_i^2=T_1+\left(\frac{\gamma_iT_1-b_{11}}{b_{12}}\right)^2 \tag{20}$$

To solve the inhomogenous system in Eqs. (11) and (12), we construct a solution $\bar{\hat{\mathbf{s}}}$ of the form

$$\bar{\hat{\mathbf{s}}}=F_1(x)\mathbf{e}_1+F_2(x)\mathbf{e}_2 \tag{21}$$

where F_i is a linear combination of the elementary solutions $e^{\pm\sqrt{\gamma_i}x}$ and $Ae^{\sqrt{\gamma_i}x}+Be^{-\sqrt{\gamma_i}x}$ and A and B are step functions with jumps at $x=0$, 1, chosen so that F is everywhere continuous, decays at ∞, and satisfies the following jump conditions:

$$\frac{\partial F_i}{\partial x}(1^+)-\frac{\partial F_i}{\partial x}(1^-)=-\frac{1}{2\pi pl_i^2}\left(\frac{\gamma_iT_1-b_{12}}{b_{12}}\right)=-\frac{\beta_i}{l_i^2} \tag{22}$$

where

$$\beta_i=\frac{1}{2\pi p}\left(\frac{\gamma_iT_1-b_{12}}{b_{12}}\right) \tag{23}$$

and

$$\frac{\partial F_i}{\partial x}(0^+)-\frac{\partial F_i}{\partial x}(0^-)=\frac{\lambda}{l_i^2}(F_1(0)+F_2(0)) \tag{24}$$

Eqs. (22) and (24) follow by substituting Eq. (21) into Eq. (13), using the eigen-equation [Eq. (15)] and the orthogonality condition [Eq. (19)] and then integrating the resulting equation about a small neighborhood of $x=1$ and $x=0$, respectively. Assuming that $\Re(\sqrt{\gamma_i})>0$, we obtain

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$$F_i(x) = \begin{cases} A_i e^{x\sqrt{\gamma_i}} & x < 0 \\ A_i e^{-x\sqrt{\gamma_i}} + \frac{\beta_i}{2\sqrt{\gamma_i} l_i^2}\left(e^{(x-1)\sqrt{\gamma_i}} - e^{-(x+1)\sqrt{\gamma_i}}\right) & 0 \le x \le 1 \\ A_i e^{-x\sqrt{\gamma_i}} + \frac{\beta_i}{2\sqrt{\gamma_i} l_i^2}\left(e^{(1-x)\sqrt{\gamma_i}} - e^{-(x+1)\sqrt{\gamma_i}}\right) & x > 1 \end{cases} \tag{25}$$

where

$$A_1 = \frac{1}{l_1^2 \Delta}\left[\left(\frac{\lambda\beta_2}{l_2^2} + 2\sqrt{\gamma_2}\right)\beta_1 e^{-\sqrt{\gamma_1}} - \frac{\lambda\beta_2}{l_2^2} e^{-\sqrt{\gamma_2}}\right] \tag{26}$$

$$A_2 = \frac{1}{l_2^2 \Delta}\left[\frac{-\lambda\beta_1}{l_1^2} e^{-\sqrt{\gamma_1}} + \left(\frac{\lambda}{l_1^2} + 2\sqrt{\gamma_1}\right)\beta_2 e^{-\sqrt{\gamma_2}}\right] \tag{27}$$

and

$$\Delta = 4\sqrt{\gamma_1\gamma_2} + 2\lambda\left(\frac{\sqrt{\gamma_1}}{l_2^2} + \frac{\sqrt{\gamma_2}}{l_1^2}\right) \tag{28}$$

The total stream depletion ΔQ is given by

$$\Delta Q = \lambda \int_{-\infty}^{\infty} s_1(0, y, t) dy = 2\pi\lambda \hat{s}_1(0, 0, t)$$

Therefore, the Laplace transform of ΔQ is given by

$$\Delta \bar{Q} = 2\pi\lambda \bar{\hat{s}}_1(0, 0, p) \tag{29}$$

Acknowledgments

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Notation

The following symbols are used in this paper:

L = distance of well from stream;
Q = pumping rate;
$s_1 = s_1(x, y, t)$ = drawdown in aquifer 1;
$s_2 = s_2(x, y, t)$ = drawdown in aquifer 2;
$\mathcal{S}_1$ = specific yield in aquifer 1;
$\mathcal{S}_2$ = storativity in aquifer 2;
T_1 = transmissivity in aquifer 1;
T_2 = transmissivity in aquifer 2;
K' = aquitard hydraulic conductivity;
B' = aquitard thickness;
λ = streambed conductance;
λ_{eff} = effective streambed conductance;

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