Unsteady Stream Depletion from Ground Water Pumping

by Bruce Hunt[a]

Abstract

A solution is obtained for stream flow depletion created by pumping from a well beside a stream. This solution assumes that streambed penetration of the aquifer and dimensions of the streambed cross section are all relatively small. It also assumes that the streambed is clogged and that a linear relationship exists between the outflow seepage through the streambed and the change in piezometric head across the semipervious clogging layer. The solution is general enough to include the earlier solutions of Theis, Glover and Balmer, and Hantush. A solution is also obtained for the drawdown at any point within the aquifer, and it is suggested that this solution might be matched with experimental field data to obtain estimates for aquifer and streambed leakage parameters.

Introduction

Pumping from a well beside a stream lowers ground water levels and reduces surface water flow within the stream. In smaller streams this decrease in flow can be large enough to create harmful effects upon the stream and its wildlife. An understanding of the interaction between ground and surface water in this problem would allow engineers to specify well locations and pumping schedules that would minimize these harmful effects.

The first unsteady solution for this problem was obtained by Theis (1941). As shown in Figure 1, the river edge was modeled as an infinitely long straight line with zero drawdown, the stream was assumed to completely penetrate a homogeneous aquifer, and changes in free surface elevations were assumed to be small enough to allow use of the linearized form of the equations that are derived from the Dupuit approximation. Theis (1941) obtained the solution in the form of an integral, which he evaluated with an infinite series. Thirteen years later, this same solution was rewritten by Glover and Balmer (1954) in terms of the complimentary error function, erfc, as follows:

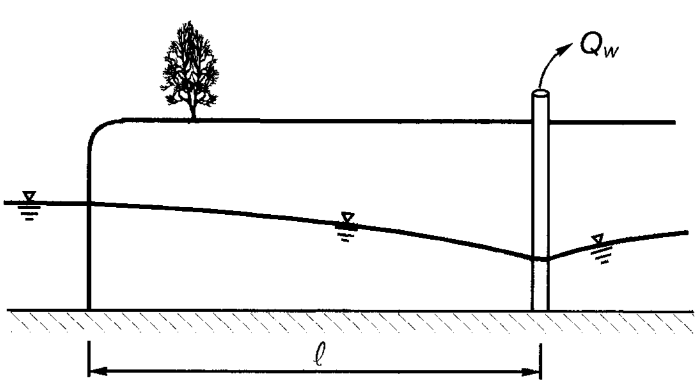

Figure 1. The problem considered by Theis (1941).

$$\frac{\Delta Q}{Q_w} = \text{erfc}\left(\sqrt{\frac{S\ell^2}{4T\,t}}\right) \tag{1}$$

where ΔQ is the stream depletion flow rate; Q_w is the constant flow rate abstracted at the well from $t = 0$ to $t = \infty$; S is the aquifer storage coefficient, specific yield, or effective porosity; T is the aquifer transmissivity; t is time; and ℓ is the shortest distance between the well and stream edge.

A second problem was solved by Hantush (1965) for a streambed lined with semipervious material. This problem differed from the problem considered by Theis (1941) only by the inclusion of a vertical layer of semipervious material along the stream edge, as shown in Figure 2. The Hantush solution is given by

$$\frac{\Delta Q}{Q_w} = \text{erfc}\left(\sqrt{\frac{S\ell^2}{4T\,t}}\right) - \exp\left(\frac{T\,t}{SL^2} + \frac{\ell}{L}\right)\text{erfc}\left(\sqrt{\frac{T\,t}{SL^2}} + \sqrt{\frac{S\ell^2}{4T\,t}}\right) \tag{2}$$

where L is a stream leakance that has dimensions of length and is defined as a combination of the aquifer permeability, K, the permeability, K′, and thickness, b′, of the semipervious layer.

$$L = \frac{K}{K'}\,b' \tag{3}$$

Equation 2 reduces to Equation 1 when $L \to 0$, which is equivalent to letting $b' \to 0$ for a fixed value of K/K′. Hantush (1965) also suggested that the effects of partial stream penetration might be approximated by using an "effective" value of ℓ that exceeds the measured value of ℓ in the physical problem. A solution obtained herein will show that this approximation is not correct.

Jenkins (1968) elaborated on some of the specific numerical details that are needed when applying Equation 1 to specific prob-

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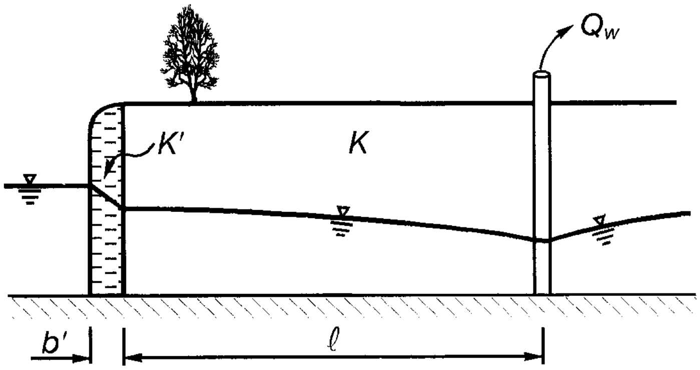

Figure 2. The problem considered by Hantush (1965).

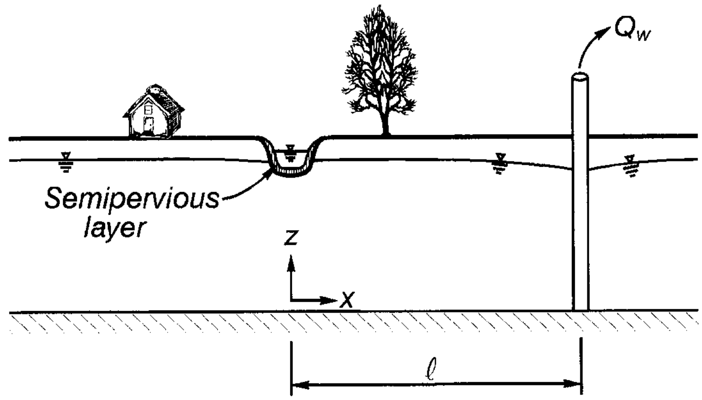

Figure 3. Definition sketch for the problem considered herein.

lems. In particular, superposition and time translation were used to calculate solutions from Equation 1 for intermittent pumping schedules. Wallace et al. (1990) carried out a similar analysis for cyclic pumping of wells. Spalding and Khaleel (1991) and Sophocleous et al. (1995) used numerical models to assess the result of simplifying assumptions that were made to obtain Equations 1 and 2. Streambed clogging and partial stream penetration were found to create some of the more significant errors in applications of Equations 1 and 2.

A solution will be obtained herein that considers the effects of streambed clogging and partial stream penetration. A definition sketch for this problem is shown in Figure 3. The calculation of this solution will assume that

- The ratio of vertical to horizontal velocity components is small (the Dupuit approximation)
- The aquifer is of infinite extent and is homogeneous and isotropic in all horizontal directions
- Drawdowns are small enough compared with saturated aquifer thicknesses to allow the governing equations to be linearized
- The streambed cross section has horizontal and vertical dimensions that are small compared to the saturated aquifer thickness, and the stream extends from $y = -\infty$ to $y = \infty$ along $x = 0$
- The well flow rate, Q_w, is constant for $0 < t < \infty$
- Changes in water surface elevation in the river created by pumping are small compared with changes created in the water table elevation on the aquifer side of the semipervious layer
- Seepage flow rates from the river into the aquifer are linearly proportional to the change in piezometric head across the semipervious layer.

Problem Formulation

The first three assumptions listed in the previous paragraph lead to the following partial differential equation (e.g., Hunt 1983):

$$T\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right) = S\frac{\partial h}{\partial t} + w \tag{4}$$

where h is the water table elevation above the horizontal (x, y) plane and w is the specific discharge at the free surface. The Dupuit approximation assumes that the ratio of vertical to horizontal velocity is small, but vertical velocities at the top and bottom boundaries must be included in the continuity equation to ensure that mass is conserved. For the problem under consideration, w is given by

$$w = Q_w\,\delta(x - \ell)\,\delta(y) - \lambda(H - h)\,\delta(x) - R \tag{5}$$

where δ is the Dirac delta function; Q_w is the constant well flow rate; H is the elevation of the free surface in the river; R is the specific discharge from rainfall and/or irrigation; and λ is a constant of proportionality between the seepage flow rate per unit distance (in the y direction) through the streambed and the difference between river and ground water levels at $x = 0$. Thus, the integration of w in Equation 5 over the streambed surface gives the total outflow seepage through the streambed.

$$\Delta Q = \lim_{\epsilon \to 0} \int_{-\infty}^{\infty}\int_{-\epsilon}^{\epsilon} w\,dx\,dy = -\lambda\int_{-\infty}^{\infty} [H - h(0, y, t)]\,dy \tag{6}$$

Equations 4 through 6 can also be written for the piezometric head, $h_0(x, y, t)$, that would exist if $Q_w = 0$. Then, since T, S, λ, R, and H have the same values in both sets of equations, substraction of corresponding equations leads to the following set of equations with the drawdown, ϕ, as the new dependant variable:

$$T\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) = S\frac{\partial \phi}{\partial t} - Q_w\,\delta(x - \ell)\,\delta(y) + \lambda\phi\delta(x) \tag{7}$$

$$\Delta Q = \lambda\int_{-\infty}^{\infty} \phi(0, y, t)dy \tag{8}$$

where $\phi = h_0(x, y, t) - h(x, y, t)$. The constant λ has units of velocity, and $\phi(x, y, t)$ and ΔQ are the drawdown and stream depletion rate, respectively, caused by pumping from the well. In other words, the linearity of Equations 4 through 6 has allowed an application of the superposition principle to isolate the effects of well pumping from all other perturbations in the system. The problem formulation is completed by adding an initial condition and a boundary condition at infinity.

$$\phi(x, y, 0) = 0 \tag{9}$$

$$\lim_{(x^2 + y^2) \to \infty} \phi(x, y, t) = 0 \tag{10}$$

The solution domain for this problem consists of the entire (x, y) plane, which contrasts with the semi-infinite domain used for the solutions obtained by Theis (1941), Glover and Balmer (1954), and Hantush (1965).

Problem Solution

If the Laplace transform of ϕ is denoted by

$$\bar{\phi}(x, y, p) = \int_0^{\infty} \phi(x, y, t)\, e^{-pt}\, dt \tag{11}$$

and if the Fourier transform of $\bar{\phi}$ is denoted by

$$\bar{\Phi}(x, \alpha, p) = \int_{-\infty}^{\infty} \bar{\phi}(x, y, p)\, e^{i\alpha y}\, dy \tag{12}$$

then the successive use of these two transforms on Equations 7 and 10 and use of the initial condition (Equation 9) gives an ordinary differential equation and boundary conditions for $\bar{\Phi}$.

$$\frac{d^2\bar{\Phi}}{dx^2} - \beta^2\bar{\Phi} = -\frac{Q_w}{pT}\delta(x - \ell) + \frac{\lambda}{T}\bar{\Phi}\,\delta(x) \tag{13}$$

$$\lim_{x \to \pm\infty} \bar{\Phi}(x, \alpha, p) = 0 \tag{14}$$

The parameter β in Equation 13 is given by

$$\beta = \sqrt{\alpha^2 + pS/T} \tag{15}$$

Since $\delta(x - \ell)$ and $\delta(x)$ vanish when x is not equal to ℓ and zero, respectively, and since $\bar{\Phi}$ vanishes at both plus and minus infinity, the solution of Equations 13 and 14 is given by

$$\begin{aligned} \bar{\Phi} &= C_1 e^{\beta x} && (-\infty < x < 0) \\ &= C_2\cosh(\beta x) + C_3\sinh(\beta x) && (0 < x < \ell) \\ &= C_4 e^{-\beta x} && (\ell < x < \infty) \end{aligned} \tag{16}$$

By integrating Equation 13 with respect to x across two narrow regions containing x = 0 and x = ℓ, we see that $\bar{\Phi}$ is continuous at these two points and that $d\bar{\Phi}/dx$ has discontinuities of $\lambda\bar{\Phi}(0, \alpha, p)/T$ and $-Q_w/(pT)$ at x = 0 and x = ℓ, respectively. These four conditions lead to the following solution for C_i:

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \frac{Q_w}{pT(2\beta + \lambda/T)} \begin{bmatrix} e^{-\beta\ell} \\ e^{-\beta\ell} \\ \left(1 + \dfrac{\lambda}{\beta T}\right) e^{-\beta\ell} \\ \cosh(\beta\ell) + \left(1 + \dfrac{\lambda}{\beta T}\right)\sinh(\beta\ell) \end{bmatrix} \tag{17}$$

The inversion of Equation 16 to calculate $\phi(x, y, t)$ is a relatively difficult problem. The stream depletion, however, can be obtained with far less difficulty. Taking the Laplace transform of Equation 8 gives

$$\overline{\Delta Q} = \lambda\int_{-\infty}^{\infty} \bar{\phi}(0, y, p)\, dy \tag{18}$$

Therefore, setting x = α = 0 in Equation 12 and comparing the result with the right side of Equation 18 shows that

$$\overline{\Delta Q} = \lambda\,\bar{\Phi}(0, 0, p) = \frac{Q_w\, e^{-\ell\sqrt{pS/T}}}{p\left(1 + \dfrac{2}{\lambda}\sqrt{pST}\right)} \tag{19}$$

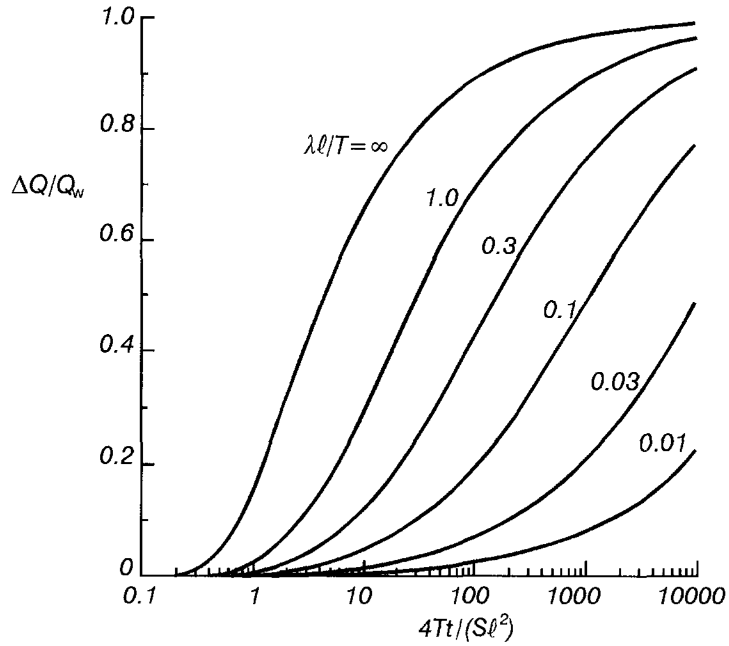

Figure 4. A plot of the solution given by Equation 20.

where Equations 16 and 17 have been used to calculate $\Phi(0, 0, p)$. Thus, calculation of ΔQ only requires inversion of a Laplace transform. In fact, an inversion formula given in Abramowitz and Stegun (1964) allows Equation 19 to be inverted in the following form:

$$\frac{\Delta Q}{Q_w} = \mathrm{erfc}\left(\sqrt{\frac{S\ell^2}{4Tt}}\right) - \exp\left(\frac{\lambda^2 t}{4ST} + \frac{\lambda\ell}{2T}\right)\mathrm{erfc}\left(\sqrt{\frac{\lambda^2 t}{4ST}} + \sqrt{\frac{S\ell^2}{4Tt}}\right) \tag{20}$$

The right side of Equation 20 approaches zero and one as t approaches zero and infinity, respectively.

A comparison of Equations 2 and 20 shows that they have a remarkably similar form. In particular, Equations 2 and 20 become identical if we choose

$$L = 2\frac{T}{\lambda} \tag{21}$$

In other words, the Hantush solution for a fully penetrating streambed and a semi-infinite aquifer can be used to describe flow depletion from a slightly penetrating streambed in an infinite aquifer by modifying the stream leakance, L. Hantush (1965), as noted previously, chose incorrectly to modify ℓ. It is, however, much more straightforward to work directly with Equation 20 since all of the variables and parameters that appear in this equation have a clearly defined physical meaning.

The solution given by Equation 20 has been plotted in Figure 4. This solution contains both the Hantush solution, if λ is replaced with the combination of variables given by Equation 21, and the Glover-Balmer solution, since Equation 20 reduces to Equation 1 when $\lambda\ell/T = \infty$. In principle, the plot in Figure 4 can be used to estimate the leakage coefficient, λ, from field data if $\Delta Q/Q_w$ and $4Tt/(S\ell_2)$ are known. If the parameters λ, T, and S are all unknown, then a semi-log plot of the field data ($\Delta Q/Q_w$ versus log(t)) might be used with Figure 4 and a match-point method (as described, for example, by Hunt [1983]) to obtain two equations for these three unknowns. A third equation would have to be obtained, perhaps by estimating S, to calculate a solution for λ, T, and S. However, this approach is likely to be unworkable in practice because of difficulties in obtaining accurate measurements for ΔQ. The procedure described in the next section may prove to be a more practical way to estimate λ, T, and S.

Drawdown Calculations

Drawdowns can be calculated from Equations 16 and 17 for the entire infinite solution domain. The following expression for $\bar{\Phi}$ can be obtained by substituting values of C_i from Equation 17 into Equation 16:

$$\bar{\Phi} = \bar{\Phi}_1 - \bar{\Phi}_2 \qquad (-\infty < x < \infty) \tag{22}$$

$$\bar{\Phi}_1 = \frac{Q_w}{2T}\frac{e^{-|\ell - x|\sqrt{\alpha^2 + pS/T}}}{p\sqrt{\alpha^2 + pS/T}} \tag{23}$$

$$\bar{\Phi}_2 = \frac{Q_w\lambda}{4T^2}\frac{e^{-(\ell + |x|)\sqrt{\alpha^2 + pS/T}}}{p\sqrt{\alpha^2 + pS/T}\,[\sqrt{\alpha^2 + pS/T} + \lambda/(2T)]} \tag{24}$$

The expression for $\bar{\Phi}_1$ can be inverted in at least three different ways. The most difficult way is to obtain $\bar{\Phi}_1$ from the Bromwich contour integal by closing the infinite straight line parallel to the imaginary p axis with a semicircle in the left half p plane and a branch cut along the real p axis from minus infinity to the point $p = -T\alpha^2/S$. Then differentiation gives $\partial\bar{\Phi}_1/\partial t$, and the Fourier inversion integral can be used to calculate $\partial\phi_1/\partial t$. Finally, ϕ_1 can be calculated by integrating with respect to t.

A second way of calculating ϕ_1 is to write a mathematical statement for the problem in which a well at $(x, y) = (\ell, 0)$ abstracts a flow of Q_w in an infinite aquifer with no stream depletion. Then, use of Equations 11 and 12 to calculate $\bar{\Phi}_1$ shows that $\bar{\Phi}_1$ is given by Equation 23. Therefore, Φ_1 is the Theis solution for a well at the point $(\ell, 0)$.

The third, and easiest, way to calculate ϕ_1 is to notice from Equations 22 through 24 that $\bar{\Phi}_1$ is the solution for $\bar{\Phi}$ when $\lambda \to 0$. Since the stream depletion, ΔQ, is seen from Equation 8 to vanish when $\lambda \to 0$, it is immediately obvious that ϕ_1 must be the Theis solution for a well at the point $(\ell, 0)$. Thus, in all cases

$$\phi_1(x, y, t) = \frac{Q_w}{4\pi T}E_1\left[\frac{(\ell - x)^2 + y^2}{4Tt/S}\right] \tag{25}$$

where E_1 is the exponential integral that is often called the well function, W, in ground water hydrology. Abramowitz and Stegun (1964) give convenient ways of evaluating E_1 for computer calculations.

The inversion of $\bar{\Phi}_2$ is easy once $\bar{\Phi}_1$ has been inverted. The expression for $\bar{\Phi}_2$ can be rewritten in the equivalent form

$$\bar{\Phi}_2 = \frac{Q_w\lambda e^{(\ell + |x|)\lambda/(2T)}}{4T^2}\int_{|x|}^{\infty}\frac{e^{-(\ell + \xi)[\sqrt{\alpha^2 + pS/T} + \lambda/(2T)]}}{p\sqrt{\alpha^2 + pS/T}}\,d\xi \tag{26}$$

(Equation 26 can be checked by evaluating the integral and noticing that the result agrees with Equation 24.) Moving the first exponential term inside the integral allows Equation 26 to be rewritten as

$$\bar{\Phi}_2 = \frac{Q_w\lambda}{4T^2}\int_{|x|}^{\infty}e^{-(\xi - |x|)\lambda/(2T)}\left[\frac{e^{-(\ell + \xi)\sqrt{\alpha^2 + pS/T}}}{p\sqrt{\alpha^2 + pS/T}}\right]d\xi \tag{27}$$

The transform variables p and α appear only within the bracketed term on the right side of Equation 27. Therefore, ϕ_2 can be obtained by replacing the bracketed term with its inverse. But this inverse has already been given by the inverse of $\bar{\Phi}_1$ if $Q_w/(2T)$ and $|\ell - x|$ in $\bar{\Phi}_1$ and ϕ_1 are replaced with unity and $(\ell + \xi)$, respectively. Thus, the inverse of $\bar{\Phi}_2$ is

$$\phi_2(x, y, t) = \frac{Q_w\lambda}{8\pi T^2}\int_{|x|}^{\infty}e^{-(\xi - |x|)\lambda/(2T)}E_1\left[\frac{(\ell + \xi)^2 + y^2}{4Tt/S}\right]d\xi \tag{28}$$

Finally, it is convenient to change the integration variable from ξ to θ by setting $\theta = (\xi - |x|)\lambda/(2T)$ in Equation 28 to obtain

$$\phi_2(x, y, t) = \frac{Q_w}{4\pi T}\int_0^{\infty}e^{-\theta}E_1\left[\frac{(\ell + |x| + 2T\theta/\lambda)^2 + y^2}{4Tt/S}\right]d\theta \tag{29}$$

The integral on the right side of Equation 29 must be evaluated numerically. This is a straightforward calculation since the integrand decays exponentially as θ becomes large and has no singularities on the integration interval when t is greater than zero.

Equations 22, 25, and 29 can be used to obtain an expression for the drawdown that is valid everywhere in the (x, y) plane. The result is

$$\phi(x, y, t) = \frac{Q_w}{4\pi T}\left\{E_1\left[\frac{(\ell - x)^2 + y^2}{4Tt/S}\right] - \int_0^{\infty}e^{-\theta}E_1\left[\frac{(\ell + |x| + 2T\theta/\lambda)^2 + y^2}{4Tt/S}\right]d\theta\right\}$$

$$(-\infty < x < \infty, \ -\infty < y < \infty, 0 < t < \infty) \tag{30}$$

As noted previously, Equation 30 reduces to the Theis solution for a well at $(x, y) = (\ell, 0)$ when $\lambda \to 0$. A more interesting case occurs, however, when $\lambda \to \infty$. This limit gives

$$\lim_{\lambda \to \infty}\phi = \frac{Q_w}{4\pi T}\left\{E_1\left[\frac{(\ell - x)^2 + y^2}{4Tt/S}\right] - E_1\left[\frac{(\ell + |x|)^2 + y^2}{4Tt/S}\right]\right\} \tag{31}$$

Thus, when $x > 0$ so that $|x| = x$, Equation 31 gives the drawdown from an abstraction well at $(\ell, 0)$ and a recharge well at the image point $(-\ell, 0)$. On the opposite side of the stream, where $x < 0$ and $|x| = -x$, this equation gives a zero drawdown for $-\infty < x \le 0$. This is the same solution used by Theis (1941) and Glover and Balmer (1954) to obtain the stream depletion caused by a well beside a stream with zero drawdown. (It was noted previously that Equation 20 also reduces to the solution obtained by Theis [1941] and Glover and Balmer [1954] when $\lambda \to \infty$.) One of the interesting aspects of this result is that the solution has been obtained herein in the context of a slightly penetrating stream rather than the fully penetrating stream that was originally assumed by these earlier workers.

Dimensionless drawdown contours are plotted in Figure 5 from Equation 30 for fixed dimensionless values of t and λ. The

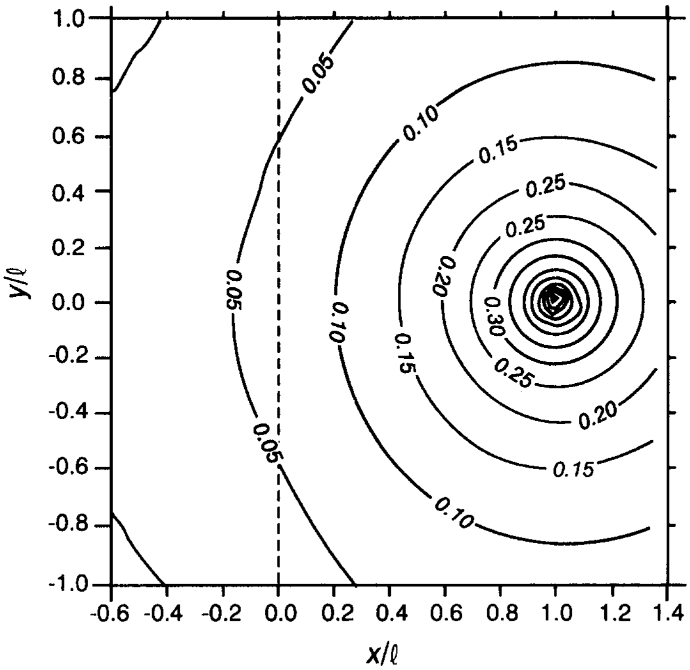

Figure 5. Drawdown contours showing values of $\phi T/Q_w$ for $Tt/(S\ell^2)$ = 1 and $\lambda\ell/T$ = 1. The stream location is shown with a dashed line.

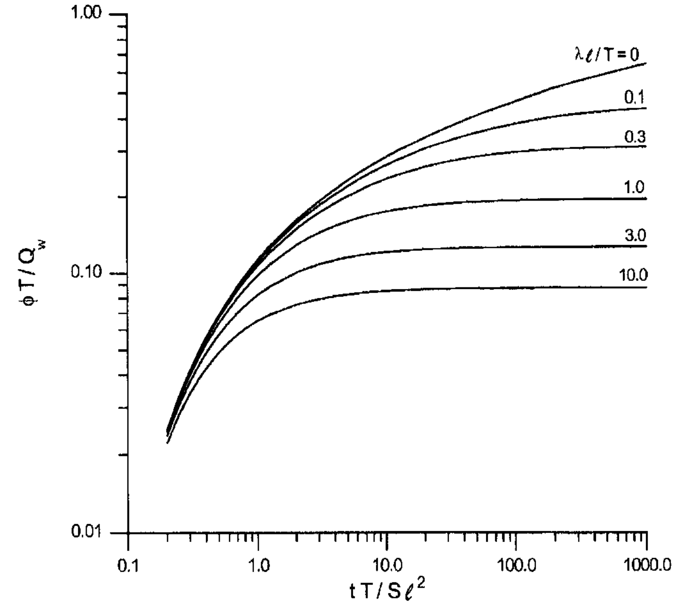

Figure 6. Dimensionless drawdowns in an observation well at $(x/\ell, y/\ell)$ = (0.2, 0).

result is probably of limited practical use, although it does show that Equation 30 gives a physically reasonable behavior and that finite drawdowns occur on both sides of the stream. A more useful result, however, is obtained by using this equation to make a log-log plot of $\phi T/Q_w$ versus $tT/(S\ell^2)$ for fixed values of x/ℓ and y/ℓ, and a range of different values of $\lambda\ell/T$. An example is shown in Figure 6 for $(x/\ell, y/\ell) = (0.2, 0)$. If experimental measurements of ϕ and t were made in an observation well at this point, then a log-log plot of ϕ versus t might be used with Figure 6 and the match-point method to calculate T, S, and λ. This approach, which has yet to be tested with field data, is likely to be the most practical way to obtain accurate estimates for these parameters.

Since all of the governing equations are linear with coefficients that are independent of time, the principles of superposition and time translation can be used to obtain solutions in which Q_w is constant for a finite time anywhere within the range $0 < t < \infty$ and zero for all other values of t. Hydrologists have used this procedure for many years in unit hydrograph theory, and Jenkins (1965) illustrated the use of these principles for some applications of Equation 1.

Numerical techniques can be used to obtain solutions under more general conditions than were used to obtain Equations 20 and 30. For example, if enough field data is available, it is possible to take into account variations in permeability or transmissivity and aquifer anisotropy. In principle, it is even possible to consider problems in which vertical velocities are relatively large and in which stream channels have finite dimensions. Examples of numerical solutions are given in Spalding and Khaleel (1991) and Sophocleous et al. (1995). However, there are practical limits to the amount of time and money that can be spent gathering field data and constructing and using numerical models. For these reasons, there will probably continue to be uses for closed-form solutions like those obtained herein.

Conclusions

A solution has been obtained for stream depletion created by pumping from a well beside a stream. This solution assumes that streambed penetration of the aquifer and dimensions of the streambed cross section are all relatively small. It also assumes that the streambed is clogged and that a linear relationship exists between the outflow seepage through the streambed and the change in piezometric head across the semipervious clogging layer. It has been shown that a plot of this solution is general enough to contain solutions calculated earlier by Theis (1941), Glover and Balmer (1954), and Hantush (1965). Use of the Hantush solution (which describes the depletion from a fully penetrating, clogged streambed) for a slightly penetrating clogged streambed requires modification of the stream leakance, L, rather than the distance, ℓ, between the stream edge and well, which is contrary to the modification suggested by Hantush and used by present day analysts. An expression has also been obtained for drawdowns at any point within the aquifer, and it has been suggested that this solution might be matched with experimental field data to obtain estimates for aquifer and streambed leakage parameters.

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