Delft University of Technology

SC42145 Robust Control

PART 1 SISO Analysis and Control Design

The objective of this part of the assignment is to design a SISO Controller for the floating wind turbine. Since the goal of this SISO Controller is to pitch the blades in order to increase the rotational velocity of the turbine and acquire the new desired generator speed, the control input to the SISO Controller is the pitch angle β, which is in radians. The output to be controlled is the generator speed ω_r which is in radians per second. The state space system provided is:

$$\dot{x} = \begin{bmatrix} -0.4220 & -0.2204 & 0 & -0.2204 & 0 \\ 0.0233 & -0.0109 & -0.0400 & -0.0096 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 \\ 0.1455 & -0.0598 & 0 & -0.1651 & -10.8232 \\ 0 & 0 & 0 & 1.0000 & 0 \end{bmatrix} x + \begin{bmatrix} -0.0799 \\ -0.0067 \\ 0 \\ -0.0420 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} x + 0u$$

where

$$x = \begin{bmatrix} \omega_r \\ \dot{z}_1 \\ z_1 \\ \dot{z}_2 \\ z_2 \end{bmatrix}, u = \beta, y = \omega_r.$$

Using the MATLAB command **ss2tf**, we can represent this state space system as a transfer function. The corresponding transfer function of the plant is:

$$G(s) = \frac{-0.079878(s^2 - 0.007693s + 0.04)(s^2 + 0.0492s + 10.82)}{(s + 0.4104)(s^2 + 0.02113s + 0.04101)(s^2 + 0.1664s + 10.85)} \tag{1}$$

The poles of this plant are located at [- 0.0832 + 3.2936i, - 0.0832 - 3.2936i, - 0.4104, - 0.0106 + 0.2022i, - 0.0106 - 0.2022i]. The zeroes of the plant are [- 0.0246 + 3.2897i, - 0.0246 - 3.2897i, 0.0038 + 0.2000i, 0.0038 - 0.2000i].
We have to design a reference tracking controller K(s) that can achieve the highest possible bandwidth while remaining within the design requirements. The design requirements are:

- Small settling time.
- Overshoot<1%.
- No steady state error.

The structure of the system(Controller+Plant) is as shown in Figure 1. where r is the reference input,

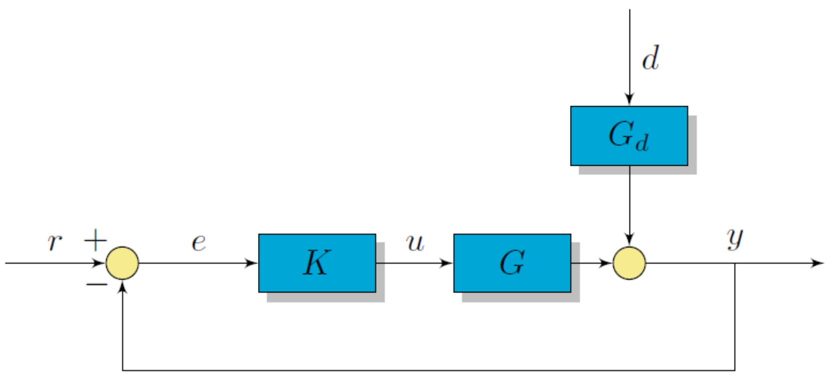


Figure 1: The Control Architecture

y is the output, e is the control error, which is r-y. This e serves as the input to the controller K. The output of the controller K is u, which is the control input to the plant G. d is the disturbance input, and G_d is the disturbance model at the output.

1 QUESTION 1

The open-loop Bode plot of the the plant is as shown in Figure 2. The Gain Margin G_m is 14.5dB at

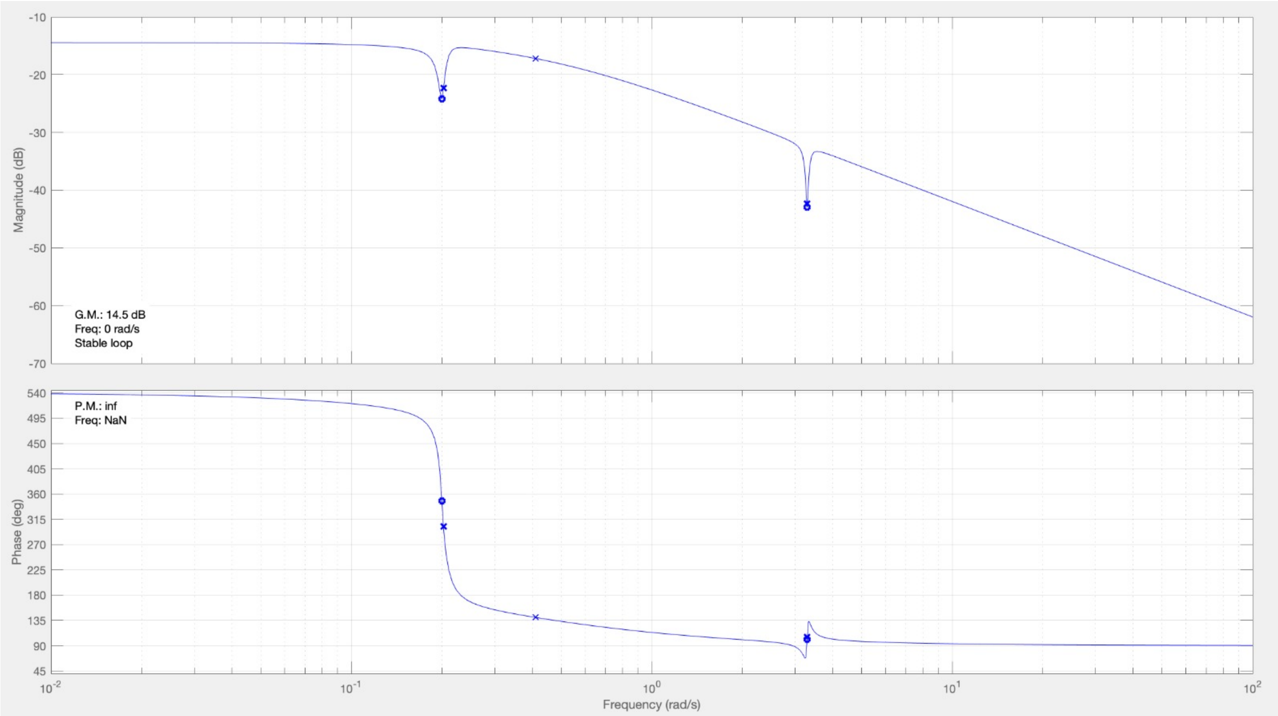


Figure 2: Open-Loop Bode Plot of Plant

Phase Crossover Frequency $\omega_{cp} = 0\ rad/s$. The Phase Margin P_m is ∞, and hence the Gain Crossover Frequency ω_{cg} is ∞. Note that the 540° mark in the phase plot corresponds to the $-180°$.

The Pole-Zero map of the plant is as shown in Figure 3. We have a total of 5 poles and 4 zeroes in the system. 1 pole is on the real axis, and the rest of the poles are complex conjugate poles. All the poles of the plant are in the left half plane. All of the zeroes are complex conjugate, but 2 of the zeroes are in the left half plane, and the other 2 are in the right half plane.

Since all the poles of the plant are in the left half plane, we can conclude that the plant is stable. Another indication of stability and the robustness is the positive gain margin and infinite phase margin.

The feature in this system that hinders us from achieving a high bandwidth is the right half plane zeroes present at [0.0038 + 0.2000i, 0.0038 - 0.2000i]. These zeroes are also shown in the pole-zero map of Figure 3. Right Half Plane Zeroes introduce a phase lag of $-90°$, causing the phase to fall quickly, as can be seen from Figure 2, and thus lowering the bandwidth of the system. This characteristic is difficult, if not impossible, to compensate. Thus, in case of the right half plane zeroes, the frequencies provide bandwidth limits on the pitch to generator speed loop, and this is the reason we cannot achieve a bandwidth of > 0.1Hz.

Figure 4 shows the bode plot of the plant(in green), as well as the closed-loop sensitivity(in red) and complementary sensitivity(in blue) functions. The closed-loop bandwidth of the plant is the frequency value where the magnitude plot of the sensitivity function first crosses the -3dB mark from below. Alternatively, closed-loop bandwidth of the plant is the highest frequency value where the magnitude plot of the complementary sensitivity function crosses the -3dB mark from above. In Figure 4, we can see the sensitivity curve crosses the -3dB mark from above at infinite frequency. The physical interpretation of this is that the system has a very poor response and will have trouble tracking various reference signals without excessive error.

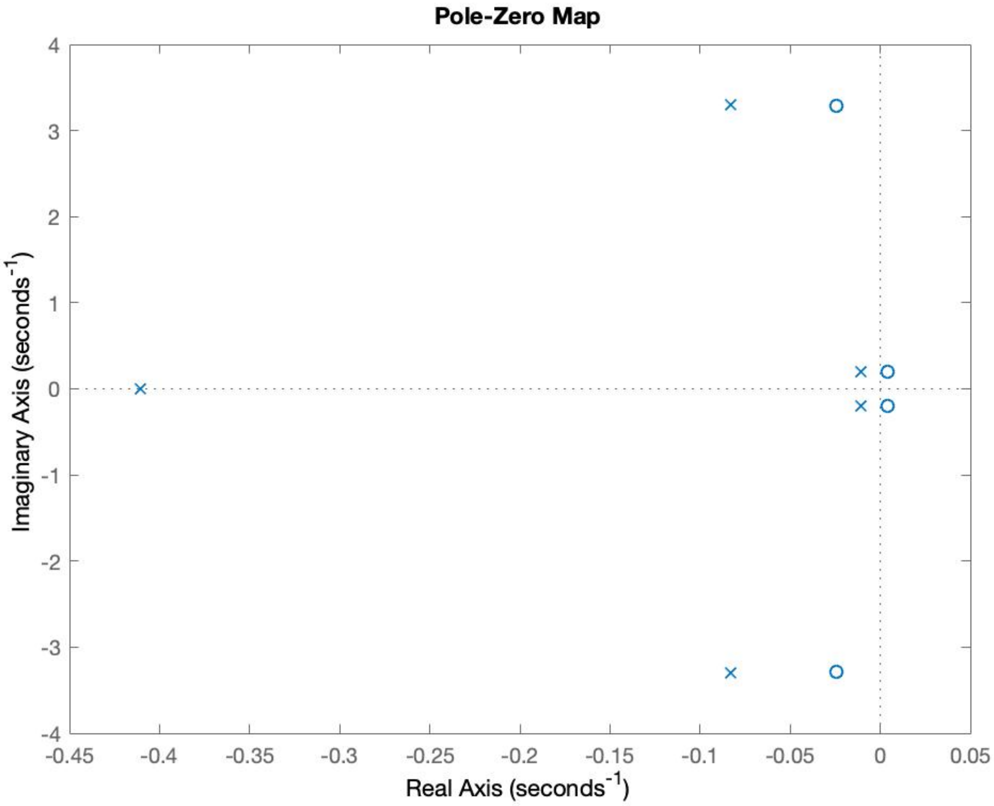


Figure 3: Pole-Zero Map of Plant Plot

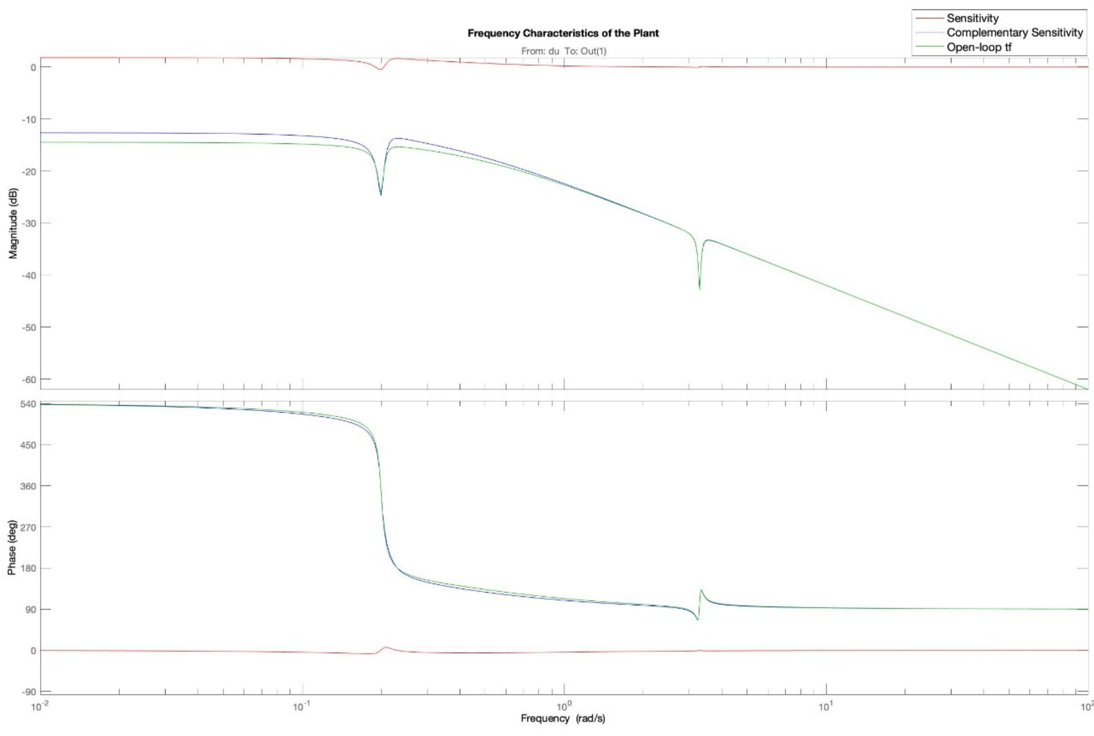


Figure 4: Frequency Characteristics of the Plant

2 QUESTION 2

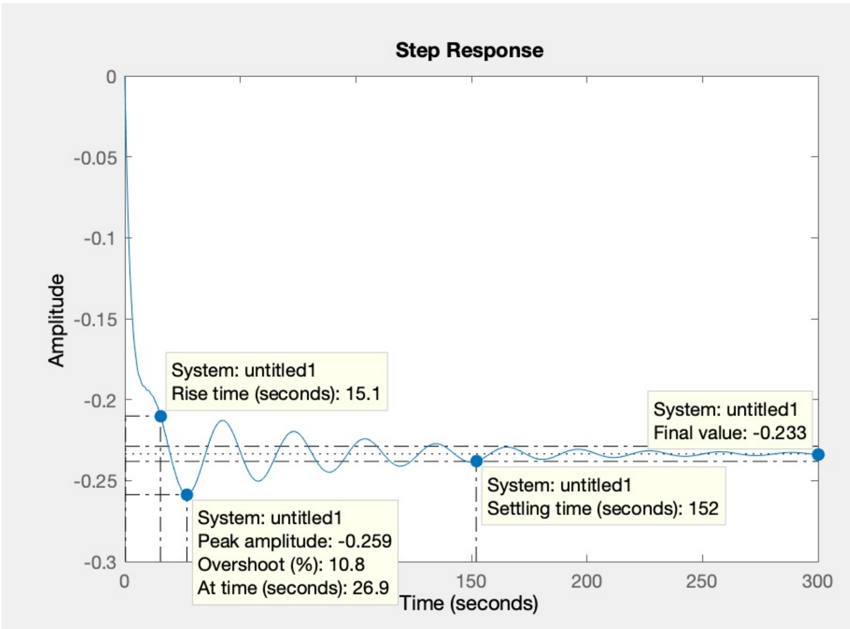
Since the plant does not have any poles at 0, the type number of the plant is 0. Using the final value theorem, i.e.

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} F(s)$$

The steady state value can be found using the MATLAB command **dcgain**. This also corresponds to the low frequency gain of the open loop bode plot. The steady state value for the closed loop system with a step reference input is -0.2335. This shows that the steady state error is very large.
From the sensitivity function of the plant in Figure 4, since the bandwidth is infinite, the response of the system is extremely slow, which indicates a very large settling time.
The overshoot of the closed loop plant to a step response is 10.788%, which is much higher than required. The overshoot of the system is dependent on the phase margin, and the lower limit of the phase margin can be calculated from the peak value of the sensitivity function. There is a close relation between the maximum peaks and the gain and phase margin. If M_S is the peak of the sensitivity function,

$$GM \geqslant \frac{M_S}{M_S - 1}, PM \geqslant 2arcsin(\frac{1}{2M_S}) \geqslant \frac{1}{M_S} \tag{2}$$

where GM is the Gain Margin and PM is the Phase Margin. Figure 5 shows the step input response

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struct with fields:

        RiseTime: 15.1007
    SettlingTime: 152.0831
     SettlingMin: -0.2587
     SettlingMax: -0.2107
       Overshoot: 10.7888
      Undershoot: 0
            Peak: 0.2587
        PeakTime: 26.8641
```

Figure 5: Step Response and Time Domain Characteristics of the Plant

characteristics of the closed-loop plant. To match the design requirements and achieve reference tracking with maximum bandwidth, we need to design a controller K. To achieve a small settling time, we should also have a small rise time, which is achieved by increasing the closed-loop bandwidth. For achieving the desired overshoot, we manipulate the phase margin of the system. For minimizing the

steady state error, we need to ensure that the low frequency gain of the bode plot magnitude response is as desired.
Since the steady state value the closed-loop system converges to a negative value, any gains that we use in our controller must be negative.

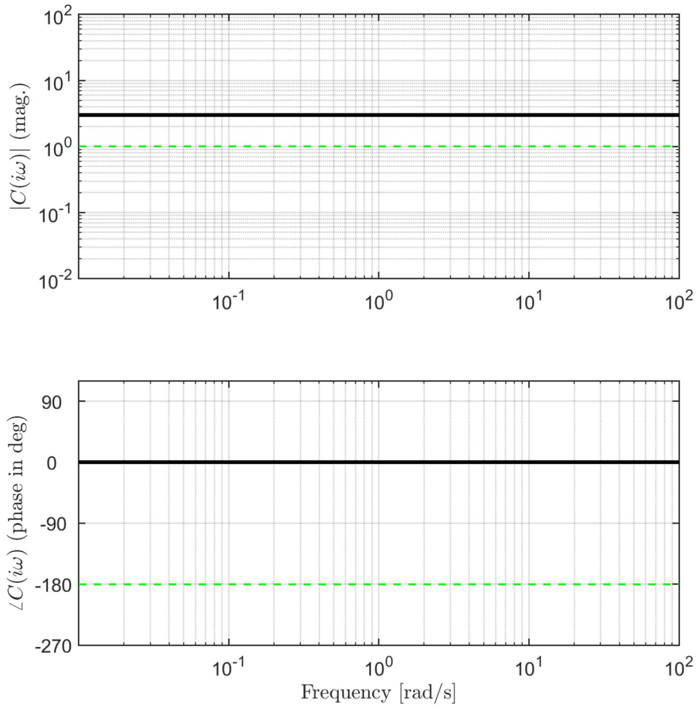

Figure 6: Bode Plot of a Generic Proportional Controller

We will first attempt to use just a proportional controller with our plant. The bode plot of a generic proportional controller is as shown in Figure 6. The proportional controller contributes a constant magnitude, and no phase. When combined with the plant, this means that the proportional controller 'shifts' the magnitude plot of the system. The proportional term moves the magnitude of the frequency response of the open loop up or down and hence is used to set the cross-over frequency of the open loop. This frequency is very important as it is directly related to the bandwidth of the system. Since we want a greater bandwidth and a low rise time, we can increase the value of the proportional gain K_p as long as the system is stable. As the value of K_p increases(in this case, becomes more negative), the rise time of the system improves, thus also increasing the bandwidth. The K_p value of the system destabilises it when K_p crosses around -16.3. When K_p drops below around -16.3, the gain margin of the system becomes negative, making it unstable. Also, within the stable values, the steady state value of the system is not 1, as desired by the step response input. Thus, the role of K_p has been to improve the transient response of the system, but it fails to settle at the desired steady state value.
We now demonstrate this by showing the step response, bode plots, gain and phase margin, and bandwidth for 4 different K_p values of -5, -10, -16 and -17. Figure 7, 9, 11 and 13 show the step responses of the system for the 4 values of K_p. Figure 8, 10, 12 and 14 show the bode and sensitivity plots for the 4 values of K_p.

As can be seen from the bode plot in Figure 14, since the gain margin is negative, the system is unstable. From values -5 to -16, the bandwidth steadily increases, the rise time decreases, the settling time increases, the gain margin decreases, and thus, so does the stability. This implies that K_p comes at the cost of gain and stability. The steady state values attained are not 1, hence all the 4 plots show a steady state error. Also, the settling time is too high, and does not match our requirements. Table 1 summarizes the results obtained from these simulations.

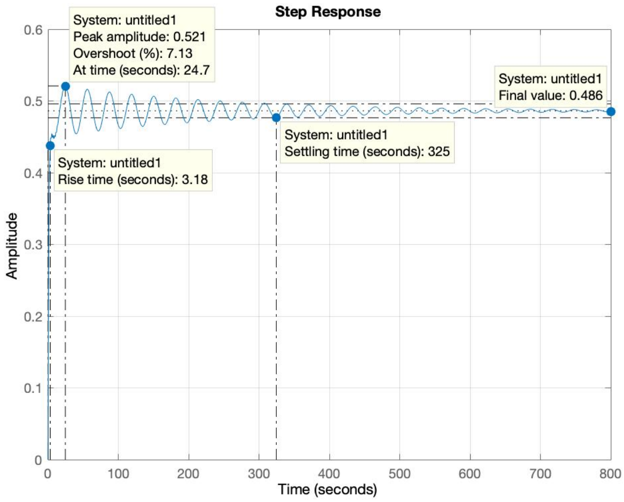

Figure 7: Step Response of System when K_p=-5

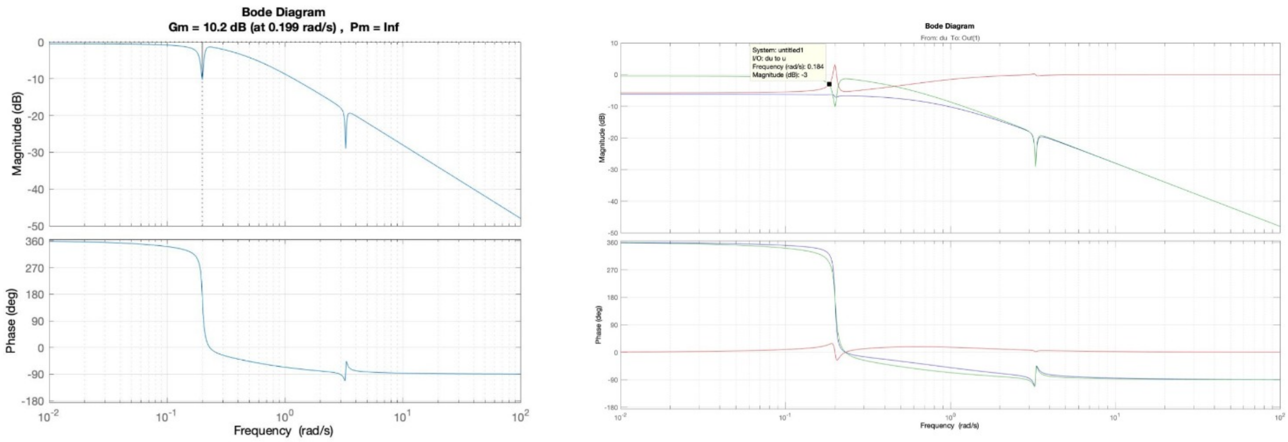

Figure 8: Bode Plot and Sensitivity of System when K_p=-5

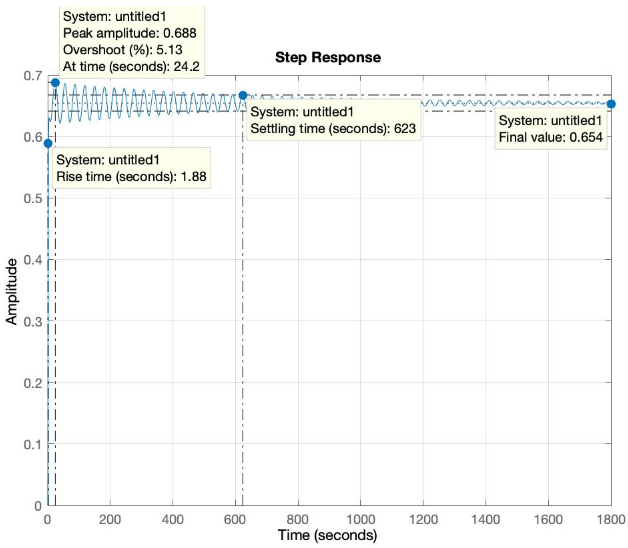

Figure 9: Step Response of System when K_p=-10

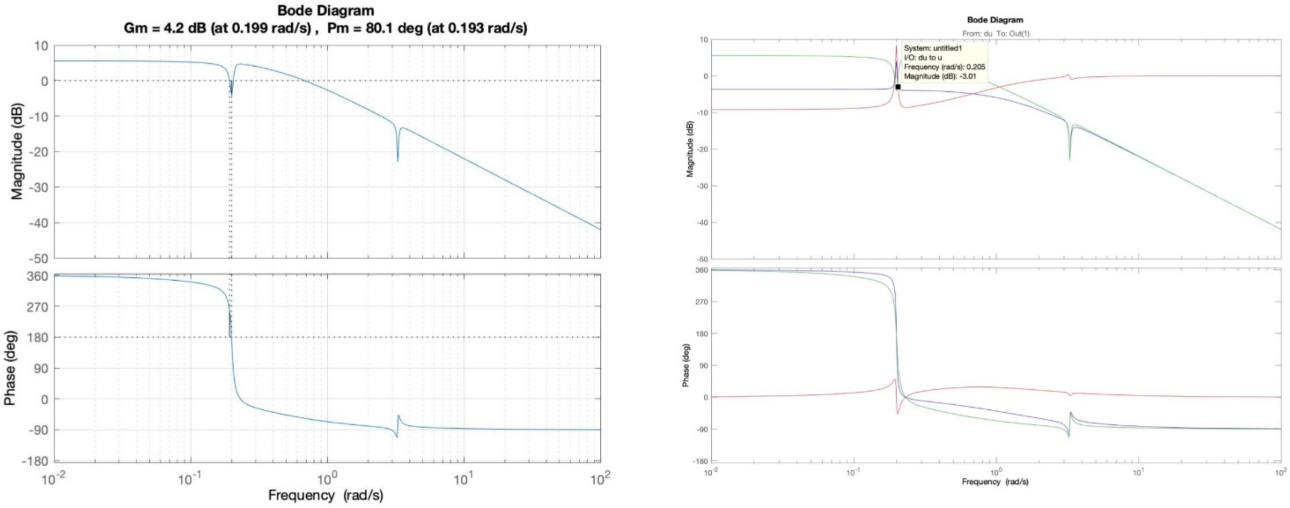

Figure 10: Bode Plot and Sensitivity of System when K_p=-10

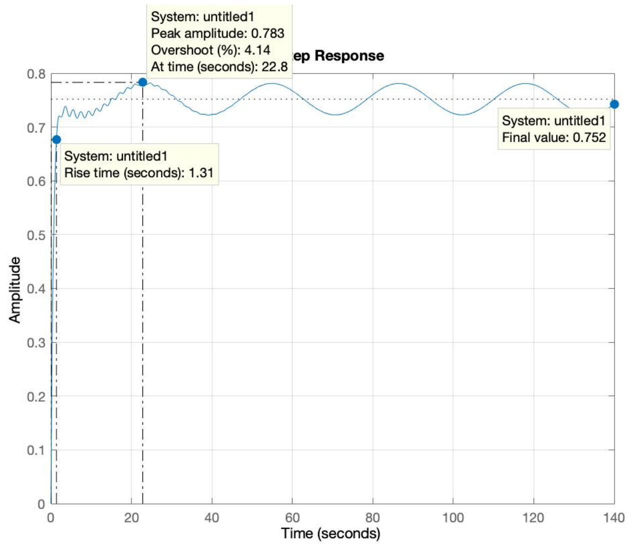

Figure 11: Step Response of System when K_p=-16

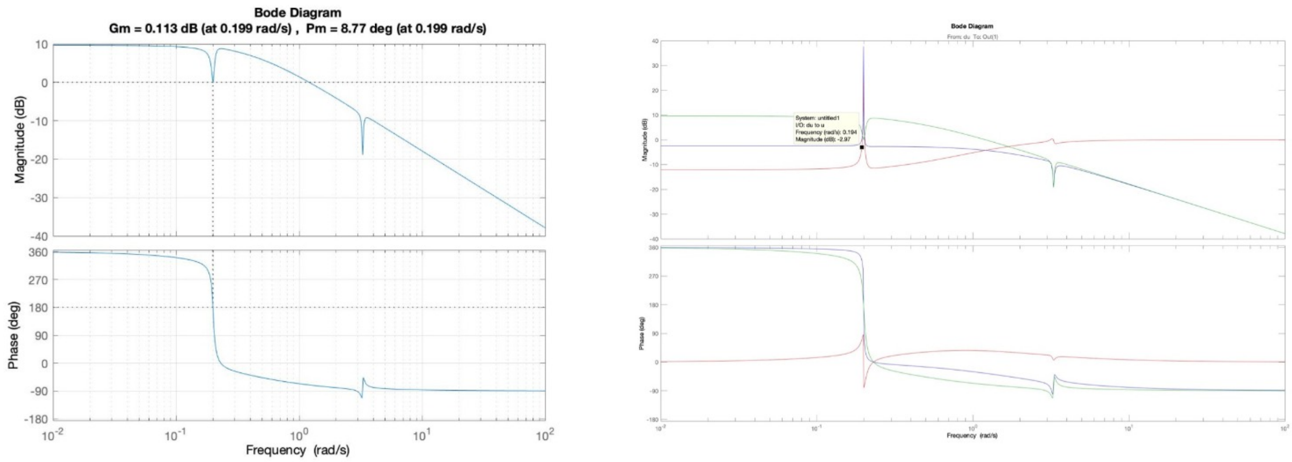

Figure 12: Bode Plot and Sensitivity of System when K_p=-16

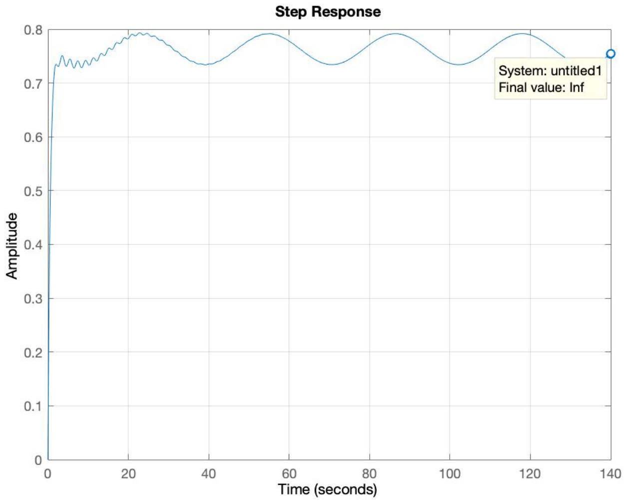

Figure 13: Step Response of System when K_p=-17

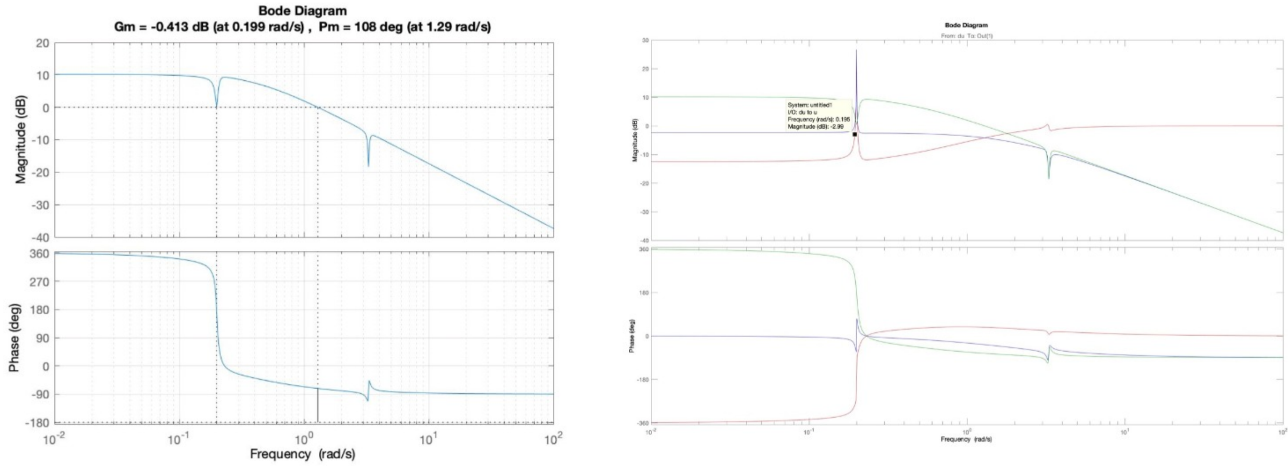

Figure 14: Bode Plot and Sensitivity of System when K_p=-17

Table 1: Frequency and Time Domain Characteristics for the Loop Transfer Function

Properties	$K_p = -5$	$K_p = -10$	$K_p = -16$	$K_p = -17$
Rise Time(s)	3.18	1.88	1.31	NaN
Overshoot(%)	7.13	5.13	4.14	NaN
Settling Time(s)	325	623	NaN	NaN
Steady State Value	0.486	0.654	0.752	∞
Gain Margin(dB)[@rad/s]	10.2[@0.199]	4.2[@0.199]	0.133[@0.199]	-0.413[@0.199]
Phase Margin(deg)[@rad/s]	∞	80.1[@0.193]	8.77[@0.199]	108[@1.296]
Stability	Stable	Stable	Stable	Unstable
Bandwidth(rad/s)	0.184	0.196	0.200	0.195

Since the proportional controller had obvious drawbacks and did not satisfy our requirements, we will now use an integral controller only(of the form $\frac{K_i}{s}$) with our plant. The bode plot of a generic integral controller is as shown in Figure 15. The integral term is used to ensure a zero steady-state error. The integral controller forces the open loop magnitude to be very high at very small frequencies. These small frequencies correspond to t $\rightarrow \infty$ in the the time domain. This effect forces the closed loop to have a zero steady state error. Another way of looking at this is that the

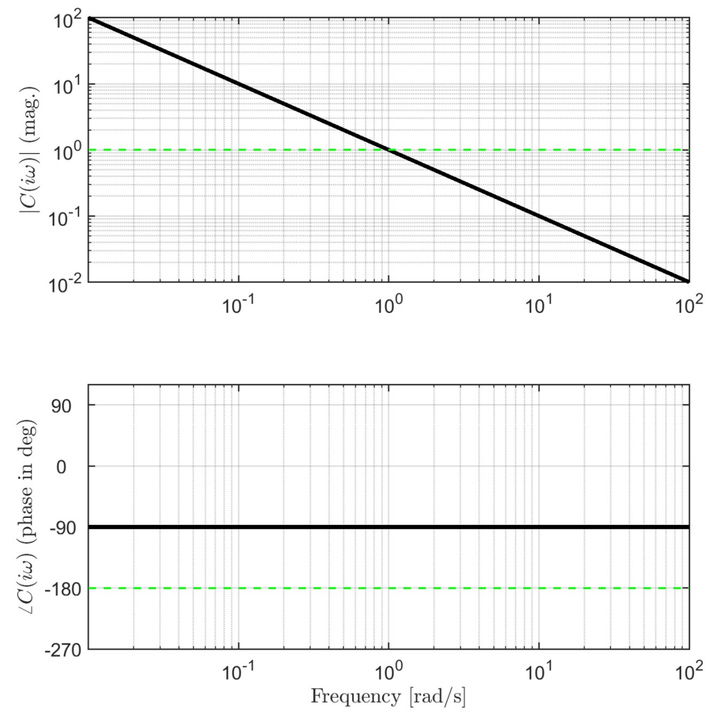

Figure 15: Bode Plot of a Generic Proportional Controller

integral controller makes the system a type 1 system, and thus a step input response ensures 0 steady state error according to the final value theorem. Ideally, the integrator should only affect the loop response at low frequencies. It should raise the response at low frequencies but not affect the loop at high frequencies. The integral controller adds gain and comes with a phase lag of $-90°$. // We want a greater bandwidth, low rise time, low settling time, with no steady state error, while we stay within the overshoot limit. We can increase the value of the integral gain K_i as long as the system is stable. As the value of K_i increases(in this case, becomes more negative), the rise time of the system improves, thus also increasing the bandwidth. The K_i value of the system destabilises it when K_i crosses around -1.73. When K_i drops below around -1.73, the gain margin of the system becomes negative, making it unstable. For the stable systems, the steady state value is always 1, as desired, thus the integral controller successfully eliminates steady state error from the system. When the K_i value is less(around -0.25), the resultant system satisfies all the requirements we desire(overshoot less than 1%, no steady state error and small settling time. However, the bandwidth is found to be very low as compared to the proportional controller. Since we should also be achieving the highest possible bandwidth, we cannot just use an integral control for this.
We now demonstrate this by showing the step response, bode plots, gain and phase margin, and bandwidth for 3 different K_i values of -0.25, -0.46, and -1.74. Figure 16, 18 and 20 show the step responses of the system for the 3 values of K_i. Figure 17, 19 and 21 show the bode and sensitivity plots for the 3 values of K_i.

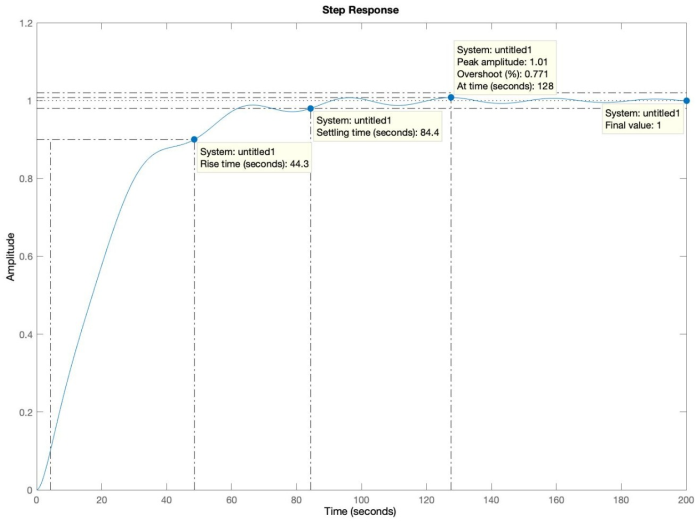

Figure 16: Step Response of System when K_i=-0.25

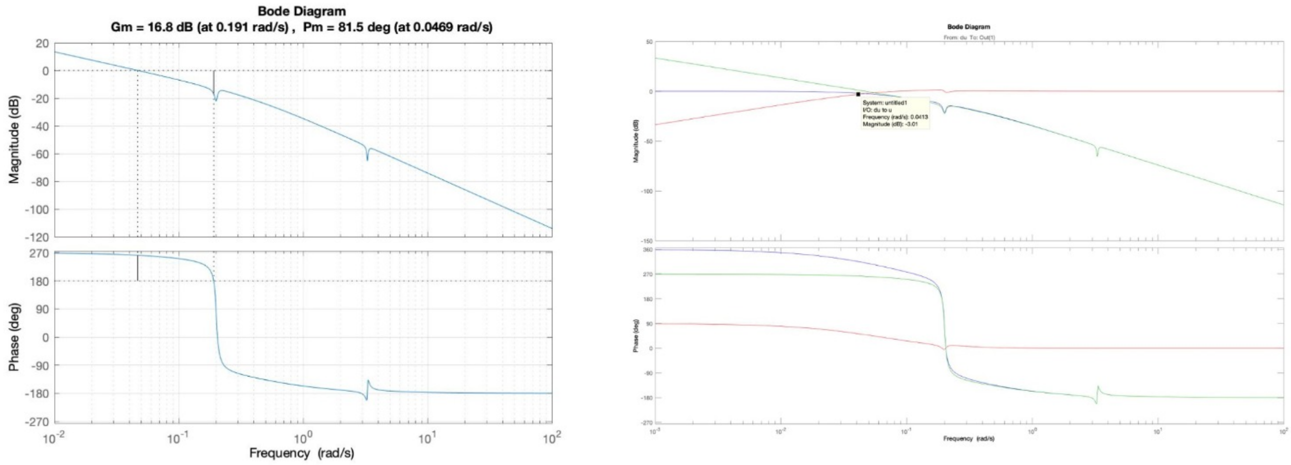

Figure 17: Bode Plot and Sensitivity of System when K_i=-0.25

Table 2: Frequency and Time Domain Characteristics for the Loop Transfer Function

Properties	$K_i = -0.25$	$K_i = -0.46$	$K_i = -1.74$
Rise Time(s)	44.3	19.3	NaN
Overshoot(%)	0.77	3.91	NaN
Settling Time(s)	84.4	113	NaN
Steady State Value	1	1	∞
Gain Margin(dB)[@rad/s]	16.8[@0.191]	11.5[@0.191]	-0.012[@0.191]
Phase Margin(deg)[@rad/s]	81.5[@0.0469]	74[@0.0853]	-0.0692[@0.191]
Stability	Stable	Stable	Unstable
Bandwidth(rad/s)	0.0413	0.0692	0.181

As can be seen from the bode plot in Figure 21, since the gain margin is negative, the system is unstable. From values -0.25 to -0.46, the bandwidth steadily increases, although it is very low as compared to the proportional controller bandwidth values, the rise time decreases, the settling time increases, the gain margin and phase margin decrease, and thus, so does the stability. This implies that K_i comes at the cost of gain, phase and stability. The steady state values attained are 1, hence

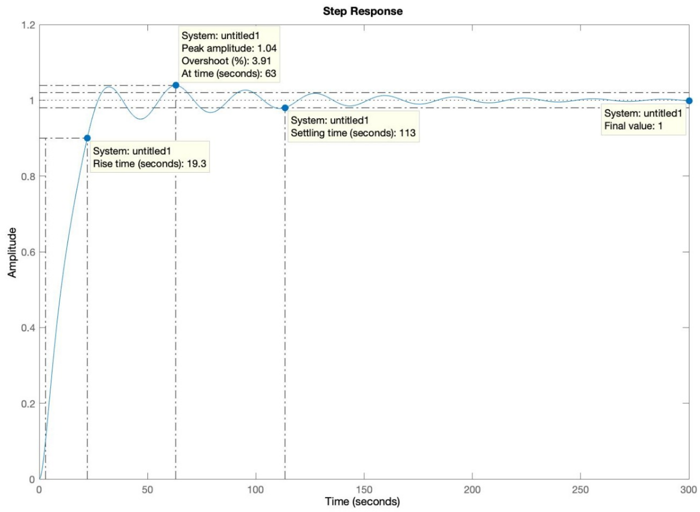

Figure 18: Step Response of System when K_i=-0.46

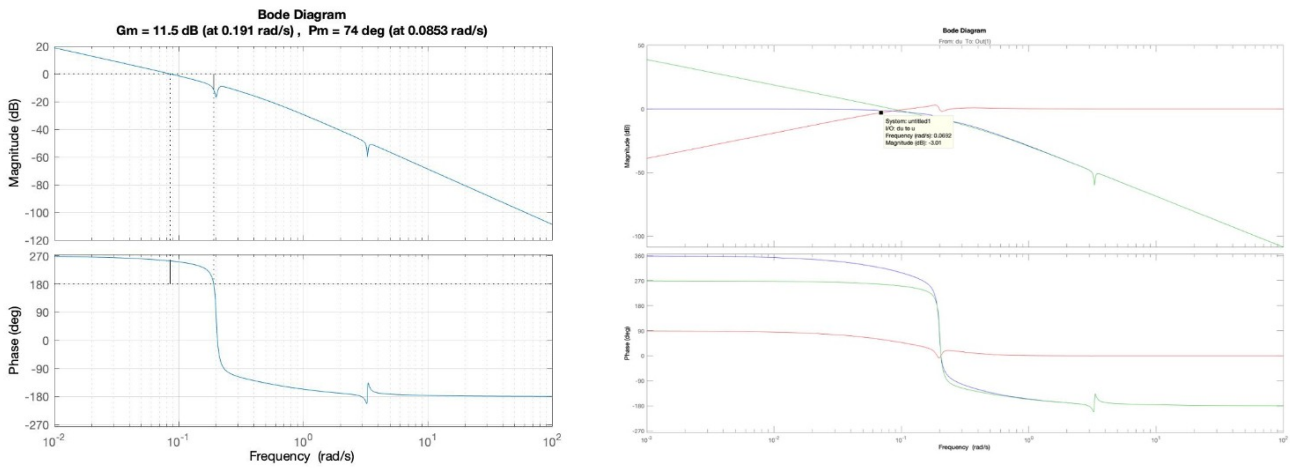

Figure 19: Bode Plot and Sensitivity of System when K_i=-0.46

satisfying the requirement. Table 2 summarizes the results obtained from these simulations. The K_i of -0.25 satisfies all our requirements. We now combine the proportional and integral controller to design a Proportional Integral(PI) Controller.

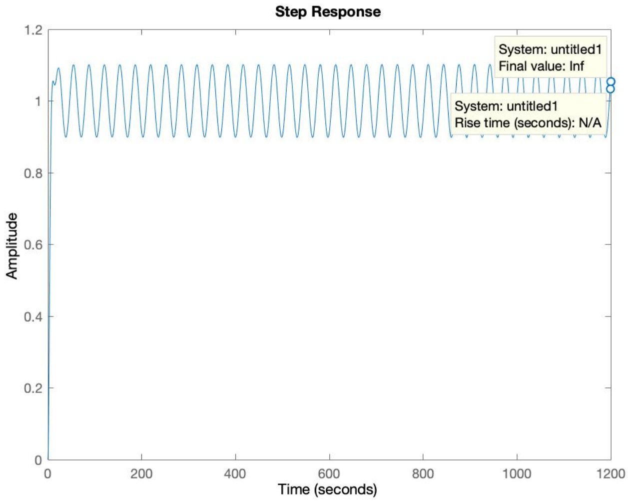

Figure 20: Step Response of System when K_i=-1.74

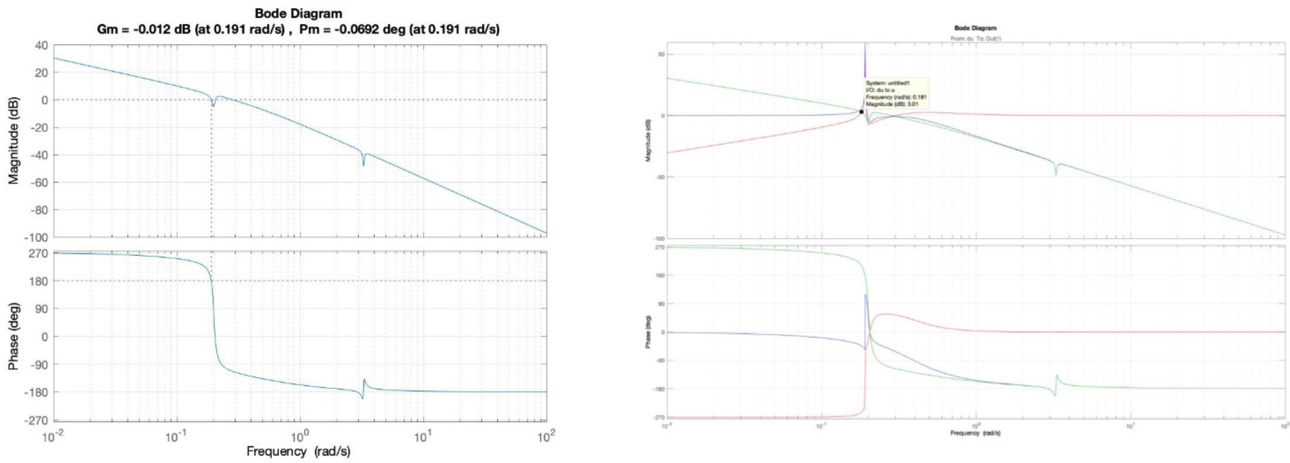

Figure 21: Bode Plot and Sensitivity of System when K_i=-1.74

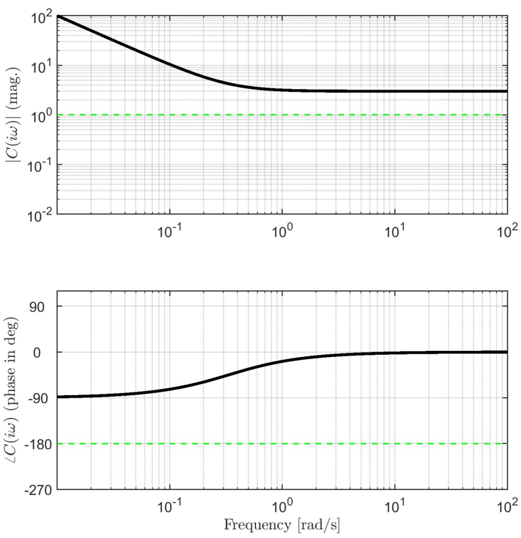

Figure 22: Bode Plot of PI Controller

The PI Controller combines the best attributes of the proportional and integral control. At lower frequencies, the PI Controller behaves like just an I controller, with phase lag and high gain, while at higher frequencies, we observe a behavior similar to a proportional controller, i.e. 0 phase and constant gain. This can be seen from the bode plot of a generic PI controller as shown in Figure 22. We perform the PI Controller Tuning in Question 3.

3 QUESTION 3

Now that we are using a PI Controller, we need to determine the values for K and T_i. The PI Controller is of the form $K(1+\frac{1}{T_i s})$, where T_i is the reset time of the controller. This is the parallel form of the controller, where K_p=K, and $K_i = K/T_i$. Our first is to determine an approximate value of T_i. To do this, we choose T_i as the largest time constant of the process to be cancelled. The constant representation of our plant is:

$$G(s) = \frac{-0.18929(1-0.03846(5s)+(5s)^2)(1+0.01496(0.304s)+(0.304s)^2)}{(1+2.437s)(1+0.1044(4.938s)+(4.938s)^2)(1+0.05051(0.3035s)+(0.3035s)^2)}$$

We can see that the largest time constant in the denominator of our plant is 4.938. We will initially assume this as our T_i, and will assume a K=-1, to visualize the approximate performance of our untuned controller. Therefore, our controller is now $-\frac{(s+0.2025)}{s}$. We now simulate the step response, bode plots and sensitivity functions to assess our system performance, as shown in Figure 23 and Figure 24.

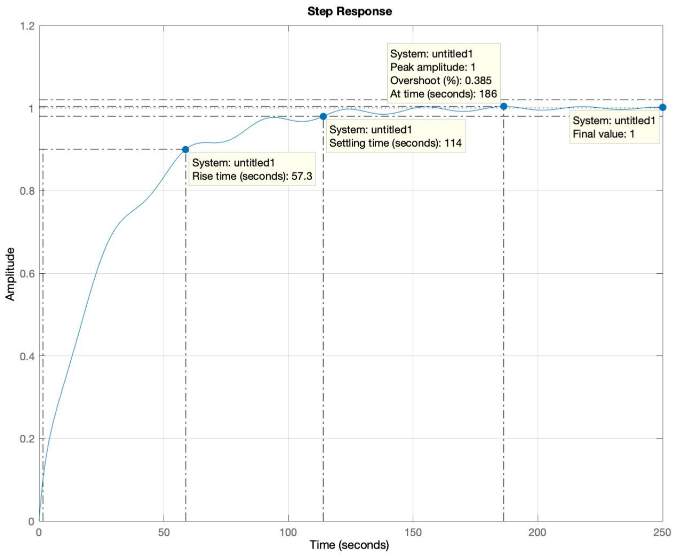

Figure 23: Step Response of System when K=-1, T_i=4.938

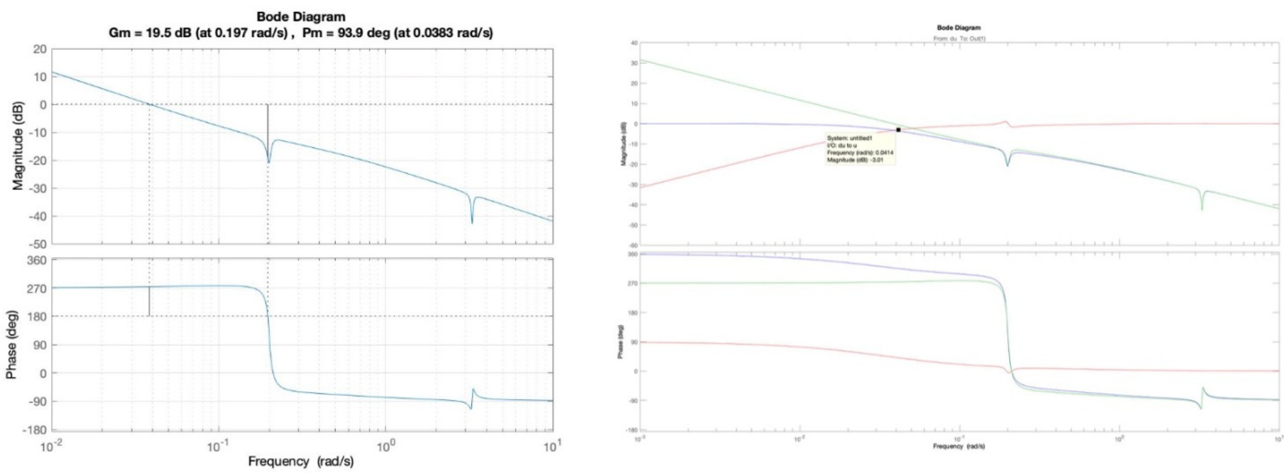

Figure 24: Bode Plot and Sensitivity of System when K=-1, T_i=4.938

We can see that this controller satisfies all our requirements. However, the settling time of 113.9255s

can still be decreased. The Overshoot is 0.3851%, the GM and PM are 19.5dB and 93.9°, and the bandwidth is 0.039rad/s. Thus, we have significant room(gain and phase margin) to improve our system and stay within the design requirements. Our next step is to decrease the reset time T_i to 4s, in order to improve the settling time, and the bandwidth.
Therefore, our controller with K=-1, and T_i=4 is now $-\frac{(s+0.25)}{s}$. We now simulate the step response, bode plots and sensitivity functions to assess our system performance, as shown in Figure 25 and Figure 26. We can see that this controller also satisfies all our requirements. The settling time is

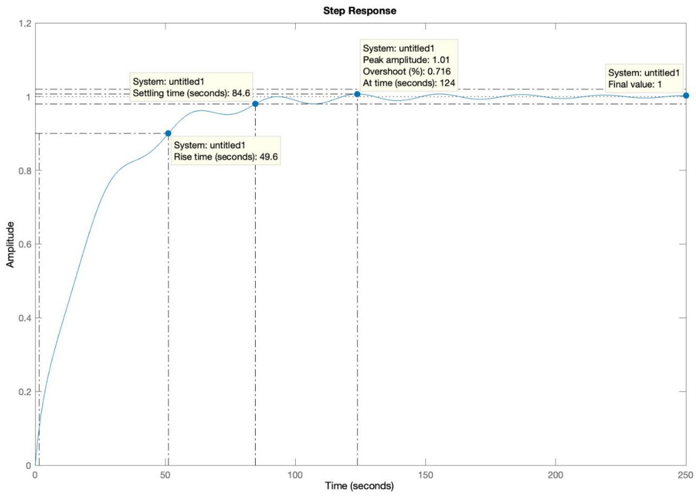

Figure 25: Step Response of System when K=-1, T_i=4

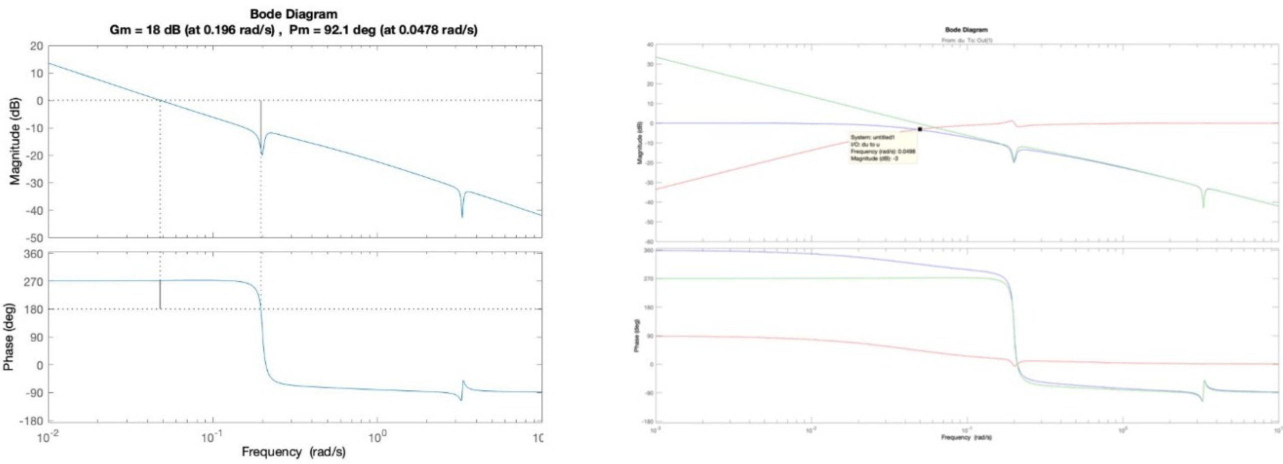

Figure 26: Bode Plot and Sensitivity of System when K=-1, T_i=4

now 84.608s which is a huge improvement. The Overshoot is 0.7152%, the GM and PM are 18dB and 92.1°, and the bandwidth is 0.0511rad/s. The bandwidth has also increased. Since overshoot and GM, PM still permit us to improve the system performance while remaining stable, we now increase the value of K, keeping T_i as it is. We now make K=-1.1. Therefore, our controller with K=-1.1, and T_i=4 is now

$$-\frac{1.1(s+0.25)}{s}$$

We now simulate the step response, bode plots and sensitivity functions to assess our system performance, as shown in Figure 27, Figure 28 and Figure 29. We can see even that this controller satisfies all our requirements. The settling time is now 82.9476s which is an improvement. The Overshoot is

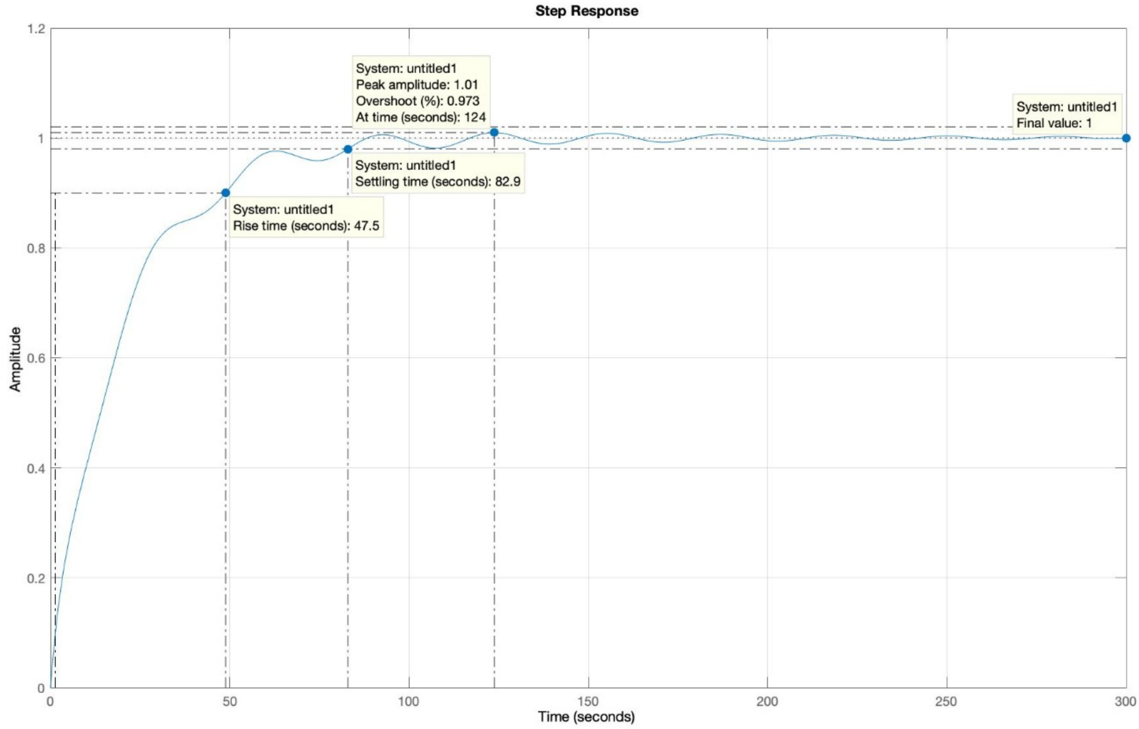

Figure 27: Step Response of System when K=-1.1, T_i=4

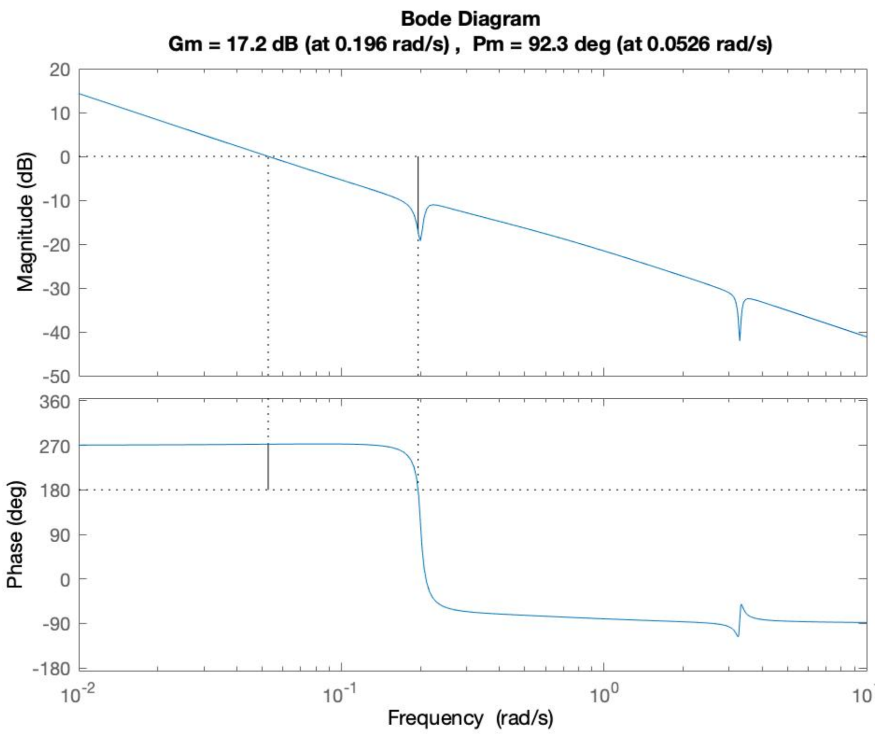

Figure 28: Bode Plot of System when K=-1.1, T_i=4

0.9729%, the GM and PM are 17.2dB and 92.3°, and the bandwidth is 0.0542rad/s. The bandwidth has also increased a little. Any further attempt at improving this system will carry the overshoot above 1%, which will violate our design constraints. Thus, we conclude this to be the final and nearly

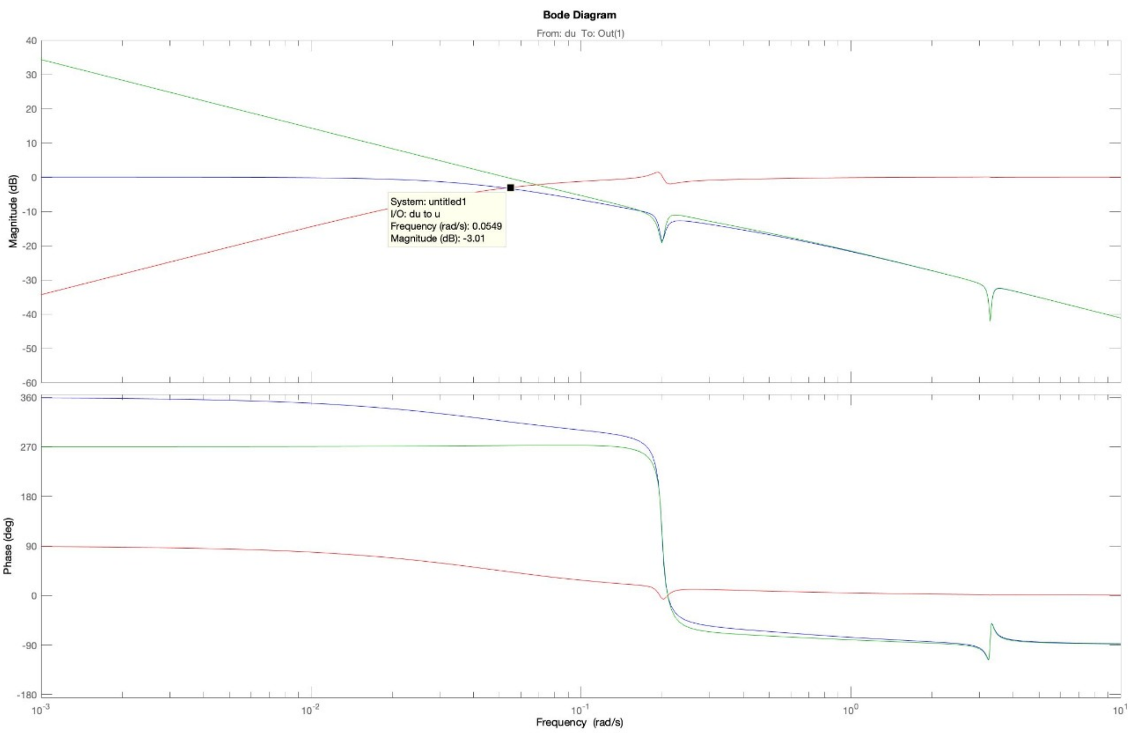

Figure 29: Sensitivity of System when K=-1.1, T_i=4

perfect controller for the given system. The summary of the performance of the 3 controllers used during tuning are given in Table 3. **minreal**() computes the minimal realization of the system by

Table 3: Frequency and Time Domain Characteristics for the Loop Transfer Function

Properties	$K = -1, T_i = 4.938$	$K = -1, T_i = 4$	$K = -1.1, T_i = 4$
Rise Time(s)	57.26	49.6474	**47.4665**
Overshoot(%)	0.3851	0.7152	**0.9729**
Settling Time(s)	113.9255	84.608	**82.9476**
Steady State Value	1	1	**1**
Gain Margin(dB)[@rad/s]	19.5[@0.197]	18[@0.196]	**17.2[@0.196]**
Phase Margin(deg)[@rad/s]	93.9[@0.0383]	92.1[@0.0478]	**92.3[@0.0526]**
Stability	Stable	Stable	**Stable**
Bandwidth(rad/s)	0.039	0.0511	**0.0542**

cancelling pole-zero pairs in transfer functions. However, our system does not have any pole-zero cancellations, and thus the transfer function we use is the minimal realization.
To summarize, the PI Controller with K=-1.1 and T_i=4, when combined with the plant transfer function, provides a minimal settling time of 82.9476 seconds, an overshoot of 0.9729%, which is less than 1%, and no steady state error, as the steady state value(DC Gain) is 1 for the step input, as desired. The gain margin is 17.2dB, and the phase margin is 92.3°Thus, we have successfully designed a reference tracking controller that achieves the highest possible bandwidth of 0.0542rad/s(0.0086Hz).

4 QUESTION 4

Now we use our previously designed SISO PI Controller for output disturbance rejection. The disturbance input is a step input with the disturbance plant state space system being:

$$\dot{x} = \begin{bmatrix} -0.4220 & -0.2204 & 0 & -0.2204 & 0 \\ 0.0233 & -0.0109 & -0.0400 & -0.0096 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 \\ 0.1455 & -0.0598 & 0 & -0.1651 & -10.8232 \\ 0 & 0 & 0 & 1.0000 & 0 \end{bmatrix} x + \begin{bmatrix} 0.2204 \\ 0.0096 \\ 0 \\ 0.0598 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} x + 0u$$

where

$$x = \begin{bmatrix} \omega_r \\ \dot{z}_1 \\ z_1 \\ \dot{z}_2 \\ z_2 \end{bmatrix}, u = d, y = \omega_r.$$

d is the disturbance input, which is a step input in this case. Using the MATLAB command **ss2tf**, we can represent this state space system as a transfer function. The corresponding transfer function of the plant is:

$$G_d(s) = \frac{0.22037(s^2 + 0.00128s + 0.04)(s^2 + 0.1053s + 10.82)}{(s + 0.4104)(s^2 + 0.02113s + 0.04101)(s^2 + 0.1664s + 10.85)} \tag{3}$$

The poles of this plant are located at [- 0.0832 + 3.2936i, - 0.0832 - 3.2936i, - 0.4104, - 0.0106 + 0.2022i, - 0.0106 - 0.2022i]. The zeroes of the plant are [- 0.0526 + 3.2894i, - 0.0526 - 3.2894i, - 0.0006 + 0.2000i, - 0.0006 - 0.2000i]. Refer to Figure 1, to see the architecture of output disturbance rejection. This can also be summarised in Figure 30.

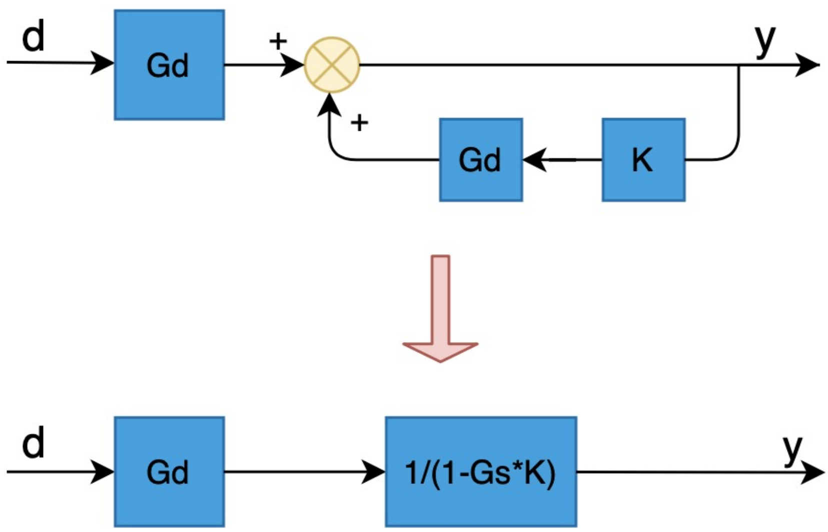

Figure 30: Closed-Loop Control Architecture and Equivalent Open Loop Representation

We now apply step input d, to the plant $G_d(s)(1 - G(s)K)^{-1}$, and analyze the step response, as shown in Figure 31. Since this is an step output disturbance rejection setup, the system must converge to the value 0 at steady state, thus demonstrating that the step disturbance, which has value 1, has been

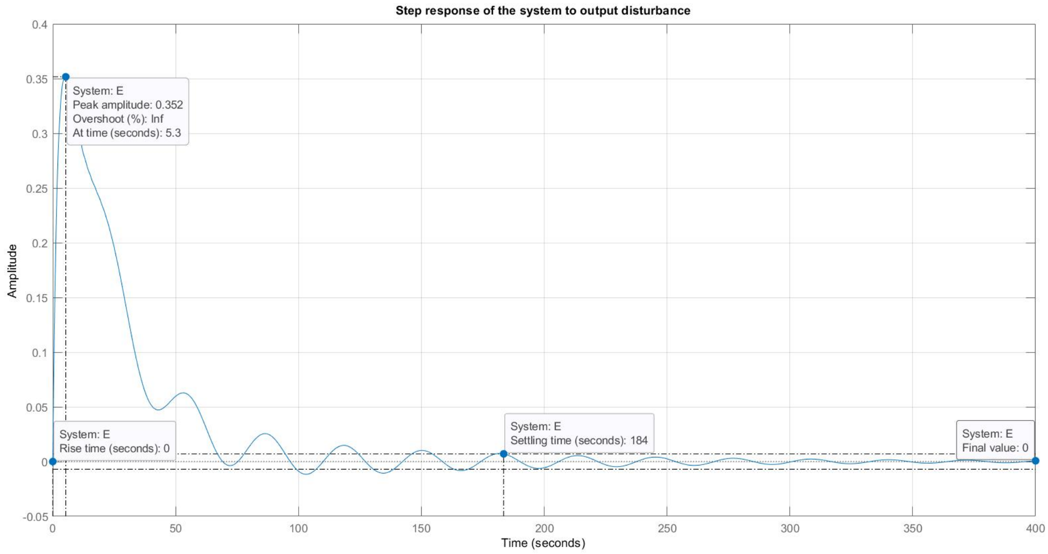

Figure 31: Step Response to Output Disturbance

rejected successfully.

The DC Gain(Steady State Value) of the response is 0, as desired. However, due to the disturbance plant, there is a very high peak initially, which is undesirable. The system eventually reject the disturbance and converges to 0, but due to the spike, all of the system parameters take a hit. Therefore, the controller to be used for output disturbance rejection is unsatisfactory and surely needs to be altered.

In Figure 32, we vary the parameters K and T_i to show trends in the step response for output disturbance rejection. We observe that keeping the same value of T_i and increasing K improves the

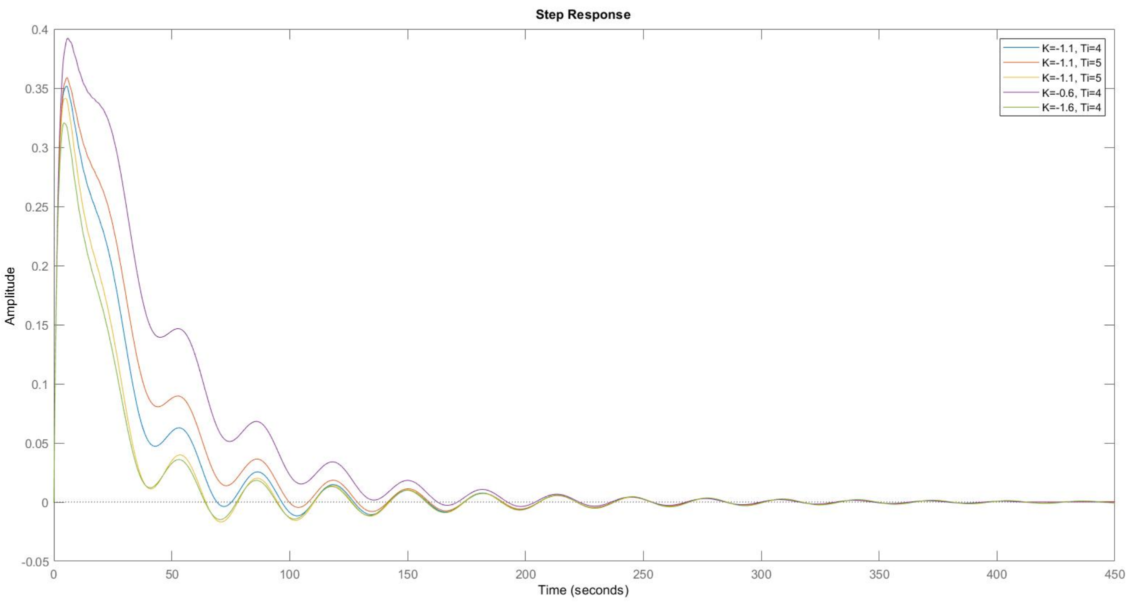

Figure 32: Various Step Responses to Output Disturbances

response of the system to disturbances. Thus, improvements could be made in this direction. The ultimate objective of designing a controller is balancing its performance with respect to reference tracking as well as output disturbance rejection.